FEATURE

ESTIMATING RECOVERABLE RESERVES – DEVELOPMENTS IN APPLIED GEOSTATISTICS

AIG Seminar, November 1989

by Vivienne Snowden

INTRODUCTION

So much work has been done in recent years using geostatistics to solve practical problems that we no longer have to apologise for nor justify the method. The method is as good as the practitioner, which goes, after all, for any technical application. There have been enough case studies now to show that almost any orebody, simple or complex in structure, which has a definable spatial continuity, will be amenable to geostatistical evaluation given that it is suitably sampled.

Early kriging techniques catered only for calculating the average grade of a fixed volume of ground which, in structurally complex orebodies would not reflect ultimate selective mining of the orebody but today techniques do cater for estimating recoverable reserves from exploration drilling.

The most useful tool, at the exploration stage, is the semivariogram. This quantifies the range and direction of grade continuity and lends support to geological interpretation. It can highlight potential problems due to insufficiently close spaced drilling or, indeed incorrectly orientated drilling. The relationship between variability and block sizes, which is quantified by the semivariogram, allows one to interpret the degree of selectivity possible for given selective mining units. The classification of the resource or reserve is affected by the range of influence of the semivariograms.

This paper presents the current arguments revolving around the most appropriate kriging technique to use.

DESCRIPTION OF KRIGING TECHNIQUES

Ordinary kriging (OK) was developed by Matheron in the 1960's to address the estimation of average block grades by weighting surrounding samples according to the semivariogram. The two assumptions for OK are:

1. Stationarity (i.e. no drift or trend)
2. Normal distribution

Where ore values are lognormally distributed and the proportional effect is therefore present the values are transformed (or normalised) by taking logs, establishing the semivariogram for the logs, kriging then retransforming back into the natural values, (**ordinary log normal kriging, OLNK**).

Later techniques were developed to address the problem of how to build an ore reserve model from wide-spaced data to estimate recoverable selective mining reserves. **Indicator Kriging (IK), probability kriging (PK)** and **multigaussian kriging (MK)** were developed to estimate the distribution of grades within a block. These followed after attempts to promote disjunctive kriging (DK) encountered severe drawbacks because its estimated probability density function may take negative values (Verly, 1983).

Indicator kriging

Indicator kriging is an estimation method developed by Switzer (9177) and Journel (1983) to address the problem of how to weight outliers in skewed distributions such as found in precious metal deposits so that they cause no bias. It gives an estimate of the distribution of grades within a block, unlike ordinary kriging (OK) which gives only the average grade.

The basis of the technique is a binomial transformation such that, for any given cutoff grade, Z_c, sample values are assigned an indicator value of 1 if the sample is below the cutoff and 0 if the sample is above this cutoff.

i.e. Indicator, I = 1 if sample $\leq$ Zc

= 0 if sample > Zc

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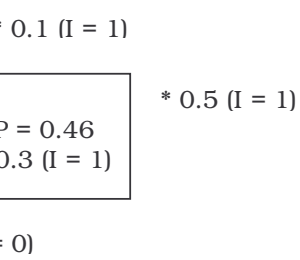
These transformed values from the input point data which are kriged to obtain the probability, P, of a block being below the cutoff grade, for example using a cutoff, Z_c = 1.0:



An indicator value of 0.46 implies that 46% of that block is likely to be below the cutoff grade on which the indicator transformation was based.

Subsequent kriging runs for different cutoff grades gives a different percentage below cutoff for the same block. Hence one can develop a grade/tonnage curve from which one can estimate the ore grade and ore tonnes above any specific cutoff grade within each block. The percentage below cutoff gives the cumulative frequency of grades. From this can be derived the histogram of grades. A simple calculation will give the average grade above a specific cutoff. Naturally the more indicator cutoffs addressed, the more definition there will be along the grade/tonnage curve.

In order to estimate the block indicator one needs a semivariogram. This is based on the indicator values for a given cutoff. There are as many semivariograms as there are cutoff grades. It is obvious that the semivariogram parameters will vary according to the cutoff under consideration.

Lemmer (1985) studied the effects on the semivariogram for increasing cutoffs (Z_c) and found an important relationship. The semivariogram of grades is linearly related to the indicator semivariogram for any given cutoff grade, Z_n.

$$\gamma_I(h, z_n) = A + B\gamma_Z(h)$$

where z = sample grade
z_n = cutoff grade
h = lag
A = intercept
B = slope

This relationship is best defined for a particular cutoff grade termed the mononodal cutoff, Z_{nc}:

$Z_{nc} = M + 0.5\sigma\Gamma_3$ where {M = mean
{σ = standard deviation
{Γ_3 = skewness

Indicator semivariograms, especially the mononodal cutoff semivariogram, are reportedly well behaved and resistant to outliers. Hence $\gamma(h)$ can be derived from the relationship above if the parameters A and B can be defined.

In practice according to Kim et al (1987b) it is found that both the sill and the range generally increase as the cutoff grade increases, until the median cut-off grade is reached. After that both the sill and range decrease. Kim et al (1987a and b) report that they use the median indicator semivariogram during indicator kriging.

Kim et al (1987a) compared OK and IK, obtaining an absolute semivariogram for OK by transformation from a logarithmic semivariogram. Their results, based on a single cutoff grade, suggest that IK is an improvement on OK, giving a 5% underestimation of contained gold compared with 25% overestimation by OK. However they warn that their test is not comprehensive in that only 1 cutoff grade was compared.

The later paper by Kim et al (1987b) compares IK with lognormal kriging and probability kriging. They conclude that all three methods are viable tools with the comments that:

1. IK is easy to apply.
2. PK, though more involved, gives superior performance.
3. LK is viable provided the data are truly lognormal and the underlying assumptions of the grade distribution are valid.

Probability Kriging (PK)

This is more sophisticated than IK and is also distribution free. Samples are sorted by increasing grade and assigned their respective cumulative frequencies. This is called a rank order transform.

For a given indicator cutoff value both the binomial indicator and the rank order transform is calculated. The semivariogram of the indicator and of the rank order transform variable as well as the cross semivariogram between the indicator and the transformed variable are required.

Co-kriging is performed for a discrete number of cutoffs, n, and 2n + 2 semivariograms in all are required to implement PK. Kim et al (1987b) report that PK gave 0.2% difference, IK, -10% and OLNK -6% difference from actual production in their mean estimation errors for a performance comparison on a gold-silver deposit in Nevada.

Their data was not truly lognormal even using a three parameter lognormal transformation.

Multigaussian Kriging (MK)

MK is another method for estimating recoverable reserves (Verly (1983) and Verly and Sullivan (1985)). It is distribution dependent but is general in that the grade distribution can have any shape. Grades are transformed into a normal score using hermite polynomials.

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The semivariogram of the transformed data is used to krige these scores. An indicator function is set up and the proportion of grades is equal to the proportion of normal scores higher than the transformed cutoff grade. The grade tonnage relationship is built up accordingly.

Lognormal Shortcut Method

Kim et al (1987b) compare OLNK with IK and PK. In order to obtain recoverable reserves the ore grade is assumed to be lognormally distributed with the mean equal to the kriged estimate and the variance equal to the sum of the dispersion and kriging variances.

$\sigma^2 d = \sigma^2 N - \sigma^2 N$ where v = SMU and V = block

The proportion of ore above the cutoff grade and the average ore grade is computed for each block.

This technique is termed the Lognormal Shortcut Method (David, 1988) and can be used to estimate recoverable reserves within blocks kriged by either OK or OLNK.

Selective mining estimates are made by assuming lognormality of SMU grades within kriged blocks. The kriging variance + dispersion variance and the kriged average grade represent the distribution of SMU's within the kriged block.Hence, for any cutoff grade, the lognormal recovery functions are used to calculate the ore grade and ore tonnes above cutoff as shown below:

Tonnage above cutoff

$$T_C = G\left(\frac{1}{\beta} L_n\left(\frac{X_c}{m_o}\right) + \frac{\beta}{2}\right)$$

Proportion above cutoff

$$Q_C = G\left(\frac{1}{\beta} L_n\left(\frac{X_c}{m_o}\right) + \frac{\beta}{2}\right)$$

Grade above cutoff

$$m_C = (Q_c / T_c) m_o$$

Where x_c = cutoff
Mo = mean
G = Function of standard normal distribution
σ^2 = absolute variance

Logarithmic Variance $\beta^2 = L_n (\sigma^2 / M_o^2 + 1)$

PRECIOUS METAL DEPOSITS – EXPERIENCE IN AUSTRALIA

Skewed precious metal deposits have been found to be well represented by the lognormal distribution, particularly if care is taken to remove the spike of near zero values in the histogram from outside the orebody. In these deposits OLNK gives a good representation of the basic orebody geometry and the average grade per block is well defined.

The most important aspect of applying OLNK is the accurate determination of the relevant grade distribution. It is important to recognise the mineralization indicator grade above which the data is lognormally distributed during semivariogram calculation. This is effectively the removal of the spike of near-zero values which do not represent the orebody as such but are merely due to drilling through barren ground. If these are not removed the variance of the distribution predicted by the semivariogram is inflated. Lognormal kriging is sensitive to the total variance and care should be taken in defining the variance correctly.

In practice the mineralization indicator grade is recognised by an inflection near the origin on the cumulative frequency curve. Commonly in 3-5 g/t Au orebodies this value is of the order of 0.1 or 0.2 g/t. Logarithmic semivariograms are the most robust of all semivariograms run on skewed data as the effect of high grade outliers is reduced. Usually the nugget effect is found to be fairly high, about 30-50% of the total sill.

Lognormal kriging using all the data but using the logarithmic semivariogram based only on grades above the mineralization indicator grade is a very good spatial estimator, that is it defines the ore zone geometry much better than ordinary kriging would do. If the distribution parameters are carefully defined as described above it also gives a good measure of the average grade in a block.

The lognormal shortcut for recoverable reserves has proven itself in numerous deposits to be an effective tool. Knowledge of the mining method is required in order to estimate the effective selective mining unit size, (SMU) that is the smallest mineable unit on which ore/waste decisions are based. The dispersion variance, i.e. variance of SMU's within a block, is required in order to estimate the recoverable reserves above a specific cutoff within a kriged block.

The distinct advantage over IK is that it is a one step calculation. Once the average block grade and kriging variance are calculated, the distribution of SMU's is known. For IK this distribution is built up by an IK run for each of a large number of cutoffs. Of course this enhances the flexibility in that it will work for any distribution. If data is known to be lognormal however, OLNK is a perfectly good estimator and more efficient than IK. Its success is linked to a sound appreciation of the physical mining process linked with a good understanding of the grade distribution.

USES AND ABUSES

The underlying safety precaution in reserve estimation is to understand the geology of the deposit. This is often difficult in the early stages of exploration. Semivariogram analysis, as a tool for the geologist, is a powerful aid in confirming his hypothesis. If it is impossible to gauge geological continuity one can not expect the semivariogram to show good structural definition.

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Sometimes the geological continuity is excellent but the semivariogram is pure nugget effect. This situation means that sample grades are randomly distributed at the scale at which samples are spaced. Classical statistics should then be used to estimate global averages since no sample to sample correlation exists. If, however, a semivariogram is defined and has a given range of influence, the confidence in estimation is improved by kriging as opposed to other weighting techniques.

There are tendencies in industry to ignore geostatistics if there is difficulty in obtaining a semivariogram. For example block grades are often estimated by polygons or inverse distance weighting even if the semivariogram is pure nugget effect. This flaunts the observation of ***no*** correlation. Nothing more than a global estimate is valid in this situation. If there is no correlation, spatial estimation is a farce.

Another common approach is to obtain spatial parameters from semivariograms then apply these within an ellipsoidal search using inverse distance weighting because it is quicker (or easier) than kriging. However this will not give the optimum result as kriging uses sample weighting calculated to give the best unbiased estimate.

Of course if the wrong kriging method is used it can do more harm than good. The essential rule is to understand the grade distribution within the orebody and apply the relevant estimation technique for the style of mining.

Bulk Mining

Normal distribution	:	OK
Lognormal distribution	:	OLNK

Selective Mining

Normal distribution	:	OK with LN Shortcut
	:	OLNK with LN Shortcut
Highly skewed, not	:	IK
Lognormal	:	PK
	:	MG

In practice IK is not always applied in the manner described theoretically in the literature. For example, instead of applying all the indicator cutoffs to build up the recovery function, the cutoff of interest is used to krige the ore proportion. Then the actual grades are kriged to obtain a grade estimate. The average block grade is accepted as the grade above cutoff for the indicator ore proportion. This means that the average grade would be used for ***any*** ore proportion. Obviously this does not make sense as the grade should increase as the ore proportion decreases. Ideally the grade above cutoff must be obtained from the grade/tonnage curve built up by kriging the indicator grades for a range of indicator cutoffs. It cannot be estimated for a simple kriging run. Applied correctly it is a viable non-linear estimator.

Probability kriging is not yet widely applied. The software is not common and the complexity of the method puts operational staff off. The available literature does, however, praise it as a distribution-free (non-parametric) non-linear estimator based entirely on the available sample data and, although it needs extra time and effort, is justifiably superior in performance.

Provided data are lognormally estimated, OLNK is both quick and easy as logarithmic semivariograms are generally easier to obtain and one-pass kriging is all that is required. A good knowledge of the grade distribution is essential. The lognormal shortcut gives recoverable reserves for a range of cutoffs in a single run.

MG kriging tends to have been developed in the diamond industry where extremely skewed data are common and is not as yet commonly used in other industries.

COMMENT

Geostatistics is the perfect tool for the earth sciences where spatial correlation is the basis for exploration and evaluation. It quantifies geological interpretation and re-inforces it when used in estimation. Many case studies have highlighted the importance not only of understanding the geological processes and physical mining constraints but also of understanding the statistical and geostatistical features. Mining is multidimensional and it is important to incorporate all the various disciplines in order to achieve the very best solution possible.

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NOTE: The feature article in Newsletter No. 3 will address the important issue of ASSAY DISTRIBUTIONS and the selection of appropriate resource/reserve estimation methods