AN INTRODUCTION TO CUT-OFF GRADE ESTIMATION

SECOND EDITION

BY JEAN-MICHEL RENDU

BLENDING CONSTRAINTS WASTE BLOCK CAVING STOCK
CLOSURE GRADE-TONNAGE PRICES MINE CUT-OFF LEAC
UNDERGROUND NPV SOCIOECONOMIC METAL EQUIVALEN
OPEN PIT OPPORTUNITY POLYMETALLIC PROCESS PRO
RECLAMATION SELECTIVITY STOPES UTILITY NSR VA
RECOVERY NPV MILL CUT-OFF LEACHING BREAKEVEN
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DEDICATION

I am dedicating this book to my wife Karla and my two sons, Yannick and Mikael. Life with a husband and father who spent too much time traveling to remote mines all over the world, and then returned home to work long hours in front of his computer, was not without challenges and disappointments. I am grateful for their patience, understanding, and unquestioning love.

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Preface to 2014 Edition

The first edition of *An Introduction to Cut-off Grade Estimation* was published in 2008. Since then, many changes have occurred in the world economy. Prices and costs have changed significantly. The commodity megatrend that prevailed during the last decade changed management's emphasis from cost control to growth in production. But this trend reached its inevitable end in 2012, resulting in a return to a high priority being given to efficiency and cost controls. The increasingly demanding political and socioeconomic environment has altered the way decisions are made.

Such changes should not alter the validity of a reference manual. However, the need to update *An Introduction to Cut-off Grade Estimation* became apparent as a result of feedback received from those who read the first edition and who attended cut-off grade courses I have given over the years. It became apparent that the information was not always presented in a rational fashion. Some fundamental concepts, such as those related to opportunity costs and how those costs should be accounted for in decision making, needed clarification. The focus of the 2008 edition was on metal mining, and, over time, many questions have been raised concerning application to industrial minerals, iron ore, and coal.

This new edition has attempted to take this feedback into account. Chapters have been reorganized and their content clarified. The relationship between optimization of net present value, capacity constraints, and opportunity cost is explained in greater detail. A new section has been added that discusses blending strategies applicable to coal and iron ore mines, as well as in an increasing number of metal mines. The objective was to make this new edition easier to read and of greater practical interest to practitioners operating in a wider range of situations. In addition, the bibliography has been greatly expanded. You, the readers, will decide whether the effort that went into this new edition was justified.

Many individuals contributed to this edition. I am particularly grateful to Professor Roussos Dimitrakopoulos of McGill University, Canada, who not only reviewed and commented on early drafts of this book, but also gave me the opportunity to give short courses on cut-off grade estimation at McGill University and in Perth, Australia. The challenging questions raised by the short-course attendees resulted in many of the changes made in this new edition. I also thank Professor Joao Felipe Costa of Universidade Federal do Rio Grande do Sul, Brazil, who also reviewed early drafts and helped correct

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some of the errors I made. I also recognize Steven Hoerger and Fred Seymour whom I have had the pleasure to work with for many years, and who contributed to the early development of ideas included in both the first and second editions. I also thank the Society for Mining, Metallurgy & Exploration, who accepted the manuscript, as well as Jane Olivier, manager of book publishing, and Diane Serafin, senior editor, who made publication possible.

Bringing this book to successful publication required the effort of many more individuals not listed here, but they know who they are and I am grateful to all of them. The responsibility for any errors, lapses, or absences of clarity that may remain from the first edition or that I may have introduced in the second edition is entirely mine.

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Preface to 2008 Edition

This book started with a desire to understand how to answer an apparently simple but actually complex question faced by all those responsible for the development and operation of mines: How do we determine which cut-off grade should be used to separate material that should be processed from that which should be sent to the waste dump? The answer appears straightforward: If it is profitable to process one metric ton of material, this ton should be processed. But what is *profitable*? The cut-off grade has a direct bearing on the tonnage of material mined, the tonnage and average grade of material processed, the size of the mining operation, and consequently capital costs, operating costs, environmental and socioeconomic impacts. Should we maximize cash flow, net present value, the life of the mining operation, the return to shareholders? How do we take into account economic, environmental, social, political, ethical and moral values, objectives, and regulations?

Somewhat surprisingly, only one other book has been written exclusively on the subject of cut-off grade estimation: *The Economic Definition of Ore: Cut-Off Grades in Theory and Practice* by Ken Lane, published in 1988. Lane's book was and will remain the standard for mathematical formulation of solutions to cut-off grade estimation when the objective is to maximize net present value. Concepts first formulated by Lane were used as the foundation of this book.

Considerable progress has been made in the last twenty years to improve mine planning and optimize cut-off grades. Increasingly complex algorithms have been developed, and better, easier to use computer programs have been written to assist engineers and economists in analyzing mine plans, testing the options, and improving production schedules. Computer programs have become easier to use, but the assumptions made by those who write the programs are often lost to the end user. With this book I am hoping to bridge the gap between theory and practice, the ivory tower and engineers in the field, by describing the fundamental principles of cut-off grade estimation and providing concrete examples.

This book started as notes written during the last thirty years. Eventually these notes turned into an introductory short course. Each time I gave the course, more and more questions were asked, concerning increasingly complex situations, demanding more practical examples and challenging the assumptions made. Each question resulted in corrections, additions, and more chapters. I am extremely thankful to all those who helped me in this respect.

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They include too many individuals over too many years to be listed here. They know who they are and I would not have continued this work without their probing and their interest in the subject. I am particularly grateful to Ernie Bohnet who kept on motivating me when I doubted that I had a story to tell or that there would be sufficient interest in continuing this effort to make it worthwhile. It is because of Ernie that I completed this book. I also want to thank the Society for Mining, Metallurgy & Exploration, and Jane Olivier, who accepted the manuscript and brought it to publication in record time. None of these people, of course, can be blamed for any errors or lapses that I may have made and for which I am fully responsible.

My first book, *An Introduction to Geostatistical Methods of Mineral Evaluation*, was published in 1978 with the objective to clarify the already arcane science of geostatistics. It is only fitting that *An Introduction to Cut-Off Grade Estimation* be published, with similar objectives, in 2008, exactly thirty years later.

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1 Introduction

A *cut-off grade* is generally defined as the minimum amount of valuable product or metal that one metric ton (i.e., 1,000 kg) of material must contain before this material is sent to the processing plant. This definition is used to distinguish material that should not be mined or should be wasted from that which should be processed. In complex geological environments, impurities may have to be considered to define the cut-off grade. Cut-off grades are also used to decide the routing of mined material when two or more processes are available, such as heap leaching and milling. Cut-off grades are used to decide whether material should be stockpiled for future processing or processed immediately.

The need to separate material being mined according to its physical and chemical properties before it is processed was well understood by historical miners. In the year 1556, Georgius Agricola reminded the readers of *De Re Metallica* that

> *Experienced miners, when they dig ore, sort the metalliferous material from earth, stones, and solidified juices before it is taken from the shafts and tunnels, and they put the valuable metal in trays and the waste into buckets.... To smelt waste together with an ore involves a loss, for some expenditure is thrown away, seeing that out of earth and stones only empty and useless slags are melted out, and further, the solidified juices also impede the smelting of the metals and cause loss.*

Agricola thus pointed out that, for economic reasons, both chemical and physical properties of material being mined must be taken into account, including impurities and deleterious elements (*solidified juices*) before this material is processed.

Cut-off grades are calculated by comparing costs and benefits. In simple geological and metallurgical environments, a single number, such as a minimum metal content, is sufficient to define the cut-off grade. In more complex situations, the quality of the material being processed, including the

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amount of deleterious elements, must be taken into account. Sales contracts may impose quality requirements that can only be satisfied by stockpiling and blending material according to quality.

In most situations, costs and recoveries, and therefore cut-off grades, vary with the geological characteristics of the material being mined. Grade is usually the most important factor but may not be the only one. If material is sent to a waste dump, the acid-generating potential of this material may have a direct impact on costs related to environmental controls. Sulfide content may be a critical—even overriding—factor for material sent to a roasting or flotation plant. Clay content may have a deleterious effect on the recovery and throughput of a leaching plant. Phosphorous and silica grades must be considered in selecting iron ore. Ash and sulfur content influence decisions in coal mines.

The cut-off grade defines the profitability of a mining operation as well as the mine life. A high cut-off grade can be used to increase short-term profitability and the net present value (NPV) of a project, thus possibly enhancing the benefit to shareholders and other financial stakeholders, including the government and local communities. However, increasing the cut-off grade is also likely to decrease the life of the mine. A shorter mine life can reduce time-dependent opportunities such as those offered by price cycles. A shorter mine life can also result in higher socioeconomic impact with reduced long-term employment and decreased benefits to employees and local communities.

Increased cut-off grades may be considered to reduce political risk by ensuring a higher financial return over a shorter time period. The cut-off grade may be increased when metal prices go up if this is needed to strengthen the financial position of the company and reduce the risk of failure when metal prices fall. Conversely, cut-off grades may be decreased during periods of high prices to increase mine life and keep high-grade material available to maintain profitability when prices fall. Cut-off grades may also be constrained by economic or technical performance criteria imposed by bank loans and other financial institutions.

In some instances, a conscious decision might be made to increase the mining capacity while keeping the processing capacity constant. This allows an increase in cut-off grade. Some of the lower-grade material may be stockpiled for processing at a later date. Stockpiling may have a number of consequences—some positive (such as increased useful life of processing facilities) and others negative (such as increased environmental risk and decreased metallurgical recovery of stockpiled material).

Public reporting of reserves, which are dependent on the cut-off grade, is subject to the rules and regulations of the various stock exchanges and other

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regulatory agencies. A link exists between published reserves and generally accepted accounting practices. Reserves enter into the calculation of capital depreciation, company book value, unit cost of production, and taxes. A link also exists between published reserves and the value that the financial market gives to a mining company. For some commodities there is a fairly widely held but arguably incorrect belief that this link is primarily a function of the magnitude of the reserves while quality is of lesser significance. Low cut-off grades may be considered desirable by those calculating or publicly reporting reserves if personal bonuses are a function of the magnitude of the published reserves. As a result of these various links—some desirable, some not—there can be a desire to maximize the published reserves by using the lowest technically, financially, and legally defendable cut-off grade. However, one must always keep in mind that reserves are published for the purpose of informing investors and other stakeholders, and that processes and controls should be put in place to eliminate the influence of factors that could result in publication of misleading estimates.

Both outsider and insider stakeholders have an interest in the cut-off grade and the reserves deriving from it. Outsiders include shareholders, financial institutions, local communities, environmentalists, regulators, governmental and nongovernmental agencies, suppliers, and contractors and buyers of the product being sold. Insiders include company management and employees. The board of directors represents the interests of the shareholders and is often composed of both insiders and independent outsiders. Cut-off grades are and should be primarily calculated taking into account only technical and economic constraints. However, the often-conflicting interests and objectives of the many stakeholders must be understood and prioritized if the best decision is to be made concerning cut-off grade determination.

The technical literature includes many publications on estimation and optimization of cut-off grades. The most comprehensive reference is Kenneth F. Lane's *The Economic Definition of Ore: Cut-Off Grades in Theory and Practice* (Lane 1988). The objective most commonly accepted in cut-off grade optimization studies is optimization of the NPV of future cash flows. To reach this objective one must take into account space-related variables (such as the geographic location of the deposit and its geological characteristics), as well as time-related variables (including the order in which the material will be mined and processed and the resulting cash flow). The time–space nature of the problem is quite complex; consequently, so are the proposed mathematical solutions to cut-off grade optimization. The bibliography provides detailed references to some of these solutions. This book attempts to explain

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basic concepts in a simple fashion, making them accessible to mine managers, analysts, geologists, mining engineers, accountants, and other practitioners.

The book is divided into several chapters. General concepts are introduced in Chapter 2, including those of utility function, breakeven cut-off grade, and opportunity cost. Each one of these concepts is then analyzed for its application in increasingly complex situations, starting with Chapter 3 in which estimation of breakeven cut-off grades is discussed. Examples are given for how to separate ore from waste, choose between different processes, analyze polymetallic deposits, and develop stockpiling strategies.

The less intuitive but fundamental concept of opportunity cost is discussed in Chapters 4 and 5. Opportunity costs result from constraints that are present in all mining operations. As shown in Chapter 4, ignoring capacity constraints can result in underestimation of cut-off grades and the processing of uneconomic material. Formulas are developed and examples given to illustrate the relationship between opportunity costs and cut-off grades. Opportunity costs decrease over time, and cut-off grades should decrease accordingly. Constraints imposed by the geology of the deposit are discussed in Chapter 5. Capacity constraints and geologic constraints must be taken into account jointly to determine the applicable cut-off grade. This complex relationship is illustrated by a number of examples.

Chapter 6 shows that while the same fundamental concepts can be used to estimate cut-off grades in any situation, there are significant differences in the application of these concepts to different mining methods. The costs that should be included in cut-off grade calculations are further discussed in Chapter 7. Situations are considered in Chapter 8 where there is a need to stockpile and blend material to satisfy constraints imposed by technical or marketing requirements. New methods are introduced to optimize blending depending on the objective to be reached.

Closing remarks can be found in Chapter 9, which is followed by Appendix A, a list of symbols, and a bibliography. Appendix A contains additional examples and solutions to problems referenced in the main text of this book.

Measurements are usually given using the International System of Units (SI), such as metric tons (t), grams (g), or meters (m). However, examples are included that also make use of other units, such as ounces (oz) for gold or pounds (lb) for copper, which are commonly used in specific industries.

One metric ton is equal to 1,000 kg (kilograms), or 2,205 lb. The word "ounce" refers to a troy ounce of gold and is equal to 31.1035 g. The word "short ton," commonly used in the United States, is equal to 2,000 lb.

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2 General Concepts

Choosing a cut-off grade is equivalent to choosing the value of a geologically defined parameter or set of parameters that will be used to decide whether one metric ton (1 t) of material should be sent to one process or another.

MATHEMATICAL FORMULATION: UTILITY FUNCTION

Let x be the value of the parameter(s) that must be taken into account to determine the destination to which the material should be sent. In simple cases, a single parameter may be sufficient to define the destination such as copper grade or gold grade. In other cases, a set of parameters may have to be considered such as copper and gold grades, sulfide content, clay content, and percentage of deleterious elements.

The value, or *utility**, of sending one metric ton of material with parameter value (grade) x to destination 1 (process 1) is $U_1(x)$. The utility of sending the same material to destination 2 (process 2) is $U_2(x)$. The destination that should be chosen is that which results in the highest utility $U_{max}(x)$. Figure 2-1 shows the relationship between $U_{max}(x)$, $U_1(x)$, and $U_2(x)$. The cut-off grade x_c is the value of x for which

$$U_1\left(x_c\right) = U_2\left(x_c\right)$$

If $U_2(x)$ exceeds $U_1(x)$ for x greater than x_c, then all material for which x is greater than x_c should be sent to process 2. Figure 2-1 shows how the cut-off

* The term *utility* is used in decision theory to represent the satisfaction gained from following a given course of action (Raiffa and Schlaifer 2000). This satisfaction is a function of preferences and values specific to the decision maker. The utility of a given cut-off grade strategy is a measure of the extent to which this strategy reaches the mining company's objectives.

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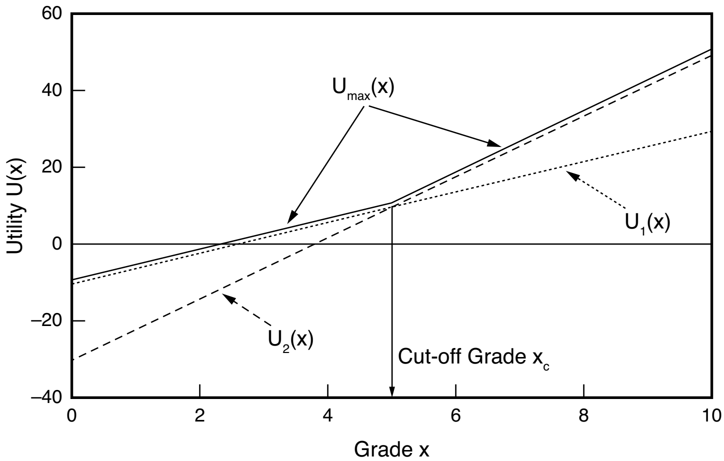

Figure 2-1 Utility maximization and graphical determination of cut-off grade

grade can be graphically determined by finding the intersection between $U_1(x)$ and $U_2(x)$.

As indicated in the introduction, the choice of a cut-off grade is governed primarily by financial objectives. However, the consequences of choosing a given cut-off grade are complex and not all of a financial nature. When estimating cut-off grades, all controlling variables must be taken into account. To facilitate this process, the utility U(x) of sending material of grade x to a given process is expressed as the sum of three parts:

$$U(x) = U_{dir}(x) + U_{opp}(x) + U_{oth}(x)$$

In this equation, $U_{dir}(x)$ represents the direct profit or loss that will be incurred from handling one metric ton of material of grade x. $U_{opp}(x)$ represents the opportunity cost or benefit of changing the processing schedule by adding one metric ton of grade x to the material flow. This opportunity cost only occurs when there are constraints that limit how many tons can be processed at a given time. Other factors, which must be taken into account in the calculation of cut-off grades but may not be quantifiable, are represented by $U_{oth}(x)$.

CUT-OFF GRADE AND GRADE–TONNAGE RELATIONSHIP

The cut-off grade determines the tonnage and average grade of material delivered to a given process and therefore the amount of product sold. In first

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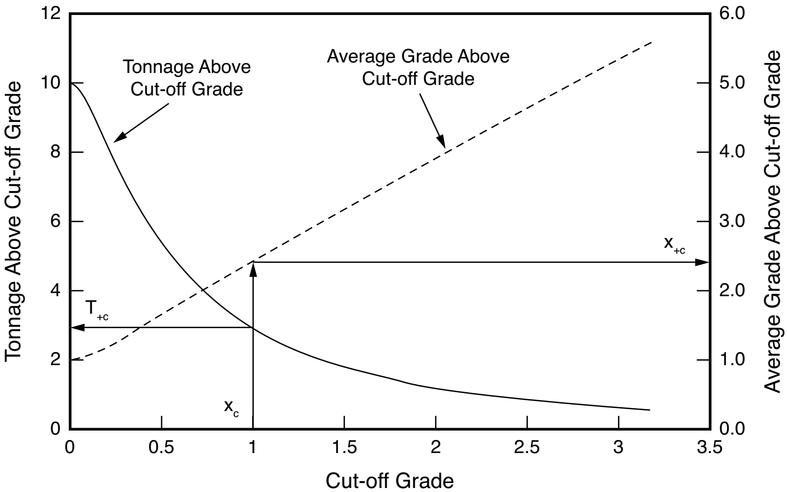

Figure 2-2 Example of grade–tonnage curve: tonnage and grade above cut-off grade

approximation, if T_{+c} represents the tonnage and x_{+c} symbolizes the average grade of material above the cut-off grade x_c, the revenue from sales is equal to $T_{+c} \cdot x_{+c} \cdot r \cdot V$ where r is the proportion of valuable product recovered during processing and V is the market value of the product sold. The cut-off grade also determines the tonnage of material mined that will not be processed. Figure 2-2 shows the relationship among cut-off grade and tonnage and average grade above cut-off grade. The curves on this graph are known as the grade–tonnage curves. Grade–tonnage curves are used extensively throughout this document to illustrate the impact that different cut-off grade strategies have on the economics of a mining operation. Another relationship that is useful in optimizing cut-off grades is shown in Figure 2-3. This figure shows the quantity Q_{+c} of valuable product contained in material above cut-off grade as a function of the cut-off grade:

$$Q_{+c} = T_{+c} \cdot x_{+c}$$

As an example, when considering a copper mine, the grade x, the cut-off grade x_c, and the average grade above cut-off x_{+c} are expressed in % Cu; the tonnage T_{+c} is expressed in metric tons; and the quantity Q_{+c} is expressed in metric tons or pounds of copper (1 t equals 2,205 lb). In a gold mine, the grades are expressed in grams per metric ton (g/t) and the quantity of metal in metric tons or ounces of gold (1 troy ounce equals 31.1035 g).

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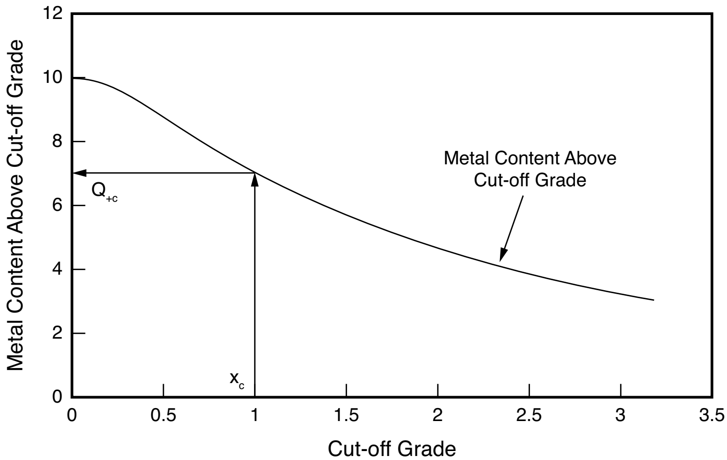

Figure 2-3 Quantity of valuable product contained above cut-off grade

DIRECT PROFIT AND LOSS: $U_{dir}(x)$

Direct profits or losses associated with one metric ton of material, $U_{dir}(x)$, are estimated by taking into account only costs and revenues that can be directly assigned to mining this material, processing it, and selling the final product. Breakeven cut-off grades are calculated taking into account only costs and revenues directly resulting from mining and processing (or wasting) one metric ton of material at grade x. The impact, if any, on total tons mined, production schedule, and cost per metric ton is ignored. The grade–tonnage relationship is also ignored.

Mathematical Formulation

The direct profit or loss $U_{dir}(x)$ expected from processing one metric ton of material of grade x is $U_{ore}(x)$ expressed as follows:

$$U_{ore}(x) = x \cdot r \cdot (V - R) - (M_o + P_o + O_o)$$

where

- x = average grade
- r = recovery, or proportion of valuable product recovered from the mined material
- V = value of one unit of valuable product
- R = refining, transportation, cost of sales, and other costs incurred per unit of valuable product

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M_o = mining cost per metric ton processed
P_o = processing cost per metric ton processed
O_o = overhead cost per metric ton processed

If the valuable product is a concentrate, V is the value of one unit of metal contained in the concentrate. For example, V can be the copper price expressed in dollars per pound of copper, or the gold price expressed in dollars per troy ounce of gold. r is the percentage of metal in concentrate that will be paid for. R includes transportation and refining costs, and other deductions and penalties to be deducted from V. When concentrate is sold to a smelter, the applicable values of V, r, and R may be negotiated between seller and buyer and specified in a smelter contract.

If the material is to be wasted, the value of $U_{dir}(x)$ is $U_{waste}(x)$, expressed as follows:

$$U_{waste}(x) = -(M_w + P_w + O_w)$$

M_w and O_w are mining and overhead costs, respectively, per metric ton of waste. P_w is the cost of processing one metric ton of waste as may be needed to avoid potential water contamination and acid generation, and to satisfy other applicable regulatory and environmental requirements. The cut-off grade between ore and waste is x_c such that $U_{ore}(x_c) = U_{waste}(x_c)$. Solving this equation gives the following result:

$$x_c = \left[(M_o + P_o + O_o) - (M_w + P_w + O_w)\right] / \left[r \cdot (V - R)\right]$$

The cut-off grade between ore and waste is equal to the difference between the cost of mining and processing ore and waste, $(M_o + P_o + O_o) - (M_w + P_w + O_w)$, divided by the value of one unit of metal contained in one metric ton of material processed $r \cdot (V - R)$.

Precious Metal Example

To illustrate how these formulas are used to calculate the cut-off grade, consider an open pit gold mining operation characterized as follows:

- For ore being processed: r = 90%, V = \$1,400 per ounce of gold, R = \$25.00 per ounce, M_o = \$4.50 per metric ton mined and processed, P_o = \$75.00 per metric ton processed, and O_o = 20% of operating costs.
- For wasted material: $M_w + P_w$ = \$5.00, and O_w = 20% of operating costs.

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The cut-off grade between ore and waste is calculated as follows:

$$x_c = [1.20 \cdot (4.50 + 75.00) - 1.20 \cdot 5.00] / [0.90 \cdot (1{,}400 - 25)]$$
$$= 0.072 \text{ oz/t} = 2.25 \text{ g/t}$$

NET PRESENT VALUE: NPV_i

The optimal cut-off grade strategy is that which maximizes the enterprise-specific objective function. Such a function nearly always includes, but is not limited to, net present value optimization. The net present value (NPV_i) of a project is today's cash equivalent of the cash flow that is expected to be generated by this project, assuming that money can be invested or borrowed at a specified discount rate i (e.g., i = 10%).

Mathematical Formulation

If n is the life of a project measured in years, and the (positive or negative) cash flow generated in year k is C_k (k = 0, 1, 2, ..., n−1), the net present value of the project using the discount rate i is

$$NPV_i = C_0 + C_1/(1+i) + C_2/(1+i)^2 + \ldots + C_{n-1}/(1+i)^{n-1}$$
$$NPV_i = \sum_{k=0,n-1} C_k/(1+i)^k$$

Note that in this formula, and throughout this book, the first year (year 0) is not discounted.

Example of NPV Calculation

Consider a project that is expected to generate the following cash flow over seven years. The discount rate to be used is i = 15%. Yearly cash flows are in millions of dollars.

$$C_{k,\,k=0,6} = \{-200,\ 20,\ 100,\ 120,\ 100,\ 50,\ -20\}$$

The net present value is calculated in Table 2-1. The discounted cash flow for a given year is equal to the undiscounted cash flow multiplied by the discount factor for this year. The total undiscounted cash flow is \$170 million, whereas the discounted cash flow—the project net present value—is \$45.30 million.

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Table 2-1 Calculation of net present value using a 15% discount rate

Year, k	Undiscounted Cash Flow, millions, C_k	Discount Factor, $1/(1 + i)^k$	Discounted Cash Flow, millions, $C_k/(1 + i)^k$
0	($200.00)	1.0000	($200.00)
1	$20.00	0.8696	$17.39
2	$100.00	0.7561	$75.61
3	$120.00	0.6575	$78.90
4	$100.00	0.5718	$57.18
5	$50.00	0.4972	$24.86
6	($20.00)	0.4323	($8.65)
Total	$170.00 undiscounted		$45.30 discounted

NPV of Constant and Perpetuity Cash Flows

In some situations a constant cash flow must be generated over a finite number of years to justify an investment such as sustaining capital. Some situations also exist where constant yearly costs are expected to perpetuity, such as environmental remediation costs. Examples will be encountered throughout this book. In such situations, simplifying formulas can be used to calculate net present values.

If a project is expected to generate a constant cash flow C over n years, the following equation can be used to calculate the net present value:

$$NPV_i = C\left[1 + 1/(1+i) + 1/(1+i)^2 + \ldots + 1/(1+i)^{n-1}\right] = C\,g(i,n)$$

where $g(i,n) = [1 - (1 + i)^{-n}]\,(1 + i)/i$

If the cash flow is expected to be constant to perpetuity, the net present value is

$$NPV_i = C(1+i)/i$$

Proof of these formulas is given in Examples 1 and 2 in Appendix A for constant cash flows and cash flows to perpetuity, respectively. Table 2-2 shows how the net present value of a $1.00 constant cash flow varies depending of the life of the project and the discount rate. For example, when calculated at a 7% discount rate, the net present value of a yearly $1.00 cash flow over five years has a net present value of $4.39. The same cash flow over an infinite period has a net present value of $15.29.

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Table 2-2 Calculation of net present value of a constant $1.00 cash flow over n years

Discount Rate, i	Project Life (n years)				
	5	10	15	20	Infinity
	$NPV_i = g(i,n)$				$NPV_i = (1 + i)/i$
5%	$4.55	$8.11	$10.90	$13.09	$21.00
7%	$4.39	$7.52	$9.75	$11.34	$15.29
9%	$4.24	$7.00	$8.79	$9.95	$12.11
11%	$4.10	$6.54	$7.98	$8.84	$10.09
13%	$3.97	$6.13	$7.30	$7.94	$8.69
15%	$3.85	$5.77	$6.72	$7.20	$7.67

OPPORTUNITY COSTS AND BENEFITS: $U_{opp}(x)$

Ken Lane (1964, 1988) was first to demonstrate the importance of opportunity costs in cut-off grade estimation. *Opportunity cost* is defined as the change in a project net present value that results from changing the production schedule.

Capacity Constraints and Opportunity Cost

Consider a project for which the net present value of future cash flows NPV_i was calculated using the discount rate i. Assume that material was added to the production schedule that was not included in the original schedule. Also assume that there are production constraints, the implication being that by adding new material, the originally planned production schedule is postponed or slowed down. The net present value of the delayed production schedule is NPV'_i, less than the original net present value NPV_i. By definition, the *opportunity cost* of changing the production schedule is equal to the decrease in net present value, $NPV'_i - NPV_i$. Adding new material to the production schedule is economically justified only if the net present value of the cash flow generated by this new material exceeds the opportunity cost of changing the original production schedule.

Consider a mining operation in which, under current operating conditions, the processing plant capacity is fully utilized. If new material is added to the original production schedule and given priority in the processing plant, the material originally scheduled for processing will be delayed. The duration t of this delay is the time needed to process the new material. The net present value of the originally scheduled production will be reduced from NPV_i to NPV'_i:

$$NPV'_i = NPV_i/(1+i)^t$$

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The opportunity cost of postponing mining of the already scheduled material is equal to the loss in NPV:

$$NPV'_i - NPV_i = NPV_i\left[(1+i)^{-t} - 1\right]$$

The new material should be mined or processed only if its net present value exceeds the opportunity cost. An example is given in Example 3 in Appendix A, which shows how to calculate the opportunity cost of mining a peripheral deposit when this deposit is given priority over previously scheduled production.

Application to Cut-off Grade Estimation

The cut-off grade between two options is obtained by comparing the utility as well as the advantages and disadvantages of choosing one option rather than the other. The conclusions are valid only if all costs and benefits are taken into account.

Opportunity costs or benefits, $U_{opp}(x)$, may result from mining and processing one metric ton of material not previously scheduled for processing. No opportunity cost is incurred if the mine, mill, refining facilities, and volume of sales are not capacity constrained and if adding one more metric ton to the process has no impact on previously expected cash flows. If there is a capacity constraint, the opportunity cost includes the cost of displacing material already scheduled for processing and postponing treatment of this material.

Consider a project whose net present value of future cash flows NPV_i was calculated on the basis of currently planned production. According to the current plan, there is no spare capacity in the processing plant. If one metric ton of material is added to the capacity-constrained processing plant, treatment of the originally scheduled material is postponed by the time needed to process the additional ton. Processing one metric ton of material takes t units of time, where t is the inverse of the yearly plant capacity. Adding one new metric ton of material not currently planned to be processed will decrease the net present value of future cash flows from NPV_i to $NPV'_i = NPV_i / (1 + i)^t$. The difference, $NPV'_i - NPV_i$, is the opportunity cost of processing one more metric ton of material:

$$U_{opp}(x) = NPV_i\left[(1+i)^{-t} - 1\right]$$

In this equation, the time t is very small, as it is equal to the inverse of the yearly capacity of the constrained material flow. Calculus can be used to demonstrate the following simplified formula for the opportunity cost:

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$$U_{opp}(x) = -i \cdot t \cdot NPV_i$$

A comparison of the exact formula $NPV_i\,[1 - (1 + i)^{-t}]$ and its first-order approximation $i{\cdot}t{\cdot}NPV_i$ is given in Example 4 in Appendix A. This difference is less than 10% for a discount rate i = 15%, and decreases for smaller discount rates.

The discount rate i is expressed as a percentage per year; the time t is expressed in years per unit of capacity-constrained material; the net present value is expressed in dollars. The opportunity cost is therefore expressed in dollars per unit of capacity-constrained material. The opportunity cost must be added to the direct cost of the process that is capacity constrained.

If the mine is capacity constrained and a decision is made to mine an additional metric ton of material that was not scheduled to be mined, t is the time needed to mine this ton and the opportunity cost must be added to the mining cost M_o or M_w. If one new ton of ore is sent to a capacity-constrained mill, t is the time needed to mill this metric ton, and the opportunity cost must be added to the processing cost P_o. If the refining process is capacity limited, t is the time needed to refine the concentrate produced from one metric ton of material at grade x, and the opportunity cost must be added to the refining cost R.

Application to an Underground Gold Mine

Consider an underground gold mine in which the net present value of future cash flows is NPV_i = $800,000,000. This value was calculated using the discount rate i = 15%. The mineshaft is capacity constrained, with a maximum haulage capacity of 1,500,000 t/yr. Consideration is given to mining low-grade material on the periphery of high-grade stopes. This material was not included in calculation of the above-mentioned net present value. The time needed to mine and deliver one metric ton of material to the surface is t = 1/1,500,000 year. The opportunity cost of adding one metric ton to the production schedule is

$$\begin{aligned} U_{opp}(x) &= -15\% \cdot \$800{,}000{,}000/1{,}500{,}000 \\ &= -\$80.00 \text{ per metric ton of ore mined} \end{aligned}$$

The following parameters apply to ore being processed:

r = 90%

V = $1,400 per ounce of gold recovered

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$R =$ \$20.00 per ounce of gold recovered
$M_o =$ \$250.00 per metric ton mined
$P_o =$ \$85.00 per metric ton processed
$O_o =$ 20% of operating costs

If only direct costs and revenues are taken into account, the cut-off grade between ore and waste is

$$x_c = [1.20 \cdot (250.00 + 85.00)] / [0.90 \cdot (1{,}400.00 - 20.00)]$$
$$= 0.324 \text{ oz/t} = 10.07 \text{ g/t}$$

Adding the \$80.00 opportunity cost to the mining cost results in a 2 g/t increase to the cut-off grade:

$$x_c = [1.20 \cdot (250.00 + 85.00) + 80.00] / [0.90 \cdot (1{,}400.00 - 20.00)]$$
$$= 0.388 \text{ oz/t} = 12.07 \text{ g/t}$$

At this stage of the mine life, low-grade peripheral material should be mined only if it averages at least 12.07 g/t. When the mine approaches the end of its life, the net present value of future cash flows will decrease toward zero and so will the opportunity cost $U_{opp}(x)$. The cut-off grade will then decrease to reach the marginal cut-off grade, 10.07 g/t.

Multiple examples are given later that show how opportunity costs must be taken into consideration depending on prevailing situations.

OTHER COSTS AND BENEFITS: $U_{oth}(x)$

Cut-off grades play a critical role in defining tonnages mined and processed, average grade of mill feed, and project economic feasibility. In addition to the quantifiable financial impact that cut-off grade changes may have, other costs and benefits must be taken into account, although their impacts are often not easily quantifiable. Consideration must be given not only to changes in net present value and cash flow—as measured by $U_{dir}(x)$ and $U_{opp}(x)$—but also to all other impacts, $U_{oth}(x)$, including those of an environmental, socioeconomic, ethical, or political nature.

Costs and benefits to all stakeholders must be evaluated, including shareholders, financial institutions, analysts, employees, customers, suppliers, local communities, government, nongovernmental agencies, and future generations. Senior management must decide how to balance the needs, interests,

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and requirements of these stakeholders. Practical guidelines must be developed, including guidelines for cut-off grade determination, to ensure that the projects are designed to reach the company's often conflicting objectives. Maximizing shareholder value is often quoted as a company's primary objective. However, other objectives must be taken into account, which include recognition of responsibilities toward all stakeholders.

CUT-OFF GRADE AND BLENDING STRATEGY

When determining cut-off grades, the situation most commonly encountered is one where the decision whether a material type—defined by its physical and chemical properties—should be mined and processed is made solely on the basis of these properties. This does not take into account more complex situations where blending of different material types is needed to obtain a product that can be economically processed and sold. A material type may have no economic value if considered on its own, but it is needed for blending with other material to obtain an economically valuable product. For example, in gold mines where refractory ore is treated by roasting, material with a high sulfide content and low gold content may have no value if considered on its own. Yet the calorific value of this material may be needed to economically extract the gold contained in high-grade, low-sulfide ore.

Circumstances also occur where one material type does not satisfy constraints imposed by sales contracts, but blending with other material will result in a product that exceeds contractual requirements. This can be the case in a coal mining operation where one seam has a low calorific value and high ash content and another seam has a high calorific value and low ash content. By appropriately mixing the two products, it may be possible to increase the tonnage of salable coal. Similarly, the product from one iron mine may not be salable on its own, but a salable product may be obtained by blending with the product of another mine that exceeds contractual requirements.

A typical situation happens where stockpiles are available with known tonnages and geochemical properties. In which proportion should the stockpiles be blended to satisfy specified quality requirements? A similar situation occurs when two or more mines are being designed whose products will have to be blended before being processed or sold. What should be the mining rate for each mine, and which cut-off grades should be applied to obtain a material blend that maximizes profits? Different blending strategies are discussed in this book.

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Breakeven Cut-off Grade

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Minimum or breakeven cut-off grades are those that apply to situations where only direct operating costs are taken into account. Capacity constraints are ignored. Cash flows are not discounted. Opportunity costs are not taken into consideration and neither are other consequences, financial or otherwise, that changing the cut-off grade may have on mining and processing schedules and cash flows.

CUT-OFF GRADE BETWEEN ORE AND WASTE

Consider material for which the decision has already been made that it will be mined, so the remaining question is whether it should be sent to the processing plant or wasted.

Mathematical Formulation

Using notations introduced previously, the utility of mining and processing one metric ton of ore-grade material, $U_{ore}(x)$, can be written as follows:

$$U_{ore}(x) = x \cdot r \cdot (V - R) - (M_o + P_o + O_o)$$

where

- x = average grade
- r = proportion of valuable product recovered from the mined material
- V = value of one unit of valuable product
- R = refining costs, defined as costs that are related to the unit of valuable material produced
- M_o = mining cost per metric ton of ore
- P_o = processing cost per metric ton of ore
- O_o = overhead cost per metric ton of ore

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The utility of mining and wasting one metric ton of waste material, $U_{waste}(x)$, can be written as follows:

$$U_{waste}(x) = -(M_w + P_w + O_w)$$

where

M_w = mining cost per metric ton of waste
P_w = processing cost per metric ton of waste, as may be needed to avoid potential water contamination and acid generation
O_w = overhead cost per metric ton of waste

The minimum cut-off grade is the value x_c of x for which

$$U_{ore}(x_c) = U_{waste}(x_c)$$

$$x_c = [(M_o + P_o + O_o) - (M_w + P_w + O_w)] / [r \cdot (V - R)] \qquad \text{(EQ 3-1)}$$

In this formula, the numerator represents the difference between mining, processing, and overhead costs incurred when treating the material as ore and those incurred when treating the same material as waste. In the denominator, the metal recovery r must be that which applies to material of grade x_c, which is not necessarily equal to the average recovery for all material sent to the processing plant.

The relationship between utility functions and cut-off grade is graphically represented in Figure 3-1.

Internal or Mill Cut-off Grade

If material must be mined, it should be processed if its grade is high enough to pay for processing costs even if it does not pay for mining costs.

If the costs of mining and shipping material to the waste dump or to the primary crusher are the same ($M_o = M_w$) and there are no significant additional costs in processing waste ($P_w = 0$ and $O_w = 0$), this cut-off grade is only a function of mill costs and recoveries and is independent of mining costs. Equation 3-1 can be written as follows:

$$x_c = [P_o + O_o] / [r \cdot (V - R)] \qquad \text{(EQ 3-2)}$$

This cut-off grade applies to material located within the limits of an open pit mine or an underground stope. This material is internal to the current mine plan and must be mined. The only question is whether it should be

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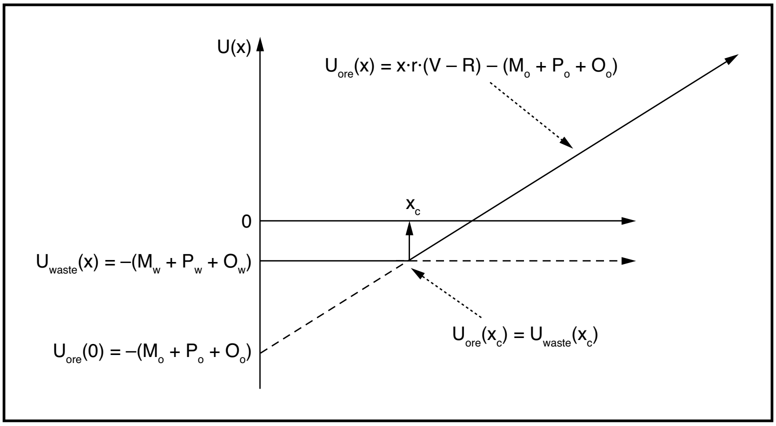

Figure 3-1 Relationship between utility functions and cut-off grade

processed. For these reasons it is often called *internal* or *mill* cut-off grade. In this formula the overhead cost O_o is that which applies to processing only.

External or Mine Cut-off Grade Without Waste Stripping

If the option is to leave the material in the ground at no cost, it should be mined only if its grade is high enough to pay for both mining and processing costs.

$$U_{waste}(x) = 0$$
$$U_{ore}(x) = x \cdot r \cdot (V - R) - (M_o + P_o + O_o)$$

The cut-off grade between ore and waste is

$$x_c = (M_o + P_o + O_o) / [r \cdot (V - R)] \quad \text{(EQ 3-3)}$$

This cut-off grade includes the mining cost and is often referred to as the *external* or *mine* cut-off grade*. The overhead cost O_o is that which applies to both mining and processing.

External or Mine Cut-off Grade with Waste Stripping

In the previous formula (Equation 3-3), the external or mine cut-off grade applies to one metric ton of material of grade x that is exposed to the surface

* Other terms used in open pit mines include *in-pit* cut-off grade for material within the pit limits (the *internal* or *mill* cut-off grade) and *ex-pit* cut-off grade for material not included in the pit limits (the *external* or *mine* cut-off grade).

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but had not been included in the mine plan; this material can be mined without waste stripping. This next example considers the case where some waste stripping is needed. The strip ratio is defined as follows:

$$s = \text{metric tons of waste per metric ton of ore}$$

If this material is not mined, the utility is zero:

$$U_{waste}(x) = 0$$

If the decision is made to mine the mineralized material, the utility is

$$U_{ore}(x) = x \cdot r \cdot (V - R) - (M_o + P_o + O_o) - s \cdot (M_w + P_w + O_w)$$

The cut-off grade is

$$x_c = \left[(M_o + P_o + O_o) + s \cdot (M_w + P_w + O_w)\right] / \left[r \cdot (V - R)\right] \qquad \text{(EQ 3-4)}$$

CUT-OFF GRADES IN OPEN PIT MINES

The following example shows how to calculate mine and mill cut-off grades, with and without waste stripping, in an open pit copper mine. The mine operating parameters are characterized as follows:

- r = 86% (includes mill and smelter recovery)
- V = \$3.20 per pound of payable copper
- R = \$0.60 per pound of payable copper (includes freight, smelting, and refining)
- M_o = \$3.00 per metric ton of ore mined
- P_o = \$10.00 per metric ton of ore processed
- O_o = \$1.70 per metric ton of ore processed
- M_w = \$3.30 per metric ton of waste mined
- P_w = \$0.50 per metric ton of waste mined (includes environmental control and remediation)
- O_w = \$0.40 per metric ton of waste mined

The opportunity cost of classifying one metric ton of material as ore or waste is as follows (1 t = 2,205 lb):

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$$\begin{aligned} U_{ore}(x) &= x \cdot r \cdot (V - R) - (M_o + P_o + O_o) \\ &= x \cdot 0.86 \cdot (3.20 - 0.60) \cdot 2{,}205 - (3.00 + 10.00 + 1.70) \\ &= 4{,}929x - \$14.70 \\ U_{waste}(x) &= -(M_o + P_o + O_o) \\ &= -(3.30 + 0.50 + 0.40) \\ &= -\$4.20 \end{aligned}$$

For material internal to the pit limits, the *mill* cut-off grade is

$$x_c = (14.70 - 4.20)/4{,}929 = 0.21\%\ \text{Cu} \qquad \text{(EQ 3-5)}$$

For material external to the pit limits, the *mine* cut-off grade is

$$x_c = 14.70/4{,}929 = 0.30\%\ \text{Cu} \qquad \text{(EQ 3-6)}$$

The cut-off grades calculated in Equations 3-5 and 3-6 could have been obtained directly by substituting the appropriate values in Equations 3-1 and 3-3. The difference in cut-off grade, 0.09% Cu, represents the amount of additional copper needed to pay for the cost of mining material that was not scheduled to be mined. Figures 3-2 and 3-3 give graphical representations of the utility functions used to calculate the mill and mine cut-off grades, respectively.

In the previous calculations leading to Equation 3-6, the external or mine cut-off grade is that which applies to one metric ton of material of grade x that is exposed to the surface but had not been included in the mine plan. This material can be mined without waste stripping. This next formula considers the case where some waste stripping is needed to mine this material. The ore to waste strip ratio is

s = 1.5 t of waste per metric ton of mineralized material

The utility of mining and processing the mineralized material is

$$\begin{aligned} U_{ore}(x) &= x \cdot r \cdot (V - R) - (M_o + P_o + O_o) - s \cdot (M_w + P_w + O_w) \\ &= x \cdot 0.86 \cdot (3.20 - 0.60) \cdot 2{,}205 - (3.00 + 10.00 + 1.70) \\ &\quad -1.5 \cdot (3.30 + 0.50 + 0.40) \\ &= 4{,}929x - \$21.00 \end{aligned}$$

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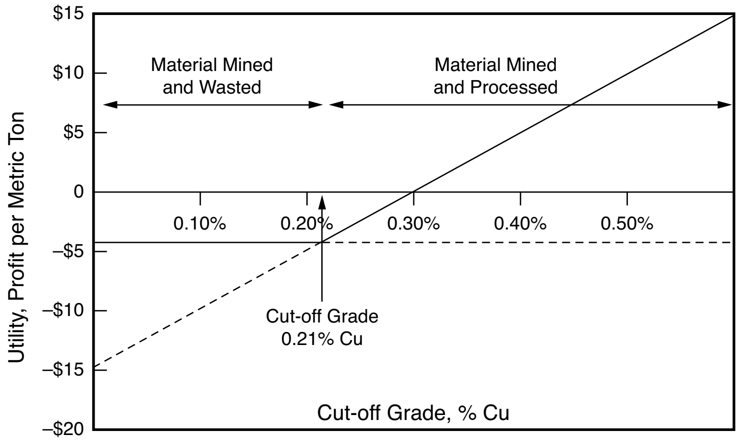

Figure 3-2 Graphical estimation of internal (mill) cut-off grade

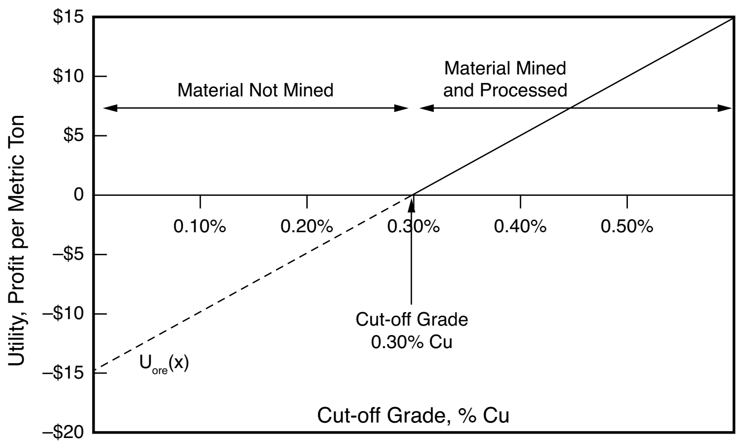

Figure 3-3 Graphical estimation of external (mine) cut-off grade

The corresponding cut-off grade is

$$x_c = 21.00/4{,}929 = 0.43\%\ \text{Cu} \qquad \text{(EQ 3-7)}$$

This cut-off grade could have been calculated directly by substituting the appropriate values in Equation 3-4. Figure 3-4 gives a graphical representation of the utility functions used to calculate the mine cut-off grade with a 1.5 strip ratio.

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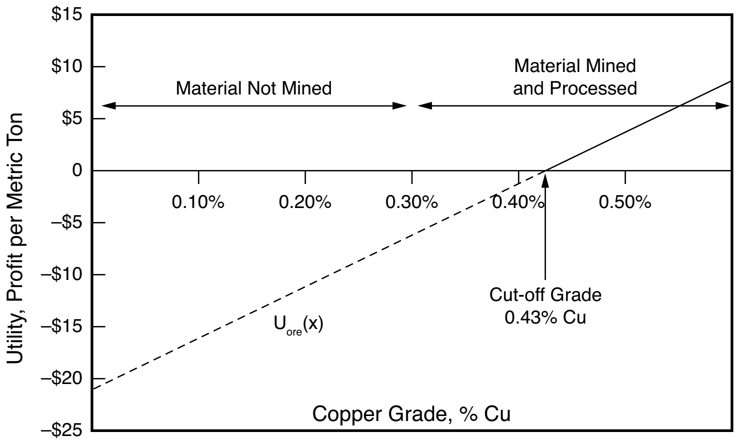

Figure 3-4 Graphical estimation of internal (mill) cut-off grade with strip ratio

CUT-OFF GRADES IN UNDERGROUND MINES

General Considerations

Capacity constraints are common in underground mines. These may include constraints imposed by ore body geometry, geotechnical conditions, shaft and haulage capacities, ventilation requirements, mining method, size and type of mining equipment, health and safety regulations, and other restrictions that limit production from a stope, a mine section, or the mine as a whole. Furthermore, many decisions made in an underground mine are capital intensive and take a significant amount of time to implement and result in positive cash flow.

In such an environment, capacity constraints have a significant effect on the cut-off grades. Opportunity costs must be taken into account to quantify how the project net present value is modified over time by changing cut-off grade and mine plan. In this section, only undiscounted cash flows are taken into account, without discounting. Opportunity costs will be discussed in Chapter 4.

The minimum conditions that must be satisfied to justify development of a new stope and how to determine the locations of stope boundaries, are two aspects of underground mining that are discussed in the next sections (see Figure 3-5).

Minimum Tonnage and Grade

A minimum grade is occasionally quoted when referring to the average grade that a stope must exceed before it is considered for mining. Strictly speaking,

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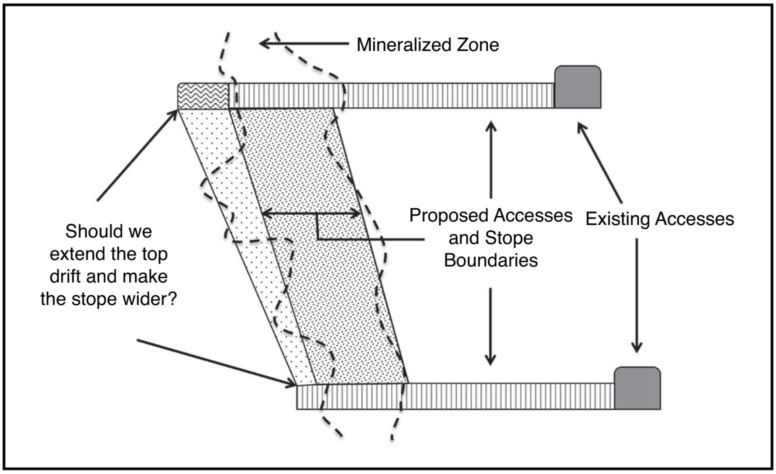

Figure 3-5 Estimation of average grade and cut-off grade in an underground mine

this is not a cut-off grade but an average grade, which must be linked to a tonnage. The minimum stope average grade depends on the size of the stope, its location with respect to existing facilities, ease of access, geotechnical conditions, and other stope-specific characteristics. This average grade is that for which the cost of developing and mining the stope is expected to be less than the profit made by processing the ore and selling the final product. This calculation must be made on a discounted basis, taking all physical constraints into account.

Optimization of Stope Size

When designing a stope, one must take into account the constraints imposed by mining method and geotechnical conditions. One must also determine whether lower-grade material located along the boundary of the stope should be included in the stope. Such material should be mined only if the expected value of the recoverable product it contains exceeds all incremental costs, including mining, haulage, processing, backfilling, and other costs. The minimum cut-off grade that defines boundary material which should be mined is the *mine cut-off grade*, and is estimated using a formula similar to that for material at the bottom of an open pit mine (Equation 3-3):

$$x_c = (M_o + P_o + O_o)/[r \cdot (V - R)]$$

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As an example, consider an underground gold mine where the incremental mining cost is $120.00 per metric ton, the mill processing cost is $80.00 per metric ton, and the mining and processing overhead is $30.00 per metric ton. The mill recovery is 90%. Given a gold price of $1,400 per ounce and a sales cost of $80.00 per ounce, the minimum cut-off grade to be considered to locate a stope boundary is (1 troy ounce = 31.1035 g)

$$x_c = 31.1035 \cdot [120.00 + 80.00 + 30.00] / [0.90 \cdot (1{,}400.00 - 80.00)] = 6.02 \text{ g/t}$$

This cut-off grade is the lowest average grade that a stope increment must have before it is considered for mining. Both planned and unplanned dilution must be taken into account. The mining cost should include the incremental development cost needed to make the stope wider. Opportunity costs such as those imposed by haulage capacity constraint should be taken into account, which will increase the cut-off grade. Opportunity costs will be discussed in Chapter 4.

If low-grade material must be mined because it is located within a stope or within other planned openings such as shafts, raises, drifts, crosscuts, and so forth, a lower cut-off grade can be used to determine whether this material should be wasted or processed. For such material, blasting and haulage costs will be incurred whether the material is treated as ore or waste. Only incremental costs need be considered. The minimum cut-off grade is estimated using the following general formula:

$$x_c = \left[\left(M_o + P_o + O_o \right) - \left(M_w + P_w + O_w \right) \right] / \left[r \cdot (V - R) \right]$$

If ore and waste mining costs, including overhead, are the same ($M_o = M_w$) and waste processing costs are negligible ($P_w = 0$), this formula can be written as follows:

$$x_c = \left(P_o + O_o \right) / \left[r \cdot (V - R) \right]$$

The mill cut-off grade is recognized here (Equation 3-2). In this formula, the overhead cost O_o is that which applies to processing only. Applicable opportunity costs, which in this case are likely to be only those imposed by mill capacity constraints, should also be taken into account, which will increase the cut-off grade.

As an example, consider exploration drifts in an underground gold mine. While mostly in waste, these drifts periodically encounter mineralized zones, and the decision must be made whether this material should be wasted or

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sent to the processing plant. Mining costs are $M_o = M_w$ = \$350.00 per metric ton, and mining overhead costs are O_w = \$70.00 per metric ton, regardless of whether the material is hauled to the waste dump or the mill. The milling cost is P_o = \$79.00 per metric ton, with a processing overhead cost of \$12.00 per metric ton. The total overhead cost per metric ton of ore, including mining and processing, is O_o = \$70.00 + \$12.00 = \$82.00. The gold price is \$1,450 per ounce, and the cost of sales is \$90.00 per troy ounce. The mill recovery is 89%.

The minimum cut-off grade that should be used to send material to the mill is

$$x_c = 31.1035 \cdot \begin{bmatrix} (350.00 + 79.00 + 82.00) \\ -(350.00 + 70.00) \end{bmatrix} \Big/ \left[0.89 \cdot (1,450.00 - 90.00)\right]$$

$$= 31.1035 \cdot (79.00 + 12.00) / \left[0.89 \cdot (1,450.00 - 90.00)\right]$$

$$= 2.34 \text{ g/t}$$

CUT-OFF GRADE BETWEEN TWO PROCESSES

Mathematical Formulation

If two processes are available to treat the same material, cut-off grades must be calculated to separate waste from ore being processed and to decide to which one of the two processes the ore should be sent. How to decide whether material should be processed or wasted was discussed previously. To decide between two processes, the utility of sending material of grade x to process 1 must be compared with that of sending the same material to process 2. Mining costs, including haulage cost to the processing plant, may vary depending on the process. Processing costs will be different and so will metallurgical recoveries and overhead costs. If the product sold is a function of the process being used, even the revenue per metric ton produced may differ. The cut-off grade between two processes is calculated using the following formulas, where subscripts refer to the process number:

$$U_1(x) = x \cdot r_1 \cdot (V - R_1) - (M_{o1} + P_{o1} + O_{o1})$$

$$U_2(x) = x \cdot r_2 \cdot (V - R_2) - (M_{o2} + P_{o2} + O_{o2})$$

$$U_1(x_c) = U_2(x_c)$$

$$x_c = \left[(M_{o1} - M_{o2}) + (P_{o1} - P_{o2}) + (O_{o1} - O_{o2})\right] / \left[r_1 \cdot (V - R_1) - r_2 \cdot (V - R_2)\right]$$

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In this formula, the numerator is the difference between costs incurred if one metric ton of material is sent to process 1 and costs incurred if the same metric ton is sent to process 2. The denominator is the difference between revenues resulting from sending one metric ton of material containing one unit of valuable product (e.g., one ounce of contained gold or one pound of contained copper) to process 1 and revenues resulting from sending the same metric ton to process 2.

Precious Metal Example

Consider a gold mining operation from which material can be sent to either of two processing facilities, a leach pad (process 1) or a mill (process 2). The costs associated with this operation are as follows:

M_w = \$3.00 mining cost per metric ton of waste
P_w = \$0.80 processing cost per metric ton of waste
O_w = \$0.40 overhead cost per metric ton of waste
M_{o1} = \$5.00 mining cost per metric ton of ore leached
M_{o2} = \$5.50 mining cost per metric ton of ore milled
P_{o1} = \$9.00 processing cost per metric ton leached
P_{o2} = \$35.00 processing cost per metric ton milled
O_{o1} = \$1.20 overhead cost per metric ton leached
O_{o2} = \$4.50 overhead cost per metric ton milled
r_1 = 60% average leach recovery
r_2 = 97% average mill recovery
V = \$1,600 per ounce of gold sold
R_1 = \$50.00 per ounce of gold sold
R_2 = \$50.00 per ounce of gold sold

The utilities of sending one metric ton of material to the waste dump, the leach pad, or the mill are calculated below. Dollars per ounce was converted to dollars per gram (1 troy ounce = 31.1035 g) to obtain cut-off grades in grams per metric ton.

$$U_{waste}(x) = -(M_w + P_w + O_w) = -\$4.20$$

$$U_1(x) = x \cdot r_1 \cdot (V - R_1)/31.1035 - (M_{o1} + P_{o1} + O_{o1}) = \$29.90 \cdot x - \$15.20$$

$$U_2(x) = x \cdot r_2 \cdot (V - R_2)/31.1035 - (M_{o2} + P_{o2} + O_{o2}) = \$48.34 \cdot x - \$45.00$$

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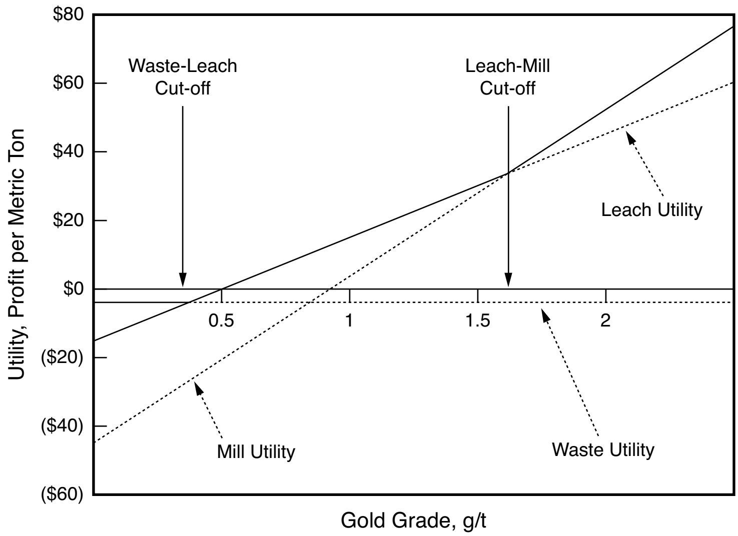

Figure 3-6 Graphical estimation of cut-off grade between wasted, leached, and milled material

These utility functions are plotted in Figure 3-6. The cut-off grades are

$$\text{Waste-leach cut-off} = (15.20 - 4.20)/(29.90)$$
$$= 0.37 \text{ g/t}$$
$$\text{Leach-mill cut-off} = (45.00 - 15.20)/(48.34 - 29.90)$$
$$= 1.62 \text{ g/t}$$

CUT-OFF GRADE WITH VARIABLE RECOVERY

Mathematical Formulation

In the previous examples, it was assumed that the recovery achieved in the processing plant was a constant. For many processes and deposits, the recovery r is a function r(x) of the head grade x. The value of $U_{ore}(x)$ must then be written as follows:

$$U_{ore}(x) = x \cdot r(x) \cdot (V - R) - (M_o + P_o + O_o)$$

The value of $U_{waste}(x)$ remains independent of x:

$$U_{waste}(x) = -(M_w + P_w + O_w)$$

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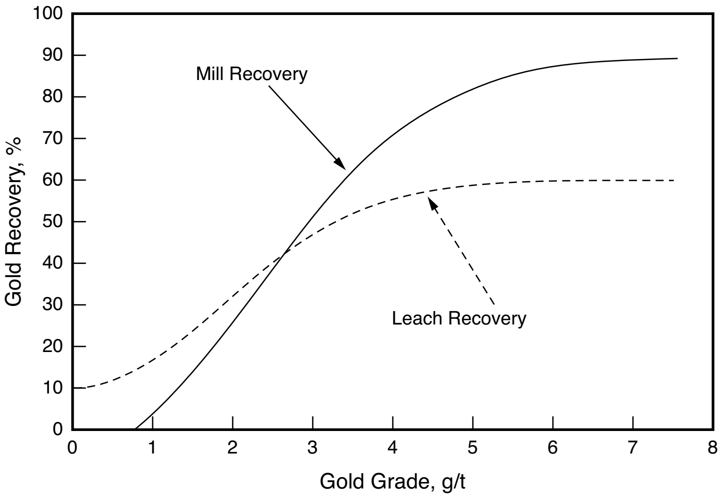

Figure 3-7 Relationship between recoveries and average grade

Calculating the cut-off grade requires finding the value of x such that $U_{ore}(x) = U_{waste}(x)$.

Nonlinear Recovery: A Precious Metal Example

Consider a gold mining operation from which material can be sent to either of two processing facilities, a leach pad or a mill. Mining, processing, and overhead costs are $4.20 for waste, $15.20 for leach, and $45.00 for mill. The gold price is $1,600 per ounce from which must be deducted a $50.00-per-ounce sales cost. The maximum gold recoveries are 60% for leaching and 90% for milling. Significantly lower recoveries are achieved for low-grade material. Figure 3-7 shows the relationship between recoveries and grade, as determined from metallurgical testing and historical production statistics.

Figure 3-8 shows the profit per metric ton that will be made depending on whether material of grade x is wasted ($U_{waste}(x) = -\$4.20$), sent to the leach pad ($U_1(x) = x \cdot r_1 \cdot \$49.83 - \$15.20$), or processed in the mill ($U_2(x) = x \cdot r_2 \cdot \$49.83 - \$45.00$). ($49.83 is the maximum value of one gram of gold contained, assuming 100% recovery: (V – R)/32.1035). Figure 3-8 also shows how the cut-off grade can be determined by graphical method. The relationship between the utility of leaching or milling material and the average grade of this material is no longer linear. The optimal process for material of grade x is that for which the utility is highest. The cut-off grades are the grades at which the curves intersect. If a constant 60% recovery for leached material and 90% recovery for milled material had been assumed, the waste-leach

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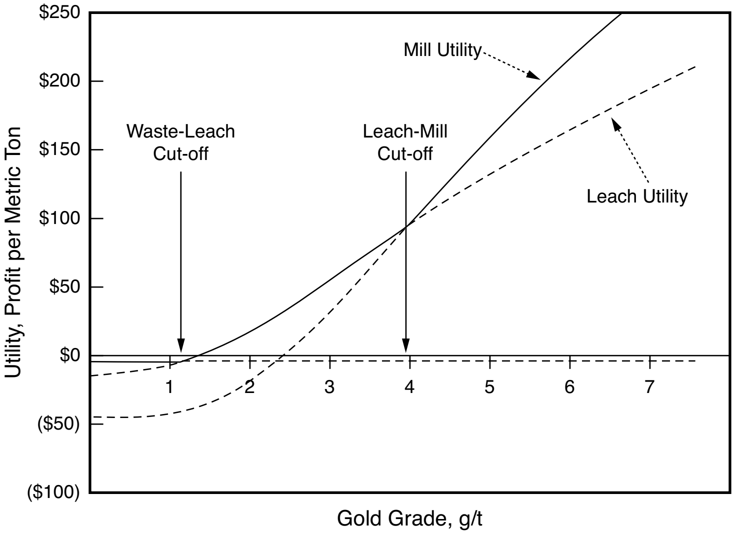

Figure 3-8 Graphical estimation of cut-off grade between wasted, leached, and milled material with variable recoveries

cut-off would have been estimated at 0.38 g/t and the leach-mill cut-off at 1.99 g/t. When variable recoveries are taken into account, the cut-offs are substantially higher, 1.16 g/t and 3.95 g/t respectively.

CONSTANT TAIL MODEL OF VARIABLE RECOVERY

Mathematical Formulation

A model often used to represent the relationship between plant recovery and average grade of plant feed is the constant tail model. This model assumes that a fixed amount of metal cannot be recovered, whatever the grade of the material sent to the plant. If x is the average grade of one metric ton of material and c is the fixed amount that cannot be recovered, the recoverable amount is

$$x \cdot r(x) = r_c \cdot (x - c)$$

where

x = average grade of material sent to process
$r(x)$ = plant recovery if head grade is x
r_c = constant recovery after subtracting constant tail
c = constant tail

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The recovery function is as follows:

$$\text{If } x < c:\ r(x) = 0$$
$$\text{If } x > c:\ r(x) = r_c \cdot (1 - c/x)$$

The cut-off grade is determined as follows:

$$U_{waste}(x) = -(M_w + P_w + O_w)$$
$$U_{ore}(x) = x \cdot r(x) \cdot (V - R) - (M_o + P_o + O_o)$$
$$= r_c \cdot (x - c) \cdot (V - R) - (M_o + P_o + O_o)$$
$$x_c = [(M_o + P_o + O_o) - (M_w + P_w + O_w)] / [r_c \cdot (V - R)] + c$$

If x'_c is the cut-off grade with constant recovery r_c, the cut-off grade with constant tail is

$$x_c = x'_c + c$$

Constant Tail Model: A Precious Metal Example

Consider a gold mining operation where material is being milled. Mining, processing, and overhead costs are \$4.20 for waste and \$45.00 for mill. The gold price is \$1,600 per ounce from which must be deducted a \$50.00-per-ounce charge.

As a result of metallurgical tests, the mill recovery was determined to be best represented using the constant tail model with parameters r_c = 90% and c = 0.50 g/oz:

$$\text{If } x < c:\ r(x) = 0$$
$$\text{If } x > c:\ r(x) = r_c \cdot (1 - c/x) = 0.90 \cdot (1 - 0.50/x)$$

The relationship between recovery r(x) and average grade x is shown in Figure 3-9. The relationships between $U_{ore}(x)$ and $U_{waste}(x)$ and grade x are calculated as follows:

$$U_{waste}(x) = -(M_w + P_w + O_w) = -4.20$$
$$U_{ore}(x) = x \cdot r(x) \cdot (V - R) - (M_o + P_o + O_o)$$
$$= 0.90 \cdot (x - 0.50) \cdot (1{,}600 - 50.00) / 31.1035 - 45.00$$
$$= 44.85x - 67.43$$

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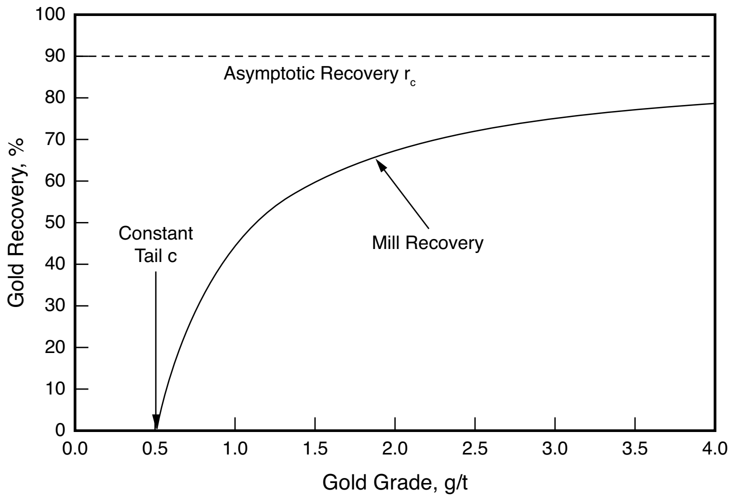

Figure 3-9 Relationship between recovery and average grade with constant tail

These relationships are depicted in Figure 3-10. The cut-off grade between ore and waste is x_c such that $U_{ore}(x) = U_{waste}(x)$:

$$x_c = (67.43 - 4.20)/44.85 = 1.41 \text{ g/t}$$

BREAKEVEN CUT-OFF GRADE AND POLYMETALLIC DEPOSITS

Polymetallic deposits are defined as deposits that contain more than one metal of economic value. The formulas that must be used to calculate the utility of sending one metric ton of material to a given destination or process must consider the contribution of each metal. The decision whether one metric ton of material should be wasted or sent to the processing plant can no longer be made on the basis of grade alone. Dollar values must be calculated for each possible process, and the cut-off between ore and waste must be expressed in dollar terms.

General Considerations

Consider a metric ton of material that contains two valuable metals, copper and gold. Let x_1 and x_2 be the copper and gold grades, respectively. The processing plant consists of crushing, grinding, and flotation circuits. A copper concentrate is produced, which is sold to a smelter. The flotation plant recovery is r_1 for copper and r_2 for gold. Mining, processing, and overhead costs

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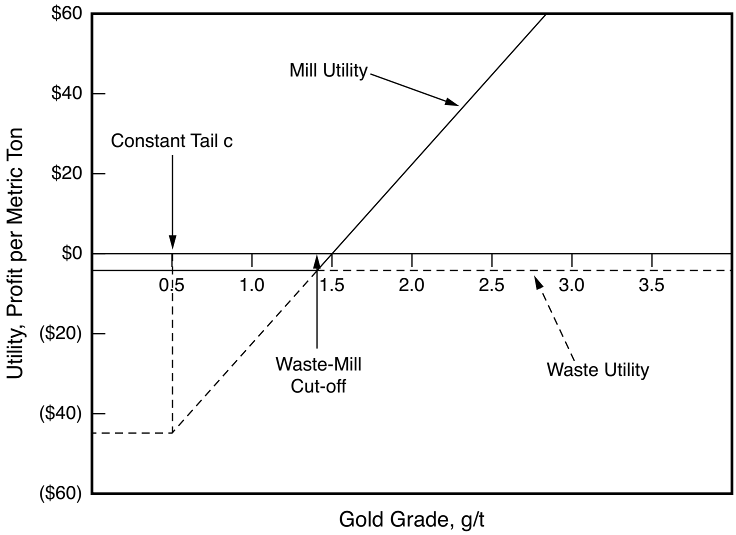

Figure 3-10 Graphical estimation of cut-off grade between wasted and milled material with constant tail

associated with one metric ton of material sent to the flotation plant are M_o, P_o, and O_o, respectively. The corresponding costs per metric ton of waste are M_w, P_w, and O_w. According to the smelter contract, the value received for sale of the concentrate is p_1 = 96.5% of the value of the copper contained in the concentrate after deducting d_1, and p_2 = 99% of the gold contained. Smelter cost deductions are C_s per metric ton of concentrate. The concentration ratio K is the number of metric tons of material that must be processed to produce one metric ton of concentrate. The cost of shipping one metric ton of concentrate to the smelter is C_t. Metal prices are those quoted on the London Metal Exchange, V_1 and V_2 for copper and gold, respectively. Hence the value of one metric ton of material sent to the flotation plant:

$$U_{ore}(x_1,x_2)=(x_1r_1-d_1)p_1V_1+(x_2r_2)p_2V_2-C_s/K-C_t/K-(M_o+P_o+O_o)$$

If the same metric ton is sent to the waste dump, the corresponding costs are

$$U_{waste}=-(M_w+P_w+O_w)$$

The material should be sent to the processing plant if

$$U_{ore}(x_1,x_2)>U_{waste}$$

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These formulas show that many factors enter into the calculation of the cut-off between ore and waste. Processing costs and recoveries are likely to be dependent not only on metal content but also on other geological characteristics such as mineralogy, hardness, clay content, and degree of oxidation, which change depending on the area of the deposit being mined. Smelter contracts heavily penalize concentrates that are found to contain excessive amounts of specified deleterious elements. All these factors must be taken into account when estimating the cut-off value applicable to one metric ton of mineralized material.

Because the value of one metric ton of material is a function of more than one grade, it is no longer meaningful to talk about a "cut-off grade." Historically, this multidimensional problem was reduced to a one-dimensional problem by defining a "metal equivalent." With the advance of computers and the ease of use with which complex mathematical calculations can be made, one now talks of *cut-off values*, which are expressed in dollar terms and require calculation of a net smelter return. Net smelter return and metal equivalent are discussed in the following sections.

Calculation of Cut-off Grades Using Net Smelter Return

For polymetallic deposits, the utility of sending one metric ton of material to the smelter is best expressed in terms of net smelter return, or NSR. The *net smelter return* is defined as the return from sales of concentrates, expressed in dollars per metric ton of ore, excluding mining and processing costs.

Mathematical Formulation

In the previous copper-gold example, the net smelter return of one metric ton of ore with copper grade x_1 and gold grade x_2 is

$$NSR(x_1, x_2) = (x_1 r_1 - d_1) p_1 V_1 + (x_2 r_2) p_2 V_2 - C_s/K - C_t/K$$

The utility of sending this metric ton of ore to the processing plant is

$$U_{ore}(x_1, x_2) = NSR(x_1, x_2) - (M_o + P_o + O_o)$$

Using NSR values simplifies the calculation of cut-off grades. The NSR cut-off between processing and wasting one metric ton of material is NSR_c, obtained by setting $U_{ore}(x_1, x_2)$ equal to U_{waste}:

$$NSR_c = (M_o + P_o + O_o) - (M_w + P_w + O_w)$$

In polymetallic deposits, cut-offs should not be expressed in terms of minimum metal grade; they should be expressed in terms of minimum NSR.

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Calculation of NSR Cut-off: A Copper-Molybdenum Example

Consider a copper molybdenum mining operation. In this section, the subscript 1 refers to copper and subscript 2 refers to molybdenum. Hence x_1 is the copper grade and x_2 is the molybdenum grade. The following parameters characterize the operation:

r_1 = 89% copper flotation plant recovery
p_1 = 96.5% copper smelting recovery
r_2 = 61% molybdenum flotation plant recovery
p_2 = 99% molybdenum roasting recovery
V_1 = \$1.20 value of one pound of copper sold
V_2 = \$6.50 value of one pound of molybdenum sold
R_1 = \$0.065 refining cost per pound of copper
K = 72 metric tons of ore must be processed to produce one metric ton of concentrate
$C_s + C_t$ = \$145 smelting and freight costs per metric ton of concentrate
R_2 = \$0.95 conversion, roasting, and freight costs per pound of molybdenum
M_o = \$1.00 mining cost per metric ton of ore milled
P_{o1} = \$3.00 mill processing cost per metric ton milled
P_{o2} = \$0.15 incremental molybdenum processing cost per metric ton milled
O_o = \$0.50 overhead cost per metric ton milled
M_w = \$1.00 mining cost per metric ton wasted
P_w = \$0.05 processing cost per metric ton wasted
O_w = \$0.05 overhead cost per metric ton wasted

The NSR of one metric ton of material with average grade x_1, x_2 is calculated as follows:

$$\begin{aligned}
NSR(x_1,x_2) &= x_1 r_1 p_1 (V_1 - R_1) \\
&\quad + x_2 r_2 p_2 (V_2 - R_2) - (C_s + C_t)/K \\
&= 0.89 \cdot 0.965 \cdot (1.20 - 0.065) \cdot 2{,}205 \cdot x_1 \\
&\quad + 0.61 \cdot 0.99 \cdot (6.50 - 0.95) \cdot 2{,}205 \cdot x_2 - 145.00/72 \\
&= 2{,}149 x_1 + 7{,}390 x_2 - 2.014
\end{aligned}$$

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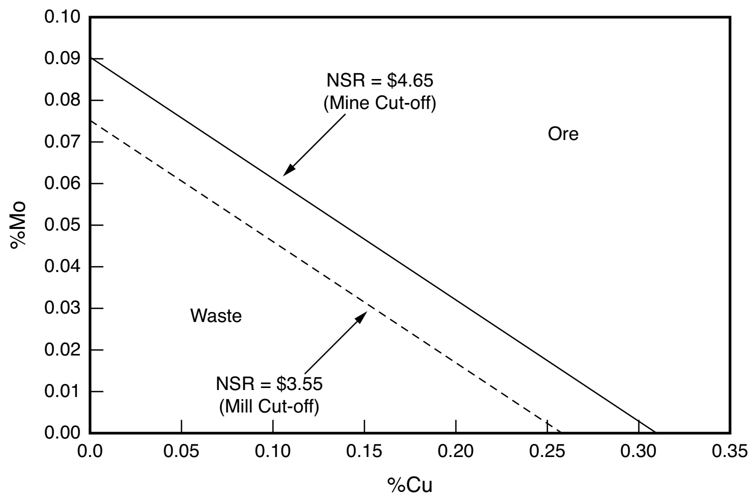

Figure 3-11 Relationship between cut-off NSR and metal grades

Therefore, the NSR of one metric ton of ore averaging x_1 = 0.45% Cu and x_2 = 0.035% Mo is $10.24. For material that must be mined but can be either wasted or processed, the cut-off NSR (*mill* or *internal* cut-off NSR) is NSR_c calculated as follows:

$$\begin{aligned} NSR_c &= (P_{o1} + P_{o2} - P_w) + (O_o - O_w) + (M_o - M_w) \\ &= (3.00 + 0.15 - 0.05) + (0.50 - 0.05) + (1.00 - 1.00) \\ &= \$3.55 \text{ per metric ton} \end{aligned}$$

For material that need not be mined (*mine* or *external* cut-off NSR), NSR_c is calculated as follows:

$$\begin{aligned} NSR_c &= P_{o1} + P_{o2} + O_o + M_o \\ &= 3.00 + 0.15 + 0.50 + 1.00 \\ &= \$4.65 \text{ per metric ton} \end{aligned}$$

The relationship between NSR_c, x_1, and x_2 is shown in Figure 3-11.

Calculation and Reporting of Metal Equivalent

Before computers became widely used, it was common practice to talk about polymetallic deposits in terms of metal equivalent. If one metric ton of material contains two metals, copper and molybdenum, with average grade x_1 and

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x_2 respectively, the corresponding copper equivalent is defined as the copper grade x_{1e} that one metric ton must contain to produce the same revenue, assuming no molybdenum.

The revenue generated by mining and processing one metric ton of material with copper grade x_1 and molybdenum grade x_2 is NSR(x_1, x_2). The revenue generated by mining and processing one metric ton of material with copper grade x_{1e} and no molybdenum is NSR(x_{1e}, 0.0). The copper equivalent is obtained by solving the following equation:

$$NSR\left(x_{1e}, 0.0\right) = NSR\left(x_1, x_2\right)$$

A molybdenum equivalent can be calculated instead of a copper equivalent. The molybdenum equivalent is the molybdenum grade x_{2e} that satisfies the following equation:

$$NSR\left(0.0, x_{2e}\right) = NSR\left(x_1, x_2\right)$$

In the previous copper-molybdenum example, the net smelter return was expressed as follows:

$$NSR\left(x_1, x_2\right) = x_1 r_1 p_1\left(V_1 - R_1\right) + x_2 r_2 p_2\left(V_2 - R_2\right) - \left(C_s + C_t\right)/K$$

Hence,

$$NSR\left(x_{1e}, 0.0\right) = x_{1e} r_1 p_1\left(V_1 - R_1\right) - \left(C_s + C_t\right)/K$$

The copper equivalent is

$$x_{1e} = x_1 + x_2\left[r_2 p_2\left(V_2 - R_2\right)\right]/\left[r_1 p_1\left(V_1 - R_1\right)\right]$$

Similarly, the molybdenum equivalent is

$$x_{2e} = x_2 + x_1\left[r_1 p_1\left(V_1 - R_1\right)\right]/\left[r_2 p_2\left(V_2 - R_2\right)\right]$$

Using the information listed previously concerning prices, cost, and recoveries, the copper and molybdenum equivalents can be calculated as follows:

$$x_{1e} = x_1 + x_2\left(7{,}390/2{,}149\right) = x_1 + 3.439x_2$$
$$x_{2e} = x_2 + x_1\left(2{,}149/7{,}390\right) = x_2 + 0.291x_1$$

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The copper equivalent of material averaging x_1 = 0.45% Cu and x_2 = 0.035% Mo is 0.57% Cu-equivalent. The molybdenum equivalent of the same material is 0.166% Mo-equivalent.

In practice, because of the complexity of the formulas to be used to correctly estimate the value of one metric ton of material, and because equivalence changes with metal price, recoveries, and refining costs, grade equivalence is rarely a useful tool in calculation of cut-off grades. Quoting the amount of metal equivalent contained in a deposit is of little use to investors. Publication of reserves in terms of metal equivalence is generally not accepted by regulatory agencies unless additional disclosures are made, including publication of the average grade of each metal and explanation of the formula used to calculate metal equivalence.

OPPORTUNITY COST OF NOT USING THE OPTIMUM CUT-OFF GRADE

If the optimum cut-off grade is not used, material is sent to a destination where the profit is less than could be made otherwise or the loss incurred is greater than necessary. Figure 3-12 shows the opportunity cost incurred per metric ton when a leach-mill cut-off grade of 3 g/t is used although the optimal cut-off grade is 4 g/t. The loss is represented by the difference between the utility of the chosen process and that of the optimal process for the same average grade. Figure 3-13 shows the opportunity cost incurred per metric ton when a leach-mill cut-off grade of 5 g/t is used.

Mathematical Formulation

Let $U_1(x)$ be the utility of leaching one metric ton of material of grade x and $U_2(x)$ be the utility of milling the same metric ton. These utilities can be written as follows (in these equations the cost of sales R is included in V, and the overhead costs O_o are included in M_o, P_{o1}, and P_{o2}):

$$U_1(x) = x \cdot r_1 \cdot V - (M_o + P_{o1})$$
$$U_2(x) = x \cdot r_2 \cdot V - (M_o + P_{o2})$$

The optimal cut-off grade is

$$x_c = (P_{o1} - P_{o2}) / [(r_1 - r_2)V]$$

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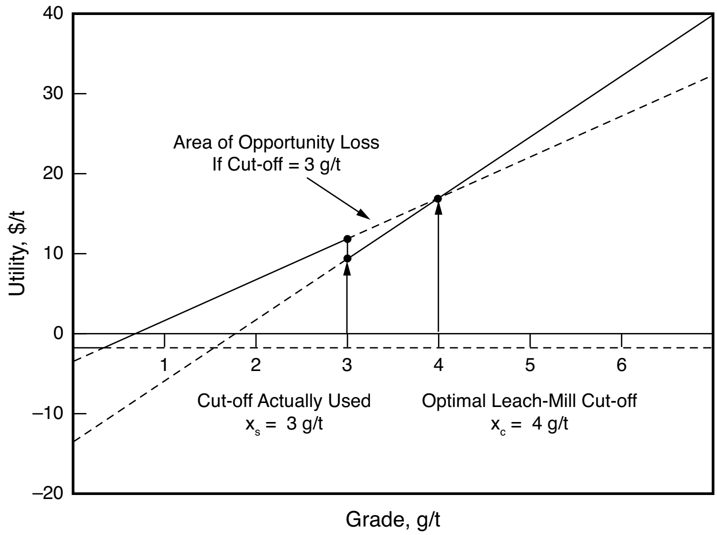

Figure 3-12 Opportunity cost of using a cut-off grade lower than the optimal cut-off grade

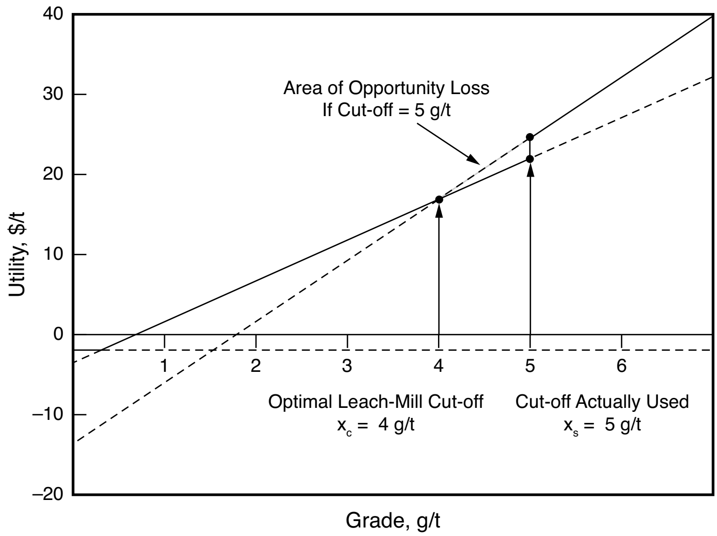

Figure 3-13 Opportunity cost of using a cut-off grade higher than the optimal cut-off grade

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Let x_s be the selected cut-off grade, which is lower than the optimal cut-off grade x_c (Figure 3-12). Material with grade x between x_s and x_c is being milled but, ideally, should be leached. For each metric ton of grade x between x_s and x_c, the opportunity cost is

$$U_2(x) - U_1(x) = x \cdot (r_2 - r_1) \cdot V - (P_{o2} - P_{o1})$$

Integrating this formula from $x = x_s$ to $x = x_c$ we obtain the total opportunity cost:

$$\text{total opportunity cost} = \left[Q(x_s) - Q(x_c)\right] \cdot (r_2 - r_1) \cdot V - \left[T(x_s) - T(x_c)\right] \cdot (P_{o2} - P_{o1})$$

In this formula, $T(x_s) - T(x_c)$ is the tonnage of material with average grade between x_s and x_c, and $Q(x_s) - Q(x_c)$ is the quantity of metal contained in this material. One could avoid this opportunity cost by increasing the mill capacity by a tonnage amount equal to $T(x_s) - T(x_c)$. Such an increase in capacity is justified if the cost of such an increase is expected to be less than the total opportunity cost.

Similar equations are applicable if x_s is higher than x_c and material that should be milled is leached (Figure 3-13):

$$\text{total opportunity cost} = \left[Q(x_c) - Q(x_s)\right] \cdot (r_1 - r_2) \cdot V - \left[T(x_c) - T(x_s)\right] \cdot (P_{o1} - P_{o2})$$

An example of how to estimate the opportunity cost is given in Example 5 in Appendix A.

CUT-OFF GRADE AND LOW-GRADE STOCKPILES

Consideration may be given to stockpiling low-grade material instead of wasting it if such material is not currently economic to process but metal prices are expected to be higher at a later date. Stockpiling low-grade material may also be considered when capacity constraints prevent current processing of material that otherwise could be processed economically. To decide whether material of grade x should be wasted or stockpiled, one must compare the utility of wasting $U_{waste}(x)$ with that of stockpiling $U_{stp}(x)$. The cut-off grade between stockpile and waste is the value x_c of x for which $U_{stp}(x) = U_{waste}(x)$.

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The utility of wasting material of grade x is

$$U_{waste}(x) = -(M_w + P_w + O_w)$$

To calculate the utility of stockpiling, one must take into consideration stockpiling costs, and the cost of retrieving material from the stockpile and processing it at a later date. In addition, metallurgical recoveries of stockpiled material may differ from those of freshly mined material, and the price of the product sold may be different from that prevailing when the decision to stockpile is made:

$$\begin{aligned} U_{stp}(x) = -(M_{stp} + P_{stp} + O_{stp}) \\ -\text{NPV(future costs of stockpile maintenance)} \\ -\text{NPV(future re-handling and processing costs)} \\ +\text{NPV(future revenues from sales)} \end{aligned}$$

where

M_{stp} = current mining costs per metric ton delivered to the low-grade stockpile

P_{stp} = current costs of stockpiling material that will be processed later, including the cost per metric ton of extending the stockpile area if required

O_{stp} = current overhead costs associated with mining and stockpiling

NPV(future costs of stockpile maintenance) = net present value of yearly costs that will be incurred to maintain stockpiled material in an environmentally safe fashion until it is processed

NPV(future re-handling and processing costs) = net present value of the one-time costs that will be incurred when the material is retrieved from the stockpile and processed

NPV(future revenues from sales) = net present value of revenues expected from sales when processed material is sold

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At the time the product recovered from the stockpiles is sold, the revenues are $x \cdot r_{stp} \cdot (V_{stp} - R_{stp})$, where r_{stp} is the expected recovery at the time of processing, V_{stp} is the dollar value of the product sold at the time it is sold, and R_{stp} is the corresponding cost of sales per unit of product sold. The recovery r_{stp} may be less or higher than that which would apply to the same material if processed when mined. Sulfide material is likely to oxidize during stockpiling. If a sulfide flotation process is to be used, oxidation will result in lower recovery. Conversely, if an oxide leach process is to be applied to material that was not fully oxidized when mined, stockpiling may result in enhanced recovery.

There are obvious difficulties in using these formulas, the main one being that future costs and revenues are difficult or impossible to estimate with accuracy. Furthermore, because processing of stockpiled material is likely to occur late in the mine life, the net present value of future revenues is likely to be small compared with costs incurred at the time of mining and to maintain the stockpile over time. For this reason, stockpiling low-grade material is often a strategic decision that takes into account expectations of future increases in metal prices, benefits associated with lengthening the mine life, and good management of mineral resources. An example of how cut-off grades can be estimated can be found in Example 6 in Appendix A.

Special Case: Low-Cost Stockpiles

The previous formulas are of a general nature and apply to stockpiles intended to be processed in the foreseeable future that are located in specially prepared areas. Some situations allow low-grade material to be stockpiled on top of waste dumps with no additional cost. Such stockpiles may or may not be processed at a future date. The utility of stockpiling can then be written:

$$\begin{aligned} U_{stp}(x) = &-\left(M_w + P_w + O_w\right) \\ &-\text{NPV(future re-handling and processing costs)} \\ &+\text{NPV(future revenues from sales)} \end{aligned}$$

The cut-off grade between waste and stockpile is the value of x for which

$$\text{NPV(future revenues from sales)} = \text{NPV(future re-handling and processing costs)}$$

Because sales and stockpile re-handling happen at the same time, discounting can be ignored and the cut-off grade is obtained by setting the expected revenues from sales equal to the expected re-handling and processing costs.

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The expected revenues from sales are $x \cdot r_{stp} \cdot (V_{stp} - R_{stp})$. The cut-off grade between waste and stockpile is

$$x_c = \frac{(\text{expected re-handling and processing costs})}{r_{stp} \cdot \left(V_{stp} - R_{stp}\right)}$$

Net present values do not enter in this equation. If the assumptions concerning prices, costs, and recoveries that will prevail at the time of processing are independent of time, the cut-off grade will also be independent of when the stockpile is processed.

Special Case: High-Cost Waste Dumps

Other situations occur where the cost of maintaining mineralized waste dumps is as high as that of stockpiling low-grade material for future processing. However, waste dumps must be maintained for life, whereas low-grade stockpiles will be processed. If the expected life of the stockpile is n years and C is the yearly cost of maintaining either the stockpile or the waste dump, the net present value of maintenance costs is*

$$P_{waste} = C\, g(i,n) + C(1+i)/i$$
$$P_{stp} = C\, g(i,n)$$

C g(i,n) is the net present value of a constant cash flow C over n years. C (1 + i)/i is the net present value of a constant cash flow C to perpetuity. The cut-off grade between waste and stockpile can be written as follows:

$$x_c = \frac{(\text{expected re-handling and processing costs}) - C(1+i)/i}{r_{stp} \cdot \left(V_{stp} - R_{stp}\right)}$$

If the assumptions concerning prices, costs, and recoveries that will prevail at the time of processing are independent of time, the cut-off grade will also be independent of when the stockpile is processed.

* It was shown earlier that the net present value of a constant cash-flow C over n years is $NPV_i = C\, g(i,n)$, where $g(i,n) = [1 - (1 + i)^{-n}]\,(1 + i)/i$ while the net present value to perpetuity is C (1 + i)/i.

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CUT-OFF GRADE AND OPTIMIZATION OF PROCESSING PLANT OPERATING CONDITIONS

In this section a method is developed to optimize a copper mining operation where mining capacity is fixed, but the capacity of the processing plant can be changed by changing grind size. Depending on the metallurgical properties of the ore, using a coarser grind will increase plant throughput while reducing cost per metric ton processed and decreasing recovery. Conversely, a finer grind can decrease plant capacity, increase processing cost and recovery.

Mathematical Formulation

The following notations are used in this section:

r = processing plant recovery
V = value of copper contained in concentrate, after deduction for smelter loss, freight, smelting, and refining costs
P_o = cost per metric ton of ore processed, including overhead
x_c = cut-off grade
T_{+c} = tonnage above cut-off grade to be processed in one year
Q_{+c} = quantity of copper to be processed in one year
x_{+c} = average grade above cut-off grade

Since mining capacity and costs are fixed, the utility function that must be optimized to estimate the economically optimal grind size is only a function of mill operations and can be written as follows:

$$U(T_{+c}) = Q_{+c} \cdot r(T_{+c}) \cdot V - T_{+c} \cdot P_o(T_{+c})$$

where

$U(T_{+c})$ = utility of running the plant at T_{+c} capacity for one year
$r(T_{+c})$ = processing plant recovery, if plant capacity is T_{+c}
$P_o(T_{+c})$ = cost per metric ton of ore processed, if plant capacity is T_{+c}

Q_{+c} is also a function of T_{+c}. Both Q_{+c} and T_{+c} are functions of the cut-off grade x_c. The optimal plant capacity is that for which $U(T_{+c})$ reaches a maximum, and is calculated by setting the first derivative of $U(T_{+c})$ equal to zero:

$$\begin{aligned} dU(T_{+c})/dT_{+c} &= 0 \\ &= dQ_{+c}/dT_{+c} \cdot r(T_{+c}) \cdot V - P_o(T_{+c}) \\ &\quad + Q_{+c} \cdot dr(T_{+c})/dT_{+c} \cdot V - T_{+c} \cdot dP_o(T_{+c})/dT_{+c} \end{aligned}$$

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If the tonnage processed is changed by a small amount dT_{+c} because of a small change in cut-off grade x_c, the amount of copper contained is increased from $Q_{+c} = T_{+c} \cdot x_{+c}$ to $Q_{+c} + dQ_{+c} = T_{+c} \cdot x_{+c} + dT_{+c} \cdot x_c$. Hence, $dQ_{+c} = dT_{+c} \cdot x_c$ and the optimal plant capacity is T_{+c} such that

$$x_c \cdot r(T_{+c}) \cdot V - P_o(T_{+c}) + Q_{+c} \cdot dr(T_{+c})/dT_{+c} \cdot V - T_{+c} \cdot dP_o(T_{+c})/dT_{+c} = 0$$

If the recovery r and the processing cost P_o were independent of T_{+c}, this equation would be easily solved for x_c:

$$x_c = P_o(T_{+c})/\left[r(T_{+c}) \cdot V\right] = P_o/[r \cdot V]$$

In such a situation, the applicable cut-off grade is the *mill cut-off grade*.

The term $Q_{+c} \cdot dr(T_{+c})/dT_{+c} \cdot V$ represents the change in the value of the product sold in one year, which results from the change in recovery. The term $T_{+c} \cdot dP_o(T_{+c})/dT_{+c}$ represents the change in operating cost per year, which results from the change in processing cost per metric ton.

In this formulation of the problem, it was assumed that the value V of the product sold is independent of the tonnage processed. This may not be the case if the quality of the concentrate varies with tonnage processed and head grade. It was also assumed that recovery is only a function of tonnage processed and is independent of head grade. More complex equations would apply if these assumptions could not be made.

Example: Optimization of Grinding Circuit in a Copper Mine

This example illustrates how plant capacity can be optimized when mine plans are fixed; no major change can be made to the processing plant, but the plant capacity can be increased by changing grind size. Mine production is fixed for at least one year, and the tonnage, grade, and metal content of copper-bearing material expected to be mined during this one-year period is as shown in Table 3-1 and illustrated in Figures 3-14 and 3-15.

The ore is to be processed in a flotation plant. The mill was designed to operate at the rate of 39.5 million metric tons per year with an average copper recovery of 95%. Under these conditions, the mill operating costs are $5.24 per metric ton. When mine plans were finalized for the coming year, the expected value of product sold was $1.00 per pound of copper in concentrate, and the following mill cut-off grade was used for planning:

$$x_c = 5.24/(0.95 \cdot 1.00 \cdot 2,205) = 0.25\% \text{ Cu}$$

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Table 3-1 Grade–tonnage relationship for the coming year of mining

Cut-off, % Cu	Tonnage, million metric tons	Average grade, % Cu	Copper Content: thousand metric tons Cu	Copper Content: million lb Cu
0.15%	53.7	0.335%	180	397
0.16%	52.6	0.340%	179	395
0.17%	51.4	0.344%	177	390
0.18%	50.1	0.348%	174	384
0.19%	48.8	0.352%	172	378
0.20%	47.5	0.355%	168	372
0.21%	46.0	0.360%	165	365
0.22%	44.4	0.365%	162	357
0.23%	42.8	0.370%	159	349
0.24%	41.2	0.375%	155	341
0.25%	39.5	0.381%	150	332
0.26%	37.7	0.387%	146	322
0.27%	35.9	0.393%	141	311
0.28%	34.1	0.399%	136	300
0.29%	32.1	0.406%	131	288
0.30%	30.2	0.413%	125	275
0.31%	28.2	0.421%	119	262

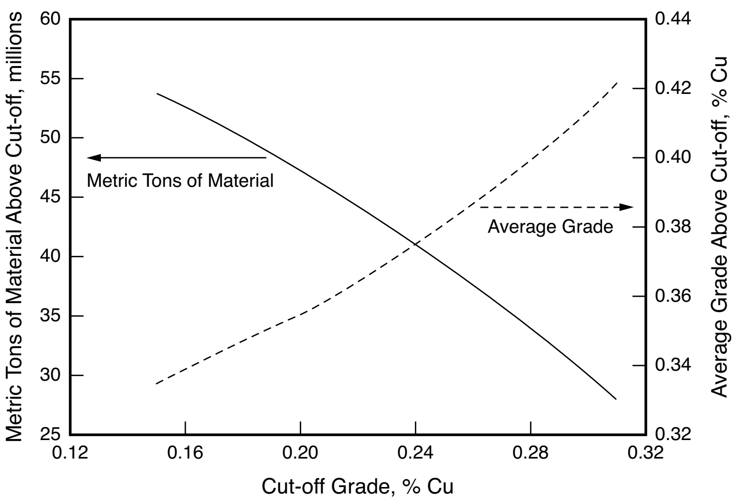

Figure 3-14 Next year expected relationship between tonnage and grade above cut-off grade

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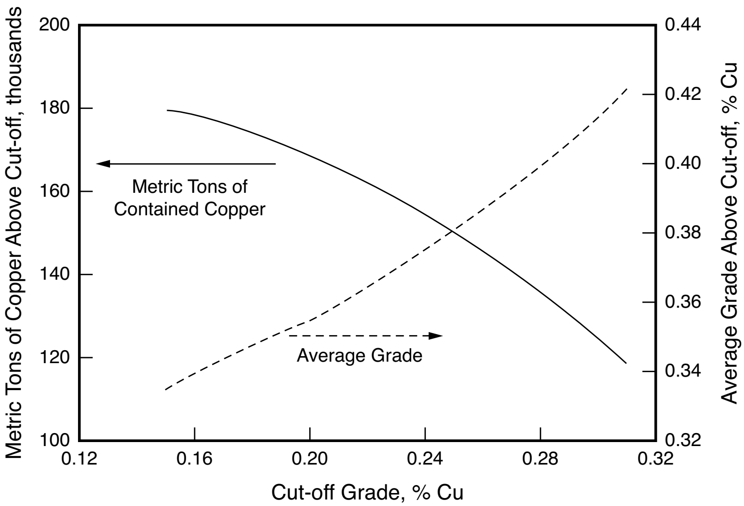

Figure 3-15 Next year expected relationship between metal content and grade above cut-off grade

As shown in Table 3-1, this cut-off grade implies that the mill feed will be 39.5 million metric tons averaging 0.381% Cu and containing 332 million pounds of copper. The value of the material sent to the mill, based on $1.00 per pound of recoverable copper and excluding mining costs, was expected to be

$$\begin{aligned} U(T_{+c}) &= Q_{+c} \cdot r(T_{+c}) \cdot V - T_{+c} \cdot P_o(T_{+c}) \\ &= 332 \cdot 0.95 \cdot 1.00 - 39.5 \cdot 5.24 \\ &= \$108 \text{ million} \end{aligned}$$

Because of an unexpected increase in copper price, the mining company is investigating whether short-term changes could be made to the mill feed and throughput, which would result in increased utility. The copper price is now expected to be $1.50 per pound of copper in concentrate instead of the $1.00 that was used for planning. The mine plan cannot be changed for at least one year, and only changes in operating conditions can be made to the processing plant. One option is to operate the mine and mill as planned while selling the concentrate at the higher price. The value of the material sent to the mill, excluding mining costs, would increase from $108 million to

$$\begin{aligned} U(T_{+c}) &= 332 \cdot 0.95 \cdot 1.50 - 39.5 \cdot 5.24 \\ &= \$266 \text{ million} \end{aligned}$$

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Alternatively, one could consider a decrease in cut-off grade. At $1.50 per pound of copper in concentrate, the minimum cut-off grade is

$$x_c = 5.24/(0.95 \cdot 1.50 \cdot 2{,}205) = 0.17\%\ \text{Cu}$$

Table 3-1 shows that 51.4 million metric tons of ore would be mined above this cut-off grade, averaging 0.344% Cu. Under current operating conditions, the mill can only process 39.5 million metric tons. The higher-grade tons could be sent to the mill and the lower-grade tons could be stockpiled. But such an approach is likely to increase short-term costs without increasing revenues from concentrate sales. No advantage is taken of the higher copper price.

Another option would consist of increasing mill throughput by increasing grind size. The result would be a decrease in operating cost per metric ton. However, this is expected to result in a decrease in mill recovery. It has been determined that the mill operating costs are 55% fixed costs and 45% inversely proportional to the tonnage processed:

$$P_o(T_{+c}) = 2.88 + 93.1/T_{+c}$$

This relationship between operating cost per metric ton and tonnage processed per year is shown in Figure 3-16. The relationship between copper recovery and mill throughput is shown in Figure 3-17. This relationship is represented by the following equation:

$$r(T_{+c}) = -0.000232(T_{+c})^2 + 0.01362T_{+c} + 0.7729$$

The function to be optimized is

$$U(T_{+c}) = Q_{+c} \cdot r(T_{+c}) \cdot V - T_{+c} \cdot P_o(T_{+c})$$

In this equation, $Q_{+c} \cdot r(T_{+c}) \cdot V$ represents the value of copper in concentrate, and $T_{+c} \cdot P_o(T_{+c})$ represents the processing costs. The relationship between $U(T_{+c})$ and the cut-off grades (which defines T_{+c}) is easily calculated using Table 3-1 and the three preceding equations. The results are summarized in Table 3-2 and shown in Figure 3-18. The highest return is $272 million, $6 million higher than the $266 million calculated previously when plant capacity was kept at 39.5 million metric tons per year. This highest return is reached by increasing the plant capacity to approximately 44.4 million metric tons per year.

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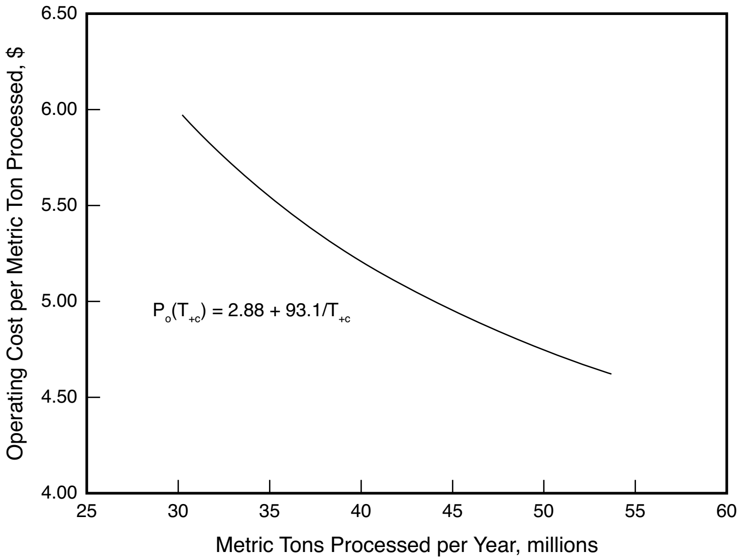

Figure 3-16 Relationship between mill operating cost per metric ton and tonnage processed per year

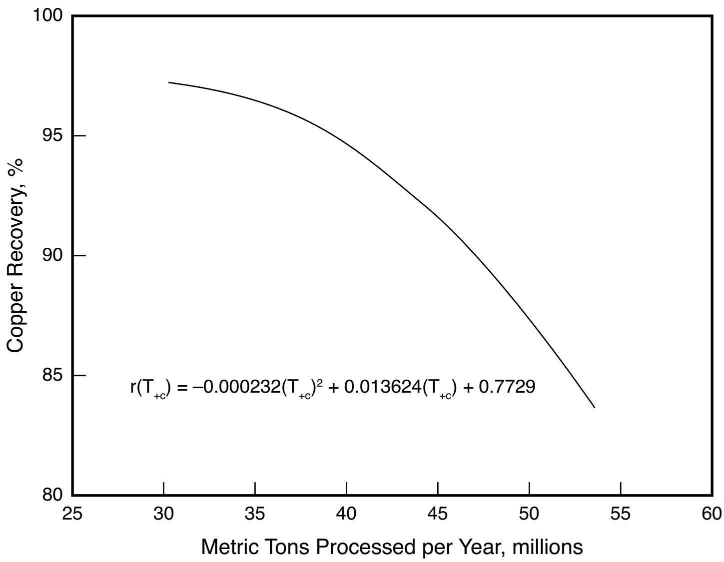

Figure 3-17 Relationship between copper recovery and tonnage processed per year

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Table 3-2 Calculation of $U(T_{+c})$ for various cut-off grades and corresponding tonnages of mill feed T_{+c}

V	$/lb	$1.50	$1.50	$1.50	$1.50	$1.50	$1.50
x_c	% Cu	0.20%	0.21%	0.22%	0.23%	0.24%	0.25%
T_{+c}	million metric tons	47.5	46.0	44.4	42.8	41.2	39.5
x_{+c}	% Cu	0.355%	0.360%	0.365%	0.370%	0.375%	0.381%
Q_{+c}	million pounds Cu	372	365	357	349	341	332
$r(T_{+c})$	%	89.65%	90.86%	92.04%	93.09%	94.03%	94.90%
$P_o(T_{+c})$	$/t	$4.84	$4.90	$4.98	$5.06	$5.14	$5.24
$Q_{+c} \cdot r(T_{+c}) \cdot V$	million $/yr	$500	$497	$493	$487	$481	$473
$-T_{+c} \cdot P_o(T_{+c})$	million $/yr	($230)	($226)	($221)	($216)	($212)	($207)
$U(T_{+c})$	million $/yr	$270	$271	$272	$271	$269	$266

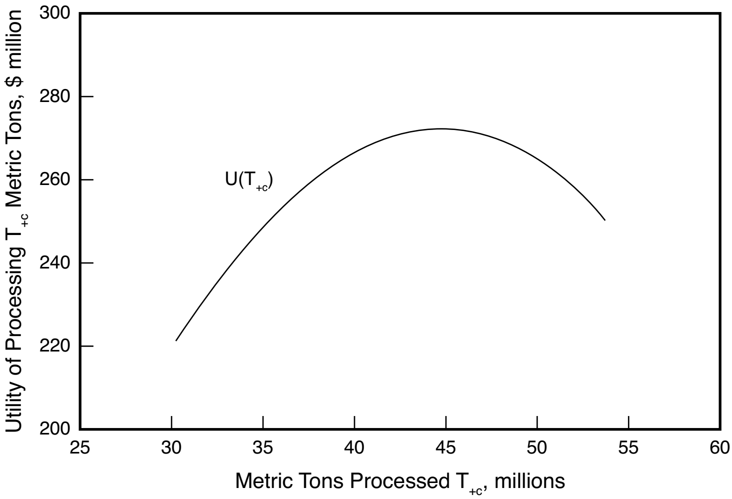

Figure 3-18 Relationship between utility $U(T_{+c})$ and tonnage of mill feed T_{+c}

An alternative method of calculating the optimum processing rate consists of solving the following equation:

$$dU(T_{+c})/dT_{+c} = 0.0$$

which can be written as

$$x_c \cdot r(T_{+c}) \cdot V - P_o(T_{+c}) + Q_{+c} \cdot dr(T_{+c})/dT_{+c} \cdot V - T_{+c} \cdot dP_o(T_{+c})/dT_{+c} = 0.0$$

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Table 3-3 Calculation of $dU(T_{+c})/dT_{+c}$ for various cut-off grades and corresponding tonnages of mill feed T_{+c}

x_c	% Cu	0.20%	0.21%	0.22%	0.23%	0.24%	0.25%
$x_c \cdot r(T_{+c}) \cdot V - P_o(T_{+c})$	\$/t	\$1.09	\$1.41	\$1.72	\$2.03	\$2.32	\$2.61
$Q_{+c} \cdot dr(T_{+c})/dT_{+c} \cdot V$	\$/t	(\$4.70)	(\$4.23)	(\$3.74)	(\$3.27)	(\$2.81)	(\$2.34)
$-T_{+c} \cdot dP_o(T_{+c})/dT_{+c}$	\$/t	\$1.96	\$2.02	\$2.10	\$2.18	\$2.26	\$2.36
$dU(T_{+c})/dT_{+c}$	\$/t	(\$1.65)	(\$0.80)	\$0.08	\$0.94	\$1.77	\$2.62

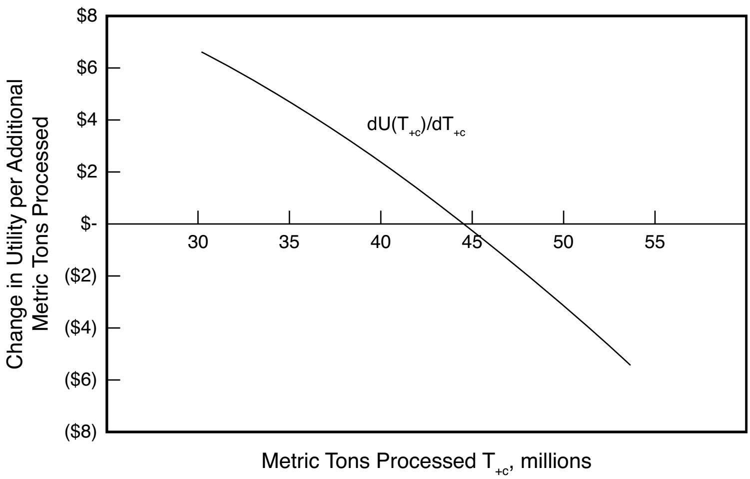

Figure 3-19 Relationship between incremental utility $dU(T_{+c})/dT_{+c}$ and tonnage of mill feed T_{+c}

In this equation, $x_c \cdot r(T_{+c}) \cdot V - P_o(T_{+c})$ represents the change in value assuming that recovery and costs remain constant; $Q_{+c} \cdot dr(T_{+c})/dT_{+c}$ is attributable to the change in recovery; $-T_{+c} \cdot dP_o(T_{+c})/dT_{+c}$ is attributable to the change in costs.

The derivatives of $P_o(T_{+c})$ and $r(T_{+c})$ are easily calculated:

$$dP_o(T_{+c})/dT_{+c} = -93.1/(T_{+c})^2$$

$$dr(T_{+c})/dT_{+c} = -0.000464 \cdot T_{+c} + 0.01362$$

The relationship between $dU(T_{+c})/dT_{+c}$ and the cut-off grades is easily calculated using Table 3-2 and the three preceding equations. The results are summarized in Table 3-3 and plotted in Figure 3-19. The optimal return is obtained if the tonnage of mill feed is set slightly less than 45 million metric tons per year, the point where $dU(T_{+c})/dT_{+c} = 0.0$ (Figure 3-19). Setting the cut-off grade at 0.22% Cu will reach this objective, producing 44.4 million

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metric tons of mill feed (Table 3-2). The average mill head grade will be 0.365% Cu. Increasing the tonnage from 39.5 million metric tons to 44.4 million metric tons will be achieved by decreasing recovery from 95% to 92%. This loss in recovery will be more than compensated by a decrease in operating costs from $5.24 to $4.98 per metric ton.

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4

Capacity Constraints and Opportunity Costs

Capacity constraints have a direct influence on the cut-off grade. Opportunity costs are associated with each constraint, which implies that the cut-off grade must be higher than the marginal cut-off grade, if the primary objective is to maximize net present value. Decisions that are made without fully understanding the effects, direct and indirect, of changing cut-off grades can have dire consequences. This was the case when very low marginal cut-off grades were first introduced in low-grade gold leaching operations.

WHEN MARGINAL ANALYSIS NO LONGER APPLIES: A GOLD LEACHING OPERATION

In the 1990s, many gold mining companies significantly increased the tonnage of material placed on their leach pads by lowering the cut-off grade. Marginal analysis of leaching costs indicated that already low cut-off grades, often less than 0.5 g/t, could be further reduced, sometimes down to 0.2 g/t. The expectation was that, with more ounces being placed on the leach pad, the amount of gold recovered would increase on a monthly basis, as well as cumulatively over time, while the cost per metric ton placed would decrease. The results initially obtained were often disappointing. The tonnage of material added by lowering the cut-off grade was large, resulting in a short-term decrease in recovery, which in the worse cases meant a decrease in revenue instead of the expected increase. In addition, the long-term impact of adding large tonnages of very low-grade material to a leach pad was not fully understood. In some cases, the result seemed to be a decrease in overall pad recovery, not only postponing short-term revenues but showing no increase in cumulative revenues over the life of the project. On a discounted basis, the benefit of lowering the cut-off grade was significantly less than expected, if not negative.

To illustrate this point, consider a gold mining operation where the total tonnage of ore and waste material scheduled to be mined in the coming year was 10 million metric tons. This material was characterized by the

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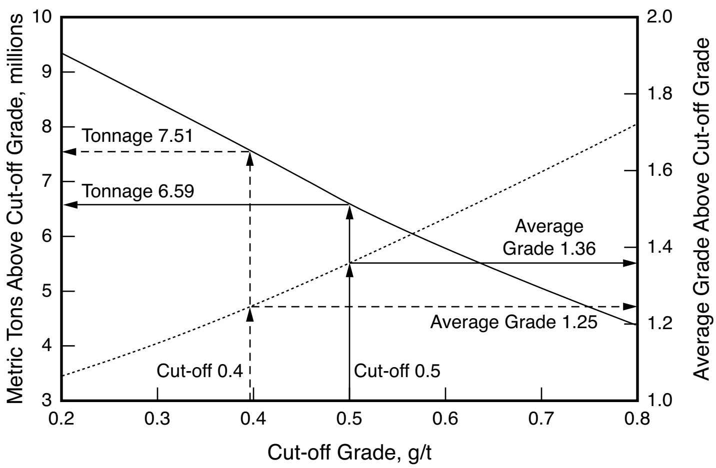

Figure 4-1 Estimation of tonnage and grade above cut-off grade

grade–tonnage curve shown in Figure 4-1. Initially, the cut-off grade was set at 0.50 g/t, which corresponded to 6.59 million metric tons of leach-grade material averaging 1.36 g/t and containing 288,000 oz of gold. The leach recovery was expected to be 65%, resulting in the production of 187,000 oz in the coming year.

A review of the previous year's operating costs showed that the cut-off grade could be lowered to 0.40 g/t if the recovery could be maintained at 65%. Laboratory tests confirmed that recovery was independent of grade, and the decision was made to lower the cut-off grade and add the lower-grade material to the pad.

After two months of operation, managers realized that the gold production target for the year was not going to be met. If nothing changed, the amount of gold sold was going to be less than expected before the cut-off grade was decreased. Management immediately requested a review of the situation. The results of this review were as follows:

- Metallurgical tests confirmed no decrease in recovery for lower-grade material.
- Metallurgical tests and review of past operational conditions showed that the amount of gold recovered was an increasing function of the solution ratio, defined as the metric tons of cyanide solution used per metric ton of material placed on the pad. This relationship was as illustrated in Figure 4-2.

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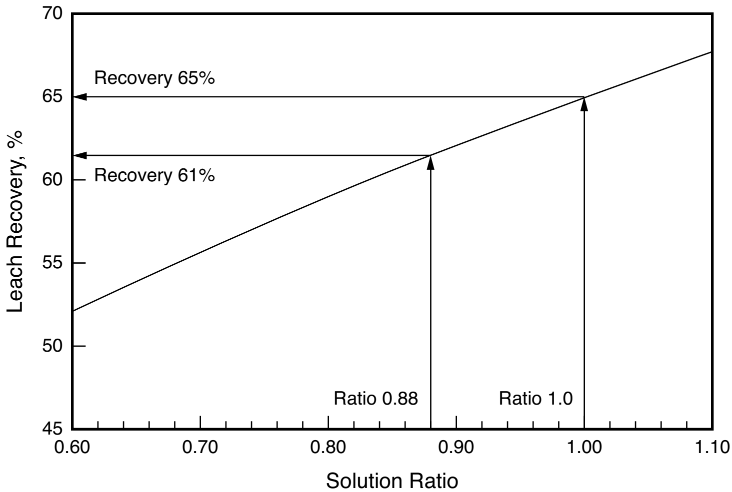

Figure 4-2 Relationship between leach recovery and solution ratio

- Provided that a four-month leaching cycle was adhered to, the original solution ratio was 1:1, as needed to reach 65% recovery.
- Lowering the cut-off grade to 0.40 g/t increased the tonnage to be placed on the pad from 6.59 to 7.51 million metric tons and decreased the average grade from 1.36 to 1.25 g/t (Figure 4-1). The ounces placed increased from 288,000 to 302,000 oz, a 5% increase.
- Because no change was made to the amount of solution placed on the pad, the increase in tonnage from 6.59 to 7.51 million metric tons resulted in a decrease in solution ratio from 1.0 to 0.88. The expected recovery should have been 61% instead of 65% (Figure 4-2).
- This 6% decrease in recovery exceeds the expected 5% increase in ounces placed on the pad. The total metal recovered during the year should have been expected to decrease from 187,000 to 184,000 oz.

Ignoring the relationship between leach recovery and solution ratio was equivalent to ignoring a capacity constraint. The corresponding opportunity cost was ignored, and consequently the cut-off grade was underestimated. Lowering the cut-off grade to 0.40 g/t might have been justified if a cost-effective method of increasing the recovery had been put in place. One option was to increase the volume of fresh solution placed on the pad, which would require changes in pond size, pipes, pumps, and the capacity of the carbon columns or Merrill-Crowe plant used to process the solution. Another option

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was to recycle the pregnant solution on the pad, which would increase the solution-to-ore ratio without incurring some of the high costs associated with the first option. All changes to the leach plant had to take into account constraints imposed by operating permits and other conditions (technical, environmental, or legal) that would limit the options available to solve the problem.

Assume that, for environmental and permitting reasons, the size of the leach plant could not be increased. Which approach should have been used to determine the optimal cut-off grade? Taking into account the low operating costs, this optimal cut-off grade was likely to be less than 0.5 g/t (which was determined on the basis of higher costs) but more than 0.4 g/t (which used the lower costs but ignored the operating constraint). An iterative approach could be used that consists of decreasing the cut-off grade by small successive increments and fully assessing the economic consequences until no further decrease is justified.

1. Assume that the cut-off grade is lowered from the current 0.50 g/t to 0.48 g/t.
2. Estimate the increase in tonnage and ounces that will be placed on the pad as a result of the lower cut-off grade.
3. Calculate the corresponding decrease in solution ratio and leach recovery.
4. Calculate the resulting change in total gold recovered, taking into account the increase in gold placed and decrease in recovery.
5. Compare the change in expected gold sold with the corresponding change in cost of operation. Differences between the cost of wasting material and placing it on the pad should be taken into account.
6. If the change in revenue from sales exceeds the change in costs, the cut-off grade can be reduced to 0.48 g/t. The analysis should then be repeated assuming a lower 0.46 g/t cut-off. The optimum cut-off is that for which the change in revenue is equal to the change in cost.

CAPACITY CONSTRAINTS AND INCREASE IN CUT-OFF GRADE

All mining operations are subject to capacity constraints. These constraints can be in the mine itself, in the processing plant, in the refinery, or in the volume of material that can be sold. A consequence is that if additional material is added to the currently expected production, this material will postpone

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mining, processing, refining, or sale of this production. The net present value of the currently expected cash flow will be decreased. This decrease, which is by definition the *opportunity cost* of adding the new material, must be taken into account when calculating cut-off grades.

Mathematical Formulation

The mathematical equations that are used to determine opportunity costs were discussed in chapter 2. They can be summarized as follows. NPV_i is the net present value of the currently expected cash flow, and t is the time by which this cash flow will be delayed by adding new material to the production schedule. The opportunity cost of adding this material is

$$U_{opp}(x) = NPV_i\left[(1+i)^{-t} - 1\right]$$

The cut-off grade is estimated by comparing the cost of adding one additional metric ton of material to the production schedule, with the value of the product recoverable from this material. A new metric ton added to the production schedule must pay for all direct costs associated with this ton, and for the opportunity cost. When t is a very small fraction of one year, as is the case when only one metric ton of material is being considered, the opportunity cost is estimated as follows:

$$U_{opp}(x) = -i \cdot t \cdot NPV_i$$

The time t that is needed to mine, process, or sell one unit of material is equal to the inverse of the constrained capacity. If the constraint is in the mine, the capacity is measured in metric tons mined per year, t is the time needed to mine one metric ton of material, and the opportunity cost is measured in dollars per metric ton mined. This cost must be added to the mining costs M_o and M_w. If the constraint is in the mill, t is the time needed to process one metric ton of material, and the opportunity cost must be added to the processing cost P_o. If the constraint is in the refining capacity—which in the case of a copper mine is expressed in pounds of copper per year—t is the time needed to refine one pound of copper, and the opportunity cost must be added to the refining cost R.

The following equations, defined previously, will also be used in this section:

Utility of mining 1 t of ore: $U_{ore}(x) = x \cdot r \cdot (V - R) - (M_o + P_o + O_o)$

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Utility of mining 1 t of waste: $U_{waste}(x) = -(M_w + P_w + O_w)$

Mine cut-off grade: $x_c = [(M_o + P_o + O_o) - (M_w + P_w + O_w)] / [r \cdot (V - R)]$

Mill cut-off grade: $x_c = (M_o + P_o + O_o) / [r \cdot (V - R)]$

Cut-off Grade in a Capacity-Constrained Copper Mine

In this section, a copper mine example is used to illustrate the relationship between capacity constraints, opportunity cost, and cut-off grade. A preliminary feasibility study was completed, which included specific assumptions concerning the mine, mill, and refinery capacities. The results of the feasibility study are summarized in Tables 4-1 and 4-2.

Results of Project Feasibility Study

The project is characterized by a set of operational parameters, both technical and financial, that are summarized in Table 4-1. These parameters are used to calculate a yearly cash flow, C = $115.68 million, which is assumed to be constant over the ten-year life of the project. Cash flow calculation and project net present value are summarized in Table 4-2. Given a constant cash flow per year over ten years (n = 10), and a discount rate of 12% (i = 12%), the project net present value can be calculated using this formula:

$$NPV_i = C\, g(i,n) = C\left[1-(1+i)^{-n}\right]\cdot(1+i)/i = 6.328\ C$$

The project net present value is $732.06 million.

Breakeven Cut-off Grades

Assuming no capacity constraint, the utility of sending one metric ton of material of grade x to the process plant is

$$U_{ore}(x) = x\cdot 0.84\cdot(2.95-0.75)-(4.50+9.00+2.20) = 1.848x-15.70$$

If one metric ton is sent to the waste dump, the utility is

$$U_{waste}(x) = -(4.00+0.80+0.65) = -5.45$$

The mine and mill breakeven cut-off grades are calculated as follows:

Mine breakeven cut-off grade: $x_c = 15.70/(1.848 \cdot 2{,}205) = 0.385\%$ Cu

Mill breakeven cut-off grade: $x_c = (15.70 - 5.45)/(1.848 \cdot 2{,}205) = 0.252\%$ Cu

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Table 4-1 Copper mine operating and financial assumptions

General operating conditions		
Mine capacity	72,000,000	metric tons mined per year
Mill capacity (flotation)	50,000,000	metric tons processed per year
Refining capacity	480,000,000	pounds of copper refined per year
Average grade	0.52% Cu	average grade of ore processed
Fixed costs (unallocated)	$100,000,000	$/yr
Variable costs		
M_w	$4.00	$/waste metric ton mined
P_w	$0.80	$/waste metric ton mined
O_w	$0.65	$/waste metric ton mined
M_o	$4.50	$/ore metric ton mined
P_o	$9.00	$/ore metric ton mined
O_o	$2.20	$/ore metric ton mined
Average grade, recovery, and revenue per pound of copper		
x	0.55% Cu	average grade of ore mined
r	84%	copper recovery (flotation and smelter)
V	$2.95	copper price per pound of recovered copper
R	$0.75	freight and smelting cost per pound of recovered copper
Discount rate and mine life		
i	12%	discount rate
n	10	mine life (years)

Cut-off Grades with Mill Capacity Constraint

If the mill capacity is limited to 50 million metric tons of ore per year, adding one new metric ton of material to the mill feed will postpone the currently expected cash flow by t years, where t is the inverse of the mill capacity. The corresponding opportunity cost is

$$U_{opp}(x) = -i \cdot t \cdot NPV_i = -0.12 \cdot 732{,}060{,}000/50{,}000{,}000$$
$$= -\$1.76 \text{ per metric ton processed}$$

The mine and mill cut-off grades are calculated adding this cost to the processing cost:

$$\text{Mine cut-off grade: } x_c = (15.70 + 1.76)/(1.848 \cdot 2{,}205) = 0.428\% \text{ Cu}$$

$$\text{Mill cut-off grade: } x_c = (15.70 + 1.76 - 5.45)/(1.848 \cdot 2{,}205)$$
$$= 0.295\% \text{ Cu}$$

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Table 4-2 Yearly cash flow and net present value of copper project

Yearly cash flow		
Metric tons mined	72	million metric tons per year
Metric tons ore	50	million metric tons per year
Metric tons waste	22	million metric tons per year
Cost per metric ton ore	\$15.70	\$/t mined and processed
Cost per metric ton waste	\$5.45	\$/t mined and wasted
Cost per year ore	\$785.00	million \$/yr
Cost per year waste	\$119.90	million \$/yr
Total operating cost	\$904.90	million \$/yr
Fixed cost per year	\$100.00	million \$/yr
Total cost per year	\$1,004.90	million \$/yr
Ore grade	0.55%	Cu
Copper contained	606.38	million lb Cu/yr
Copper recovered	509.36	million lb Cu/yr
Revenue per pound sold	\$2.20	\$/lb
Revenue from sales	\$1,120.58	million \$/yr
Net cash flow	\$115.68	million \$/yr (C)
Net present value		
Discount rate	12%	%/yr (i)
Mine life	10	years (n)
NPV factor g(i,n)	6.328	$g(i,n) = [1 - (1 + i)^{-n}] \cdot (1 + i)/i$
Net present value	\$732.06	$NPV_i = C\ g(i,n)$

To compensate for the mill opportunity cost, both the mine and mill cut-off grades must be increased by 0.043% Cu.

Cut-Off Grades with Mine Capacity Constraint

If the mine capacity is limited to 72 million metric tons of ore per year, mining one new metric ton of material that was not scheduled to be mined will postpone the currently expected cash flow by t years, where t is the inverse of the mine capacity. The corresponding opportunity cost is

$$U_{opp}(x) = -i \cdot t \cdot NPV_i = -0.12 \cdot 732{,}060{,}000/72{,}000{,}000$$
$$= -\$1.22 \text{ per metric ton mined}$$

The mine and mill cut-off grades are calculated adding this cost to the mining cost:

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Mine cut-off grade: $x_c = (15.70 + 1.22)/(1.848 \cdot 2{,}205) = 0.415\%$ Cu

Mill cut-off grade: $x_c = [(15.70 + 1.22) - (5.45 + 1.22)]/(1.848 \cdot 2{,}205)$
$= 0.252\%$ Cu

The mill breakeven cut-off grade is equal to the marginal cut-off grade. The reason is that the cost of processing one metric ton of material after it has been mined is independent of the mining cost.

Cut-off Grades with Refinery Capacity Constraint

If the refinery has a limited capacity of 500 million pounds of copper per year, adding one more pound of copper will postpone the currently expected copper production by t years, where t is the inverse of the refinery capacity. The corresponding opportunity cost is

$$U_{opp}(x) = -i{\cdot}t{\cdot}NPV_i = -0.12 \cdot 732{,}060{,}000/500{,}000{,}000$$
$$= -\$0.176 \text{ per pound refined}$$

The mine and mill cut-off grades are calculated adding this cost to the cost per pound:

Mine cut-off grade: $x_c = 15.70/[0.84 \cdot (2.95 - 0.75 - 0.176) \cdot 2{,}205]$
$= 0.419\%$ Cu

Mill cut-off grade: $x_c = (15.70 - 5.45)/[0.84 \cdot (2.95 - 0.75 - 0.176) \cdot 2{,}205]$
$= 0.273\%$ Cu

To compensate for the refinery opportunity cost, the mine and mill cut-off grades must both be increased by the same percentage: (2.95 – 0.75)/(2.95 – 0.75 – 0.176) = 8.7%.

DECREASING CUT-OFF GRADE OVER TIME: A STRATEGIC OBJECTIVE

In most situations, the expected net present value of future cash flows decreases over time. When the project approaches the end of its life, little material remains to be mined and the net present value of future cash flows approaches zero, or it may become negative as reclamation costs exceed generated cash flows.

Consider the copper mine described previously. The yearly cash flow was expected to be constant at \$115,680,000 per year. Table 4-3 shows how the

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Table 4-3 Calculation of remaining net present value in years 0 to 3

	Year when remaining NPV_i is calculated (i = 12%)							
	0		1		2		3	
Project Year	Remaining Cash Flow	Discount Factor	Remaining Cash Flow	Discount Factor	Remaining Cash Flow	Discount Factor	Remaining Cash Flow	Discount Factor
0	115.68	1.000						
1	115.68	0.893	115.68	1.000				
2	115.68	0.797	115.68	0.893	115.68	1.000		
3	115.68	0.712	115.68	0.797	115.68	0.893	115.68	1.000
4	115.68	0.636	115.68	0.712	115.68	0.797	115.68	0.893
5	115.68	0.567	115.68	0.636	115.68	0.712	115.68	0.797
6	115.68	0.507	115.68	0.567	115.68	0.636	115.68	0.712
7	115.68	0.452	115.68	0.507	115.68	0.567	115.68	0.636
8	115.68	0.404	115.68	0.452	115.68	0.507	115.68	0.567
9	115.68	0.361	115.68	0.404	115.68	0.452	115.68	0.507
10	0		0		0		0	
NPV_i	$732.05		$690.34		$643.62		$591.29	

net present value of future cash flows should be calculated for the first three years of the mine life. In year 0, this NPV is $732.05 million; in year 1, it is $690.34 million; in year 2, it is $643.62 million. In year 9, the last year of operation, the NPV is equal to a one-year undiscounted cash flow, $115.68 million. In year 10, there is no production and the cash flow is zero. Figure 4-3 shows the relationship between the NPV of future cash flows and the year when this NPV is calculated.

As the NPV of future cash flows decreases, the opportunity cost decreases and so does the cut-off grade. The relationships between NPV_i, the opportunity cost $i \cdot t \cdot NPV_i$, and the mill and mine cut-off grades are calculated in Table 4-4. The capacity constraints are assumed to be the same throughout the mine life:

- Mill capacity: 50 million metric tons processed per year
- Mine capacity: 72 million metric tons mined per year
- Refinery capacity: 500 million lb of copper per year

The cut-off grades are estimated using the equations developed earlier:

- If the mill is capacity constrained, t = 1/50,000,000 yr/t and
  Mine cut-off grade: $x_c = (15.70 + i \cdot t \cdot NPV_i)/(1.848 \cdot 2{,}205)$
  Mill cut-off grade: $x_c = (15.70 + i \cdot t \cdot NPV_i - 5.45)/(1.848 \cdot 2{,}205)$

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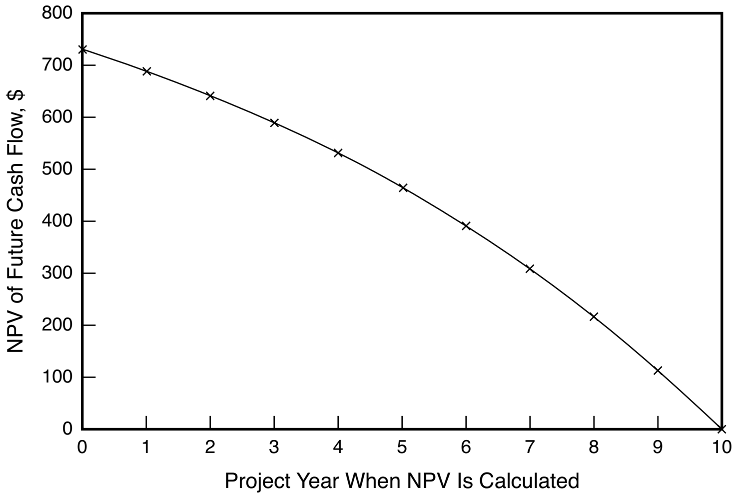

Figure 4-3 Relationship between NPV of future cash flow and year when NPV is calculated

- If the mine is capacity constrained, t = 1/72,000,000 yr/t and
  Mine cut-off grade: $x_c = (15.70 + i{\cdot}t{\cdot}NPV_i)/(1.848 \cdot 2{,}205)$
  Mill cut-off grade: $x_c = (15.70 - 5.45)/(1.848 \cdot 2{,}205)$
- If the smelter is capacity constrained, t = 1/500,000,000 yr/lb and
  Mine cut-off grade:
  $x_c = 15.70/[0.84 \cdot (2.95 - 0.75 - i{\cdot}t{\cdot}NPV_i) \cdot 2{,}205]$
  Mill cut-off grade:
  $x_c = (15.70 - 5.45)/[0.84 \cdot (2.95 - 0.75 - i{\cdot}t{\cdot}NPV_i) \cdot 2{,}205]$

Figure 4-4 shows how the mine cut-off grade changes in function of the year when it is calculated, depending on whether constraints are on mine, mill, or refinery capacity. Figure 4-5 shows the same information for the mill cut-off grade. In this example, the mill capacity constraint is that which has the highest cost, as measured by the amount of copper needed to maintain the project net present value.

CUT-OFF GRADE BELOW MARGINAL CUT-OFF GRADE

Situations arise when it is justified to use a cut-off grade below the marginal cut-off grade. At the end of the mine life, the net present value of future cash flows may be negative, when reclamation costs are exceeding expected

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Table 4-4 Calculation of opportunity cost and cut-off grade per year

Capacity Constraint		Mill Constraint, 50 million metric tons processed			Mine Constraint, 72 million metric tons mined			Refinery Constraint, 500 million lb copper		
			Cut-off Grade, % Cu			Cut-off Grade, % Cu			Cut-off Grade, % Cu	
Year	NPV_i, $ million	Opportunity Cost, $	Mine	Mill	Opportunity Cost, $	Mine	Mill	Opportunity Cost, $	Mine	Mill
0	732	1.76	0.43%	0.29%	1.22	0.42%	0.25%	0.18	0.42%	0.27%
1	690	1.66	0.43%	0.29%	1.15	0.41%	0.25%	0.17	0.42%	0.27%
2	644	1.54	0.42%	0.29%	1.07	0.41%	0.25%	0.15	0.41%	0.27%
3	591	1.42	0.42%	0.29%	0.99	0.41%	0.25%	0.14	0.41%	0.27%
4	533	1.28	0.42%	0.28%	0.89	0.41%	0.25%	0.13	0.41%	0.27%
5	467	1.12	0.41%	0.28%	0.78	0.40%	0.25%	0.11	0.41%	0.27%
6	394	0.94	0.41%	0.27%	0.66	0.40%	0.25%	0.09	0.40%	0.26%
7	311	0.75	0.40%	0.27%	0.52	0.40%	0.25%	0.07	0.40%	0.26%
8	219	0.53	0.40%	0.26%	0.36	0.39%	0.25%	0.05	0.39%	0.26%
9	116	0.28	0.39%	0.26%	0.19	0.39%	0.25%	0.03	0.39%	0.25%
10	0	0.00	0.39%	0.25%	0.00	0.39%	0.25%	0.00	0.39%	0.25%

revenues from operations. The opportunity cost changes sign from negative to positive: postponing expenditures increases the remaining project net present value. A cut-off grade below the marginal cut-off grade may be justified, provided the loss incurred in mining and processing this material is less than the benefit of postponing reclamation expenditures.

At the end of the mine life, if C_R is the expected reclamation cost, the net present value of this cost is $-C_R$. If reclamation is postponed by one year, this net present value is increased from $-C_R$ to $-C_R/(1 + i)$. The opportunity cost is

$$C_R\left[1-1/(1+i)\right]=C_R\, i/(1+i)$$

The cut-off grade that can be used to postpone reclamation by one year can be below the marginal cut-off grade. The only requirement is that the cash flow C generated by mining low-grade material during the last year, added to the corresponding opportunity, is positive:

$$C+C_R\, i/(1+i)>0$$
$$C>-C_R\, i/(1+i)$$

The cash flow C can be negative. If the cost of reclamation is $15 million and i = 10%, the maximum loss that can be justified to postpone reclamation by one year is $15 \cdot (0.1/1.1)$ = $1.36 million.

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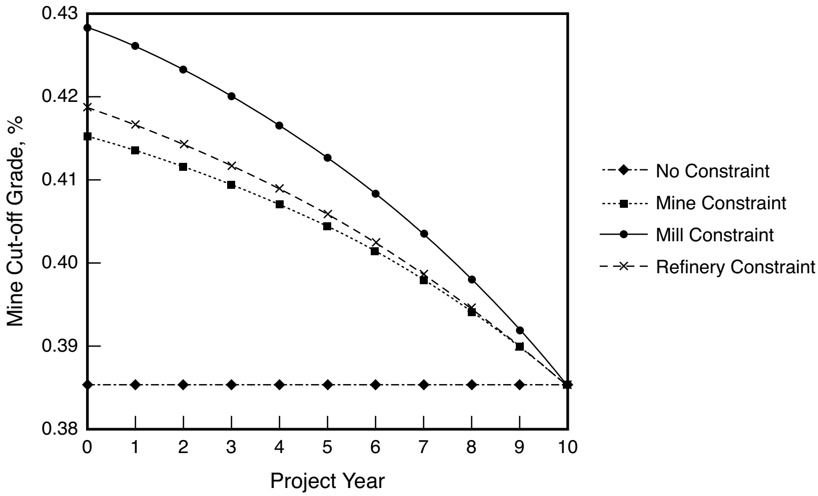

Figure 4-4 Relationship between mine cut-off grade and project year

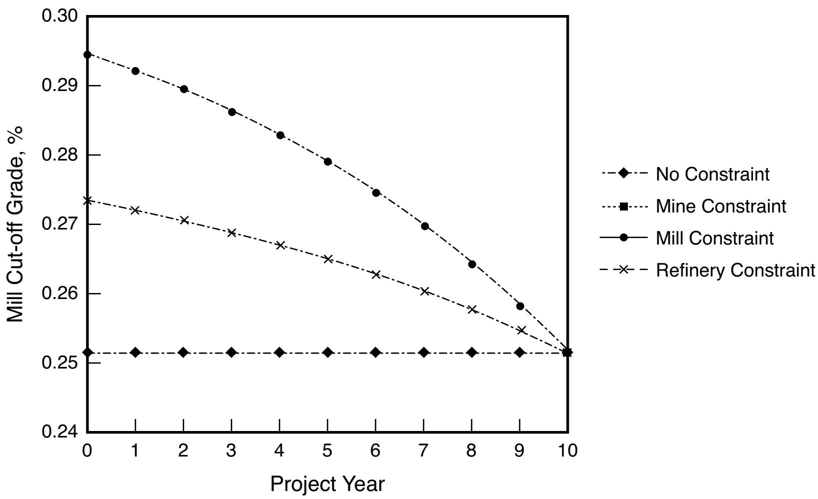

Figure 4-5 Relationship between mill cut-off grade and project year

Postponing reclamation costs is justified if maximizing net present value is the only objective. Other financial, legal, environmental, and socioeconomic factors may be taken into account that will influence the decision when to close the mine. Other circumstances occur where using a cut-off grade below the marginal cut-off grade is justified. This may be the case when the tonnage of material exceeding the marginal cut-off grade is less than the

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plant capacity. Several options must then be considered, which have different technical and financial consequences:

- Run the plant below capacity. This will increase the cost per metric ton processed. In some circumstances, higher recovery is achievable when reducing the mill throughput. How to optimize plant capacity was discussed previously.
- Run the plant as a batch plant, with periodic shutdowns. The cost of shutdowns must be considered to optimize cut-off grade.
- Run the plant at capacity using a cut-off grade below the marginal cut-off grade. If the tonnage needed to fill the plant is relatively small, this option is likely to be the best one.

CUT-OFF GRADE OPTIMIZATION WITH OPPORTUNITY COSTS: PRACTICAL CONSIDERATIONS

The formula $U_{opp}(x) = -i \cdot t \cdot NPV_i$ is useful to determine whether cut-off grades and net present value have been optimized. The cut-off grade calculated using opportunity costs must be compared with that currently used in the mine plan. If the former is higher than the latter, consideration should be given to increasing the cut-off grade. This may be possible by increasing mine capacity or changing plant capacity. Conversely, if the planned cut-off grade is lower than the cut-off grade calculated using the opportunity cost, indications are that facilities might have been overdesigned. Constraints imposed by the geology of the deposit have a major influence on the determination of cut-off grades and optimum mine and mill capacities. This influence will be discussed in Chapter 5.

Cut-off grades calculated from cash flows that have not been optimized are also not optimal. An iterative approach must be used to maximize the net present value. For example, one could first calculate a cash flow using a fixed cut-off grade such as the marginal cut-off grade. From this cash flow, cut-off grades could be reestimated using opportunity costs. But new cut-off grades imply new mine plans, new cash flows, and therefore new opportunity costs, which must be used to, once again, reestimate the cut-off grades. This lengthy process must be repeated until the net present value is optimized. Fortunately, algorithms have been developed and continue to be improved to facilitate simultaneous optimization of mine plans, production schedules, and material allocation, with implied optimization of cut-off grade strategies (see bibliography). Computer programs are available that render complex theoretical solutions accessible to mining engineers and other practitioners.

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While solutions applicable to open pit environments are more advanced, underground mines are given increasing attention.

Before changing the cut-off grade, all costs and benefits likely to result from this change must be carefully examined. Complex relationships occur between the space-dependent (and sometimes time-dependent) geological and geotechnical properties of the deposit, the time-dependent economic variables (capital and operating costs, metal prices, environmental and socioeconomic impact), and technical constraints, which, to some extent, can be controlled (mining and processing methods, operational capacity). These factors must all be taken into account to optimize cut-off grades, yearly production, and cash flows. No simple solutions exist for this complex multidimensional optimization problem.

Cut-off grades that were estimated to be optimal at the time the original mine plan was developed must be continuously reexamined. Current and expected costs and prices, mine and mill performance, and environmental and socioeconomic conditions will change over time, resulting in changes in future cash flow and opportunity costs. Maximizing net present value tends to give no value to actions where consequences will only be felt at the end of the mine life. For example, actions may have to be taken throughout the life of a project to minimize future costs of reclamation and environmental compliance. The cost of these actions may be significant from a net present value point of view, but the resulting savings, which will be incurred at the end of the mine life, may have no impact on the net present value. Similarly, stockpiling low-grade material may increase costs throughout the mine life, but revenues resulting from processing these stockpiles will only be realized at the end of the mine life. Maximizing net present value should never be the sole guide to decision making.

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5

Geologic Constraints and Opportunity Costs

As mentioned previously, constraints imposed by the geology of the deposit play a critical role in defining the optimal cut-off grade. The tonnage and grade of material that can be made available for processing is continuously changing during the life of the mine. The influence of geologic constraints is analyzed in conjunction with other constraints such as those on mine or mill capacity, and on volume of sales.

CUT-OFF GRADE WHEN PROCESSING CAPACITY IS FIXED

Ideally, a new mine should be designed such that mining capacity and processing capacity are perfectly balanced and the planned cut-off grades are optimum given the expected cash flow. In practice, this situation occurs only on paper, when the project is designed. As soon as operations start, imbalances invariably appear. The actual processing plant capacity exceeds or falls below that expected. The mining capacity is higher or lower than planned. Mine and mill capacities are no longer balanced, new constraints appear, and the cut-off grade must be changed accordingly. The cut-off grade must also take into account differences between expected and actual costs, productivities, recoveries, and market value of product sold. When a new project is designed, mine and mill capacities and corresponding cut-off grades are chosen primarily to optimize financial objectives. Once mine and mill facilities are built, physical constraints become the main drivers and studies must be completed to determine whether removing these constraints is financially justified.

Relationship Between Mine Capacity and Cut-off Grade

In this section, it will be assumed that the capacity of the processing plant is fixed and cannot be changed. The only change that can be made is to the mining capacity. What is the impact of a change in mining capacity on the cut-off grade and the grade of the material sent to the plant?

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- Consider an increase in mining capacity, defined as tonnage mined per year.
- This increase requires an increase in mining capital cost, and is likely to result in an increase in total mine operating costs per year. However, it is also likely to result in decreased mining and overhead costs per metric ton mined.
- Given that the processing capacity is fixed, the cut-off grade must be increased to keep the tonnage sent to the mill constant. The average grade of mill feed will increase and so will the quantity of product sold.
- The mine life will decrease.
- In some instances, the lower-grade material, which is not processed, will be stockpiled. Stockpiling of low-grade material was discussed previously.

To decide whether an increase in mining capacity is justified, the expected net impact on the utility of the project must be assessed, taking into account the following factors:

- Increased capital cost of new mining capacity
- Decreased mine unit operating cost
- Increased plant head grade and increased metal sales per year
- Loss of low-grade material or delayed processing of some of this material
- Reduced mine life and resulting socioeconomic and political impact
- Reduced project life and decreased political risk, if applicable
- Change in environmental impact

A simple example follows. Consider a mining operation in which the plant was designed to process an average of 250,000 metric tons per month, or 3 million metric tons per year. The grade–tonnage relationship corresponding to the mineralized material expected to be mined during the coming year is shown in Figure 5-1. At the current mining capacity, the 3 million metric ton plant capacity is consistent with a cut-off grade of 0.74 g/t and a mill feed average grade of 1.56 g/t.

Consideration is being given to increasing the mining capacity by 50%, and the impact such a change would have on the coming year is being investigated. If the mining capacity is increased by 50%, the material currently scheduled to be mined in one year will be mined in eight months. Since the mill

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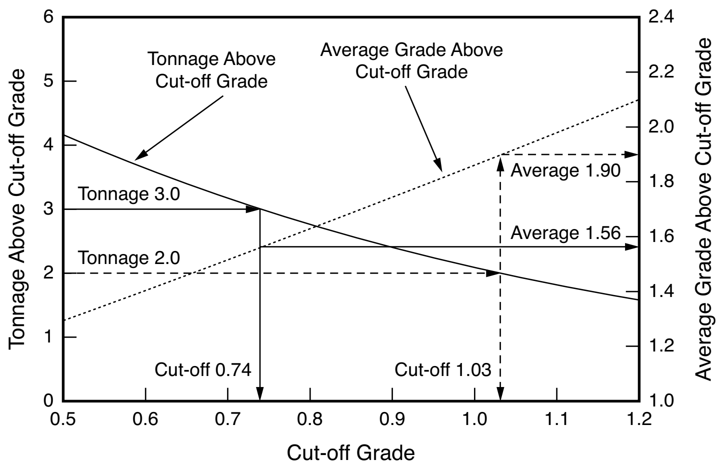

Figure 5-1 Estimation of cut-off grade assuming fixed processing capacity

can process only 250,000 metric tons a month, it will only consume 2 million metric tons during this eight-month period. This tonnage corresponds to a cut-off grade of 1.03 g/t and an average mill feed head grade of 1.90 g/t (Figure 5-1). Consideration should be given to stockpiling the material between 1.03 g/t and a cut-off grade somewhat higher than 0.74 g/t. This material should be considered for re-handling and processing at a later date.

The proposed 50% increase in mining capacity may or may not be optimal. To be economically justified, an increase in mining capacity must take into account financial, technical, environmental, permitting, and other constraints imposed by deposit size and shape, mining method, size of equipment, safety and environmental regulations, and other parameters. Depending on the limitations imposed by these constraints, an iterative approach is best suited to mining capacity optimization. Such an approach can consist of the following steps:

1. Assume a 1 million metric ton increase in mining capacity (or some other increase that is technically achievable).
2. Calculate the resulting decrease in mine life.
3. Estimate the increase in cut-off grade and resulting higher mill head grade that is consistent with the increase in mining capacity and fixed processing capacity.
4. Estimate the increase in mine capital and yearly operating costs needed to increase the mining capacity. Calculate the corresponding

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discounted incremental mining costs (DIMC) for the remaining life of the project.

5. Estimate the increase in mill production per year (units of product sold) and calculate the corresponding discounted incremental revenues (DIR).
6. If low-grade material is to be stockpiled, the net present value of this material should also be taken into account.
7. If the DIR exceed the DIMC, this analysis should be repeated assuming an additional 1-million-metric-ton increase in mining capacity.
8. The optimal mining capacity is that for which DIR equals DIMC.

In this previous discussion, it was assumed that the increase in mine capacity could be achieved without changing mine selectivity. The grade–tonnage curve did not change. The volumes being mined remained the same, but these volumes were mined faster. This situation will occur if more equipment of the same size is added to an open pit mine with no change to the pit design, or if more stopes are put in production simultaneously without changing the underground mining method or the stope design. Some situations occur where the assumptions of constant grade–tonnage curve cannot be made. In open pit mines, larger trucks and loading equipment, increased bench height, and wider spacing between blast holes can be used to increase capacity. The result is a decrease in selectivity, resulting in a new grade–tonnage curve. Similarly, underground production can be increased by using a different mining method, which may result in lower selectivity joined with significantly lower costs per metric ton. The impact of selectivity on the grade–tonnage curve, the cut-off grade, and the mill feed average grade will be discussed at the end of Chapter 6. Increasing the mining capacity will not necessarily result in a higher head grade if this increase is realized by significantly decreasing mine selectivity.

Using Opportunity Cost as a Guide to Mine Capacity Optimization

The approach to mine capacity optimization previously described can be extremely time-consuming. An alternative approach is presented here, which can be used to rapidly converge toward the desired solution, provided there is little variability in the geologic properties of the deposit being mined.

Consider an open pit gold mine operating under the following conditions:

- Total tonnage mined: T_1 = 4.5 million t/yr
- Waste mined: T_{w1} = 1.5 million t/yr

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- Ore mined and processed: T_{+c1} = 3.0 million t/yr
- Cut-off grade: x_{c1} = 0.74 g/t
- Average grade above cut-off: x_{+c1} = 1.56 g/t

Costs associated with waste material:

- M_w = \$3.50 per metric ton of waste
- P_w = \$0 per metric ton of waste
- O_w = \$0.88 per metric ton of waste

Costs associated with processed material:

- M_o = \$3.20 per metric ton of ore
- P_o = \$15.00 per metric ton of ore
- O_o = \$4.55 per metric ton of ore
- Plant recovery: r = 87%
- Gold price: V = \$1,400 per ounce of gold (1 troy ounce = 31.1035 g)
- Cost of sales: R = \$50 per ounce of gold

The grade–tonnage curves representing one year of production are shown in Figure 5-2. The marginal cut-off grades are as follows:

Mill marginal cut-off grade:

$$x_c = \left[\left(M_o + P_o + O_o\right) - \left(M_w + P_w + O_w\right)\right] / \left[r(V - R)/31.1035\right] = 0.49 \text{ g/t}$$

Mine marginal cut-off grade:

$$x_c = \left(M_o + P_o + O_o\right) / \left[r(V - R)/31.1035\right] = 0.60 \text{ g/t}$$

These marginal cut-off grades are significantly lower than the 0.74 g/t cut-off grade used in the current operation. The difference reflects constraints imposed by the plant's capacity.

The total tonnage available for mining is 22.5 million metric tons. The deposit will be mined out during the five-year life of the mine. Because of low variability in the deposit geology, the yearly cash flow (YCF) is expected to be constant over a five-year mine life. This cash flow is estimated as follows:

$$YCF_1 = T_{+c1} x_{+c1} r(V - R) - T_{+c1}\left(M_o + P_o + O_o\right) - T_{w1}\left(M_w + P_w + O_w\right) = \$102 \text{ million per year}$$

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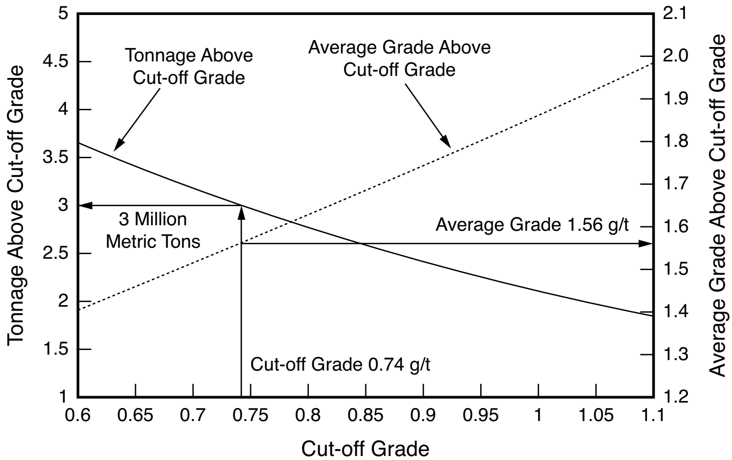

Figure 5-2 Grade–tonnage curves for material mined in one year at 4.5 t/yr

The net present value of this cash flow, calculated at a discount rate i = 10%, and a five-year mine life is NPV_{i1} = \$425 million. The opportunity cost of the mill capacity constraint is $i \cdot t \cdot NPV_{i1} = 0.10 \cdot 425/4.5$ = \$9.44 per metric ton processed, which is equivalent to $9.44 \cdot 31.1035/[0.87 \cdot (1{,}400 - 50)] = 0.25$ g/t. When taking the opportunity cost into consideration, the mill cut-off grade is raised from 0.49 g/t to 0.74 g/t, which happens to be the cut-off grade used in the current mine plan.

Consideration is being given to increasing mine capacity by 25%, from 4.5 to 5.625 million metric tons a year, thus reducing the mine life from five to four years. The mill capacity remains the same, at 3 million metric tons a year. The amount of waste material is increased from 1.5 million metric tons to 2.625 million metric tons, calculated as follows:

- Total tonnage mined: T_2 = 5.625 million t/yr
- Ore mined and processed: T_{+c2} = 3.0 million t/yr
- Waste mined: T_{w2} = 2.625 million t/yr

The tonnage-grade curves that represent current (4.5 million metric tons) and proposed (5.625 million metric tons) yearly production are shown in Figure 5-3. The proposed tonnage curve is obtained by multiplying the current one by 1.25. The proposed grade curve is identical to the current one. From these curves, one gets the cut-off grade to be used to fill the plant, and the average grade above cut-off grade is as follows:

- Cut-off grade: x_{c2} = 0.90 g/t

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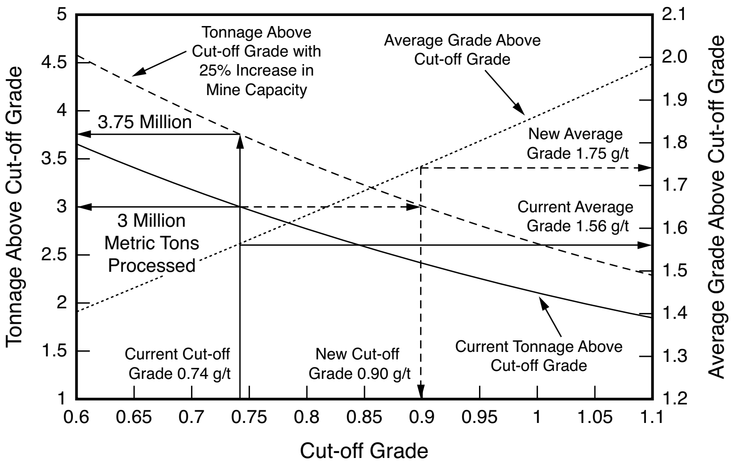

Figure 5-3 Current and proposed grade–tonnage curves for material mined in one year

- Average grade above cut-off: $x_{+c2} = 1.75$ g/t

Assuming that the costs per metric ton remain the same, the proposed yearly cash flow is estimated at

$$YCF_2 = T_{+c2}x_{+c2}r(V-R) - T_{+c2}(M_o + P_o + O_o) - T_{w2}(M_w + P_w + O_w)$$
$$= \$119 \text{ million per year}$$

As a result of a higher head grade, this yearly cash flow is significantly higher than the \$102 million currently expected. The net present value of this cash flow, calculated at a discount rate i =10% and a reduced life of four years is NPV_{i2} = \$415 million. This net present value is less than the \$425 million estimated at the current mining rate. The reason is that the life of the project has been reduced from five years to four. The difference in net present value would be even more significant if, as it should be, the cost of expanding the mine capacity was taken into account. This cost would occur in year 0 and would not be discounted.

The marginal cut-off grades are significantly lower than the cut-off grades used to fill the plant. This is more so when the mine capacity is increased. The net present value of both scenarios would be significantly improved if low-grade material was stockpiled and processed at the end of the mine life, after mining is completed and the higher-grade material has been processed. More low-grade material is available for stockpiling when the mine

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capacity is increased, and stockpiling will have a greater economic impact in this situation.

Alternative mine plans should be considered, all of which should include a stockpiling option. One alternative is to lower the mine capacity and therefore the mill head grade, thus increasing the mine life but decreasing the yearly cash flow. Another option, if feasible, would consist of increasing the mill capacity. The optimal solution is likely to be that where declining cut-off grades are used, starting with 0.74 g/t during the current year and reaching 0.49 g/t at the end of the mine life.

CUT-OFF GRADE WHEN MINING CAPACITY IS FIXED

In the previous analysis, a fixed processing capacity was assumed to be the case. Now consider the situation where the mining capacity is fixed but an increase in plant capacity is being proposed. A lower cut-off grade is needed to balance the mining capacity with the plant capacity. Increasing the mill capacity has the following impacts:

- The tonnage processed per year is increased.
- The tonnage mined is not changed. The cut-off grade must be decreased to keep the processing plant full.
- The average grade of material sent to the mill decreases, but the metal content of this material increases.
- More metal is recovered, resulting in higher revenues from sales.
- The capital cost of plant expansion must be taken into account.
- The plant operating costs are likely to increase per unit of time (cost per year) but should decrease per unit of production (cost per metric ton processed).

The optimal plant capacity is that which maximizes the total utility of the project, taking into account financial impact (increased capital cost, decreased unit operating cost, increased revenue from sales), as well as socioeconomic, environmental, political, and other impacts.

For example, consider a mining operation in which the plant was designed to process an average of 250,000 metric tons per month, or 3 million metric tons per year. The grade–tonnage relationship corresponding to the mineralized material expected to be mined at the current capacity is shown in Figure 5-4. At the current mining capacity, this plant capacity is consistent with a cut-off grade of 0.74 g/t, corresponding to a mill feed average grade of 1.56 g/t.

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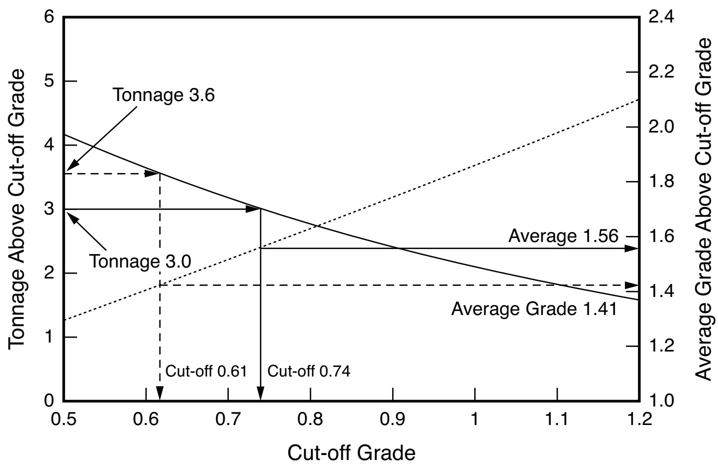

Figure 5-4 Estimation of cut-off grade assuming fixed mining capacity

Management is considering increasing the size of the processing plant by 20% and investigating the impact that such a change would have on the coming year. If the processing capacity is increased by 20% and the mining capacity is kept constant, the cut-off grade must be decreased to 0.61 g/t to supply 3.6 million metric tons to the mill (Figure 5-4), and the average grade of mill feed will decrease to 1.41 g/t. The gold content of the material processed will increase from 2.51 million oz to 2.73 million oz. This cut-off grade calculation only takes into account capacity constraints and is independent of the economics of the project. The increase in plant capacity must be justified not only by the increase in material processed, but also taking into account capital cost requirements, possible changes (increase or decrease) in recovery, a likely decrease in operating costs, and all other direct and indirect costs and benefits.

The proposed 20% increase in processing capacity may or may not be optimal. To be economically justified, an increase in plant capacity must take into account financial, technical, environmental, permitting, and other constraints imposed by the size of the available processing equipment, limitations on tailings dam expansion, maximum permitted dust emission, and other parameters. Depending on the limitations imposed by these constraints, an iterative approach is best suited to plant capacity optimization. This approach can consist of the following steps:

1. Assume a 1-million-metric-ton increase in processing capacity (or some other increase that is technically achievable).

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2. Estimate the decrease in cut-off grade and resulting lower mill head grade that is consistent with the fixed mining capacity and higher processing capacity.
3. Estimate the increase in mill capital and yearly operating costs needed to increase the processing capacity. Calculate the corresponding discounted incremental processing cost (DIPC) for the remaining life of the project.
4. Estimate the increase in mill production per year (units of product sold) and calculate the corresponding discounted incremental revenue (DIR).
5. If DIR exceeds DIPC, this analysis should be repeated, assuming an additional 1-million-metric-ton increase in processing capacity.
6. The optimal processing capacity is that for which DIR equals DIPC.

CUT-OFF GRADE WHEN VOLUME OF SALES IS FIXED

In this analysis, it is assumed that the volume of sales is fixed. This may be because all products are sold under contracts that specify the volume that will be bought on a yearly basis. Perhaps the market is small and the amount of product that can be sold is limited. Or it might be that management specifies the amount to be produced from a given operation for reasons external to the operation under consideration.

Fixed Sales with No Mining or Processing Constraint

In this example it is assumed that the recovery achieved in the processing plant is independent of tonnage processed and plant head grade. In such a situation, requiring a fixed volume of sales is equivalent to requiring a fixed quantity of metal (or other salable product) delivered by the mine to the processing plant. This quantity Q_{+c} is equal to the tonnage delivered T_{+c} multiplied by the average grade of plant feed x_{+c}:

$$Q_{+c} = T_{+c} \cdot x_{+c}$$

Consider a gold mining operation that has been requested to supply 4 t of gold (130,000 oz) to the processing plant over a one-year period (Q_{+c} = 4.0 t of gold). Consider three scenarios:

1. There is no constraint on either mine or plant capacity. This is usually the case only during the feasibility study.

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2. The mine capacity is fixed, but the plant capacity is not.
3. The plant capacity is fixed, but the mine capacity is not.

If neither the mine nor the processing plant is capacity constrained, the number of possible cut-off grades is theoretically infinite. A high cut-off grade will result in a high average grade above cut-off grade x_{+c}. The higher the cut-off grade, the lower the capacity T_{+c} of the processing plant that is needed to keep sales at the required level. But in the case of an open pit mine, a higher cut-off grade will require mining more metric tons per year. In the case of an underground mine, smaller stopes may have to be designed to eliminate peripheral low-grade material, and low-grade stopes may have to be rejected.

When neither mine nor plant capacity is fixed, cut-off grade optimization requires analysis of several feasible solutions: low cut-off grade and large plant size or high cut-off grade and smaller plant size. Technical constraints, including constraints imposed by the geology of the deposit, will reduce the number of feasible options. Higher cut-off grades will result in lower capital costs for the plant and likely higher operating costs, while the impact on mine capital and operating costs will be a function of the geological properties of the deposit and the applicable mining methods. Cut-off grade optimization requires estimation of capital and operating costs and cash flow analysis for each feasible solution.

Fixed Sales and Fixed Processing Rate with No Mining Constraint

Cut-off grade determination becomes easier if, in addition to the constraint on the amount of metal processed, one adds a constraint on either plant or mine capacity. First assume that the plant capacity, defined as tonnage processed per year, is fixed. With both tonnage processed T_{+c} and metal content Q_{+c} being fixed, the plant head grade x_{+c} is calculated as follows:

$$x_{+c} = Q_{+c} / T_{+c}$$

If it is known what material can be mined in the coming months, one can determine the cut-off grade needed to reach the necessary average grade and the mining rate needed to reach the necessary tonnage of mill feed T_{+c}.

Consider the gold mine that was asked to supply 4 t of gold to the processing plant during the coming year. In addition, assume that the capacity of the processing plant is fixed at 2 million t/yr. To satisfy these constraints, the head grade must be

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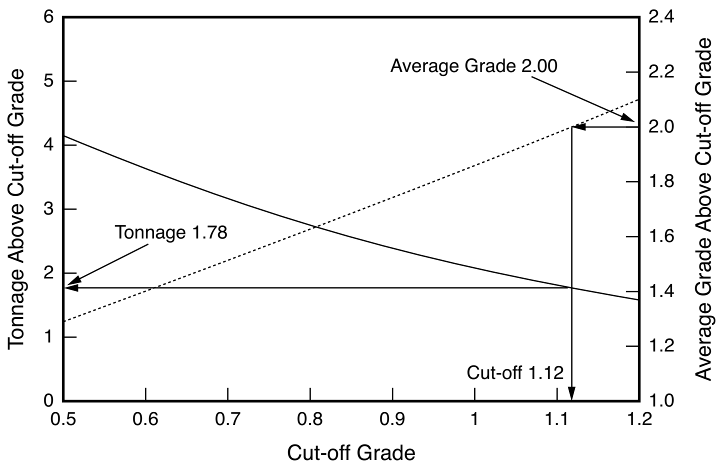

Figure 5-5 Estimation of cut-off grade and tonnage given an average grade

$$
\begin{aligned}
x_{+c} &= Q_{+c}/T_{+c} \\
&= (4{,}000{,}000 \text{ g/yr})/(2{,}000{,}000 \text{ t/yr}) \\
&= 2.00 \text{ g/t}
\end{aligned}
$$

A preliminary mine plan was developed during which 6 million metric tons of material, both ore and waste, would be mined. The corresponding grade–tonnage relationship is shown in Figure 5-5. From this relationship, one determines that the cut-off grade needed to get an average grade of 2.0 g/t is 1.12 g/t. There are only 1.78 million metric tons of mill feed above cut-off grade in this preliminary mine plan. Since the mill capacity is 2 million t/yr, this material will be processed in 10.7 months, calculated as follows:

$$(12 \text{ months/yr})\cdot(1.78 \text{ million metric tons})/(2.0 \text{ million t/yr}) = 10.7 \text{ months}$$

Six million metric tons are scheduled to be mined in this preliminary mine plan. To mine this tonnage in 10.7 months, the mining rate must be 6.0/10.7 = 560,000 t/month or 6.7 million t/yr.

In conclusion, for the mine to send 4 t of gold per year to a plant that has a capacity of 2 million t/yr, a total of 6.7 million metric tons must be mined every year and a cut-off grade of 1.12 g/t must be used. The plant head grade will be 2.0 g/t.

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Fixed Sales and Fixed Mining Rate with No Processing Constraint

Now consider the case where the mine capacity is constrained at 6 million t/yr and the metal content of the material to be sent to the mill is set at 4 t of gold per year. A yearly mine plan was developed in which 6 million metric tons are to be mined. The corresponding grade–tonnage relationship is shown in Figure 5-5. From the values of T_{+c} and x_{+c} shown in this figure, one can calculate the metal content of material above cut-off grade $Q_{+c} = T_{+c} \cdot x_{+c}$ and plot this metal content as a function of cut-off grade x (Figure 5-6).

Figure 5-6 shows the relationship between cut-off grade and quantity of metal above cut-off grade, as scheduled to be mined in the current mine plan. Because the quantity of metal to be processed is Q_{+c} = 4.0 t of gold, the cut-off grade must be 0.97 g/t. The tonnage and average grade of material above this cut-off grade can be determined using the grade–tonnage relationship (Figure 5-7):

$$T_{+c} = 2.20 \text{ million metric tons}$$

$$x_{+c} = 1.82 \text{ g/t}$$

Given that 6 million metric tons of material are scheduled to be mined in the coming year and that the mine must send 4 t of gold to the processing plant, a cut-off grade of 0.97 g/t must be used, resulting in 2.20 million metric tons of material being sent to the processing plant, averaging 1.82 g/t. This can only be achieved if the plant capacity is at least 2.20 million t/yr.

RELEASING CAPACITY CONSTRAINTS: A BASE METAL EXAMPLE

The previous analyses have shown the sometimes deterministic effect that geologic constraints imposed by nature, combined with capacity constraints, have on cut-off grades. In this section, an example is given for how to determine the economic feasibility of releasing capacity constraints under defined geologic conditions.

Consider a copper mining and processing operation. Mine and mill capacities are 79 million metric tons and 39.5 million t/yr, respectively. The copper resources included in that part of the deposit scheduled to be mined in the coming year are listed in Table 5-1. The cut-off grade for mill feed is 0.25% Cu. The reserves to be mined and processed in the coming year are 39.5 million metric tons averaging 0.381% Cu and containing 150,000 metric tons of copper (332 million lb of copper).

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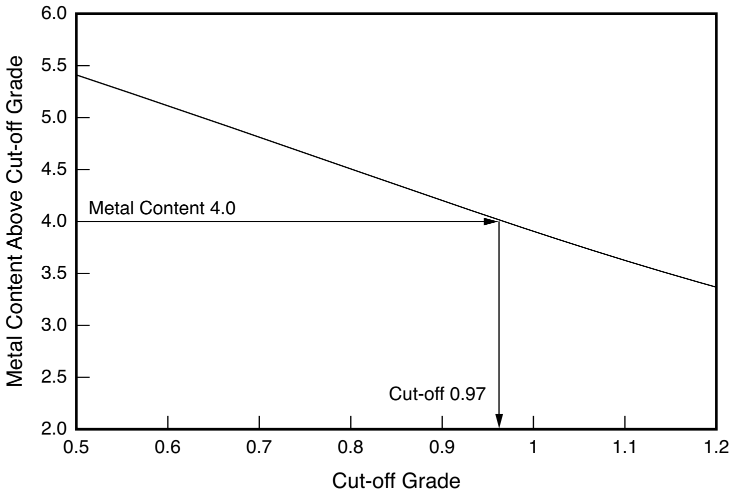

Figure 5-6 Estimation of cut-off grade given the required metal content of mine feed

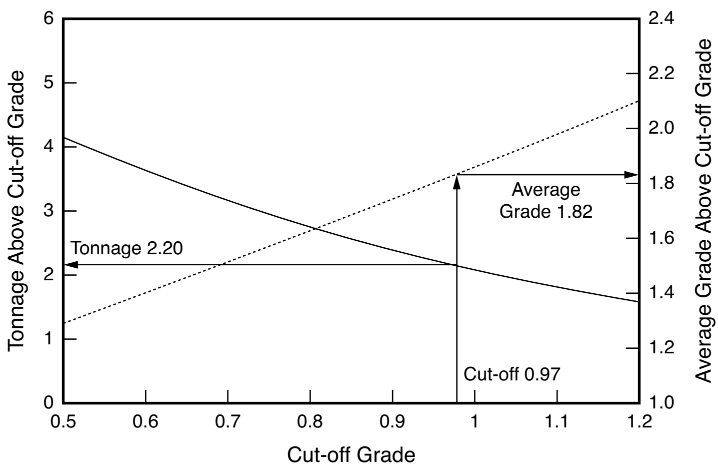

Figure 5-7 Estimation of tonnage and average grade above cut-off grade

Management wishes to assess the sensitivity of the project to changes in mine, mill, or smelter capacity. Four cases are to be considered:

- **Case 1:** Assume that the mine capacity is increased by 10%, from 79 to 86.9 million metric tons, but the mill capacity remains fixed at 39.5 million t/yr. The 79 million metric tons that were scheduled to be mined in one year, including the resources shown in Table 5-1,

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Table 5-1 Copper resources contained in material scheduled to be mined

Cut-off Grade, % Cu	Minable Tonnage, million metric tons	Minable Grade, % Cu	Minable Copper Content: thousand metric tons Cu	Minable Copper Content: million lb Cu
0.15%	53.7	0.335%	180	397
0.16%	52.6	0.340%	179	395
0.17%	51.4	0.344%	177	390
0.18%	50.1	0.348%	174	384
0.19%	48.8	0.352%	172	378
0.20%	47.5	0.355%	168	372
0.21%	46.0	0.360%	165	365
0.22%	44.4	0.365%	162	357
0.23%	42.8	0.370%	159	349
0.24%	41.2	0.375%	155	341
0.25%	**39.5**	**0.381%**	**150**	**332**
0.26%	37.7	0.387%	146	322
0.27%	35.9	0.393%	141	311
0.28%	34.1	0.399%	136	300
0.29%	32.1	0.406%	131	288
0.30%	30.2	0.413%	125	275
0.31%	28.2	0.421%	119	262

will be mined in 1/1.1 = 0.91 years (10.9 months). During this period, the mill can only process 35.9 million metric tons. From Table 5-1, one sees that to send only 35.9 million metric tons to the processing plant, the cut-off grade must be increased to 0.27% Cu. The mill head grade will be 0.393% Cu. Assuming that the same average grade can be maintained over one year, 39.5 million metric tons of ore will be processed at an average grade 0.393% Cu, containing 155,000 t of copper.

- **Case 2:** Assume that the capacity of the flotation plant is increased by 10%, from 39.5 to 43.5 million metric tons, but the mine capacity is unchanged at 79 million t/yr. The resources available to feed the mill in one year remain as shown on Table 5-1. To supply 43.5 million metric tons to the mill, the cut-off grade must be lowered to 0.225% Cu. The mill head grade will average 0.367% Cu, resulting in 160,000 t of copper being processed.

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Table 5-2 Cut-off grades, and mine and mill capacities required to satisfy specific capacity requirements

	Cut-off Grade, % Cu	Tonnage Milled, million metric tons	Average Grade, % Cu	Copper Content		Tonnage Mined, million metric tons
				thousand metric tons Cu	million lb Cu	
Base Case						
Values	0.250%	39.5	0.381%	150	332	79.0
Case 1: Increase mining rate by 10%. Keep processing rate at same level.						
Values	0.270%	39.5	0.393%	155	342	86.9
Difference from base case	8%	0%	3%	3%	3%	10%
Case 2: Increase processing rate by 10%. Keep mining rate at same level.						
Values	0.225%	43.5	0.367%	160	352	79.0
Difference from base case	–10%	10%	–4%	6%	6%	0%
Case 3: Increase copper produced by 10%. Keep mining rate at same level.						
Values	0.210%	46.0	0.360%	165	365	79.0
Difference from base case	–16%	16%	–6%	10%	10%	0%
Case 4: Increase copper produced by 10%. Keep milling rate at same level.						
Values	0.305%	39.5	0.418%	165	364	107
Difference from base case	22%	0%	10%	10%	10%	36%

- **Case 3:** Management wishes to determine under which conditions 10% more copper could be sent to the processing plant if mine capacity remains fixed at 79 million metric tons. The copper content of processed material must increase from 150,000 to 165,000 t. From Table 5-1 it can be seen that the cut-off grade must be decreased to 0.21% Cu, resulting in 46.0 million metric tons of ore being sent to the mill, averaging 0.360% Cu. If the mining rate is not changed, a 10% increase in copper processed can only be achieved by decreasing the average grade by 6% and increasing the tonnage milled by 16%.
- **Case 4:** Management wishes to determine under which conditions 10% more copper could be sent to the processing plant if mill capacity remains fixed at 39.5 million metric tons. To increase the copper content of mill feed from 150,000 to 165,000 t, the mill head

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grade must be increased from 0.381% Cu to 165,000/39,500,000 = 0.418% Cu. Table 5-1 shows that, to reach this average grade, it is necessary to use a cut-off grade of 0.305% Cu. There are only 29.2 million metric tons above this cut-off grade. Given the mill capacity of 39.5 million metric tons, this ore will be consumed in 29.2/39.5 = 0.74 year (8.88 months). The mining rate must therefore be increased from 79 million t/yr to 79/0.74 = 107 million t/yr. If the processing rate is not changed, a 10% increase in copper processed can only be reached by increasing the average grade by 10% and increasing the tonnage mined by 36%.

The results are summarized in Table 5-2. Each case is compared with the base case, in which 79 million metric tons are mined and 39.5 million metric tons are processed.

This simple example shows procedures that can be used to calculate cut-off grades taking into account geologic constraints (as summarized in Table 5-1) and technical constraints, including mine, mill, or production capacities. No attempt was made to assess whether the proposed solutions were economically feasible or justified. To do so would require completing this analysis not only over one year (as done in this example) but for the life of the mine. Implementing any of the mining and processing plans summarized in Table 5-2 would require additional capital expenditures; would change operating costs; might shorten the mine life (cases 1 and 4); could justify stockpiling of low-grade material (case 4); and might require other operational changes. All these changes would result in changes in cash flow, which would have to be quantified to decide on the best option.

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Cut-off Grade and Mine Planning

6

While there are obvious differences, there are also many similarities between questions that must be answered when designing open pit and underground mines. These similarities are illustrated in the examples that follow. Different mining methods are considered, including open pit, selective underground, and block and panel caving.

OPEN PIT MINE: ECONOMIC VALUATION OF A PUSHBACK

In this section, a situation is analyzed where a pushback is considered to be added to an open pit mine. To make this decision and optimize the cut-off grade, two mine plans should be considered:

1. The first plan assumes that the pushback is not mined. The net present value of future cash flows is NPV_{i1}.
2. The second plan assumes that the pushback is mined. The cash flow generated by this plan can be divided in two parts: the cash flow corresponding to the material originally scheduled to be mined and the cash flow generated by the pushback.

Under the second mine plan, the net present value of the material that was initially scheduled to be mined is likely to decrease from NPV_{i1} to NPV_{i2}. The pushback net present value is NPV'_i. The net present value of the second mine plan is $NPV_{i2} + NPV'_i$. The pushback should be mined only if NPV'_i exceeds the difference $NPV_{i1} - NPV_{i2}$:

$$NPV'_i > NPV_{i1} - NPV_{i2}$$

The difference $NPV_{i1} - NPV_{i2}$ represents the opportunity cost of mining the pushback. The pushback should only be mined if its net present value NPV'i is not only positive but also exceeds the opportunity cost. The reason why there is an opportunity cost to adding a pushback to an existing mine

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plan is because there are capacity constraints. Mine capacity, mill capacity, or volume of sales is constrained. These capacities must be shared between the material originally scheduled to be mined and the new material coming from the pushback. To release the capacities that are required to mine the pushback, the following changes might be considered:

- Reduce the rate at which the original material is being mined, and allocate the spare mining capacity to the pushback.
- Increase the cut-off grade of direct mill feed to release mill capacity for material coming from the pushback.
- Delay processing of lower-grade material by stockpiling it.

These actions will decrease the net present value from NPV_{i1} to NPV_{i2}. Alternatively, additional capital could be invested, including that needed to increase the mine or mill capacity. This capital cost reduces the net present value NPV'_i of the pushback.

The net present value NPV'_i of the pushback is calculated from the value of each block included in the pushback and can be expressed as follows:

$$NPV'_i = -I + \Sigma\, U_{jk} / (1 + i)^k$$

I = capital cost needed to mine and process the pushback

U_{jk} = utility of mining block j in year k

i = discount rate

$U_{jk} = U_{jk,dir} + U_{jk,opp} + U_{jk,oth}$ where

$U_{jk,dir}$ = direct utility of mining block j in year k

$U_{jk,opp}$ = opportunity cost of mining block j in year k

$U_{jk,oth}$ = other utility of mining block j in year k

If there is no capacity constraint, all blocks that will generate a positive cash flow when processed ($U_{jk,dir} > 0$) should be processed when mined. The cut-off grade is independent of when the block is mined. The pushback can be mined without changing when, or whether, other material is being mined outside the pushback. The net present value of the material that was originally scheduled to be mined remains unchanged, $NPV_{i2} = NPV_{i1}$. The pushback's mining schedule is only a function of the sequence to be followed to mine the pushback and is independent of any other mining activity that may be taking place. The decision to mine the pushback is only a function of whether or not its net present value NPV'_i is positive.

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However, optimization of mine and mill operations implies balancing capital and operating costs, which invariably results in capacity constraints and nonzero opportunity costs: $U_{jk,opp} > 0$. Nonzero opportunity costs result in time delays, higher cut-off grades, fewer blocks being processed, and, therefore, lower pushback net present value. A pushback for which NPV'_i is positive if capacity constraints are ignored may have a negative NPV'_i if these constraints are taken into account. Ignoring capacity constraints may result in mining pushbacks that should not be mined and designing a pit that is larger than it should be.

There are two dependent sets of constraints to be taken into account when evaluating a pushback: constraints that apply if the pushback is mined as a stand-alone operation and constraints that result from having to share capacity with the material originally scheduled to be mined. Considering a pushback as a stand-alone operation is rarely, if ever, a reasonable assumption. It may be possible to dedicate mining equipment entirely to the pushback. This capital cost must be included in NPV'_i and in cut-off grade optimization. Even under these circumstances, mine schedules and cut-off grades must take into account process capacity constraints. The plant is at least in part utilized by material previously scheduled to be mined, and taking this capacity constraint into account will reduce the value of the pushback.

When more constraints are added to the pushback, its value NPV'_i decreases. Releasing these constraints can be achieved by assigning capacity to the pushback that was previously utilized elsewhere. However, this will increase the constraints on the material originally scheduled to be mined and increase the opportunity cost $NPV_{i1} - NPV_{i2}$. The optimal capacity assignment is that where further change will result in a decrease in NPV_{i2} that exceeds the increase in NPV'_i. Finding this optimum can be time-consuming and may require use of specialized open pit mine-design software.

UNDERGROUND MINE: ECONOMIC VALUATION OF A STOPE

Situations similar to those previously described in open pit mines are also encountered in underground mines. A stope should be mined if the net present value of generated cash flow is positive. All costs and benefits must be taken into account, as well as when these costs and benefits are realized. This includes the cost of stope development (such as access drifts and crosscuts); the cost of waste mining, stockpiling, and re-handling; the cost of ore mining, stockpiling, re-handling, and processing; and all costs allocated to

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low-grade stockpiles, if any. Revenues include those incurred from processing ore directly sent to the mill, as well as those realized at a later date from low-grade stockpiles.

If there is no capacity constraint, all material that can generate a positive cash flow, if processed when mined, will be processed. But project optimization invariably results in capacity constraints, such as those imposed by shaft and drift haulage capacity, ventilation, maximum speed of development, or mining method. These constraints result in nonzero opportunity costs and higher cut-off grades. When capacity constraints are taken into account, the size of some stopes is likely to be reduced, and some stopes will no longer be considered economically minable.

Shifting capacity from other parts of the mine can be used to release constraints on the stope and increase the stope net present value (NPV'i). For example, one may assign shaft capacity to material produced by the new stope by slowing down mining from other stopes. However, more constraints will result, which apply to material previously scheduled to be mined. If NPV_{i1} was the net present value of this material before the new stope was taken into account, and NPV_{i2} its net present value when the new constraints are imposed, NPV_{i2} is less than NPV_{i1}. The difference $NPV_{i1} - NPV_{i2}$ is the opportunity cost of mining the new stope. Adding the new stope to the production schedule is justified only if

$$NPV'_i > NPV_{i1} - NPV_{i2}$$

When more constraints are added to a stope, its value NPV'_i decreases. Releasing these constraints can be achieved by assigning capacity to the stope that was previously utilized elsewhere. However, this will increase the constraints on the material originally scheduled to be mined and increase the opportunity cost $NPV_{i1} - NPV_{i2}$. The optimal capacity assignment is that where further change will result in a decrease in NPV_{i2} that exceeds the increase in NPV'_i. Finding this optimum can be time-consuming and may require use of specialized underground mining software.

SIMILARITIES BETWEEN OPEN PIT AND UNDERGROUND MINE PLANNING

As shown in the previous discussions, there are many similarities between questions concerning open pit and underground mines, and the approach that must be followed to answer these questions. Here are some of these questions:

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- How do capacity constraints influence cut-off grade and cash flow?
- Which cut-off grade should be used to separate waste material, stockpiled material, and material sent to the processing plant?
- Should a pushback be mined in an open pit mine or a stope be mined in an underground mine?
- Should low-grade material at the bottom of a pushback or surrounding a stope be mined or left in the ground?
- If low-grade material must be mined, should it be wasted, stockpiled, or processed?
- How should the time difference between mining, stockpiling, processing, and selling material be taken into account in designing open pit and underground mines?

BLOCK AND PANEL CAVING

When a block or panel caving mining method is used, estimation of cut-off grades must take into account the limited flexibility that operators have in controlling the grade of material pulled. Cut-off grades are used to determine the location and size of a block or panel, and to decide when pulling material from a drawpoint should be stopped. Cut-off grades are not likely to play a significant role, if any, when waste or low-grade material is encountered in the block.

Constraints Imposed by Block and Panel Caving

Many factors must be taken into account when designing a block in addition to the geotechnical properties of the deposit and the continuity of mineralization. Ideally, blocks are located in relatively high-grade areas that can be mined without significant internal or external waste dilution, the drawpoints and production levels are located in lower-grade or waste areas, and the block boundaries are located near lower-grade or waste zones. Internal and external waste or low-grade dilution will occur, which must be taken into account when locating blocks and drawpoints. When ore is drawn, waste is mixed with higher-grade material, thus eliminating the opportunity to mine waste selectively.

The rate at which material is pulled from drawpoints should match the natural rate of caving. The material should be drawn in a uniform fashion across drawpoints. Production cannot be stopped in one drawpoint without affecting surrounding drawpoints. If a drawpoint containing waste is

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surrounded by other high-grade drawpoints, mining waste cannot be stopped. However, if the waste drawpoint is located on the periphery of the block being mined, this drawpoint can be stopped. Production is stopped when waste indicates that the entire ore column has been pulled.

Productivity is dependent on a high rate of production, which is not conducive to selective mining of ore and waste material. The capital cost of underground and surface infrastructure needed to handle waste separately from ore-grade material is likely to be high. Attempting selective mining is likely to increase the mine operating costs. For these reasons, some block caving operations have chosen to send all material mined to the processing plant, whatever the grade.

Marginal Cut-off Grade and Drawpoint Management

Once a block has been developed and the infrastructure is in place (including drifts, haulage facilities, drawpoints, ventilation, etc.), the utility of mining and processing one metric ton of material is

$$U_{dir}(x) = x \cdot r \cdot (V - R) - (M_o + P_o + O_o)$$

where

- x = average grade
- r = recovery, or proportion of salable product recovered from the mined material
- V = value of one unit of salable product
- R = refining, transportation, and other costs of sales that are expressed in dollars per unit of salable product
- M_o = mining cost per metric ton processed
- P_o = processing cost per metric ton processed
- O_o = overhead cost per metric ton processed

The minimum grade that can be mined and processed at a profit is x_{c1} such that $U_{dir}(x_{c1}) = 0$:

$$x_{c1} = [M_o + P_o + O_o] / [r \cdot (V - R)]$$

This cut-off grade should be used to decide whether production from a drawpoint should be stopped because of excessive lateral dilution or because the entire ore column has been mined.

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Marginal Cut-off Grade and Block Design

Incremental analysis must be used to determine the optimal size and location of a block. To decide whether a new row of drawpoints should be added along the periphery of a block, one must first estimate the tonnage T and average grade x of the material that will be pulled from these drawpoints, taking dilution into account. If one considers only operating costs and ignores the capital and opportunity cost of adding one row of drawpoints, this row should be added if the average grade x exceeds the cut-off grade x_{c1} calculated previously.

If the average grade of the last row of drawpoints is equal to x_{c1}, the cash flow generated from these drawpoints will not justify the capital cost of developing them. In addition, development of a larger block by addition of peripheral drawpoints will delay production from what could have been a smaller block. The cut-off grade applicable to the last row of drawpoints must take into account capital and opportunity costs.

Influence of Capital Cost and Discount Rate

Additional capital expenditures are needed to develop one more row of drawpoints. This capital cost I must be recovered from profits generated by the drawpoints. On an undiscounted basis, the profit made from mining and processing T metric tons of material with average grade x is $T \cdot [x \cdot r \cdot (V - R) - (M_o + P_o + O_o)]$. This profit must be greater than or equal to the capital cost I. The cut-off grade applicable to this last row of drawpoints is determined by adding the capital cost per metric ton I/T to the operating costs M, P, and O:

$$x_{c2} = \left[M_o + P_o + O_o + I/T\right] / \left[r \cdot (V - R)\right]$$

where

I = capital cost incurred to develop a new row of drawpoints

T = tonnage to be mined from the new row of drawpoints

The requirement of a minimum rate of return should be taken into account in calculating the cut-off grade. The following additional notations are used:

i = minimum rate of return (discount rate)

n = number of years during which material will be pulled from the new drawpoints

Then make the simplifying assumption that the tonnage mined and corresponding average grade will be the same every year, T/n and x, respectively.

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The yearly cash flow YCF expected to be generated from the new drawpoints is

$$YCF = (T/n) \cdot \left[x \cdot r \cdot (V - R) - (M_o + P_o + O_o) \right]$$

The net present value of this cash flow (NPV'_i) is calculated as follows*:

$$NPV'_i = YCF \cdot \left[1 + 1/(1+i) + 1/(1+i)^2 + \ldots + 1/(1+i)^{n-1} \right] = YCF\, g(i,n)$$

where $g(i,n) = [1 - (1+i)^{-n}]\,(1+i)/i$.

The minimum cut-off grade applicable to the new row of drawpoints is x_{c3} such that the net present value of generated cash flows NPV'_i is equal to the capital investment I:

$$NPV'_i = I$$

$$(T/n) \cdot \left[x_{c3} \cdot r \cdot (V - R) - (M_o + P_o + O_o) \right] \cdot g(i,n) = I$$

$$x_{c3} = \left[(M_o + P_o + O_o) + n / g(i,n) \cdot (I/T) \right] / \left[r \cdot (V - R) \right]$$

Opportunity Cost

In addition to increasing capital costs, increasing the size of a block can delay production that could be pulled from a smaller block. Assume that a small, presumably high-grade block has been designed and that a production schedule has been developed accordingly. The net present value NPV_i of future cash flows expected to be generated from mining this block was calculated using the discount rate i. If t is the time by which production from the smaller block will be delayed to allow development of one more row of drawpoints, the corresponding opportunity cost is

$$U_{opp}(x) = -t \cdot i \cdot NPV_i$$

This opportunity cost represents a decrease in net present value, which must be added to the capital cost of adding the new drawpoints. Taking this cost into account, the cut-off grade is as follows:

* The notation NPV'_i is used to make the distinction between the net present value of previously scheduled operations, NPV_i, and the net present value of the proposed additional row of drawpoints, NPV'_i. The first year of production is not discounted.

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$$x_{c4} = \left[M_o + P_o + O_o + n/g(i,n)\cdot\left(I + t\cdot i\cdot NPV_i\right)/T\right]/\left[r\cdot(V - R)\right]$$

where

M_o = mining cost per metric ton processed

P_o = processing cost per metric ton processed

O_o = overhead cost per metric ton processed

$g(i,n)$ = $[1 - (1 + i)^{-n}]\,(1 + i)/i$

n = number of years during which material will be pulled from the new row of drawpoints

i = minimum rate of return (discount rate)

I = capital cost incurred to develop the new row of drawpoints

t = time by which previously scheduled production will be delayed

NPV_i = net present value of previously scheduled production

T = tonnage to be mined from the new row of drawpoints

r = recovery, or proportion of salable product recovered from the mined material

V = value of one unit of salable product

R = refining, transportation, and other costs of sales that are expressed in dollar per unit of salable product

Example: Block Caving Copper Mine Operation

Consider a copper mine that is exploited by the block caving method. The deposit is scheduled to be mined using macroblocks 150 m high. The drawpoints are located on a 20-m grid. The current operating conditions are as follows:

- Direct mining cost: $9.00 per metric ton of ore
- Processing cost: $7.00 per metric ton of ore
- General and administrative cost: $1.60 per metric ton of ore
- Freight, smelting, and refining: $0.50 per pound of payable copper

The mill recovery is 87% and the smelter recovery is 96%, including transportation losses. The long-term copper price is assumed to be $2.30 per pound of payable copper (1 t = 2,205 lb).

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Using the above notations, this can be written as follows:

M_o = \$9.00
P_o = \$7.00
O_o = \$1.60
V = \$2.30
R = \$0.50
r = 0.87 · 0.96 = 83.52%

The marginal cut-off grade for an existing drawpoint is

$$x_{c1} = [9.00 + 7.00 + 1.60] / [83.52 \cdot (2.30 - 0.50) \cdot 2{,}205] = 0.53\%\ \text{Cu}$$

The cost of developing a new drawpoint, including access and other development costs, is I = \$240,000. The specific gravity of the ore is 2.5 t/m^3, and the average tonnage contained in one column is T = 20 · 20 · 150 · 2.5 = 150,000 t. On an undiscounted per-metric-ton basis, the cost of drawpoint development is

$$I/T = \$240{,}000/150{,}000 = \$1.60\ \text{per metric ton}$$

The amount of copper needed to recover this cost is

$$x_{c2} - x_{c1} = \$1.60 / [83.52 \cdot (2.30 - 0.50) \cdot 2{,}205] = 0.05\%\ \text{Cu}$$

The cut-off grade to be used to pay for operating costs and undiscounted capital cost per metric ton is

$$x_{c2} = 0.53 + 0.05 = 0.58\%\ \text{Cu}$$

The life of a macroblock is estimated at n = 10 years. An i = 15% rate of return is expected for all investments. The capital recovery needed per metric ton to obtain this rate of return is n/g(i,n) · (I/T):

$$g(i,n) = \left[1 - (1 + 0.15)^{-10}\right] \cdot (1 + 0.15)/0.15 = 5.772$$
$$n/g(i,n) \cdot (I/T) = (10/5.77) \cdot 1.60 = \$2.77\ \text{per metric ton}$$

The amount of copper needed to recover this cost is

$$x_{c3} - x_{c1} = \$2.77 / [83.52 \cdot (2.30 - 0.50) \cdot 2{,}205] = 0.08\%\ \text{Cu}$$

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The cut-off grade to be used to pay for operating costs and discounted capital cost per metric ton is

$$x_{c3} = 0.53 + 0.08 = 0.61\%\ \text{Cu}$$

The macroblock under consideration is scheduled to go into production immediately. As designed, the macroblock is composed of 15 columns for a total tonnage of 15 · 150,000 = 2,250,000 t. The average grade of this material is 2.00% Cu, and the total value of the ore contained is

$$2{,}250{,}000 \cdot \left[0.02 \cdot 0.8352 \cdot (2.30 - 0.5) \cdot 2{,}205 - (9.00 + 7.00 + 1.60)\right] = \$109{,}570{,}896$$

Assuming a uniform cash flow per year over ten years, the net present value of this cash flow is

$$NPV_i = (109{,}570{,}896/10) \cdot g(i,n) = 10{,}957{,}090 \cdot 5{,}772 = \$63{,}269{,}762$$

Consideration is being given to adding one more drawpoint to the macroblock. The time needed to develop this drawpoint is estimated at 12 days. This will delay copper production from the macroblock by 12 days at an opportunity cost:

$$t \cdot i \cdot NPV_i = (12/365) \cdot 0.015 \cdot 63{,}269{,}762 = \$311{,}867$$

This cost is incurred today but must be recovered over the life of the project. It must be added to the capital cost I. To ensure a 15% rate of return, the amount needed per year is

$$n/g(i,n) \cdot (311{,}867/T) = (10/5.77) \cdot 2.08 = \$3.60 \text{ per metric ton}$$

The amount of copper needed to recover this cost is

$$x_{c4} - x_{c3} = \$3.60/\left[83.52 \cdot (2.30 - 0.50) \cdot 2{,}205\right] = 0.11\%\ \text{Cu}$$

The cut-off grade to be used to pay for operating costs, discounted capital cost per metric ton, and discounted opportunity cost is

$$x_{c4} = 0.61 + 0.11 = 0.72\%\ \text{Cu}$$

This cut-off grade is significantly higher than the marginal cut-off grade x_{c1} = 0.53% Cu. If a lower discount rate was acceptable, such as i = 6% instead of 15%, the cut-off grade would be reduced to x_{c4} = 0.64% Cu. To fully take

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into account all costs and benefits associated with adding one more drawpoint to an existing macroblock, one should also look at consequences other than those considered here. These may include the opportunity cost of delaying production from other parts of the operation, lowering the head grade, and increasing the mine life.

RELATIONSHIP BETWEEN MINE SELECTIVITY, DEPOSIT MODELING, ORE CONTROL, AND CUT-OFF GRADE

In the previous examples, it was assumed that the grade–tonnage relationship which characterizes the deposit is independent of the mining capacity. However, in many instances, changes in mining capacity are accompanied by changes in mining method, size of mining equipment, bench height, stope dimensions, drill hole spacing, ore control method, and other parameters that determine mine selectivity and the shape of the grade–tonnage curve. These changes must be taken into account in establishing the likely effect that changes in mining capacity and cut-off grade will have on mill feed and reserves.

As an example, consider a gold deposit for which the total resources above a zero cut-off grade are calculated at 20 million metric tons averaging 10 g/t. The geology of the deposit is such that either open pit or underground mining methods can be used. Figures 6-1 and 6-2 both show the grade–tonnage relationships corresponding to the open pit and underground mining methods.

The open pit cut-off grade was estimated at 3.0 g/t. The amount of material that could be mined above this cut-off grade was 15.2 million metric tons, averaging 12.6 g/t and containing 6.1 million oz (solid lines in Figure 6-1). If the high-selectivity model had been used to evaluate the open pit option, the reserves would have been erroneously estimated at 11.1 million metric tons, averaging 16.9 g/t and containing 6.0 million oz (dotted lines in Figure 6-1).

The underground cut-off grade was estimated at 7.0 g/t. The amount of material that could be mined above this cut-off grade was 6.5 million metric tons, averaging 25.6 g/t and containing 5.3 million oz (solid lines in Figure 6-2). If the low-selectivity model had been used to determine the feasibility of the underground mining method, the reserves would have been erroneously estimated at 8.9 million metric tons, averaging 18.2 g/t and containing 5.2 million oz (dotted lines in Figure 6-2).

The errors made when using the open pit model to evaluate the underground resources or the underground model to evaluate the open pit resources are summarized in Table 6-1. Although this table represents an extreme case, it clearly shows that changes in both mining method and cut-off grade must be

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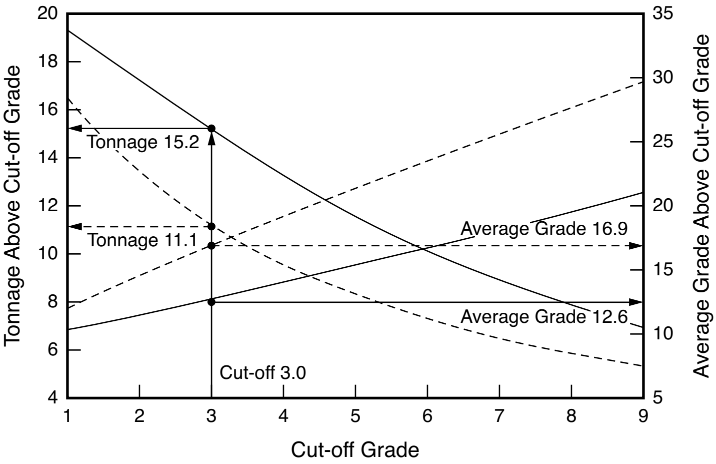

Figure 6-1 Application of open pit low-selectivity cut-off grade to low- and high-selectivity models (solid lines show the low-selectivity model; dotted lines show the high-selectivity model)

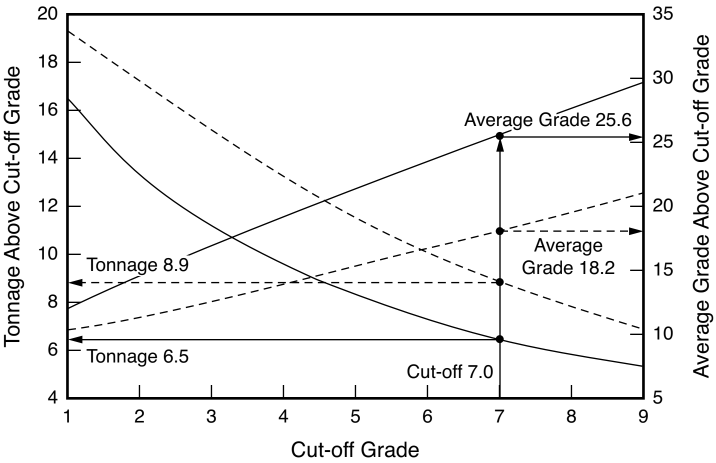

Figure 6-2 Application of the underground high-selectivity cut-off grade to high- and low-selectivity models (solid lines show the high-selectivity model; dotted lines show the low-selectivity model)

evaluated jointly. Appropriate deposit models must be used, which reflect the conditions that are expected to prevail when these changes are made. When assessing the impact that changes in mining capacity may have on mill head grades, one must take into account not only changes in cut-off grades but also changes in the grade–tonnage curve. The grade–tonnage curve will remain

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Table 6-1 Influence of deposit model and cut-off grade on estimated mineral resources

	Open Pit Mine Cut-off, 3.0 g/t			Underground Mine Cut-off, 7.0 g/t		
Deposit Model	Metric Tons, millions	Grade, g/t	Ounces, millions	Metric Tons, millions	Grade, g/t	Ounces, millions
High selectivity	11.1	16.9	6.0	6.5	25.6	5.3
Low selectivity	15.2	12.6	6.1	8.9	18.2	5.2
Correct model	15.2	12.6	6.1	6.5	25.6	5.3
Error if correct model not used	–27%	34%	–2%	36%	–29%	–3%

the same only if no change is made to mining method, ore control practices, and size of mining equipment.

A computer-generated deposit model is the foundation on which mine plans are developed, cut-off grades are optimized, and the tonnage and average grade of material processed are determined. For the results of a feasibility study to be meaningful, the deposit model must reflect the geological properties of the deposit. In addition, the relationship between cut-off grade, tonnage, and average grade above cut-off grade, which is implied by the deposit model, must be the same as that which will be realized when the deposit is mined.

The deposit model must be developed taking into account the mining method that will be used and how selective this method will be. Different models are usually needed for open pit and underground mines, for bulk mining and selective mining, for block caving and cut-and-fill. Selectivity is a function not only of the geology of the deposit and the mining method but also of bench height and blast hole spacing, stope design, type and size of mining equipment, and ore control method. The significance of these factors must be assessed when developing the deposit model.

It is not sufficient to make realistic selectivity assumptions when developing the deposit model and optimizing the cut-off grade. These assumptions must be respected when the deposit is being mined. Otherwise, the tonnage and average grade of material mined and processed will differ from that estimated when the project feasibility study was completed. In practice, changes will occur during the life of the mine, which will change selectivity. Such changes may include changing mining method, using smaller or higher bench heights, designing larger or smaller stopes, changing the equipment size, and modifying ore control practices. Whenever such changes are made, one must question whether they will change the grade–tonnage curve sufficiently to require development of a new deposit model.

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7 Which Costs Should Be Included in Cut-off Grade Calculations?

Mining engineers face a significant challenge when determining which costs should be included in a cut-off grade calculation. Collaboration between engineers and accountants is necessary to ensure that meaningful numbers are used and that all applicable costs are included. In this chapter, some general principles concerning costs and how they should be treated in the estimation of cut-off grades are discussed, which are illustrated by specific examples.

GENERAL CONSIDERATIONS

Costs can be divided between fixed and variable. Fixed costs are expenses for which the total does not change in proportion to the level of activity within the relevant time period or scale of production. By contrast, variable costs change in relation to the level of activity. In cut-off grade calculations, costs incurred when drilling, sampling, blasting, loading, crushing, and grinding the ore as well as during flotation, concentrate drying, filtering and shipping, smelting and refining, and so forth, are usually considered variable costs. These costs are directly related to the production capacity. Initial capital expenditures, equipment depreciation, general administration, property taxes, marketing, public relations, government relations, and so on, are considered fixed costs. To the extent that fixed and variable costs are properly defined, cut-off grade optimization requirements only take variable costs into consideration.

It is important to realize that fixed costs are fixed only within a certain range of activity or over a certain period of time. If significant changes are made to the cut-off grade that require expansion of a leach pad or tailings dam, costs related to such expansions can no longer be considered fixed. If the life of mine is extended or shortened beyond the current expected life, general and administrative (G&A) costs will change. These changes should be taken into account in the cut-off grade calculation by allocating their cost to that part of the operation (mine, mill, leach plant, concentrator, smelter, refinery, etc.) that drives the change.

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Sunk costs are expenses that were incurred in the past and do not change with the level of activity. Once a mine is in full production, the costs incurred for pre-stripping, shaft sinking, plant construction, and original infrastructure are sunk. Such costs are not taken into account when deciding whether the cut-off grade should be changed. However, during a project feasibility study, all costs, including the initial capital cost, have an influence on the cut-off grade. The cut-off grade determines the tonnage, grade, and location of material available for processing, which in turn drive mine and plant size, capital and operating costs, and financial performance. Conversely, operating costs are a critical input in the determination of the minimum cut-off grade. Different cut-off grade profiles, including cut-off grades that decrease over time, will require different mine plans and capital costs, and will result in better or worse financial performance. An iterative process must be used to determine the combination of cut-off grades, size of operation, and resulting capital and operating costs that will best satisfy the company's objectives.

Balancing initial and sustaining capital costs, operating costs, and cut-off grades is a critical part of a project feasibility study. If all assumptions made during the feasibility study, including those related to the geology of the deposit, the production capacity, the cost of operations, and the value of the product sold, remained true during the entire life of the mine, the cut-off grades would remain as planned. No cut-off grade change could be justified since plans were optimized and changes would reduce the value of the project. Decreasing the cut-off grade would require that additional lower-grade material be processed, which could not be achieved without either increasing the size of the plant or decreasing the net present value of future cash flows. Conversely, increasing the cut-off grade above that planned would result in underutilization of available capacity.

In practice, operating conditions differ from those assumed during the feasibility study, the geological properties of the deposit differ from those initially expected, capacities are either not reached or exceeded, mine and mill are no longer balanced, costs and the value of products sold are better or worse than expected, and cut-off grades must be continuously reestimated.

A company's financial objectives are likely to include expectation of a minimum return on investment, which cut-off grade calculations must take into account. If the time needed to mine one metric ton of material, process it, recover a salable product, and get a return from the sale of this product exceeds one year, costs and revenues should be discounted at the company-specified rate. While sunk costs do not influence cut-off grades, the cost of future sustaining capital expenditures must be included in the cut-off grade

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calculation to ensure that all material processed covers the capital invested, including a specified minimum return on investment.

HOW SPECIFIC COSTS INFLUENCE CUT-OFF GRADE CALCULATIONS

A few examples follow that illustrate how specific costs influence the cut-off grade. It is assumed that the mining company expects a minimum 15% return (i = 15%) on all investments:

- *Stockpiling of low-grade material.* This was discussed previously. The decision to stockpile material is more often than not a strategic decision rather than a decision based solely on expected cash flows and net present value.
- *Leaching operation.* In leaching operations, capital and operating costs, metal recovery, and metal sales occur over several years, and the cut-off grade might have to be determined using discounted cash flows. Consider a gold leaching operation with a five-year leach cycle. The total gold recovery is expected to reach 60%. Recovery and cost per year are given in Table 7-1. The waste-leach cut-off grade is to be estimated using a gold price V = \$1,150 per ounce, a cost of sales of \$50 per ounce, and a discount rate of 15%. The expected yearly revenue per ounce of gold contained in the material being leached is r (V – R), where r varies from year to year. This revenue is shown in Table 7-1, column 4. The last three columns in Table 7-1 show the discount factor 1/(1 + i), the discounted cost per metric ton, and the discounted revenue per ounce contained. The cut-off grade is calculated as the ratio of total costs over total revenues per ounce. If cash flows are not discounted, this cut-off grade is 0.47 g/t. To obtain a 15% return on investment the cut-off grade must be increased to 0.52 g/t.
- *Sustaining capital.* Sustaining capital is capital expenditures that must be incurred on a periodic basis to maintain production at the current level. For example, new trucks may have to be bought every eight years, leach pad expansions may be needed every four years, tails dam lifts may be added every seven years. Let I be the total cost of this investment and n its expected useful life in years. The cut-off grade should be high enough to ensure a minimum return on investment (i = 15%). This is achieved by including the cost of capital in the cut-off grade calculation. Let C_I be the cost per year that must be recognized to recover the investment I over n years at the specified

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Table 7-1 Calculation of cut-off grade in a gold leach operation

		Undiscounted		Discounted		
Year	Recovery	Cost per Metric Ton	Revenue per Ounce Contained	Discount Factor	Cost per Metric Ton	Revenue per Ounce Contained
0	15%	$6.00	$165	1.00	$6.00	$165
1	15%	$1.00	$165	0.87	$0.87	$143
2	12%	$1.00	$132	0.76	$0.76	$100
3	10%	$1.00	$110	0.66	$0.66	$72
4	8%	$1.00	$88	0.57	$0.57	$50
Total	60%	$10.00	$660	—	$8.85	$531
Cut-off grade	oz/t	—	0.0152	—	—	0.0167
	g/t	—	0.47	—	—	0.52

discount rate i. The net present value of this constant cost is $C_I\, g(i,n)$, where $g(i,n) = [1 - (1 + i)^{-n}] \cdot (1 + i)/i$. This cost must satisfy the following equation:

$$C_I\, g(i,n) = I$$

Therefore, the cost per year that should be included in the cut-off grade calculation is

$$C_I = I/g(i,n)$$

If i = 15% and n = 8, the cost of capital is $C_I = 0.19$ I per year. If no minimum return on investment was specified, the cost of capital would be $C_I = I/n = 0.13$ I per year. In the cut-off grade calculation, costs per year must be converted to costs per unit of production. These costs must be added to mining costs if the sustaining capital is for mining equipment, to leaching costs if it is for leach pad expansion, and to milling costs if it is for a tailings dam. Incremental capital expenditures, which are optional and must be justified by a feasibility study, should not be considered as sustaining capital.

- *Incremental capital expenditures.* Such expenditures may be required to maintain production beyond the planned life or to reach a higher level of production. The cost of these incremental capital expenditures must be taken into account in deciding whether the life should be extended or production should be increased. Different cut-off grade strategies should be considered. Once the decision is made to

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make the investment, incremental capital expenditures are sunk, and only remaining capacity constraints should influence cut-off grade decisions.

- *Closure and reclamation costs.* At the end of a mine life, one-time closure costs are incurred. These costs, C_R, decrease the net present value of a project's future cash flow and therefore the opportunity cost $i \cdot t \cdot NPV_i$. At the beginning of the mine life, the influence of closure costs on net present value is small, but it increases when the project matures. It was shown previously that if closure can be postponed by one year by mining low-grade material, the corresponding cash flow C could be negative. The only constraint is that $C > -C_R\, i/(1 + i)$.
- *Cost to perpetuity.* In some circumstances (e.g., if a water treatment plant must operate forever after closing a mining operation), a project generates a constant negative cash flow to perpetuity. It was shown previously that if C is a yearly cash flow to perpetuity, the net present value of this cash flow is $NPV_i = C(1 + i)/i$. This net present value is constant over time and can be considered as a fixed cost that does not influence the cut-off grade. However, this cost decreases the net present value of the project and should be taken into consideration when completing a feasibility study.
- *Overhead costs.* G&A and other overhead costs must also be divided between fixed and variable costs. Variable G&A costs must be included in all cut-off grade calculations. Fixed G&A costs, usually expressed on a per-year basis, must be included if the change in cut-off grade will change the mine life. This will be the case whenever one of the processes is capacity constrained. The fixed part of overhead costs can no longer be considered as fixed because lowering the cut-off grade will require extending the mine life. These costs must be expressed on a per-unit-of-production basis (by dividing costs per year by production per year) and added to the unit cost of the capacity-constrained process.

OTHER COSTS AND BENEFITS

Cut-off grades play a critical role in defining tonnages mined and processed, average grade of mill feed, life of mine, cash flows, and all major characteristics of a mining operation. In addition to the economically quantifiable financial impact that cut-off grade changes may have, other costs and benefits must be taken into account even though their impact is not easily quantifiable.

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Consideration must be given not only to changes in cash flow and net present value—as measured by $U_{dir}(x)$ and $U_{opp}(x)$—but also to all other impacts, $U_{oth}(x)$, including those of an environmental, socioeconomic, ethical, or political nature. Costs and benefits to all stakeholders must be evaluated. For most mining operations, the following stakeholders must be taken into account:

- Shareholders, who supply the capital needed for the operation and expect a return on their investment
- Banks, who contribute to the supply of financial resources the mining company needs to operate or expand
- Analysts, who advise the investing community
- Employees and their families
- Users of the final product sold by the mining operation, whether it is coal, gold, copper concentrate, iron ore, processed metal, or industrial minerals
- Suppliers, from whom the mining operation purchases equipment, energy, consumables, supplies, services, or expertise
- Local communities, including neighbors of the mining operation
- The local, regional, federal, or country governments, who are responsible for the welfare of their citizens and benefit from the taxes levied from the mining company. These governments must plan for new infrastructure, roads, health, education, and entertainment; increases in traffic, crime, and prostitution; and higher demand for water, food, and housing. They also have a fiduciary duty to ensure appropriate exploitation of national resources.
- Future generations, which will live with the long-term impact, good or bad, of the mining operation
- Nongovernmental organizations whose mission, self-appointed or otherwise, is to defend the interests of some of the above stakeholders

Senior management decides how to balance the needs, interests, and requirements of the different stakeholders. Those in charge of mine planning must be given practical guidelines for cut-off grade determination to ensure that the projects are designed to reach the company's objectives. Maximizing shareholder value (including minimizing shareholder liability) is often quoted as a company's primary objective. However, a company's objectives must include recognition of responsibilities toward all stakeholders, not only the shareholders.

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Higher cut-off grades may increase short-term profitability and enhance return to shareholders and other financial stakeholders. Higher cut-off grades may shorten the payback period, thus reducing political risk of creeping or outright nationalization. But reduced mine life reduces time-dependent opportunities, such as those offered by price cycles. Conversely, lower cut-off grades may increase project life with longer economic benefit to all stakeholders, including shareholders, employees, local communities, and government. Longer mine life may result in more stable employment, less socioeconomic disruption to local communities, and more stable tax revenues to government. Lower cut-off grades imply fuller consumption of mineral resources, which may present political advantages or may be required by law. All stakeholders may have to choose between higher financial returns over shorter time periods or lower returns over longer time periods. Using high but decreasing cut-off grades early in the mine life and stockpiling low-grade material for later processing can help balance financial returns and mine life.

One method of optimizing cut-off grades while taking into account unquantifiable costs and benefits consists of evaluating the project under a variety of constraints imposed on discount rate, mine or mill capacity, volume of sales, capital or operating costs, and so forth. Changes in the opportunity cost of imposing these constraints, $U_{opp}(x)$, are compared with the corresponding changes in other costs, $U_{oth}(x)$. The optimal cut-off grade is that for which the marginal (and quantifiable) increase in opportunity cost is equal to the corresponding marginal (but subjective) decrease in other costs.

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8 Blending Strategy

In an increasingly complex environment, blending strategies may be critical in maximizing the profitability of an operation or set of operations. For example, a metallurgical process may require that the chemical properties of the mill feed fall within specific ranges. Sales contracts may impose limits on the quality of the product being sold and the contaminants it contains. Environmental regulations may impose constraints on the mill feed, as needed for environmental emission control.

In a gold mine where refractory ore is treated by roasting, there might be different parts of the deposit with different geochemical properties. Material with high sulfide content and low gold content may have no economic value if considered on its own. Conversely, processing material with high gold content and low sulfur content may be problematic. However, adding the calorific value of the high-sulfide material to the gold value of the high-gold material may result in a highly profitable operation.

In a coal mining operation, one seam may have a low calorific value and high ash content, and another seam a high calorific value and low ash content. By appropriately mixing the two products, it may be possible to increase the tonnage of salable coal that satisfies contractual agreements on both sulfur grade and calorific value. Similarly, the product from one iron mine may have no economic value if considered on its own, but a salable product may be obtained by blending it with iron ore from another mine that exceeds contractual requirements.

Blending strategies applicable in relatively simple situations, such as when there are only two or three stockpiles, are discussed in this chapter. The objective is to give the reader a high-level understanding of the strategic aspect of stockpiling and blending. Although practitioners are likely to encounter more complex situations, the solution to such problems falls outside the scope of this book.

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Table 8-1 Properties of concentrates available for blending

Material Source	Index i	Tonnage, Kt T_i	Grade: Sulfur, % x_{1i}	Grade: Gold, g/t x_{2i}	Metal Content: Sulfur, Kt $T_i x_{1i}$	Metal Content: Gold, kg $T_2 x_{2i}$
Concentrate 1	1	20	3.50	40	0.70	800
Concentrate 2	2	40	1.20	90	0.48	3,600
Total	—	60	1.97	73.3	1.18	4,400

Table 8-2 Percentage of available tonnage and metal content in each concentrate

Material Source	Index i	Tonnage, %	Metal Content: Sulfur, %	Metal Content: Gold, %
Concentrate 1	1	33	59	18
Concentrate 2	2	67	41	82
Total	—	100	100	100

BLENDING TWO MATERIAL TYPES

Consider a gold roasting operation that has access to two sources of concentrate, a 20,000 t high-sulfide, low-gold stockpile (concentrate 1) and a 40,000 t low-sulfide, high-grade stockpile (concentrate 2). The concentrate properties are summarized in Table 8-1. Table 8-2 shows that 82% of the gold is in concentrate 2. The following notations are used:

T_i = tonnage of concentrate i, i = 1,2

x_{1i} = sulfur grade of concentrate i, i = 1,2

x_{2i} = gold grade of concentrate i, i = 1,2

T_o = tonnage of roaster feed

x_{1o} = sulfur grade of roaster feed

x_{2o} = gold grade of roaster feed

p_i = proportion of concentrate i in roaster feed, i = 1,2

t_i = tonnage of concentrate i in roaster feed, i = 1,2

Blending to Maximize Gold Grade in Roaster Feed

To satisfy fuel requirements, the roaster needs feed averaging at least 1.5% sulfur. An objective could be to maximize the blended gold grade while satisfying the sulfur requirements. Figure 8-1, called a blending diagram, shows the relationship between sulfur grade and gold grade for the two concentrates and all possible concentrate blends. Potential roaster feeds are represented by

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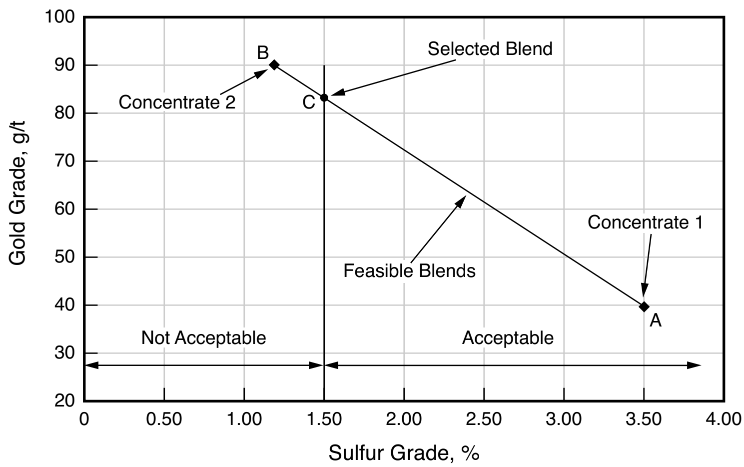

Figure 8-1 Feasible blends for two concentrate stockpiles

a point on the diagram: concentrate 1 is at point A, and concentrate 2 is at point B. The only feasible grades that can be obtained by blending concentrates 1 and 2 are those along the line AB. If only blends that contain more than 1.5% sulfur are acceptable, the blend that maximizes the gold grade is that which exactly satisfies the sulfur constraint; it is represented by point C on the blending diagram.

If p_1 is the percentage of concentrate 1 in the blend, and p_2 the percentage of concentrate 2, the optimal blend is obtained by solving the following equations:

$$p_1 x_{11} + p_2 x_{12} = 1.5\%$$
$$p_1 + p_2 = 1$$

The concentrates should be fed to the roaster in the following proportions:

$$p_1 = (1.5\% - 1.2\%)/(3.5\% - 1.2\%) = 13\%$$
$$p_2 = 1 - 13\% = 87\%$$

Note that p_1 and p_2 are equal to the ratio of the line segments on the blending diagram: p_1 = CB/AB and p_2 = AC/AB. After blending, the gold and sulfur grades of the concentrate delivered to the roaster are

$$x_{10} = p_1 x_{11} + p_2 x_{21} = 1.5\% \text{ sulfur}$$
$$x_{20} = p_1 x_{21} + p_2 x_{22} = 83.5 \text{ g/t}$$

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The total tonnage of material sent to the roaster is T_o, composed of t_1 metric tons of concentrate 1 and t_2 metric tons of concentrate 2, which must satisfy the following equations:

$$T_o = t_1 + t_2$$
$$t_1 = T_o p_1$$
$$t_2 = T_o p_2$$

Neither t_1 nor t_2 can exceed the total tonnages, T_1 and T_2, available in each stockpile. Therefore, the maximum value of T_o is the smallest value of T_i/p_i.

$$T_1/p_1 = 20/0.13 = 154 \text{ thousand metric tons}$$
$$T_2/p_2 = 40/0.87 = 46 \text{ thousand metric tons}$$
$$T_o = \text{minimum } (154, 46) = 46 \text{ thousand metric tons}$$
$$t_1 = 13\% \cdot 46 = 6 \text{ thousand metric tons}$$
$$t_2 = 87\% \cdot 46 = 40 \text{ thousand metric tons}$$

The gold content of the roaster feed is

$$t_1 x_{11} + t_2 x_{21} = 6 \cdot 40 + 40 \cdot 90 = 3{,}840 \text{ kg, or 123 thousand ounces of gold}$$

Blending to Maximize Gold Content in Roaster Feed

In the previous example, the objective was to maximize the gold grade in the roaster feed while maintaining a sulfur grade not less than 1.5%. This resulted in 13% of stockpile 1 and 87% of stockpile 2 containing 3,840 kg of gold being sent to the roaster. This example considers the economically more meaningful objective of maximizing the amount of gold sent to the roaster while still satisfying the requirement that the sulfur content be not less than 1.5%.

The gold and sulfur contents vary with the proportion of each stockpile sent to the roaster. Using the following formula, one can calculate the maximum tonnage T_o of material sent to the roaster for different proportions p_1 of concentrate 1:

$$T_o = \text{minimum } \left(T_1/p_1, T_2/p_2\right)$$

where p_1 and p_2 must be positive and their sum must be such that $p_1 + p_2 = 1$. Once the tonnage T_o has been calculated for a given p_1, it is possible to deduce the tonnages t_1 and t_2 of each stockpile sent to the roaster:

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$$t_1 = T_o p_1$$
$$t_2 = T_o p_2$$

Total tonnages and tonnages originating from each stockpile are plotted in Figure 8-2. The sulfur and gold average grades of the blended material are obtained using the following equations (Figure 8-3):

$$x_{10} = p_1 x_{11} + (1 - p_1) x_{21}$$
$$x_{20} = p_1 x_{21} + (1 - p_1) x_{22}$$

The corresponding sulfur and gold contents, $T_o x_{10}$ and $T_o x_{2o}$, respectively, are shown in Figure 8-4. The blend that maximizes the gold content is composed of 33% concentrate 1 and 67% concentrate 2. This results in 60,000 t of material being sent to the roaster, averaging 1.97% sulfur and 73.3 g/t gold and containing 4,400 kg of gold. Figure 8-4 shows that when the proportion of concentrate 1 decreases below 33%, the decrease in gold content is slow while the decrease in sulfur content is very sharp. These trends are reversed above 33%. An understanding of these relationships is fundamental if optimal blends are to be achieved under different cost, recovery, geochemical, or operational conditions.

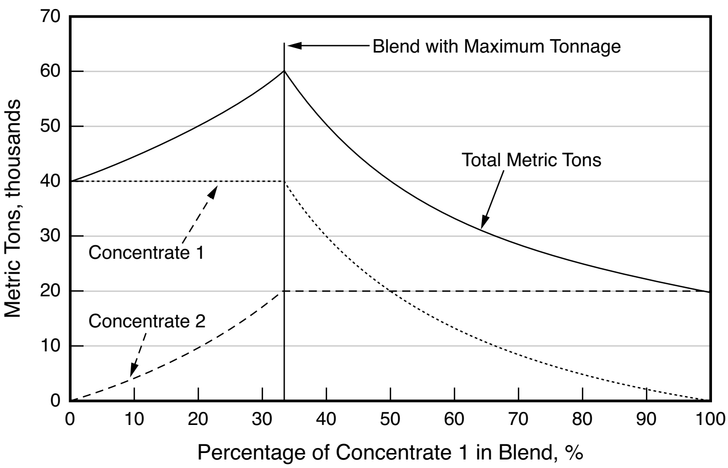

Figure 8-2 Feasible tonnage of blended material, as a function of the percentage p_1 of stockpile 1

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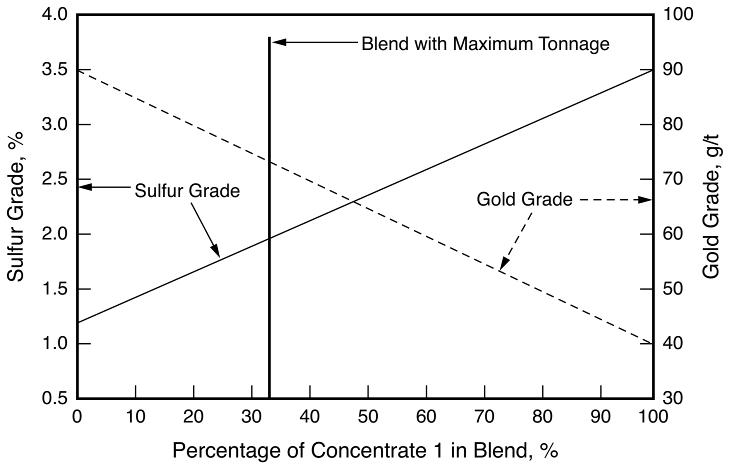

Figure 8-3 Average grade of blended material, as a function of the percentage p_1 of stockpile 1

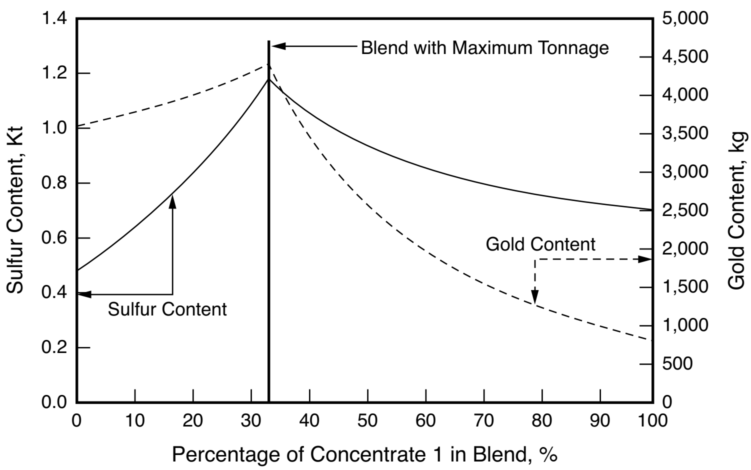

Figure 8-4 Metal content of blended material, as a function of the percentage p_1 of stockpile 1

BLENDING THREE MATERIAL TYPES

In the previous example, the case was considered where only two stockpiles were available for blending. Now consider three stockpiles. Each stockpile is characterized by its tonnage and two grades, as shown on Table 8-3. If all stockpiles were blended to get a single product, the result would be seven

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Table 8-3 Stockpiles available for blending

Stockpile	Index i	Stockpile Tonnage, T_i	Stockpile Grade 1, x_{1i}	Stockpile Grade 2, x_{2i}	Contained Metal 1, $T_i x_{1i}$	Contained Metal 2, $T_i x_{2i}$
1	1	2,000	1.00	7.0	2,000	14,000
2	2	1,000	2.00	12.0	2,000	12,000
3	3	4,000	0.80	15.0	3,200	60,000
Total		7,000	1.03	12.3	7,200	86,000

Table 8-4 Percentage of available tonnage and metal content in each stockpile

Stockpile	Index i	Tonnage, %	Metal 1, %	Metal 2, %
1	1	29	28	16
2	2	14	28	14
3	3	57	44	70
Total		100	100	100

thousand metric tons of material with grade 1 equal to 1.03 and grade 2 equal to 12.3. To get this result, the stockpiles would have to be blended in the following proportions (Table 8-4): 29% from stockpile 1, 14% from stockpile 2, and 57% from stockpile 3.

The blending diagram in Figure 8-5 shows the relationship between grade 1 and grade 2 for the three stockpiles. The three points, A, B, and C, represent the stockpiles. The only grade combinations that can be obtained by blending the three stockpiles are those located within the boundaries of the triangle ABC, defined as the feasible blends domain.

Blending to Reach Specific Grades

Consider the case where a blend needs to be obtained with specific grades: x_{10} for grade 1 and x_{20} for grade 2. To obtain these grades, the stockpiles much be blended in the following proportions, p_1, p_2, and p_3, such that

$$p_1 x_{11} + p_2 x_{12} + p_3 x_{13} = x_{10}$$
$$p_1 x_{21} + p_2 x_{22} + p_3 x_{23} = x_{20}$$
$$p_1 + p_2 + p_3 = 1$$

In addition, p_1, p_2, and p_3 must be positive, a condition that will be satisfied if the point with coordinates x_{10}, x_{20} falls within the ABC triangle in Figure 8-5. The solution to these equations is easily calculated:

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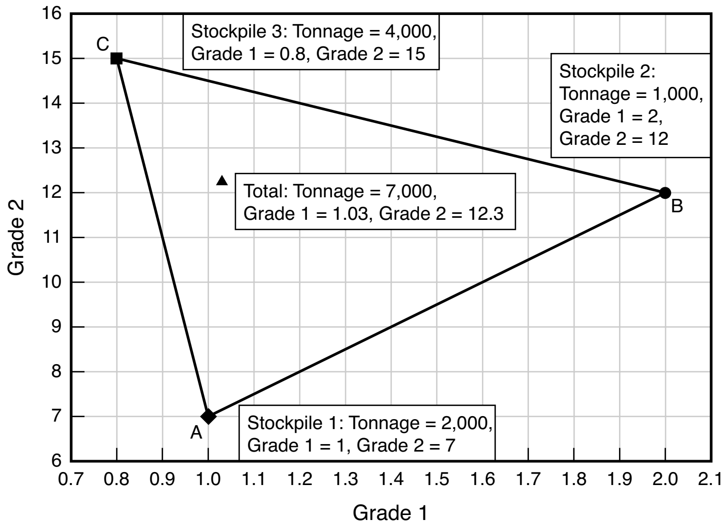

Figure 8-5 Feasible blends domain for three stockpiles

$$p_1 = A_1/B_1$$
$$A_1 = (x_{10} - x_{13})(x_{22} - x_{23}) - (x_{20} - x_{23})(x_{12} - x_{13})$$
$$B_1 = (x_{11} - x_{13})(x_{22} - x_{23}) - (x_{21} - x_{23})(x_{12} - x_{13})$$
$$p_2 = A_2/B_2$$
$$A_2 = (x_{10} - x_{13})(x_{21} - x_{23}) - (x_{20} - x_{23})(x_{11} - x_{13})$$
$$B_2 = (x_{12} - x_{13})(x_{21} - x_{23}) - (x_{22} - x_{23})(x_{11} - x_{13})$$
$$p_3 = 1 - p_1 - p_2$$

For example, if x_{10} = 1.20 and x_{20} = 11.0, then p_1 = 40.0%, p_2 = 26.7%, and p_3 = 33.3%. Knowing these proportions, one needs to calculate the maximum tonnage of blended material that can be produced and the tonnage of each stockpile that will be used (and therefore the tonnage that will remain in stockpiles) to reach this blended tonnage.

The maximum tonnage of blended material is T_0 made of t_1 metric tons of stockpile 1, t_2 metric tons of stockpile 2, and t_3 metric tons of stockpile 3:

$$T_0 = t_1 + t_2 + t_3$$

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Table 8-5 Material blended and processed, and material remaining in stockpile

Material Type	Index i	Tonnage, t T_i	Percentage, p_i	Grade 1, x_{1i}	Grade 2, x_{2i}
Material blended and processed					
Stockpile 1	1	1,500	40.00%	1.0	7.0
Stockpile 2	2	1,000	26.67%	2.0	12.0
Stockpile 3	3	1,250	33.33%	0.8	15.0
Total	0	3,750	100.00%	1.2	11.0
Material remaining in stockpiles					
Stockpile 1	1	500	—	—	—
Stockpile 2	2	0	3.10%	2.0	12.0
Stockpile 3	3	2,750	96.90%	0.8	15.0
Total	0	3,250	100.00%	0.8	14.9

The tonnage t_i that can be taken from stockpile i cannot exceed the total tonnage T_i of material in stockpile i. Using the previous example, the maximum value of T_0 is the smallest of

$$T_1/p_1 = 2{,}000/40.0\% = 5{,}000 \text{ t}$$
$$T_2/p_2 = 1{,}000/26.7\% = 3{,}750 \text{ t}$$
$$T_3/p_3 = 4{,}000/33.3\% = 12{,}000 \text{ t}$$

Therefore, T_0 = 3,750 t. The material consumed and that remaining in stockpiles is summarized in Table 8-5. Stockpile 2 was entirely consumed, but 3,250 t remain in stockpile, of which 500 t are in stockpile 1 and 2,750 t in stockpile 3.

Using Linear Algebra to Determine Stockpile Blends

The following equations were used to determine how three stockpiles should be blended to obtain specific average grades:

$$p_1x_{11} + p_2x_{12} + p_3x_{13} = x_{10}$$
$$p_1x_{21} + p_2x_{22} + p_3x_{23} = x_{20}$$
$$p_1 + p_2 + p_3 = 1$$

These equations can be written using matrix notations:

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$$\begin{pmatrix} x_{11} & x_{12} & x_{13} \\ x_{21} & x_{22} & x_{23} \\ 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} p_1 \\ p_2 \\ p_3 \end{pmatrix} = \begin{pmatrix} x_{10} \\ x_{20} \\ 1 \end{pmatrix}$$

If $\mathbf{X}$ is the stockpile grade matrix, $\mathbf{p}$ the proportion vector, and $\mathbf{x_0}$ the target grade vector, the proportions $\mathbf{p}$ are calculated as follows:

$$\mathbf{Xp} = \mathbf{x_0}$$
$$\mathbf{p} = \mathbf{X^{-1}x_0}$$

The vector $\mathbf{p}$ is calculated by multiplying the inverse of the matrix $\mathbf{X}$ with the vector $\mathbf{x_0}$. Using the previous numerical example*:

$$\begin{pmatrix} p_1 \\ p_2 \\ p_3 \end{pmatrix} = \begin{pmatrix} 1.0 & 2.0 & 0.8 \\ 7 & 12 & 15 \\ 1 & 1 & 1 \end{pmatrix}^{-1} \begin{pmatrix} 1.2 \\ 11 \\ 1 \end{pmatrix}$$
$$= \begin{pmatrix} -0.333 & -0.133 & 2.267 \\ 0.889 & 0.022 & -1.044 \\ -0.556 & 0.111 & -0.222 \end{pmatrix} \begin{pmatrix} 1.2 \\ 11 \\ 1 \end{pmatrix}$$
$$= \begin{pmatrix} 40.0\% \\ 26.7\% \\ 33.3\% \end{pmatrix}$$

These equations do not guarantee that the proportions pi are all between 0 and 1. Indeed if, in the blending diagram, the point defined by the vector $\mathbf{x_0}$ is outside the ABC feasible area, proportions greater than 1 and less than 0 will be calculated.

BLENDING TO MAXIMIZE TONNAGE

There are circumstances where the objective is to maximize the tonnage of blended material subject to specified grade constraints. The tonnages that can be obtained by blending are those that correspond to points falling within the

* The matrix inversion was performed using the Microsoft Excel function MINVERSE.

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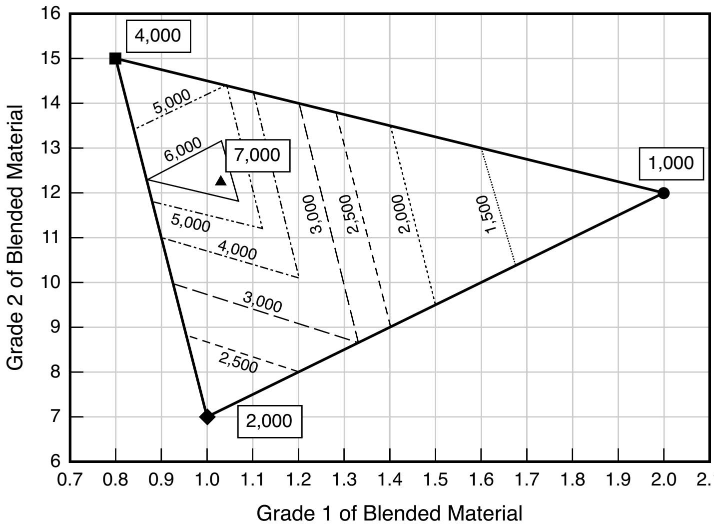

Figure 8-6 Grade–tonnage relationship and constant tonnage (Iso-T) lines of blended material

ABC feasible domain shown in Figure 8-5. To understand the relationship between grade constraints and tonnages, one must understand how tonnages vary within the feasible domain. Using the method previously described, the tonnage T_o can be calculated for all points within the feasible domain. The results are shown in Figure 8-6. One obtains a two-dimensional grade–tonnage surface, akin to the more familiar one-dimensional grade–tonnage curve. The constant-tonnage curves, defined as *Iso-T lines*, are composed of line segments that are parallel to the sides of the ABC triangle. This property of the Iso-T lines is explained in Example 7 in Appendix A.

Blending to Maximize Metal Content

There are circumstances, such as the one presented in the two-stockpiles example, where the objective is to maximize the metal content of the blended material, subject to specific grade constraints. For example, the objective might be to maximize the gold content, subject to the sulfur grade being within specified limits. Optimizing metal content requires understanding how this content varies within the feasible domain of blended material. Using the same example as previously mentioned, the metal content $Q_{20} = T_o x_{20}$ was calculated for all points within the feasible domain, and the results were contoured as shown in Figure 8-7. A surface is obtained that represents the relationship between gold content and both sulfur and gold grades. This

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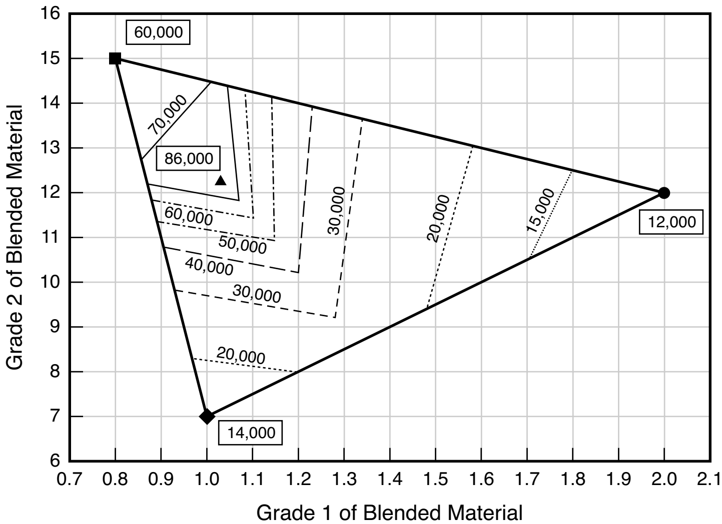

Figure 8-7 Grade–metal content relationship and constant metal content (Iso-Q) lines of blended material (metal 1)

relationship can be represented by contour lines, defined as *Iso-Q lines*, along which the gold quantity is constant. These lines are composed of segments that are no longer parallel to the sides of the ABC triangle.

Blending to Satisfy Ratio Constraints

Some circumstances occur where the constraints are on variables, such as ratios, that cannot be averaged between stockpiles. For example, in a nickel laterite operation, there might be a requirement on the silica-to-magnesia ratio. The ratio of a blended product is not equal to the weighted average of the individual stockpile ratios. Using the same example as above, one can calculate the ratio between the two grades (grade 2 divided by grade 1) and plot it as shown in Figure 8-8. The optimal blend might be that which maximizes one of the variables while maintaining the ratio between two other variables within specified boundaries.

REMARKS CONCERNING AN INCREASINGLY COMPLEX BLENDING PROBLEM

The methods described previously can be used to optimize blending of any number of stockpiles when the difference between the number of stockpiles and the number of grade or quality constraints is exactly equal to 1. It was

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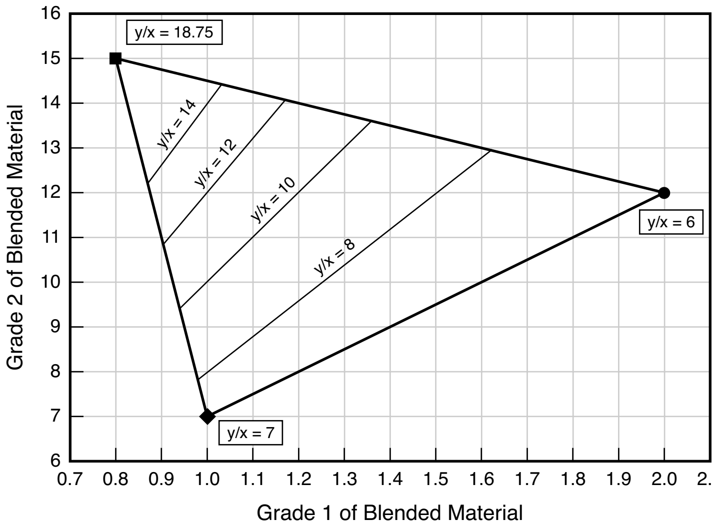

Figure 8-8 Grade–metal ratio relationship and constant ratio (Iso-R) lines of blended material

also assumed that the objective function was to maximize tonnage or metal content, subject to specified constraints on grade or quality. When these conditions are not satisfied, defining an optimal blending strategy can become increasingly complex, and mathematical programming methodologies have to be considered, which fall outside the scope of this book (see bibliography).

In the previous examples, it was assumed that the stockpiles already existed. A similar situation would be that where consideration is given to blending production from mines with fixed yearly production, such that tonnage, grade, and quality of material available for blending are known. In such situations, the feasible domain is fully defined by the tonnages and grades/qualities available from the stockpiles or from the different mines. When considering new mines that are not yet operating, a stockpiling strategy needs to be developed to optimize the feasibility of the project. Depending on the geological properties of the deposit, a number of stockpiling options are likely to be feasible, each stockpile being defined by both upper and lower cut-off grades. Depending on deposit geology, mine schedule, and cut-off grades, there is likely to be a very large number of feasible stockpile options. On the blending diagram, the feasible domain defined by the stockpiles must contain the target point where the requirements imposed on the blended product are satisfied. In addition, the stockpiles should be such that when blended in

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the required proportions, they are consumed in their entirety. Solving this problem, known as the inverse blending problem, can be extremely complex. A number of mine planning optimization tools have been developed to assist engineers in developing stockpiling and corresponding cut-off grade strategies that maximize specified objective functions during the life of a mining operation.

The previous examples also assume a deterministic environment, where the stockpile properties are known and constant throughout the life of the stockpile. In practice, there is always uncertainty concerning stockpile properties. A blending solution that is optimal under deterministic assumptions is not likely to be acceptable in the presence of uncertainty. A small variation in stockpile properties may result in a blended product that does not satisfy specified requirements. In such situations, the multivariate distributions of the properties of the different sources of material must be considered to develop a stockpiling and blending strategy that is flexible and can adapt to change. This strategy might be quite different from the deterministic solution. The complexity of the blending problem also increases significantly when constraints are on variables that cannot be averaged, such as ratios between two grades. Considerable research is dedicated to finding solutions to these complex situations. Development of appropriate algorithms and related computer programs is aimed at bringing practical solutions to practitioners (see the bibliography).

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9

Closing Remarks

The cut-off grade determines the tonnage and average grade of material processed and is critical in the determination of the economic feasibility of a project. All consequences of choosing a cut-off grade must be taken into account, including technical, economic, legal, environmental, social, and political as illustrated by the following fundamental equation:

$$U(x) = U_{dir}(x) + U_{opp}(x) + U_{oth}(x)$$

This equation represents the utility of sending material of grade x to a given destination. The optimal cut-off grade between two options is that where the utility $U_1(x)$ of one option is equal to the utility $U_2(x)$ of the other option.

$U_{dir}(x)$ represents direct revenues and costs. If only this term is taken into account, the cut-off grade between two options is

$$x_c = \left[\left(M_1 + P_1 + O_1\right) - \left(M_2 + P_2 + O_2\right)\right] / \left[\left(r_1 - r_2\right)\left(V - R\right)\right]$$

$U_{opp}(x)$ represents the opportunity cost of mining or processing material that was not scheduled to be mined or processed, and there are operating constraints. This cost is proportional to the net present value of future cash flows, and inversely proportional to the size of the constraints imposed on production:

$$U_{opp}(x) = -t \cdot i \cdot NPV_i$$

Because the net present value of future cash flows decreases over time, optimization of project economics is usually best achieved with high cut-off grades at the beginning of the mine life and lower cut-off grades at the end.

$U_{oth}(x)$ represents other costs that must be taken into account that are often not quantifiable in dollar terms. These costs can be highly significant

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and imply that net present value is typically not the only objective to be considered when optimizing cut-off grades.

Cut-off grade optimization is an iterative process. When planning a new mining operation, a cut-off grade profile must be chosen to define operating conditions, including mining rate, processing rate, volume of sales, capital and operating costs, cash flow, and net present value. Because the optimal cut-off grade is a function of operating costs and net present value, the first mine plan is not likely to be optimal. Once a cash flow has been determined under a specific set of assumptions—including assumptions related to cut-off grades—the underlying cut-off grade strategy must be reevaluated. The sensitivity of the project economics must be analyzed as a function of changes in cut-off grades.

As a feasibility study progresses over time, the properties of the deposit are better understood as a result of additional drilling and improved modeling; mining and processing methods are better defined; constraints on production are quantified; capital and operating costs are refined; and socioeconomic, environmental, and other costs are better understood. All these changes should be taken into account when determining the cut-off grade.

The determination of cut-off grades must also consider all future costs. During the feasibility study, these costs include the capital costs of building the site. When the mine and mill are operating, these costs are sunk and should no longer be taken into consideration. The cut-off grade strategy that optimizes net present value is independent of sunk costs. During the feasibility study there are no capacity constraints, but capital expenditures are taken into account to optimize cut-off grades. When the mine is operating, sunk costs should no longer be taken into account to optimize cut-off grades. However, there are capacity constraints, which are the results of decisions made during the feasibility study. If the estimates made during the feasibility study, including those related to mine and mill capacities, hold true when the mine is in operation, the optimal cut-off grade strategy remains unchanged. This strategy is the same at the end of the feasibility study when capital costs are taken into account to optimize the size of the facilities, and during operations when capacity constraints are imposed by the built facilities.

But operating conditions are never identical to those assumed during the feasibility study. These conditions change continuously:

- The actual mine and mill capacities differ from those which were expected before construction.
- The mill recovery and optimal operating conditions are different from planned.

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- The deposit properties differ from those in the model that was used during the feasibility study.
- Costs are different from expected.
- Prices of products sold are different.

Consequently, cut-off grades should be continuously reviewed and changed as conditions demand.

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Appendix A

EXAMPLE 1. NET PRESENT VALUE OF CONSTANT CASH FLOW: PROOF OF FORMULA

Problem Statement

If a project is expected to generate a constant cash flow C over n years (C_k = C for k = 0, 1, ..., n − 1), the net present value of this cash flow is

$$NPV_i = C\left[1 + 1/(1+i) + 1/(1+i)^2 + \ldots + 1/(1+i)^{n-1}\right]$$

Show that

$$1 + 1/(1+i) + 1/(1+i)^2 + \ldots + 1/(1+i)^{n-1} = g(i,n)$$

where $g(i,n) = [1 - (1 + i)^{-n}] \cdot (1 + i)/i$.

Solution

Define a = 1/(1 + i). Then

$$1 + 1/(1+i) + 1/(1+i)^2 + \ldots + 1/(1+i)^{n-1} = 1 + a + a^2 + a^3 + \ldots + a^{(n-2)} + a^{(n-1)}$$

If the right-hand side of this equation is multiplied by (1 − a), the following result is obtained:

$$\left[1 + a + a^2 + a^3 + \ldots + a^{(n-2)} + a^{(n-1)}\right] \cdot (1-a) = 1 - a^n$$

Therefore,

$$1 + a + a^2 + a^3 + \ldots + a^{(n-2)} + a^{(n-1)} = (1 - a^n)/(1-a)$$

Given that a = 1/(1 + i), this equation can be written as follows:

$$1 + 1/(1+i) + 1/(1+i)^2 + \ldots + 1/(1+i)^{n-1} = \left[1 - (1+i)^{-n}\right] \cdot (1+i)/i$$

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EXAMPLE 2. NET PRESENT VALUE OF PERPETUITY CASH FLOW: PROOF OF FORMULA

Problem Statement

Show that if a project is expected to generate a constant cash flow C to perpetuity, its net present value is $NPV_i = C\ (1 + i)/i$.

Solution

A proof was given previously that if a project generates a constant cash flow C for n years, its net present value is $NPV_i = C\ g(i,n)$, where

$$g(i,n)=\left[1-(1+i)^{-n}\right]\cdot(1+i)/i$$

If n goes to infinity, then $(1 + i)^n$ goes to infinity and $(1 + i)^{-n}$ goes to zero. The limit of g(i,n) is

$$g(i,n)=(1+i)/i$$

EXAMPLE 3. OPPORTUNITY COST OF MINING A PERIPHERAL DEPOSIT

Problem Statement

Consider the open pit mine shown in Figure A-1. Two parts of the deposit remain to be mined:

1. A “main pit” that requires mining low-grade and waste material for two years, followed by eight years of mining higher-grade material. The cash flow generated during the first two years is expected to be negative at $45 million per year. During the next eight years, a positive cash flow will be generated equal to $250 million per year.
2. A “mini-pit” that can be mined in one year and is expected to generate a positive cash flow of $90 million.

Under current conditions, the main pit can only be mined if the mini-pit area can be used as a waste dump. This leaves two options: Mine only the main pit and condemn the mini-pit (option 1); or mine the mini-pit first followed by the main pit (option 2). Assuming a 15% discount rate, what is the opportunity cost of mining the mini-pit first? Which one of the two options would you recommend? If there were other locations where waste could be dumped, a third option would be to mine the main pit first and the mini-pit last (option 3). Which option would you recommend?

Repeat this analysis using a 5% discount rate.

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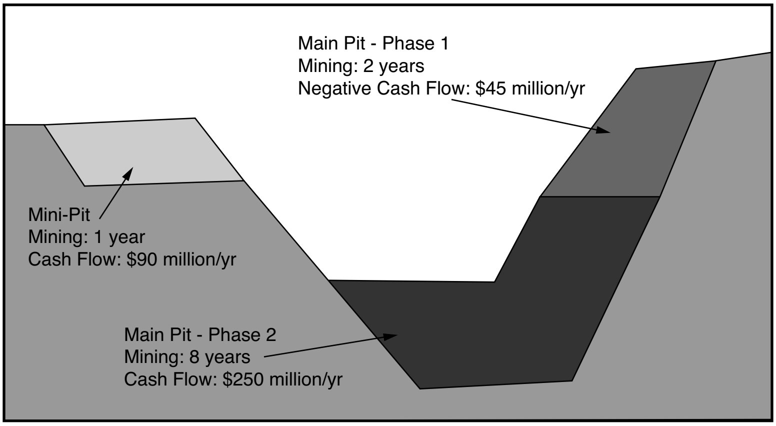

Figure A-1 Schematic representation of main pit and mini-pit

Solution

The net present values of each option are calculated in Table A-1. To show the influence that each pit has on the other, the expected main pit and mini-pit undiscounted cash flows are shown separately. The discount factors $1/(1 + i)^{k-1}$ have been calculated for each year k using a discount rate i = 15%. The net present value of each option is the sum of the product of undiscounted yearly cash flows with a discount factor. Under option 1, only the main pit is mined with net present value NPV_i = \$891 million. Under option 2, mining the main pit is postponed by one year. The main pit net present value is decreased to NPV'_i = \$775 million. The opportunity cost of mining the mini-pit first is \$891 – \$775 = \$116 million. The net present value of the mini-pit is only \$90 million, not enough to pay for the opportunity cost. The NPV of option 2 is \$865 million, less than that of option 1. If the choice is between options 1 and 2, option 1 is that which should be chosen. Under option 3, the net present value of the mini-pit is \$22 million, and the total net present value is \$913 million. If feasible, option 3 is the best choice.

If a 5% discount rate is used, the results are as shown in Table A-2. The opportunity cost of mining the mini-pit first (option 2) is \$69 million. The mini-pit's net present value is \$90 million. Option 2 is better than option 1. However, option 3 is still the best option.

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Table A-1 Cash flow and net present value of options 1 to 3 (discount rate i = 15%)

Year k	Discount Factor $1/(1+i)^k$	Option 1 Main Pit	Option 1 Mini-Pit	Option 2 Main Pit	Option 2 Mini-Pit	Option 3 Main Pit	Option 3 Mini-Pit
0	1.000	($45)	—		$90	($45)	—
1	0.870	($45)	—	($45)	—	($45)	—
2	0.756	$250	—	($45)	—	$250	—
3	0.658	$250	—	$250	—	$250	—
4	0.572	$250	—	$250	—	$250	—
5	0.497	$250	—	$250	—	$250	—
6	0.432	$250	—	$250	—	$250	—
7	0.376	$250	—	$250	—	$250	—
8	0.327	$250	—	$250	—	$250	—
9	0.284	$250	—	$250	—	$250	—
10	0.247	—	—	$250	—		$90
Net present value per pit		$891	$0	$775	$90	$891	$22
Opportunity cost		$0	—	($116)	—	$0	—
Total net present value		$891		$865		$913	

Table A-2 Cash flow and net present value of options 1 to 3 (discount rate i = 5%)

	Option 1 Main Pit	Option 1 Mini-Pit	Option 2 Main Pit	Option 2 Mini-Pit	Option 3 Main Pit	Option 3 Mini-Pit
Net present value per pit	$1,451	$0	$1,382	$90	$1,451	$55
Opportunity cost	$0	—	($69)	—	$0	—
Total net present value	$1,451		$1,472		$1,506	

EXAMPLE 4. SIMPLIFIED EQUATION TO ESTIMATE OPPORTUNITY COST: VERIFICATION OF FIRST-ORDER APPROXIMATION

Problem Statement

Show that for small values of t, the difference between the exact formula $[1 - (1 + i)^{-t}]\ NPV_i$ and its first-order approximation $i \cdot t \cdot NPV_i$ is less than 10%. Compare the two formulas for t varying from zero to two hundredths of a year (approximately one week) and i = 15%.

Solution

Figure A-2 shows the relationship between $1 - (1 + i)^{-t}$ and $i \cdot t$ as a dotted line. The maximum value of $i \cdot t$ is $0.02 \cdot 15\% = 0.003$. The discount factor

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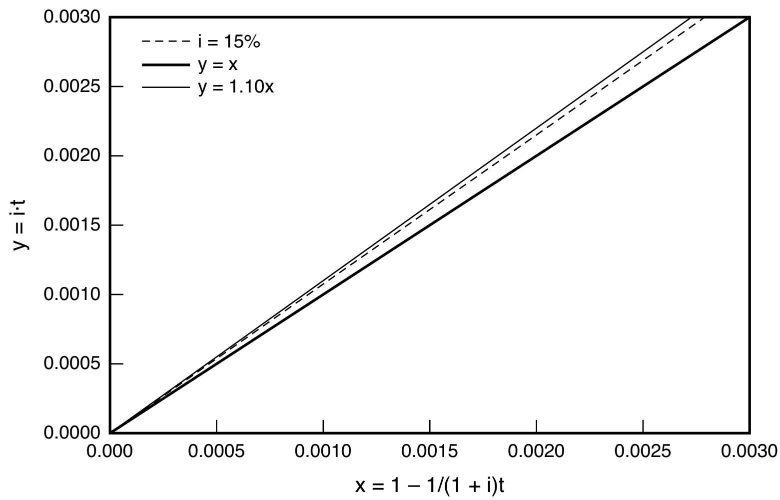

Figure A-2 Comparison of $1 - (1 + i)^{-t}$ and $i \cdot t$ for i = 15% and t less than 0.02 years

calculated using the first-order approximation, $i \cdot t$, is less than 10% higher than that using the exact equation, $1 - (1 + i)^{-t}$. This difference decreases for smaller values of i.

EXAMPLE 5. OPPORTUNITY COST OF NOT USING THE OPTIMAL CUT-OFF GRADE IN A COPPER MINE

Problem Statement

Consider a copper mine in which the material can be either leached (process 1) or sent to a flotation plant (process 2). The relevant operating conditions are summarized as follows:

Leach recovery:	$r_1 = 57\%$
Mill recovery:	$r_2 = 89\%$
Leach processing cost:	P_{01} = \$2.50 per metric ton leached (including overhead)
Mill processing cost:	P_{02} = \$29.00 per metric ton milled (including overhead)
Copper price:	V = \$2.50 per pound of salable copper (after cost of sales)

Demonstrate that the cut-off grade x_c between leach and mill is 1.50% Cu.

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The mill capacity is such that it can only accept material exceeding x_s = 1.60% Cu. The tonnage and metal content of material above the two cut-off grades are as follows:

Above x_c = 1.50% Cu: 10.842 million metric tons containing 588.84 million lb of copper

Above x_s = 1.60% Cu: 9.606 million metric tons containing 546.59 million lb of copper

What is the opportunity cost of sending material averaging between x_c and x_s to the leach pad instead of the mill?

Solution

The cut-off grade between mill and leach is

$$x_c = (P_{o1} - P_{o2}) / [(r_1 - r_2)V]$$

$$x_c = (29.00 - 2.50) / [(89\% - 87\%) \cdot 2.5 \cdot 2{,}205] = 1.50\%\ \text{Cu}$$

A change in cut-off grade from x_c = 1.50% Cu to x_s = 1.60% Cu results in moving 1.236 million metric tons containing 42.25 million lb of copper from the mill to the leach pad. The result is a savings in processing costs and a loss of revenue:

Cost savings: $[T(x_s) - T(x_c)] \cdot (P_{o2} - P_{o1})$ = 1.236 · (29.00 – 2.50)
= $32.75 million

Revenue loss: $[Q(x_s) - Q(x_c)] \cdot (r_2 - r_1) \cdot V$ = 42.25 · (89% – 57%) · 2.5
= $33.80 million

Total opportunity cost = 33.80 – 32.75 = $1.05 million

For the mill to accept all material above 1.50% Cu, its capacity would have to be increased by 13%. Such an increase would be justified only if it could be achieved for less than $1.05 million.

EXAMPLE 6. STOCKPILING AND CUT-OFF DETERMINATION IN A COPPER MINE

Problem Statement

Consider an open pit copper mine in which material can be sent to a flotation plant, a stockpile, or a waste dump. The parameters that apply to each option are listed in Table A-3. You are asked to do the following:

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Table A-3 Copper mine operating parameters

Milled material		
M_o	\$1.90	Mining cost per metric ton of ore mined and shipped to the mill, including mine overhead
P_o	\$10.00	Processing cost per metric ton of ore, including process overhead
r	86%	Copper recovery for fresh material
Wasted material		
M_w	\$2.00	Mining cost per metric ton of waste mined and placed on waste dump, including mine overhead
P_w	\$0.05	Yearly cost per metric ton of waste dumped, including stockpile maintenance and water treatment plant operating cost
Stockpiled material		
M_{stp1}	\$2.50	Mining cost per metric ton of material mined and stockpiled, including overhead and capital cost per metric ton for pad preparation
P_{stp}	\$0.10	Yearly cost per metric ton of material stockpiled, including stockpile maintenance and water treatment plant operating cost
M_{stp2}	\$1.00	Cost per metric ton to retrieve material from stockpile and ship it to the mill, including environmental remediation and overhead
dr	–3%	Loss of recovery due to oxidation during life of stockpile
Life of mine		
n	5	Life of stockpile in years, including year 0 when ore is mined and year 4 when stockpile is processed
Value of recovered copper		
V	\$3.00	Value of one pound of copper recovered, net of shipping and smelting costs, and other costs of sale
Discount rate		
i	10%	Discount rate per year

- Calculate and plot the utility of each option as a function of the copper grade.
- Calculate the cut-off grade between waste, stockpile, and mill. Interpret the results.
- Which cut-off grades would apply to the stockpile if the mill capacity is such that only material exceeding 0.35% Cu can be processed?

Solution

Table A-4 shows the mining and processing costs per year for waste dump, direct mill, and stockpile options. In this table, the \$0.10 yearly cost per ton of material stockpiled applies to years 1 to 4, whereas the \$1.00 cost of retrieving one ton of material from stockpile is only incurred in year 4. The \$0.05 yearly

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Table A-4 Mining and processing costs per year for material wasted, milled, and stockpiled

	Mining Cost per Metric Ton			Processing Cost per Metric Ton		
Year	**Waste**	**Mill**	**Stockpile**	**Waste**	**Mill**	**Stockpile**
0	($2.00)	($1.90)	($2.50)	—	($10.00)	—
1	($0.05)	—	($0.10)	—	—	—
2	($0.05)	—	($0.10)	—	—	—
3	($0.05)	—	($0.10)	—	—	—
4	To infinity	—	($1.10)	—	—	($10.00)

Table A-5 Undiscounted and discounted costs per metric ton mined, and revenues per pound processed

	Total Cost per Metric Ton			Dollar per Pound of Cu Contained			Discount Factor, $1/(1 + i)^n$
Year	**Waste**	**Mill**	**Stockpile**	**Waste**	**Mill**	**Stockpile**	
0	($2.00)	($11.90)	($2.50)	—	$2.58	—	1.000
1	($0.05)	—	($0.10)	—	—	—	0.909
2	($0.05)	—	($0.10)	—	—	—	0.826
3	($0.05)	—	($0.10)	—	—	—	0.751
4	To infinity	—	($11.10)	—	—	$2.49	0.683
Total	Undiscounted			Undiscounted			
	($7.00)	($11.90)	($13.90)	$0.00	$2.58	$2.49	
	Discounted			Discounted			
	($2.55)	($11.90)	($10.33)	$0.00	$2.58	$1.70	

cost per metric ton of waste applies to perpetuity. Table A-5 shows the total cost (mining plus processing) per year for the three processes, and the corresponding value per pound contained in material processed. For direct mill feed, the value per pound is r V = 86% · $3.00 = $2.58, which is recovered in year 0. For material stockpiled, this value is (r + dr) V = (86% – 3%) · $3.00 = $2.49 per pound, which is recovered in year 4.

Also shown in Table A-5 are cumulative costs per metric ton and value per pound, both discounted and undiscounted, for as long as the material under consideration carries costs (to perpetuity for waste, five years for stockpiled material, one year for direct mill feed). For stockpiled material, the undiscounted value is based on the assumption that the $0.05 yearly cost will apply to 100 years, starting in year 1. The discounted value of the $0.05 cost to perpetuity is $0.05 · (1 + i)/i = 11 · $0.05 = $0.55.

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The utility of sending one metric ton of material to a given process is equal to the net present value of the cash flow generated by this ton. For example, if one metric ton of material of grade x is sent to the stockpile, the net present value of this decision is $NPV_{stp} = -\$10.33 + \$1.70 \cdot 2{,}205 \cdot x$, where x is expressed in % Cu and 2,205 is the number of pounds per metric ton. The three net present values can be written as follows:

For waste: $U_{waste}(x) = -\$2.55$

For direct feed: $U_{ore}(x) = -\$11.90 + 2.58 \cdot 2{,}205 \cdot x$

For stockpile: $U_{stp}(x) = -\$10.33 + 1.70 \cdot 2{,}205 \cdot x$

These lines are plotted in Figure A-3. The cut-off grades between two processes are defined by the point of intersection of the lines corresponding to these processes:

Waste-mill cut-off grade:

$$x_c = \frac{(11.90 - 2.55)}{(2.58 \cdot 2{,}205)} = 0.16\%\ \text{Cu}$$

Waste-stockpile cut-off grade:

$$x_c = \frac{(10.33 - 2.55)}{(1.70 \cdot 2{,}205)} = 0.21\%\ \text{Cu}$$

Mill-stockpile cut-off grade:

$$x_c = \frac{(11.90 - 10.33)}{[(2.58 - 1.70) \cdot 2{,}205]} = 0.08\%\ \text{Cu}$$

As one would expect, if there is no mill capacity constraint, the best decision is to waste all material below 0.16% Cu and to send material above 0.16% Cu directly to the mill. However, if the mill capacity is such that only material above 0.35% Cu can be processed, all material between 0.21% Cu and 0.35% Cu should be stockpiled and processed at a later date (Figure A-4).

EXAMPLE 7. PROPERTIES OF CONSTANT-METRIC-TONS CURVES IN BLENDING DIAGRAMS

Problem Statement

Figure A-5 shows the relationship between three stockpiles, i = 1,3. The tonnage of material in stockpile i is T_i with grade x_{1i} and x_{2i}. The first grade, x_1, is plotted on the horizontal axis, while the second grade, x_2, is plotted along the vertical axis. The stockpiles are represented by the points A, B, and C. The objective is to blend the three stockpiles to obtain a product of grade x_{10}, x_{20}. The proportions in which the stockpiles are blended is p_i, i = 1,3. The

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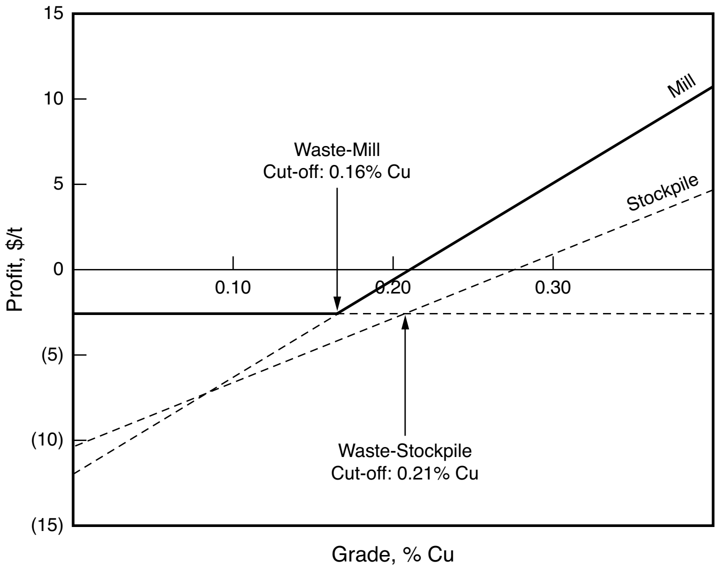

Figure A-3 Net present value of three options: best option with no mill capacity constraint

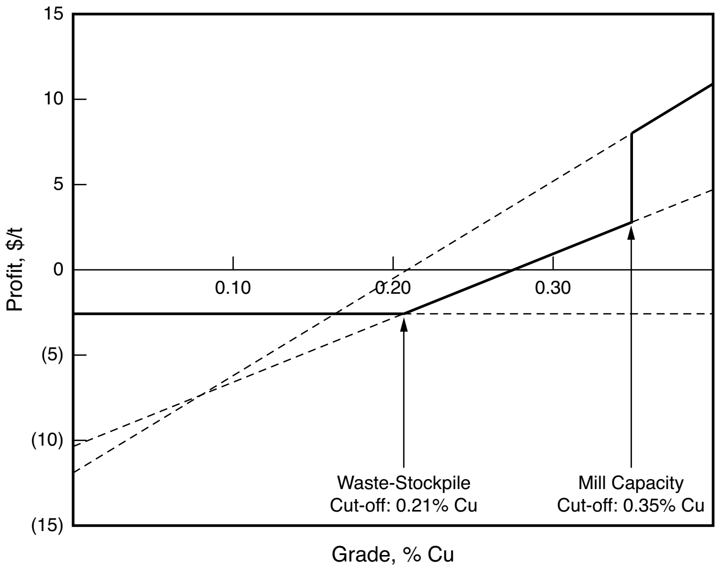

Figure A-4 Net present value of three options: best option with mill capacity constraint

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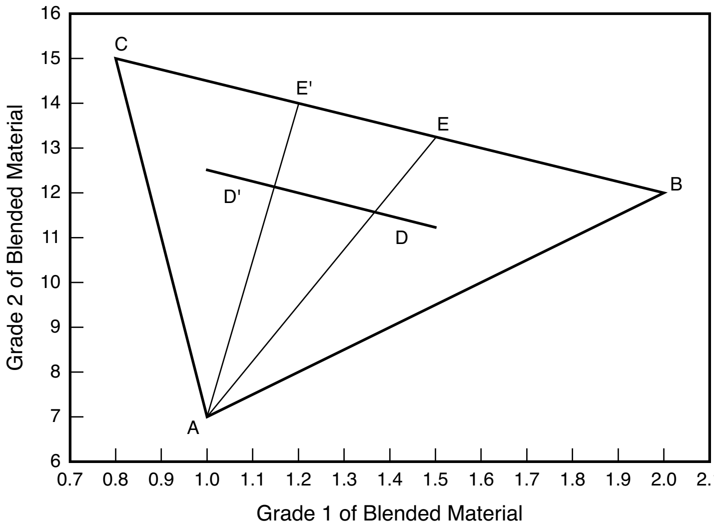

Figure A-5 Relationship between stockpile properties and lines with constant tonnage

maximum tonnage of blended material is T_0. It was shown in chapter 8 that T_0 is equal to the smallest value of the ratios T_i/p_i.

Show that, on graphs such as Figure A-5, the shapes that define blends with the same maximum tonnage (Iso-T shapes, on which T_0 is a constant) are made of line segments parallel to the lines AB, AC, or AD.

Solution

For the tonnages T_0 to be constant, the proportion pi applicable to one of the stockpiles must remain constant so that the ratio T_i/p_i remains constant. If a line is drawn from A (stockpile 1), which intersects an Iso-T line in D and the line BC in E, the proportion p_1 is equal to the ratio DE/AE (Figure A-5). If another line AE' is drawn, which intersects the same Iso-T line in D', the proportion p'_1 is equal to the ratio D'E'/AE'. If stockpile 1 is that which limits the maximum tonnage T_0, the proportions p_1 and p'_1 must be equal. The ratios DE/AE and DE'/AE' are equal only if the Iso-T line DD' is parallel to the line BC.

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List of Symbols

Symbol	Description
C	constant yearly cash flow
c	constant tail in recovery function
C_I	revenues required every year during n years to get a return on investment i on a capital investment I: $C_I = I / g(i,n)$
C_k	cash flow generated by a project in year k, k = 0, 1, 2, ..., n
C_R	expected end-of-life reclamation costs
C_s	smelter costs per metric ton of concentrate
C_t	cost of shipping one metric ton of concentrate to the smelter
d_1	metal grade deducted from recovered grade in calculation of smelter payment for metal 1
DIMC	discounted incremental mining costs
DIPC	discounted incremental processing costs
DIR	discounted incremental revenues
$dP_o(T_{+c})/dT_{+c}$	first-order derivative of $P_o(T_{+c})$ with respect to T_{+c}
dQ_{+c}/dT_{+c}	first-order derivative of Q_{+c} with respect to T_{+c}
$dr(T_{+c})/dT_{+c}$	first-order derivative of $r(T_{+c})$ with respect to T_{+c}
$dU(T_{+c})/dT_{+c}$	first-order derivative of $U(T_{+c})$ with respect to T_{+c}
$g(i,n)$	$= [1 - 1/(1 + i)^n](1 + i)/i$
I	capital cost invested
i	minimum rate of return (discount rate)
K	concentration ratio defined as number of metric tons of material that must be processed to produce one metric ton of concentrate
M	mining cost per metric ton processed
M_o	mining cost per metric ton of ore

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Symbol	Description
M_{o1}	value of M_o for process 1
M_{o2}	value of M_o for process 2
M_{stp}	current mining costs per metric ton delivered to low-grade stockpile
M_w	mining cost per metric ton of waste
n	number of years
NPV	net present value
NPV_i	net present value of future cash flows, calculated using the discount rate i
$NSR(x_1, x_2)$	net smelter return, defined as returns from selling concentrate produced from one metric ton of ore with average grades x_1, x_2, less smelting charges
NSR_c	NSR cut-off between processing and wasting one metric ton of material
O_o	overhead cost per metric ton of ore
O_{o1}	value of O_o for process 1
O_{o2}	value of O_o for process 2
O_{stp}	current overhead costs associated with mining and stockpiling one metric ton of low-grade material
O_w	overhead cost per metric ton of waste
p_1	proportion of metal 1 contained in concentrate that is paid for by smelter
p_2	proportion of metal 2 contained in concentrate that is paid for by smelter
p_i	in Chapter 8, proportion of stockpile or concentrate i in blended material
P_o	processing cost per metric ton of ore
$P_o(T_{+c})$	processing cost per metric ton of ore processed, if plant capacity is T_{+c}
P_{o1}	processing cost for process 1
P_{o2}	processing cost for process 2
P_{stp}	net present value of future costs of stockpiling material that will be processed later
P_w	processing cost per metric ton of waste
P_{waste}	net present value of future costs of wasting material

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Symbol	Description
$Q(x)$	quantity of metal in material whose grade is greater than x
Q_{+c}	quantity of metal contained in material above cut-off grade x_c: $Q_{+c} = T_{+c} \cdot x_{+c}$
R	refining, transportation, and cost of sales per unit of product sold
r	recovery, or proportion of valuable product recovered from the mined material
$r(T_{+c})$	processing plant recovery, if plant capacity is T_{+c}
$r(x)$	process recovery for material of average grade x
R_1	value of R for process 1
R_2	value of R for process 2
r_1	value of r for process 1
r_2	value of r for process 2
r_c	constant recovery after subtracting constant tail c
R_{stp}	refining, transportation, and cost of sales per unit of product sold from stockpile at the time product is sold
r_{stp}	recovery expected at the time stockpiled material is processed
s	stripping ratio, defined as tons of waste per tons of ore
T	tonnage of ore and waste to be mined
t	time, measured in years
T_{+c}	tonnage above cut-off grade
T_i	tonnage of material in stockpile or concentrate i
t_i	tonnage of stockpile or concentrate i in blended material
T_o	tonnage of material in blended material
T_w	tonnage of waste to be mined
$U(T_{+c})$	utility of running the plant at T_{+c} capacity for one year
$U(x)$	utility of sending one metric ton of material of grade x to a given process: $U(x) = U_{dir}(x) + U_{opp}(x) + U_{oth}(x)$
$U_1(x)$	utility of sending one metric ton of material of grade x to process 1
$U_2(x)$	utility of sending one metric ton of material of grade x to process 2
$U_{dir}(x)$	direct utility (profit or loss) of processing one metric ton of material of grade x

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Symbol	Description
U_{jk}	utility of mining block j in year k
$U_{jk,dir}$	direct utility of mining block j in year k
$U_{jk,opp}$	opportunity cost of mining block j in year k
$U_{jk,oth}$	other utility of mining block j in year k
$U_{max}(x)$	maximum utility resulting from sending material of grade x to either of two processes
$U_{opp}(x)$	opportunity cost or benefit of changing the processing schedule by adding one metric ton of grade x to the material flow
$U_{ore}(x)$	utility of mining and processing one metric ton of material of grade x
$U_{ore}(x_1, x_2)$	utility of sending one metric ton of material with metal grades x_1, x_2 to the processing plant
$U_{oth}(x)$	utility of other factors that must be taken into account in the calculation of cut-off grades
$U_{stp}(x)$	utility of stockpiling material of grade x
$U_{waste}(x)$	utility of mining and wasting one metric ton of material of grade x
V	value of one unit of valuable product
V_1	value of one unit of product 1, such as one pound of copper
V_2	value of one unit of product 2, such as one ounce of gold
V_{stp}	dollar value of the product recovered from stockpile at the time product is sold
x	average grade
x_{+c}	average grade above cut-off grade x_c
x_{1e}	grade equivalent expressed in terms of metal 1
x_{2e}	grade equivalent expressed in terms of metal 2
x_{1i}	average grade of metal 1 in stockpile or concentrate i
x_{2i}	average grade of metal 2 in stockpile or concentrate i
x_c	cut-off grade
x_{c1}	cut-off grade 1, taking only operating costs into account
x_{c2}	cut-off grade 2, taking into account operating costs and undiscounted capital cost per metric ton
x_{c3}	cut-off grade 3, taking into account operating costs and discounted capital cost per metric ton

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Symbol	Description
x_{c4}	cut-off grade 4, taking into account operating costs, discounted capital cost per metric ton, and opportunity costs
x_s	selected cut-off grade
YCF	yearly cash flow

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