I dedicate this book to my lovely wife, Katherine, who so unselfishly relinquished much of our time together so that I might spend nights and weekends writing this book. Also, I thank God for the many blessings that He has continued to bestow upon me and my family.

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THE TECHNOLOGY OF ARTIFICIAL LIFT METHODS

Volume 1
Inflow Performance
Multiphase Flow In Pipes
The Flowing Well

Kermit E. Brown
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Preface

When an earlier book, "Gas Lift Theory and Practice," was completed, I promised myself I would never write another book. But here is book #1 of a three-volume series. Although this book stands alone as a text and engineering source book, it is also a prelude to the second text on "Artificial Lift Methods."

In a nutshell, this first book provides all the material needed to design an artificial lift installation.

The first chapter deals with a much-neglected area of petroleum engineering – inflow performance. I thank Mike Fetkovich for permission to use much of his material in this chapter. Also, many of my graduate students, including Mansoor, Yousaf, and Kadi, contributed to this chapter.

The second chapter, dealing with multiphase flow in pipes, was co-authored by Dale Beggs. I extend thanks also to Jim Brill for his many contributions. Much is yet to be learned about multiphase flow and I am proud to be part of the University of Tulsa where research is continuing in this area under the supervision of Dr. James Brill and Dr. Dale Beggs.

Chapters 1 and 2 are used to predict the behavior of a flowing well as found in Chapter 3. Again, I thank my graduate students for their assistance, including Celio Fonseca, Gustavo Lopez, Pedro Regnauld, Hugo Marin, Victor Gomez, and Harry Hong.

As is the case in almost any text, I have leaned heavily on published material in piecing various sections together.

I thank the University of Tulsa for a good ten years and the encouragement that I have received from many people, including President Twyman.

Finally, I thank my family for their dedication and encouragement.

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Chapter 1

Inflow performance

1.1 INTRODUCTION

The inflow performance of a well represents the ability of that well to give up fluids. A typical plot is noted in Figure 1.1 and shows how the shapes of the curves may differ. For example, flowing pressure vs rate may be essentially a straight line (water drive and/or pressure above saturation pressure) or it may curve (solution gas drive and flow below the bubble point). The ability of a well to give up fluids depends to a great extent upon the type of reservoir and drive mechanism, and such variables as reservoir pressure, permeability, etc. It is common practice to assume that inflow into a particular well with constant conditions is directionally proportional to ($\bar{p}_R$). Note curve A in Fig. 1.1 which is a straight line. Normally this is true only for flowing pressures above the bubble point.

For curve A the PI of the well is constant and is represented in Fig. 1.1 by the inverse of the slope of the straight line. PI is defined as barrels of total production per day per psi of pressure drop $\left(\frac{b/d}{psi}\right)$ or symbolically $PI = J = \frac{q_o + q_w}{\bar{p}_R - p_{wf}}$ where q_o = oil flow rate, q_w = water flow rate, $\bar{p}_R$ = average static well bore pressure, p_{wf} = flowing well bore pressure. When the value of this slope is constant the well is said to have a single PI.

However, it is known that curvature exists in this line for many wells. In this case a well cannot be said to have a linear PI (straight line), because the slope varies with the variation in drawdown. (Note curves B and C in Fig. 1.1). Two field cases are shown in Figs. 1.2 and 1.3. Fig. 1.2 shows three flow tests taken on a solution gas drive well with flowing pressures below the bubble point. Fig. 1.3 shows three flow tests that exhibit a straight line with flow being above the bubble point.

The IPR (Inflow Performance) and PI are not equivalents. The IPR is the relationship between flowing pressure and rate. The PI is the first differential of the IPR in the special case where the IPR is a straight line, or is close enough to being straight that curvature can be neglected.

The ability to predict inflow performance is further complicated because the inflow performance curve and PI may also change with cumulative production and again depends upon the type of reservoir.

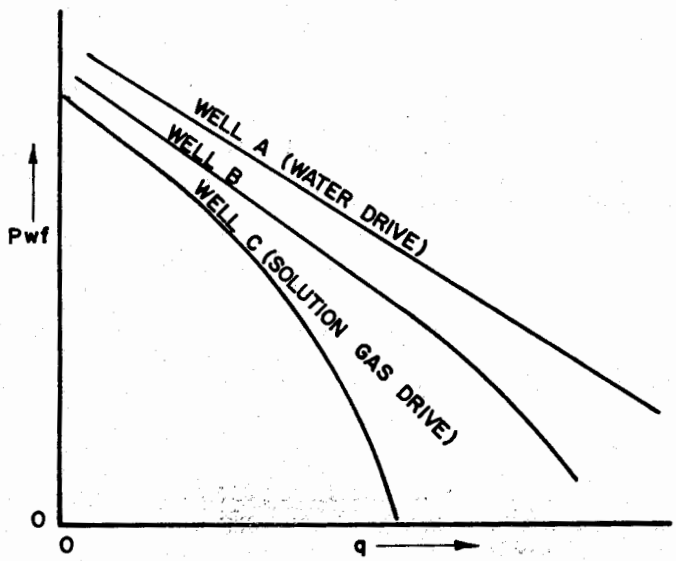


Fig. 1.1 Typical inflow performance curves.

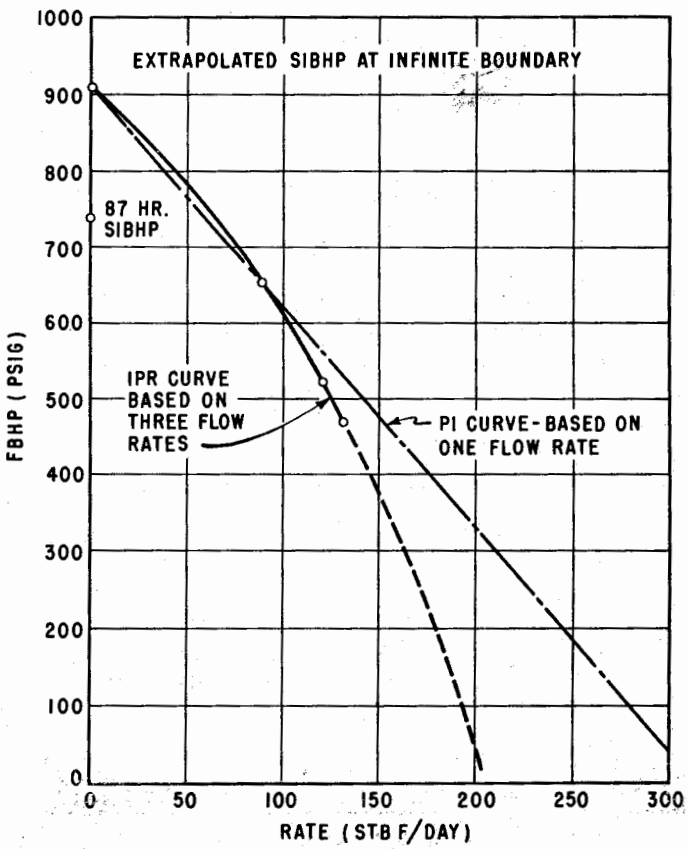


Fig. 1.2 Inflow performance curve (after ref. 3).

Data from real reservoirs[1] are noted in Fig. 1.4.

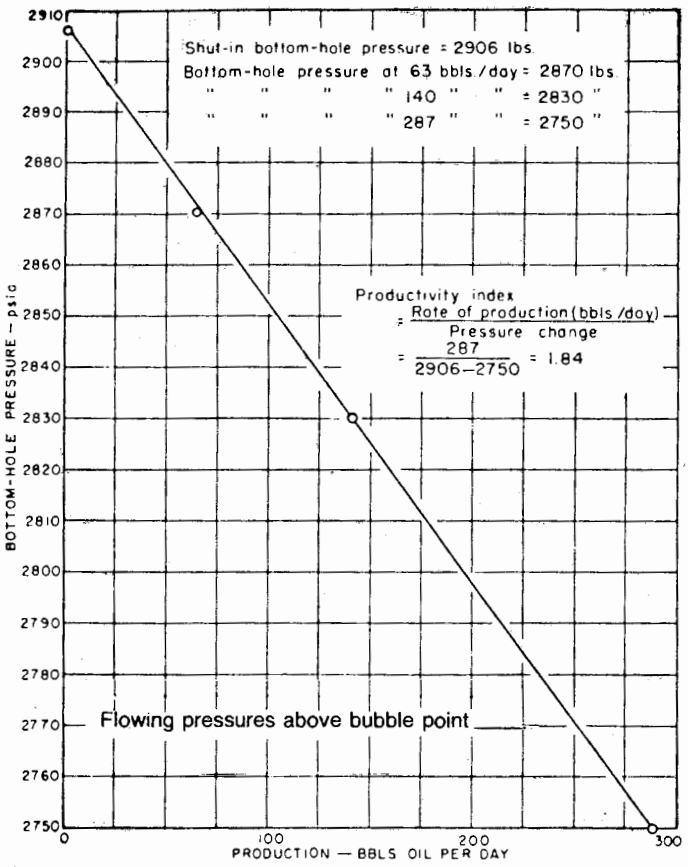


Fig. 1.3 PI test—solution gas drive reservoir.

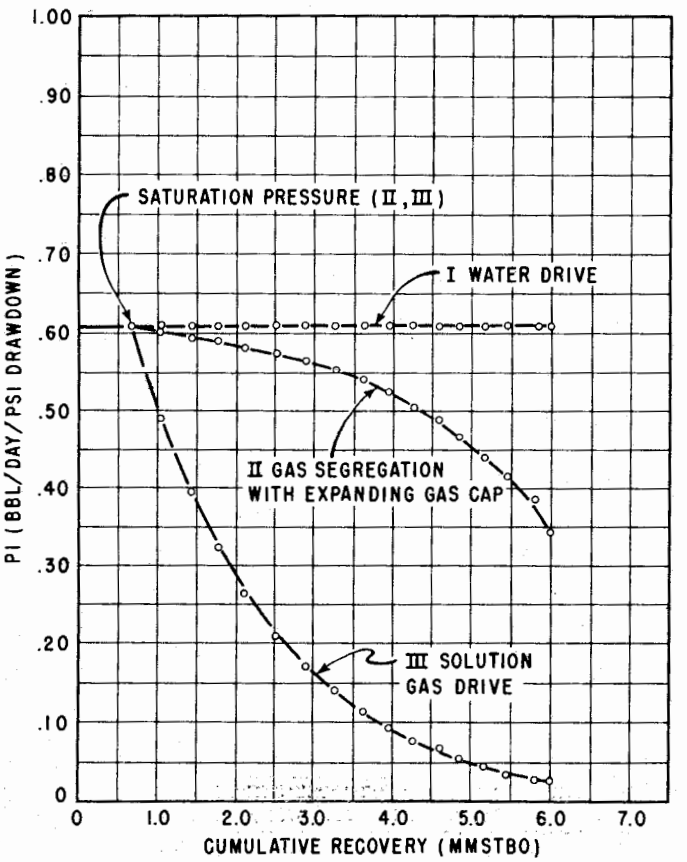


Fig. 1.4 PI vs. recovery for different types of reservoirs (after ref. 3).

For a very active water drive in which the pressure remains above the bubble point, the PI remains constant, and for a solution gas drive in which the flowing pressures are below the bubble point the PI changes rapidly. If an inflow curve is constructed at any time in the life of the reservoir of Fig. 1.4 (say 3,000,000 stk bbl oil) the q vs. p_{wf} curve will probably be a straight line for the water drive reservoir and curved for the solution gas drive reservoir. The PI will probably be high for the water drive with very little drop in average reservoir pressure at the high rates.

In order to better understand inflow performance, a brief discussion on types of reservoirs with a more detailed discussion on inflow performance follows.

1.2 TYPES OF RESERVOIRS

In order to properly design an artificial lift installation, an understanding of the reservoir drive mechanism is important. The type of reservoir will materially influence the production rate, hence the type of artificial lift installation. There are generally conceded to be three basic reservoir types with possible two- and three-way combinations of these three. A brief discussion of each follows:

1.21 Solution gas drive (Fig. 1.5)

This type of reservoir drive may also be referred to as internal gas drive, depletion drive, and/or volumetric performance. Some of the associated characteristics are:

(1) A constant volume. This means that there is no change in the initial size of the reservoir. There is no water encroachment for this particular type of drive mechanism.

(2) There is two-phase flow at pressures below the bubble point. In other words, gas coming out of solution flows along with the oil.

(3) The gas comes out of solution but does not move upward to form a gas cap. Gas bubbles formed in the oil phase remain in the oil phase, resulting in simultaneous flow of both oil and gas.

(4) Oil production is the result of the volumetric expansion of the solution gas and the volumetric expulsion of oil.

(5) This type of reservoir drive mechanism approaches a flash gas-liberation process.

In the early production life of the reservoir, oil is replaced by gas on an equal volume basis, but as pressure declines, a larger gas phase develops. More gas expansion is then required per unit volume of oil produced because of the free-flow ability of the gas phase.

It is known that creating excessive drawdown in a depletion drive reservoir results in an increased gas phase in the reservoir—an increased permeability to gas and decreased permeability to oil.

Fig. 1.5 shows typical performance curves for this type of reservoir. Of particular importance is the rapid decline in pressure, the rapid decline in PI and the increase in the gas-oil ratio.

In general, wells in this type of reservoir can expect to be low rate producers in their latter life unless they

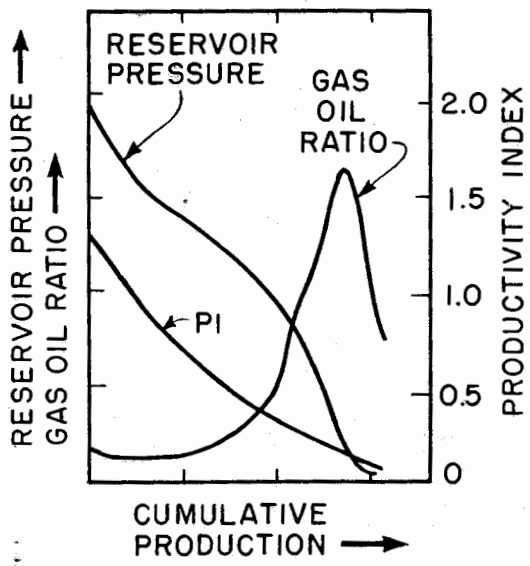


Fig. 1.5 Typical performance for a solution gas drive field.

have long pay intervals such as the Cook Inlet in Alaska. Many wells of this type will be candidates to produce less than 100 bpd or perhaps less than 25 bpd.

1.22 Water drive (Fig. 1.6)

The water drive mechanism may also be referred to as water encroachment or hydraulic control. Some of the associated characteristics are:

(1) The reservoir volume for oil does not remain constant. Water encroaches, changing the initial volume of the container (reservoir).

(2) There is a displacement of the oil by water.

(3) This reservoir type could also have a gas phase, resulting in a combination water depletion drive.

(4) There will be an optimum rate of production for this reservoir type.

Fig. 1.6 shows typical performance curves for this type of reservoir. In a very active water drive the pressure decline may be very small, and in fact, pressure may remain constant. Of great importance is the trend of the PI to remain constant over the life of the well. In turn the GOR also remains constant.

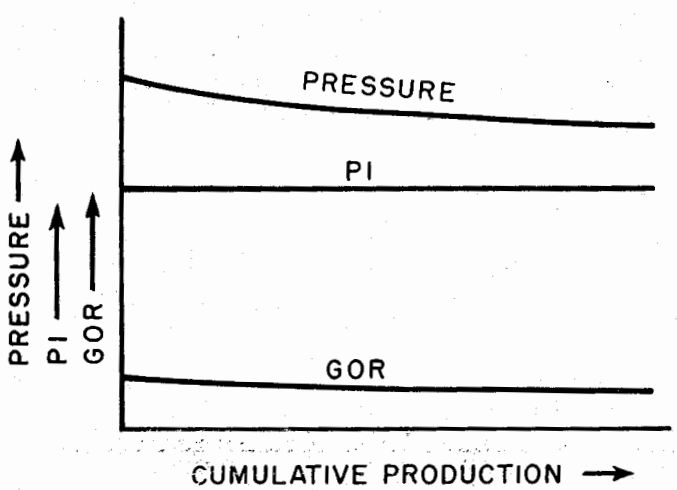


Fig. 1.6 Typical performance for a water drive field for a low production rate.

In a water drive reservoir the PI's of individual wells are normally more reliable than those of a depletion drive reservoir. Water encroachment may be such that there is very little loss in bottom hole pressure. It is generally conceded that PI information may be extrapolated linearly for drawdowns necessary to give the desired production. Probably, in most cases, the pressure remains above the bubble point. Most artificial lift installations can be designed with more reliability and confidence for water drive than for any other type of drive mechanism. However, there are instances where an increase in water-cut causes a decrease in PI.

1.23 Gas cap expansion drive (Fig. 1.7)

This type of reservoir drive mechanism may also be referred to as segregation or gravity drainage. The reservoir is in a state of segregation – an oil zone overlain by a gas cap. The drive may be further classified as to whether or not gas coming out of solution in the reservoir flows to the gas cap. A segregation drive with counterflow will have gas coming out of solution and moving to the gas cap. As production proceeds the gas cap expands and moves down, resulting in gas cap expansion drive.

Generally, the permeability of the formation determines whether or not counterflow will occur. As an estimate, it would be expected to occur for permeabilities in excess of 100 millidarcies.

The segregation drive with counterflow approaches a differential gas liberation process, defined as a process in which the gases liberated from solution in the oil when the pressure is reduced are removed from contact with the oil as rapidly as they are formed.

In Fig. 1.7 the performance curves appear to be somewhere between those for solution gas drive and water drive. In general the pressure declines fairly rapidly and the PI follows the same trend.

1.24 Summary

There are many reservoirs having combination drive mechanisms and their performance may differ considerably from the typical curves given for water drive, solution gas drive, or gas-cap expansion drive mechanisms. An effort should be made to identify the

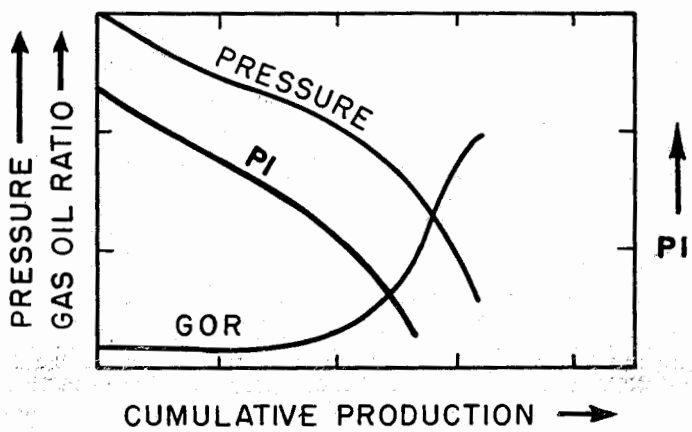


Fig. 1.7 Typical performance for "gas cap expansion drive."

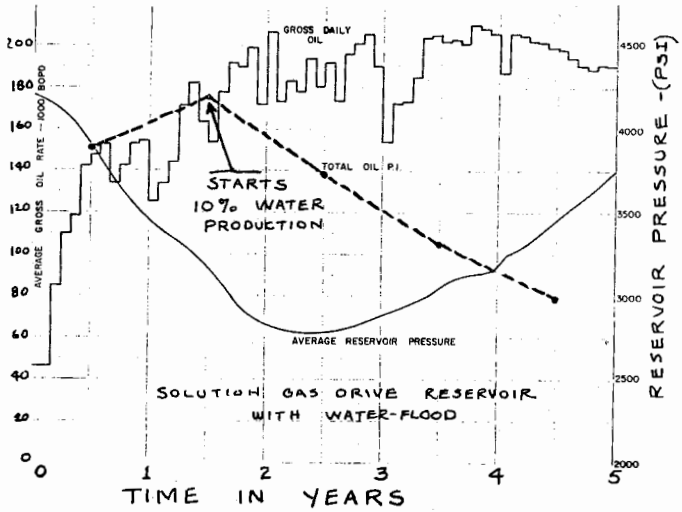


Fig. 1.8 Effect of water on total oil PI.

reservoir drive mechanism in order to permit a better determination of the ability of the well to give up fluids. There is no substitute for good data taken during the early life of the reservoir in predicting future performance.

Several field cases for reservoirs are shown in Figs. 1.8, 1.9, 1.10, and 1.11. These examples were taken from reference (1).

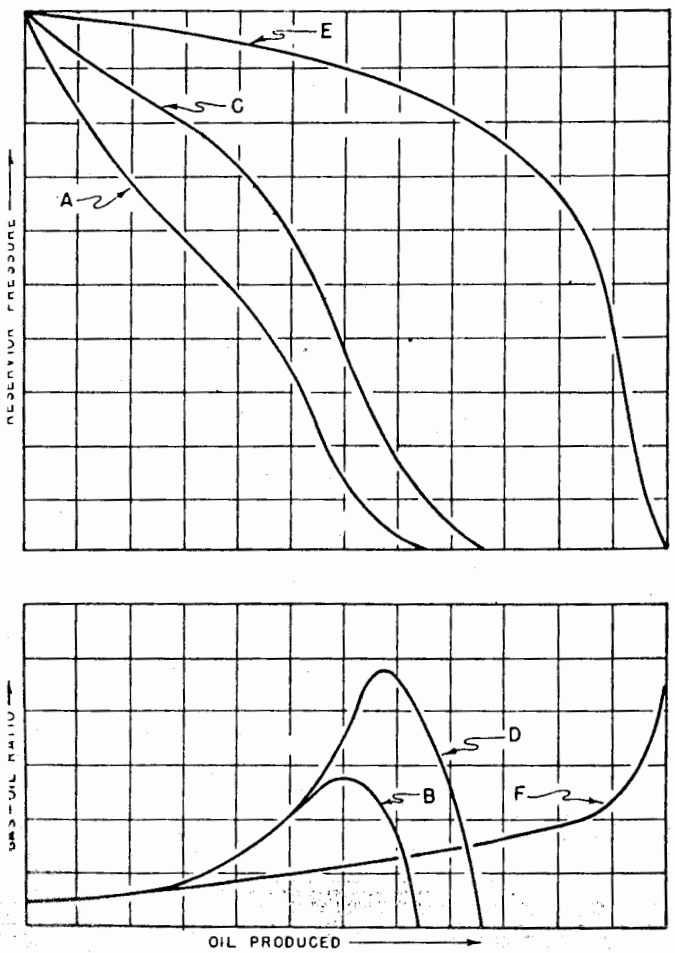


Fig. 1.9 Solution gas drive reservoirs (after ref. 1).

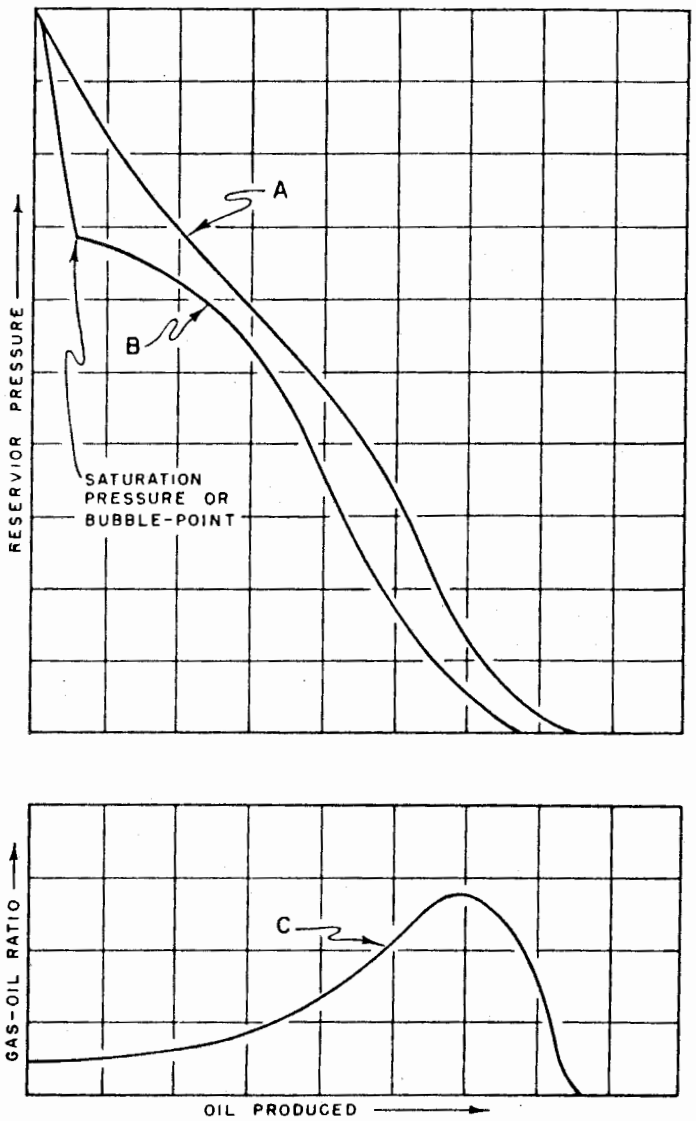


Fig. 1.10 Solution gas drive reservoirs (after ref. 1).

1.3 INFLOW PERFORMANCE RELATIONSHIPS

1.31 Introduction

In discussing inflow performance relationships we must keep in mind the type of reservoir, and the shape of the IPR curve, which is a plot of flowing pressure vs rate (Fig. 1.1). We must also recall that the inflow performance of a well is very likely to change with time and cumulative production.

1.32 Productivity index

The commonly-used term, PI (productivity index), represents one point on the inflow performance curve. The PI is defined as $q/\Delta P$ in bpd per psi pressure drop from static reservoir pressure to flowing bottom hole pressure.

In the design of artificial lift installations the productivity index is expressed in b/d of total liquid (oil + water) and is defined as:

$$J = \frac{q_o + q_w}{\bar{p}_R - p_{wf}} \qquad (1.1)$$

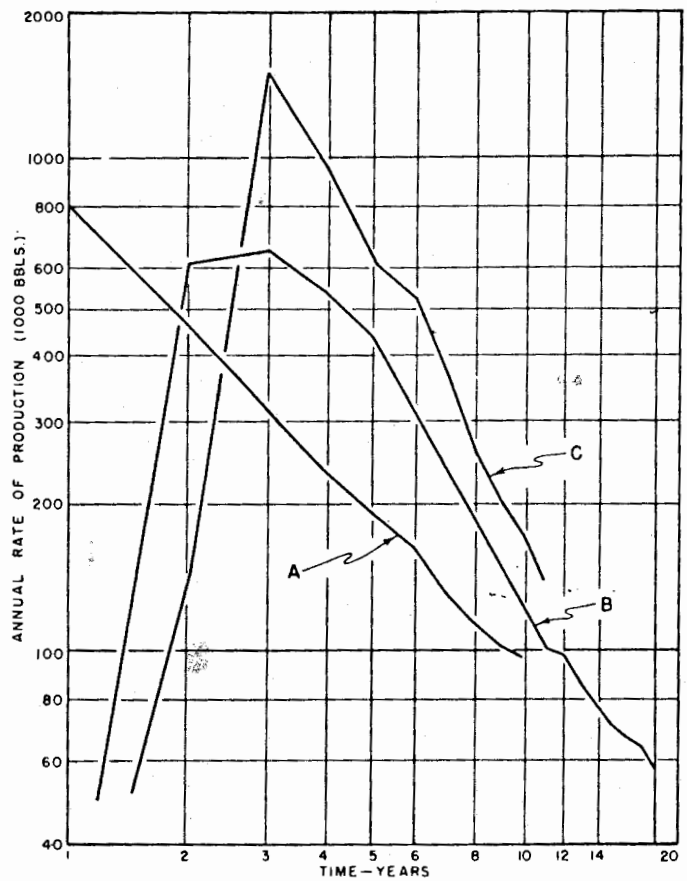


Fig. 1.11 Solution gas drive reservoirs (after ref. 1).

where:

q_o = stock tanks bbls of oil per day
q_w = stock tanks bbls of water per day
p_{wf} = bottom hole flowing pressure, psi
$\bar{p}_R$ = static pressure, psi

Fig. 1.12 shows PI in an ideal case where PI is tan $\theta = \frac{OB}{OA}$.

In Fig. 1.13 we note a case where a straight line relationship does not exist, representing flow below the bubble point pressure.

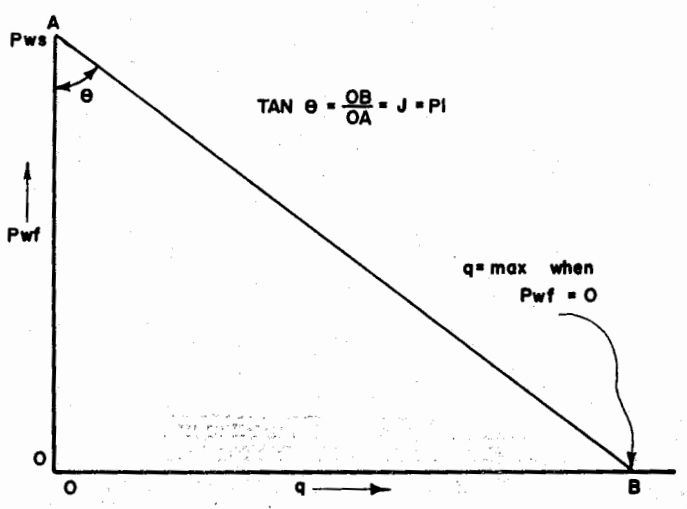


Fig. 1.12 Ideal case for PI.

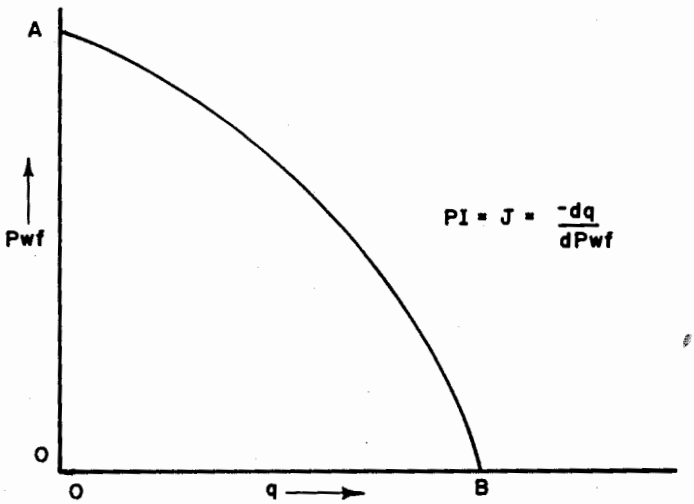


Fig. 1.13 Actual case for PI.

We note that $PI = J = \frac{-dq}{dp_{wf}}$ (1.2)

where the negative sign indicates a decreasing PI for an increasing rate.

EXAMPLE PROBLEM #1: HOW TO DETERMINE PI (LINEAR CASE)

Given: $\bar{p}_R = 2{,}400$ psi
$q_o = 200$ bpd
$q_w = 300$ bpd
$p_{wf} = 2{,}200$ psi

Find the PI assuming it to be a linear relationship (ideal case)

$$PI = J = \frac{q_o + q_w}{\bar{p}_R - p_{wf}} = \frac{200 + 300}{2{,}400 - 2{,}200} = 2.5 \text{ bpd/psi}$$

CLASS PROBLEM #1-A: TO DETERMINE PI (LINEAR CASE)

Given: $\bar{p}_R = 2{,}800$ psi
$p_{wf} = 2{,}200$ psi
$q_o = 600$ bpd
$q_w = 1{,}000$ bpd

Find the PI average for this flow test.

It is common practice to measure one or two PI's in the early life of a well, then use that same PI to estimate drawdowns necessary for greater production rates as well as assuming that the same PI exists later in the life of the well. We are probably fairly safe in doing this for wells in a water drive field where the flowing pressure is above the bubble point. But this may be in error for wells in a solution gas drive reservoir or at flow below the bubble point.

Another term sometimes used is specific PI. This is the productivity index divided by the net feet of pay. It is commonly used to compare different wells in a given field.

$$J_s = \frac{J}{h} = \frac{q_{sto}}{h(\bar{p}_R - p_{wf})} = \text{bpd/psi/ft} \qquad (1.3)$$

EXAMPLE PROBLEM #2: HOW TO DETERMINE SPECIFIC PI

Given: $\bar{p}_R = 2{,}500$ psi
$p_{wf} = 2{,}200$ psi
$q_o = 200$ bpd
$h = 20$ ft

Find the specific PI:

$$J_s = \frac{J}{h} = \frac{q_{sto}}{h(\bar{p}_R - p_{wf})} = \frac{200}{(20)(2{,}500 - 2{,}200)} = 0.0333 \frac{\text{bpd/psi}}{\text{ft}}$$

CLASS PROBLEM #2-A: TO DETERMINE SPECIFIC PI

Given: $\bar{p}_R = 3{,}000$ psi
$p_{wf} = 2{,}800$ psi
$q_o = 1{,}000$ bpd
$h = 50$ ft

Find the specific PI.

The PI refers to surface production of liquids and to andface pressure differences.

In reservoir engineering the PI based on surface oil production only is sometimes used. For flowing and artificial lift wells, it is necessary to lift the total liquid from the well, and therefore the PI equation should include the producing water/oil ratio:

$$J = \frac{q_o + q_o F_{wo}}{(\bar{p}_R - p_{wf})} \tag{1.4}$$

where F_{wo} = water oil ratio. This is the same as equation 1.1 given again as $J = \frac{q_o + q_w}{(\bar{p}_R - p_{wf})}$, and is more commonly used in this form.

The variables which affect the PI can easily be seen by deriving the PI equation using Darcy's radial flow equation. If both water and oil are flowing in the reservoir, the radial flow equations for each fluid must be used:

Oil

$$q_o = \frac{7.08 K_o h(\bar{p}_R - p_{wf})}{B_o \mu_o \ln(r_e/r_w)} \tag{1.5}$$

Water

$$q_w = \frac{7.08 K_w h(\bar{p}_R - p_{wf})}{B_w \mu_w \ln(r_e/r_w)} \tag{1.6}$$

The PI based on total liquid production can then be calculated as follows:

$$J = \frac{q_o + q_w}{(\bar{p}_R - p_{wf})} = \frac{7.08(\bar{p}_R - p_{wf})h}{(\bar{p}_R - p_{wf})\ln(r_e/r_w)}\left[\frac{K_o}{B_o\mu_o} + \frac{K_w}{B_w\mu_w}\right] = \frac{7.08h}{\ln(r_e/r_w)}\left[\frac{K_o}{B_o\mu_o} + \frac{K_w}{B_w\mu_w}\right] \tag{1.7}$$

Although the pressure terms drop out, the PI remains dependent on pressure since the viscosities and formation volume factors are functions of pressure. The effective permeabilities are also functions of the fluid saturations.

1.321 Estimated productivity index

An estimate of the PI can be obtained from the following approximation:

Let $\frac{7.08}{\ln r_e/r_w} = 1.0$

For a well bore radius of 2.5 in. this represents a radius of drainage of 247 ft, and for 3.5 in. it is 346 ft.

then: $$J = h\left[\frac{K_o}{B_o\mu_o} + \frac{K_w}{B_w\mu_w}\right] \tag{1.8}$$

For the special case of negligible water production ($K_w = 0$) the preceding equation becomes:

$$J = \frac{hK}{B_o\mu_o} \tag{1.9}$$

EXAMPLE PROBLEM #3: HOW TO ESTIMATE PI

An 8,000 ft well on 40 acre spacing has a net pay thickness of 20 ft. The following additional data are known:

$\bar{p}_R = 4{,}000$ psi
$p_{wf} = 3{,}800$ psi
$r_w = 3$ in. (0.25 ft)
$B_o = 1.25$
$B_w = 1.05$
$\mu_o = 10$ cp
$\mu_w = 0.8$ cp
$K_o = 2$ darcies
$K_w = 0.01$ darcies
$q_o = 550$ bpd
$q_w = 50$ bpd

Find: (1) the estimated productivity index neglecting water production; (2) the estimated productivity index including water production; (3) the actual productivity index of the well, based on the radial flow equation and a radius of drainage equivalent to a 40-acre circle; and (4) PI based on flow test.

Solution

(1) $J = \frac{hK}{B_o\mu_o} = \frac{(20)(2)}{(10)(1.25)} = 3.2$ b/d/psi

(2) $J = h\left[\frac{K_o}{B_o\mu_o} + \frac{K_w}{B_w\mu_w}\right] = (20)\left[\frac{2}{(10)(1.25)} + \frac{0.01}{(0.8)(1.05)}\right] = 3.44$ b/d/psi

(3) $J = \frac{7.08}{\ln(r_e/r_w)} h\left[\frac{K_o}{B_o\mu_o} + \frac{K_w}{B_w\mu_w}\right] = \frac{7.08}{\ln(r_e/r_w)}(3.44) = \frac{24.37}{\ln(r_e/r_w)}$

Determination of r_e:

$\pi r_e^2 = (40 \text{ acre})(43{,}560 \text{ sq ft/acre})$
$r_e = 745$ ft

$$J = \frac{24.37}{\ln(745/0.25)} = 3.04 \text{ bpd/psi}$$

(4) PI based on flow test $= \frac{550 + 50}{4{,}000 - 3{,}800} = 3.0$ bpd/psi

CLASS PROBLEM #3-A: TO ESTIMATE PI

Given:

Depth = 10,000 ft	$B_w = 1.04$
Spacing = 60 acres	$\mu_o = 3$ cp
h = 30 ft	$\mu_w = 0.7$ cp
$\bar{p}_R = 3{,}800$ psi	$K_o = 1.9$ darcies
$p_{wf} = 3{,}765$ psi	$K_w = .02$ darcies
Well bore diameter = 5 in.	$q_o = 500$ bpd
$B_o = 1.23$	$q_w = 25$ bpd

Find:

(1) Estimated PI neglecting water production
(2) Estimated PI including water production
(3) PI for the well based on all available data and assuming a circular drainage area for 60 acres
(4) PI based on flow test

Lewis, Horner, and Stekoll[2] showed that the productivity index could be related by the following equation:

$$PI = J = 5.9 \times 10^{-4} \frac{Kh}{\mu_o B_o} \text{ for } K = \text{md} \qquad (1.10)$$

This can be approximated as:

$$J = \frac{0.6\ Kh}{\mu_o B_o} \qquad (1.11)$$

K = darcies
h = ft
μ_o = centipoise
B_o = formation volume factor

Lewis and Horner[2] presented a certain amount of data where they compared measured values of PI against calculated values of PI. The data for calculated vs. measured productivity indices are included in Tables 1.1 and 1.2.

In Fig. 1.14 they plotted $\frac{Kh}{\mu B}$ against values of measured PI. Line A is drawn through points that represent wells in which the average reservoir pressure was relatively high at the time of test, and flow was probably above the bubble point. Line B represents data from wells with a low bottom hole pressure with flowing pressures below the bubble point. An average gas saturation of 11.5% was estimated and a relative permeability to oil of 51% was determined. Applying these values, Fig. 1.15 was prepared and now both sets of points are in agreement. Line A of Fig. 1.15 represents a correlation constant of $5.9 \times 10^{-4} \frac{Kh}{\mu B}$ where K = md, h = ft, μ = cp, and $PI = \frac{\text{bpd}}{\text{psi}}$. $B = \frac{\text{bbl}}{\text{stk bbl}}$

Figs. 1.14 and 1.15 also show two theoretical lines as calculated from the radial flow equation (equation 1.5) for values of $\frac{r_d}{r_w} = 4{,}000$, and $\frac{r_d}{r_w} = 400$. For an r_w value of 3 in. these values correspond to drainage *radii* of 1,000 and 100 ft respectively. The position of Line A is somewhat above the theoretical lines, therefore, the measured values of PI are less than would be calculated from the radial flow equation. Depending upon r_e/r_w the correlation constant may differ from 5.9×10^{-4}.

EXAMPLE PROBLEM #4: HOW TO DETERMINE PI BY METHOD OF LEWIS AND HORNER

Lewis and Horner[2] gave the following field example that was checked against the measured PI from a flowing pressure test:

Kh = 11,545 millidarcy-ft
μ_o = 0.44 cp (reservoir conditions)
B_o = 1.48

$$PI = \frac{(5.9 \times 10^{-4})(11{,}545)}{(0.44)(1.48)} = 10.5 \text{ bpd/psi}$$

TABLE 1.1
DATA FOR CALCULATING PRODUCTIVITY INDEX[a]

Well no.	Average permeability × thickness, Kh, millidarcy-ft.	Measured reservoir viscosity, μ centipoises	Measured formation-volume factor, B_o	Perforated or open hole	Measured productivity index	Calculated productivity index[b]
1	16,100	0.614	1.33	P	19.0	11.7
2	15,540	0.614	1.33	P	15.4	11.2
3	14,400	0.614	1.33	P	11.9	10.4
4	11,545	0.44	1.48	P	9.5	10.5
5	7,800	0.614	1.33	P	6.3	5.6
6	8,330	3.42	1.02	OH	0.81	0.7
7	6,910	3.42	1.02	OH	0.64	0.66
8	8,980	3.42	1.02	OH	0.62	0.77
9	5,200	3.42	1.02	OH	0.52	0.45
10	3,360	3.42	1.02	OH	0.28	0.29
11	2,500	3.42	1.02	OH	0.17	0.21
12	1,830	3.42	1.02	OH	0.12	0.16
13	575	3.42	1.02	OH	0.043	0.050
14	400	1.60	1.20	OH	0.105	0.126

[a] Points listed are plotted as circles on Figs. 1.14 and 1.15
[b] Calculated by using line A on Fig. 1.15
[c] Wells 6 through 13 had gas saturation of 11.5% of pore space when measurements were made. Effective permeability of 51% was used in calculations.

TABLE 1.2
ADDITIONAL PRODUCTIVITY INDEX DATA[a]

Well no.	Average permeability × thickness, Kh, millidarcy-ft.	Reservoir viscosity, μ Centipoises	Formation-volume factor, B_o	Measured productivity index	Calculated productivity index[c]
15	11,800	0.80[b]	1.45	6.0	6.1
16	3,179	0.80[b]	1.45	2.1	1.62
17	2,516	0.80[b]	1.45	1.7	1.30
18	15,000[b]	1.5	1.2	5.0	4.92
19	28,000[b]	1.5	1.2	6.5	9.2
20	24,000[b]	1.5	1.2	5.4	7.87
21	150,000[b]	1.5	1.2	53.0	49.0
22	70,000[b]	1.5	1.2	29.0	23.0
23	23,000[b]	2.5	1.1	5.11	4.93
24	22,000[b]	0.5[b]	1.26	16–19	20.6
25	2,630	0.53[b]	1.48[b]	1.22	1.97

[a] Points listed are plotted as triangles on Figs. 1.14 and 1.15
[b] Numerical value represents estimation required because of incomplete core recovery or lack of bottom-hole sample data. Other values, except calculated productivity index, were measured.
[c] Calculated by using line A on Fig. 1.15

The PI of this well was found to be 9.5 by conducting est on the well.

CLASS PROBLEM #4-A

Given: $K_o = 2$ darcies
$h = 40$ ft
$\mu_o = 0.8$ cp
$B_o = 1.35$

termine the PI by the method of Lewis and Horner. We have defined PI as follows:

$$J = \frac{q_L}{(\bar{p}_R - p_{wf})}$$

This represents an average PI between the static reservoir pressure and the stabilized flowing pressure for q_L. This definition assumes the PI is a straight line relationship, and in turn that a plot of q vs. p_{wf} will yield a straight line relationship. Although this may be true in many cases for flow above the bubble point, there are numerous solution gas drive reservoirs in which this will not be true. We must keep in mind that this also represents a PI test on one specific day in the life of the well. This may also change with time and cumulative recovery as discussed in the following section.

1.322 Productivity index change with time

It is known that the PI changes with cumulative recovery as noted in Fig. 1.4. There is a decided change

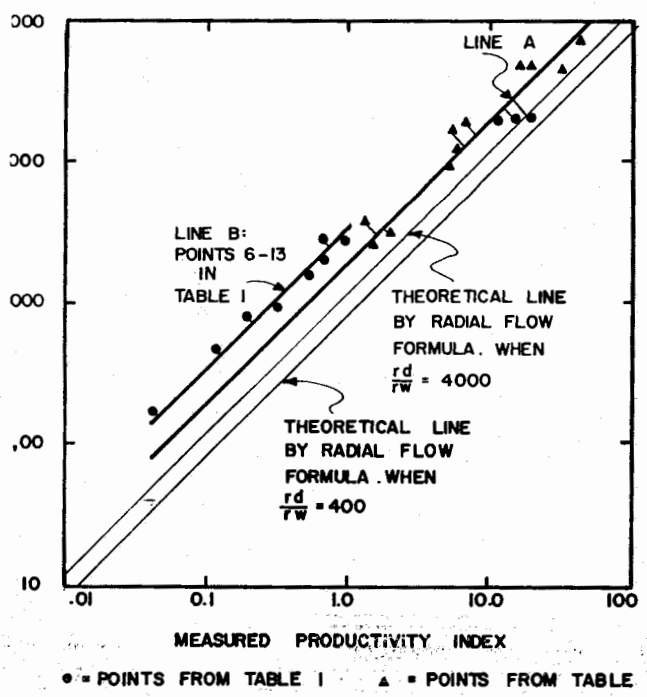


Fig. 1.14 Correlation of reservoir data with productivity index (from Lewis and Horner).

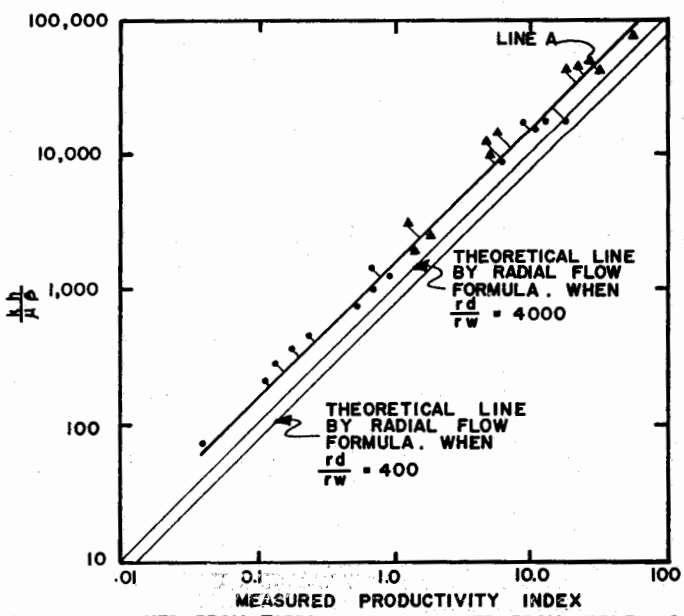


Fig. 1.15 Correlation of reservoir data with productivity index (from Lewis and Horner).

for a solution gas drive reservoir and a gas cap expansion drive. The water drive reservoir shows a constant PI with time, but also changes in some cases due to changes in permeability to oil and water, and when flow is below the bubble point pressure.

A method for predicting the PI in the future[3] was noted by making a semi-log plot of Fig. 1.4 as shown in Fig. 1.16. As noted, the water drive plot remains a straight line, and the points for the solution gas drive also give essentially a straight line. Some caution is necessary in trying to obtain meaningful PI's, that is, tests taken at the same drawdown in pressure. The PI's may vary depending upon the drawdown in pressure, and hence the flowing pressure.

The gas cap expansion also approaches a straight line relationship. This represents one method that can be utilized in predicting future PI's.

This is further illustrated for a solution gas drive reservoir in Figs. 1.17 and 1.18. In Fig. 1.17 the PI changes from 2.0 to 0.4 with less than 20,000 bbls cumulative production, and the extrapolation of this curve becomes difficult. When the same three points are plotted on Fig. 1.18 (semi-log plot) a straight line results, giving a means of predicting PI's at future dates with some degree of confidence[2]. Although this method worked very well for this particular well, it may not be applicable in other cases. It is an empirical procedure, but may very well apply to other solution gas drive wells.

Most of this change in PI is likely caused by an increased free gas saturation around the well bore which

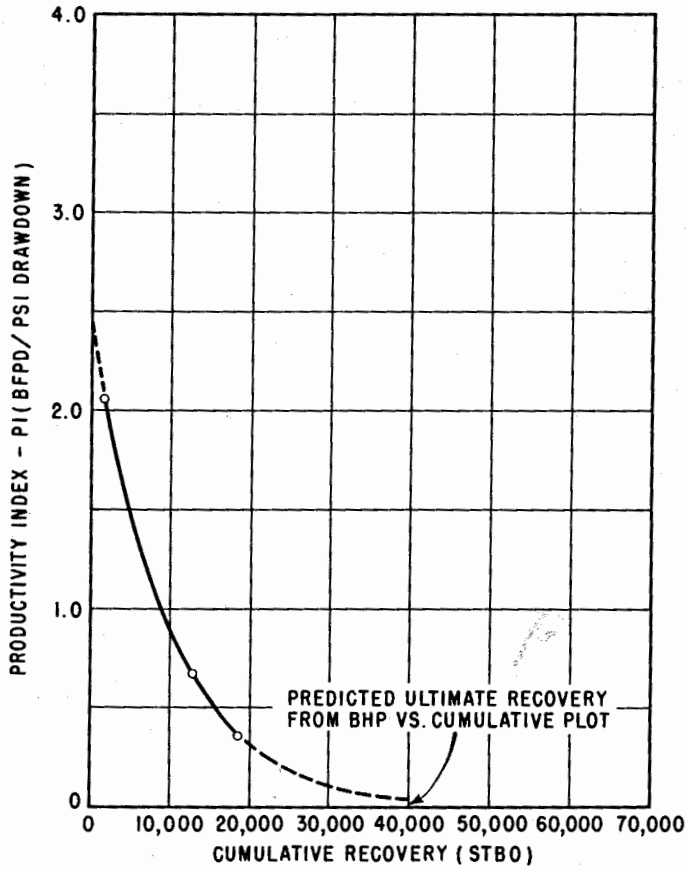


Fig. 1.17 PI change vs. cumulative recovery (after ref. 3).

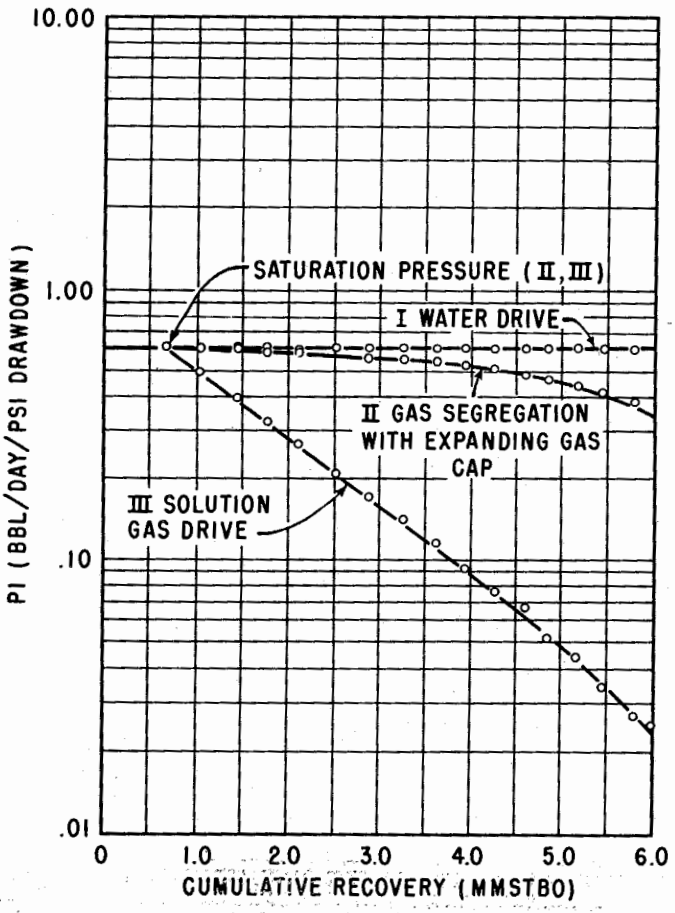


Fig. 1.16 PI vs. recovery for different types of reservoirs (after ref. 3).

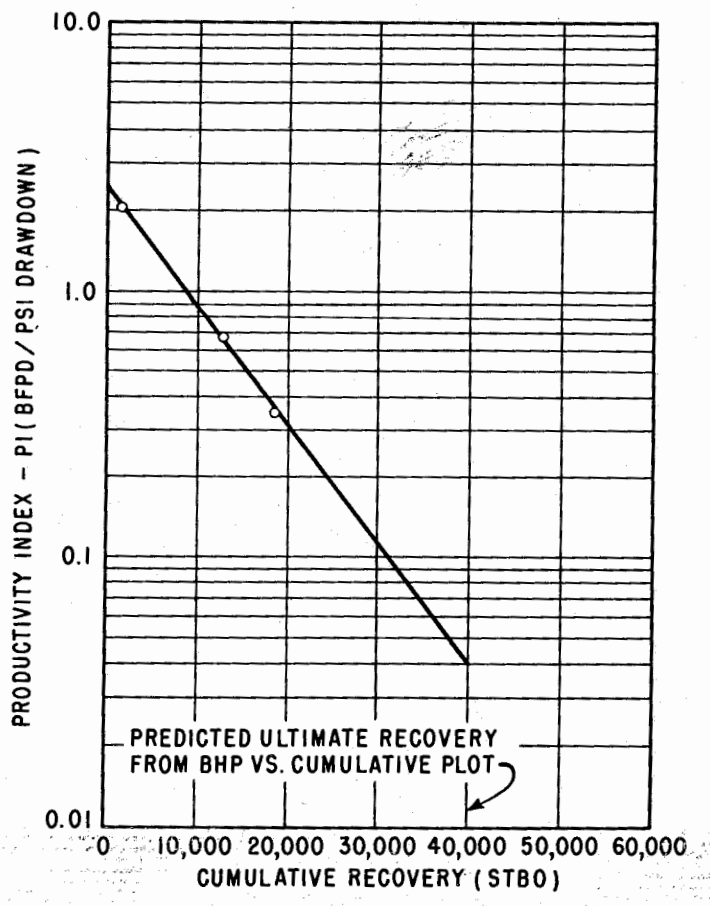


Fig. 1.18 PI change vs. cumulative recovery (after ref. 3).

ıcreases the permeability to gas and decreases the ɜrmeability to oil. Other possibilities are increased l viscosity with pressure drop below the bubble ɔint, and reduction in permeability due to formation ɔmpressibility.

33 Some early discussion on PI

Some of the first work on PI's and change in PI's as done by Evinger and Muskat[4] in 1942. This work as also discussed by Calhoun[5] in his book on reservoir ıgineering. They noted that a plot of flowing pressure ɜainst rate was not always a straight line.

Starting with the radial flow equation as follows:

$$q_o = \frac{7.08\ r\ h\ K_o\ dp}{\mu_o\ B_o\ dr} \qquad (1.13)$$

Equation 1.13 can be rearranged as follows by de-ning $K_{ro} = \frac{K_o}{K}$:

$$\frac{q_o}{7.08\ Kh}\int_{r_w}^{r_e}\frac{dr}{r} = \int_{p_w}^{p_e}\frac{K_{ro}}{\mu_o B_o}\,dp \qquad (1.14)$$

here $\frac{dr}{r}$ can be integrated between limits and the ɜrm $\frac{K_{ro}}{\mu_o B_o}$ can be evaluated as a function of pressure ɑd integrated.

At any one time we have a constant gas oil ratio ɜ given by the following equation:

$$R = \frac{K_g}{K_o}\,\frac{\mu_o}{\mu_g}\,\frac{B_o}{B_g} + R_s \qquad (1.15)$$

here R = current producing gas oil ratio and R_s = gas ı solution at current pressure.

For a given R value a plot of oil saturation vs pres-ıre can be made such as Fig. 1.19.

From Fig. 1.19 the values of K_g/K_o can be deter-ıined from the oil saturation and the values of $\frac{\mu_g}{\mu_o}$, B_o, B_g, and R_s can be determined by knowing the pressure, temperature, and fluid properties. Once we have the pressure vs. saturation correlation, the K_{ro} values can be determined for the saturation at a given pressure and the term $\frac{K_{ro}}{\mu_o B_o}$ can be evaluated and plotted against pressure as in Fig. 1.20.

The right hand side of equation 1.14 can now be integrated graphically by taking the area between values of p_1 and p_2 under the curve as shown in Fig. 1.20. Therefore, the equation then appears as follows:

$$q_o = \frac{7.08\ Kh}{\ln\frac{r_e}{r_w}}\ (\text{area under curve}) \qquad (1.16)$$

Evinger and Muskat[4] stated that the PI could be expressed in terms of three parameters; (1) the producing gas oil ratio, (2) the reservoir pressure, and (3) the pressure gradient in the well system.

The following equation expresses oil flow in the reservoir:

$$q_o = \frac{7.08\ Kh}{\ln\frac{r_e}{r_w}}\int_{p_w}^{p_e}\frac{K_{ro}}{\mu_o B_o}\,dp \qquad (1.17)$$

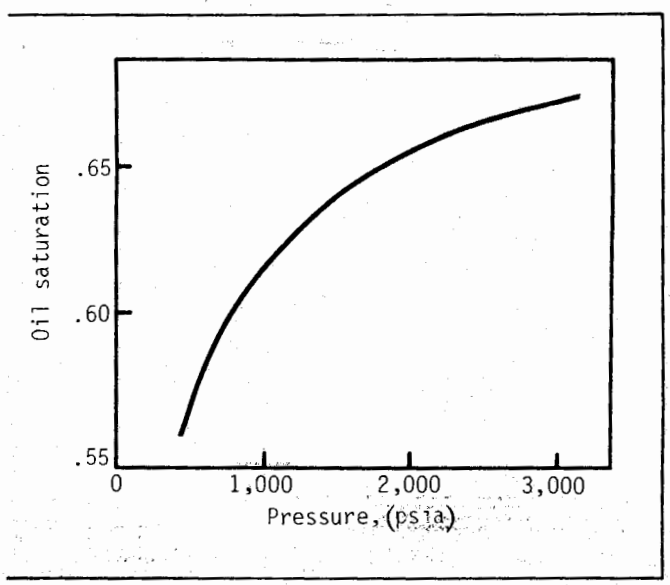
The integral can be evaluated as shown in Fig. 1.21 by finding the area under the curve between any two pressure points. The PI can then be determined from the equation:

$$PI = \frac{q_o}{\bar{p}_R - p_{wf}} = \frac{7.08\ Kh\ (\text{area under curve})}{(\bar{p}_R - p_{wf})\ln\frac{r_e}{r_w}} \qquad (1.18)$$

Several factors are noted from an examination of equation 1.18, and Fig. 1.21:

(1) The PI will not double if $(\bar{p}_R - p_{wf})$ doubles because the area under the curve will not double.

(2) If a constant value of $(\bar{p}_R - p_{wf})$ is taken at a high pressure as compared to a low pressure, the area will be greater at the high pressure. Therefore the PI will be greater at higher reservoir pressures and low drawdowns.



ig. 1.19 Oil saturation vs. pressure (after Calhoun).

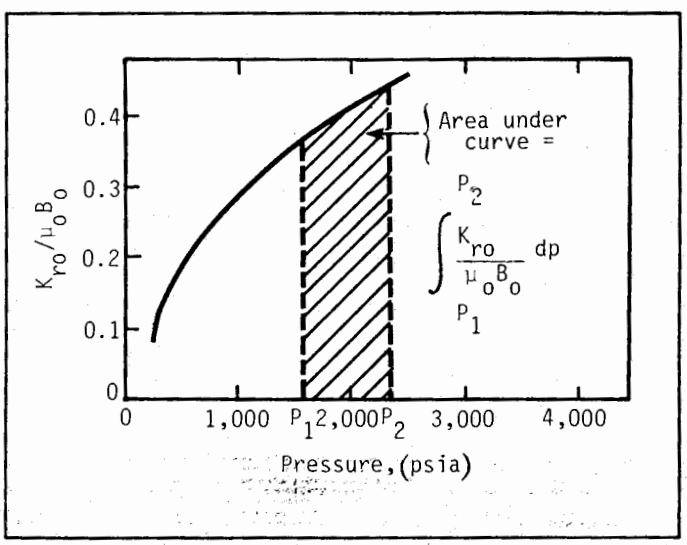


Fig. 1.20 Evaluation of the integral (after Calhoun).

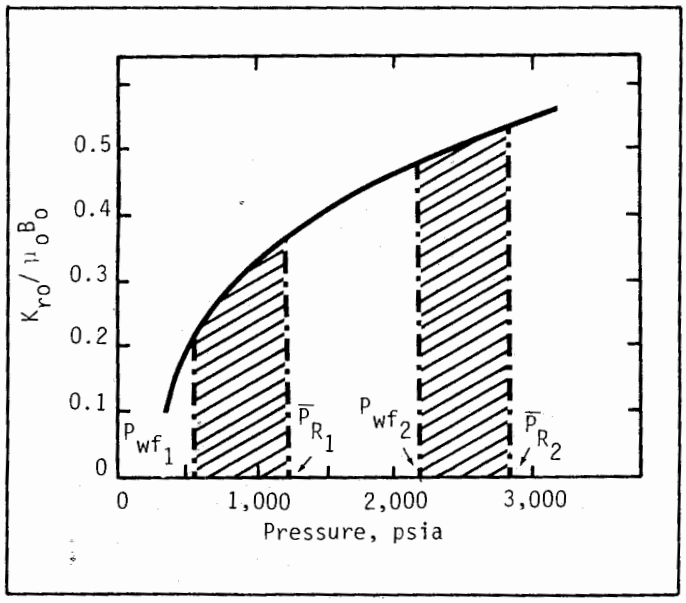


Fig. 1.21 Pressure vs. $\frac{K_{ro}}{\mu_o B_o}$ (after Calhoun).

(3) The PI value will depend on the producing gas oil ratio. Each gas-oil ratio value means a different steady state system for which a different curve will apply. Calhoun showed how the function changes with R values (Fig. 1.22).

Calhoun[5] also prepared Fig. 1.23 which shows the dependence of PI on reservoir pressure and pressure drawdown for one gas-oil ratio only. A similar figure would be needed for each different gas-oil ratio.

The theoretical Productivity Index of a radial system for steady state flow can be expressed as follows:

$$PI = \frac{7.08\ Kh}{(\bar{p}_R - p_{wf})\left(\ln \frac{r_e}{r_w}\right)} \int_{\bar{p}_{wf}}^{\bar{p}_R} \frac{K_{ro}}{\mu_o B_o} dp \qquad (1.19)$$

Evinger and Muskat[4] suggested that in order to use the PI value as a means of comparison, the comparisons be made for $(\bar{p}_R - p_{wf})$ equal to zero, that is,

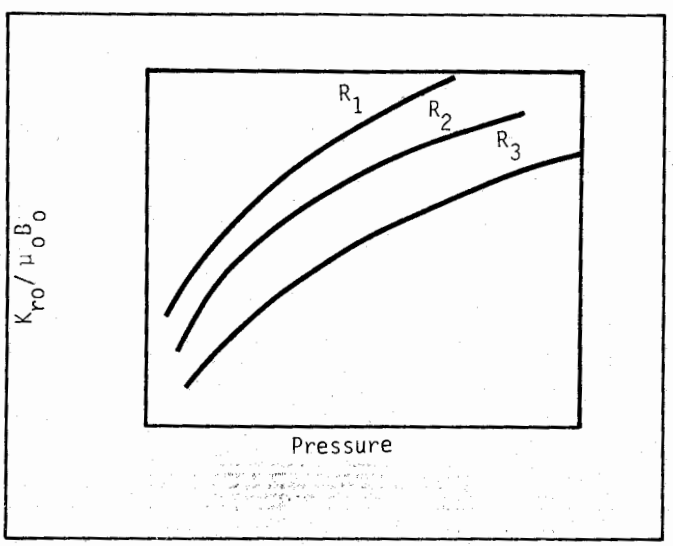


Fig. 1.22 Effect of gas in solution (after Calhoun).

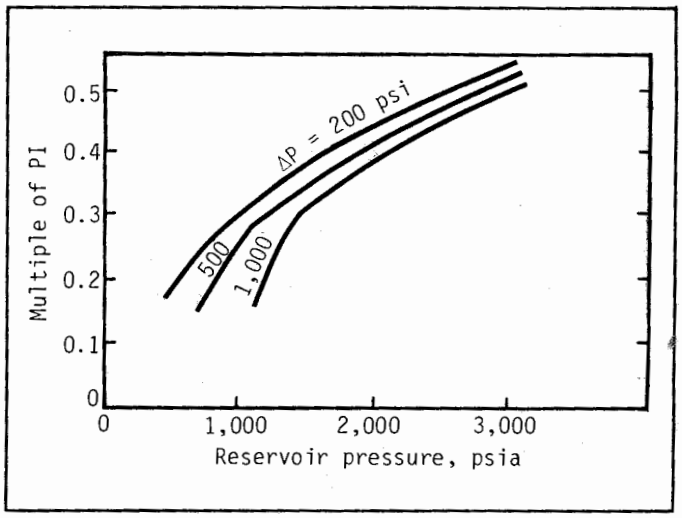


Fig. 1.23 Effect of ΔP on PI (after Calhoun).

for p_{wf} approaching $\bar{p}_R$ as a limit. Then the PI equation becomes:

$$PI = \frac{7.08\ Kh}{\ln \frac{r_e}{r_w}} \left(\frac{K_{ro}}{\mu_o B_o}\right)_{\bar{p}_R} \qquad (1.20)$$

In order to compare PI values at two different times we have:

$$\frac{(PI)_1}{(PI)_2} = \frac{\left(\frac{K_{ro}}{\mu_o B_o}\right)_1}{\left(\frac{K_{ro}}{\mu_o B_o}\right)_2} \qquad (1.21)$$

It is then possible to evaluate the PI of a solution gas drive well. In order to do this we must evaluate the oil saturation at some future time. By knowing the pressure we can obtain B_o, and μ_o, and K_{ro} is defined from the saturation. Calhoun[5] showed a typical decline in PI based on this analysis (Fig. 1.24).

Fig. 1.25 shows how the PI varies at different pres-

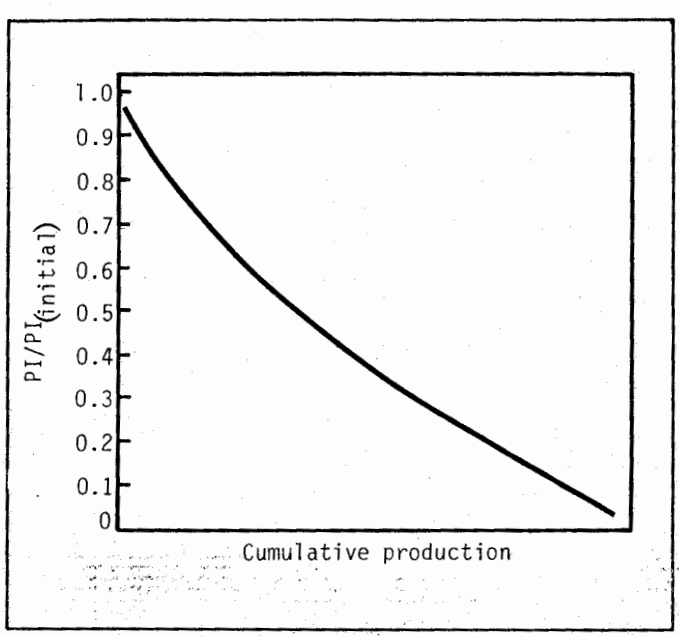
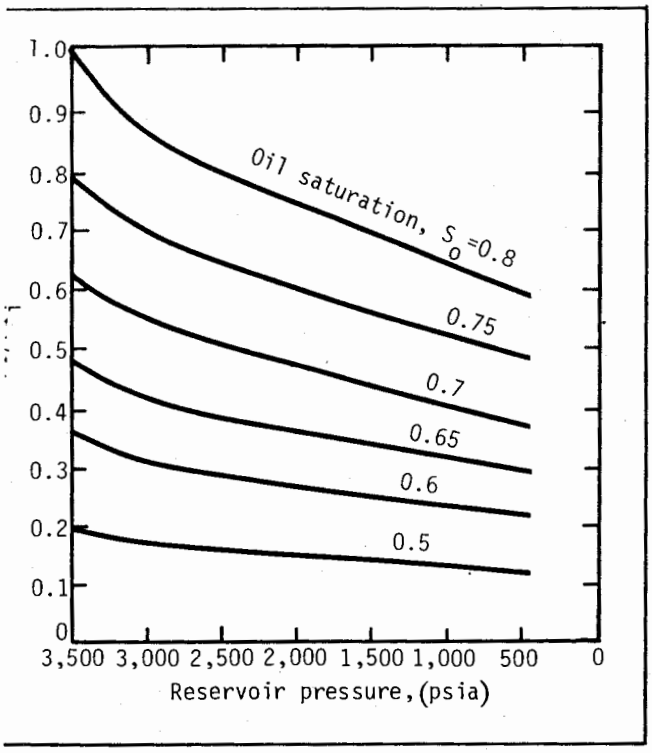


Fig. 1.24 PI vs. cumulative production (after Calhoun).



. 1.25 *PI vs. pressure (after Calhoun).*

res for constant saturations. Therefore if the pres-
re and saturation are known we could obtain PI/PI_i
if the PI and pressure were known saturation could
estimated. Fig. 1.25 was prepared for a particular
ɜervoir having an original oil saturation of 80%
d an original pressure of 3,500 psi.
The method proposed by Evinger and Muskat has
en used extensively in field cases and has proven
be very good in many cases. It is recommended
at this procedure be evaluated along with other
ɔcedures described in this chapter. It may very well
ɔve to be sufficiently accurate for prediction of PI's
d inflow curves. Calhoun[5] gives an excellent dis-
ssion on this procedure.

ɜ4 Inflow performance curves

ɜ41 Introduction

The PI not only changes with time or cumulative
oduction but is also subject to change with increased
awdown at any one specific time in the life of the
ɜll. If we measure several PI's in a well during a
ecific time interval, a relationship will be obtained
tween rate and flowing pressure which normally is
ɔt linear for a solution gas drive field. This phe-
ɔmenon may be attributed to one or more of the
llowing factors:
(1) Increased gas saturation with subsequent lower-
g of permeability to oil near the well bore as a result
reduced reservoir pressure near the well bore at
gher producing rates.
(2) Change from laminar to turbulent flow in some
ɔw capillaries near the well bore at increased pro-
ɪcing rates.
(3) Exceeding critical flow rates through pores at
rmation face in the well bore. These pores act as orifices and when the critical rate is exceeded, increased drawdowns have a diminished effect on increasing rates.

This plot of q vs. p_{wf} is called inflow performance and was first used by Gilbert in describing well performance[6]. A typical plot is noted in Fig. 1.1 and differs depending upon the type of reservoir.

1.342 Vogel's work

A publication by Vogel in January 1968[7] offered a solution in determining an inflow performance curve for a solution gas drive field for flow below the bubble point. By the use of a computer, he calculated IPR curves for wells producing from several ficticious solution-gas drive reservoirs that covered a wide range of oil PVT properties and reservoir relative permeability characteristics. He made several assumptions such as circular, radial uniform flow with a constant water saturation. He neglected gravity segregation and his solution is valid for two-phase flow in the reservoir only. Vogel[7] showed how rate vs. flowing bottom hole pressure as a function of cumulative recovery changed in Fig. 1.26. As noted, the result is a progressive deterioration of the IPR's as depletion proceeds with time in a solution-gas drive reservoir.

Vogel[7] also presented Fig. 1.27 which shows the effect of viscosity and gas-oil ratio (GOR). Curve B used a crude oil with about 1/2 the viscosity of the crude for Curve A. Also, the crude of Curve B used a GOR about twice that of the crude for Curve A.

He plotted all the IPR's as "dimensionless IPR's." The pressure for each point on an IPR curve is divided by the maximum or shut-in pressure for that particular curve, and the corresponding production rate is

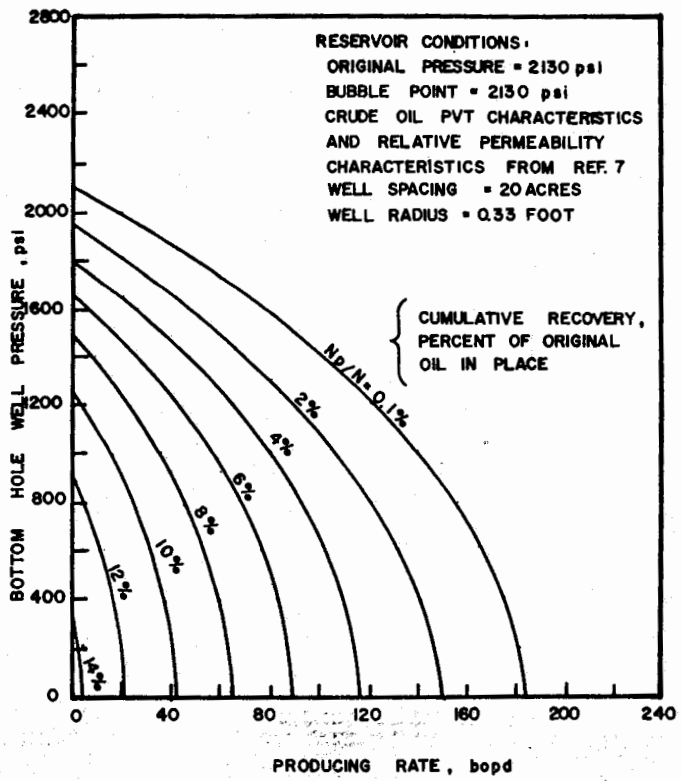


Fig. 1.26 Computer calculated inflow performance relationships for a solution gas drive reservoir (after ref. 7).

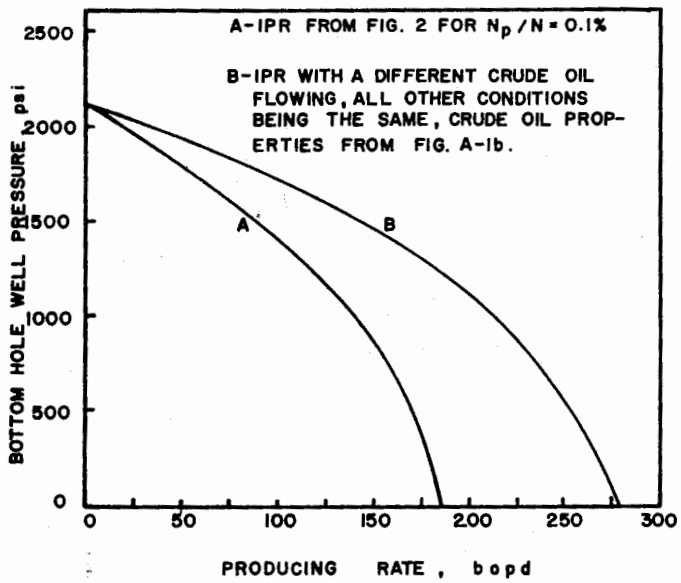


Fig. 1.27 IPR's showing effect of viscosity and gor (after Vogel).

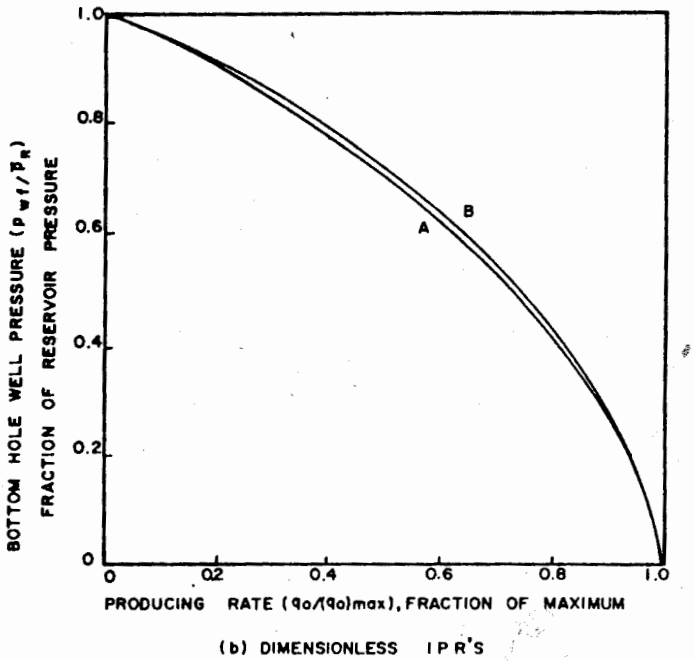


Fig. 1.29 Effect of crude oil properties on IPR's (after Vogel).

divided by the maximum (100% drawdown) producing rate for the same curve. When this is done, the curves from Fig. 1.26 can be replotted as shown in Fig. 1.28.

It is then apparent that with this type of construction the curves are remarkably similar throughout most of the producing life of the reservoir. He also noted that the same dimensionless plot of Fig. 1.27 gave IPR's that were similar as in Fig. 1.29.

Before constructing his final curve he made calculations for more viscous crudes, varying GORs, varying relative permeabilities, different well spacings, fractured wells, and for wells with skin effect. In summary, his calculations for 21 reservoir conditions resulted in IPR's generally exhibiting a similar shape. One exception was a well for skin effect in which the IPR approached a straight line. The more viscous crudes and reservoirs above the bubble point also showed significant deviation. However, curvature was apparent.

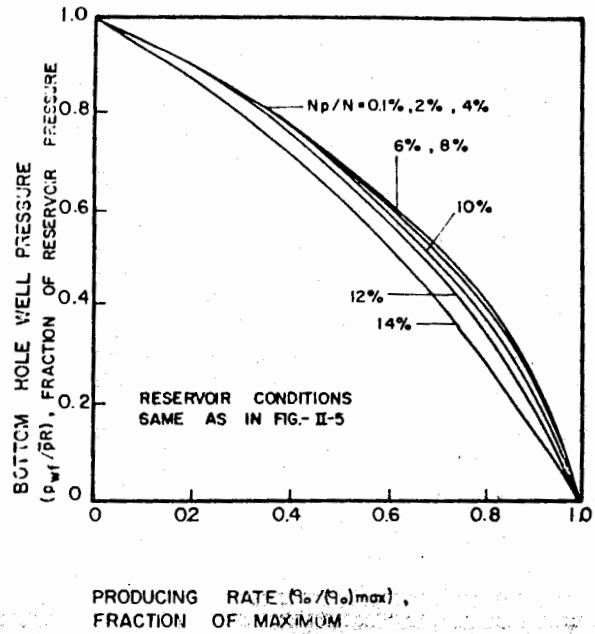


Fig. 1.28 Dimensionless inflow performance relationships for a solution gas drive reservoir (after Vogel).

Vogel's work[7] resulted in his construction of a reference curve (Fig. 1.30) which is all that is needed from his paper to construct an IPR curve from one flowing test on a well. This curve should be regarded as a general solution of the solution-gas drive reservoir flow equations with the constants for particular solutions depending upon an individual reservoir, and for flowing pressures below the bubble point.

The equation of the curve of Fig. 1.30 is:

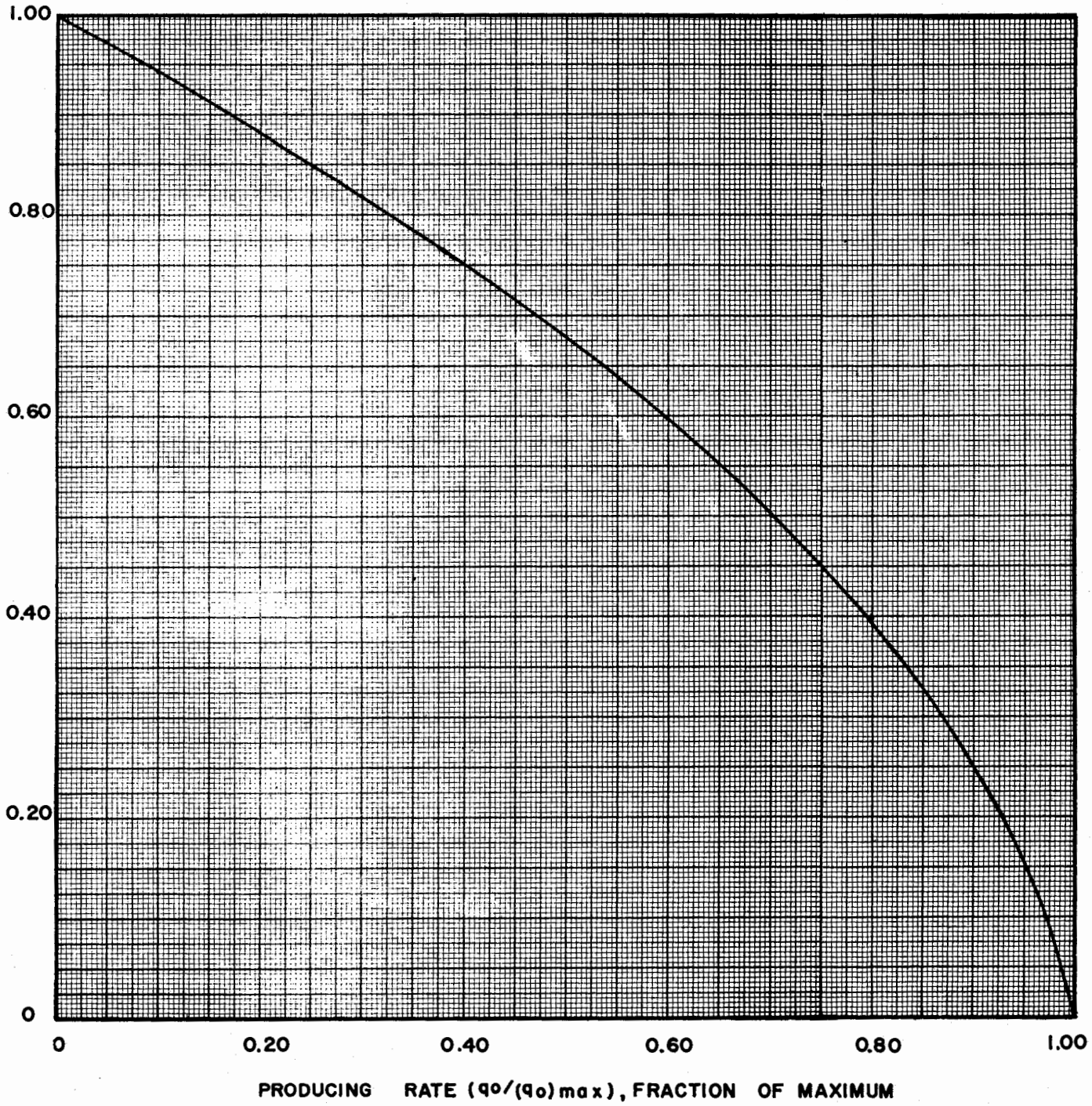
$$\frac{q_o}{(q_o)_{max}} = 1 - 0.20\left(\frac{p_{wf}}{\bar{p}_R}\right) - 0.80\left(\frac{p_{wf}}{\bar{p}_R}\right)^2 \qquad (1.22)$$

where q_o is the producing rate corresponding to a given well intake pressure p_{wf}, $\bar{p}_R$ the corresponding reservoir pressure, and $(q_o)_{max}$ is the maximum (100% drawdown) producing rate. $\bar{p}_R$ is the average reservoir pressure.

For comparison, the relationship for a straight-line IPR is:

$$\frac{q_o}{(q_o)_{max}} = 1 - \frac{p_{wf}}{\bar{p}_R} \qquad (1.23)$$

Vogel compared the reference curve of Fig. 1.30 with those calculated on the computer. The curve matches more closely the IPR curves for early stages of depletion than the IPR curves for later stages of depletion. In this way, the percent error is less when dealing with the higher producing rates in the early stages of depletion. The percentage error becomes greater in the later stages of depletion, but here production rates are low and, as a consequence, absolute errors would be less. These comparisons can be found in Fig. 11 of his original paper.[7] Maximum error in the use of the curve will occur when well tests are made at very low producing rates and low drawdowns and then an attempt is made to extrapolate to 100% drawdowns. Vogel states that most errors should not



g. 1.30 Inflow performance relationship for solution—gas drive reservoirs (after Vogel).

xceed 10%. I have personally consulted with many ngineers that have used this curve and they have een very pleased with the results.

Vogel[7] also made the following observations:

(1) This procedure would not be considered correct here other types of drive exist. However, some ngineers have used this procedure for other types nd combinations of drive mechanisms with good esults. For wells producing with a flowing pressure elow the bubble point, his work may very well be ood.

(2) Since the reference curve is for the two-phase ow of oil and gas only, it would not be considered alid when three phases (oil, gas, and water) are flowing. However, again some engineers have noted its continued accuracy for three-phase flow.

(3) Its comparison to single-phase liquid flow and single-phase gas flow is noted in Fig. 1.31.

(4) The conclusions are based on computer solutions involving several simplifying assumptions and additional comparisons with field data are needed. However, I feel that this represents the best solution to date and is more accurate than assuming a linear relationship.

The reference curve of Fig. 1.30 is very simple to use. All that is needed is one flow test of flowing bottom hole pressure vs rate and the static reservoir pressure. From the ratio of $p_{wf}/\bar{p}_R$ (ordinate) a value of $q_o/(q_o)_{max}$

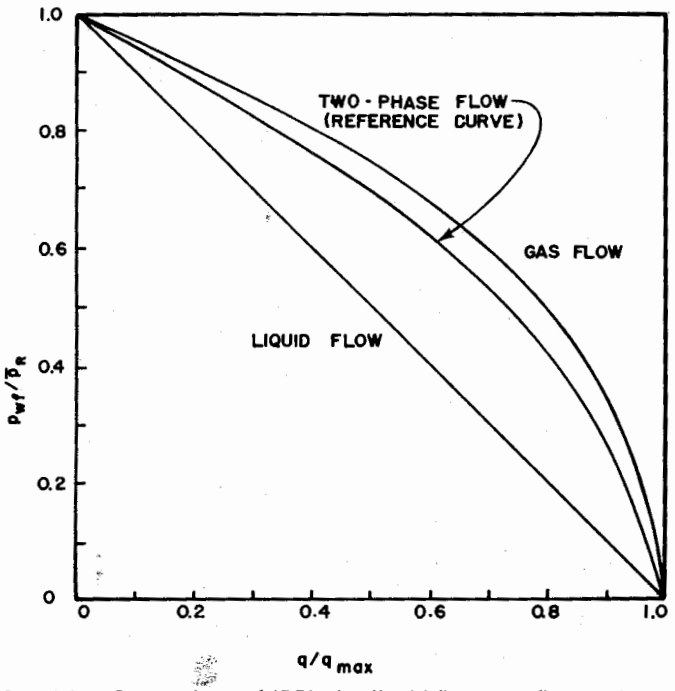


Fig. 1.31 Comparison of IPR's for liquid flow, gas flow and two-phase flow (after Vogel).

can be found and $(q_o)_{max}$ determined. Once $(q_o)_{max}$ has been determined a value of q for any p_{wf} can be found and the construction of an inflow performance curve is then possible. Solutions are offered in the following examples.

EXAMPLE PROBLEM #5: HOW TO DETERMINE MAXIMUM FLOW RATE FOR SOLUTION GAS DRIVE WELL

The following test was conducted on a solution gas drive well:

Given data: $\bar{p}_R = 2{,}000$ psi
(After Vogel[7]) $p_{wf} = 1{,}500$ psi
$q_o = 65$ bpd

Find: (a) $(q_o)_{max}$
(b) q_o for $p_{wf} = 500$ psi

Solution:

(a) $\frac{p_{wf}}{\bar{p}_R} = \frac{1{,}500}{2{,}000} = 0.75$

From Fig. 1.30 we find $\frac{q_o}{(q_o)_{max}} = 0.40$

Then $\frac{65}{(q_o)_{max}} = 0.40$

$$(q_o)_{max} = \frac{65}{0.40} = 162 \text{ bpd}$$

The well makes 162 bpd for $p_{wf} = 0$ which is essentially impossible from a practical point of view.

(b) Find q_o for $p_{wf} = 500$ psi; $\frac{p_{wf}}{\bar{p}_R} = 0.25$

From Fig. 1.30 we find $\frac{q_o}{(q_o)_{max}} = 0.90$. We now know $(q_o)_{max}$ therefore $\frac{q_o}{162} = 0.90$

$$q_o = (0.90)(162) = 146 \text{ bpd}$$

CLASS PROBLEM #5-A

Given data: $\bar{p}_R = 2{,}500$ psi, $p_b = 2{,}500$ psi
$p_{wf} = 1{,}800$ psi
$q_o = 100$ bpd

Find: (a) $(q_o)_{max}$
(b) q_o for $p_{wf} = 1{,}000$ psi

CLASS PROBLEM #5-B

Given data: $\bar{p}_R = 3{,}000$ psi, $p_b = 3{,}000$ psi
$p_{wf} = 2{,}500$ psi, $q_o = 500$ bpd

Find: (a) $(q_o)_{max}$
(b) q_o for $p_{wf} = 1{,}200$ psi
(c) Find $(q_o)_{max}$ if a linear relationship is assumed.

EXAMPLE PROBLEM #6: HOW TO CONSTRUCT IPR CURVE FROM ONE FLOW TEST

Construct a complete Inflow Performance Curve for the data given in Example Problem #5.

This problem is solved by assuming various flowing pressures and determining the corresponding flow rates. We will make use of the information found previously.

Prepare a table as follows:

Assumed p_{wf}	q	Remarks
0	162	Previously calculated
1,500	65	Given
500	146	Previously calculated
2,000	0	Given

Additional values of p_{wf} are assumed as necessary and the corresponding flow rates determined. This information is then plotted as in Fig. 1.32. It should be kept in mind that this represents an inflow performance curve for the test date and will differ at a later date. However, it will probably retain the same general shape but will begin with a lower static reservoir pressure. A plot of this type was shown in Fig. 1.26.

CLASS PROBLEM #6-A: TO FIND MAXIMUM FLOW RATE AND FLOW RATE FOR ANY FLOWING PRESSURE

Given data: $\bar{p}_R = 2{,}300$ psi
$p_{wf} = 1{,}400$ psi
$q_o = 95$ bpd

Find: (a) $(q_o)_{max}$
(b) q_o for $p_{wf} = 700$ psi
(c) construct a complete IPR curve of q vs. p_{wf}

CLASS PROBLEM #6-B.

(a) Construct an IPR curve for Class Problem #5-B.
(b) Construct an IPR curve for Class Problem #5-B assuming a linear relationship.

1.343 Standing's extension of Vogel's work to account for damaged or improved wells

The initial work of Vogel assumed a flow efficiency of 1.00 and did not account for wells that were damaged

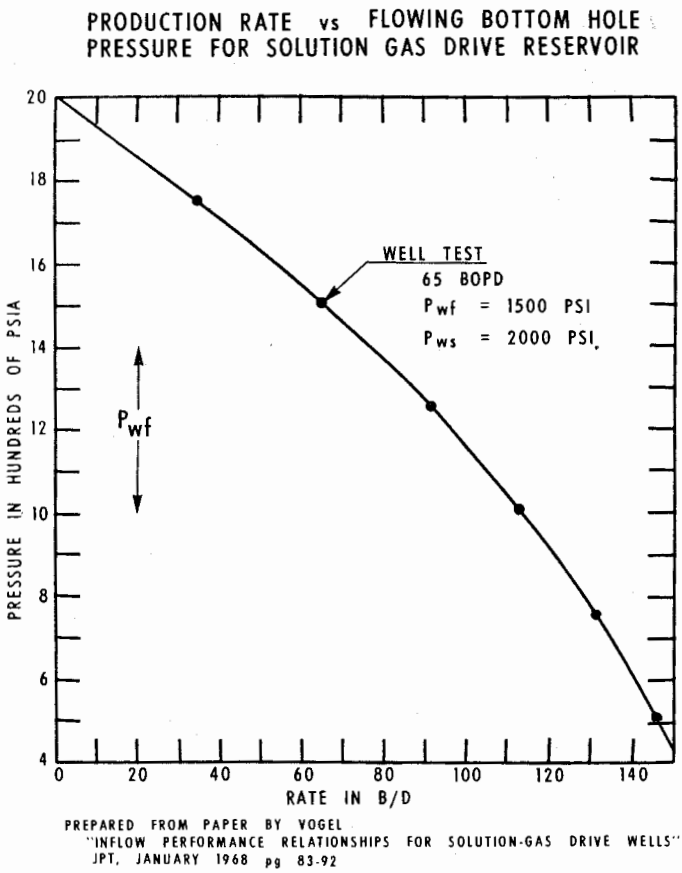


Fig. 1.32 Constructed IPR curve.

or improved. Standing[8] proposed a companion chart to account for conditions where the flow efficiency was not equal to 1.00.

In Fig. 1.33 flow efficiency is defined as:

$$FE = \frac{\text{Ideal drawdown}}{\text{Actual drawdown}} = \frac{\bar{p}_R - p'_{wf}}{\bar{p}_R - p_{wf}}$$

where $p'_{wf} = p_{wf} + \Delta p_{skin}$

substituting:

$$FE = \frac{\bar{p}_R - (p_{wf} + \Delta p_{skin})}{\bar{p}_R - p_{wf}} = \frac{\bar{p}_R - p_{wf} - \Delta p_{skin}}{\bar{p}_R - p_{wf}} \quad (1.24)$$

which is the ratio of useful pressure drop across the system to total pressure drop. For a well draining a cylindrical volume:

$$FE = \ln \frac{0.47 r_e}{r_w} \Big/ \left[\ln \frac{0.47 r_e}{r_w} + S \right] \quad (1.25)$$

where S is the dimensionless skin factor. Some confusion exists when we think that FE also expresses the well's flow rate with damage to the flow rate without damage. Only when the IPR curve is a straight line (undersaturated liquids) is FE defined by equation 1.24 also equal to the ratio of flow rates, where equation 1.24 is:

$$FE = \frac{\bar{p}_R - p_{wf} - \Delta p_{skin}}{\bar{p}_R - p_{wf}} \quad (1.24)$$

As noted in Fig. 1.33, an undamaged well would flow at rate q for a flowing pressure of p'_{wf} while the damaged well must flow at the lower pressure of p_{wf} in order to produce the same rate q.

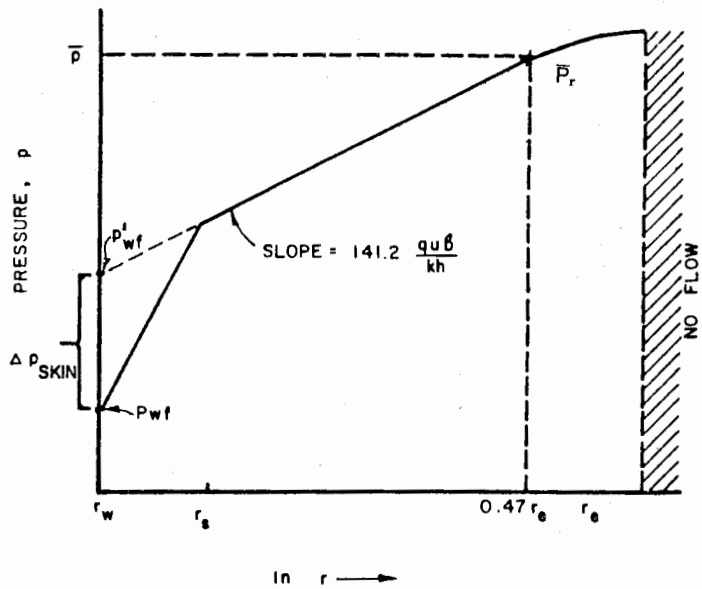


Fig. 1.33 Pressure profile of damaged wells producing by solution gas drive (after ref. 8).

The Δp_{skin} is thus seen to be the difference between p'_{wf} and p_{wf}. Fig. 1.34 shows the region of added resistance to flow near the well-bore. There may be many factors which cause or control this added resistance including invasion of the zone by mud or "kill-fluids," swelling of shale, and others. This may also represent a region of improvement after a stimulation treatment.

The determination of Δp_{skin} is made by first determining S (skin factor) from a standard pressure build-up test on a well as in Fig. 1.35.

Δp_{skin} was defined by Van Everdingen[9] as:

$$\Delta p_{skin} = S \frac{q\mu}{2\pi Kh} \quad (1.26)$$

where S = skin factor or:

$$S = \frac{K_e - K_a}{K_e} \ln \frac{r_a}{r_w} \quad (1.27)$$

where "a" denotes the zone of altered permeability and "e" denotes the zone of unaltered permeability.

The standard equation for determining skin effect is:

$$S = 1.151 \left[\frac{p_{(1hr)} - p_{wf}}{m} - \log \frac{K}{\phi \mu c r_w^2} + 3.23 \right] \quad (1.28)$$

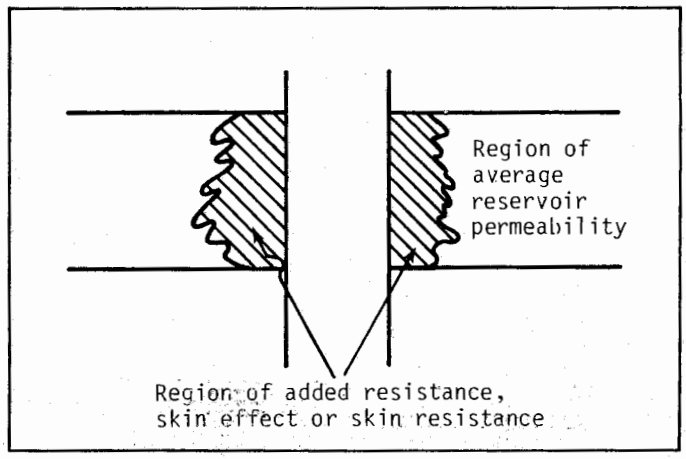


Fig. 1.34 Skin effect.

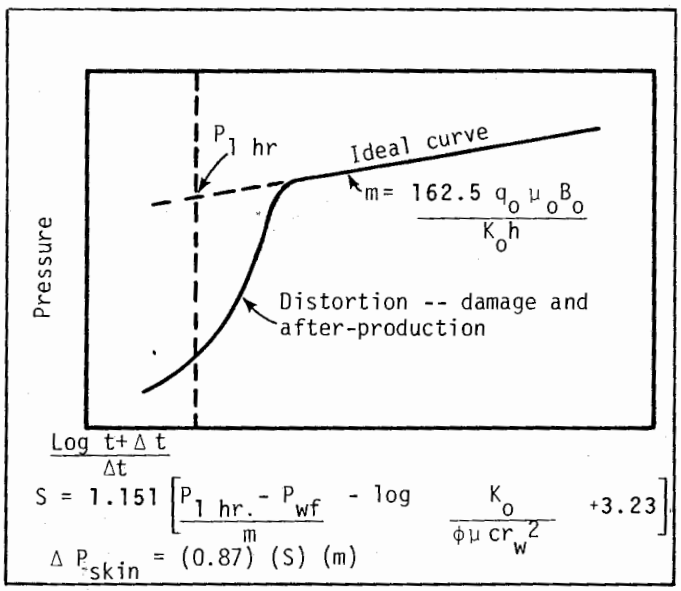


Fig. 1.35 Method of determining ΔP skin.

This is for Δt small

$$\text{where } \frac{t+\Delta t}{\Delta t} > 1 \text{ and } \bar{p}_R = p_{1hr}$$

We could use a time other than 1 hour and this would only change the constant 3.23. For example, for $\Delta t =$ 10 hours the constant would be $(3.23 - \log 10) = 2.23$.

We may also recall that:

$S = +$ indicates damage
$S = 0$ indicates that $K_a = K_e$
$S = -$ indicates improvement and that values of -3 to -5 are common for a fractured well.

One precaution: the p_{1hr} must come from the straight line portion of the curve and may have to be extrapolated backwards.

The value of Δp_{skin} is then calculated from:

$$\Delta p_{skin} = 0.87Sm \tag{1.29}$$

m = slope from straight line portion of the pressure build-up curve, determined from the following equation:

$$m = \frac{162.5\, q_o\, \mu_o B_o}{K_o h} \tag{1.30}$$

recalling that:

$$\Delta p_{skin} = S\frac{q\mu}{2\pi Kh} \tag{1.26}$$

and

$$m = \frac{2.303\, q\mu}{2\pi Kh} \tag{1.30}$$

then $\Delta p_{skin} = 0.87Sm$. This is the method for determining Δp_{skin} which is used in the following equation:

$$FE = \frac{\bar{p}_R - p_{wf} - \Delta p_{skin}}{\bar{p}_R - p_{wf}} \tag{1.24}$$

Another method for determining FE is from two flow tests and is given in Appendix A.

Sometimes we define a completion efficiency as:

$$CE = \frac{(Kh)_{PI}}{(Kh)_{BU}} \tag{1.31}$$

$$\text{where } Kh_{PI} = 1{,}000\, B_o \mu_o PI \tag{1.32}$$

and Kh_{BU} is determined from a pressure build-up test.

Also in Fig. 1.33 we see that the effective drainage radius stabilizes at approximately $0.472r_e$, and it appears that the average reservoir pressure has been reached at this distance.

From the radial flow equation we have:[32]

$$\ln\left(\frac{r_d}{r_w}\right) = \frac{2\pi Kh}{q\mu}(\bar{p}_R - p_{wf}) \tag{1.33}$$

$$\text{also} \qquad \bar{p}_R = p_i - \frac{q\mu r_w^2 t_D}{\pi r_r^2 hK} \tag{1.34}$$

$$\text{where} \qquad t_D = \frac{Kt}{\phi\mu r_w^2 C} \tag{1.35}$$

we can eliminate $\bar{p}_R$

$$\ln\left(\frac{r_d}{r_w}\right) = \frac{2\pi Kh}{q\mu}(p_i - p_{wf}) - 2t_D\left(\frac{r_w}{r_r}\right)^2 \tag{1.36}$$

$$p_{(t_D)} = \frac{2\pi Kh(p_i - p_{wf})}{q\mu} \tag{1.37}$$

$$\ln\left(\frac{r_d}{r_w}\right) = p_{(t_D)} - 2t_D\left(\frac{r_w}{r_r}\right)^2 \tag{1.38}$$

$$p_{(t_D)} = \left(\frac{r_w}{r_r}\right)^2 + 2t_D\left(\frac{r_w}{r_r}\right)^2 - 0.75\ln\left(\frac{r_r}{r_w}\right) \tag{1.39}$$

letting $\left(\frac{r_w}{r_r}\right)^2 = 0$

$$\text{then} \qquad \ln\left(\frac{r_d}{r_w}\right) = \ln\left(\frac{r_r}{r_w}\right) - 0.75 \tag{1.40}$$

$$\ln\left(\frac{r_d}{r_r}\right) = 0.75, \text{ or } \left(\frac{r_d}{r_r}\right) = e^{-0.75} = 0.472$$

then

$$r_d = 0.472r_r \text{ (for liquid system)} \tag{1.41}$$

Standing[5] constructed Fig. 1.36 which shows IPR curves for flow efficiencies between 0.5 and 1.5. Several things can be obtained from this plot:

(1) The maximum rate possible for a well with damage.

(2) The maximum rate possible if the damage is removed and FE = 1.0.

(3) The maximum rate possible if the well is stimulated and improved.

(4) The determination of the flow rate possible for any flowing pressure for different values of FE.

(5) The construction of IPR curves to show rate vs. flowing pressure for damaged and improved wells.

This extension of Vogel's work is very useful in being able to determine whether or not stimulation is profitable.

Fig. 1.36 can be slightly confusing if not studied carefully. The abscissa is the ratio of the producing rate divided by the producing rate with no damage. That is, each value that is read from the curves is a value to calculate $(q_o)_{max}$ with FE corrected to 1.00.

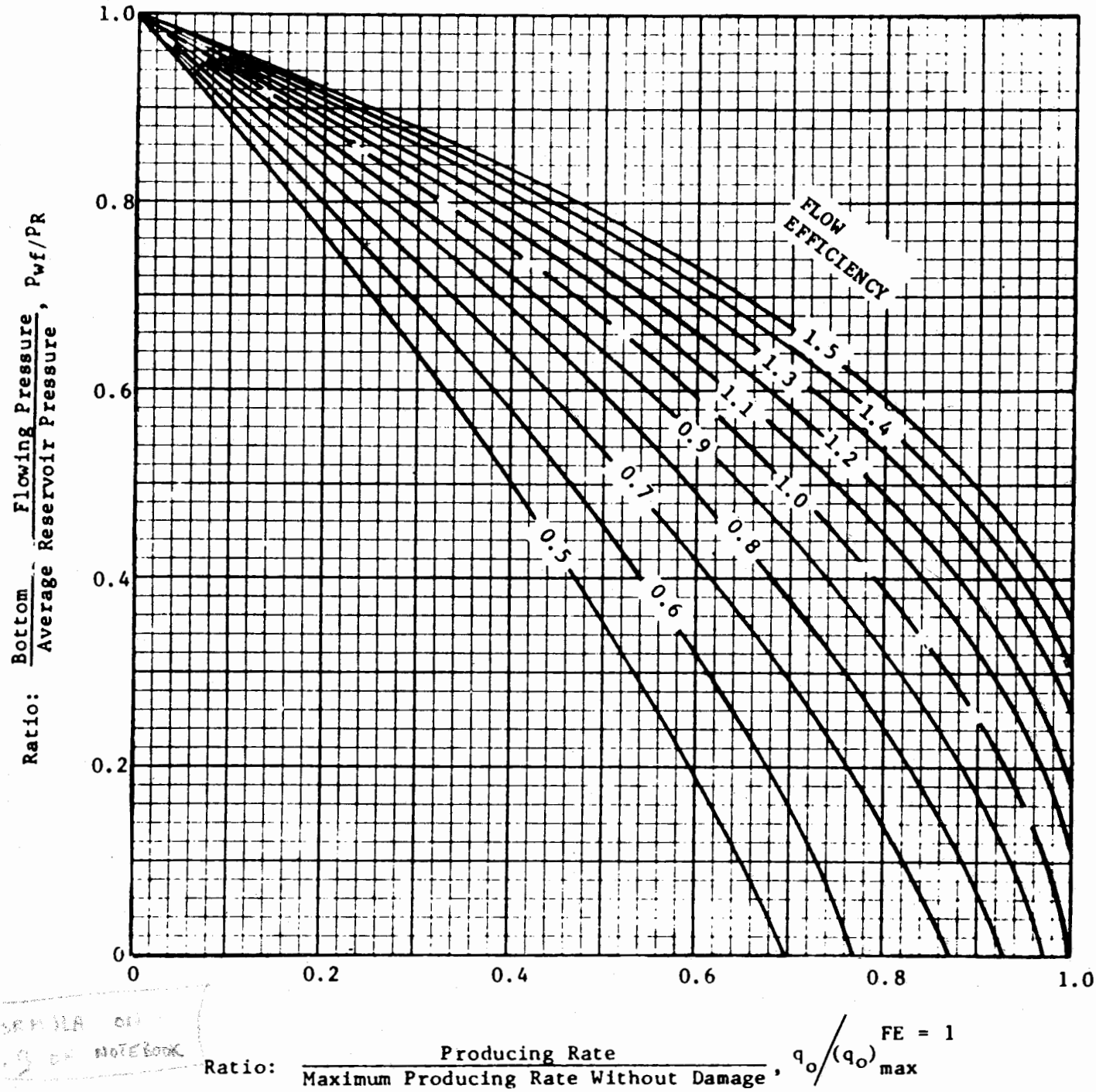


Fig. 1.36 IPR curves for damaged wells producing by solution—gas drive (after ref. 8).

For example, assume:

$q = 70$ bpd, $p_R = 1{,}000$, $p_{wf} = 700$, and FE = 0.6. The ratio of $p_{wf}/\bar{p}_R = \frac{700}{1{,}000} = 0.7$ and we can read $q_o/(q_o)_{max} = 0.295$ from Fig. 1.36 from the 0.6 curve. Then $\frac{70}{(q_o)_{max}} = .295$ or $(q_o)_{max} = 237$ bpd. However the 237 bpd is corrected for FE = 1.00.

Another way to look at this problem is to find the flowing pressure at which the well would make 70 bpd with no damage. The total Δp is $1{,}000 - 700 = 300$ psi. However only 0.6 of it, or $(0.6)(300) = 180$ psi, occurs across the formation and $300 - 180 = 120$ psi = damage or skin effect. The flowing pressure would be $1{,}000 - 180 = 820$ psi for 70 bpd in an undamaged well, which required $p_{wf} = 700$ psi for a damage of 0.6 to make the same rate of 70 bpd. The curves of Fig. 1.36 have been prepared in this manner.

We need to keep in mind that, normally, in order to use Fig. 1.36, first determine the maximum flow rate possible without damage (FE = 1.00). Once this has been done, additional requirements are relatively simple.

For example given the following information:

$q_o = 70$ bpd
$\bar{p}_R = 2{,}400$ psi
$p_{wf} = 1{,}800$ psi
FE = 0.7

our first requirement is to find the maximum flow rate possible assuming the well has no damage (FE = 1.00).

Find $(q_o)_{max}$ if FE is corrected to 1.00

$$\frac{q_o}{(q_o)_{max}}\Bigg|_{FE=0.7}^{FE=1.00} = 0.281 \quad \text{or}$$

$$(q_o)_{max}^{FE=1.00} = 70/0.281 = 249 \text{ bpd}$$

$(q_o)_{max}^{FE=0.7} \neq 249$ bpd, but is a value less and is explained later.

A further illustration may help to explain Standing's chart and the way it was developed. Using the data of the preceding problem a pressure profile for the FE = 0.7 well producing 70 bpd would appear like that shown in Fig. 1.37.

As FE = 0.7

$$0.7 = \frac{\bar{p}_R - p_{wf} - \Delta p_{skin}}{\bar{p}_R - p_{wf}}$$

$$\Delta p_{skin} = (1 - 0.7)(\bar{p}_R - p_{wf}) = 180 \text{ psi}$$

$$p'_{wf} = 1{,}800 + 180 = 1{,}980 \text{ psi}$$

or the ideal pressure drawdown is 2,400 − 1,980 = 420 psi compared to an actual drawdown of 600 psi. The $FE = \frac{\text{ideal drawdown}}{\text{actual drawdown}} = 420/600 = 0.7$. p'_{wf} is the wellbore flowing pressure that the well would have, at the 70 bpd rate, if it *were not damaged.* This value is then used in Vogel's relationship to get a value of $q_o/(q_o)_{max}$ for the *equivalent undamaged well.* Because $(q_o)_{max}$ now represents the equivalent *undamaged* maximum rate, Standing uses the nomenclature of $(q_o)_{max}^{FE=1}$. More specifically:

$\frac{p'_{wf}}{\bar{p}_R} = \frac{1{,}980}{2{,}400} = 0.825$ which is the equivalent undamaged well point as found on Fig. 1.36.

In order to use the curves of Standing we need:

$\frac{p_{wf}}{\bar{p}_R} = \frac{1{,}800}{2{,}400} = 0.75$ which is the actual damaged well point on Fig. 1.36 and read $q_o/(q_o)_{max}^{FE=1.0} = 0.281$. This value can be solved for from Vogel's relationship (equation 1.22).

$$q_o/(q_o)_{max}^{FE=1.0} = 1 - 0.2\left(\frac{p'_{wf}}{\bar{p}_R}\right) - 0.8\left(\frac{p'_{wf}}{\bar{p}_R}\right)^2$$

$$= 1 - 0.165 - 0.544 = 0.290$$

This value of 0.290 differs slightly from 0.281 obtained from the chart. The answer for $(q_o)_{max}$ using the value as obtained from the equation is: $(q_o)_{max} = \frac{70}{0.290} =$ 241 bpd instead of 249 bpd as obtained from the curves of Fig. 1.36. We will use the value of 249 bpd as read from the curves in working the following illustrations.

Our next requirement is to find the maximum flow rate from the damaged well. Using the same data in this series we want to find the maximum rate possible for this well under the present producing conditions of FE = 0.7.

$(q_o)_{max}$ for FE = 1.00 = 249 bpd (as read from curve)

The maximum rate occurs when $p_{wf} = 0$ then $\frac{p_{wf}}{\bar{p}_R} = 0$ and from Fig. 1.36 and FE = 0.7 curve we find $q_o/(q_o)_{max} = 0.87$. Then (0.87)(249) = 216 bpd. An additional clarification is offered here (Note Fig. 1.38):

$$p'_{wf} = p_{wf} + (1 - FE)(\bar{p}_R - p_{wf}) \qquad (1.33)$$
$$= 0 + (0.3)(2{,}400) = 720 \text{ psi}$$

From Vogel's relationship, the equivalent undamaged well rate ratio is:

$$q_o/(q_o)_{max}^{FE=1} = 1 - 0.2\left(\frac{720}{2{,}400}\right) - 0.8\left(\frac{720}{2{,}400}\right)^2 = 0.868$$

Therefore, $(q_o)_{max}^{FE=0.7} = (0.868)\,(q_o)_{max}^{FE=1}$

$$= (0.868)(241) = 209 \text{ bpd}$$

As noted, this does not check exactly with the values selected from Fig. 1.36. However, they should check, and therefore represent errors in construction of the curves.

$(q_o)_{max}^{FE=0.7} \neq (0.7)\,(q_o)_{max}^{FE=1}$ because of the non-linear IPR relationship for solution gas drive reservoir systems.

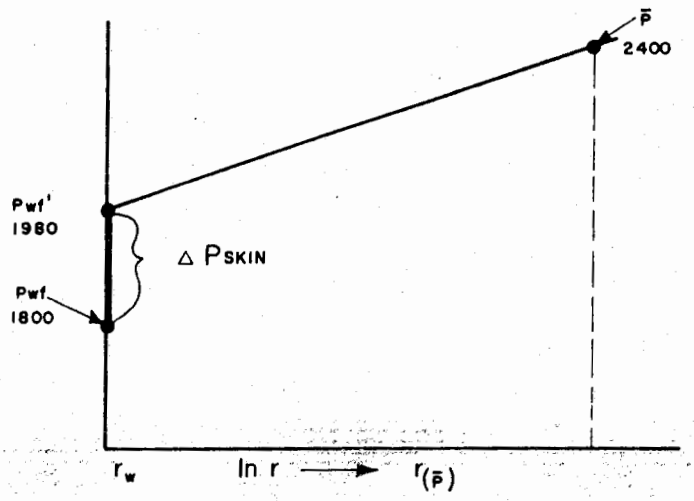


Fig. 1.37

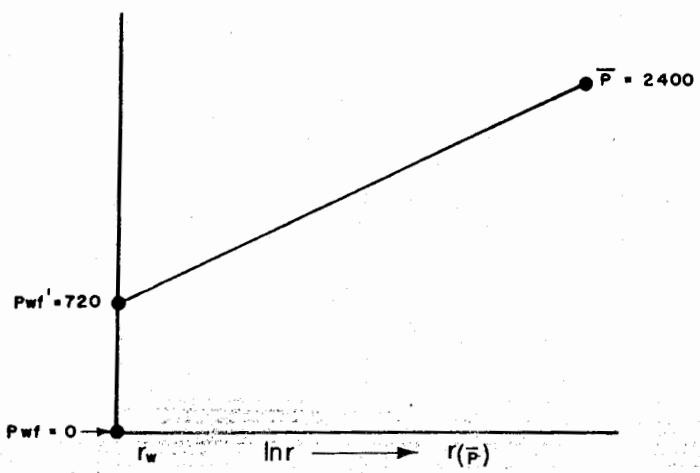


Fig. 1.38

$(q_o)_{max}$ for FE = 0.7 occurs at $p_{wf} = 0$. The same flow rate occurs at $p'_{wf} = 720$ psi for the undamaged well.

Therefore:

$$(q_o)_{max}^{FE=0.7} = (q_o)_{p_{wf}=0}^{FE=0.7} = (q_o)_{p'_{wf}}^{FE=1.00} = 720 \text{ psi}$$

$$\frac{p'_{wf}}{\bar{p}_R} = \frac{720}{2{,}400} = 0.3$$

We read from the curve of Fig. 1.36 (FE = 1.00) a value of

$$0.868 = \frac{(q_o)'^{FE=1.00}_{p_{wf}=720}}{(q_o)_{max}^{FE=1.00}}$$

$$\text{Then } (q_o)'^{FE=1.00}_{p_{wf}} = 0.868\,(q_o)_{max}^{FE=1.00} =$$

$$(q_o)_{max}^{FE=0.7} \neq (0.7)\,(q_o)_{max}$$

As an additional explanation we can define FE to be the ideal drawdown divided by the actual drawdown. For the previous example:

$$FE = \frac{\text{ideal drawdown}}{\text{actual drawdown}} = \frac{\text{ideal drawdown}}{2{,}400 - 1{,}800}$$

or ideal drawdown = (0.7) (600) = 420 psi giving an ideal flowing pressure of 1,980 psi for the same flow rate of 70 bpd.

The equivalent undamaged well point on Fig. 1.36 is located on the FE = 1.00 line where 720/2,400 = 0.3, whereas the actual well point is located on the FE = 0.7 line where 0/200 = 0 and the value of $q_o/(q_o)_{max}$ = 0.868.

Our next requirement is to find the maximum flow rate if the well is improved.

Assume that a stimulation job is performed on the above well and that FE is increased to 1.3. What is the maximum rate possible?

$(q_o)_{max}$ for FE = 1.00 = 249 bpd (from curve) for $p_{wf} = 0$ then $\frac{p_{wf}}{\bar{p}_R} = 0$ and from Fig. 1.36 on the FE = 1.3 curve (by extrapolation) $q_o/(q_o)_{max}$ = 1.1 then $(q_o)_{max}$ = (249) (1.1) = 274 bpd.

The extrapolation of the curves is not recommended since they appear to give erroneous results for values on the abscissa greater than 1.00. The solution by the equation does not appear correct either.

Fortunately, in practice we normally do not need values of $q_o/(q_o)_{max}$ greater than 1.00 and the curves and equation handle these problems in a satisfactory manner.

EXAMPLE PROBLEM #7: HOW TO FIND FLOW RATE POSSIBLE FROM DAMAGED WELL

Given data: $q_o = 70$ bpd
$\bar{p}_R = 2{,}400$ psi
$p_{wf} = 1{,}800$ psi
FE = 0.7

Find: (a) Maximum flow rate possible from this well under condition of FE = 0.7.
(b) Find q_o when $p_{wf} = 1{,}200$ psi for this well under present condition.

Solution: (a) First we must find $(q_o)_{max}$ for FE = 1.00

$$\frac{p_{wf}}{\bar{p}_R} = \frac{1{,}800}{2{,}400} = 0.75$$

From Fig. 1.36, reading on the FE = 0.7 curve gives a value of

$$q_o/q_{max} = 0.281$$

$$(q_o)_{max}^{FE=1.00} = \frac{70}{.281} = 249 \text{ bpd}$$

Keep in mind that 249 bpd is the maximum flow rate possible from this well for no damage, that is, FE = 1.00.

We now find $(q_o)_{max}$ for the well under present conditions of FE = 0.7. $(q_o)_{max}$ for FE = 1.00 = 249 bpd (as read from curve). The maximum rate occurs when $p_{wf} = 0$ then $\frac{p_{wf}}{\bar{p}_R} = 0$ and from Fig. 1.36 and FE = 0.7 curve we find $q_o/(q_o)_{max} = 0.87$. Then (0.87) (249) = 216 bpd. $(q_o)_{max}^{FE=0.7}$ = 216 bpd.

(b) Find q_o for $p_{wf} = 1{,}200$ psi (FE = 0.7)

$$\frac{p_{wf}}{\bar{p}_R} = \frac{1{,}200}{2{,}400} = 0.5$$

From Fig. 1.36 and on the FE = 0.7 curve we read $q_o/q_{max} = .523$

$$(q_o)_{p_{wf}\,=\,1{,}200} = (0.523)\,(249) = 130 \text{ bpd}$$

For FE = 1.00, q_o = 174 bpd and for 1.3, q_o = 205 bpd.

CLASS PROBLEM #7-A

Given: $q_o = 100$ bpd (FE = 0.6)
$\bar{p}_R = 2{,}200$ psi
$p_{wf} = 2{,}000$ psi

Find: (a) $(q_o)_{max}$ for FE = 0.6
(b) q_o for $p_{wf} = 1{,}000$ psi and FE = 0.6

CLASS PROBLEM #7-B

Given: $q_o = 500$ bpd (FE = 0.7)
$\bar{p}_R = 2{,}600$ psi
$p_{wf} = 2{,}200$ psi

Find: (a) $(q_o)_{max}$ for FE = 0.7
(b) q_o for $p_{wf} = 1{,}500$ psi and FE = 0.7

EXAMPLE PROBLEM #8: HOW TO FIND FLOW RATE POSSIBLE FOR WELL HAVING STIMULATION TREATMENT (SHOWS IMPROVEMENT)

Given: $q_o = 70$ bpd
$\bar{p}_R = 2{,}400$ psi
$p_{wf} = 1{,}800$ psi
FE = 0.7

Find: (a) $(q_o)_{max}$ for FE = 1.3
(b) q_o for $p_{wf} = 1{,}200$ psi and FE = 1.3

(a) First we must find $(q_o)_{max}$ for FE = 1.00

$$\frac{p_{wf}}{\bar{p}_R} = \frac{1{,}800}{2{,}400} = 0.75$$

From Fig. 1.36 and reading on the FE = 0.7 curve we read a value of:

$$q_o/(q_o)_{max} = 0.281$$

$$(q_o)_{max}^{FE=1.00} = \frac{70}{0.281} = 249 \text{ bpd}$$

Keep in mind that 249 bpd is the maximum flow rate possible from this well for no damage, that is, FE = 1.00.

For $p_{wf} = 0$ then $\frac{p_{wf}}{\bar{p}_R} = 0$ and from Fig. 1.36 on the FE = 1.3 curve (by extrapolation) we read $q_o/q_{max} = 1.1$ then $(q_o)_{max}^{FE=1.3} = (1.1)\ (249) = 274$ bpd. Since the curves are not accurate in this range the answer is somewhat questionable.

(b) Find q_o for $p_{wf} = 1{,}200$ psi and FE = 1.3

$$\frac{p_{wf}}{\bar{p}_R} = \frac{1{,}200}{2{,}400} = 0.5$$

From Fig. 1.36 and the FE = 1.3 curve, we read

$$q_o/q_{max} = 0.825$$

$$q_o = (0.825)\ (249) = 205 \text{ bpd}$$

CLASS PROBLEM #8-A

Given: $q_o = 100$ bpd
$\bar{p}_R = 2{,}200$ psi
$p_{wf} = 1{,}800$ psi
FE = 0.6

Find q_o for $p_{wf} = 1{,}500$ psi for FE values of 0.6, 1.0, and 1.4.

Find: (a) $(q_o)_{max}$ for FE = 1.4
(b) q_o for $p_{wf} = 1{,}000$ psi and FE = 1.4

CLASS PROBLEM #8-B

Given: $q_o = 500$ bpd
$\bar{p}_R = 2{,}700$ psi
$p_{wf} = 2{,}400$ psi
FE = 0.7

Find: (a) $(q_o)_{max}$ for FE = 1.3
(b) q_o for $p_{wf} = 1{,}500$ psi and FE = 1.3

CLASS PROBLEM #8-C

Given: $\bar{p}_R = 3{,}000$ psi
$p_{wf} = 2{,}500$ psi
$q_o = 500$ bpd
FE = 0.7

(a) For $p_{wf} = 1{,}000$ psi find the flow rate possible from this well under present conditions of FE = 0.7.

(b) Find q_o from this well for FE = 1.00, FE = 1.20, FE = 1.40 for a flowing pressure of 1,500 psi.

EXAMPLE PROBLEM #9: HOW TO FIND THE FLOWING PRESSURE NECESSARY FOR SET RATE IF DAMAGE IS REMOVED

Given data on well test:

$q_o = 120$ bpd
$p_{wf} = 1{,}200$ psi
$\bar{p}_R = 2{,}000$ psi
FE = 0.70

Find the flowing pressure at which the well will make 120 bpd for no damage (FE = 1.00).

Of the (2,000 − 1,200) = 800 psi drawdown, only 0.7 of it occurs across the formation, that is (0.7) (800) = 560 psi. Therefore for $p_{wf} = 2{,}000 - 560 = 1{,}440$ psi the well will make 120 bpd with FE = 1.00.

CLASS PROBLEM #9-A

$q_o = 200$ bpd
$\bar{p}_R = 2{,}300$ psi
$p_{wf} = 1{,}900$ psi
FE = 0.6

Find the flowing pressure at which the well will make 200 bpd for no damage (FE = 1.00).

EXAMPLE PROBLEM #10: HOW TO FIND FLOWING PRESSURE AND TO VERIFY RATE

Given data: $\bar{p}_R = 2{,}000$ psi
$p_{wf} = 1{,}000$ psi
$q_o = 100$ bpd
FE = 0.60

Find $(q_o)_{max}$: $\frac{p_{wf}}{\bar{p}_R} = \frac{1{,}000}{2{,}000} = 0.5$

From Fig. 1.36 $\frac{q_o}{(q_o)_{max}} = 0.461$: $(q_o)_{max} = 217$ bpd for FE = 1.00.

Find p_{wf} necessary for 217 bpd when FE = 1.00,

$$p_{wf} = 2{,}000 - (0.6)\ (1{,}000) = 1{,}400 \text{ psi.}$$

Now, verify that 217 bpd is correct for FE = 1.00

$$\frac{p_{wf}}{\bar{p}_R} = \frac{1{,}400}{2{,}000} = 0.70$$

From Fig. 1.36 $q_o/(q_o)_{max} = 0.461$ and $(q_o)_{max} = \frac{100}{0.461} =$ 217 bpd (verified).

EXAMPLE PROBLEM #11: HOW TO CONSTRUCT IPR CURVES FOR DIFFERENT VALUES OF FE

We will use the example of Standing to determine the average IPR for the well with existing damage and find its flow capability if stimulation increased FE to 1.3.

The following data was given including three flow tests:

Test number	p_{wf}, psig	q_o, bpd
1	1,440	172
2	1,200	315
3	1,015	345

$\bar{p}_R = 1{,}850$ psig; FE = 0.7

Required:

Construct IPR curves for FE = 0.7 and FE = 1.3.

In order to obtain a value for $(q_o)_{max}$ for FE = 1.00 the three tests are averaged as follows:

(1) Test number	(2) $p_{wf}/\bar{p}_R$	(3) $q_o/(q_o)_{max}$ FE = 1	(4) q_o	(5) $(q_o)_{max}$ FE = 1
1	0.78	0.26	172	660
2	0.65	0.39	315	808
3	0.55	0.49	345	704
			Average	724

Column (3) or $q_o/(q_o)_{max}$ in the table was obtained from the FE = 0.7 curve of Fig. 1.36. Column (5) or $(q_o)_{max}$ is found by dividing the q_o of column (4) by the value of $q_o/(q_o)_{max}$ in column (3). For example in Test 1, $(q_o)_{max} = 172/0.26 = 660$ bpd. The average value of 724 bpd is then used for $(q_o)_{max}$ when FE 1.0.

In order to construct IPR curves the following table is prepared for $\bar{p}_R$ = 1,850 psi and $(q_o)_{max}$ = 724 bpd.

At this point we can assume values of p_{wf} or values of $p_{wf}/\bar{p}_R$. It is more logical for ease in reading Fig. 1.36 to assume values of $p_{wf}/\bar{p}_R$.

		FE = 0.7	
Assumed $p_{wf}/\bar{p}_R$	Calculated p_{wf}	$q_o/(q_o)_{max}$ FE = 1.00	q_o (for FE = 0.7)
0.8	1,480	0.23	167
0.6	1,110	0.44	319
0.4	740	0.61	441
0	0	0.87	630

The same type of table is prepared for FE = 1.3.

		FE = 1.3	
Assumed $\bar{p}_{wf}/\bar{p}_R$	Calculated p_{wf}	$q_o/(q_o)_{max}$ FE = 1.00	q_o (for FE = 1.3)
0.8	1,480	0.41	297
0.6	1,110	0.72	521
0.4	740	0.91	659
0	0	1.10 (extrapolated)	

This information is then plotted in Fig. 1.39.

This type of information and IPR curves can be extremely beneficial in determining the feasibility of a well stimulation treatment. For example, note in Fig. 1.39 that for p_{wf} = 800 psi the well makes 420 bpd under its present damaged producing conditions when FE = 0.7, compared to 630 bpd if the FE can be changed to 1.3. It is not difficult to calculate the rapid "pay-out" time for this treatment.

For a solution gas drive reservoir these tests would need to be repeated, perhaps each year, depending upon the drop in pressure and other changing conditions. However, they should retain the same general shape.

CLASS PROBLEM #11-A: TO CONSTRUCT IPR CURVES FOR DIFFERENT VALUES OF FE

The following tests were taken on a well:

Test no.	p_{wf}	q_o, bpd	
1	800	140	
2	750	170	$\bar{p}_R$ = 1,680 psi
3	850	130	FE = 0.60

Construct IPR curves for FE = 0.60, FE = 1.00, and FE = 1.40.

1.344 Predicting future inflow performance curves

Inflow performance curves are normally constructed to show the flow rates possible vs. certain flowing bottom hole pressures. However, it is also desirable to have a plot of this type for future predictions. In order to construct such curves, we need information as to changes in bottom hole pressure, good production records, etc. Two methods will be presented here—one theoretical and the other empirical.

1.3441 Standing's extension of Vogel's work to predict future IPR curves

Standing[10] extended the work of Vogel for solution gas drive reservoirs. He gave the following development and example problem as to how future IPR curves can be developed from a current productivity index value, and other pertinent well information.

The equation for Vogel's curve is as follows:

$$\frac{q_o}{(q_o)_{max}} = 1 - 0.2\left(\frac{p_{wf}}{\bar{p}_R}\right) - 0.8\left(\frac{p_{wf}}{\bar{p}_R}\right)^2 \qquad (1.22)$$

This equation can be rearranged to:

$$\frac{q_o}{(q_o)_{max}} = \left(1 - \frac{p_{wf}}{\bar{p}_R}\right)\left[\left(1 + 0.8\left(\frac{p_{wf}}{\bar{p}_R}\right)\right)\right] \qquad (1.42)$$

The productivity index of a well can be defined as:

$$J = \frac{q_o}{\bar{p}_R - p_{wf}} \qquad (1.43)$$

By combining equations 1.42 and 1.43 we have:

$$J = \frac{(q_o)_{max}}{\bar{p}_R}\left(1 + 0.8\,\frac{p_{wf}}{\bar{p}_R}\right) \qquad (1.44)$$

If we assume fluid saturations to be the same everywhere in the reservoir, which is analogous to "zero drawdown," then:

$$J^* = \lim_{p_{wf} \to \bar{p}_R} J \qquad (1.45)$$

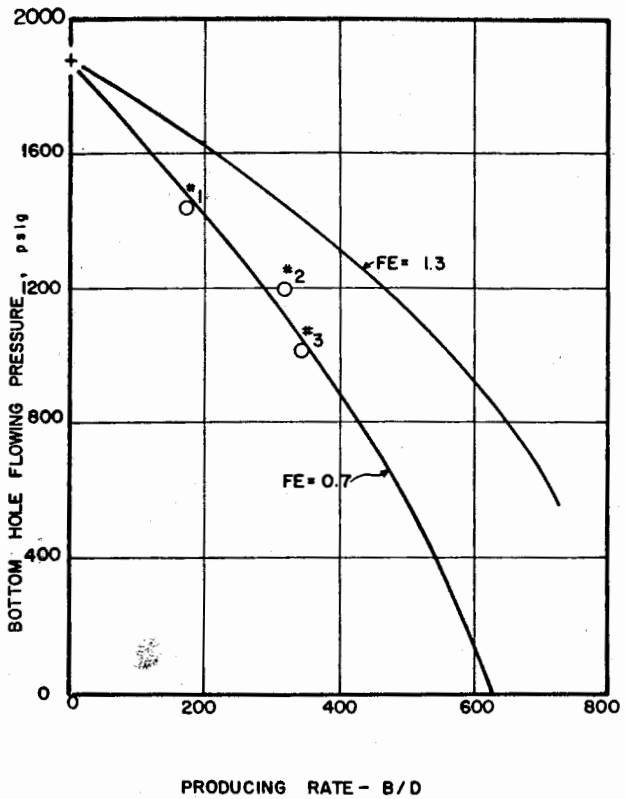


Fig. 1.39 Average IPR curves for example well calculations (after ref. 8).

or applying this same analogy to equation 1.44:

$$J^* = \frac{1.8(q_o)_{max}}{\bar{p}_R} \tag{1.46}$$

The relation of J to J* is then:

$$\frac{J}{J^*} = \frac{1}{1.8}\left(1 + 0.8\,\frac{p_{wf}}{\bar{p}_R}\right) \tag{1.47}$$

J* can also be arrived at from the following equation:

$$J^* = \frac{7.08(10^{-3})\,K_o\,(\bar{S}_o, \bar{S}_g)\,h}{\mu_o(\bar{p}_R)\,B_o\,(\bar{p}_R)\,[\ln \frac{r_e}{r_w} - 3/4]} \tag{1.48}$$

By combining equations 1.22 and 1.46 we can eliminate the term $(q_o)_{max}$:

$$q_o = \frac{J^*\bar{p}_R}{1.8}\left[1 - 0.2\left(\frac{p_{wf}}{\bar{p}_R}\right) - 0.8\left(\frac{p_{wf}}{\bar{p}_R}\right)^2\right] \tag{1.49}$$

If J_p^* is the present day value and J_f^* is the future value, then from the ratio of future and present values of equation 1.48:

$$J_f^* = J_p^*\left(\frac{K_{ro}}{\mu_o B_o}\right)_f \Big/ \left(\frac{K_{ro}}{\mu_o B_o}\right)_p \tag{1.50}$$

Procedure to develop future IPR curves is:

(1) Calculate J* from equation 1.47 and present known value of J from a well test.

(2) Adjust J_p^* to J_f^* by means of equation 1.50. K_{ro} values must be obtained from an appropriate means.

(3) Calculate future IPR's by means of equation 1.49. It is necessary to assume values of p_{wf} and calculate the corresponding values of q.

EXAMPLE PROBLEM #12: HOW TO PREDICT FUTURE IPR CURVES

Standing[10] gave the following example problem:

Given data:

40-acre spacing. Residual oil saturation = 15%

J = 0.92. Interstitial water saturation = 20%

q = 400 bpd

$\bar{p}_R$ = 2,250 psig

	Present	Future
$\bar{p}_R$	2,250 psig	1,800 psig
p_{wf}	1,815 psig	–
μ_o at $\bar{p}_R$	3.11 cp	3.59 cp
B_o at $\bar{p}_R$	1.173	1.150
Avg. oil saturation	0.768	0.741
K_{ro}	0.815	0.685

The purpose is to develop an IPR curve for the future time when the pressure is 1800 psig. A brief discussion on the determination of relative permeabilities is in order. Fig. 1.40 sketches the drainage oil relative permeability curve for the condition of 20% interstitial water and 15% residual oil. If K_{ro} is defined in terms of the effective oil permeability at interstitial water saturation (no gas saturation), the value must be 1 at $S_L = 1$. $S_L = S_{wi} + S_o$. This is the upper starting point of the K_{ro} curve. The lower end of the curve is at $S_{Lr} = 0.35$. $S_{Lr} = 0.35 = S_{wi} + S_{or}$. At residual oil saturation of 15%, K_{ro} becomes zero. This is the beginning and end point of the K_{ro} curve, and now we need to determine the shape of the curve between the two limits.

From the work of A. T. Corey,[11] we find that the *shape* of the drainage curve of the wetting phase (oil is considered wetting in preference to gas in the gas-oil-water-rock system) takes the mathematical form:

$$K_{ro} = \left(\frac{S_L - S_{Lr}}{1 - S_{Lr}}\right)^n \tag{1.51}$$

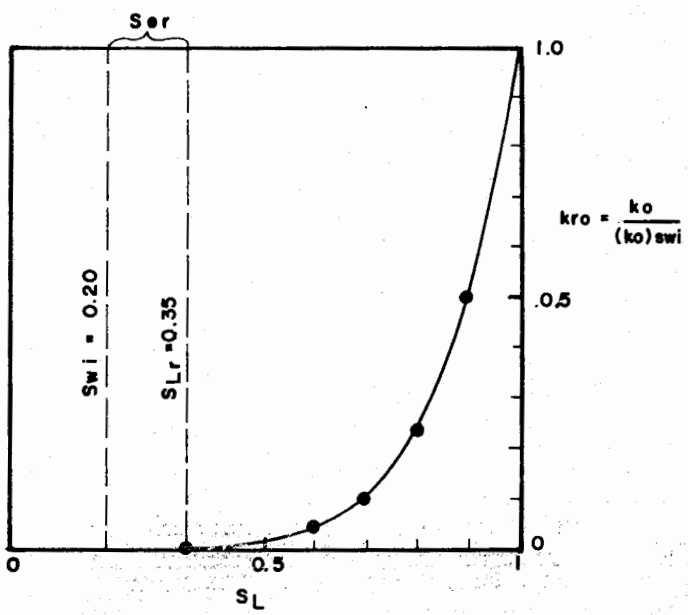


Fig. 1.40 Relative permeability curve for example problem.

where n = constant depending on the pore size distribution of the rock. For consolidated sandstones and nonvuggular limestones n is approximately 4. Unconsolidated sandstones have a lower value of n. In units of oil saturation, we have:

$$S_L = S_o + S_{wi} \tag{1.52}$$

$$S_{Lr} = S_{or} + S_{wi} \tag{1.53}$$

Subtracting 1.53 from 1.52 gives

$$S_L - S_{Lr} = S_o - S_{or} \tag{1.54}$$

also

$$1 - S_{Lr} = 1 - S_{or} - S_{wi} \tag{1.55}$$

So the Corey type relationship of equation 1.51 now becomes, for this particular sand:

$$K_{ro} = \left(\frac{S_o - S_{or}}{1 - S_{or} - S_{wi}}\right)^4 = \left(\frac{S_o - 0.15}{0.65}\right)^4 \tag{1.56}$$

By knowing both end points, we can now calculate a few points to fill in the curve:

S_o	K_{ro}	S_L	S_g
0.80	1.00	1.00	0
0.70	0.512	0.90	0.10
0.60	0.229	0.80	0.20
0.50	0.084	0.70	0.30
0.40	0.022	0.60	0.40

These are plotted on Fig. 1.40.

The next step is to apply this to the IPR relationships. Equation 1.48 is the radial flow equation for a closed outer boundary written in terms of the PI. This equation also has the conditions that fluid saturations are the same throughout the drainage volume and that fluid properties are evaluated at the average pressure. K_{ro} is saturation dependent. It is obtained by means of a Tarner gas depletion calculation to get S_o at a given pressure and application of the equation above to obtain K_{ro}.

In calculating a PI for the well at any future time, we need only to know the values for the future pressure and saturation. We then write equation 1.48 for the present time and the future time and obtain a ratio. This is usually referred to as J_{rel} or relative PI, and is equation 1.50 which may be written as follows:

$$J_{rel} = \frac{J_f}{J_p} = \left(\frac{K_{ro}}{\mu_o B_o}\right)_f \Big/ \left(\frac{K_{ro}}{\mu_o B_o}\right)_p \tag{1.57}$$

Solution:

$$(1)\ J_p^* = \left[\frac{1.8}{1 + 0.8\left(\frac{1{,}815}{2{,}250}\right)}\right](0.92) = 1.01 \text{ bpd/psi} \tag{1.47}$$

$$(2)\ J_f^* = 1.01\left(\frac{0.685}{3.59 \times 1.150}\right)\Big/\left(\frac{0.815}{3.11 \times 1.173}\right)$$

$$= 0.750 \text{ bpd/psi} \tag{1.50}$$

(3) Prepare table for future IPR by assuming values of p_{wf} and calculating corresponding rate where:

$$q_o = \frac{J_f^* \bar{p}_R}{1.8}\left[1 - 0.2\left(\frac{p_{wf}}{\bar{p}_R}\right) - 0.8\left(\frac{p_{wf}}{\bar{p}_R}\right)^2\right] \tag{1.49}$$

(1) p_{wf}	(2) $p_{wf}/\bar{p}_R$	(3) q_o
1,800	1.0	0
1,600	0.89	143
1,400	0.78	270
1,200	0.67	383

Prepare a plot of p_{wf} (Col. 1) vs. q_o (Col. 3) as shown in Fig. 1.41. The inherent error in this type of extension is prediction of correct future relative permeabilities.

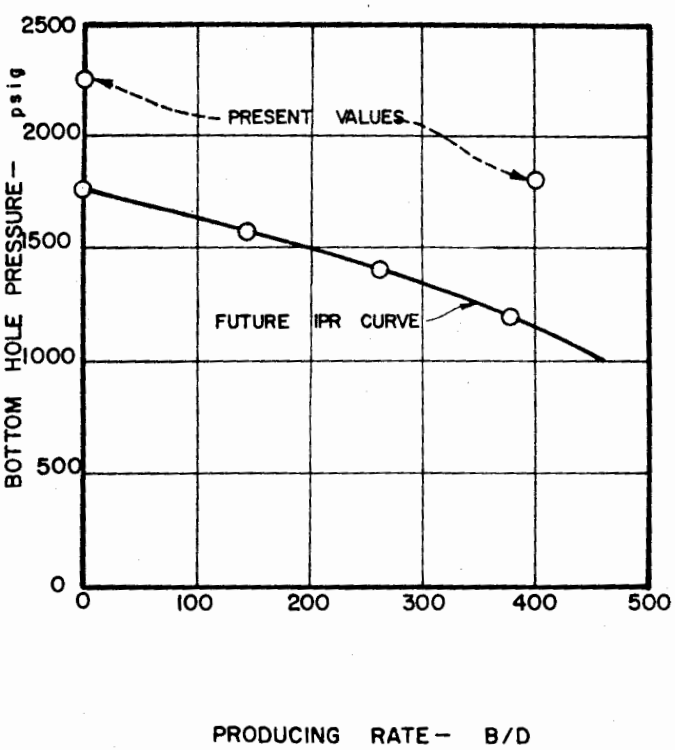


Fig. 1.41 Calculated IPR curve for condition of $\bar{p}$ = 1800 psig (after ref. 10).

CLASS PROBLEM #12-A

Given data:

	Present	Future
$\bar{p}_R$	2,500 psia	2,000 psia
p_{wf}	2,300 psia	1,500 psia
q_o	400 bpd	—
μ_o	3 cp	3.6 cp
B_o	1.18	1.16
Oil saturation	0.76	0.67
Residual oil saturation		20%
Residual water saturation		22%
Formation of Sandstone		

Find q_o when the static pressure drops to 2,000 psia and for a flowing pressure of 1,500 psia.

CLASS PROBLEM #12-B: TO PREDICT FUTURE IPR CURVE

Given data: (Solution gas drive well)

	Present	Future
$\bar{p}_R$	1,800 psi	1,500 psi
p_{wf}	1,600 psi	
μ_o at $\bar{p}_R$	4 cp	4.5 cp
B_o at $\bar{p}_R$	1.16	1.145
Avg. oil saturation	0.75	0.72
Residual oil saturation		18%
Interstitial water saturation		22%

60-acre spacing

$J = 1.25$

$q = 250$ bpd (all oil)

Construct an IPR curve for the future time when $\bar{p}_R = 1{,}500$ psi.

K_{ro} values may be calculated from:

$$K_{ro} = \left(\frac{S_o - S_{or}}{1 - S_{or} - S_{wi}}\right)^4, \text{ and average oil}$$

saturations may be determined by material balance calculations.

1.3442 Practical solution for House Mountain field—Canada

Eickmeier[12] presented a study of the House Mountain field located about 130 miles northwest of Edmonton, Canada. He predicted future performance of this rapidly declining field by making use of inflow performance relationship curves. These curves were used to predict production performance, determine need for pressure maintenance, and plan future artificial lift requirements.

The pressure in this field was declining at 100 psi/month. A test on one well showed a PI of 0.21 bpd/psi after the well had produced 15,000 bbl. If he assumed a straight line extrapolation of rate vs. flowing pressure a value of 580 bpd would be obtained for $p_{wf} = 300$ psi (assumed pumped-off pressure). However, after a cumulative production of 70,000 bbl, the well was only capable of producing 155 bpd. After two years of production the well was incapable of pumping 64 bpd (allowable).

It is quite evident that any straight line extrapolation of PI would give misleading results and would predict a much higher production rate than possible from the well.

Unit negotiations and preliminary waterflood investigations were being considered and therefore good estimates of maximum production rates from each well and the entire field were needed.

In order to prepare inflow performance curves (IPR), the pressure decline rate was established from various tests, and was found to be declining at a rate inversely proportional to porosity footage. Pressures were measured where possible, production tests were taken, gradient curves were used to estimate flowing pressures in some cases, and dynagraph cards were obtained from pumping wells at different values of cumulative production. This information is shown in Fig. 1.42 and a family of IPR curves is shown in Fig. 1.43. Finally, a set of generalized IPR curves for the House Mountain field were developed and are shown in Fig. 1.44 with an example problem.

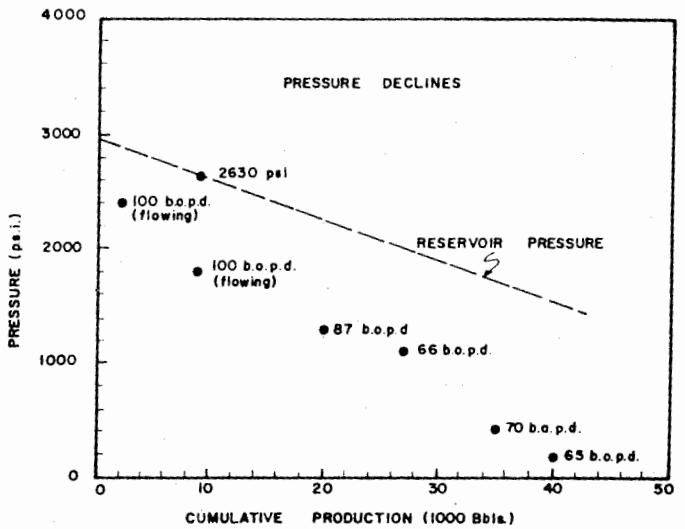


Fig. 1.42 Pressure vs. cumulative production (after ref. 12).

Eickmeier[12] was more interested in showing how the IPR's would collapse with decreasing reservoir pressure. He did not use Vogel's work in establishing the shape of his curves but the shape was determined from available data. He took into account the IPR curvature at a given static pressure as well as the change in shape of the IPR curve with declining pressure.

1.3443 Another procedure for predicting PI's into the future

This method was originally proposed by Shell Oil Company[3] and discussed by Brown.[13]

It was originally determined by Gilbert, et al.[6] that a surface producing rate vs. pressure drawdown will generally plot a straight line on log-log paper and that this line will deviate very little from a 45° slope.

In order to utilize this procedure a series of at least

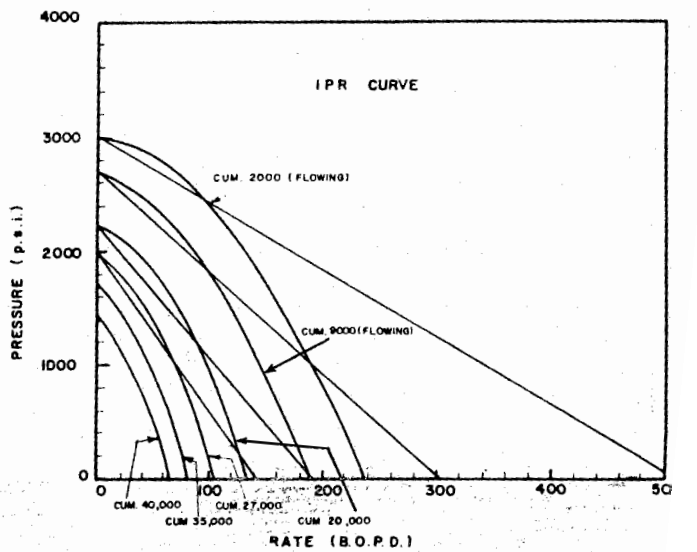


Fig. 1.43 Pressure vs. rate (after ref. 12).

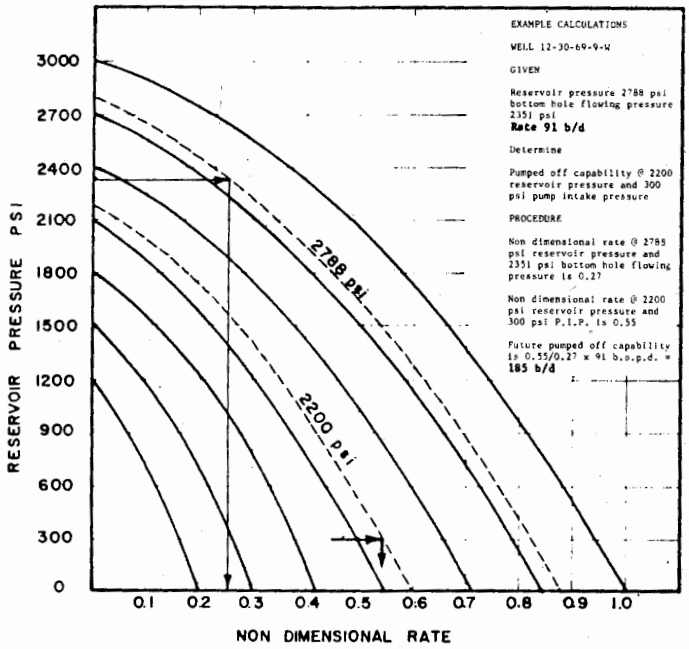


Fig. 1.44 Generalized IPR curves (after ref. 12).

3 tests on a well are necessary, that is, one test of 3 different flow rates vs. the corresponding flowing bottom hole pressures, and a measure of the static bottom hole pressure. The cumulative production of the well should also be available at that time. With this one set of tests the slope of the line on a plot of rate vs. drawdown can be established on log-log paper and will generally be very close to 1.0.

Once this has been established other lines having the same slope and for different cumulative recoveries can be drawn on the same plot.

Additional information needed is a record of the average PI vs. cumulative recovery normally giving a straight line on semi-log paper (refer to Fig. 1.46). This permits the prediction of PI's into the future at different cumulative recoveries. A plot of production rate against cumulative recoveries for various drawdowns in pressure can be made giving a means of predicting rate at future cumulative recoveries depending on the drawdown.

And finally an inflow performance curve of flowing pressure vs. rate at any one cumulative recovery can be made. This represents an empirical procedure that has been proven in the field.

Procedure

In order to offer a working procedure to utilize the preceding information on inflow performance relationships over the life of a well producing below saturation pressure from a solution gas reservoir the following step-wise procedure was taken from the Shell Oil Company[3]:

It is necessary to run a pressure flow rate test as well as having information on the change in PI vs. cumulative recovery.

(1) Plot PI vs. cumulative recovery as shown in Fig. 1.45 from tests obtained during the life of the well. These tests should be conducted at or near the same drawdown in pressure.

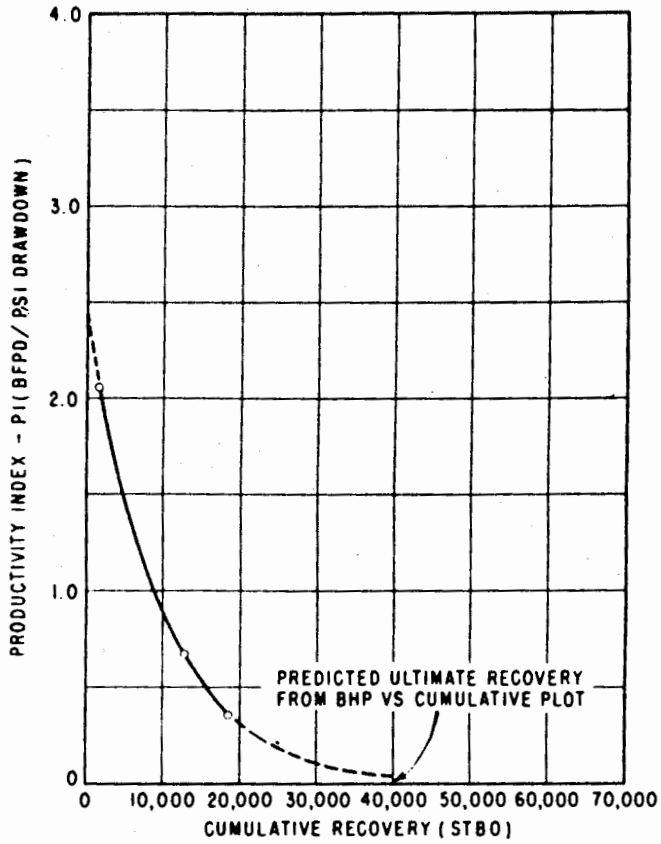


Fig. 1.45 PI change vs. cumulative recovery (after ref. 3).

In this example we have the following 3 points given:

PI	Cumulative recovery, bbl
2.1	2,000
0.7	13,000
0.35	18,400

(2) Plot the data given in Step (1) and Fig. 1.45 on semi-log paper as noted in Fig. 1.46. A straight line is obtained from which the PI at the economic limit can be obtained, as well as the necessary drawdown for the production rate at the economic limit. If we assume an economic limit of 40,000 STB, we find the PI from Fig. 1.46 to be 0.04 bpd/psi. The necessary drawdown in pressure for this rate of 4 bpd $= \frac{q}{PI} = \frac{4}{0.04} = 100$ psi. This can then be used as a reference drawdown pressure to prepare a flow rate vs. cumulative production curve. This does not necessarily coincide with the drawdown in pressures which were used in each test for PI's at different cumulative recoveries. Some error probably occurs in calculating drawdown at an economic limit rate if the drawdown differs appreciably from the test drawdowns.

(3) Run an IPR survey consisting of three stabilized flow rates and a BHP buildup survey. The following test was conducted and plotted in Fig. 1.47.

q_o	p_{wf}
0	910
85	650
120	520
130	480

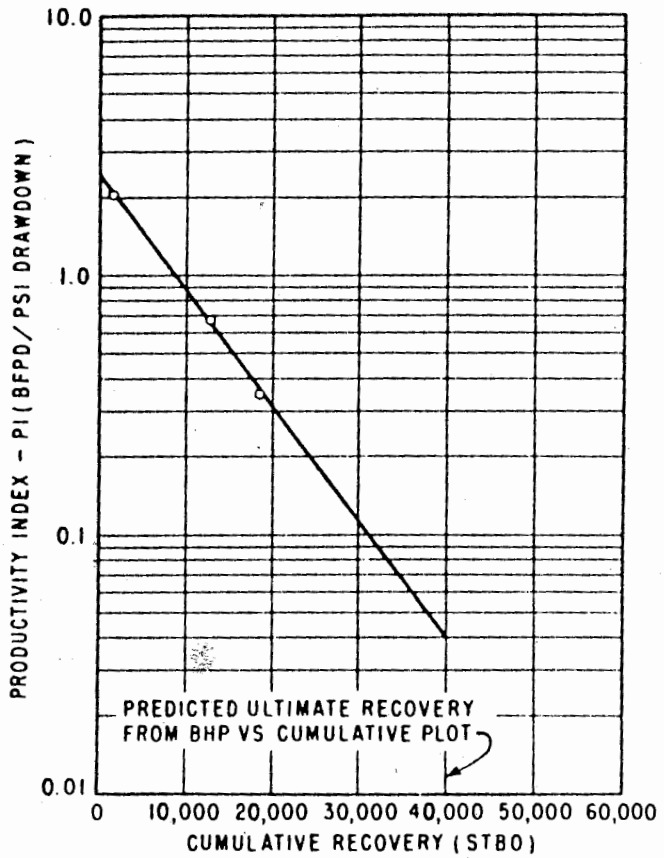


Fig. 1.46 PI change vs. cumulative recovery (after ref. 3).

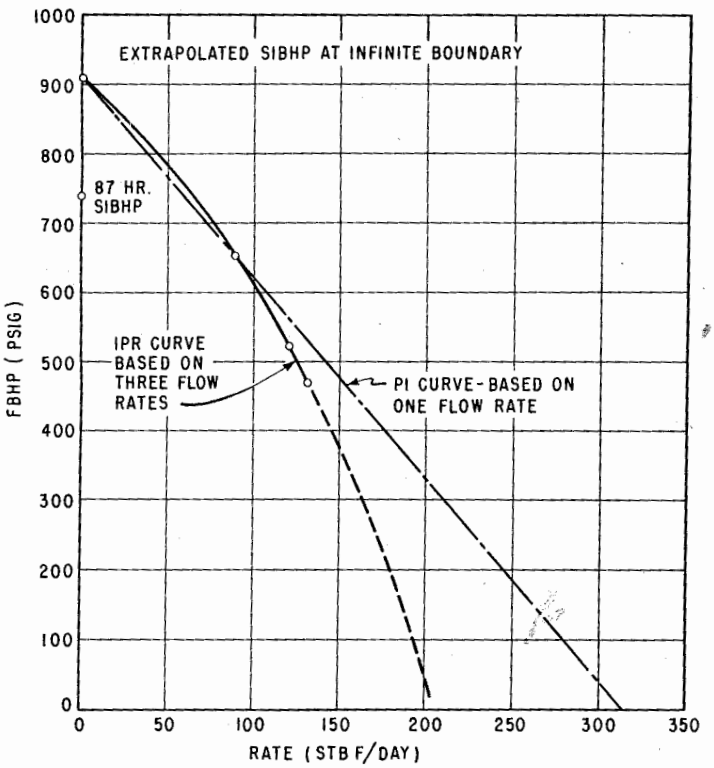


Fig. 1.47 Inflow performance curve (after ref. 3).

(4) Plot results of IPR survey on coordinate paper such as Fig. 1.47 to verify that survey results conform to a smooth curve which is generally concave to the origin. Results which deviate much from the example might invalidate the survey and should be used only with caution.

(5) Plot results of IPR survey on log-log paper in the manner of line (1) in Fig. 1.48. This should be a straight line with a slope near 45°. In this case it is not 45°. The test was taken at a cumulative recovery of 18,400 bbls. Note that the PI is decreasing with drawdown as expected in this type of reservoir.

(6) Predict PI performance for the well at various cumulative oil recoveries from saturation pressure to the ultimate recovery as estimated by conventional methods. If reliable PI's taken below saturation pressure are available for other cumulative oil recoveries, this information can be plotted as shown in Fig. 1.46. This will provide a reasonable approximation of PI performance over the life of the well. (The PI measured for the least production rate of the IPR survey will provide one control point for this plot.)

(7) Choose a reference drawdown pressure to obtain a rate at reference drawdown control point at the producing economic limit. This may be arbitrary as long as it is of low magnitude. However, a good practice is to estimate the producing economic limit and, by using the predicted PI at economic limit from a plot such as Fig. 1.46 calculate the drawdown pressure which is required to produce the economic limit rate. In this case we used 4 bpd for a PI of 0.04 giving a drawdown of 100 psi.

(8) Construct estimated IPR performance curve at depletion in the manner of line (2) in Fig. 1.48. This is a line drawn through the reference drawdown pressure of 100 psi and corresponding rate at ultimate recovery of 4 bpd with an assumed slope of 45°. Note that it does not have the same slope as line (1). I would suggest drawing line (2) parallel to line (1) since line (1) shows the well behavior from field data.

(9) Plot producing rate of 4 bpd at reference drawdown pressure of 100 psi vs. cumulative oil recovery on semilog paper as illustrated by Fig. 1.49. Use rate of 4 bpd at reference drawdown of 100 psi at ultimate recovery as one control point and rate of 46 bpd at the

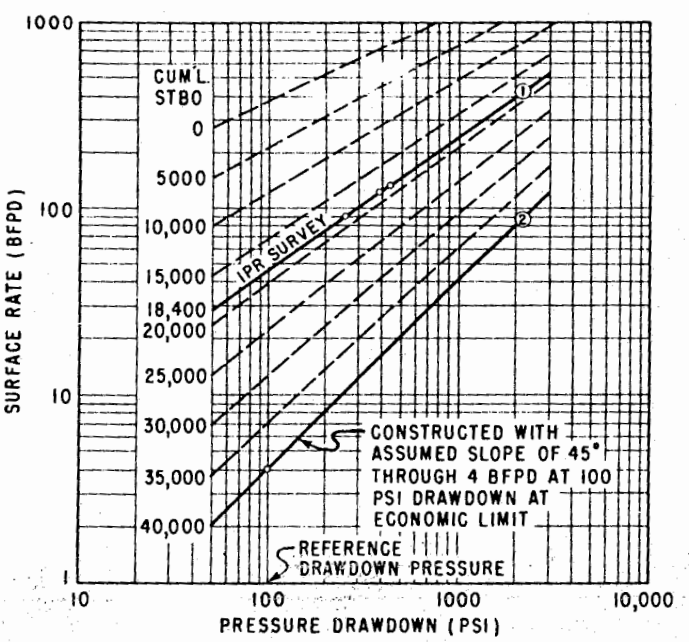


Fig. 1.48 Rate vs. pressure drawdown (after ref. 3).

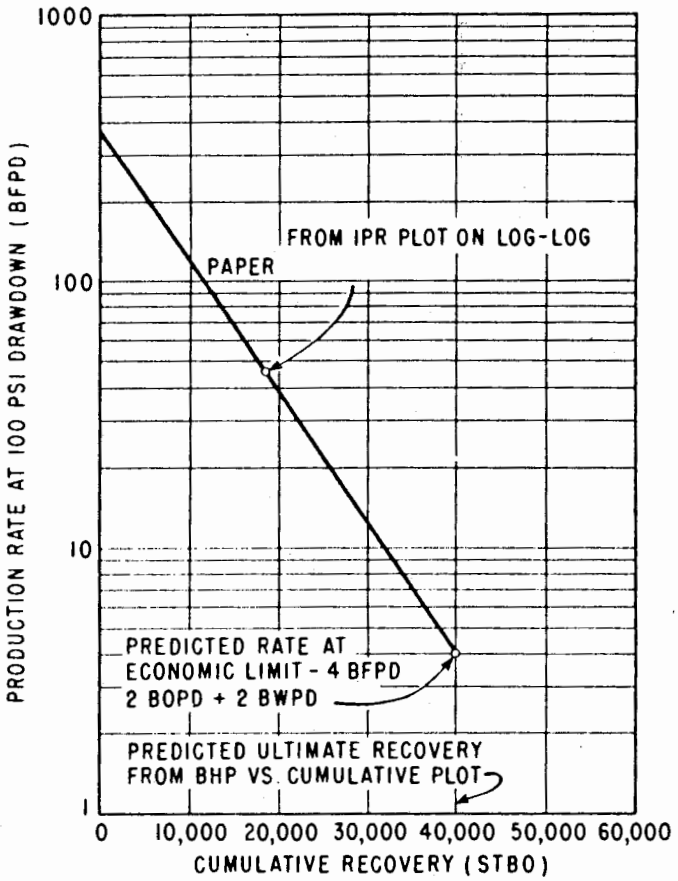


Fig. 1.49 Estimated rates for a 100 p.s.i. drawdown.

same reference drawdown of 100 psi as determined from the IPR curve (line (1) in Fig. 1.48) as the second control point. Draw a straight line as illustrated.

From this line, determine various rates at the reference drawdown pressure of 100 psi for other cumulative oil recoveries and plot these points as control points for predicted IPR performance at various cumulative oil recoveries. Construct IPR curves at other cumulative oil recoveries as illustrated by dashed lines in Fig. 1.48 making use of rates obtained from Fig. 1.49 for 100 psi drawdown. I would suggest that all these lines be drawn parallel to line (1) of Fig. 1.48 unless other IPR surveys are available at other cumulative recoveries.

(10) From Fig. 1.48 an IPR curve of p_{wf} vs. q can be constructed at any cumulative recovery thus permitting the determination of future production rates for varying flowing bottom hole pressures. In order to do this, the static bottom hole pressure must be known at the cumulative recovery. The following reservoir pressure data is available:

$\bar{p}_R$	Cumulative recovery
1,280	0
900	18,400
700	30,000

This data plots a straight line on semilog paper and gives the following static pressures at the requested cumulative recoveries.

Cumulative recovery, bbl	$\bar{p}_R$, psi
10,000	1,030
18,400	900
30,000	700

From Fig. 1.48 and the corresponding $\bar{p}_R$ and cumulative recovery, we can prepare tables 1.3, 1.4, and 1.5 of p_{wf} vs. q for each cumulative recovery and static pressure:

TABLE 1.3 (N_p = 10,000)

$\bar{p}_R$	Δp	q	p_{wf}
1,030	100	125	930
1,030	300	230	730
1,030	500	320	530
1,030	800	420	230

TABLE 1.4 (N_p = 18,400)

$\bar{p}_R$	Δp	q	p_{wf}
900	100	48	800
900	300	100	600
900	500	150	400
900	700	195	200

TABLE 1.5 (N_p = 30,000)

$\bar{p}_R$	Δp	q	p_{wf}
700	100	13	600
700	200	24	500
700	400	42	300
700	500	52	200

This information is then plotted as in Fig. 1.50.

EXAMPLE PROBLEM #13

(As given by Salas)[14]

Given:

Flowing tests were taken on a well when the cumulative production was 45,000 bbl, and $\bar{p}_R$ = 1,600 psi.

Rate, bpd	Flowing pressure, psi
200	1,200
250	1,000
300	750

The following additional information had been obtained during the life of the well:

$\bar{p}_R$	PI	Cumulative production, bbl
2,800	5.0	0
2,200	1.8	20,000
2,000	1.0	30,000
1,600	.46	45,000

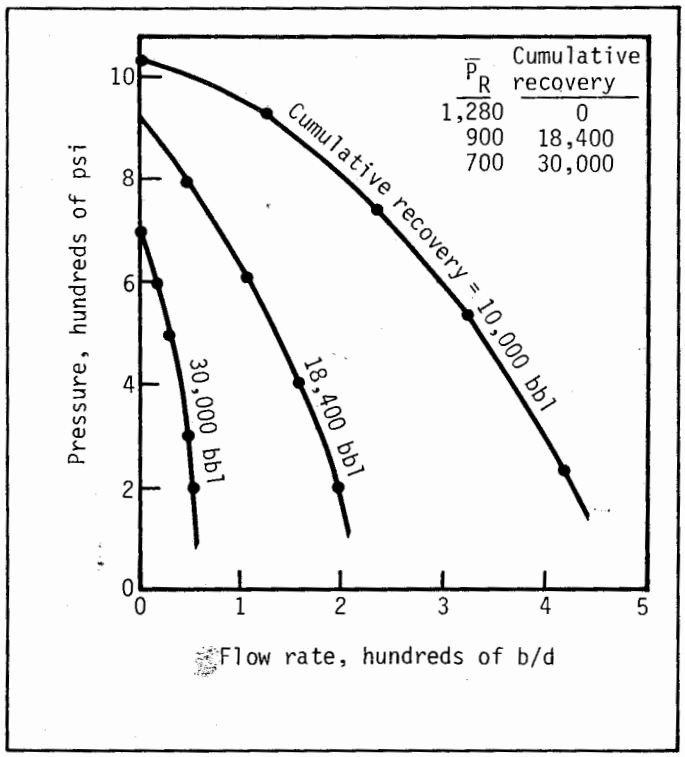


Fig. 1.50 Inflow performance curves.

All tests were conducted at pressure drawdowns between 300 and 400 psi.

It is desired to find the following information:

(1) PI at an assumed economic limit of 100,000 bbl of production.

(2) The drawdown necessary to produce an assumed economic limit rate of 8 bpd.

(3) Construct a curve of rate vs. flowing pressure on rectangular coordinate paper to note the shape of the curve.

(4) Make a log-log plot of rate vs. drawdown for a cumulative production of 45,000 bbl. This line reflects changing PI's depending upon drawdown. For example, for $\Delta p = 100$, PI = 1.00.

(5) Construct curves having the same slope of step (4) on the same plot for cumulative productions of 0, 20,000, 30,000, etc. bbl.

(6) With the reference drawdown pressure as determined in step (2) construct a plot of rate vs. cumulative production.

(7) Determine the production rate possible at different cumulative recoveries for the reference drawdown pressure.

(8) Construct IPR curves for cumulative recoveries of 20,000, 45,000, and 60,000 bbl.

Solution

(1) In order to determine the PI at the economic limit of 100,000 bbl we should prepare Figs. 1.51 and 1.52. From them we can read the PI to be 0.025 bpd/psi at a cumulative production of 100,000 bbl.

(2) The drawdown necessary to produce the economic limit is found as follows:

$$\text{Drawdown} = \frac{8}{\text{PI}} = \frac{8}{0.025} = 320 \text{ psi.}$$

(3) The plot of rate vs. flowing pressure is noted in Fig. 1.53. (Only as a matter of interest).

(4) A plot of rate vs. pressure drawdown is noted in Fig. 1.54 with the line for a cumulative production of 45,000 bbl being plotted first from given data.

(5) Additional lines having the same slope are plotted for recoveries of 0, 20,000, 30,000, 60,000, 80,000, and for 100,000 bbl on Fig. 1.54.

(6) With the reference drawdown pressure of 320 psi, a plot of rate vs. cumulative production is shown in Fig. 1.55. This shows that initially the well is capable of producing 1,600 bpd with 320 psi drawdown but declines to 8 bpd after the production of 100,000 bbl. Other lines for other pressure drawdowns can be included on the same plot.

(7) The rates possible for the reference drawdown pressure of 320 psi for different cumulative productions are as follows:

Cumulative production, bbl	PI	q (for $\Delta p = 320$ psi)
0	5.0	1,600
20,000	1.8	576
30,000	1.0	320
60,000	0.21	67
80,000	0.072	23

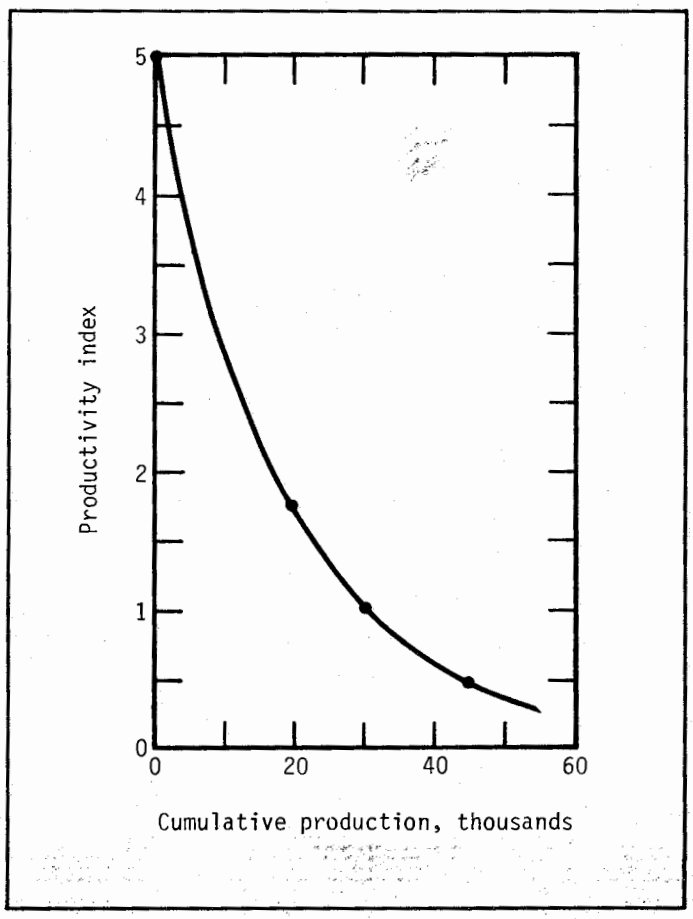


Fig. 1.51 PI vs. cumulative production.

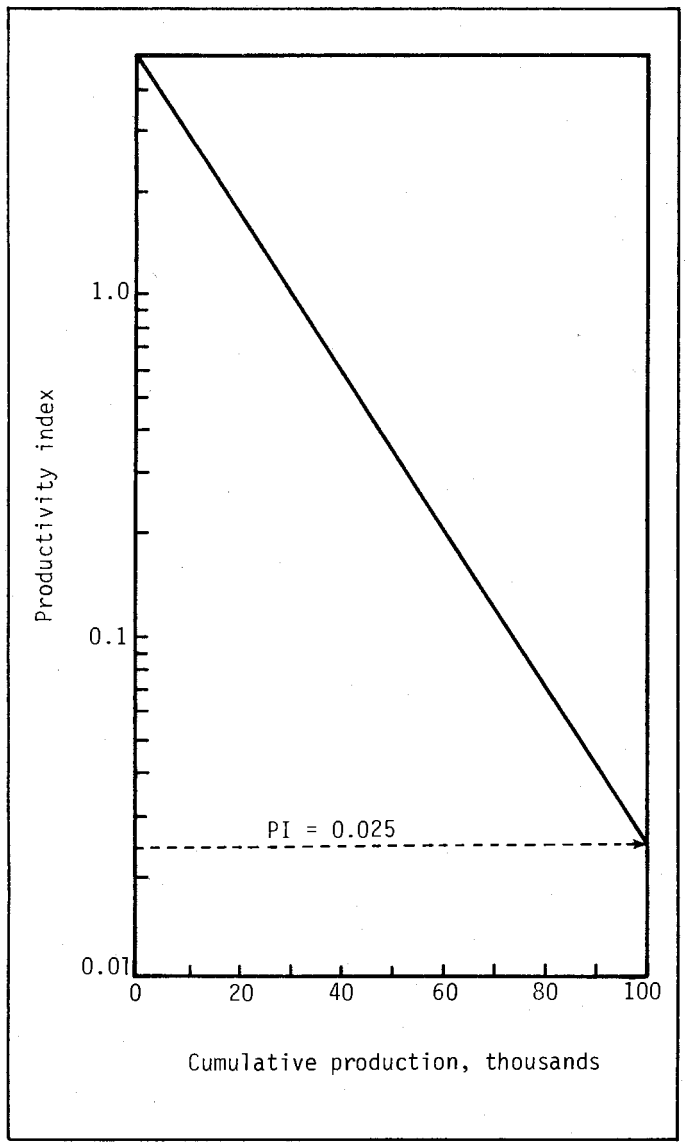


Fig. 1.52 PI vs. cumulative production.

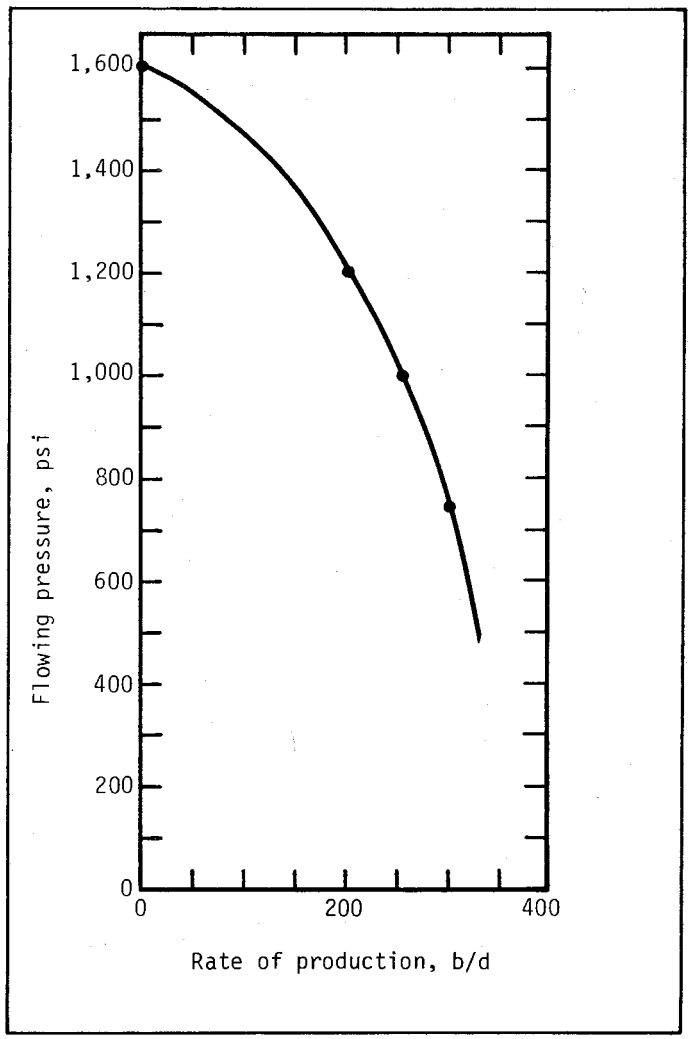


Fig. 1.53 Flowing pressure vs. rate.

CLASS PROBLEM #13-A

The following information was obtained on a well:

PI	Cumulative production, bbl
10	0
3.8	20,000
1.4	40,000
0.5	60,000

In addition, the following test was run on the well at a cumulative production of 40,000 bbl and $\bar{p}_R = 1{,}800$ psi.

Rate, bpd	Flowing pressure, psi	PI
0	1,800	—
324	1,620	1.8
560	1,400	1.4
746	1,100	1.07

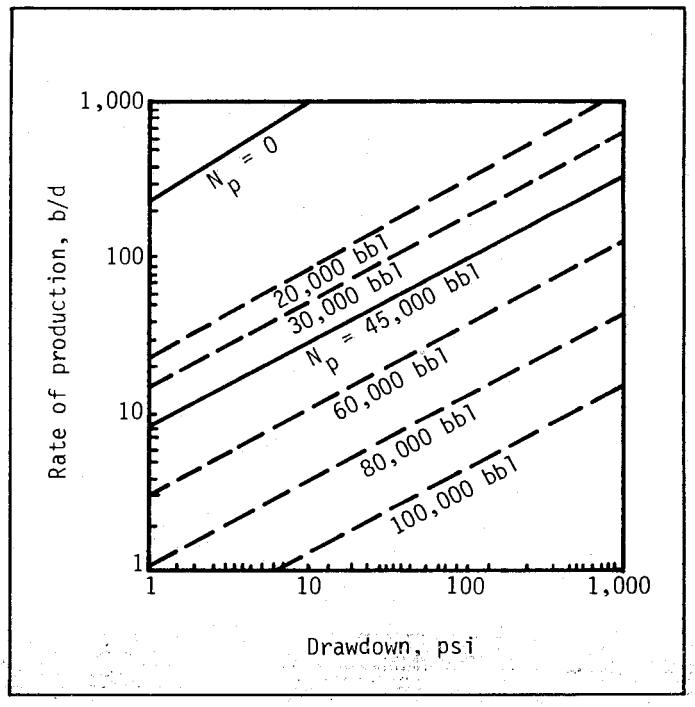


Fig. 1.54 Rate vs. drawdown.

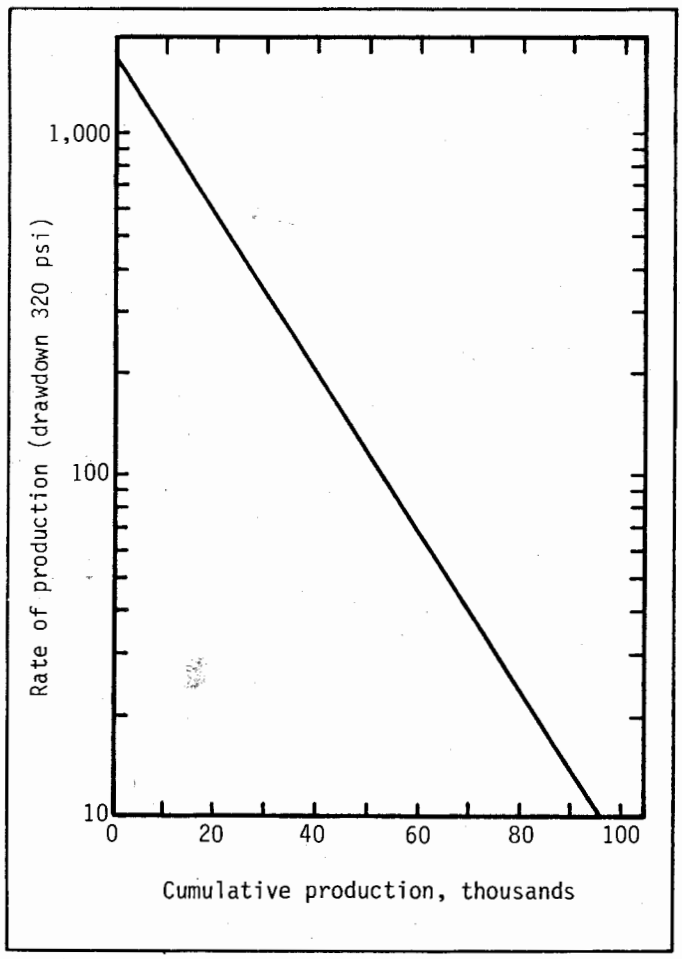


Fig. 1.55 Rate vs. cumulative production.

Find:

(a) PI at economic limit of 130,000 bbl.
(b) The drawdown necessary to produce an economic limit rate of 10 bpd.
(c) Prepare a log-log plot of rate vs. drawdown for a cumulative production of 40,000 bbl and for the economic limit of 130,000 bbl. Include cumulative recoveries of 0, 20,000, 40,000, 60,000, 80,000, 100,000 and 130,000 bbl.
(d) Prepare a plot of rate vs. cumulative production for cumulative productions of 40,000 and 100,000 bbl for the reference drawdown pressure as determined in (b).
(e) Prepare a plot of rate vs. cumulative production for drawdowns of 100, 400 and 800 psi.
(f) Prepare a table of rate vs. cumulative recoveries of the drawdown determined in (b).

CLASS PROBLEM #13-B

Given:
The following production tests were taken on a well:

Test #1 – Cumulative production = 0
Static pressure = 3,500 psi

Rate, bpd	Flowing pressure, psi
1,140	3,050
1,860	2,500
1,150	2,800

Test #2 – Cumulative production = 400,000 bbl
Static pressure = 2,900 psi

Rate, bpd	Flowing pressure, psi
700	2,300
1,300	1,550
1,000	1,940

A recent test on this well showed a rate of 570 bpd for a flowing pressure of 1,700 psi and a static pressure of 2,620 psi. Find:
(a) The maximum production rate possible from this well for a drawdown of 2,620 psi.
(b) What is the flowing pressure required for a production rate of 800 bpd?

CLASS PROBLEM #13-C

Another well in the same field as the well of Class problem #12-B was found to be producing 540 bpd with a drawdown of 1,100 psi.
Find the rate of production for a flowing pressure of 1,500 psi and a static pressure of 3,100 psi.

CLASS PROBLEM #13-D

Another well in the same field as the well of Class problem #12-B had the following tests:

Cumulative production, bbl	Static pressure, psi	Flowing pressure, psi	Rate, bpd
0	3,400	2,300	1,850
200,000	3,100	2,050	1,350

Using a pressure drawdown of 500 psi prepare the IPR curve (pressure flow rate diagram) for this well when the cumulative production is 500,000 bbl.

1.35 Isochronal and flow after flow testing of oil wells

1.351 Introduction

Multipoint testing of gas wells is considered to be an accepted procedure with "flow after flow" and isochronal testing being the two basic methods used.
A publication by Fetkovich[15] discussed the multipoint testing of oil wells. Although the multipoint testing of oil wells is not commonly used, a multipoint test was reported by T. V. Moore[16] on a well in the Yates field in 1930. The idea was to establish an open flow potential for an oil well similar to the procedure used for gas wells. The gas well testing has been well described.[17,18]

Although gas well testing procedures have been described, a discussion here will serve as a review. This discussion represents again the work of Fetkovich as presented to the Mid-Continent Section of the SPE of AIME in Tulsa, Oklahoma.[19] Much of the material is taken directly from his paper.

I want to give credit for this section on "Isochronal and flow after flow testing of oil wells" to Fetkovich[15,19] since much of the material is taken directly from his work, which represents one of the very fine contributions to our literature.

1.352 Gas well testing

"There are two basic types of multipoint tests:
1. *Flow after flow test*[17] (No shut-in between flows)
   a. Normal sequence (Fig. 1.56)
   b. Reverse sequence (Fig. 1.57)
2. *Isochronal test* (Well is shut-in between flows)
   a. True isochronal[2] (Fig. 1.58)
   b. Modified isochronal[3] (Fig. 1.59)"

1.3521 Flow after flow tests (see Figs. 1.56 and 1.57)

The flow after flow test starts from a shut-in condition after which a series of increasing flows (normal sequence) or decreasing flows (reverse sequence) is conducted. No (or very small) shut-in periods occur between each of the flows. Generally this is time to change an orifice plate. Flow times are usually arbitrary, or they can be set by a regulatory body when conducted for that purpose. As a general rule flow times are from 3 to 4 hours each.

If "stabilized" flows are obtained, the test may be considered good. This condition is generally obtained in high permeability reservoirs. Stabilization is defined in the Interstate Oil Compact Commission (IOCC) Manual[20]: "A constant flowing wellhead pressure or static column wellhead pressure and rate of flow for a period of at least 15 minutes shall constitute stabilization. . . ." If a well is tubing capacity limited, a pseudo-stabilization can occur if one uses only flowing tubing pressure as the criterion. Pseudo-stabilization can also occur as a result of flowing tubing temperature increase. Therefore, bottom hole or static column pressure stabilization is preferable for this definition.

This standard means for testing gas wells for many years was the so-called U.S. Bureau of Mines back pressure test (flow-after-flow). For this method to be valid a good well that stabilized fairly quickly was desirable. The procedure was as follows:

(1) Obtain $\bar{p}_R$ with pressure gage or calculate.

(2) Place the well on a flow rate with choke at the surface and flow for 3 to 4 hours, at which time the flowing pressure and the rate was constant. Generally this could be done at the surface without having a pressure gage at the bottom of the well.

(3) Obtain 4 different flow rates.

(4) Plot flow rate vs. $(\bar{p}_R^2 - p_{wf}^2)$ on log-log paper, which should be a straight line.

(5) Obtain the absolute open flow potential by reading q when $p_{wf} = 0$, that is, at $\bar{p}_R^2$.

1.3522 Isochronal tests (see Figs. 1.58 and 1.59)

Fetkovich noted that the isochronal method of multipoint testing gas wells is the only certain way of obtaining reliable performance curves. Each flow starts from a comparable shut-in condition. The shut-in must be close enough to a fully built up condition that any pressure rise still occurring will not affect pressure during drawdown of the subsequent flow; i.e., no prior transits exist during any flow period.

Although the flow periods for an isochronal test are usually of equal duration, they need not be. However, when a performance curve is plotted, data from flow periods of the *same duration* are plotted to obtain the *correct value of the slope, n* (Fig. 1.60). Note that rates and pressures at a *specific* time are plotted—*not average rate.*

The word isochronal implies equal times, and the isochronal test is based on the principle that the drainage radius established during a flow period is a function only of *dimensionless time* and is independent of the flow rate; i.e., for equal flow times the same drainage radius is established for different rates of flow. It follows that an isochronal test would yield a valid performance curve if conducted as either a constant rate or constant flowing pressure test. In fact, many low permeability gas well tests that exhibit severe rate declines on test are really constant wellbore pressure cases and should be analyzed as such. (In a paper by Winestock and Colpitts[22] their rate decline data analyzed as constant pressure case gives the same permeability value as a buildup test.) *A constant rate is not required for a valid isochronal test.* If one is attempting to short-cut the isochronal test using superposition, then and only then could a constant rate flow condition be required—but only for the purpose of using superposition.

For maximum information and minimum confusion Fetkovich has recommended the isochronal test method when multipoint tests are required, particularly on wildcat or initial development wells. Once the basic characteristics of the reservoir and fluid properties have been defined from valid isochronal tests, one should consider the possibility of reducing testing time without sacrificing information. The number of flows, and flow and shut-in times, can often be reduced and shut-in periods even eliminated in some cases.

Fetkovich commented on test procedures. Whenever possible bottom hole pressure gauges should be used. Surface pressures should be recorded with a dead weight tester and measured on both the tubing and annulus along with flowing temperatures. The frequency of taking the surface drawdown and buildup data should be sufficient for type curve analysis, i.e., early time data is critical for this analysis. Similarly, with about the same frequency, flow rate data should be recorded and reported. A constant wellbore pressure analysis or a Winestock and Colpitts analysis ($\Delta p^2/Q$ vs time) may be required.

Plotting and analysis of the test data, drawdown, buildup, and back-pressure curves on site during the test are rather critical to obtaining valid tests. Most important of all, the well must be cleaned up prior to

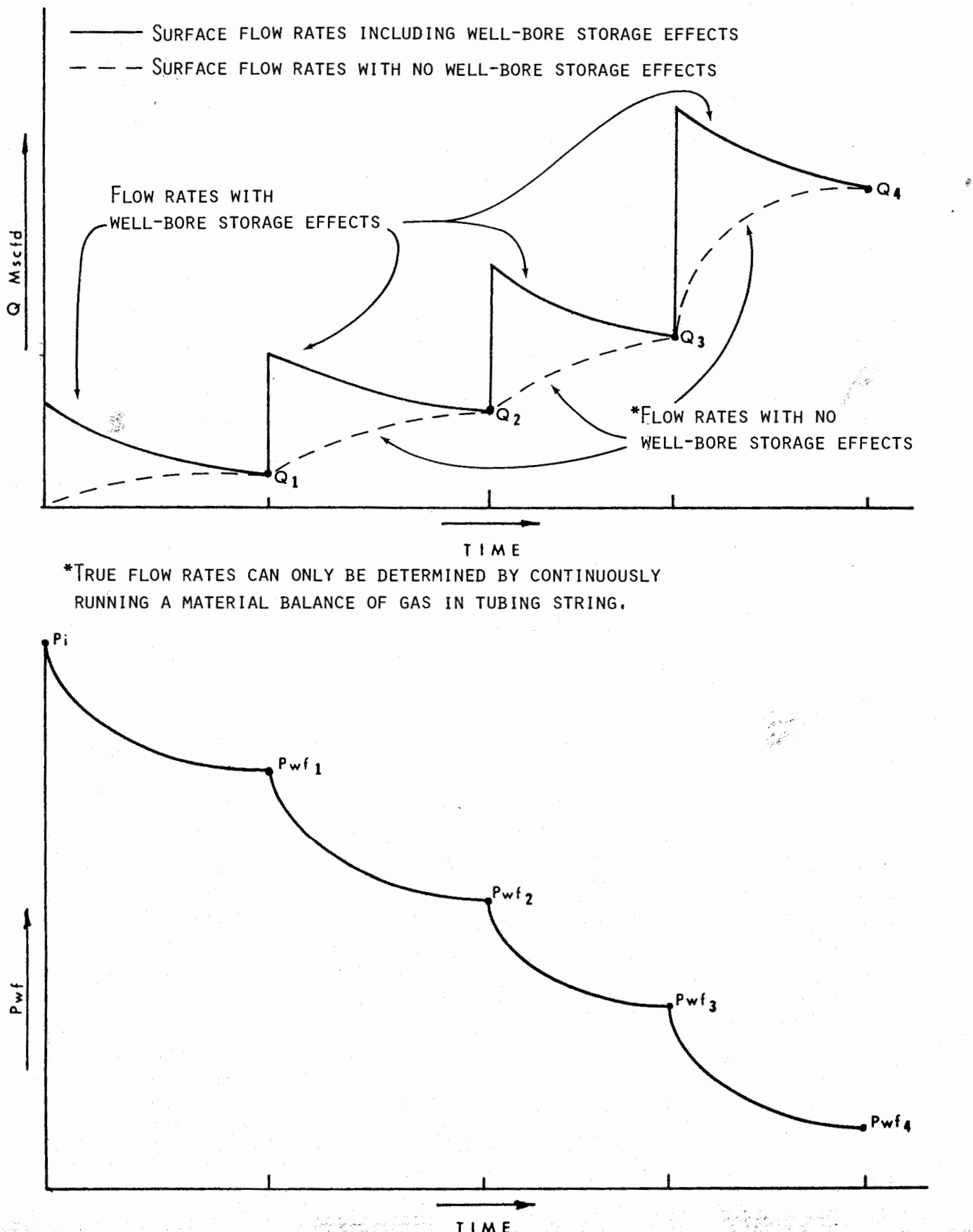


Fig. 1.56 Flow after flow (normal sequence) (after Fetkovich).

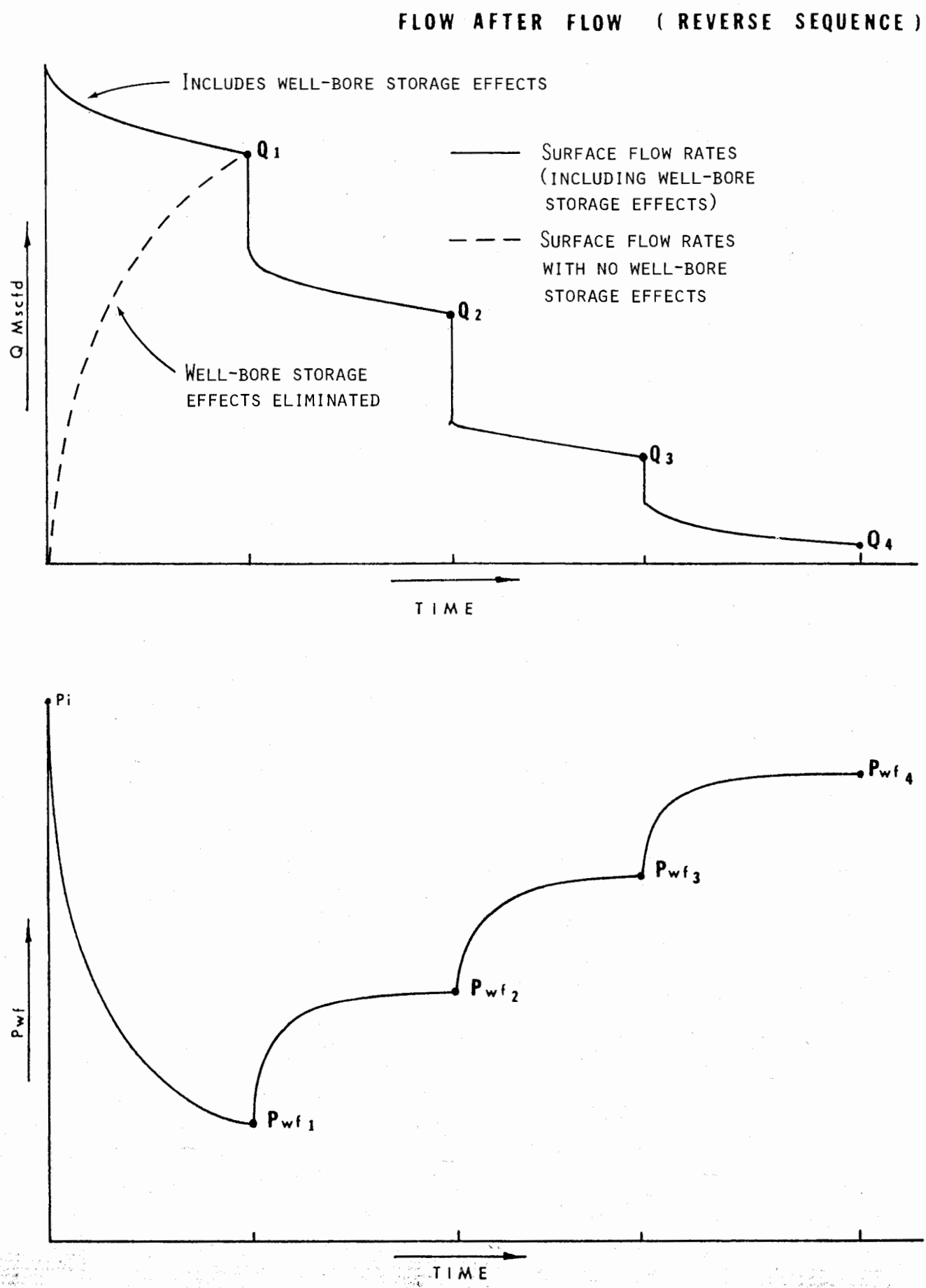
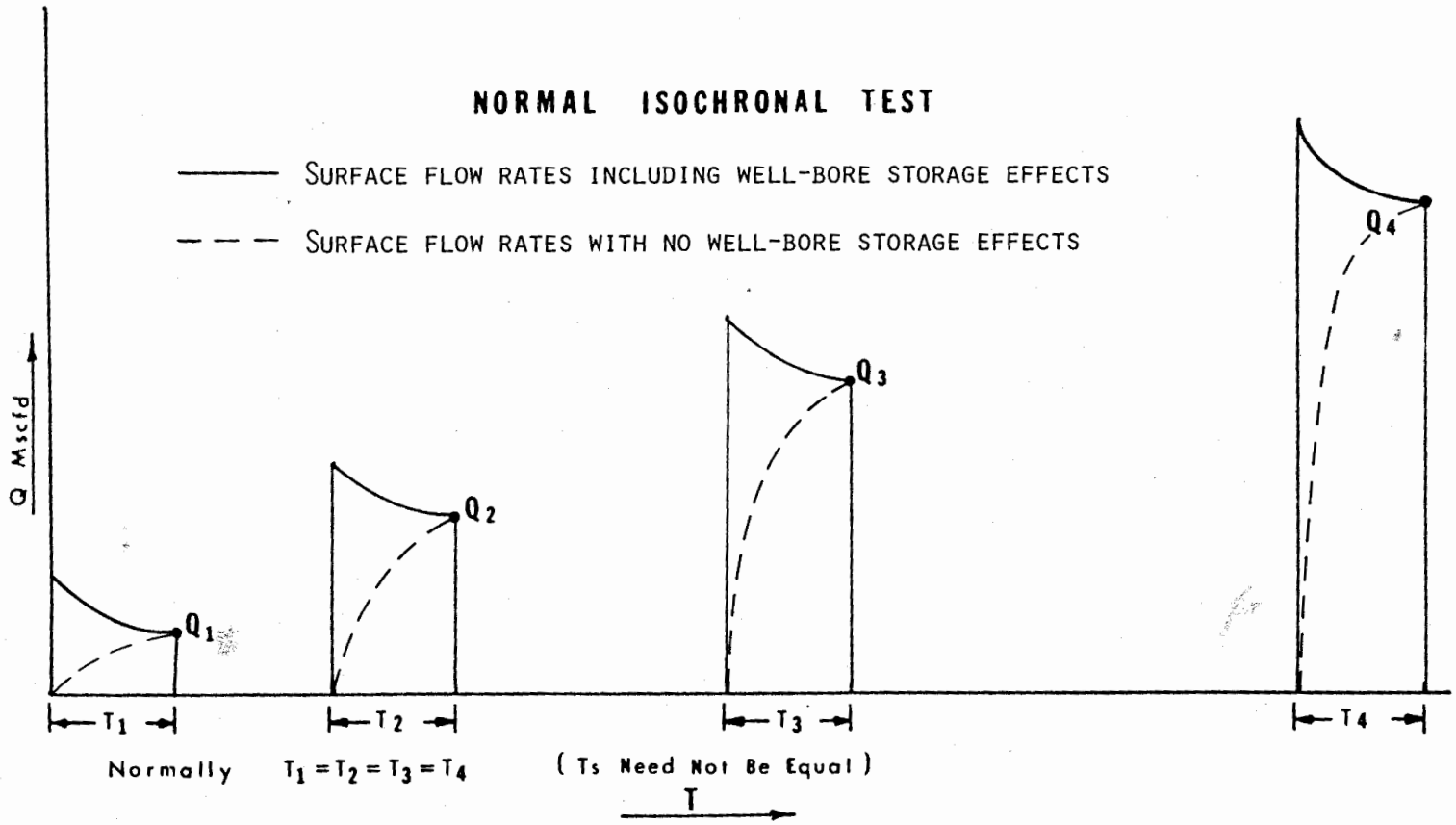


Fig. 1.57 Flow after flow (reverse sequence) (after Fetkovich).



T_1, T_2, T_3 AND T_4 MUST BE OF LONG ENOUGH DURATION TO ELIMINATE WELL-BORE STORAGE EFFECTS. BEWARE OF SHORT ISOCHRONAL TESTS!

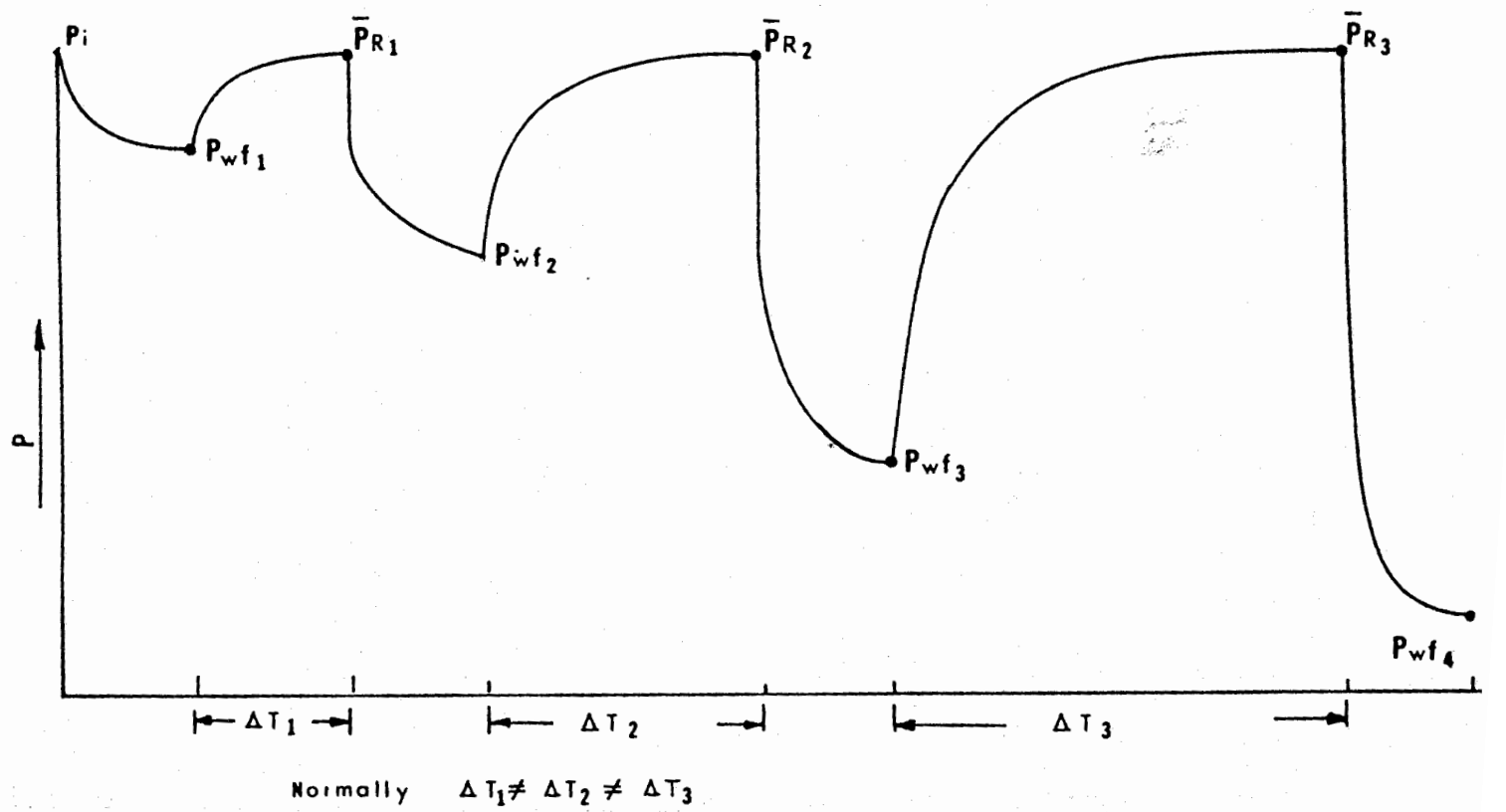
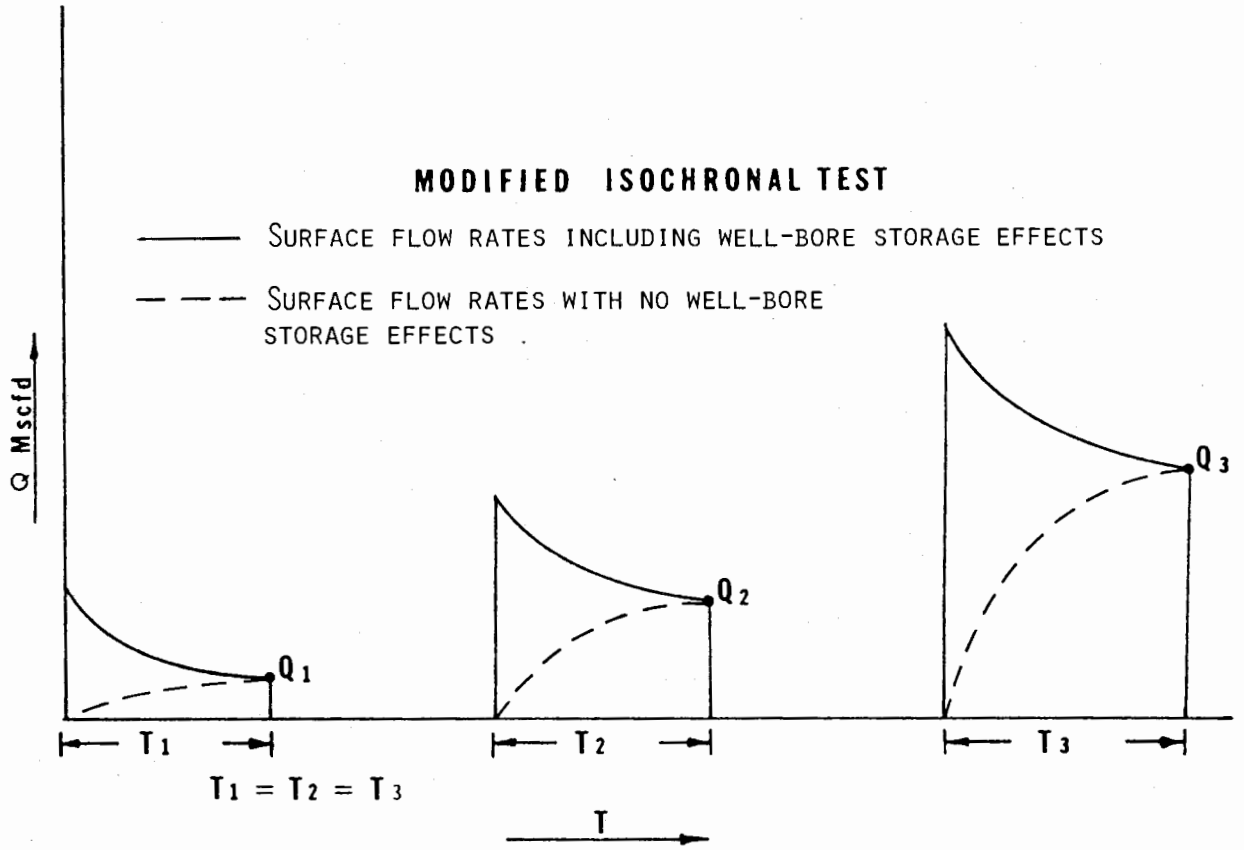


Fig. 1.58 Normal isochronal test (after Fetkovich).



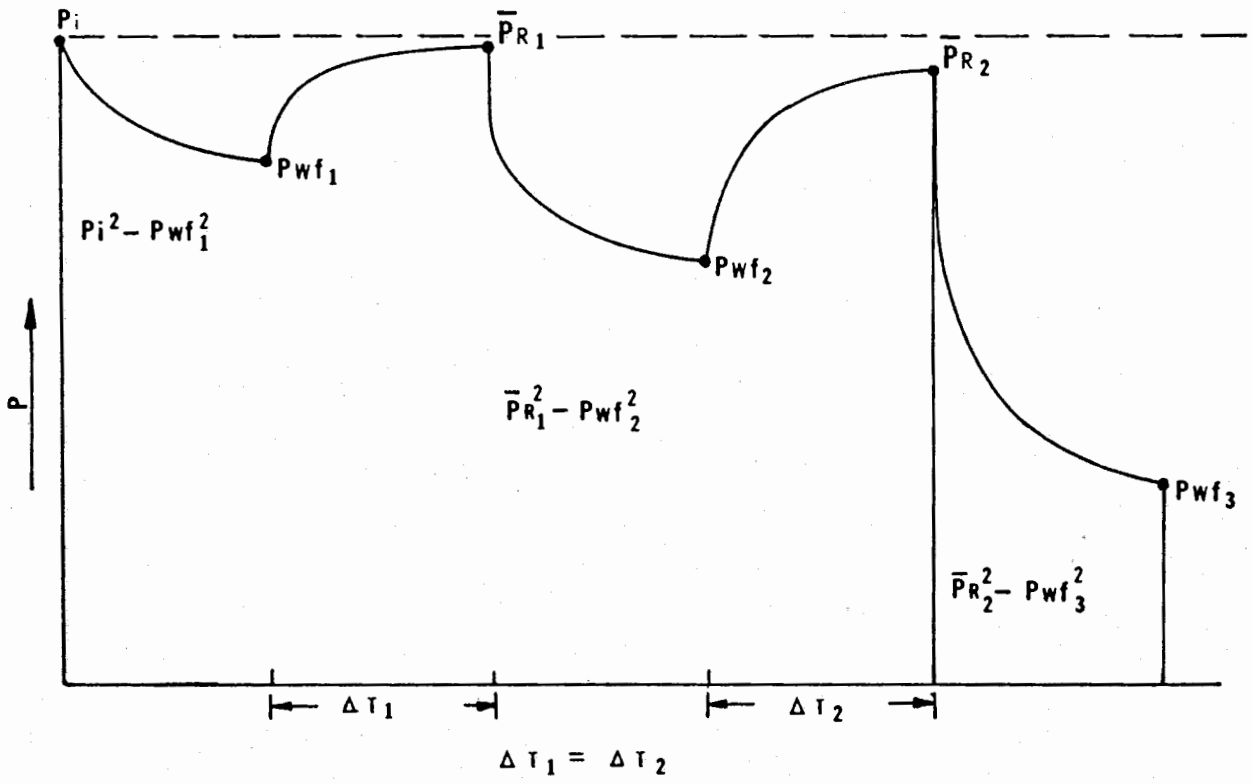


Fig. 1.59 Modified isochronal test (after Fetkovich).

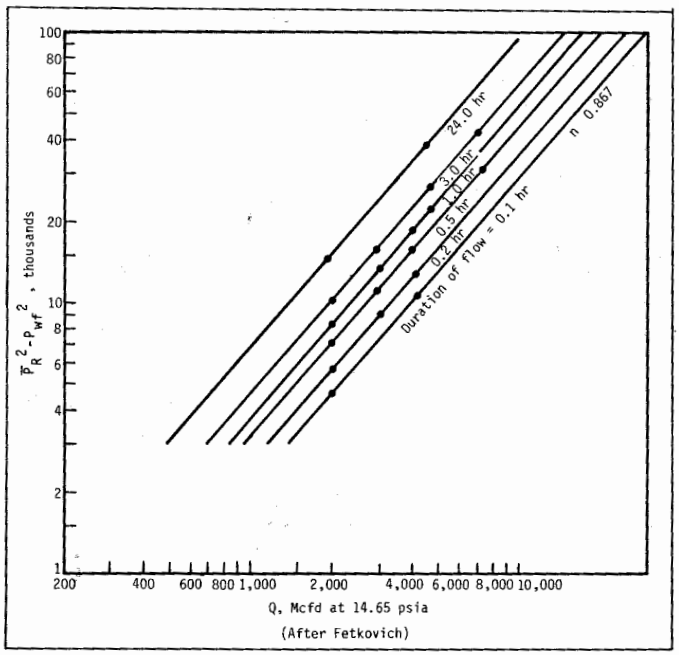


Fig. 1.60 Isochronal test (after Fetkovich).

conducting the test. The importance of a clean up flow is dramatically illustrated by the test results obtained on Well C, Fig. 1.61. As a general rule for selecting rates one should attempt to flow the well at or near the expected continuous sales rate. If sand, water coning, or other problems could develop, now is the time to find out.

Following is a typical isochronal test procedure used on a wildcat or early development well:

1. Initial flow (± 15 min)
2. Initial shut-in (± 2 hrs)
3. Clean up flow at maximum separator capacity (± 10 hrs)
4. Shut-in period (± 12 hrs)
5. Flow at ± ¼ maximum rate (± 6 hrs)
6. Shut-in period (± 9 hrs)
7. Flow at ± ½ maximum rate (± 6 hrs)
8. Shut-in period (± 9 hrs)
9. Flow at ± ¾ maximum rate (± 6 hrs)
10. Shut-in period (± 9 hrs)
11. Flow at maximum rate (± 6 hrs)

The above time periods are subject to change depending on an on-site analysis of the initial data.

The following procedure is given for testing a well by this method:

(1) Obtain $\bar{p}_R$ in some standard manner.

(2) Open the well on a specified choke size. The flow periods may be as short as 10 minutes but one hour is recommended. Record the flow rate and pressure at specific time intervals during this flow period such as every 15 minutes.

(3) Close the well in, allowing the static pressure to return to its original value (or nearly so).

(4) Open the well on another choke size (different flow rate). Obtain flow rate vs. pressure information at the same time intervals as noted in step (2).

(5) Repeat the procedure as desired. These tests can be taken on different days or even months or years.

(6) This information is plotted on log-log paper as described previously and a series of parallel lines with the same values of n at different time intervals will occur. (Note Fig. 1.60).

(7) One flow test can be conducted for a long period of time to find stabilized conditions (up to 15 days).

The different performance curves one could obtain on the same well from an increasing or decreasing sequence multipoint test, and an isochronal test is demonstrated by the results shown[21] in Fig. 1.62.

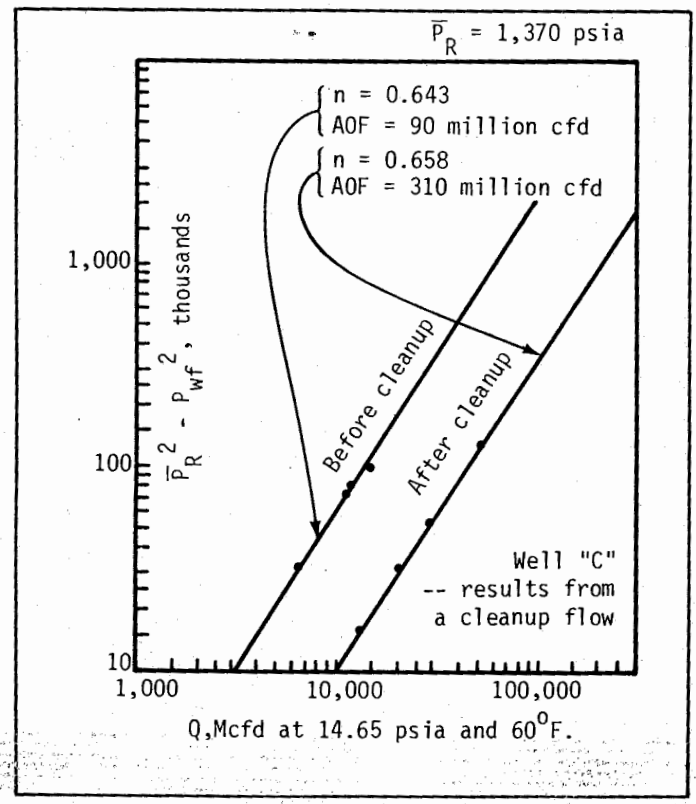


Fig. 1.61 Effect of cleanup (after Fetkovich).

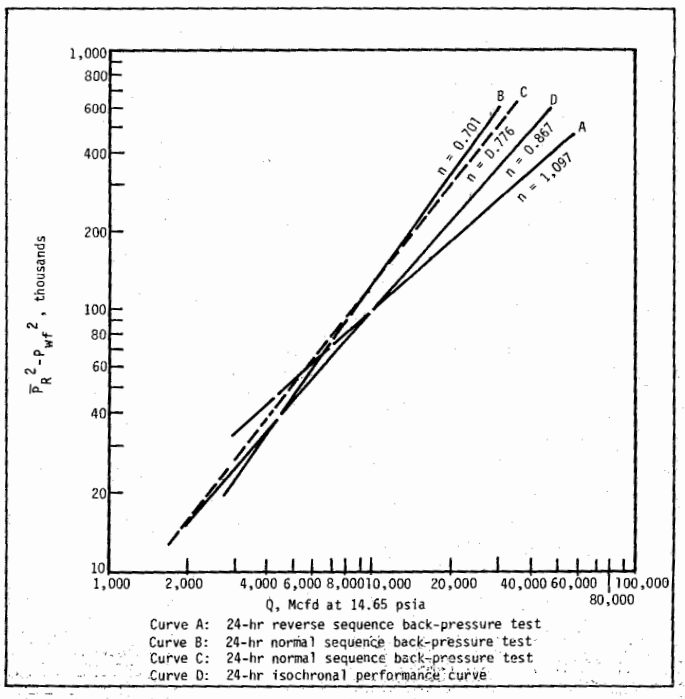


Fig. 1.62 Effect of test method (after Cullender).

These types of results are normally limited to tests conducted in low permeability reservoirs.

1.3523 Modified isochronal tests

Fetkovich noted that in very low permeability reservoirs it may require days to obtain a final buildup pressure even after relatively short periods of flow (2 to 3 hrs). In an attempt to shorten testing time, the modified isochronal test was proposed.[21] It is conducted with shut-in periods equal to the flow periods. The unstabilized shut-in pressures are used to calculate the difference in pressure relationship used with the next flow rate. Fetkovich noted that this method of testing has never been adequately justified either theoretically or by field comparisons with *true* isochronal tests.

What little discussion published justifying this method theoretically has been based on the assumption that flowing pressure behavior with time (superposition) is a function of the log of time $p = f(\ln t)$. However, most low permeability wells where the modified test would be practical require stimulation (hydraulic or acid fracs) to be commercial. In these cases pressures are more likely to be a function of the square root of time, $p = f(\sqrt{t})$. Modified tests under these conditions can have flowing pressure behaviors as functions of $\sqrt{t}$, transitional, or ln t—each for different flow rates.

For maximum reservoir information purposes, Fetkovich does not recommend the modified isochronal test, nor any other method that depends on the application of superposition techniques to shorten test times for low permeability wells. If time is of such importance in low permeability formations, one can conduct one long-duration flow period (being certain to be out of wellbore storage, $\sqrt{t}$, and transitional period prior to ln t behavior) and assuming a back-pressure curve slope (n) of one. Better still, the Two Flow Method[23] of Carter, Miller, and Riley would be preferred—i.e., two isochronal points.

1.3524 Conventional well test analysis ($\bar{p}_R$ < 2,500 psi)

Gas well analysis can be divided into two pressure regions: low to medium pressure, and high pressure wells. Much of the basic theory of testing and analysis was developed from well tests with reservoir pressure levels under 2,500 psi. This resulted in the familiar back pressure curve plotting of log q vs. log p^2 and pressure build-up and drawdown analysis using p^2 vs. $\log \frac{t + \Delta t}{\Delta t}$ plot and p^2 vs. log t, respectively.

With the advent of deeper drilling, gas wells have been discovered with reservoir pressures approaching 10,000 psi. In these cases, and down to about 2,500 psi, the conventional methods of analysis break down and the real gas potential theory approach must be used.

1.353 Testing of oil wells

Fetkovich[15] presented a method of analyzing "isochronal" and "flow after flow" multipoint back-pressure tests on oil wells. He conducted tests over a wide range of conditions including permeabilities from 6 md to 1,000 md, and different types of reservoirs. Depending upon the type of reservoir, a different interpretation method is required.

He also noted that a constant rate test was not necessary. It is only necessary to have the rates and pressures at the end of each test for isochronal and flow after flow tests. This simplifies the testing procedure since a constant choke size can then be used for each flow rate. The rate should be checked over the last few minutes of the test.

Fetkovich noted also that the $(q_o)_{max}$ as obtained by Vogel differed considerably from that of Fetkovich, and further that the $(q_o)_{max}$ as determined by Vogel can be different depending upon the flow rate and corresponding flowing pressure at which the test was taken. This, he concludes, is due to the rate-dependent skin effect. Neither Vogel nor Standing's extension of Vogel's work takes into account the rate-dependent skin effect.

Another contribution by Fetkovich is his procedure for generalizing IPR curves without any flowing tests at all on the well. In other words it is possible just to start with the reservoir parameters and prepare complete IPR curves for a well at different time periods, that is, decreasing static pressures.

In studying the work of Vogel[6], Fetkovich[15] noted that wells producing below the bubble point pressure should actually behave more like a gas well. This means $(\bar{p}_R^2 - p_{wf}^2)$ vs. q_o should plot a straight line on log-log paper with an exponent near unity. The oil well back pressure curves were found to follow the same general form as that for gas wells:

$$q_o = J'_o(\bar{p}_R^2 - p_{wf}^2)^n \qquad (1.58)$$

where J'_o = productivity index (back pressure curve coefficient) in bpd/psi^{2n}.

In examining some 40 oil well tests the exponent n was found to be between 0.568 and 1.000. This checks very closely with the values found for gas well testing. He then found the customary plot of q_o vs. logΔ p^2 plot to be as good for oil wells as gas wells.

Fetkovich[15] presented Fig. 1.63, which shows Vogel's reference curve with the corresponding equations for isochronal flow.

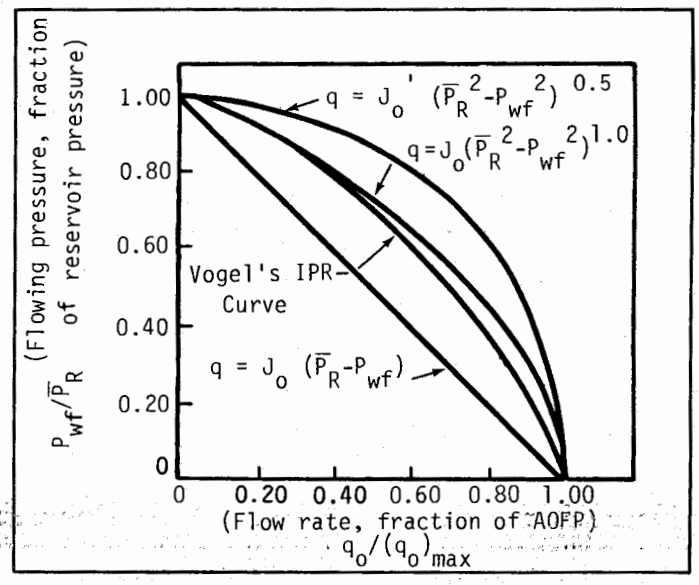


Fig. 1.63 Comparison of equations (after Fetkovich).

1.354 Basic equations and pressure functions presented by Fetkovich

The basic flow equation given by Evinger and Muskat for steady state flow is applicable for either oil or gas flow:

$$q_o = \frac{7.08Kh}{\ln\left(\frac{r_e}{r_w}\right)} \int_{p_{wf}}^{p_e} f(p)dp \qquad (1.59)$$

where f (p) can be any function of pressure. A typical pressure function is given in Fig. 1.64.

The total integral can be evaluated in two parts as follows:

$$q_o = \frac{7.08Kh}{\left[\ln\left(\frac{r_e}{r_w}\right) + S'\right]} \left[\int_{p_{wf}}^{p_b} \frac{K_{ro}(S,p)}{\mu_o B_o} dp + \int_{p_b}^{p_e} \frac{K_{ro}(p)}{\mu_o B_o} dp\right] \qquad (1.60)$$

where S' = skin effect (dimensionless).

For flow above the bubble point (single phase liquid flow), and assuming $K_{ro} = 1.0$ and evaluating $\mu_o B_o$ at the average pressure of $\frac{p_e + p_b}{2}$ the equation is written as follows:

$$q_o = \frac{7.08Kh}{\left[\ln\left(\frac{r_e}{r_w}\right) + S'\right]} \left[\int_{p_{wf}}^{p_b} \frac{K_{ro}(S,p)}{\mu_o B_o} dp + \frac{p_e - p_b}{\mu_o B_o}\right] \qquad (1.61)$$

The single phase steady-state flow equation results from integration of equation 1.59 since the pressure function is a constant.

$$q = \frac{7.08\ Kh}{\left[\ln\left(\frac{r_e}{r_w}\right) + S'\right]} \frac{(p_e - p_{wf})}{\mu B_o} \qquad (1.62)$$

Fetkovich considered the entire pressure function from p_e to 0 and he noted that f(p) could be represented approximately by two separate straight line segments and that q could be expressed as:

$$q = J'(p_b^2 - p_{wf}^2) + J(p_e - p_b) \qquad (1.63)$$

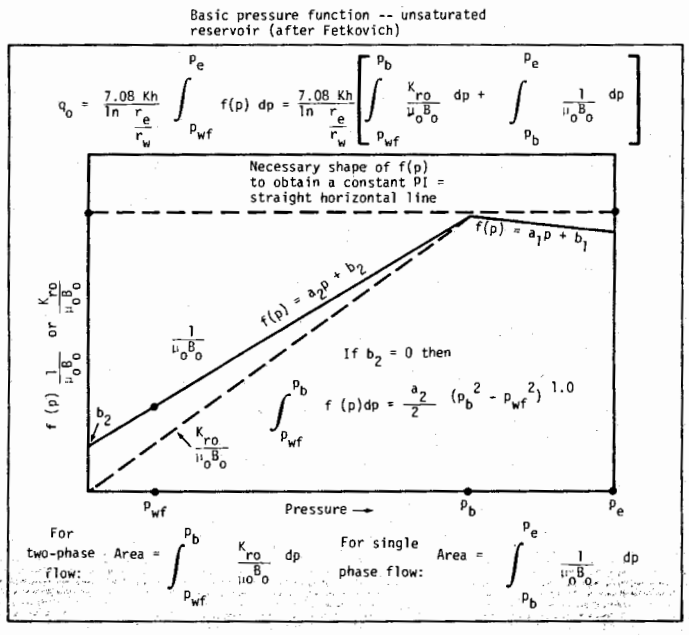


Fig. 1.64 Basic pressure function (after Fetkovich).

where:

J = regular PI (bpd/psi)
J' = PI (bpd/psi^{2n})

He noted that for drawdowns both above and below the bubble point pressure, a back pressure plot would appear as two straight line segments, and as a matter of fact the bubble point pressure could be determined from the intersection of these two lines.

He also noted that if the degree of undersaturation is slight, the two line segments may not be definable. Also he found that non-Darcy flow can exist even for all pressures above the bubble point. For all drawdowns below the bubble point pressure $J(p_e - p_b)$ of equation 1.63 is a constant while all the other terms vary non-linearly with flow pressure p_{wf}.

The overall effect results in an equation of the form:

$$q = C(p_e^2 - p_{wf}^2)^n \qquad (1.64)$$

where Fetkovich defined C as J'_o = b/d/psi^{2n}.

Equation 1.64 indicates that either a gas well or an oil well can have a slope less than 1.00 on a log q vs. log $\Delta(p^2)$ plot without non-Darcy flow existing. This was reported by Rowan and Clegg[24] for gas wells.

As p_e decreases to p_b, $n \rightarrow 1.0$ and $C \rightarrow J'$ such that for the oil well case, only the two-phase flow term remains. He then obtained the basic equation suggested from Vogel's results for $p_e \leq p_b$:

$$q_o = J'_o(p_e^2 - p_{wf}^2)^{1.0} \qquad (1.65)$$

Fetkovich[15] noted that equation 1.65 could be further generalized with an exponent, n, as follows:

$$q_o = J'_o(p_e^2 - p_{wf}^2)^n \qquad (1.66)$$

This equation was verified with numerous back-pressure tests on oil wells. Slopes much less than 1.00 were consistently obtained on wells in saturated reservoirs. This lead to the suspect of near-well-bore effects. Handy[25] made a study of the effect on PI of two phase flow in the vicinity of the well-bore. Muskat[26,27] also studied the effect of two-phase flow around the well-bore for gas condensate wells.

For constant rate transient gas flow the gas well back-pressure equation can be written as follows:

$$\frac{7.08Kh\ (p_i - p_{wf})}{q(\overline{\mu B})} = \ln\sqrt{\frac{(14.23)K_i t}{\phi(\mu C_t)_i\ r_w^2}} + S + Dq \qquad (1.67)$$

Units for equation 1.67 can be found in the example problem of section 1.3573.

Other than unique fluid properties or a pressure dependent permeability effect, the non-Darcy flow term is required to obtain an exponent, n, less than 1.00. In terms of pseudo-pressure m(p):

$$\frac{7.08\ Kh[m(p_i) - m(P_{wf})]}{q} = \ln\sqrt{\frac{14.23\ K_i t}{\phi(\mu C_t)_i\ r_w^2}} + S + Dq \qquad (1.68)$$

where m(p) can include a pressure dependent permeability

$$m(p) = \int_0^p \frac{K_{ro}(S,p)}{\mu_o B_o} dp \qquad (1.69)$$

Appendix A.2 shows the complete set of equations presented by Fetkovich. By selecting the proper equation, all possible flow conditions can be described.

1.355 Total effective skin effect

1.3551 Introduction

In order to have a grasp of the meaning and cause of skin effect we will start with the equation for total skin and explain its various components:

$$S'' = S + S(q,t) + Dq \tag{1.70}$$

where:

S'' = total effective skin effect

S = skin effect caused from a physical occurrence to the well such as drilling mud penetration causing damage, or a fracturing job causing improvement.

$S(q,t)$ is a rate and time-dependent skin effect, and is noted by a changing gas saturation in and around the well bore in particular. Therefore the K_{ro} values change depending upon saturations—the permeability to gas increases and the permeability to oil decreases with increased drawdowns in pressure.

Dq is the non-Darcy flow term and is a value peculiar to a particular well, whereby a value of "n" less than 1.0 may be obtained even in single-phase liquid flow, even though no physical damage has occurred to the well.

It is common practice in field tests to determine the well's flow ability without knowing any of the components that make up the total skin effect. That is, we may use Productivity Index or IPR curves for a well without describing skin effects. It should be noted that no stimulation or treatment can remove S(q,t) or Dq components of skin. In fact it is quite possible that stimulation may result in increasing Dq. Thus prior to treatment it is important to distinguish between the various components of skin to see if treatment is needed.

Fetkovich gave additional discussions concerning skin effects. He compared flow in and around the well bore of an oil well to that of a condensate well.

For a condensate well, for steady state distribution, and saturation S equal to 0 at $t = 0$, we can obtain the following equation:

$$r_a^2 = \frac{0.1135\, q_g^2\, \mu Z Y t}{h^2\, \phi\, K\, \bar{p}\, S_{chl}} \tag{1.71}$$

Y = Reservoir ft^3 of condensate accumulation in the reservoir per Mscf of full wellstream gas produced per psi. Y can be determined from the liquid volume data of PVT analysis. S_{chl} is the critical hydrocarbon liquid saturation to reach equilibrium (mobile liquid saturation)

q = Mscf/day, μ = cp, t = days, h = ft, r = ft, and K = Darcy

1.3552 Skin effect, S″

Reference should be made to section 1.343 for previous discussions on skin effect and ΔP_{skin}. This is the total value of skin attributed to all causes. Recall that ΔP_{skin} can be determined from S'' by the following equation:

$$\Delta P_{skin} = 0.87 S'' m$$

where m is the slope from the straight line portion of a pressure build-up curve. Also, recall that ΔP_{skin} and flow efficiency are related by the following equation given previously:

$$FE = \frac{\bar{p}_R - p_{wf} - \Delta p_{skin}}{\bar{p}_R - p_{wf}} \tag{1.24}$$

In terms of the radius of the altered zone we can find S'' as follows:

$$S'' = \frac{K_e - K_a}{2K_a} \ln\left(\frac{r_a}{r_w}\right)^2 \tag{1.72}$$

The term S'' is that total skin effect that we determine from a pressure build up curve and may reflect either damage or improvement. It includes all terms in the equation:

$$S'' = S + S(q,t) + Dq \tag{1.70}$$

1.3553 Rate and time-dependent skin, "S(q,t)"

If we substitute equation 1.71 into equation 1.72 we find:

$$S(q,t) = \frac{(K - K_a)}{2K_a} \ln\left[\frac{0.1135\, q_g^2\, \mu Z Y t}{h^2\, \phi\, K\, \bar{p}\, S_{chl}\, r_w^2}\right] \tag{1.73}$$

Equation 1.73 defines a rate and time dependent skin term that can give the appearance of non-Darcy flow. It is an approximation to determine the effects of two-phase flow in the vicinity of the well bore, and in its present form is suitable for condensate wells. A significant portion of the total skin may be attributed to S(q,t), and cannot be removed by stimulation.

An analogous behavior exists for oil wells flowing below the bubble point pressure. For a constant rate of production, the gas saturation builds up to critical gas saturation and remains constant. However, its radius increases with time until the well's drainage volume is above the critical gas saturation. This buildup of gas saturation is called "gas block," and the oil permeability decrease is constant, but its radius also increases with time. This was referred to as "pseudo-skin" by Weller.[30]

Fetkovich gave the following equation for oil wells:

$$S(q,t) = \frac{K - K_a}{2K_a} \ln\left[\frac{0.0226\, q_o^2\, B_o\, \mu_o\, X\, t}{h^2\, \phi\, K\, S_{gc}\, r_w^2}\right] \tag{1.74}$$

X = reservoir cu ft of gas evolved in the reservoir per stock tank bbl of oil produced per psi drawdown. X can be obtained from PVT data or approximated from available correlations.

S_{gc} = equilibrium or critical gas saturation

q_o = Stk bo/d

μ_o = cp

t = day

h = ft

r_w = ft

K = Darcy

Once the oil well's drainage volume exceeds the equilibrium gas saturation, equation 1.74 is no longer applicable.

1.3554 Non-Darcy flow effect D_q

D_q is the non-Darcy flow term and is difficult to describe. We can think of it as flow between laminar and turbulent, that is, flow in the transition region. The non-Darcy flow component Dq may be determined if S(q,t) and S″ are known for two different flow rates. The procedure is to plot $S'' - S(q,t)$ vs. q on coordinate paper and draw a straight line between these two points. The slope of the straight line is D and the intercept is S. This type of information can be obtained from isochronal or flow-after-flow tests.

1.3555 Value of S

S represents that part of the skin effect due to a physical occurrence in the well such as damage or improvement on a well. If this component of skin is positive then it can be removed by treatment.

1.3556 Final equation

We can now include all terms for total skin

$$S'' = S + S(q,t) + Dq \tag{1.70}$$

or:

$$\frac{7.08Kh(p_i - p_{wf})}{q(\overline{\mu B})} = \ln\sqrt{\frac{14.23\ K_i\ t}{\phi\ (\mu C_t)_i\ r_w^2}} + S + S(q,t) + Dq \tag{1.75}$$

or:

$$\frac{7.08Kh[m(p_i) - m(p_{wf})]}{q} = \ln\sqrt{\frac{14.23\ K_i\ t}{\phi\ (\mu C_t)_i\ r_w^2}} + S + S(q,t) + Dq \tag{1.76}$$

We note in the numerous equations of Fetkovich that he uses the symbol S′ which has the following definition:

$$S' = S + Dq \tag{1.77}$$

As noted, S′ does not include the rate and time-dependent skin defined as S(q,t). We can find S″ by solving equation 1.75.

$$S'' = \frac{7.08Kh(p_i - p_{wf})}{q(\overline{\mu B})} - \ln\sqrt{\frac{14.23\ K_i\ t}{\phi\ (\mu C_t)_i\ r_w^2}} \tag{1.77}$$

We can solve for S(q,t) from Equation 1.74.

$$S(q,t) = \frac{K - K_a}{2K_a} \ln\left[\frac{0.0226\ q_o^2\ B_o\mu_o\ X\ t}{h^2\ \phi\ K\ S_{gc}\ r_w^2}\right] \tag{1.74}$$

We can then solve for S′ from equation 1.70 where $S'' = S + S(q,t) + Dq$ where $S' = S + Dq = S'' - S(q,t)$. Values for S and Dq may be handled as one value, but can be found as noted in section 1.3554.

1.356 Change in performance curves with time or cumulative recovery

We have already discussed two methods for determining future IPR curves. One by Standing[10] and the other by Brown[13]. The main problem with Standing's procedure is the correct prediction of K_{ro} curves. Fetkovich noted that in solution-gas drive reservoirs, K_{ro} is approximately linear with reservoir pressure.

He proposed the following equation:

$$\frac{K_{(\bar{p}_R)}}{K_i} = \frac{\bar{p}_R}{\bar{p}_{Ri}} \tag{1.78}$$

or

$$K_{ro(\bar{p}_R)} = \frac{\bar{p}_R}{\bar{p}_{Ri}} \tag{1.79}$$

$\bar{p}_{Ri}$ is assumed to be equal to or less than the bubble point pressure.

Then $\left[\frac{K_{ro}}{\mu_o B_o}\right]_{\bar{p}R}$ plotted as a function of pressure defines a locus of values at zero drawdown. By using equation 1.65; $q_o = J'_o\ (p_e^2 - p_{wf}^2)^{1.0}$ to define drawdown and equation 1.79 to correct for depletion, $K_{ro(\bar{p}_R)} = \frac{\bar{p}_R}{\bar{p}_{Ri}}$, then we have a simple empirical equation to predict the flow rate q_o for any drawdown in pressure and for any reservoir pressure. This accounts for the change in IPR's due to depletion as well as rates. This equation is:

$$q_o = J'_{oi}\frac{\bar{p}_R}{\bar{p}_{Ri}}\ (\bar{p}_R^2 - p_{wf}^2) \tag{1.80}$$

Its more general form can be written as:

$$q_o = J'_{oi}\left(\frac{\bar{p}_R}{\bar{p}_{Ri}}\right)(\bar{p}_R^2 - p_{wf}^2)^n \tag{1.81}$$

1.357 Example problems and field test results

1.3571 Isochronal and flow after flow tests

Fetkovich presented numerous examples for the following types of reservoirs:

(1) Gas saturation above the critical or equilibrium gas saturation.

These tests were for an average gas saturation of 10–12%. The gas-oil ratios increased very little at higher pressure drawdowns with an initial solution gas-oil ratio of 684 scf/bbl. Out of 16 wells tested all but four gave values of n very close to 1.00, and showed that the plot of q_o vs $(\bar{p}_R^2 - p_{wf}^2)$ was a straight line on log-log paper.

Fig. 1.65 shows seven flow tests on one well with rates being reduced after the first four tests. All points fell on the same line. The flow points of Fig. 1.65 define a performance curve with a slope of 1.00 to its absolute open flow potential (AOFP).

Fig. 1.66 shows a good straight line and a value of n = 0.648. This showed a non-Darcy flow effect.

(2) Undersaturated reservoir. ($p_{wf} > p_b$ and $p_{wf} < p_b$)

Fetkovich presented the test results on the Phillips Ekofisk 2/4-2x well in the North Sea. Fig. 1.67 shows the results of a 6-hour isochronal test. Two straight lines were obtained on the log-log plot from which the intersection shows a bubble point pressure of 5,885 psi. If for this well a constant PI was determined from the first two tests only, an absolute open flow potential of 13,000 bpd would be indicated. The apparent potential results from assuming n = 1 = 7,000 bpd whereas the

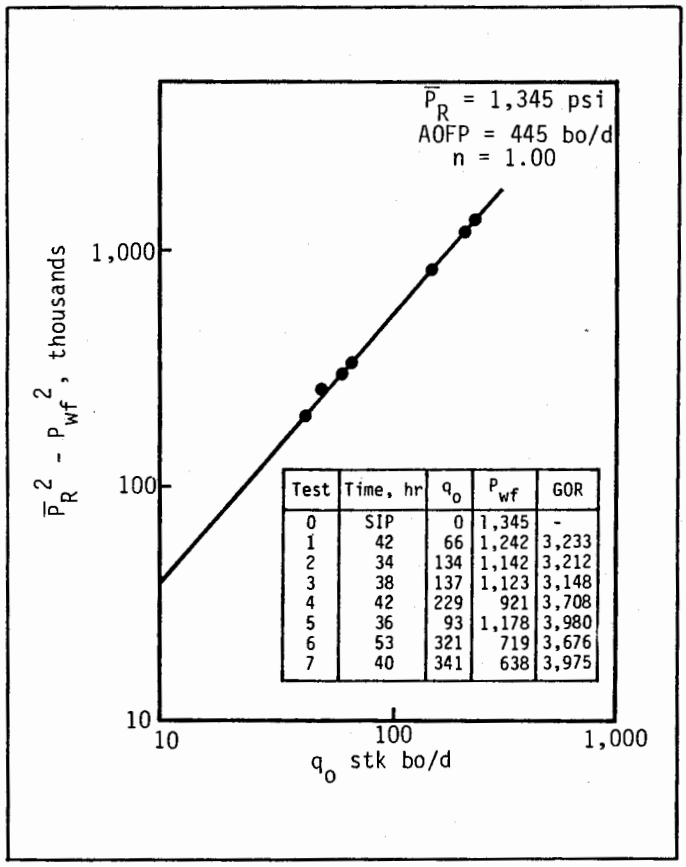


Fig. 1.65 Flow-after-flow tests (after Fetkovich).

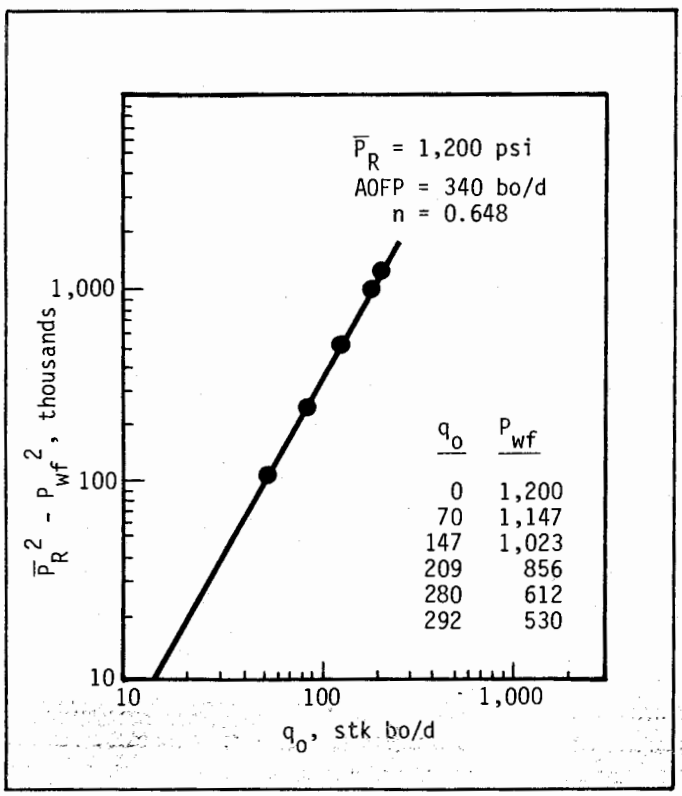


Fig. 1.66 Flow-after-flow tests (after Fetkovich).

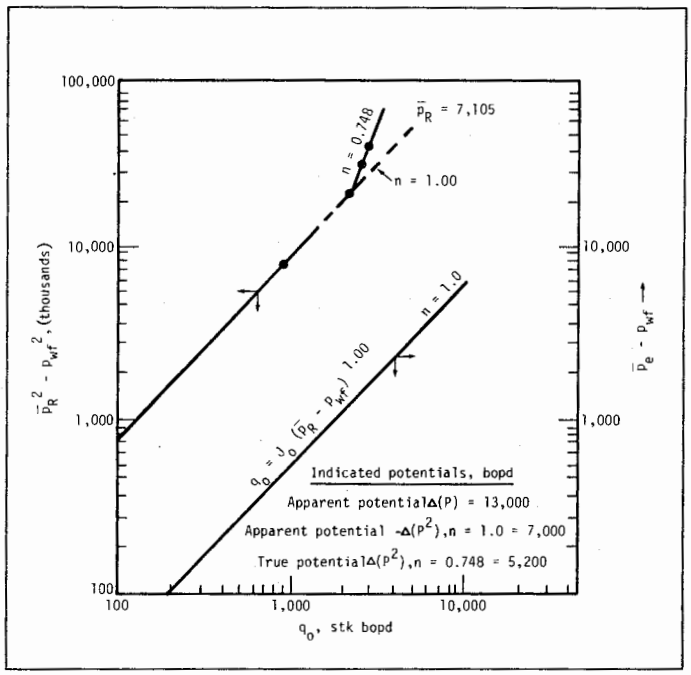


Fig. 1.67 6-hour isochronal tests (after Fetkovich).

true potential for the true value of n = 0.748 = 5,200 bpd.

Finally, a more general equation is suggested by Fetkovich:

$$q_o = J'_o (p_b^2 - p_{wf}^2)^n + J_o (p_e - p_b) \qquad (1.82)$$

(3) Saturated reservoirs.

The wells tested in this group were clean sandstones with permeabilities ranging from 130 to 2,500 md. Isochronal tests in these fields were 4-hour flow periods with a 4-hour shutin period. Values of n ranged from 0.568 to 0.875. In no case was an n value of 1.00 obtained. A typical test is shown in Fig. 1.68.

Fig. 1.69 was presented by Fetkovich to show the discrepancy in determining the absolute open flow potential by (1) the PI method: (2) Vogel's IPR curve: and (3) isochronal testing. The PI and IPR procedures both predict an AOFP that is far too high.

(4) Under-saturated reservoir. ($p_{wf} > p_b$)

Fetkovich noted that the most surprising results occurred from two wells tested in this group with flowing pressures well above the bubble point pressure (single-phase liquid flow). He found slopes of 0.813 and 0.712 indicating non-Darcy flow. Fig. 1.70 shows good alignment from eight separate flow rates and with a value of n = 0.813.

1.3572 Example problems on flow after flow and isochronal testing of oil wells

EXAMPLE PROBLEM #14: ISOCHRONAL TESTING OF OIL WELLS

Given data as noted on Fig. 1.65.

Solution:

(1) Determine $(\bar{p}_R^2 - p_{wf}^2)$ for each flow rate.

(2) Plot q_o vs. $(\bar{p}_R^2 - p_{wf}^2)$ on log-log paper. Normally it is easier to plot $(\bar{p}_R^2 - p_{wf}^2)$ in thousands. This should plot a straight line.

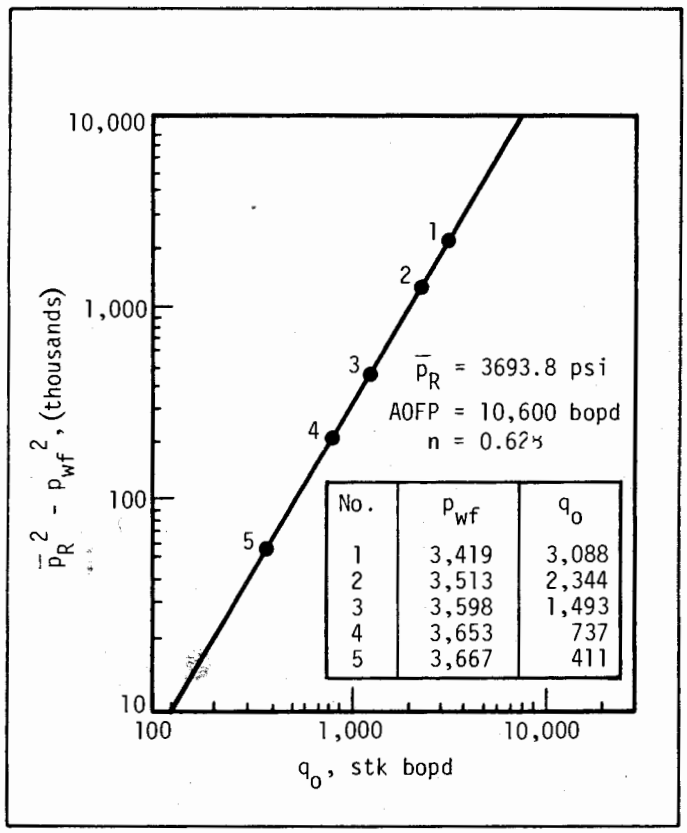


Fig. 1.68 4-hour isochronal tests (after Fetkovich).

(3) Determine the value of the exponent, n, which is the reciprocal of the slope of the equation:

$$q = C\,(\bar{p}_R^2 - p_{wf}^2)^n$$

$$n = \frac{\log q_2 - \log q_1}{\log (\bar{p}_R^2 - p_{wf2}^2)_2 - \log (\bar{p}_R^2 - p_{wf1}^2)_1}$$

For this example, n happens to be 1.00.

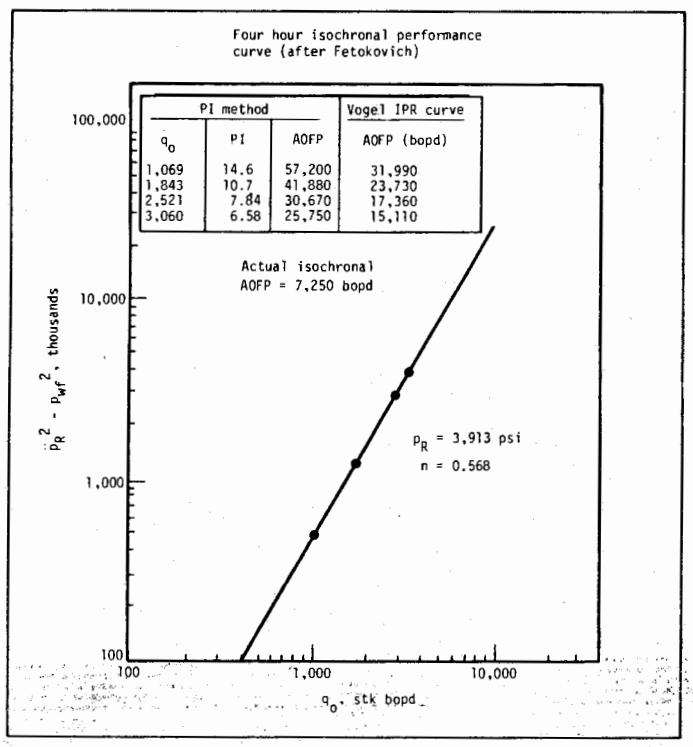


Fig. 1.69 Isochronal tests (after Fetkovich).

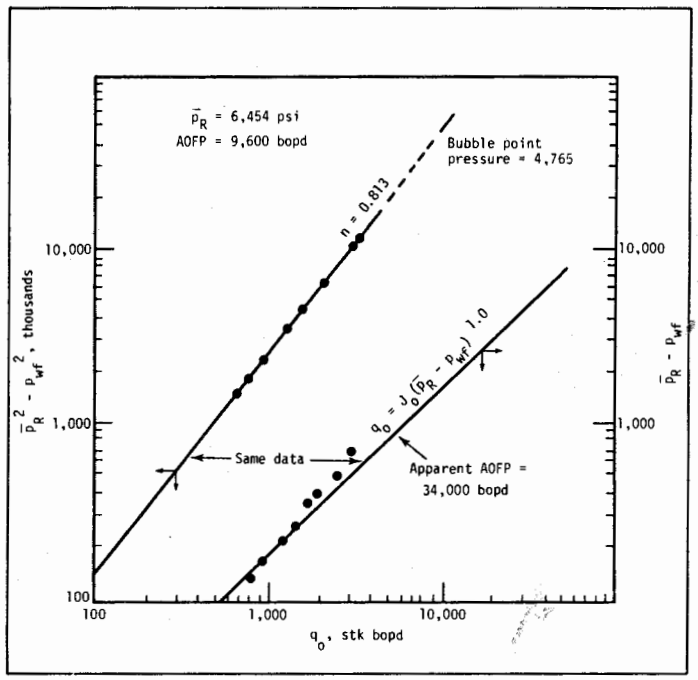


Fig. 1.70 Isochronal tests (after Fetkovich).

(4) Determine the value of C. This can be done by writing the equation for one set of data with the known value of n and solving for C. C can also be obtained by extending the line downwards until it intersects the q axis at the point where $\log (\bar{p}_R^2 - p_{wf}^2) = 0$, or $(\bar{p}_R^2 - p_{wf}^2) = 1.0$. The value of C for this example is = 0.245. This value of C is for the units of q = b/d and $(\bar{p}_R^2 - p_{wf}^2)$ in thousands of psi. Any other units change the value of C. (Defined as J_o' by Fetkovich)

(5) Write the equation for the straight line on log-log paper:

$$q = 0.245\left(\frac{\bar{p}_R^2 - p_{wf}^2}{1000}\right)^{1.00} = 0.245\left(\frac{\bar{p}_R^2 - p_{wf}^2}{1000}\right)$$

(6) Determine the absolute open flow potential (AOFP) or $(q_o)_{max}$. This can be read directly from the graph by first calculating $\left(\frac{\bar{p}_R^2 - 0}{1,000}\right)$ and reading the corresponding rate = 445 bpd. It can also be determined from the equation of step (5).

$$q = 0.245\left(\frac{\overline{1,345}^2 - 0}{1,000}\right)^{1.00} = 443 \text{ bpd}$$

CLASS PROBLEM #14-A: ISOCHRONAL TESTING (After Fetkovich[15])

Given data: $\bar{p}_R$ = 1,410 psia

Test no.	q_o	p_{wf}
0	0	1,410
1	72	1,170
2	118	1,050
3	155	888
4	208	632

Find:

(1) Absolute open flow potential, $(q_o)_{max}$.

(2) Write equation for flow rate determining the value of n and C in the equation:

$$q_o = C\,(\bar{p}_R^{\,2} - p_{wf}^{\,2})^n$$

(3) Find $(q_o)_{max}$ by method of Vogel for Test #1 and #4.
(4) Find $(q_o)_{max}$ assuming a linear PI for Test #1 and #4.

CLASS PROBLEM #14-B: ISOCHRONAL TEST ON OIL WELL (After Fetkovich[15])

Given data:

Test no.	q_o	p_{wf}	
1	2,303	4,152.5	
2	718	4,280.0	
3	2,167	4,154.2	$\bar{p}_R = 4{,}342.8$
4	1,403	4,242.2	
5	724	4,287.8	
6	372	4,313.2	

Find:

(1) Absolute open flow potential.
(2) Write equation for flow rate determining C and n.
(3) Find $(q_o)_{max}$ for method of Vogel for Test #1 and #5.
(4) Find $(q_o)_{max}$ assuming linear PI for Test #1 and #5.

CLASS PROBLEM #14-C

Given data:

q_o	p_{wf}	$\bar{p}_R = 1{,}200$ psia
0	1,200	
70	1,147	
147	1,023	
209	856	
280	612	
292	530	

Find: (1) $(q_o)_{max}$ Fetkovich method.
(2) Find values for J_o' and n

CLASS PROBLEM #14-D (After Fetkovich)

Given data: $\bar{p}_R = 3{,}693.8$ psia

Test no.	q_o	p_{wf}
1	3,088	3,419
2	2,344	3,513
3	1,493	3,598
4	737	3,653
5	411	3,667

Find: (1) $(q_o)_{max}$ by Fetkovich method.
(2) $(q_o)_{max}$ for each test by Vogel method and linear PI.

1.3573 Problems dealing with skin effect

In illustrating this type of problem we will use the example by Fetkovich and show how he obtained the various values of skin making up the total skin. The following is an example calculation of S′ and S″ for saturated reservoir.

EXAMPLE PROBLEM #15

Reservoir data

$K = 2{,}469$ md (build-up and core data)
$K_a = 1{,}284$ md, $K_{ro} = 0.52$ at 10% critical gas saturation, S_{gc}
$h = 20$ ft
$\phi = 0.21$
$S_{wc} = 0.32$
$C_t = 25 \times 10^{-6}$ psi^{-1}
$r_w = 0.33$ ft
$\mu_o = 0.27$ cp
$B_o = 1.94$ res bbl/stk bbl
$t = 0.167$ days
$X_o = 8.223 \times 10^{-3}$ res ft^3/stk bbl/psi (From PVT data).

The following additional test data was given in Table 1.6.

TABLE 1.6

Flow no.	Shut-in pressure, $\bar{p}_R$, psig	Flowing pressure, p_{wf}, psig	q_o, stk bopd	GOR, scf/stk bbl	Separator pressure, psig
		Isochronal test of 12/23/71			
1	3,680.8	3,524.3	2,308	1,211	422
2	3,672.1	3,604.0	1,452	1,309	260
3	3,670.5	3,658.4	757	1,375	139
4	3,672.9	3,665.8	419	1,383	92
SI 4HR.	3,672.9				
1	3,583.9	3,565.0	669	1,406	115
2	3,577.6	3,535.1	1,035	1,333	160
3	3,580.5	3,513.7	1,413	1,357	215
4	3,580.0	3,430.9	2,303	1,217	370
SI 4HR.	3,570.7				

Fetkovich prepared Fig. 1.71, which showed an AOFP of 10,200 bpd.

We will first find values of S(q,t) from equation 1.74 where:

$$S(q,t) = \frac{K - K_a}{2K_a} \ln\left[\frac{0.0226\, q_o^2\, B_o \mu_o\, X_o\, t}{h^2\, \phi\, K\, S_{gc}\, r_w^2}\right]$$

Checking S_1 for $q_{o1} = 2{,}308$ bpd:

$$S(q,t)_1 = \frac{2.469 - 1.284}{2(1.284)} \ln \left[\frac{(0.0226)(2308)^2(1.94)(0.27)(8.223 \times 10^{-3})(.167)}{(20)^2(0.21)(2.469)(0.10)(0.33)^2}\right]$$

$$= 0.461 \ln\left(\frac{86.595}{2.2585}\right) = 0.461 \ln (38.34)$$

$$= 0.46 \times 3.64 = 1.674$$

For $q_{o2} = 1{,}452$ bpd:

$$S(q,t)_2 = 0.461 \ln\left(\frac{16.256 \times 10^{-6}\,(1452)^2}{2.2585}\right)$$

$$= 0.461 \ln (15.17)$$
$$= 0.461 \times 2.719 = 1.25$$

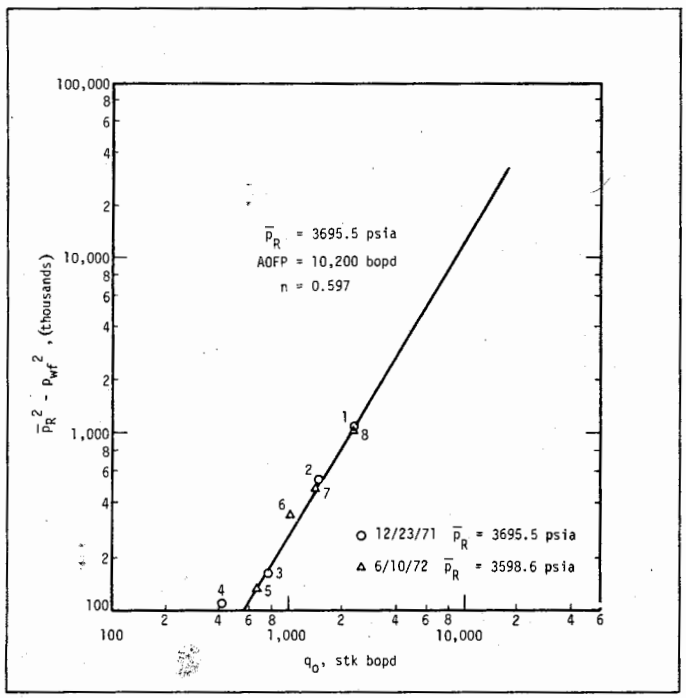


Fig. 1.71 Isochronal tests (after Fetkovich).

For $q_{o3} = 757$ bpd:

$$S(q,t)_3 = 0.461 \times \ln\left(\frac{16.256 \times 10^{-6}\,(757)^2}{2.2585}\right)$$

$$= 0.461 \times \ln(4.12)$$
$$= 0.461 \times 1.42 = 0.65$$

In order to solve for S″, recall that:

$$\frac{7.08Kh\,(p_i - p_{wf})}{q\,(\overline{\mu B})} = \ln\sqrt{\frac{14.23\,K_i\,t}{\phi\,(\mu C_t)_i\,r_w^2}} + S + S(q,t) + Dq \quad (1.75)$$

This equation is used to find the value of total effective skin effect dimensionless (S″) that is equal to:

$$S'' = S + S(q,t) + Dq$$

From equation 1.75 we find:

$$S'' = \frac{7.08Kh\,(p_i - p_{wf})}{q\,(\overline{\mu B})} - \ln\sqrt{\frac{14.23\,K_i\,t}{\phi\,(\mu C_t)_i\,r_w^2}} \quad (1.82)$$

Now we determine the value of S″ for our conditions, where:

$$P_i = 3{,}680.8 \text{ psig}$$

Then:

$$S'' = \frac{7.08Kh\,(p_i - p_{wf})}{q_o\,\overline{\mu}_o\,\overline{B}_o} - \ln\sqrt{\frac{14.23\,K_i\,t}{\phi\,(\mu C_t)_i\,r_w^2}}$$

For $p_{wf_1} = 3{,}524.3$ psig:

$$S_1'' = \frac{(7.08)(2.469)(20)(3{,}680.8 - 3{,}524.3)}{(2308)(0.27)(1.94)} - \ln\sqrt{\frac{(14.23)(2.469)(0.167)}{(0.21)(0.27)(25 \times 10^{-6})(0.33)}}$$

$$S_1'' = 667.45\,\frac{(3{,}680.8 - 3{,}524.3)}{2{,}308} - \ln(6{,}165.2)$$

$$S_1'' = 45.258 - 8.7266 = 36.53.$$

For $p_{wf_2} = 3{,}604.0$ psig:

$$S_2'' = 667.45\,\frac{(3{,}680.8 - 3{,}604.0)}{1{,}452} - \ln(6{,}165.2)$$

$$S_2'' = 35.303 - 8.7266 = 26.58$$

For $p_{wf_3} = 3{,}658.4$ psig:

$$S_3'' = 667.45\,\frac{(3{,}680.8 - 3{,}658.4)}{757} - \ln(6165.2)$$

$$S_3'' = 19.750 - 8.7266 = 11.0.$$

We now have values for S(q,t) and S″ from which we are able to solve for S′ where S′ = S + Dq.

From the equation S″ = S + S(q,t) + Dq we have S″ = S(q,t) + S′ where S′ = S + Dq. For Flow Test #1 where $q_{o_1} = 2{,}308$ bpd and $p_{wf_1} = 3{,}524.3$ psig, we found $S(q,t)_1 = 1.674$ and S″ = 36.53.

Then: $S_1' = S_1'' - S(q,t)_1$
$S_1' = 36.53 - 1.674 = 34.86$

Likewise: $S_2' = 26.58 - 1.25 = 25.33$
$S_3' = 11.0 - 0.65 = 10.35$

These values check with the results of Fetkovich as shown in Table 1.7:

TABLE 1.7
SUMMARY OF RESULTS

q_o, stk bo/d	S(q,t)	S″ = S + S(q,t) + Dq	s′ = S + Dq
2,308	1.67	36.6	34.9
1,452	1.24	26.6	25.4
757	0.64	11.1	10.4

An S′ or S″ vs. q_o plot yields S = 0 when extrapolated to q = 0. (See Fig. 1.72).

The preceding results are similar (little difference) to the values found by Fetkovich. The rate-dependent skin S(q,t) for this well is very low and can be considered insignificant as established by Fetkovich. Fig. 1.72 shows a plot of skin vs. q_o.

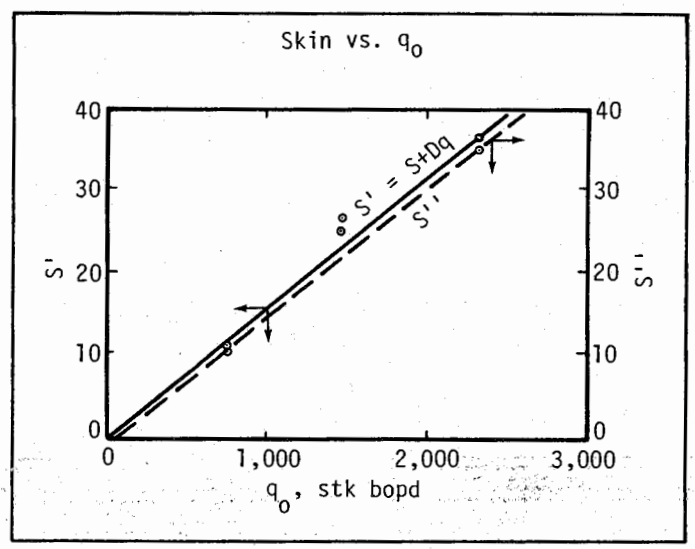


Fig. 1.72 Skin vs. q_o

1.3574 Problems dealing with future inflow performance curves

The following example illustrates the change in Performance Curves with depletion. Pressure ratio, as suggested by Fetkovich, is used to forecast rate of flow with pressure depletion.

The value of J'_{oi} can be determined using both basic reservoir variables and an initial reported flow with about equal success.

The empirical equation below is used to predict the flow rate q_o for both drawdown and reservoir pressure depletion:

$$q_o = J'_{oi}\left(\frac{\bar{p}_R}{p_{ri}}\right)(\bar{p}_R^2 - p_{wf}^2) \qquad (1.80)$$

where:

$$J'_{oi} = \frac{q_{oi}}{(\bar{p}_R^2 - p_{wf}^2)} \qquad (1.83)$$

Equation 1.83 can also be used to find the value of rate of flow:

$$q_o = J'_{oi}\,(\bar{p}_R^2 - p_{wf}^2) \qquad (1.84)$$

where:

$$J'_{oi} = \left[\frac{7.08\ K(K_{ro})s_{gc}\ h}{\left[\ln\left(\frac{r_e}{r_w}\right) - \frac{1}{2}\right](\mu_o B_o)_i\ 2p_i}\right]\frac{\bar{p}_{Ri}}{p_i} \qquad (1.85)$$

The following example problem was given by Fetkovich: EXAMPLE PROBLEM #16-A

Reservoir data:

$p_i = p_b = 2{,}075$ psia; $\phi = 0.139$; $S_{wc} = 0.177$; $h = 23.5$ ft

$r_w = 0.33$ ft; $r_e = 1{,}053$ ft (80 acres); $\mu_{oi} = 0.99$ cp; $B_{oi} = 1.33$ res bbl/stk bbl; $K = 25$ md; $S_{gc} = 0.02$ (assumed to be established rapidly); $K_{ro} = 0.444$ @ S_{gc}.

Using the values of $\bar{p}_R$ in the first part of Table 9 (paper of Fetkovich), and the value of $p_{wf} = 65$ psia, we have, as noted in Table 1.8:

TABLE 1.8

$\bar{p}_R = 1{,}708$ psia.

$\bar{p}_R$, psia	p_{wf}, psia	$\bar{p}_R^2 \times 10^3$, psia	$p_{wf} \times 10^3$, psia	$\bar{p}_R/\bar{p}_{Ri}$	$\bar{p}_R^2 - p_{wf}^2 \times 10^3$, psia²
1708	65	2,917.3	4	1.000	2,913.3
1377	65	1,896.1	4	0.8062	1,892.1
1054	65	1,110.9	4	0.6171	1,107.0
519	65	269.4	4	0.3039	265.4

In using equation 1.83 to find the value of J'_{oi}, we assume $q_{oi} = 108.8$ (from reference 28), then:

$$J'_{oi} = \frac{q_{oi}}{\bar{p}_R^2 - p_{wf}^2} = \frac{108.8\ \text{b/d}}{2913.3 \times 10^3} = 0.03735 \times 10^{-3}\ \text{bpd/psia}^2.$$

Making use of the equation 1.80:

$$q_o = J'_{oi}\left(\frac{\bar{p}_R}{\bar{p}_{Ri}}\right)\left(\frac{\bar{p}_R^2 - p_{wf}^2}{1{,}000}\right)^n$$

we can calculate the individual flow rates:

$$q_{o1} = (0.03735 \times 10^{-3})(1.0)(2{,}913.3 \times 10^3) = 108.8\ \text{bpd}$$
$$q_{o2} = (0.03735 \times 10^{-3})(0.8062)(1{,}892.1 \times 10^3) = 56.9\ \text{bpd}$$
$$q_{o3} = (0.03735 \times 10^{-3})(0.6171)(1{,}107.0 \times 10^3) = 25.5\ \text{bpd}$$
$$q_{o4} = (0.03735 \times 10^{-3})(0.3039)(265.4 \times 10^3) = 3.0\ \text{bpd}$$

In using equation 1.85, the value of J'_{oi} is calculated for each static pressure:

$$J'_{oi} = \left(\frac{7.08\ K(K_{ro})s_{gc}\ h}{\left[\ln\left(\frac{r_e}{r_w}\right) - \frac{1}{2}\right](\mu_o B_o)_i\ 2p_i}\right)\left(\frac{\bar{p}_{Ri}}{p_i}\right) \qquad (1.85)$$

For $\bar{p}_{Ri} = 1708$ psia,

$$J'_{oi} = \left(\frac{(7.08)\ (25)\ (0.444)\ (23.5)}{\left[\ln\left(\frac{1{,}305}{0.33}\right) - \frac{1}{2}\right](0.99)\ (1.33)\ (2)\ (2{,}075)}\right)\left(\frac{1{,}708}{2{,}075}\right)$$

$J'_{oi} = (0.044658)\ (0.823) = 0.03676$ bpd/thousand psia²

For $\bar{p}_{Ri} = 1{,}377$ psia:

$$J'_{oi} = (0.044658)\left(\frac{1{,}377}{2{,}075}\right) = 0.02964\ \text{bpd/thousand psia}^2$$

For $\bar{p}_{Ri} = 1{,}054$ psia:

$$J'_{oi} = (0.044658)\left(\frac{1{,}054}{2{,}075}\right) = 0.02268\ \text{bpd/thousand psia}^2$$

For $\bar{p}_{Ri} = 519$ psia:

$$J'_{oi} = (0.044658)\left(\frac{519}{2{,}075}\right) = 0.01117\ \text{bpd/thousand psia}^2$$

Applying equation 1.84:

$$q_o = J'_{oi}(\bar{p}_R^2 - p_{wf}^2) \qquad (1.84)$$

$q_{o1} = (0.03676 \times 10^{-3})\ (2{,}913.3 \times 10^{-3}) = 107.1$ bpd
$q_{o2} = (0.02964 \times 10^{-3})\ (1{,}892.1 \times 10^{-3}) = 56.1$ bpd
$q_{o3} = (0.02268 \times 10^{-3})\ (1{,}107 \times 10^{-3}) = 25.1$ bpd
$q_{o4} = (0.01117 \times 10^{-3})\ (265.4) = 2.94 \simeq 3.0$ bpd

Results are shown in the following table:

$\bar{p}_R$,	q_o, (Eqn. 1.80)	q_o (Eqn. 1.84)
1,708	108.8	107.1
1,377	56.9	56.1
1,054	25.5	25.1
519	3.0	3.0

From these results we can see that there is no great difference between the empirical equation 1.80—which takes into consideration the drawdown and the reservoir pressure depletion—and equation 1.84 which uses the value of productivity index, J'_{oi}, taking into consideration the variables of the reservoir.

Equation 1.80 is applied in the same problem to show the performance curves for different stages of depletion for different values of $\bar{p}_R$. For $\bar{p}_{Ri} = 1708$:

TABLE 1.9

p_{wf} Assume:	$\bar{p}_{Ri} = 1{,}708$ $\bar{p}_R/\bar{p}_{Ri} = 1.0$ q_o	$\bar{p}_{R2} = 1{,}377$ $\bar{p}_R/\bar{p}_{Ri} = 0.8062$ q_o	$\bar{p}_{R3} = 1{,}054$ $\bar{p}_R/\bar{p}_{Ri} = 0.6171$ q_o	$\bar{p}_{R4} = 519$ $\bar{p}_R/\bar{p}_{Ri} = 0.303$ q_o
1,500	24.9			
1,200		14.0		
1,000	71.6			
900		32.4	6.9	
600		46.0	17.3	
500	99.6			
450				0.76
350				1.67
300		54.0	23.5	
250				2.35
150				2.80
100	108.6	56.0	25.4	
50				3.00

$$q_o = J'_{oi}\left(\frac{\bar{p}_R}{\bar{p}_{Ri}}\right)(\bar{p}_R^2 - p_{wf}^2) \qquad (1.80)$$

To draw a curve for each $\bar{p}_R$, we first assume different values of p_{wf}. To find values of q_o the value of J'_{oi} is assumed to be 0.03735 bpd/thousands psia^2 as before. Example calculation:

For $\bar{p}_R = 1{,}708$ psia, assume $p_{wf} = 1{,}500$ psia:

$$q_o = (0.03735 \times 10^{-3})\left(\frac{1{,}708}{1{,}708}\right)(1708^2 - 1{,}500^2)$$

$q_o = 24.9$ bpd

Then: $q_o = J'_{oi}\left(\frac{\bar{p}_R}{\bar{p}_{Ri}}\right)(\bar{p}_R^2 - p_{wf}^2)$

$\bar{p}_{Ri} = 1{,}708$ psia

With the data of Table 1.9, a series of curves were drawn, plotting p_{wf}(psia) vs. q_o (bpd) (Fig. 1.73).

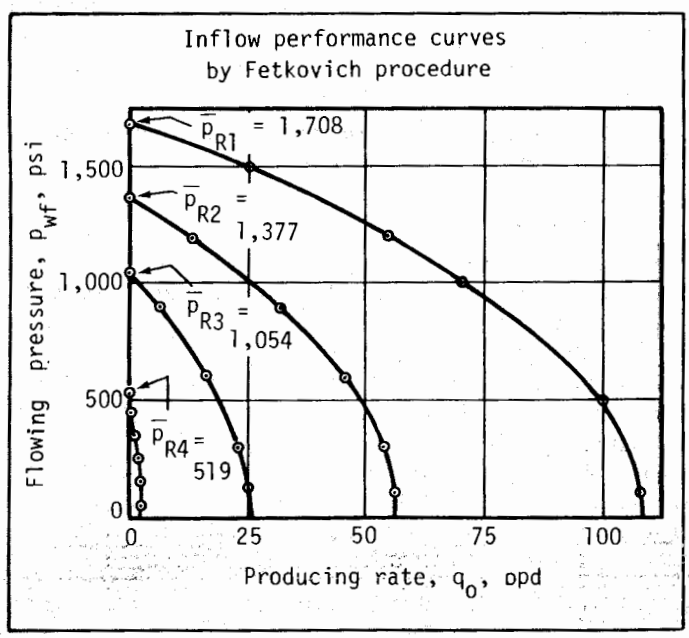


Fig. 1.73 Inflow performance curves.

EXAMPLE PROBLEM #16-B

1. Prepare future inflow performance curves for example problem #14. The equation for the production rate was found to be:

$$q_o = J'_o(\bar{p}_R^2 - p_{wf}^2)^n = 0.245\left(\frac{\bar{p}_R^2 - p_{wf}^2}{1000}\right)^{1.0}$$

where $J'_o = 0.245 = \dfrac{\text{bpd}}{\left(\dfrac{\bar{p}_R^2 - p_{wf}^2}{1{,}000}\right)^n}$, where n = 1.00.

$\bar{p}_R$ was given as 1,345 psi.

It is required to establish inflow curves for $\bar{p}_R$ values of 1,345, 1,000, and 500 psia. The following equation is applicable:

$$q_o = J'_{oi}\left(\frac{\bar{p}_R}{\bar{p}_{Ri}}\right)\left(\frac{\bar{p}_R^2 - p_{wf}^2}{1000}\right)^n$$

Solution:

For $\bar{p}_R = 1{,}345$, $q_o = 0.245\left(\dfrac{\bar{P}_R^2 - P_{wf}^2}{1000}\right)^{1.0}$ where $\dfrac{\bar{p}_R}{\bar{p}_{Ri}} = 1.00$, or $q_o = 0.245\left(\dfrac{1{,}345^2 - p_{wf}^2}{1{,}000}\right)^{1.0}$. Assume values of p_{wf} and calculate q_o:

p_{wf}	q_o
1,345	0
1,100	147
900	245
700	323
500	382

For $\bar{p}_R = 1{,}000$:

$$q_o = 0.245\left(\frac{1{,}000^2 - p_{wf}^2}{1{,}000}\right)^{1.0}\left(\frac{\bar{p}_R}{\bar{p}_{Ri}}\right)$$

Assume values of p_{wf} and calculate q_o:

p_{wf}	q_o
1,000	0
800	65
600	116
400	153

$$q_o = 0.245\left(\frac{1{,}000^2 - p_{wf}^2}{1{,}000}\right)\left(\frac{1{,}000}{1{,}345}\right)$$

Likewise for $\bar{p}_R = 500$ psia:

$$q_o = 0.245 \left(\frac{500^2 - p_{wf}^2}{1{,}000}\right)\left(\frac{500}{1{,}345}\right)$$

p_{wf}	q_o
500	0
400	8
300	15
200	19

This information is then plotted as in Fig. 1.74.

In this procedure the value of J'_o is held constant and the value of $\frac{\bar{p}_R}{\bar{p}_{Ri}}$ takes care of the decreasing productivity as the pressure changes. If sufficient test data is available, then a new J'_o could be determined at each pressure. Then the following equation would suffice:

$$q_o = J'_o (\bar{p}_R^2 - p_{wf}^2)^n$$

CLASS PROBLEM #16-A

1. Prepare future inflow performance curves for class problem #14-A for static pressures of 1,410, 1,100 and 700 psia.

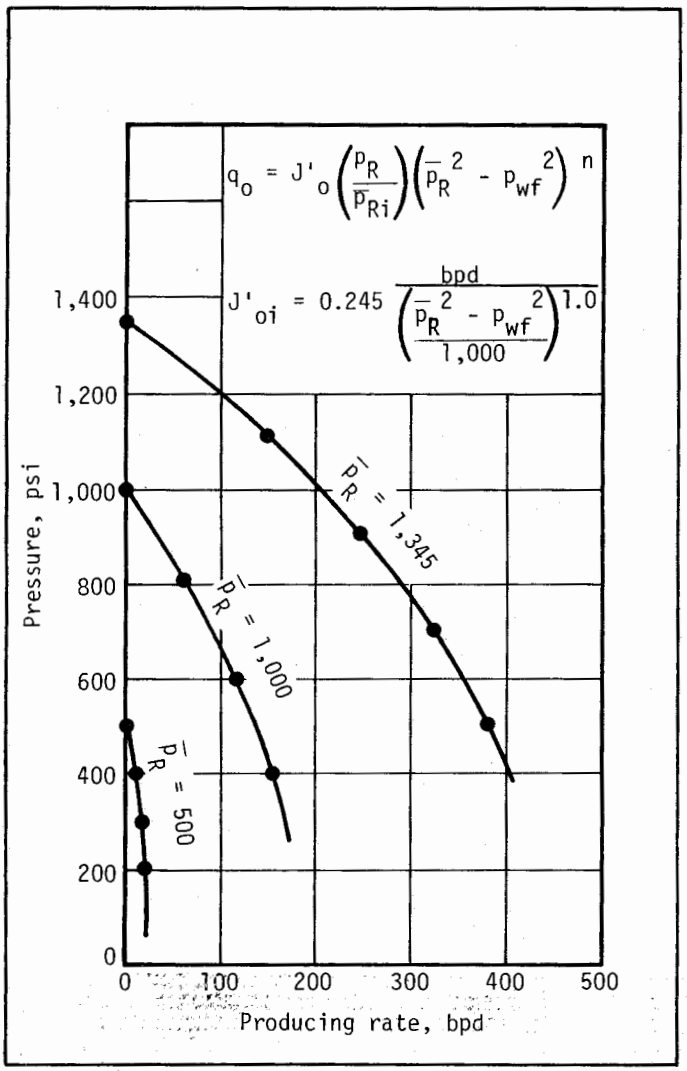


Fig. 1.74 Inflow performance curves.

2. For class problem #14-C prepare future curves for pressures of 1,200, 1,000, 750, and 400 psia.

3. For class problem #14-D prepare future inflow curves for pressures of 3,693.8, 3,000, 2,500, 1,500, 1,000, and 500 psia. Assume a bubble point pressure of 2,500 psia.

1.358 Conclusions

The following conclusions were given by Fetkovich.[15]

"The results obtained from the forty oil well multipoint back-pressure tests reported in this study, isochronal and flow after flow, leads to the following conclusions:

1. Multipoint tests for oil wells are required to accurately determine flow rates as a function of drawdown, reservoir damage, flow efficiency, and a well's true absolute openflow potential.
2. Oil wells can behave very similar to gas wells on multipoint back-pressure tests and should therefore be tested and analyzed using the same basic flow equations.
3. The exponent (n) for oil well tests determined from a log q vs. log Δ (p^2) plot was found to lie between 0.568 and 1.000, very near the limits commonly accepted for gas well back-pressure curves.
4. Flow-point alignment to establish an oil well back-pressure curve on a log q vs. log Δ (p^2) plot is as good as that normally obtained from gas well back-pressure tests.
5. A non-Darcy flow-term is generally required to account for slopes (n) less than 1 obtained on oil well back-pressure performance curves.
6. Back-pressure curve slopes less than 1 can be obtained on wells in undersaturated reservoirs without a non-Darcy flow term because of the shape of the pressure function ($K_{ro}/\mu_o B_o$).
7. In some cases, it is possible to determine the bubble-point pressure of an undersaturated reservoir from multipoint tests when a sufficient range of flow rates is taken.
8. Flow after flow tests or isochronal tests on oil wells will yield the same performance curve in high permeability reservoirs.
9. With a single data point, a simple empirical equation predicts flow rates as a function of drawdown and pressure depletion for wells in a volumetric solution-gas drive reservoir (no fluid injection). Field verification is obviously needed."
10. Future inflow performance curves can be predicted from reservoir parameters without having to take a flowing test on the well.

1.36 Comparison of methods for estimating and predicting inflow performance curves

1.361 Introduction

Yousaf[29] made a comparative study of methods to estimate and predict the inflow performance relationships and the following discussions are essentially the same as found in his thesis.

The main objective of this work was to estimate and compare the IPR curves calculated by the methods proposed by Weller,[30] Vogel[7] and Fetkovich.[15] The IPR's for hypothetical depletion-type reservoirs having different oil PVT properties and reservoir relative permeability characteristics were calculated by employing these methods. These IPR's were then used as a basis for evaluating the methods of predicting future IPR's of the reservoirs. As Vogel's dimensionless IPR equation needs a minimum of one well test in order

to construct the IPR's, Weller's results are used for this purpose. Vogel calls this result chosen to generate his IPR's as "match point."

Two hypothetical reservoirs labelled as reservoir 1 and reservoir 2 were used in his study. It is assumed that both reservoirs are presently existing at their bubble point pressure and will be producing at and below this pressure. Details of the properties of each reservoir are shown in Tables 1.10 and 1.11 and Figs. 1.75 to 1.78.

The drainage area of each of the reservoirs is assumed to be 20 acres thus having a drainage radius of 526.6 ft. The wellbore radius in each case is 0.33 ft. The value of absolute permeability of reservoir 1 is 20 md and that of reservoir 2 is 30 md. Each has a different value of critical gas saturation. Reservoir 1 has a higher viscosity range than reservoir 2. The connate water saturation ranges from 20–25% of pore volume in both reservoirs.

1.362 Weller's inflow performance relationship

Weller developed a method of calculating the inflow performance of depletion-type reservoirs, in which he used the following assumptions:

(1) The reservoir is circular and completely bounded, with a completely penetrating well at its center.

TABLE 1.10
RESERVOIR DATA FOR INFLOW PERFORMANCE RELATIONSHIP CALCULATION
Reservoir 1

Reservoir drainage area, acres	20.0
Reservoir drainage radius, ft	526.6
Reservoir thickness, ft	25.0
Porosity, fraction of bulk volume	0.15
Absolute permeability, md	20.0
Connate water saturation, fraction	0.20
Critical gas saturation, fraction	0.10
Initial oil saturation, fraction	0.80
Initial pressure, psia	2,130.0
Bubble point pressure, psia	2,130.0
Oil compressibility, psi^{-1}	2.0×10^{-5}
Well radius, ft	0.33
Skin	−3 to +3

TABLE 1.11
RESERVOIR DATA FOR INFLOW PERFORMANCE RELATIONSHIP CALCULATION
Reservoir 2

Reservoir drainage area, acres	20.0
Reservoir drainage radius, ft	526.6
Reservoir thickness, ft	30.0
Porosity, fraction of bulk volume	0.20
Absolute permeability, md	30.0
Connate water saturation, fraction	0.25
Critical gas saturation, fraction	0.04
Initial oil saturation, fraction	0.75
Initial pressure, psia	2,250.0
Bubble point pressure, psia	2,250.0
Oil compressibility, psi^{-1}	2.0×10^{-5}
Well radius, ft	0.33
Skin	−3 to +3

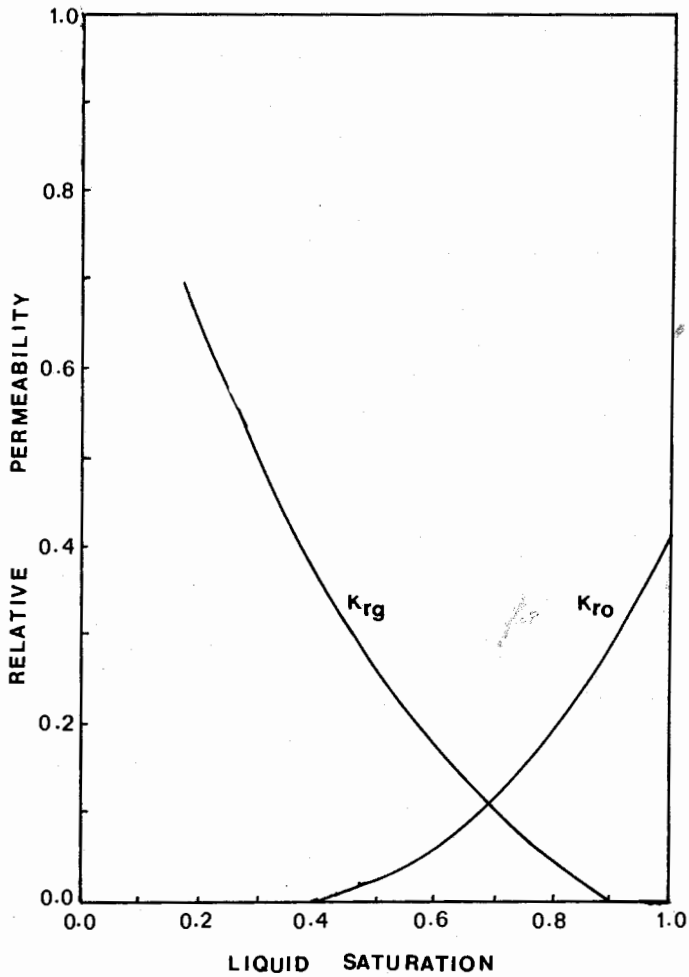


Fig. 1.75 Relative permeability curves—reservoir one (after ref. 29).

(2) The porous medium is uniform and isotropic with a constant water saturation at all points.

(3) Gravity effects can be neglected.

(4) Compressibilities of rock and water can be neglected.

(5) The composition and equilibrium are constant for oil and gas.

(6) The same pressure exists in both the oil phase and gas phase.

(7) A semisteady-state condition exists in which the tank-oil desaturation rate $\frac{\partial}{\partial t}\left(\frac{S_o}{B_o}\right)$ is the same at all points at a given instant.

The semisteady-state assumption can be used to determine the saturation in the following manner. Since all portions of the reservoir desaturate at the same rate, saturations at any time can be obtained from the initial saturation distribution by subtraction. The initial distribution is the one prevailing at the same time the semisteady-state assumption becomes valid; that is when stabilized conditions are attained.

When change of rate takes place, a disturbance is propagated outward towards the drainage boundary. This period is called the transient period during which the semisteady-state assumption is not valid. In order to obtain an initial saturation distribution from which to subtract the uniform desaturation occurring during the semisteady-state period, it is necessary to ap-

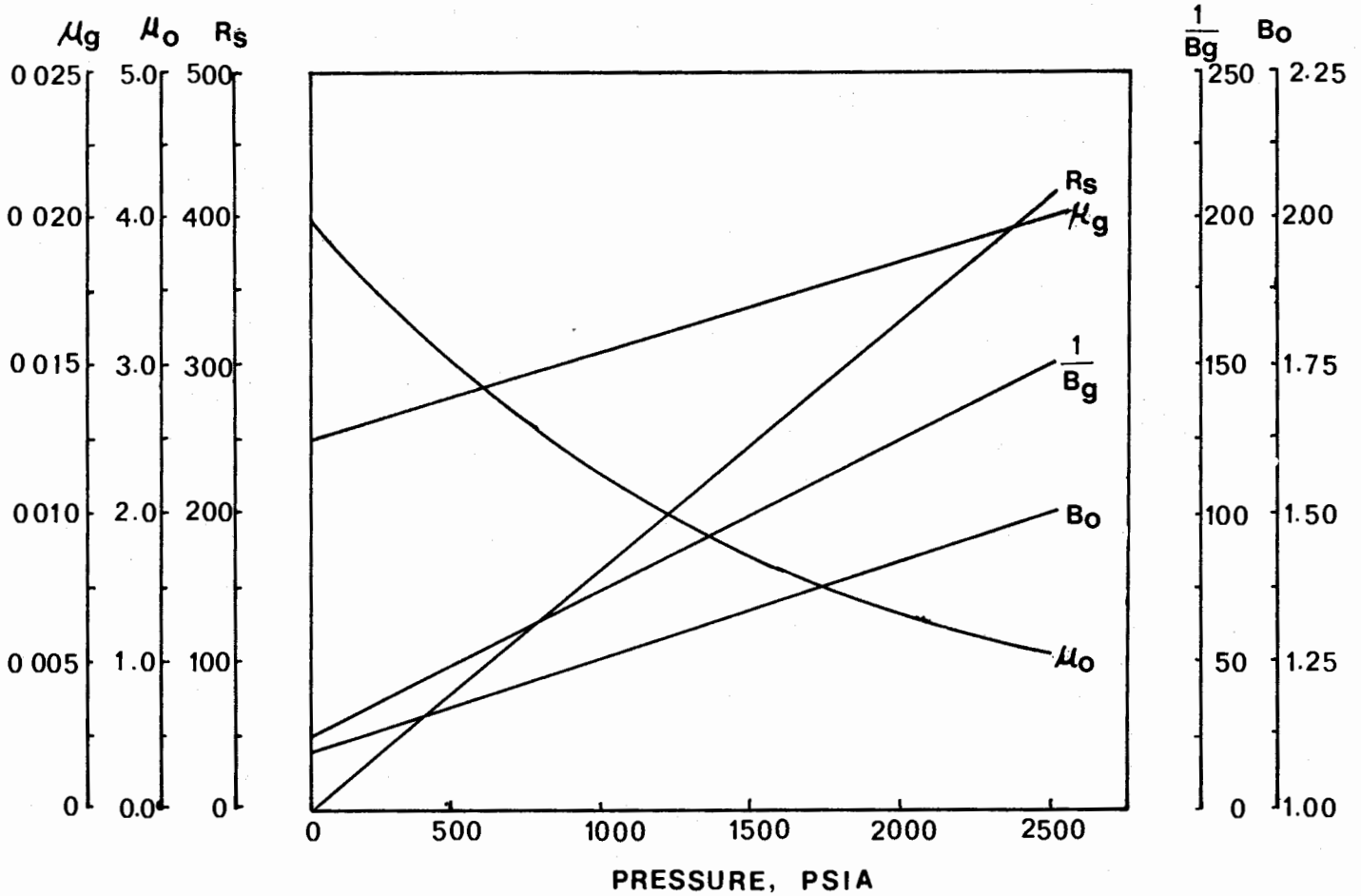


Fig. 1.76 PVT properties of the oil and gas—reservoir one (after ref. 29).

proximate the transient period by a succession of semisteady-states. Weller developed an approximate solution based on this approach.

Neglecting the gravity effects, i.e., assumption 3, we can write Darcy's law in the following manner:

$$\vec{U}_o = -\frac{1}{158.064}\frac{K_o}{\mu_o}\nabla p \qquad (1.86)$$

where $\vec{U}_o$ is a vector of Darcy flow velocity for oil. For a radially symmetric circular reservoir with a completely penetrating well (assumption 1), this becomes:

$$U_o = -\frac{1}{158.064}\frac{K_o}{\mu_o}\frac{\partial p}{\partial r} \qquad (1.87)$$

where

K_o = effective permeability to oil, md
μ_o = oil viscosity, cp
p = pressure, psia
r = radial distance from center line of well, ft.

As the reservoir is completely bounded, there is no flow across the boundary. This can be expressed in the following form:

$$U_o = \frac{B_o}{2\pi rh}\int_r^{r_e}\left[\frac{\partial}{\partial t}\left(2\pi rh\phi\frac{S_o}{B_o}\right)\right]dr \qquad (1.88)$$

Using assumption 2, ϕ and h can be treated as constants, and so:

$$U_o = \phi\frac{B_o}{r}\int_r^{r_e} r\left[\frac{\partial}{\partial t}\left(\frac{S_o}{B_o}\right)\right]dr \qquad (1.89)$$

The semisteady-state assumption states that the desaturation rate $\frac{\partial}{\partial t}\left(\frac{S_o}{B_o}\right)$ is the same at all points at a given time, and is equal to the instantaneous volumetric average. Thus:

$$U_o = \phi\frac{B_o}{r}\left(\frac{r_e^2 - r^2}{2}\right)\left[\frac{\partial}{\partial t}\left(\frac{S_o}{B_o}\right)\right]_{ave} \qquad (1.90)$$

where:

$$\left[\frac{\partial}{\partial t}\left(\frac{S_o}{B_o}\right)\right]_{ave} = -\frac{5.615\ q_o}{\pi(r_e^2 - r_w^2)h\phi} \qquad (1.91)$$

Therefore,

$$U_o = -\frac{5.615\ q_o B_o}{2\pi rh}\left(\frac{r_e^2 - r^2}{r_e^2 - r_w^2}\right) \qquad (1.92)$$

By substituting in Equation 1.87:

$$\frac{\partial p}{\partial r} = 141.244\frac{\mu_o q_o B_o}{rKK_{ro}h}\left(\frac{r_e^2 - r^2}{r_e^2 - r_w^2}\right) \qquad (1.93)$$

where:

$K_{ro} = \frac{K_o}{K}$ is the relative permeability to oil.

In order to determine this, an initial saturation distribution must be known from which the distribution, at

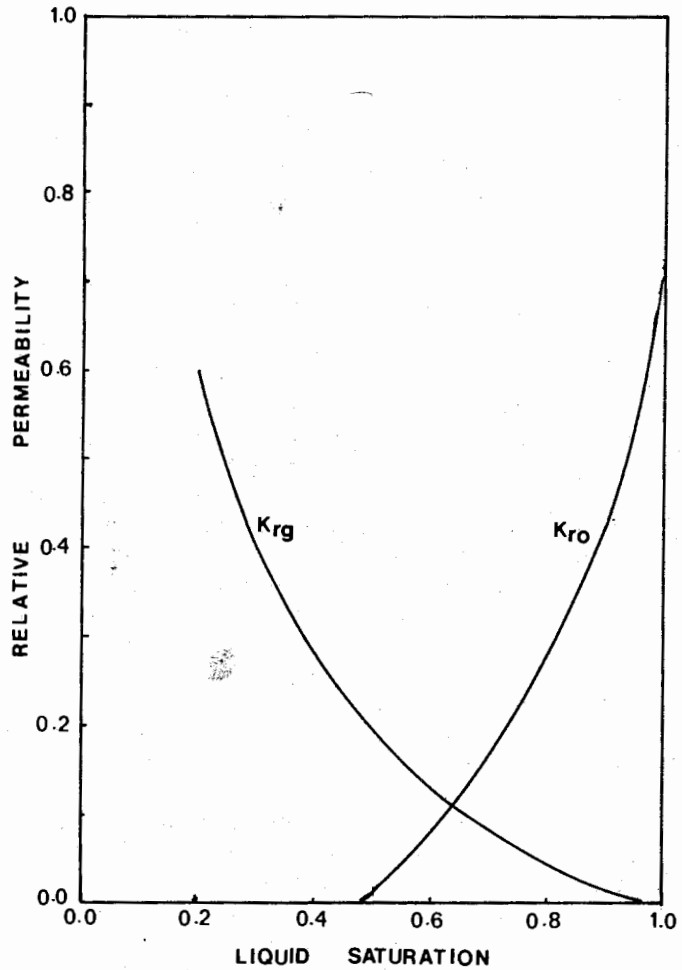


Fig. 1.77 Relative permeability curves—reservoir two (after ref. 29).

any later time, can be calculated. Weller found that the saturation distribution at the moment under consideration is:

$$\frac{S_o}{B_o} = \frac{S_{oi}}{B_{oi}}\left(1 - \frac{N_p - N_{pt}}{N}\right) - 92.58\,\frac{q_o C_o \mu_{oi}}{K_{oi} h}\ln\left(\frac{r_e + r_w}{r + r_w}\right) \tag{1.94}$$

where:

N_p = cumulative oil production
N_{pt} = oil production during transient period

$$= 25.9\,\phi\,(r_e - r_w)^2\,\frac{\mu_{oi} C_o}{K_{oi}}\,v_o$$

N = oil initially in place

Equations 1.93 and 1.94 make it possible to calculate pressure and saturation profiles with known oil PVT properties and relative permeability data in a step-by-step fashion inward from a pair of boundary values determined by the Muskat's differential material balance of the following form:

$$\frac{d\left(\frac{S_o}{B_o}\right)}{dp} = \left(\frac{S_o}{B_o}\right)\left(\frac{B_g}{5.615\,B_o}\frac{dR_s}{dp} - \frac{1}{B_o}\frac{dB_o}{dp}\right) + \left(\frac{1 - S_w}{B_o} - \frac{S_o}{B_o}\right)\left[B_g\frac{d\left(\frac{1}{B_g}\right)}{dp}\right] \tag{1.95}$$

and the following volumetric relationship:

$$\frac{S_o}{B_o} = \frac{S_{oi}}{B_{oi}}\left(1 - \frac{N_p}{N}\right) \tag{1.96}$$

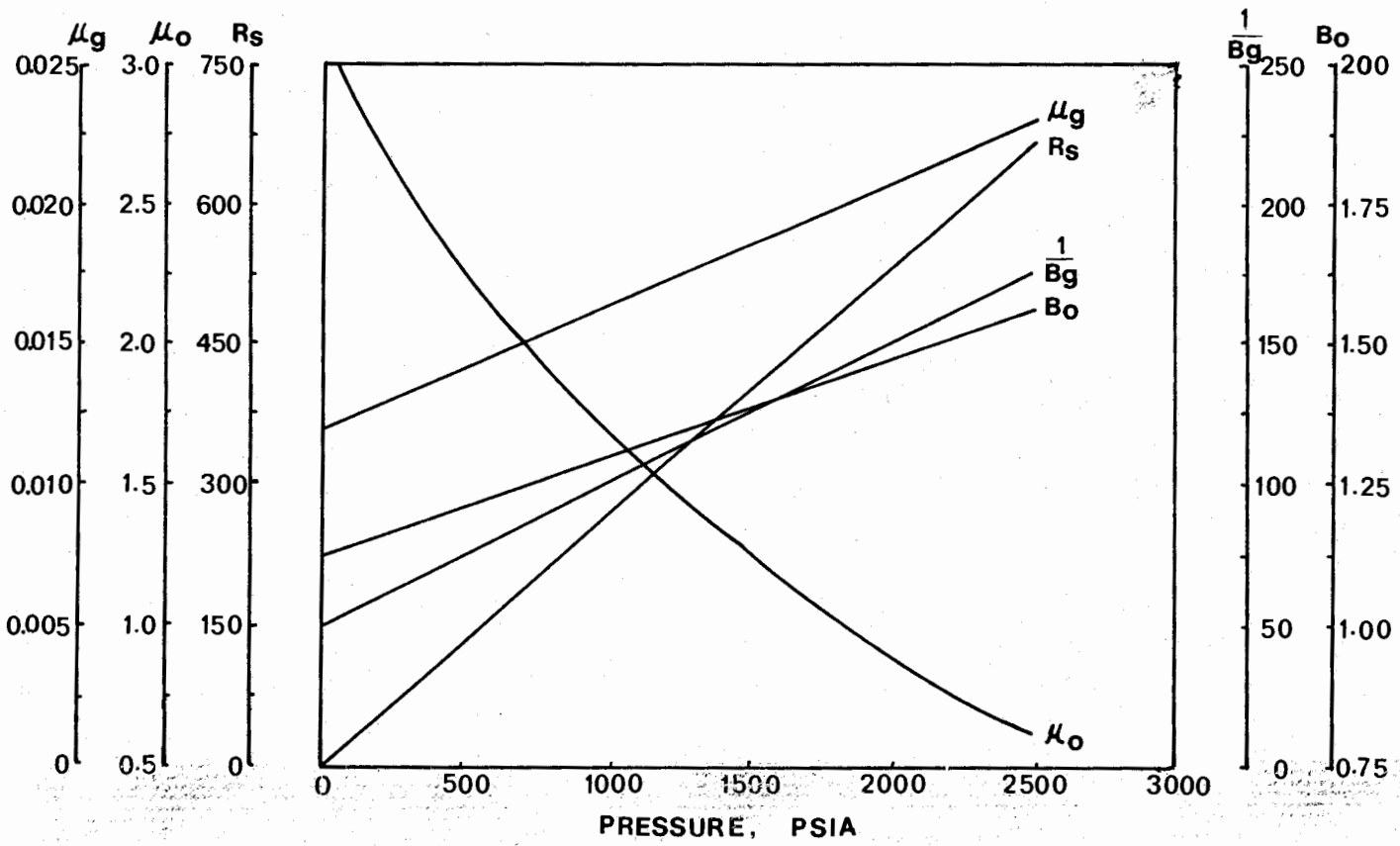


Fig. 1.78 PVT properties of the oil and gas—reservoir two (after ref. 29).

These equations were programmed for the purpose of calculating IPR's of the previously described depletion-type reservoirs. The computation is done in the following manner.

Equations 1.95 and 1.96 were solved by successive approximations to calculate average reservoir pressure and oil saturation at various fractions of the cumulative oil produced. A small incremental increase in cumulative oil production (0.001) is assigned in the present work. This average reservoir pressure and its corresponding value of oil saturation are then assumed to be the outer boundary values in order to start computations at the instant considered.

From these boundary values and equations 1.93 and 1.94, the pressure and saturation profiles are generated along the reservoir assuming a constant rate of production at the sand face. The pressure profile of the reservoir is obtained by equation 1.93. The relative permeability value (K_{ro}) used in this equation is interpolated from the relative permeability curve for that reservoir using the saturation values calculated from equation 1.94.

Pressure at the sand face is then the flowing bottom-hole pressure. To obtain IPR curves, the flowing bottom-hole pressure is calculated for different oil flow rates. The maximum flow rate is the value of flow rate at 100% drawdown ($p_{wf} = 0$).

The IPR curves obtained by this procedure for two previously described hypothetical reservoirs are shown in Figs. 1.79 and 1.81. Results of the material balance calculations for each reservoir, together with the computed maximum flow rate, are tabulated in Tables 1.12 and 1.13. These results are discussed and compared with the results obtained by the other methods at the end of this section.

The work of Vogel and Fetkovich have been discussed previously. However, the sensitivity of Vogel's IPRs to the "match points" used to generate them is discussed and shown in Fig. 1.80 and 1.82. The value of the exponent, n, in Fetkovich's flow equation was assumed to be unity for this comparative study.

1.363 Comparative evaluation of IPR curves

For the purpose of comparing the IPR curves calculated by Weller, Vogel, and Fetkovich's methods for the depletion-type reservoirs under consideration, plots were made on cartesian graph paper (Figs. 1.79 and 1.81). To observe the sensitivity of Vogel's IPRs to the "match points" used to generate them, compared to Weller's IPRs, Vogel's IPRs using three different match points at two different fractions of cumulative oil produced are shown in Figs. 1.80 and 1.82. The results of the material balance calculations and the computed maximum flow rates are tabulated in Tables 1.12 and 1.13.

It is generally observed that the IPRs calculated by Fetkovich's method give higher flow rates than Weller's and Vogel's methods at a given pressure. The difference is estimated to be 15–30%. This difference is greater at higher fractions of cumulative recovery than at lower ones. However, it must be noted that this difference depends on the value of the exponent, n, used in Fetkovich's method. The value of n is assumed to be 1.0 in the present investigation. If n could be established slightly lower than 1.0, this difference be-

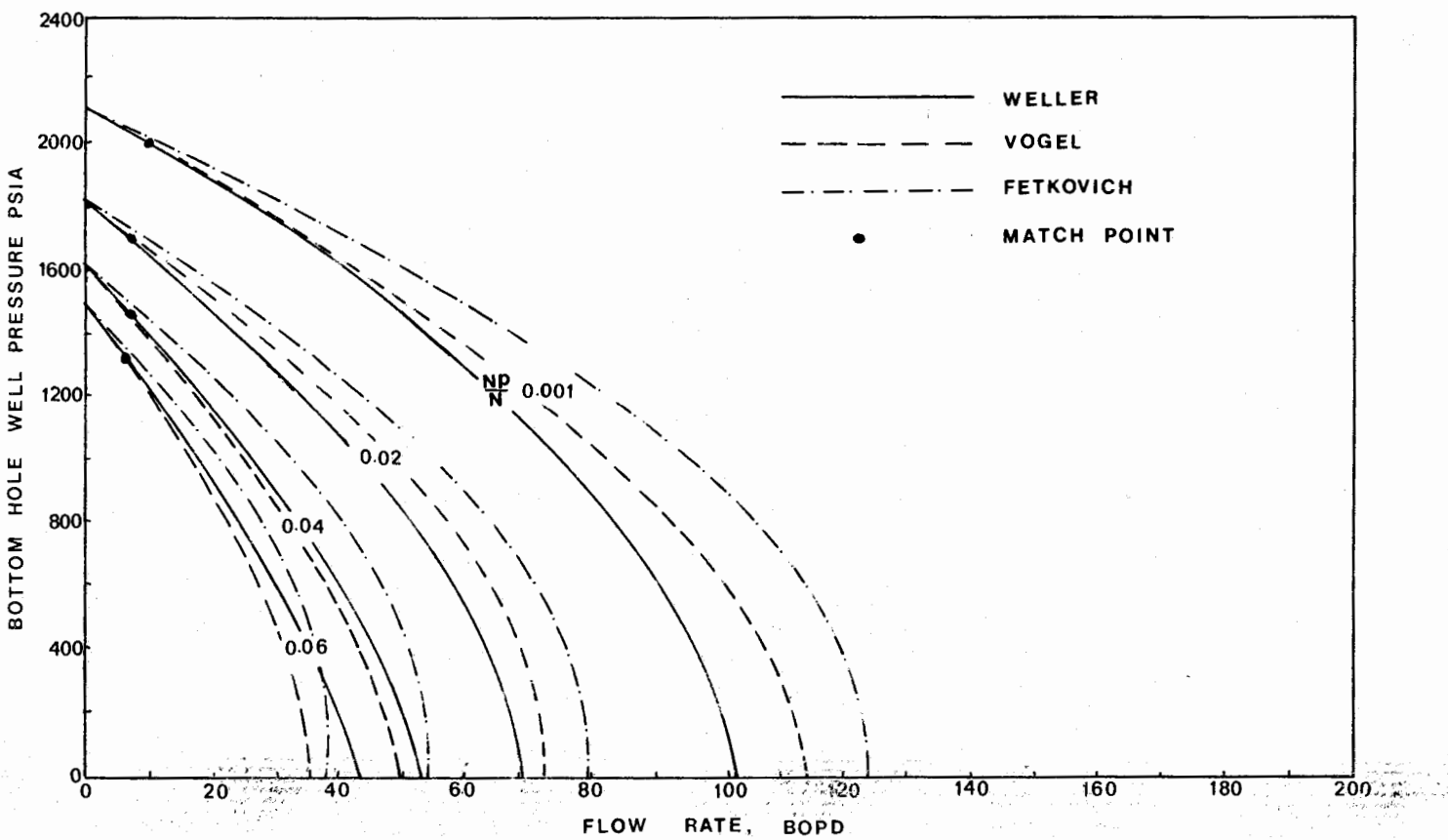


Fig. 1.79 Comparison of methods (after ref. 29).

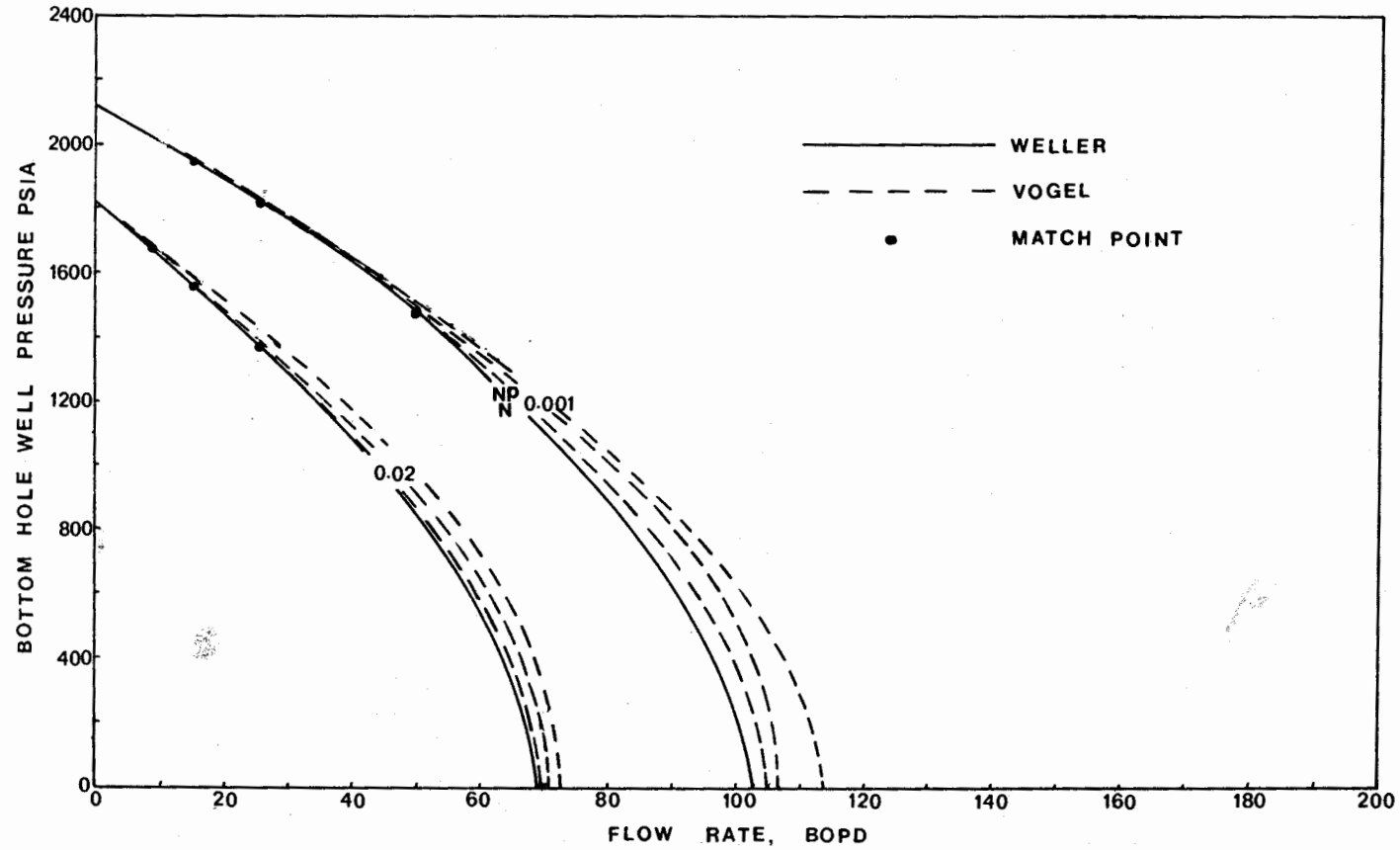


Fig. 1.80 Comparison of methods (after ref. 29).

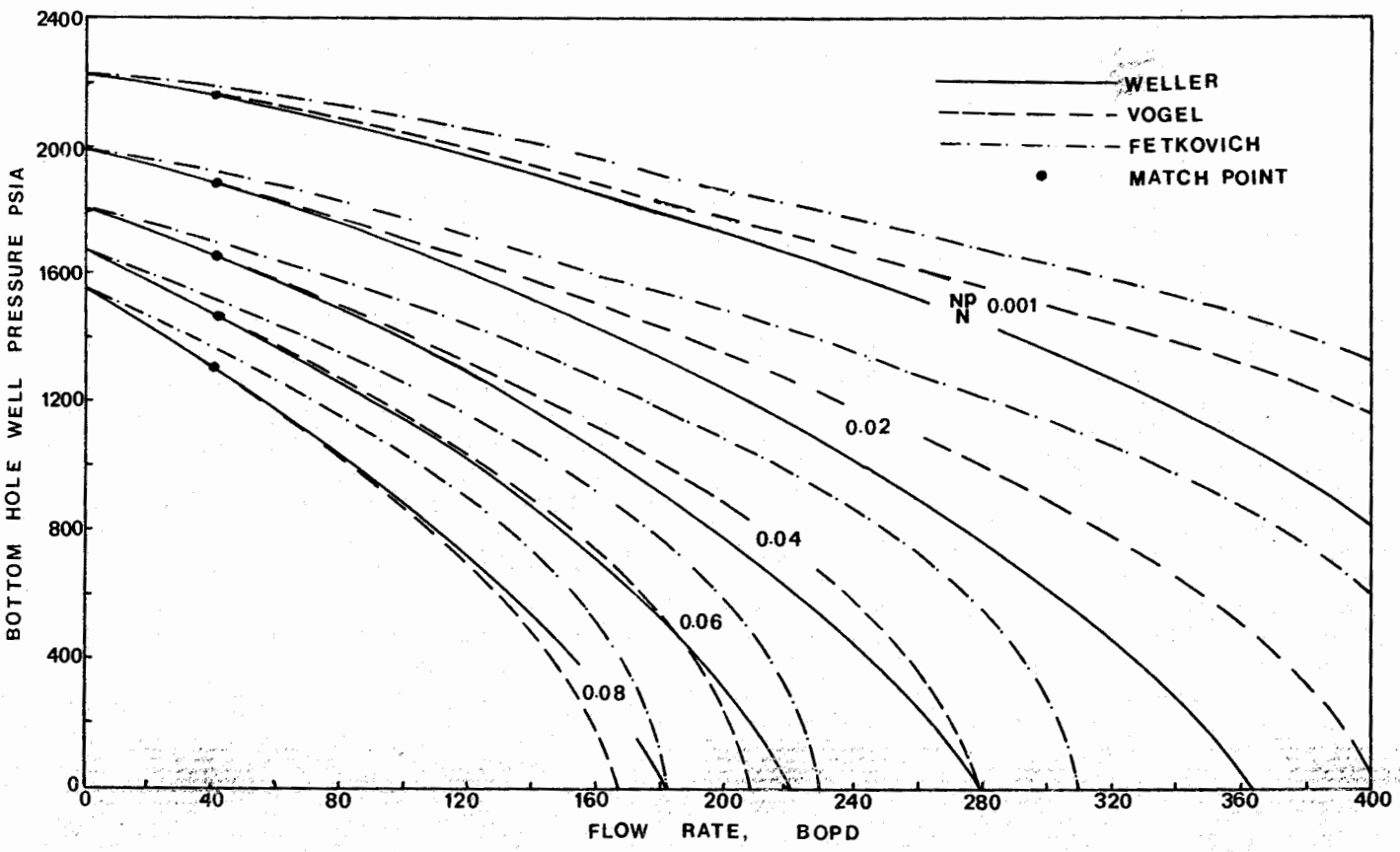


Fig. 1.81 Comparison of methods (after ref. 29).

TABLE 1.12
RESULTS OF MATERIAL BALANCE CALCULATION AND MAXIMUM FLOW RATES CALCULATED FROM WELLER, VOGEL AND FETKOVICH METHODS
Reservoir 1

Cum. oil produced, fraction	Avg. res. press., psia	Avg. oil satn., fraction	Maximum flow rates (bbl/day)					
				Vogel Match point				
			Weller	5	15	25	50	Fetkovich
0.001	2,111.0	0.800	102.5	115.5	115.0	112.9	106.7	123.7
0.010	1,954.4	0.781	83.3	92.1	89.7	87.6	81.5	99.7
0.020	1,827.0	0.760	69.7	72.9	70.8	68.7	64.5	79.1
0.030	1,727.6	0.742	60.3	59.7	57.7	55.6	55.8	64.8
0.040	1,644.7	0.726	53.2	49.6	47.6	46.5	–	54.0
0.050	1,573.1	0.711	48.1	41.5	40.6	41.3	–	45.4
0.060	1,509.8	0.698	44.3	36.3	36.7	37.8	–	38.8
0.070	1,452.8	0.686	41.1	33.4	33.9	35.2	–	35.5
0.080	1,400.6	0.676	38.2	30.7	31.5	33.1	–	32.7

TABLE 1.13
RESULTS OF MATERIAL BALANCE CALCULATION AND MAXIMUM FLOW RATES CALCULATED FROM WELLER, VOGEL AND FETKOVICH METHODS
Reservoir 2

Cum. oil produced, fraction	Avg. res. press., psia	Avg. oil satn., fraction	Maximum flow rates (bbl/day)					
				Vogel Match point				
			Weller	40	60	80	160	Fetkovich
0.001	2,235.4	0.741	503.8	594.9	584.7	574.6	562.6	647.2
0.010	2,110.4	0.722	427.4	490.3	479.8	466.6	453.5	533.3
0.020	1,997.0	0.705	364.8	402.0	389.5	377.6	366.8	443.6
0.030	1,901.4	0.690	317.6	332.9	322.3	312.6	303.7	367.6
0.040	1,817.8	0.676	280.3	280.6	271.5	264.0	260.2	310.1
0.050	1,743.2	0.662	248.2	239.3	233.2	229.8	229.4	264.8
0.060	1,655.6	0.650	222.6	209.0	205.6	204.3	206.5	227.9
0.070	1,613.7	0.638	202.8	186.2	183.9	184.2	188.2	202.8
0.080	1,556.5	0.626	182.4	166.8	165.6	167.4	173.8	181.4

comes less. A number of flow tests should be conducted to determine the actual value of n.

Vogel's dimensionless IPR equation generally gives low flow rates at a given pressure, compared to Fetkovich's method. The flow rates calculated by Vogel's IPRs depend upon the choice of the match points used to generate them. If the match points are chosen at a lower producing rate and a lower corresponding pressure drawdown, the flow rate at a given pressure will be higher at lower fractions of cumulative recovery and lower at higher fractions, compared to Weller's method. If the match points are chosen at correspondingly higher producing rates and higher pressure drawdowns, Vogel's IPRs become closer to Weller's at lower fractions of cumulative recovery and deviate from them at higher fractions. In either case the difference does not exceed 20%.

It is also observed that the viscosities of the crude oil and the absolute permeability values of the reservoir do affect the shape of the IPR curves. High oil viscosity and low permeability to oil give lower producing rates at maximum drawdown, and vice versa.

The magnitude of the producing rates at 1.13 maximum drawdown, tabulated in Tables 1.12 and 1.13, gives an idea of the comparative nature of the IPR curves.

1.364 Prediction of future IPR curves

Yousaf also made a comparative study of the Fetkovich and Standing methods for predicting future IPR curves. The investigation of skin effect due to any physical occurrence, such as damage or improvement, on the future IPR's by Fetkovich's method is also done. Both of the above procedures have been previously discussed. The results obtained are discussed in the following section.

1.3641 Application of Fetkovich's method to Vogel's dimensionless IPR

Fetkovich's[15] method of predicting future performance of depletion-type reservoirs producing below the bubble point pressure can be modified for use with Vogel's dimensionless IPR equation.

From Fetkovich's flow equation:

$$q_o = J'_o (\bar{p}_R^2 - p_{wf}^2)^n \qquad (1.58)$$

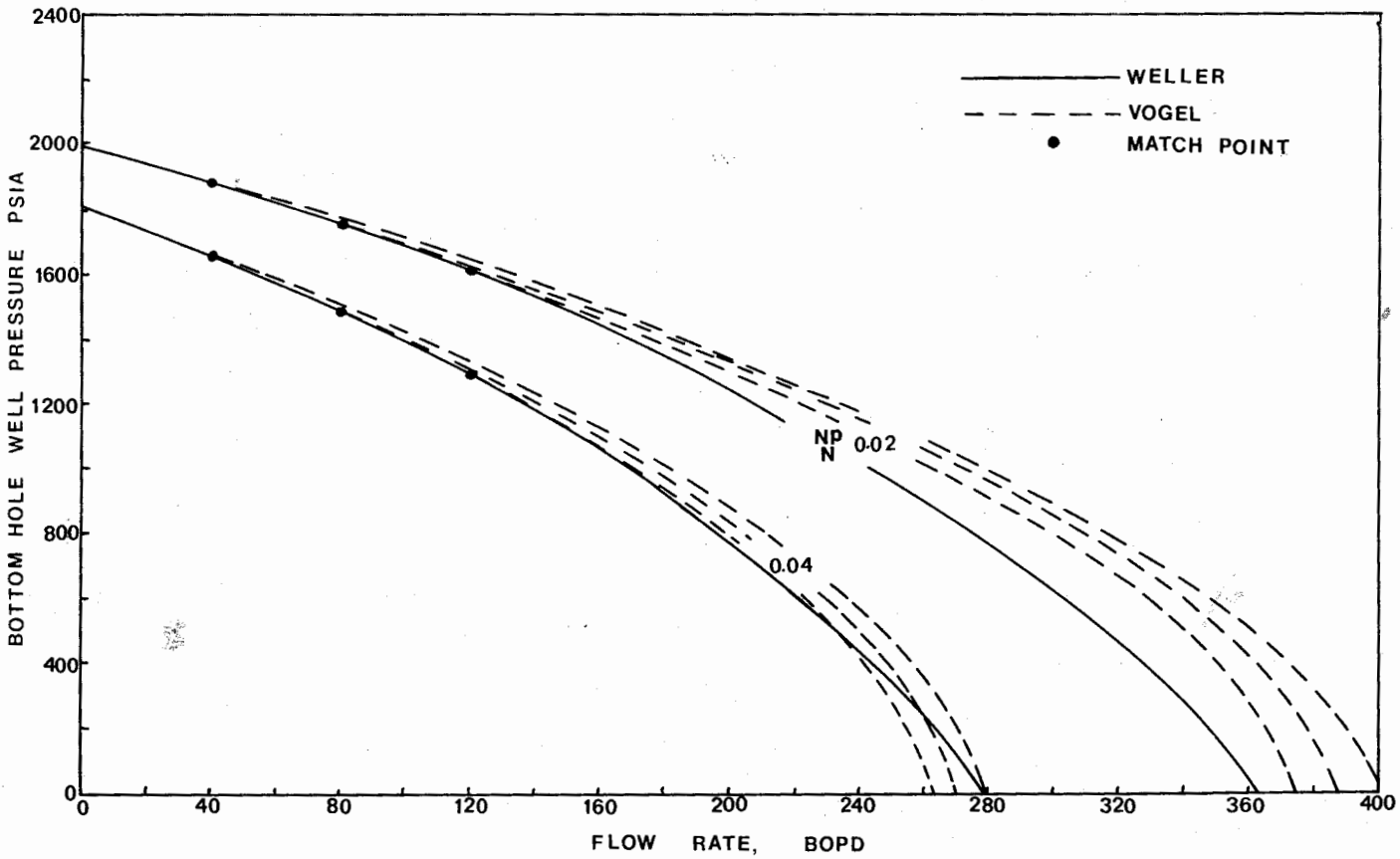


Fig. 1.82 Comparison of methods (after ref. 29).

we can write the flow equation that determines the flow rate corresponding to maximum drawdown pressure at initial condition as:

$$(q_o)_{max,i} = (J'_o)_i\,(\bar{p}_{Ri}{}^2)^n \qquad (1.97)$$

At any future pressure this equation becomes:

$$(q_o)_{max,f} = (J'_o)_i \frac{(\bar{p}_R)_f}{(\bar{p}_R)_i}\,(p_R)_f{}^{2n} \qquad (1.98)$$

Combining equation 1.97 and 1.98 and assuming that n equals 1.0, J'_o can be eliminated to give:

$$\frac{(q_o)_{max,f}}{(q_o)_{max,i}} = \left(\frac{\bar{p}_{R,f}}{\bar{p}_{R,i}}\right)^3 \qquad (1.99)$$

Equation 1.99 enables us to determine future IPRs in the following manner:

From the initial flow condition, initial values of maximum flow rate are calculated using Vogel's equation. Then at the required future average reservoir pressure, $(q_o)_{max,f}$ is calculated by equation 1.99 and IPR curves are obtained.

1.3642 Application of Standing's method to Fetkovich's flow equation

Standing's pressure function $(K_{ro}/\mu_o B_o)$ relationship can also be modified to predict future IPRs using Fetkovich's flow equation. From Fetkovich's flow equation J'_o is defined as:

$$J'_o = \frac{7.08\ Kh}{\ln(r_e/r_w) - 3/4}\left[\left(\frac{K_{ro}}{\mu_o B_o}\right)_{\bar{p}_R}\left(\frac{1}{2\,\bar{p}_R}\right)\right] \qquad (1.85)$$

At any pressure J'_o is the function of $(K_{ro}/\mu_o B_o)$ and the average reservoir pressure itself. The value of J'_o can then be adjusted at different times by accounting for the changes of $(K_{ro}/\mu_o B_o)$ and the average reservoir pressure. The ratio of J'_o in the future to that of the present (initial) can be written as:

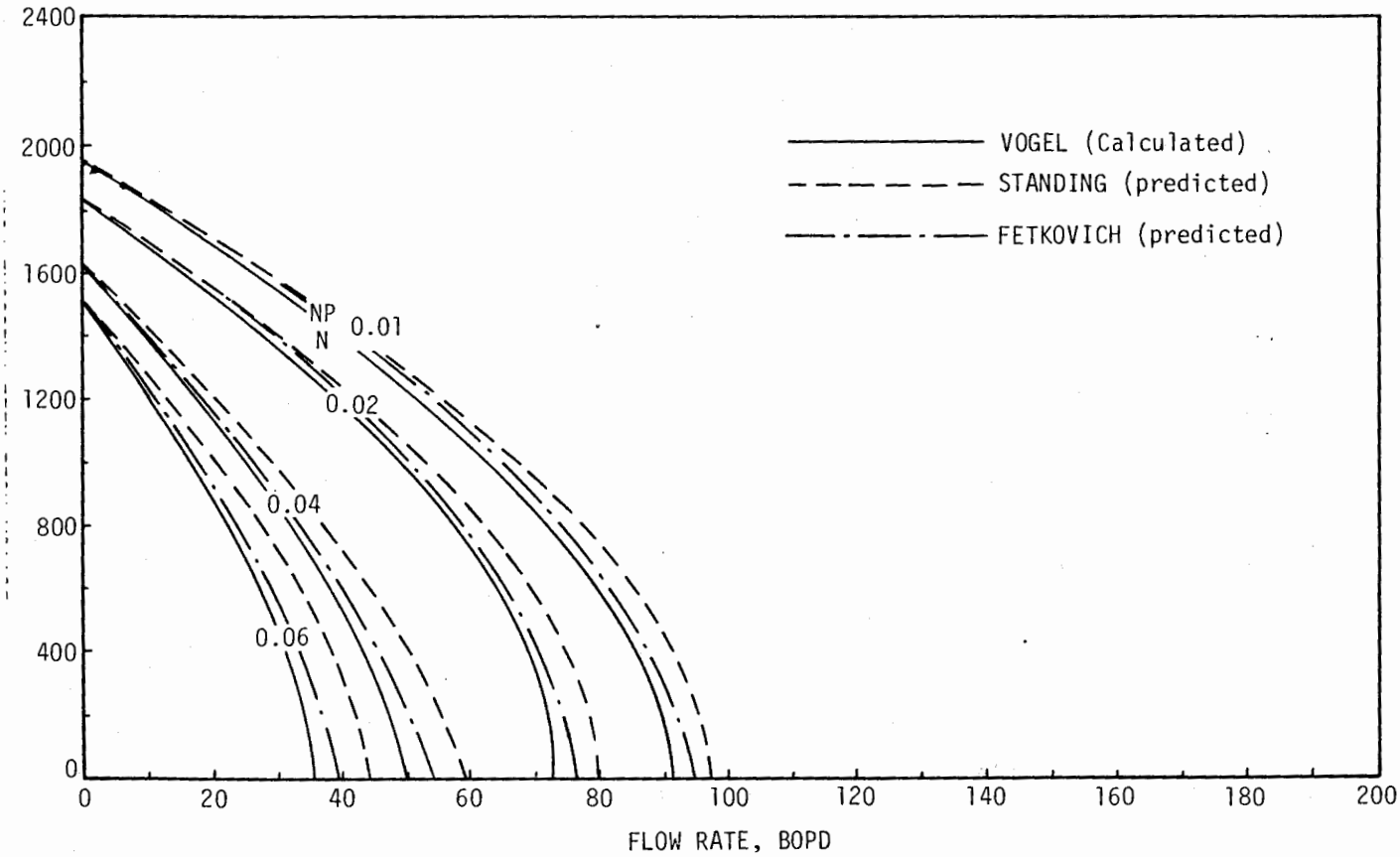
$$\frac{(J'_o)_f}{(J'_o)_i} = \frac{(K_{ro}/\mu_o B_o)_f}{(K_{ro}/\mu_o B_o)_i}\left(\frac{\bar{p}_{Ri}}{\bar{p}_{Rf}}\right) \qquad (1.100)$$

The future values of $(K_{ro}/\mu_o B_o)$ may be evaluated using either Tarner's material balance or Muskat's differential material balance as previously described in Standing's method. Once J'_o in the future is known, future IPRs may then be obtained from Fetkovich's equation.

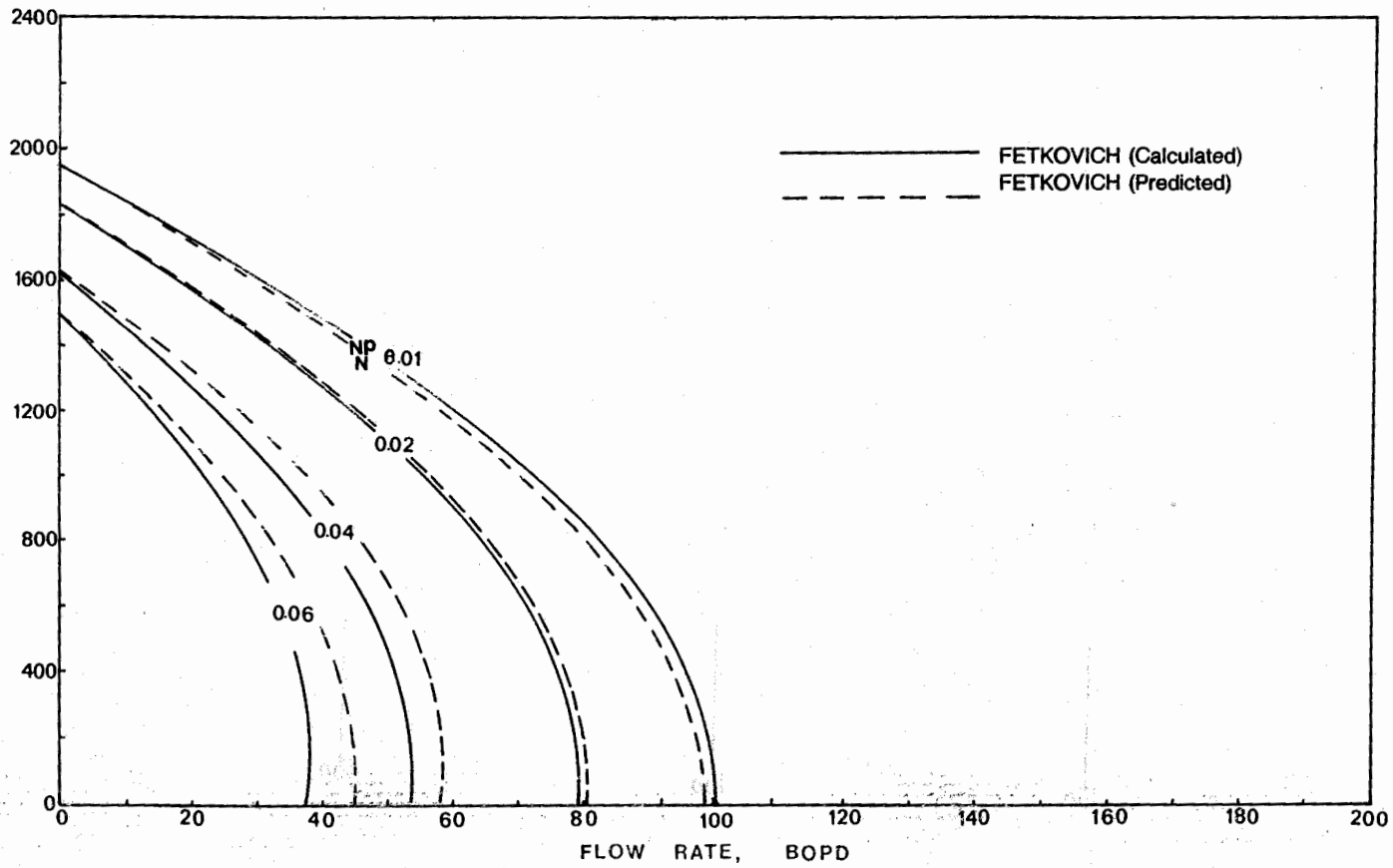
1.3643 Discussion and comparison of results

The predicted future IPRs calculated by Standing and Fetkovich's methods are shown in Figs. 1.83 to 1.86 along with the actual, i.e., present, IPRs for comparison. The effect of dimensional skin, due to any physical occurrence such as damage or improvement, on the predicted future IPRs by Fetkovich's method is shown in Figs. 1.87 to 1.90. The computed maximum flow rates by these methods, with or without skin, are tabulated in Tables 1.14 to 1.17.

It is generally observed that the flow rates predicted by Standing's method, which uses the Vogel's dimensionless IPR equation, are higher than the values calculated from flow data at various fractions of cumulative recovery. This difference does not exceed 25%



ig. 1.83 Comparison of methods (after ref. 29).



ig. 1.84 Comparison of methods (after ref. 29).

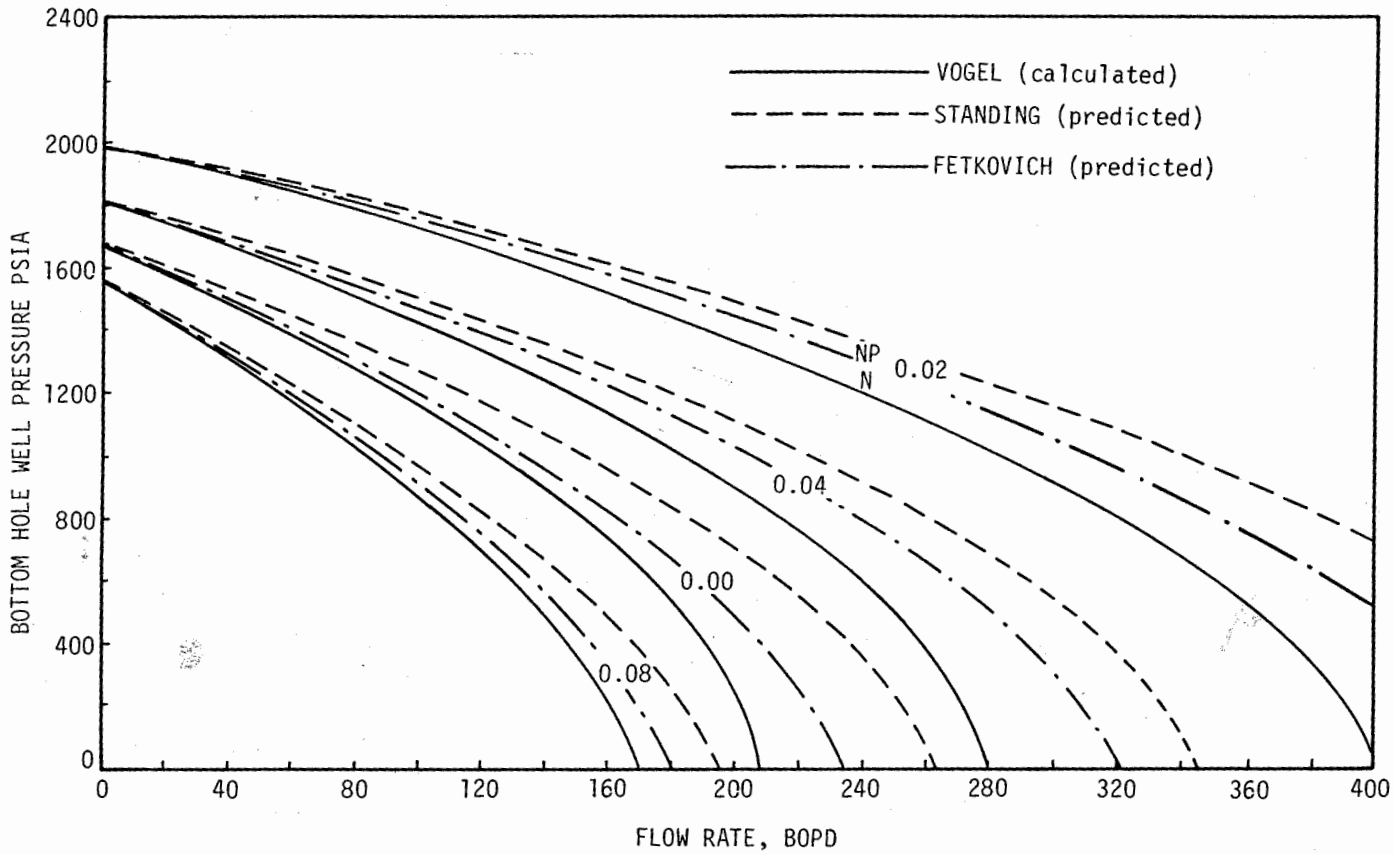


Fig. 1.85 Comparison of methods (after ref. 29).

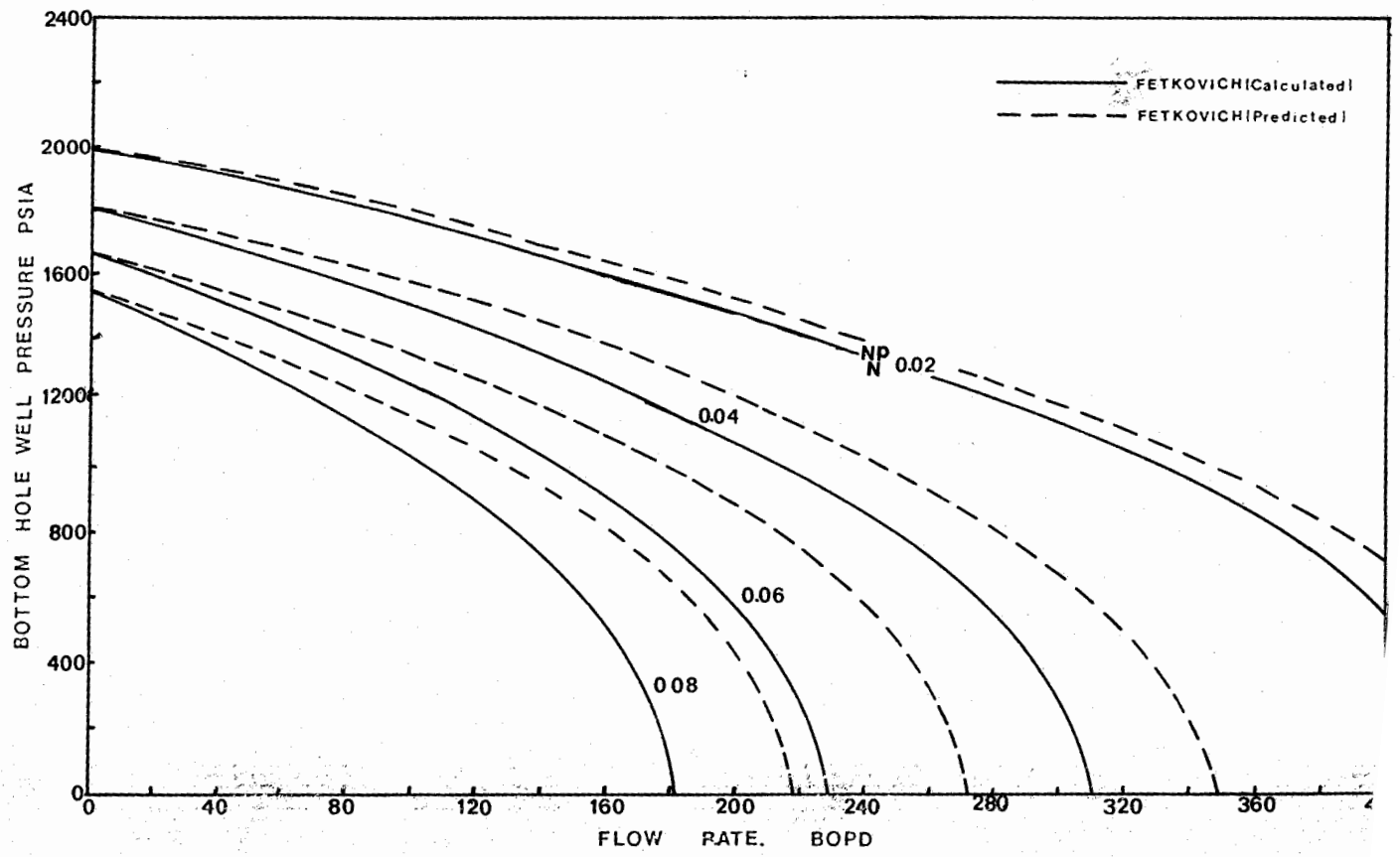


Fig. 1.86 Comparison of methods (after ref. 29).

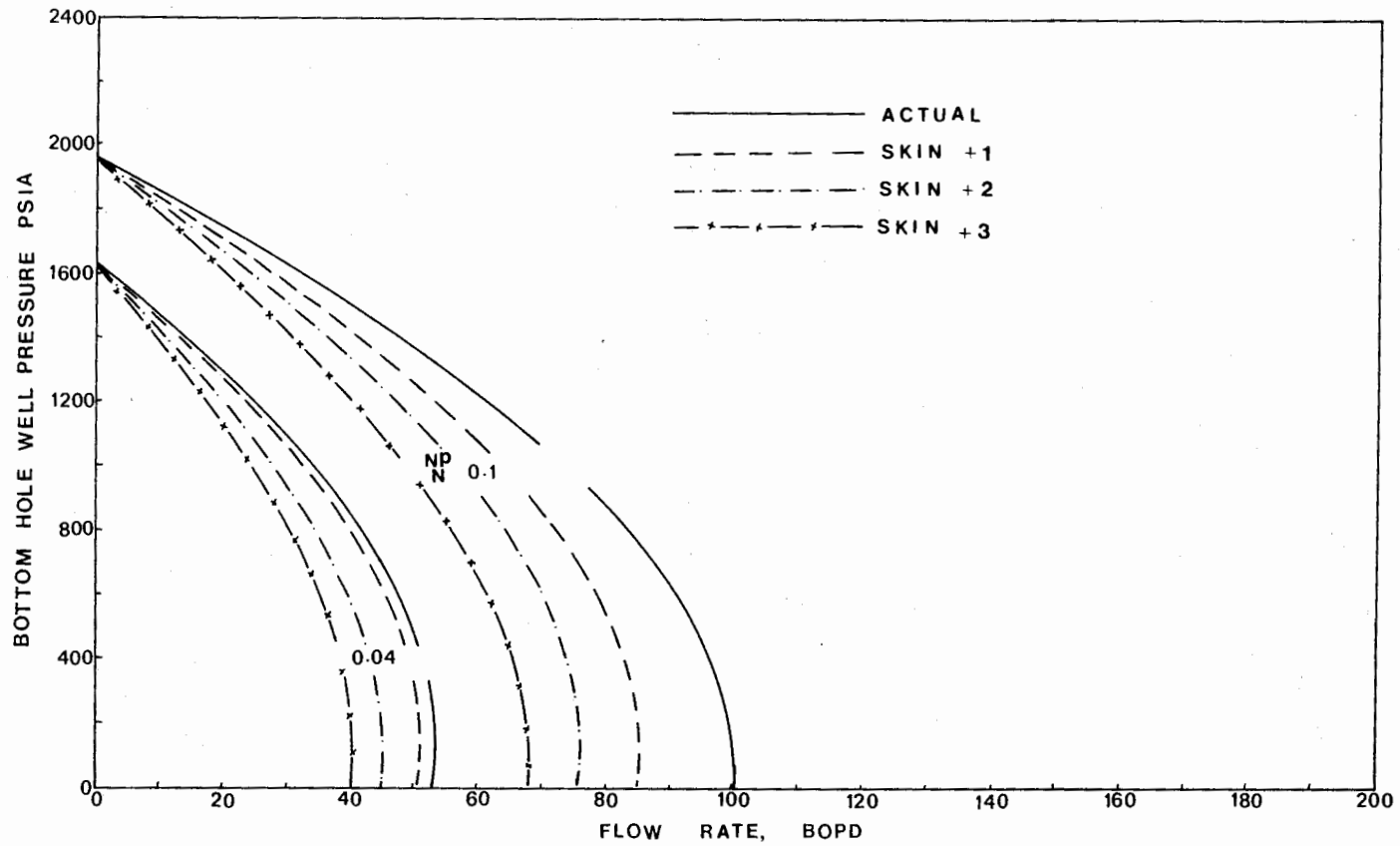


Fig. 1.87 Effect of skin (after ref. 29).

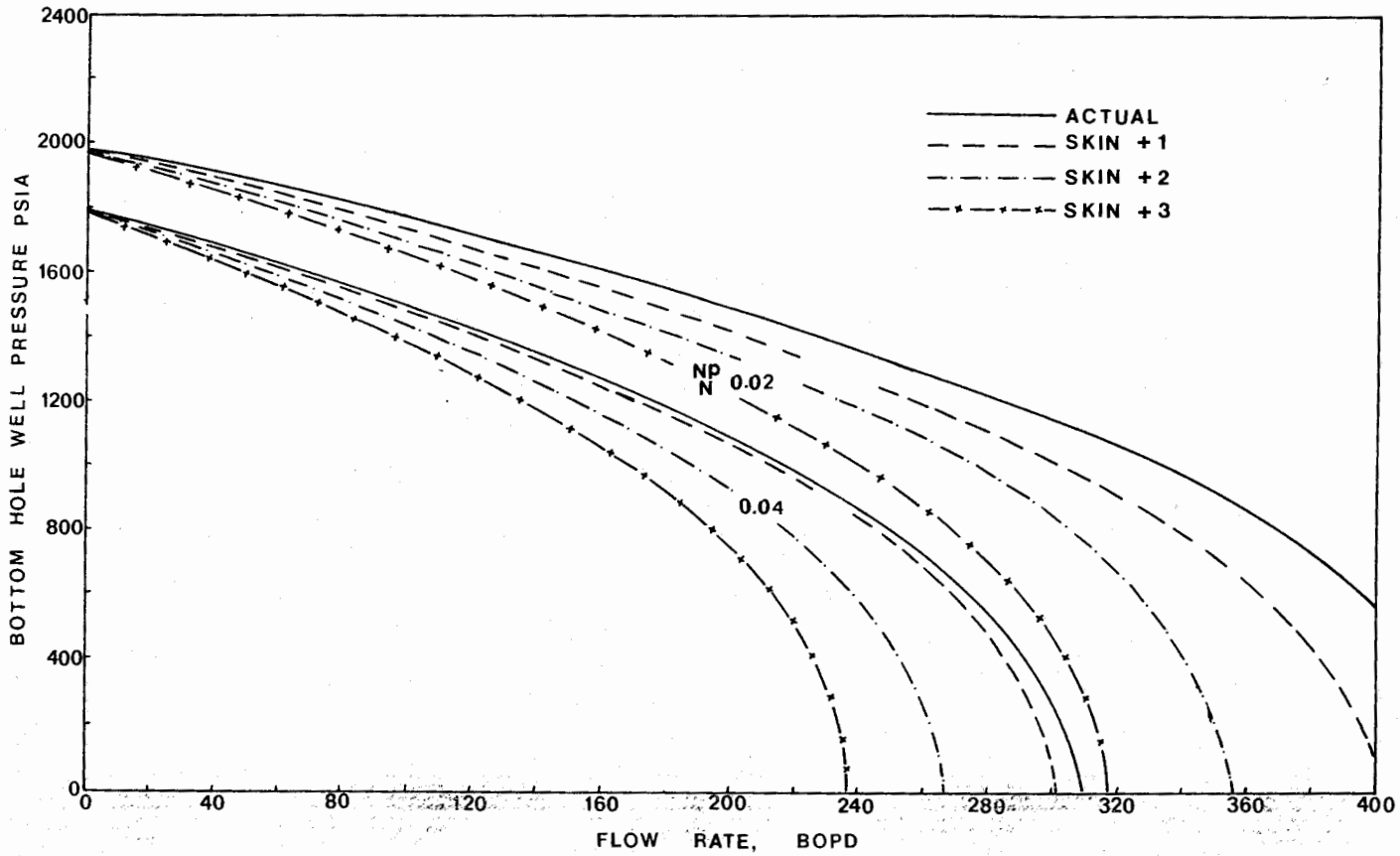


Fig. 1.88 Effect of skin (after ref. 29).

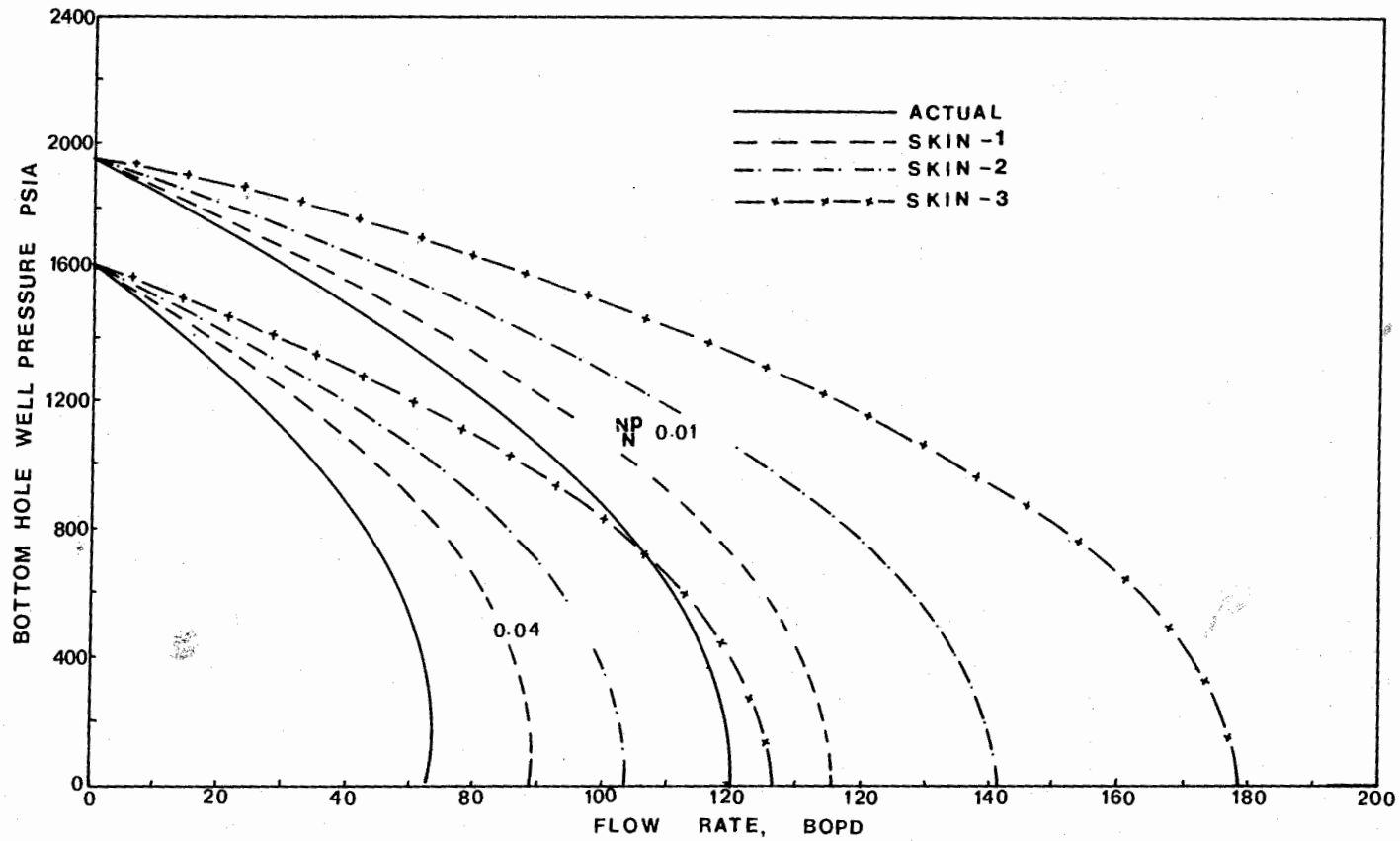


Fig. 1.89 Effect of skin (after ref. 29).

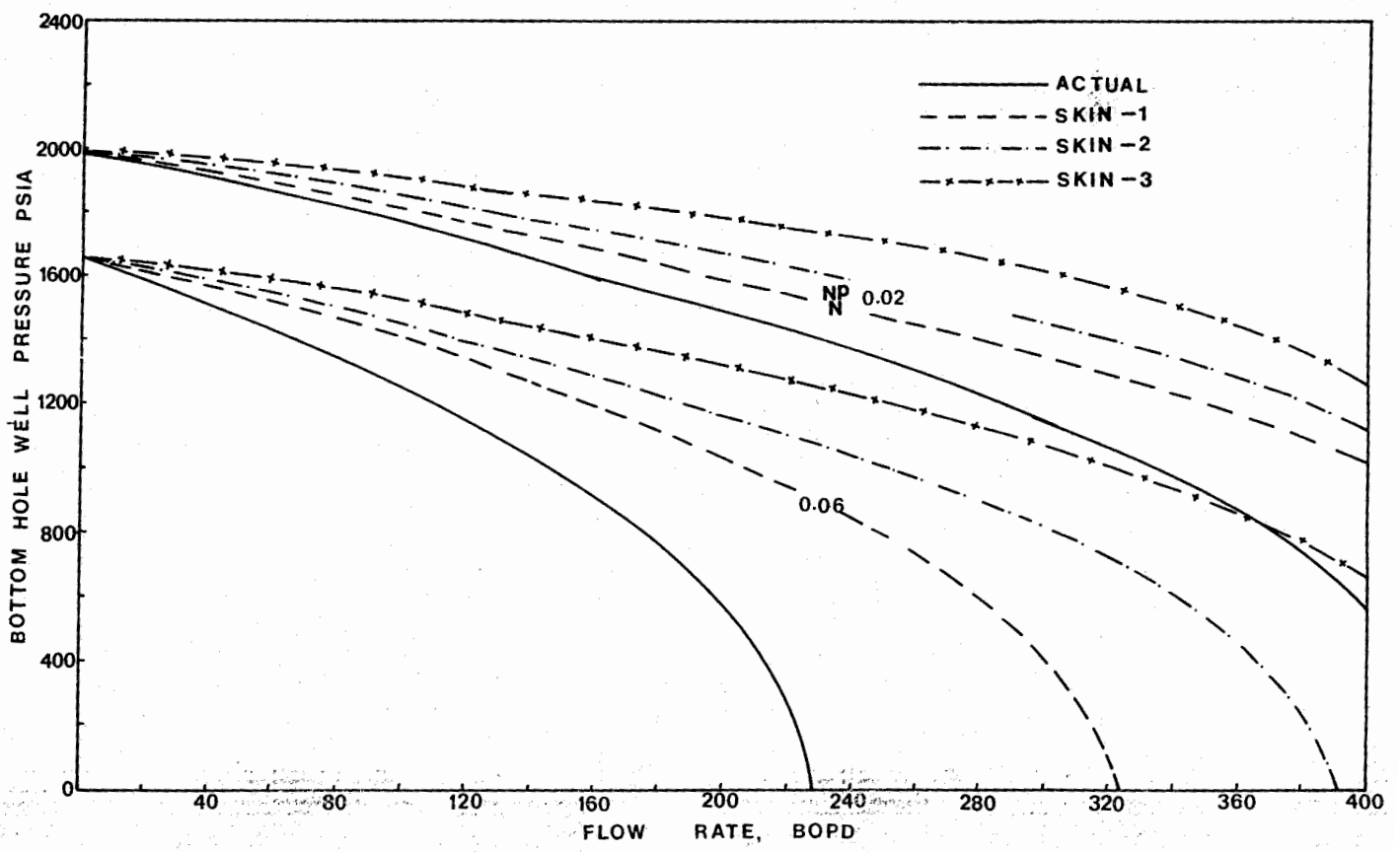


Fig. 1.90 Effect of skin (after ref. 29).

TABLE 1.14
EDICTED MAXIMUM FLOW RATES CALCULATED USING VOGEL'S IPR EQUATION
Reservoir 1

Initial test data used:
Fraction of cum. oil produced = 0.001
Average reservior pressure, psia = 2,111.0
Oil flow rate, bpd = 10.0
Flowing bottom-hole pressure, psia = 2,007.0

ım. oil oduced, action	Avg. res. pressure, psia	Calculated max. flow rate, bpd	Predicted future maximum flow rate, bpd Standing	Fetkovich
0.010	1,954.4	92.1	97.2	96.1
0.020	1,827.0	72.9	80.0	76.3
0.030	1,727.6	59.7	67.8	63.4
0.040	1,644.7	49.6	58.4	53.7
0.050	1,573.1	41.5	53.1	47.7
0.060	1,509.8	36.4	44.4	39.4

TABLE 1.16
PREDICTED MAXIMUM FLOW RATES CALCULATED USING VOGEL'S IPR EQUATION
Reservoir 2

Initial test data used:
Fraction of cum. oil produced = 0.001
Average reservoir pressure, psia = 2,235.4
Oil flow rate, bpd = 40
Flowing bottom-hole pressure, psia = 2,150.4

Cum. oil produced, fraction	Avg. res. pressure, psia	Calculated max. flow rate, bpd	Predicted future maximum flow rate, bpd Standing	Fetkovich
0.01	2,110.4	490.3	530.4	500.3
0.02	1,997.0	402.0	440.6	420.5
0.03	1,901.4	332.9	425.5	360.3
0.04	1,817.8	280.6	346.1	320.2
0.05	1,743.2	239.3	299.3	280.3
0.06	1,655.6	209.0	262.8	235.5
0.07	1,613.7	186.2	231.2	200.5
0.08	1,556.7	166.8	197.5	180.7

the actual values. The results seem to be better and thin 10% of the actual values at higher fractions cumulative recovery. It must be noted that this fference will depend upon the selection of the match int used to generate the actual IPRs.

Fetkovich's empirical formula for predicting the ture IPRs, using his flow equation, gives better sults than Standing's method. This method does t seem to overpredict or underpredict the per- mance of the reservoir at future times. The accuracy within 10 to 15% above or below the actual per- mance values. At lower fractions of cumulative covery, this method predicts higher values at a ven pressure and the reverse occurs at higher frac- ns. It is observed that a positive skin in the well ds to decrease the values of predicted future flow tes. This decrease becomes more as the skin in- eases. A negative skin, on the other hand, tends to crease the values of the predicted future flow rates. is increase is more at lower skin and vice versa. e computed results tabulated in Tables 1.14 to 1.17 compare predicted future performance with and without skin.

Standing's method of predicting future IPR's is a more viable method in that it takes into account the changing values of the pressure function ($K_{ro}/\mu_o B_o$). However, the fact that this method uses Vogel's dimensionless IPRs which vary with the selection of the match points used to generate them, coupled with the inability to define the real permeability curves for a specific well, make this method an approximation. On the contrary, in Fetkovich's method, we do not need to define an actual relative permeability curve due to the assumed linearity of the K_{ro} curve with respect to pressure. This method gives reasonably accurate future performance compared to Standing. It has another advantage in that it does not require the tedious material balance calculations since the oil saturation is not necessary to predict the future performance.

Fetkovich has shown some validity of the equations

TABLE 1.15
PREDICTED MAXIMUM FLOW RATES CALCULATED BY FETKOVICH'S METHOD USING FETKOVICH'S FLOW EQUATION
Reservior 1

Initial test data used:
Fraction of cum. oil produced = 0.001
Average reservoir pressure, psia = 2,111.0
Oil flow rate, bpd = 10.0
Flowing bottom-hole pressure, psia = 2,007.0

Cum. oil produced, fraction	Avg. res. pressure, psia	Calculated max. flow rate, bpd	Predicted future maximum flow rate, bpd Fetkovich Skin 0	+1	+2	+3	−1	−2	−3
0.01	1,954.4	99.8	98.2	85.3	75.5	67.6	115.7	140.7	179.5
0.02	1,827.0	79.1	80.2	69.7	61.6	55.2	94.5	114.9	146.6
0.03	1,727.6	64.8	67.8	58.9	52.1	46.7	79.9	97.2	123.9
0.04	1,644.7	54.0	58.5	50.9	45.0	40.3	68.9	83.9	106.9
0.05	1,573.1	45.4	51.2	44.5	39.3	35.3	60.3	73.4	93.6
0.06	1,509.8	38.8	45.3	39.3	34.8	31.2	53.3	64.8	82.8
0.07	1,452.8	35.5	40.4	35.0	30.9	27.8	47.5	57.8	73.7
0.08	1,400.6	32.7	36.2	31.4	27.8	24.9	42.6	51.8	66.1

TABLE 1.17
PREDICTED MAXIMUM FLOW RATES CALCULATED BY FETKOVICH'S METHOD USING FETKOVICH'S FLOW EQUATION
Reservoir 2

Initial test data used:
Fraction of cum. oil produced = 0.001
Average reservoir pressure, psia = 2,235.4
Oil flow rate, bpd = 40.0
Flowing bottom-hole pressure, psia = 2,150.4

Cum. oil produced, fraction	Avg. res. pressure, psia	Calculated max. flow rate, bpd	Predicted future maximum flow rate, bpd Fetkovich Skin						
			0	+1	+2	+3	−1	−2	−3
0.01	2,110.4	533.3	544.6	473.2	418.3	374.8	641.6	780.1	995.2
0.02	1,997.0	443.6	461.5	400.9	354.5	317.6	543.5	661.0	843.4
0.03	1,901.4	367.6	398.3	346.1	305.9	274.2	469.1	570.5	727.9
0.04	1,817.8	310.1	348.0	302.4	267.3	239.6	409.9	498.5	636.1
0.05	1,743.2	264.8	306.9	266.7	235.7	211.3	361.5	439.6	560.9
0.06	1,655.6	227.9	272.6	236.8	209.4	187.6	321.0	390.4	498.1
0.07	1,613.7	202.8	243.4	211.5	187.0	167.6	286.7	348.7	444.9
0.08	1,556.5	181.4	218.5	189.8	167.8	150.4	257.3	312.9	399.3

developed by Vogel and also of his proposed equation. However, actual field results are needed for verification.

On the basis of the present investigation, the use of reservoir simulators is recommended to generate IPRs. The IPRs obtained from the simulators can then be compared to those obtained by the approximate methods to see if the later are viable to be used in the field.

Investigations should be continued to do a more detailed study of the match points used to generate Vogel's IPRs. The effect of time and rate dependent skin on predicted future IPRs by Fetkovich and the IPRs of damaged wells by Standing's method should also be studied. The effect of non-darcy flow in the reservoir should be investigated.

1.365 Conclusions

Yousaf gave the following conclusions:

1. Both Vogel's dimensionless IPR equation and Fetkovich's flow equation are valid for estimating the performance of a depletion-type reservoir. Vogel's method, however, has a disadvantage in that it is sensitive to the match point used.

2. Fetkovich's flow equation generally results in higher flow rate performance curves compared to Vogel's dimensionless IPR equation.

3. Standing's method of predicting future IPRs, using Vogel's dimensionless IPR equation, gives higher flow rates than actual rates. This method requires reliable relative permeability information and also requires material balance calculations to predict oil saturation at the future average reservoir pressure. The predicted performance will also depend upon the match point used to generate actual performance.

4. Fetkovich's method of predicting future performance using his flow equation gives reasonably accurate results compared to Standing's method. The value of the exponent, n, should be established early in the life of the reservoir to form a basis for future prediction of IPR curves. This method has the advantage over Standing's method in that it does not need the tedious material balance calculations and we do not need to define an actual relative permeability curve.

5. Positive skin tends to decrease the predicted future flow rates by Fetkovich's method and negative skin tends to increase them.

1.370 Effect of watercut on IPR

1.371 Gilbert's discussion

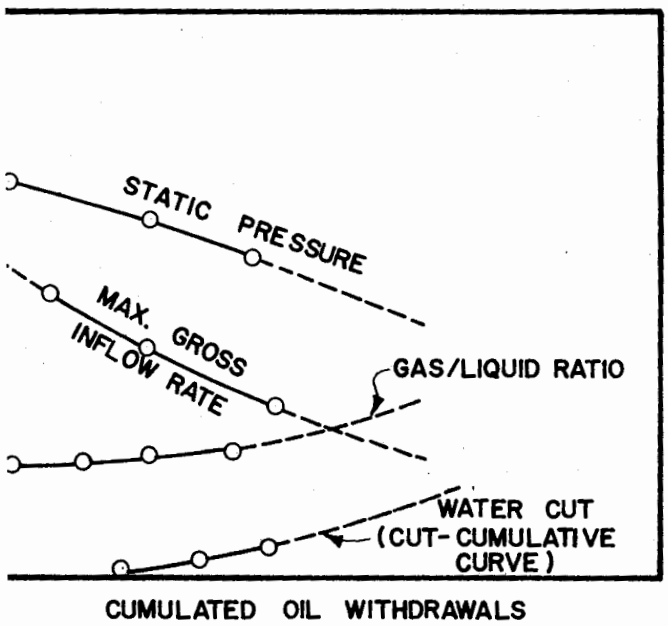
Gilbert[6] showed the effect of water-cut on IPR curves, and its relationship to other factors such as interflow rates. In Fig. 1.91 we note a typical plot for a well making some water. The static pressure, gross inflow rate, and gas liquid ratio curve are also shown.

The gross inflow rate decreases as the water cut increases which is indicative of a solution gas drive well. In most cases the gross inflow rates for active water drive wells will increase as the water cut increases.

The water entry into a well may come from a relatively low-pressure source (Fig. 1.92) or from a relatively high-pressure source (Fig. 1.93).

The differences are readily apparent as noted in Figs. 1.92 and 1.93. For the low pressure source as noted in Fig. 1.92 the flowing bottom hole pressure in the oil zone must drop to a point such that water is capable of entering. For the high pressure water source the flowing pressure must drop to a point such that oil will enter the well bore. Hence we have 100% water entry in initially opening the well for flow (Fig. 1.94).

If curves of the types in Figs. 1.92 and 1.93 are available, they are extremely valuable in estimating future oil recoveries from wells producing water.



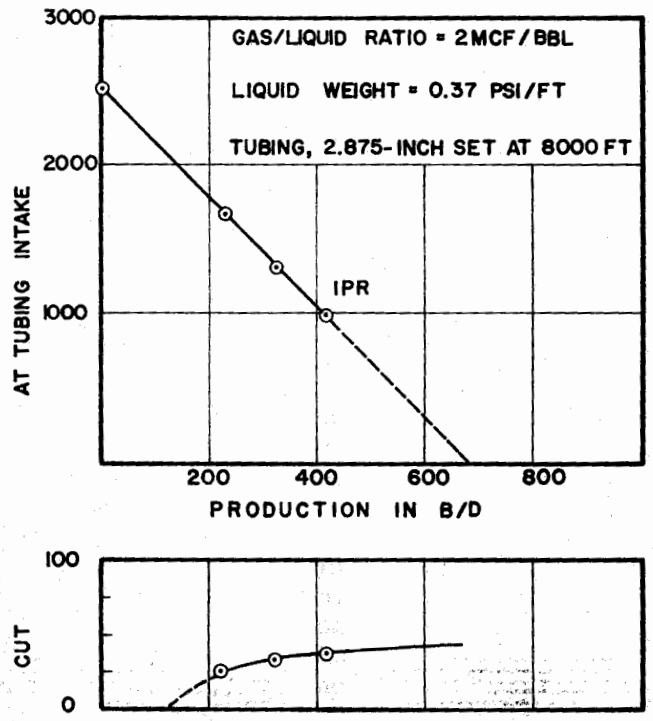
ig. 1.91 Graph of individual—well inflow—performance trends ıfter Gilbert).

In order to select the best operating gross rate at hich to produce a well, Gilbert[6] suggested the lotting of the approximate water IPR from the gross PR along with 2 or 3 water cuts as indicated in Fig. .94.

One should remember that temporary use of high ıflow rates may result in a permanent increase in ater production. Gilbert noted that this may be ıused from bad water shut-off jobs, high oil viscosi-es, slight formation dips, and edge water close to the ell bore.

Gilbert noted as follows:[6]

Differential depletion is progressive during sustained owing periods wherever the ratio of lateral permeabilities to vertical permeabilities is large; and interflow through the bore between producing layers takes place during any subsequent periods of shutdown unless a suitable mud is spotted in the producing interval. Thus any water inflow from a relatively high-pressure source tends to seek out and enter the more depleted oil layers during a shutdown period, and with permanent injury in some fields to effective oil permeabilities. After long periods of flow, a two- or three-week shutdown can cause from 10 to 40% permanent reduction of the inflow capacity of a well tapping a depletion-type reservoir even if the water cut is no greater than 5%. Thus, differential depletion is a factor requiring close consideration in many fields if the operator's equity in operating wells is to be protected."



g. 1.92 Diagram of inflow performance (after Gilbert).

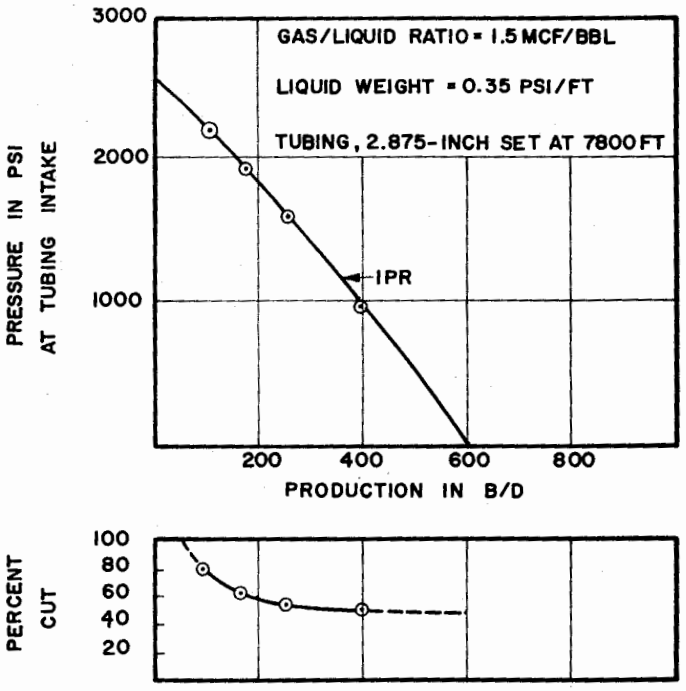


Fig. 1.93 Diagram of inflow performance with curved IPR (after Gilbert).

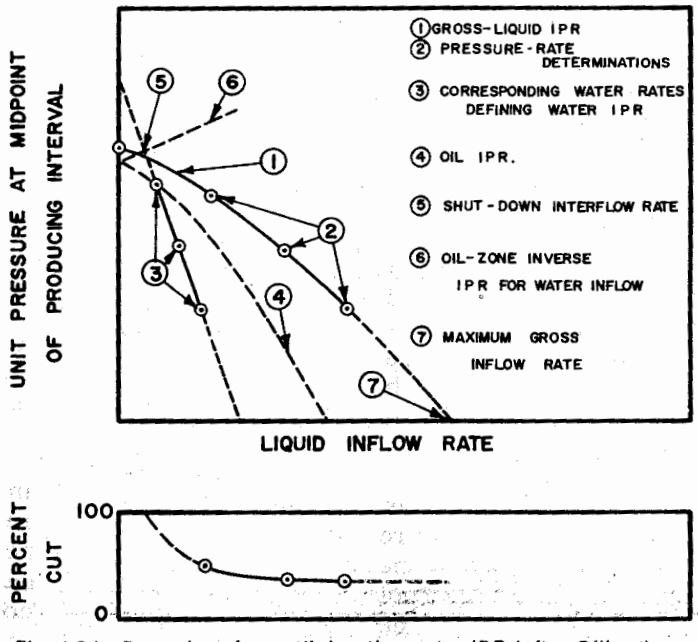


Fig. 1.94 Procedure for outlining the water IPR (after Gilbert).

1.372 Nind's discussion

Nind[31] discussed the effect of drawdown on water oil ratios for stratified formations. He gave an example for determining at the well bore, whether the pressure in the water zone is greater or less than the pressure in the oil zones. By testing the well at 3 or 4 rates giving different water cuts and from an analysis of the gross IPR it can be determined whether or not "high pressure" or "low pressure" water exists. It is important to remember that this analysis applies only to stratified or separate zones of water and oil and does not apply to an oil zone producing some water.

Nind gave the following example which illustrates his method of approach.[31]

EXAMPLE PROBLEM #17

Given data:
A series of well tests were conducted and the following information obtained:

Gross rate, bpd	Water cut, water/gross, %	Bottom-hole flowing pressure, psig
47	85	1,300
90	60	920
125	48	630
162	45	310

From the preceding tests the static pressure and PI of the oil and water zones respectively can be determined. If the well were shut in at what rate could water be expected to flow into the oil zone (or vice versa)?

Solution:
From the gross rates and the measured water cuts the water and oil rates can be determined.

Gross rate, bpd	Water cut, %	Water rate, bpd	Oil rate, bpd	P_{wf}, psig
47	85	40	7	1,300
90	60	54	36	920
125	48	60	65	630
162	45	73	89	310

Reference should be made to Fig. 1.95 which shows a plot of this data. As noted, three principal sets of data are plotted, that is, the flowing bottom hole pressure versus (1) gross rate, (2) water rate, and (3) oil rate. These three curves represent IPR curves respectively for the gross rate, water rate, and oil rate.

The static bottom hole pressure of the oil zone and water zone can be found by extending the curve for all oil and all water to a zero production rate and noting the corresponding pressure. The static bottom hole pressure of each zone is found to be:

$$\bar{p}_R \text{ (oil)} = 1{,}350 \text{ psig}$$
$$\bar{p}_R \text{ (water)} = 2{,}600 \text{ psig}$$

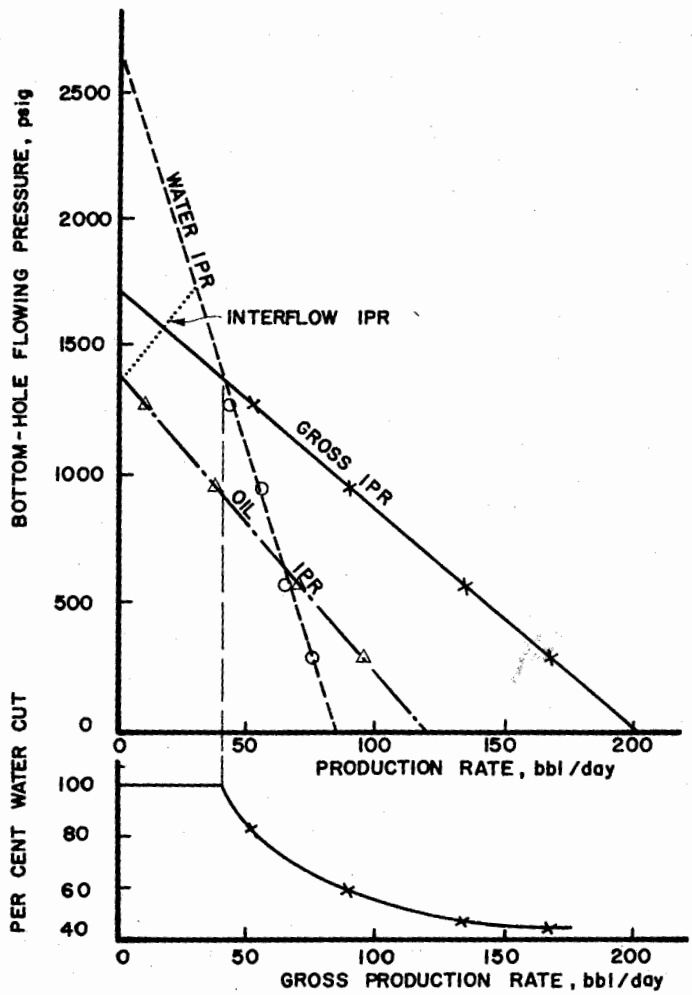


Fig. 1.95 IPR and water cut curves: high-pressure water (after Principles of Oil Well Production *by Nind. Copyright 1964, McGraw-Hill. Used with permission of McGraw-Hill Book Company).*

The bottom hole measured pressure for this well would likely stabilize at the point where the gross IPR curve was extended to q = zero, and is noted to be 1,700 psig.

In order to determine the rate at which water would flow into the oil zone under shut-in conditions we need only find that point on the water IPR line where the pressure is equal to the static pressure on the gross IPR curve (1,700 psi), and extend this point vertically downward to find a rate of 28 bpd. The interflow IPR curve equal to the gross IPR static pressure (1,700 psi) to the static pressure of the oil zone. This is noted as line 4 in Fig. 1.95.

It is interesting to note the shape of the water cut curve. We see that no oil is produced until the gross flowing pressure reaches a point below the static pressure of the oil zone (1,350 psi). The water production rate must exceed approximately 40 bpd before any oil is produced. For this well we have "high pressure" water which means that the well produces water at the low rates.

Determination of the PIs:

The average PIs over the entire flowing range can be obtained by extending the IPR lines to zero pressure and reading the corresponding rates. Although the IPR lines are straight for this example they are not necessarily straight for many wells. PIs may th

ıeed to be taken at intervals of drawdown in pressure. ?or this example:

Avg. PI of oil zone $J_o = \frac{120}{1350} = 0.089 \frac{bpd}{psi}$

$$= \frac{q_o}{(\bar{p}_{R(oil)} - o)}$$

Avg. PI of water zone $J_w = \frac{82}{2600} = 0.0315 \frac{bpd}{psi}$

$$= \frac{q_w}{(\bar{p}_{R(water)} - o)}$$

Gross PI for artificial lift purposes $= \frac{200}{1700}$

$$= 0.1178 \frac{bpd}{psi} = \frac{q_o + q_w}{(\bar{p}_{R(gross)} - o)}$$

CLASS PROBLEM #17-A

Given:

Flowing pressure	q_o	q_w
3,000	0	30
2,500	32	57
1,500	165	115
750	263	157

Construct a pressure flow rate diagram and deter-ıine the following:

(a) Oil zone static pressure
(b) Water zone static pressure
(c) Equilibrium pressure in reservoir
(d) Rate at which water will flow into oil zone under shut-in conditions
(e) PI of oil zone (average)
(f) PI of water zone (average)
(g) Gross of PI (average)

Fig. 1.96 shows the case of low pressure water. In ıis case there is no water production initially but ?ater starts when the flowing pressure in the oil zone ecomes less than the pressure in the water zone. It is robably wise to always produce at a low enough ressure to produce some water. The reason is that we o not want oil entering the water zone because we ıay never get the oil to come out of the water zone.

38 Shape of IPR curves for stratified formations

Many wells are producing from stratified formations ɔntaining layers of different permeabilities. In order ɔ illustrate the effect on the IPR curve Nind[31] gave ıe following example. Let us assume that we have 3 ɔnes with permeabilities of 10, 100, and 1 md, re-ɔectively and that there is no vertical communication etween zones, except through the well bore. Static ressures are as follows:

00 md zone ---------- 1,000 psig
10 md zone ---------- 1,200 psig
1 md zone ---------- 1,500 psig

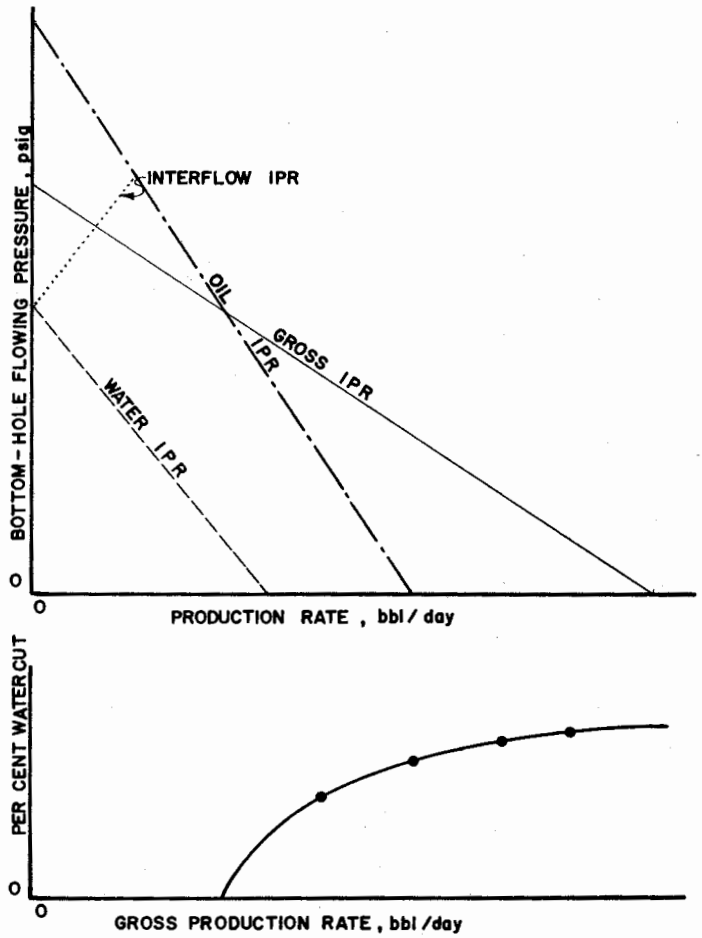


Fig. 1.96 IPR and water cut curves: low-pressure water (after Principles of Oil Well Production *by Nind. Copyright 1964, McGraw-Hill. Used with permission of McGraw-Hill Book Company).*

This example is further illustrated in Fig. 1.97 which shows the IPR curve for the 1, 10, and 100 md zones as well as showing the composite IPR.

It is interesting to note the shape of the composite IPR curve, because this is the shape exhibited when a series of flow tests is run on this well. In this case we see an improving PI with increasing production rate at the lower rates, but a deteriorating PI with higher flow rates is particularly true for solution gas drive reservoirs.

1.39 Suggested method for running a productivity index test

The following stepwise procedure is offered for obtaining data in a PI test:

(1) Shut the well in for 24 to 72 hours and obtain a pressure buildup test.

(2) With the recording pressure gage on bottom, place the well on its lowest production rate and obtain a flowing pressure recording. Ample time must be allowed for the production rate and flowing BHP to stabilize. If possible at least 48 hours should be allowed for each rate.

(3) Place the well on successive higher rates again allowing approximately 48 hours for each rate. At least three rates should be obtained.

(4) Again shut the well in and obtain a BHP buildup survey.

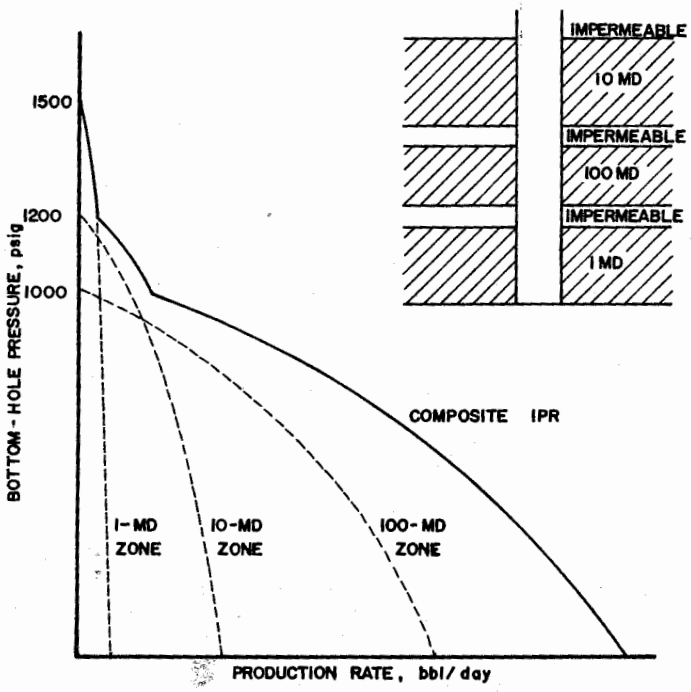


Fig. 1.97 Composite IPR for a stratified formation (after Principles of Oil Well Production *by Nind. Copyright 1964, McGraw-Hill. Used with permission of McGraw-Hill Book Company).*

In practice this represents a lot of time to conduct a test and requires a slow clock which decreases readability. In many cases only a single point can be obtained because of economics and practical operations. For example a flowing pressure survey may be required on a well. A several day stabilized flowing test is then obtained. The static pressure may be known from a previous test or may be conducted at this time. The average PI between the static and flowing pressure is then obtained. A good one point test certainly represents very valuable information for flowing and artificial lift well predictions. Unfortunately this represents the extent of information available for most of our wells.

1.310 Summary

The knowledge of the ability of a well to produce fluids is extremely important for flowing and artificial lift wells. A plot of flowing pressure vs. rate (inflow performance curves) will exhibit different shaped curves depending upon such factors as (1) type of reservoir, (2) gas oil ratios, (3) water-cut, (4) cumulative recovery and (5) stratification. In general we can expect the IPR curve for an active water drive to be a straight line and the IPR curve for a solution gas drive to curve downwards, that is, a decreasing PI with drawdown.

Vogel[7] offers a general solution to determine p_{wf} vs. rate from one flowing test for solution gas drive wells. Standing[8] shows the effect of damage and or improvement. He also gives a method for predicting IPR curves into the future for solution gas drive reservoirs. Another solution was offered by Shell Oil Company.[3]

A recent publication by Fetkovich[15] shows that oil wells behave similarly to gas wells and that isochronal testing can be applied.

The effect of water-cut on IPR curves is given. Also the effect of high and low pressure water zones is discussed. The effect of stratified zones is briefly discussed.

Finally, a method for conducting a PI test is given. Practical limitations generally limit us to one flowing test (rate and flowing pressure) and one static pressure test. This one test is invaluable for flowing and artificial lift well predictions, although it may not be sufficient to accurately predict the maximum flow rate or absolute open flow potential. At least three tests are desirable.

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Chapter 2

Multiphase flow in pipes

2.1 INTRODUCTION

Multiphase flow in pipes is defined as the concurrent movement of free gases and liquids in the pipes. Flow may be in any direction. The gas and liquid may exist as a homogeneous mixture, or the liquid may be in slugs with the gas pushing behind it. The liquid and gas may also flow parallel to each other, or other combinations of flow patterns may be present. The gas may be flowing with two liquids (normally oil and water), and the possibility exists that the two liquids may be emulsified.

The multiphase flow problem can be divided into four categories: (1) vertical multiphase flow; (2) horizontal multiphase flow; (3) inclined multiphase flow; and (4) directional multiphase flow. Fig. 2.1 shows the four multiphase flow problems.

The fluid must first enter the well-bore (inflow performance—Chapter 1). The fluids then travel through the tubing or annulus or both, to the surface. This involves either a vertical or directional multiphase flow problem. When the fluids reach the surface they may or may not pass through a "choke" before entering a horizontal or inclined flow line leading to the separator or separation facilities. The choke bean is discussed in Chapter 3, Section 3.5, and represents another multiphase flow problem. The passage of the fluids from the well-head to the separator represents either a horizontal or inclined multiphase flow problem.

As wells approach the latter stages of their flowing life, a slug flow phenomenon may develop, or the wells may be placed on a type of induced flow called stop-cocking. This type of flow also occurs in intermittent gas-lift operations. This so-called slug flow is described in this chapter.

2.11 General history of multiphase flow in pipes

The existence of multiphase flow and the problems associated with it have been recognized since 1797.

Note: Because of the length of this chapter references are found at the end of each section.

Numerous correlations and equations have been presented on the subject of multiphase horizontal and vertical flow in the technical literature. However, most of the significant contributions have been made since 1945. These contributions are presented separately under multiphase vertical, horizontal, inclined, and directional flow.

2.12 Use of multiphase flow pressure loss calculations in petroleum engineering

The application of multiphase flow correlations to predict pressure loss in pipes is extremely important to the petroleum industry. Some of the uses are:

(1) *Natural flow.* When fluids are produced from the formation into the vertical tubing and through horizontal surface flow lines, energy is dissipated. To prolong the flowing life of this well as long as possible, there is a need to minimize energy loss by proper design of the flow string.

(2) *Slim-hole completions.* For ultra-slim-hole completions, minimum casing size is installed to minimize completion costs. The casing size limits the maximum tubing size for a particular well. If the producing mechanism and the water cut are known, the maximum flow rate through tubing of a given size can be calculated with multiphase flow equations.

(3) *Dewatering gas wells.* The selection of a siphon string to unload water from gas wells, or of the production string for a gas well making water, is most important. Pressure calculations must be made to determine the proper size of the vertical tube.

(4) *Artificial lift installations.* The flowing pressure loss in vertical tubes is needed for the proper design of most artificial lift installations. Knowledge about such losses is a particular requirement for gas-lift installations where additional gas is injected in the tubing. Pressure loss is also important in artificial lift installations where the pump is set far off bottom and the well is producing with a high gas-oil ratio because pressure loss may be significant in that section of the vertical pipe below the pump.

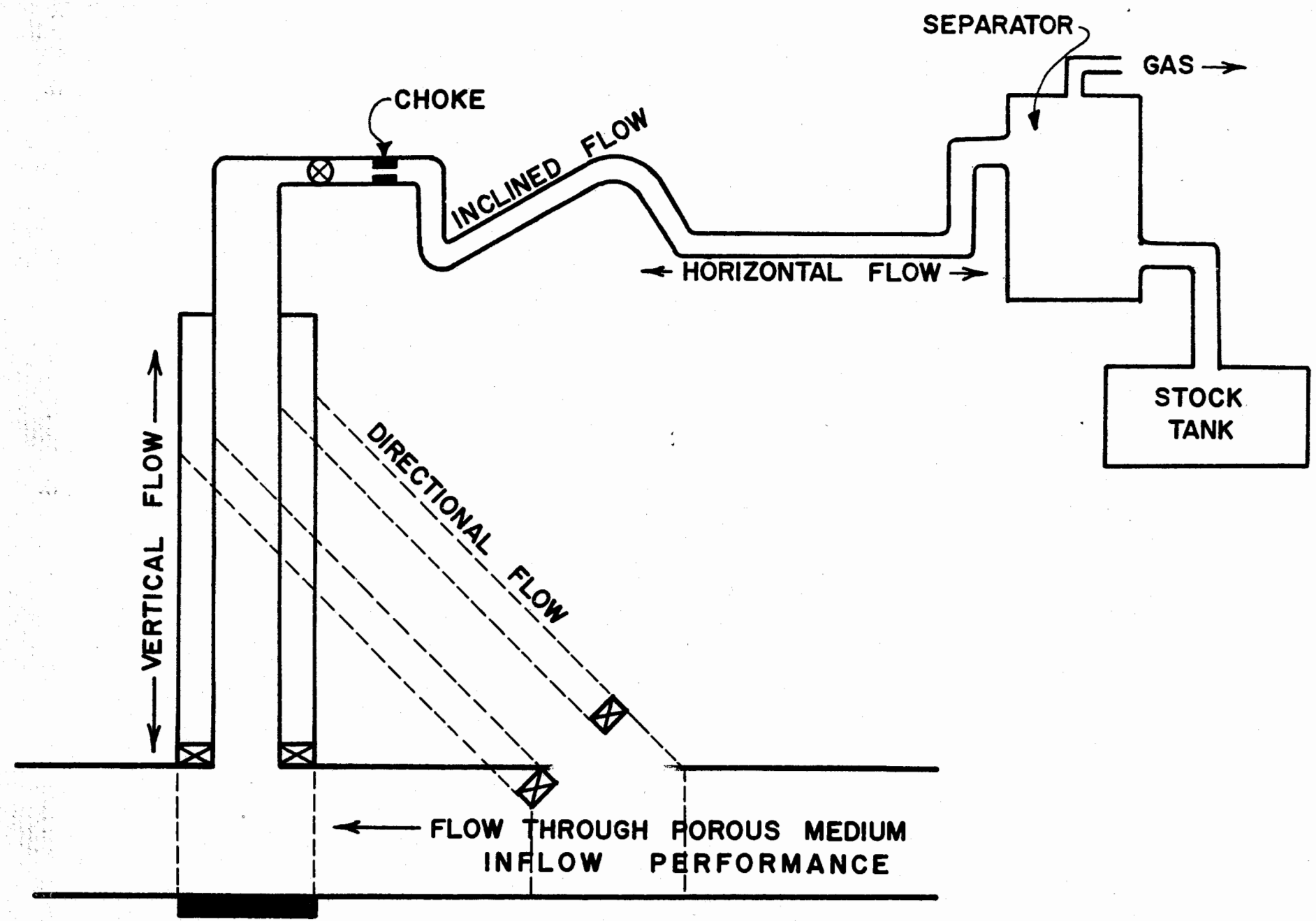


Fig. 2.1 Over-all production system.

(5) *Gathering and separation systems.* In centralized gathering and separation systems it is necessary to transport gas-liquid mixtures for relatively long distances. Correct sizing of the horizontal pipe used in these systems is important to prevent high pressure losses in the systems.

(6) *Sizing surface flow lines.* The sizing of surface flow lines for oil production is extremely important in designing for maximum allowable production. The size of the surface flow line from the wellhead to the separator combined with separator pressure establishes the flowing wellhead pressure. The flowing wellhead pressure controls the flowing bottomhole pressure which, in turn, controls the productive capacity of the well.

(7) *Sizing of transmission lines.* The prediction of pressure losses is important in the sizing of large transmission lines containing a liquid phase.

(8) *Sizing of gas lines.* Pressure loss calculations must be made when gas lines where glycol or some other chemical is being injected to prevent freezing are designed.

(9) *Tubing design in deviated wells.* The design of tubing strings for directionally-drilled wells is becoming more and more important as additional offshore wells are drilled.

(10) *Surface design for inclined flow.* The calculation of pressure losses for sizing of surface flow lines and transmission lines for inclined flow over hilly terrain, and for offshore-to-onshore facilities, is a necessity.

(11) *Heat exchanger design.* In refineries and chemical plants two-phase mixtures of petroleum fractions sometimes circulate through heat exchangers. The design of the heat exchangers involves two-phase pressure-drop correlations.

(12) *Condensate line design.* Mixtures of partially-condensed vapors flowing through condensate lines in steam and refrigeration plants are in two-phase flow. The design of these lines must take into account the additional pressure loss caused by the existence of the liquid phase.

There are other uses for multiphase flow calculations. All of the mentioned applications point out the fact that an economic problem is involved in the optimization of pipe sizes for vertical, horizontal, and inclined flow.

2.13 Objectives of this chapter

With this chapter, we hope to attain the following objectives. The first is to introduce the basic concepts and discuss the variables affecting multiphase flow. It is a vital necessity for the reader to have a proper understanding of such liquid and gas properties as density, viscosity, surface tension, etc. For this reason, these basic factors will be reviewed and related to multiphase flow.

A knowledge of gas behavior is also needed. A complete review of gases and gas laws would require too much space; consequently, a brief resume of these basic laws will be given along with example problems that show how these laws apply to multiphase flow.

A useful tool in multiphase flow calculations is dimensional analysis, often used to obtain dimensionless groups of numbers. A review of this subject, including example problems, is given.

The general energy equation is a basic requirement for background in multiphase flow and in calculations pertaining to this subject. This equation will be developed, and the terms will be individually discussed. Other equations used in single and multiphase flow will be described.

The second objective is to introduce the most significant correlations that are used for the calculation of pressure loss in multiphase vertical, horizontal, inclined, and directional flow, and to define the ranges of multiphase flow where the correlations available today predict pressure loss inaccurately.

The final objective is to make the reader aware of those areas where additional research on multiphase flow is necessary.

2.2 MATHEMATICAL AND PHYSICAL BASES FOR PRESSURE LOSS CALCULATIONS IN MULTIPHASE FLOW

This section is a brief review of: (1) units, conversions and dimensional analysis; (2) liquid properties; (3) gas properties; (4) variables such as solubility, viscosity, etc., that affect the pressure loss in multiphase flow; and (5) thermodynamic equations pertinent to multiphase flow calculations.

2.21 Conversions and dimensional analysis

2.211 Introduction

Common usage of both the English and metric systems of units in publications on multiphase flow requires an understanding of units and the procedures for converting from one system to another.

Most multiphase flow correlations utilize dimensional analysis to obtain dimensionless groups of variables for correlating parameters. Examples of such use are found in papers by Ros[1], and Hagedorn and Brown.[2] In these papers the Buckingham π Theorem was utilized to determine pertinent dimensionless groups of variables. Because many of the multiphase flow correlations are developed by using the π Theorem, a procedure for its use is given in Section 2.216.

2.212 Units

The two systems of units in common use are the metric system (CGS) and the English system. Table 2.21 shows the applicable units in these two systems.

TABLE 2.21

Symbol	Quantity	Metric (CGS) absolute system	English absolute system
L	Length	centimeter	ft
m	Mass	gram mass	lb mass (lbm)
F	Force	dyne	poundal or lb force (lb_f)
t	Time	second	second
T	Temperature	^oK	^oR
E	Energy	erg, joule, or calorie	ft-poundal, btu, or ft-lb_f
v	Velocity	cm/sec	ft/sec
a	Acceleration	cm/sec^2	ft/sec^2

We use the standard symbol "ρ" for density. Schilson and Pollard[3] explain the relationship between density and specific weight. To clarify use of these quantities, we refer to Newton's second law which states that force is directly proportional to the time rate of change of momentum. This is expressed mathematically as:

$$F = K\left[\frac{d}{dt}(mv)\right]. \tag{2.21}$$

The more common form for a constant mass is:

$$F = Kma, \tag{2.22}$$

where K is a universal positive constant with a value dependent on the units employed in Eq. 2.22.

For F in lb force, m in lb mass, and a in ft per second^2, then $K = \frac{1}{g_c}$, where g_c has the units of poundals/lb force. Poundals are equal to pounds mass times acceleration. Adopting these units, Eq. 2.22 becomes

$$F = \frac{ma}{g_c} \tag{2.23}$$

Density, ρ, is defined as:

$$\rho = \frac{m}{V}$$

In practice, we measure scale weight (W) which is the force of gravitational attraction that the earth exerts on a body. Thus, in Eq. 2.23, F = W and a = g; therefore,

$$W = \frac{mg}{g_c}$$

Specific weight is expressed mathematically as:

$$\rho_w = \frac{W}{V} \quad \text{or} \quad W = \rho_w V.$$

Then $\rho_w V = \frac{mg}{g_c}$;

or $\rho_w = \frac{g}{g_c}\frac{m}{V}$;

or $\rho_w = \frac{g}{g_c}\rho$

The ratio $\frac{g}{g_c}$ is commonly assumed to equal unity, resulting in lb force being numerically equal to lb mass. This implies that specific weight is equal to density.

The average value of g is 32.2 ft/sec^2 and, according to Schilson and Pollard,[3] varies less than one per cent at the various earth locations of interest to petroleum engineers.

2.213 Conversions

An example of a unit conversion problem follows. The real gas law is given as:

$$pV = znRT \tag{2.24}$$

In metric units:

p = pressure, atmospheres
V = volume, cubic centimeters
n = number of gram moles
T = absolute temperature, degrees Kelvin (°K)
z = compressibility factor (dimensionless)
R = universal gas constant

Using these metric units, R has a value of 82.06 with units of $\frac{(\text{atm})(\text{cu cm})}{(\text{gm-moles})(°\text{K})}$.

The problem is to convert to a value of R having common field units using p = psia; V = cu ft; T = °R; and n = lb-moles.

This problem is solved by beginning with the known value of R = 82.06 and using its corresponding correct units. For example:

$$R = 82.06\left[\frac{(\text{atm})(\text{cu cm})}{(\text{gm-moles})(°\text{K})}\right], \tag{2.25}$$

1 atmosphere = 14.7 psia,
1 cu ft = $(30.48)^3$ cu cm,
1 lb_m = 453.6 gm_m,
1°K = 1.8°R.

The units of R are then converted to the desired units in the following manner:

$$82.06\ \frac{\text{atm}}{}\ \frac{14.7\ \text{psia}}{\text{atm}}\ \frac{\text{cu cm}}{}\ \frac{\text{cu ft}}{(30.48)^3\ \text{cu cm}}\ \frac{}{453.6\ \text{gm-moles}}\ \frac{}{}\ \frac{°\text{K}}{}\ \frac{}{\text{gm-moles}}\ \frac{}{\text{1-lb mole}}\ \frac{}{°\text{K}}\ \frac{}{1.8°\text{R}}$$

and:

$$R = 10.72\left[\frac{(\text{psia})(\text{cu ft})}{(\text{lb-moles})°\text{R}}\right]. \tag{2.26}$$

The real gas law is then written as:

$pV = 10.72\ znT$.
p = psia
V = cu ft
n = lb-moles
T = °R

2.214 Determining dimensions of variables

Dimensional units are also used to find the dimensions of a particular variable, such as viscosity, and in verifying dimensionless numbers. It is important that correlations utilize dimensionless correlating groups, since non-dimensionless groups tend to lose their meaning. For example, we know that absolute viscosity is defined by:

$$\mu = \frac{F/A}{dv/dx}, \tag{2.27}$$

where: F = force
A = area
v = velocity
x = distance
μ = viscosity

For F in dynes, A in sq cm, v in cm/sec, and x in cm, μ has the units of poise.

Suppose we wish to find the dimensions of viscosity using the mass-length-time system.

$F = mLt^{-2}$
$A = L^2$
$v = Lt^{-1}$
$x = L$

Substituting these dimensions, we obtain

$$\mu = \frac{mLt^{-2}/L^2}{Lt^{-1}/L} = mL^{-1}t^{-1}.$$

According to this, viscosity has the dimensions of mass per length time.

The unit of gm-mass/cm-sec in the metric system is called a poise. Kinematic viscosity is absolute viscosity divided by density. It has the dimensions of L^2t^{-1} in both the mass-length-time and force-length-time systems, and is more difficult to use because g_c is introduced into the problem.

2.215 Solving for conversion constants to make equations dimensionally correct

Many equations used in multiphase flow calculations are derived from one set of units and must be converted to another set of units before they can be applied. To do this, a constant that keeps the equation dimensionally correct must be determined. This presents a different problem than a straight conversion, as is illustrated by the following example.

The general equation for pressure drop in single-phase liquid pipeline flow is:

$$\Delta p = \frac{\rho f L v^2}{d\ 2g_c}, \tag{2.29}$$

where Δp = pressure drop, lb_f/sq ft
ρ = density, lb_m/cu ft
f = friction factor (dimensionless)
L = length, ft
v = velocity, ft/sec
d = diameter, ft
g_c = conversion constant (32.174 lb_m ft/lb_f-sec^2).

The problem is to convert this equation to units so that:

$$\Delta p = C\frac{\rho f L q^2}{d^5}, \tag{2.210}$$

where Δp = pressure drop, psi
ρ = density, lb_m/cu ft
f = friction factor (dimensionless),
L = length, miles
q = flow rate, cu ft/sec
d = diameter, in.

There are two procedures to solve for the constant C to make this equation dimensionally correct. The first starts with an equation that is dimensionally correct, converts it to the desired units, and solves algebraically for C. The second starts with an equation that is in the units desired, converts to the units that are dimensionally correct, and the constant C will be in place. Both procedures are explained in detail by Brown.[4]

It should be remembered that in following the first procedure any constant already in the equation remains in place and does not enter into the algebraic manipulations. Starting with the equation that is dimensionally correct, we have:

$$\Delta p \frac{lb_f}{sq\ ft} = C\frac{\rho\ (lb_m/cu\ ft)\ f\ L(ft)\ v^2(ft/sec)^2}{d(ft)\ g_c(lb_m ft/lb_f\ sec^2)}.$$

In the final equation, velocity is replaced by q/A, where:

$$v^2 = \frac{q^2}{A^2} = \frac{16q^2}{\pi^2 d^4}. \tag{2.211}$$

Converting to the desired units:

$$\Delta p \frac{lb_f}{sq\ ft}\ \frac{sq\ ft}{144\ sq\ in}$$

$$= C\ \rho_f\ L\ ft \left| \frac{mile}{5280\ ft} \right| \frac{16q^2}{\pi^2\ d^4\ (12)^4 in.^4} \left| \frac{ft^4}{d\ ft\ 12\ in.} \right| \frac{1}{64.4}$$

The constants $\frac{16}{\pi^2}$ and $\frac{1}{64.4}$ remain in place.

Solving for C:

$$C = \frac{(5280)(12^4)(12)}{(144)}\left[\frac{16}{\pi^2}\ \frac{1}{64.4}\right] = 229680.$$

Substituting this constant, the equation becomes:

$$\Delta p\ (psi) = 229680\ \frac{\rho(lb/cu\ ft)\ f\ L(miles)\ q^2(cu\ ft/sec)^2}{d^5\ (in.)^5}.$$

To solve for the constant in the second procedure, we start with the desired units, convert to the units that are dimensionally correct, and obtain the constant in place, as follows:

$$\frac{\Delta p\ lb_f}{sq\ in}\ \frac{144\ sq\ in}{sq\ ft}$$

$$= \rho_f\ \frac{L(miles)\ 5280\ ft}{mile} \left| \frac{16\ q^2\ (12)^4(in.)^4}{\pi^2\ d^4(in.)^4\ ft^4} \right| \frac{12\ in.}{d\ in.\ ft} \left| \frac{1}{64.4} \right.$$

$$\Delta p = \frac{\rho\ f\ L\ q^2}{d^5}\ \frac{(5280)(16)(12)^4(12)}{\pi^2\ (144)\ (64.4)}.$$

$$\Delta p = 229680\ \frac{\rho\ f\ L\ q^2}{d^5}. \tag{2.212}$$

The advantage of this second procedure is that the constant of conversion comes out in place, thereby eliminating the manipulation of the constants $\frac{1}{2g_c}$ and $\frac{16}{\pi^2}$.

It is important to point out that when converting units, the unit equation can be treated algebraically the same way one treats the numbers. You can then check to make sure the units cancel. For example, in this problem we can write:

$$\left(\frac{\Delta p\ lb_f}{in.^2}\right)\left(\frac{144\ in.^2}{ft^2}\right)$$

$$= \left(\frac{\rho\ lb_m}{ft^3}\right)\left(\frac{f}{1}\right)\left(\frac{L\ miles}{1}\right)\left(\frac{5280\ ft}{mile}\right)\left[\frac{16}{\pi^2}\left(\frac{q\text{-}ft^3}{sec}\right)^2\left(\frac{1}{d\text{-}in.}\right)^4\right]$$

$$\left[\left(\frac{12\ in.}{ft}\right)^4\left(\frac{1}{d\text{-}in.}\right)\left(\frac{12\ in.}{ft}\right)\right]\left(\frac{lb_f\text{-}sec^2}{64.4\ ft\text{-}lb_m}\right) \tag{2.213}$$

The unit equation is:

$$\frac{lb_f}{in.^2}\left(\frac{in.^2}{ft^2}\right) = \left(\frac{lb_m}{ft^3}\right)\left(\frac{miles}{1}\right)\left(\frac{ft}{mile}\right)\left(\frac{ft^6}{sec^2}\right)\left(\frac{1}{in.}\right)^4\left(\frac{in.^4}{ft^4}\right)\left(\frac{1}{in.}\right)$$

$$\left(\frac{in.}{ft}\right)\left(\frac{lb_f\text{-}sec^2}{ft\text{-}lb_m}\right) \tag{2.214}$$

When terms are cancelled:

$$\frac{lb_f}{ft^2} = \frac{lb_f}{ft^2}$$

We thus have an automatic check to assure ourselves that the units conversion is correct. The numerical portion of the equation is:

$$144\ \Delta p = \frac{\rho f L q^2}{d^5}\,\frac{(5280)(16)(12)^4(12)}{\pi^2(64.4)} \qquad (2.215)$$

$$\text{or}\quad \Delta p = \frac{\rho f L q^2}{d^5}\,\frac{(5280)(16)(12)^3}{\pi^2(64.4)} = 229680\,\frac{\rho f L q^2}{d^5} \qquad (2.212)$$

2.216 Determining dimensionless groups

Dimensionless groups of numbers, determined by Buckingham's π Theorem, are often used in correlations for multiphase flow. Publications by Ros[1], and Hagedorn and Brown[2] contain specific examples which use the π Theorem. A rigorous proof of the theorem was made by Langhaar[5]. Other excellent discussions have been published (see Refs. 6, 7, 8, 9, 10, 11, 12 and 13). The procedure for using the π Theorem and an example problem follows:

2.2161 Stepwise procedure for use of Buckingham's π Theorem

(1) Determine the number of variables (n). These may be velocity, viscosity, density, diameter, length, and pressure loss.
(2) Determine the number of dimensions (k) included in the n variables. These may be length, mass, time, etc. In fluid flow problems, we generally find $k = 3$.
(3) Set up an $n \times k$ matrix consisting of the exponents of all variables: n for columns and dimension k for rows. Determine the rank, r, of this matrix, which in most cases will correspond to k dimension; that is, $r = k$ for the majority of our problems. Therefore, for most cases step 3 may be eliminated.
(4) Determine the number of π terms by subtracting (3) from (1); that is, $n - r$ = number of π terms, which will correspond with $n - k$ in most cases.
(5) Retain as repeating variables the number of terms equivalent to the number of units involved, k. That is, if m, L, and t are involved, we have three repeating variables in each π term, plus one non-repeating variable in each term. The repeating variables must contain all dimensions as a group. This does not mean that each term must contain all dimensions. For example, if we have dimensions of m, L, and t and three repeating variables with dimensions of L, Lt^{-1}, and $mL^{-1}t^{-1}$, this will be satisfactory, since the three together contain m, L, and t.
(6) Write out the π terms. Keep the same repeating variables in each term and include one different non-repeating variable at the end of each π term. Each repeating variable must be raised to an unknown exponential power, whereas the non-repeating variable is raised to the power of 1.

Thus:

$$\pi_1 = A_1^{a_1} \cdot A_2^{b_1} \cdot A_4^{c_1} \cdot A_3 \qquad (2.216)$$
$$\pi_2 = A_1^{a_2} \cdot A_2^{b_2} \cdot A_4^{c_2} \cdot A_5 \qquad (2.217)$$

(7) Set up dimensional equations for each term; that is, substitute the dimensions of A_1, A_2, A_3, A_4, and A_5 into (6).
(8) Equate the exponents of like dimensions in each π term and solve for the value of the exponents.
(9) Write out the final π terms. All of these are dimensionless groups of numbers.

2.2162 Example problems making use of the π Theorem

EXAMPLE NO 1.

Flow of a slightly compressible liquid through a pipe.
(1) List the variables involved:
d = diameter of pipe = L
v = velocity of liquid = Lt^{-1}
ρ = density of liquid = mL^{-3}
μ = viscosity of liquid = $mL^{-1}t^{-1}$
ΔL = length of pipe = L
Δp = pressure loss = $mL^{-1}t^{-2}$

Since we are normally concerned with the pressure loss in terms of psi/ft, we can combine ΔL and ΔP and have $\frac{dp}{dL} = mL^{-2}t^{-2}$, thereby reducing the problem to 5 variables ($n = 5$).
(2) The number of dimensions involved in the 5 variables is 3: m, L, and t.
(3) Set up an $n \times k$ matrix as follows:

$$\pi = (d)^{a_1}\,(v)^{b_1}\,(\rho)^{c_1}\,(\mu)^{d_1}\left(\frac{dp}{dL}\right)^{e_1} \qquad (2.218)$$

$$\text{or}\quad 1 = (L)^{a_1}\,(Lt^{-1})^{b_1}\,(mL^{-3})^{c_1}\,(mL^{-1}t^{-1})^{d_1}\,(mL^{-2}t^{-2})^{e_1} \qquad (2.219)$$

Collecting exponents of like terms, we have:

$$1 = m^{(c_1+d_1+e_1)}\,L^{(a_1+b_1-3c_1-d_1-2e_1)}\,t^{(-b_1-d_1-2e_1)} \qquad (2.220)$$

The exponents of m, L and t constitute three equations and their sums are zero:

$$\text{m)}\quad c_1 + d_1 + e_1 = 0 \qquad (2.221)$$
$$\text{L)}\quad a_1 + b_1 - 3c_1 - d_1 - 2e_1 = 0 \qquad (2.222)$$
$$\text{t)}\quad -b_1 - d_1 - 2e_1 = 0 \qquad (2.223)$$

From these three equations we set up the matrix as follows:

	d	v	ρ	μ	dp/dL
m)	0	0	1	1	1
L)	1	1	−3	−1	−2
t)	0	−1	0	−1	−2

Recalling that a determinant has equal columns and rows, we find that the following 3rd order determinant carved out of the matrix is not equal to zero:

$$\begin{vmatrix} 0 & 0 & 1 \\ 1 & 1 & -3 \\ 0 & -1 & 0 \end{vmatrix} = 0 + 0 + (-1) - 0 - 0 - 0 = -1 \neq 0$$

Therefore, this matrix has a rank of 3, or r = 3. If all third order determinants had been zero and any second order determinant not zero, then the rank would be 2. As noted, this value corresponds to k = 3.

(4) The number of π terms will be n − r = n − k = 5 − 3 = 2

(5) Since we have m, L, and t involved, we will retain three repeating variables. Let us select v, ρ, and μ, which in themselves contain m, L, and t.

(6) Write out the π terms, as follows:

$$\pi_1 = (v)^{a_1} (\rho)^{b_1} (\mu)^{c_1} (d)^1 \quad (2.224)$$

$$\pi_2 = (v)^{a_2} (\rho)^{b_2} (\mu)^{c_2} \left(\frac{dp}{dL}\right)^1 \quad (2.225)$$

(7) Setting up dimensional equations for each π term, we have:

For π_1, $1 = (Lt^{-1})^{a_1} (mL^{-3})^{b_1} (mL^{-1}t^{-1})^{c_1}$ (2.226)

For π_2, $1 = (Lt^{-1})^{a_2} (mL^{-3})^{b_2} (mL^{-1}t^{-1})^{c_2} mL^{-2}t^{-2}$ (2.227)

(8) Equating exponents of like dimensions in each π term, we have:
For π_1:

m) $b_1 + c_1 = 0$ (2.228)
L) $a_1 - 3b_1 - c_1 + 1 = 0$ (2.229)
t) $-a_1 - c_1 = 0$ (2.230)

Solving, we find:

$a_1 = 1$
$b_1 = 1$
$c_1 = -1$

For π_2:

m) $b_2 + c_2 + 1 = 0$ (2.231)
L) $a_2 - 3b_2 - c_2 - 2 = 0$ (2.232)
t) $-a_2 - c_2 - 2 = 0$ (2.233)

Solving, we find:

$a_2 = -3$
$b_2 = -2$
$c_2 = 1$

(9) Writing out the final terms, we have:

$$\pi_1 = d\, v^1\, \rho^1\, \mu^{-1} = \frac{dv\rho}{\mu} \quad (2.234)$$

We recognize this as the Reynold's number.

$$\pi_2 = \frac{dp}{dL} (v)^{-3} (\rho)^{-2} (\mu)^1 = \frac{\frac{dp}{dL}\mu}{v^3 \rho^2} \quad (2.235)$$

This second term (π_2) has not been found useful in pipe flow. Rather, it is better to combine π_1 and π_2 as follows:

$$(\pi_1)(\pi_2) = \left(\frac{dp/dL}{v^3 \rho^2}\right)\left(\frac{d\, v\, \rho}{\mu}\right) = \frac{d\,(dp/dL)}{v^2 \rho} \quad (2.236)$$

This is now in the form of the commonly used friction factors, where the Moody factor has a multiplying coefficient of 2 and the Fanning factor is divided by 2. If pressure is expressed as lb_f/sq ft rather than lb_m/ft-sec^2, g_c is added to render the friction factor dimensionless.

These two groups represent two out of a possible 10 for the pipe flow problem. If repeating variables are changed, it is possible to arrive at eight more dimensionless groups; however, these are not independent terms, so have little meaning.

EXAMPLE PROBLEM 2.

Ros[2] gave the following variables encountered in multiphase vertical flow: (See Table 2.22)

By applying the π Theorem to these thirteen variables, Ros arrived at 10 dimensionless groups as follows:

1. $d\sqrt{\frac{\rho_L g}{\sigma}}$
2. ϵ/d
3. ϕ
4. ρ_g/ρ_L
5. $\mu_L \sqrt[4]{\frac{g}{\rho_L \sigma^3}}$
6. $\mu_g \sqrt[4]{\frac{g}{\rho_L \sigma^3}}$
7. $v_{sL} \sqrt[4]{\frac{\rho_L}{g\sigma}}$
8. v_{sg}/v_{sL}
9. α
10. $\frac{1}{\rho_L g} \frac{dp}{dx}$

Group 5, which is $\mu_L \sqrt[4]{\frac{g}{\rho_L \sigma^3}}$, is calculated below.

By inspection we see that g, ρ_L, and σ were kept as repeating variables, therefore we will write:

$$\pi = (g)^a (\rho_L)^b (\sigma)^c \mu_L \quad (2.237)$$

$$1 = (Lt^{-2})^a (mL^{-3})^b (mt^{-2})^c\, mL^{-1}\, t^{-1} \quad (2.238)$$

Equating exponents of like terms, we have:

m) $b + c + 1 = 0$
L) $a - 3b - 1 = 0$
t) $-2a - 2c - 1 = 0$
$a = 1/4$
$b = -1/4$
$c = -3/4$

TABLE 2.22

Symbol	Quantity	Dimensions
d	Diameter	L
ϵ	Wall roughness	L
ϕ	Inclination	(1)
ρ_L	Liquid density	mL^{-3}
ρ_g	Gas density	mL^{-3}
μ_L	Liquid viscosity	$mL^{-1}t^{-1}$
μ_g	Gas viscosity	$mL^{-1}t^{-1}$
V_{SL}	Superficial liquid velocity	Lt^{-1}
V_{SG}	Superficial gas velocity	Lt^{-1}
σ	Surface tension liquid	mt^{-2}
α	Wall contact angle	(1)
g	Gravity acceleration	Lt^{-2}
dp/dx	Pressure gradient	$mL^{-2}t^{-2}$

Therefore:

$$\pi = (g)^{1/4}\,(\rho_L)^{-1/4}\,(\sigma)^{-3/4}\,\mu_L \qquad (2.239)$$

$$\pi = \mu_L \sqrt[4]{\frac{g}{\rho_L \sigma^3}} \qquad (2.240)$$

Table 2.23 is given to facilitate the selection of dimensions:

TABLE 2.23
DIMENSIONS OF ENTITIES

	Mass system	Force system
Length	L	L
Time	t	t
Mass	m	$FL^{-1}t^2$
Force	mLt^{-2}	F
Temperature	Q	Q
Specific weight	$mL^{-2}t^{-2}$	FL^{-3}
Mass density	mL^{-3}	$FL^{-4}t^2$
Angle	–	–
Pressure and stress	$mL^{-1}t^{-2}$	FL^{-2}
Velocity	Lt^{-1}	Lt^{-1}
Acceleration	Lt^{-2}	Lt^{-2}
Angular velocity	t^{-1}	t^{-1}
Angular acceleration	t^{-2}	t^{-2}
Energy, work	mL^2t^{-2}	FL
Momentum	mLt^{-1}	Ft
Power	mL^2t^{-3}	FLt^{-1}
Moment of a force	mL^2t^{-2}	FL
Dynamic coefficient of viscosity	$mL^{-1}t^{-1}$	$FL^{-2}t$
Kinematic coefficient of viscosity	L^2t^{-1}	L^2t^{-1}
Moment of inertia of an area	L^4	L^4
Moment of inertia of a mass	mL^2	FLt^2
Surface tension	mt^{-2}	FL^{-1}
Modulus of elasticity	$mL^{-1}t^{-2}$	FL^{-2}
Strain	–	–
Poisson's ratio	–	–

2.22 Liquid properties

In every multiphase flow problem, we must deal with a liquid of some type. Generally this will be water, oil, condensate, or some combination thereof. The properties of liquids, particularly viscosity and density, will greatly affect a multiphase flow pressure traverse. A brief review of liquid properties is offered in this section. Because all liquids are compared to pure water, the physical properties of pure water are given in Table 2.24.

TABLE 2.24—PROPERTIES OF WATER

Specific gravity at 20°C (68°F)	1.00	
Specific weight	62.4	lb/cu ft
	8.33	lb/gal
	350	lb/bbl
Hydrostatic pressure gradient	0.433	psi/ft
Compressibility	3×10^{-6}	psi^{-1}
Viscosity (68.4°F and 14.7 psia)	1	cp
Density (60°F and 14.7 psia)	10°	API
Surface tension (68°F and 14.7 psia)	72	dynes/cm

2.221 Liquid density

Of special interest in vertical multiphase flow problems is that part of the pressure gradient which is contributed by the liquid. This is due to the liquid density and the hydrostatic head it exerts due to gravity. For example, a column of pure water exerts a pressure gradient of 0.433 psi/ft; whereas, a column of salt water with a specific gravity (γ_w) of 1.07 exerts a pressure gradient of 0.464 psi/ft.

For multiphase flow, this gradient is lightened because the water is mixed with a gas having a much lower density. In addition, an oil of 42° API ($\gamma_o = 0.815$) has a lower density than pure water and exerts a pressure gradient of 0.352 psi/ft. Liquid density is the one property that normally has the greatest effect on pressure loss in vertical multiphase flow problems. The one exception is viscosity, especially where highly-viscous crudes are involved.

2.222 Compressibility

Changes in water volume caused by changing pressures and temperatures can be neglected in multiphase flow problems, because the water volume changes very little. For example, one bbl of water at 3,000 psi expands to 1.009 bbl at atmospheric pressure (14.7 psia). This is only a change of approximately 0.9%; therefore, it can be neglected in most practical calculations.

The change of oil volume is accounted for in the volume factor for the oil. This factor also provides for volume changes which result from gas going into solution in the oil.

2.223 Viscosity

(a) *Crude viscosity.* Very viscous crudes present a serious lift problem in vertical multiphase flow. We know, for example, how the flow properties of thick molasses compare to those of gasoline. In general, if the liquid viscosity is greater than 10 cp (approximately equivalent to a 30° API oil at 100°F), then the pressure loss per unit of pipe length for multiphase flow increases with increased viscosity of the liquid.

The viscosity of a liquid varies depending upon temperature, pressure, density, gas in solution, and to some extent on other variables, including composition. Temperature has a decided effect as shown in Figs. 2.2 and 2.3. Figs. 2.4 and 2.5 show the effect of pressure on viscosity, while Fig. 2.6 shows the effect of solution gas on viscosity.

Viscosity of oils can be correlated with density measured in °API, and Fig. 2.7 shows such a correlation by Beal.[14]

We will use the normal unit of centipoise for viscosity, recalling that pure water has a viscosity of one cp at 68.4°F and 14.7 psia. The units of viscosity are $\left(\frac{gm}{cm\text{-}sec}\right)$, called poise, and it has dimensions of m/Lt.

The numerous charts, tables, etc. required to make viscosity conversions are found in the American Society of Testing Materials Manual under ASTM Method D 446–53 (Standard Method for Conversion of Kinematic Viscosity to Saybolt Universal Viscosity); ASTM

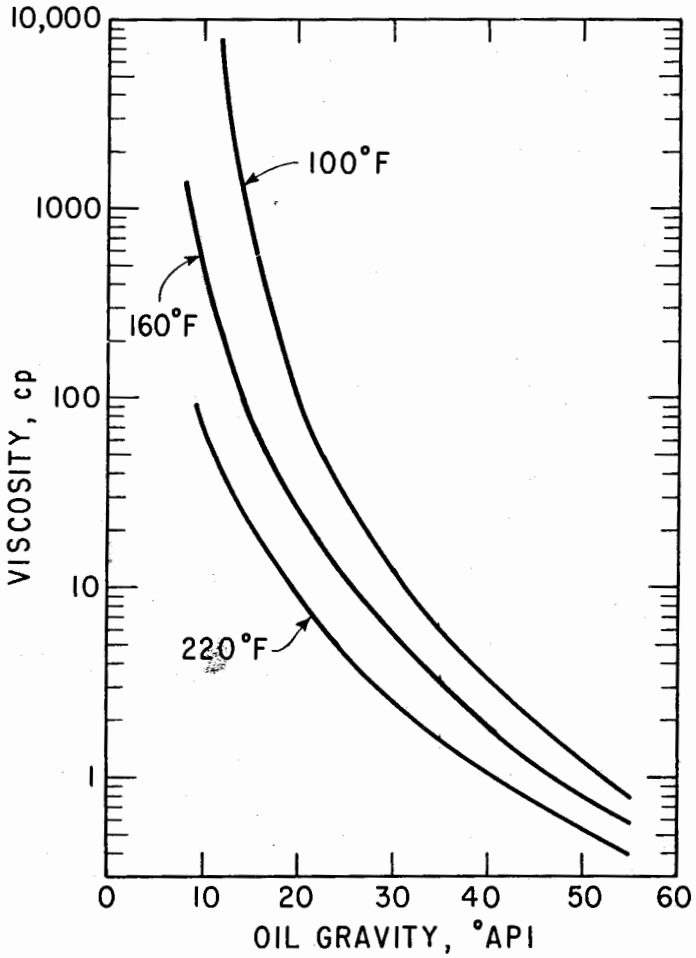


Fig. 2.2 Viscosity of gas–free crude oils at atmospheric pressure (after Beal).

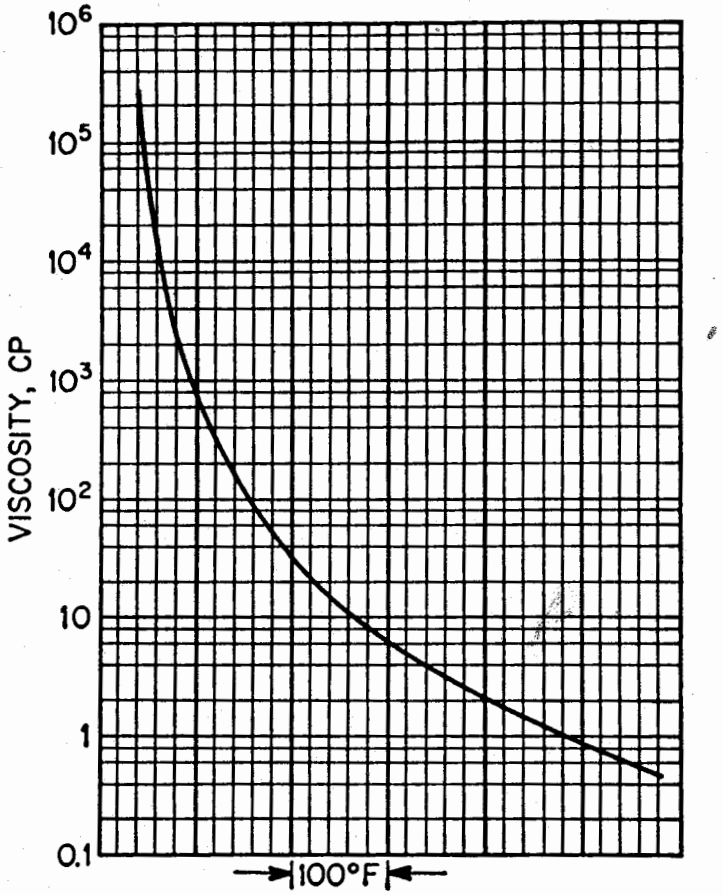


Fig. 2.3 Universal temperature-viscosity chart for crude oils (after Frick, courtesy McGraw-Hill).

Method D 666–57 (Standard Method for Conversion of Kinematic Viscosity to Saybolt Furol Viscosity); and ASTM Special Technical Publication No. 43B (ASTM Viscosity Tables for Kinematic Viscosity Conversions and Viscosity Index Calculations). See Appendix B.

Chew and Connally[15] presented an equation as follows:

$$\mu_o = A\,\mu_{oD}^{b} \tag{2.241}$$

where μ_o = live oil viscosity
μ_{oD} = dead oil viscosity

A and b = factors obtained from the Chew and Connally correlation (see Fig. 2.8). Chew and Connally gave viscosities at 100°F. The viscosity at any other temperature can be found from equations given by Beggs and Brill:[16]

$$\mu_{oD}(T) = \mu_{oD}(100°F)\left(\frac{T(°F)}{100}\right)^{X} \tag{2.242}$$

$$\text{where } X = 1.345 \ln\left[\frac{\mu_{oD}(210°F)}{\mu_{oD}(100°F)}\right] \tag{2.243}$$

Beggs and Robinson[17] presented a more recent empirical correlation for viscosity: The correlation for dead oil viscosity was developed by plotting $\log_{10}(T)$ vs. $\log_{10}\log_{10}(\mu_{oD}+1)$ on cartesian coordinates. The plots revealed a series of straight lines of constant slope. It was found that each line represented oils

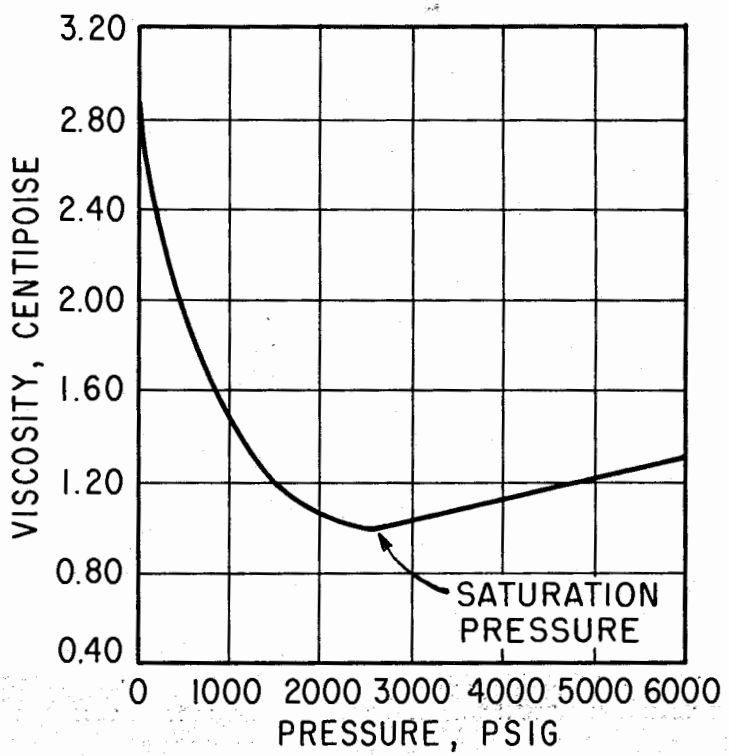


Fig. 2.4 Viscosity of liquid hydrocarbons.

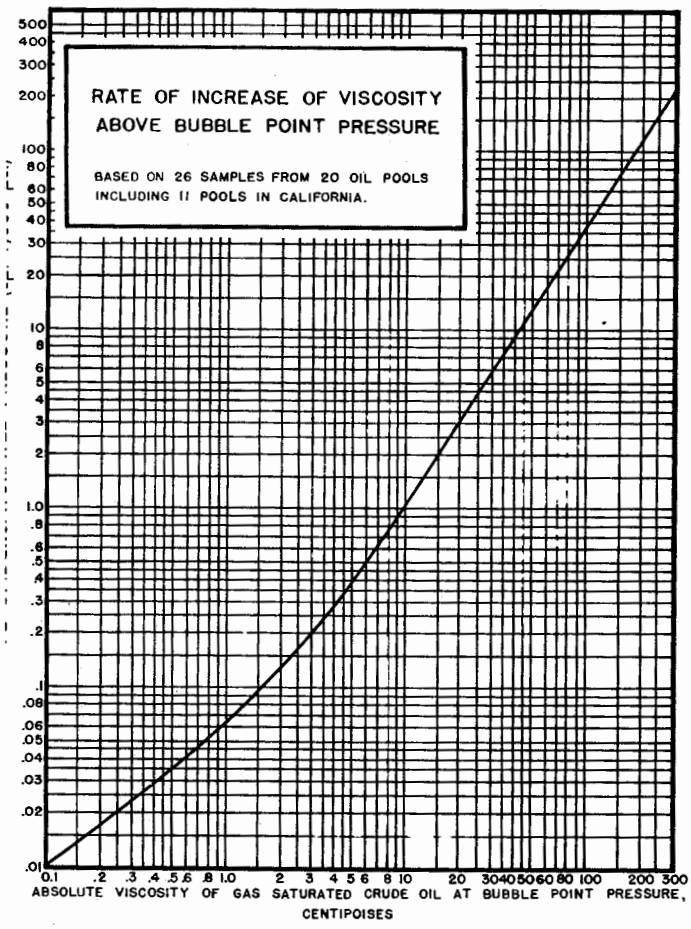


Fig. 2.5 Rate of increase of viscosity above bubble–point pressure (after Beal, courtesy Aime).

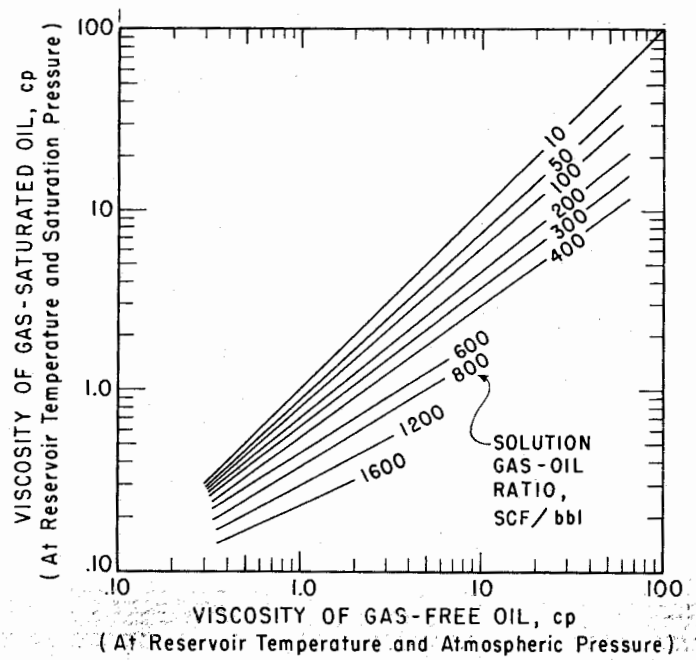


Fig. 2.6 Viscosity of gas–saturated crude oils at reservoir temperature and pressure (after Chew and Connally).

of a particular API gravity. The equation developed was:

$$\mu_{OD} = 10^{X} - 1, \qquad (2.244)$$

where:

$$X = y\,T^{-1.163}, \text{ where } T = °F$$
$$y = 10^{Z}$$
$$Z = 3.0324 - 0.02023\ °API$$

The correction of the dead oil viscosity for dissolved gas was developed by taking advantage of the fact that a linear relationship exists between $\log_{10}\mu_{oD}$ and $\log_{10}(T)$ for a particular value of dissolved gas, R_s. Live oil viscosity may be calculated from:

$$\mu = A\,\mu_{OD}^{B}, \qquad (2.241)$$

where:

$$A = 10.715\,(R_s + 100)^{-0.515}$$
$$B = 5.44\,(R_s + 150)^{-0.338}$$

The following Table 2.25 presents a statistical comparison of the correlations of Beal, Chew, and Connally, and the findings of Beggs and Robinson:

TABLE 2.25
STATISTICAL COMPARISON OF CORRELATIONS

	Average error, (percent)	Standard deviation of percent errors
Dead oil data used to develop correlation of Beggs and Robinson		
Beal	− 19.64	21.86
Beggs & Robinson	− 0.64	13.53
Dead oil data collected from literature (93 cases)		
Beal	378.46	1,568.09
Beggs & Robinson	114.27	530.00
Live oil data used to develop correlation		
Chew & Connally	25.35	35.70
Beggs & Robinson	− 1.83	27.25

(b) *Water Viscosity.* Beal[14] also showed how water viscosity varied with temperature and pressure. (Refer to Fig. 2.9) Pressure has very little effect but the viscosity ranges from 0.3 cp for temperatures above 200°F to 1.8 for low temperatures. In terms of temperature Beggs and Brill[16] gave an equation:

$$\mu_w = \exp\,(1.003 - 1.479 \times 10^{-2}T + 1.982 \times 10^{-5}T^2) \qquad (2.245)$$

where $T = °F$
and μ_w = water viscosity

Fig. 2.10 shows the viscosity of various liquids.

2.224 Surface tension

Surface tension enters into many of the correlating groups for multiphase flow; therefore, a brief discussion of this factor is included. Its relationship to the multiphase flow problem is presented in Sec. 2.243.

Surface tension measurements of liquids are usually made with the liquid surface in contact with air. The interfacial tension of two immiscible liquids,

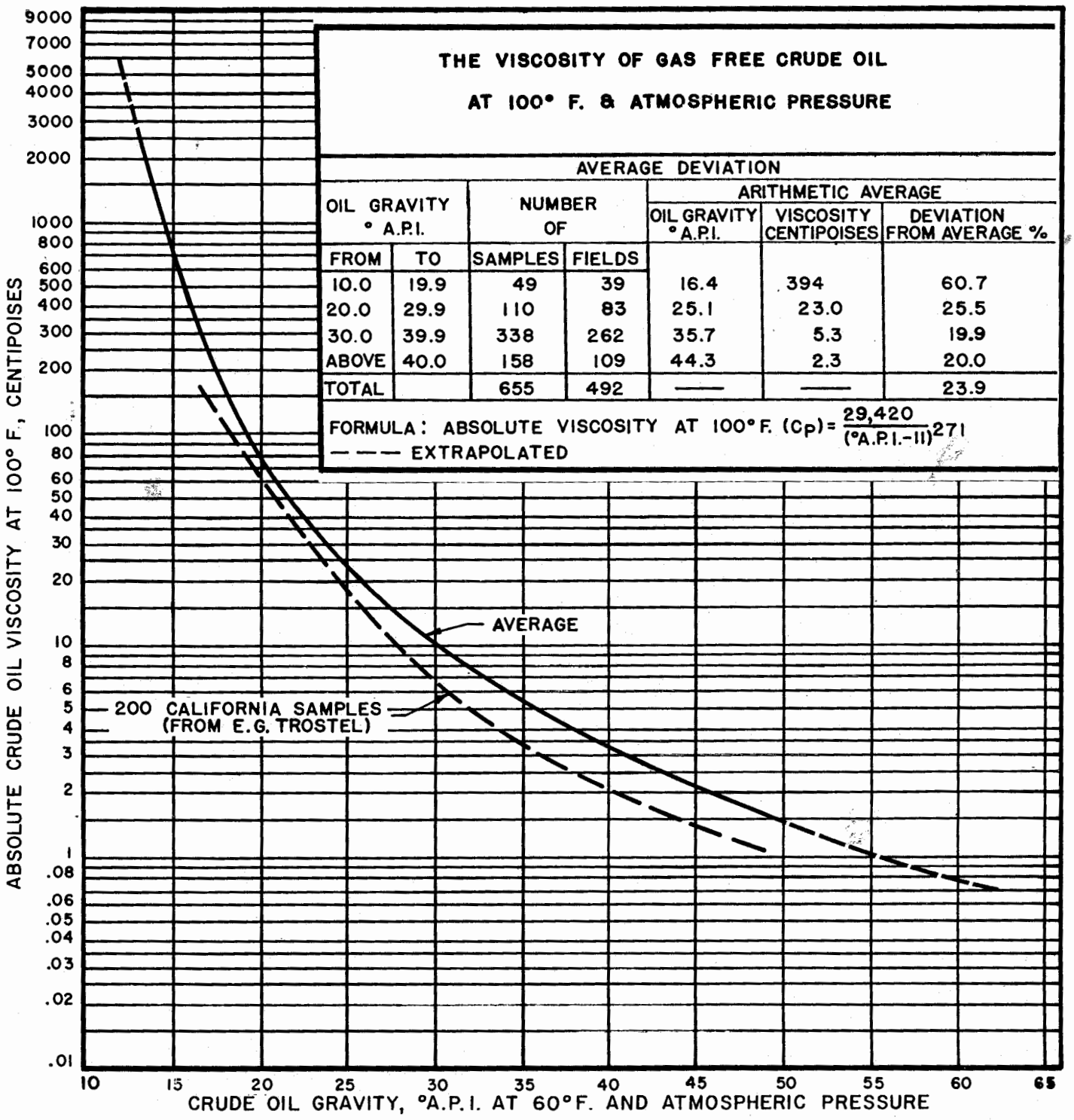


Fig. 2.7 Correlation of viscosity with °API (after Beal).

in contact with each other, is approximately equal to the difference between their individual surface tensions, as measured in contact with air.

The surface tension for water is 72 dynes/cm at 68°F and 14.7 psia. Most crudes have surface tensions at standard conditions which range from 25 to 35 dynes/cm; however, at pressures of 3,000 psia or more, surface tensions for crudes are reduced to less than 2 dynes/cm. The surface tension of hydrocarbon mixtures has been investigated by Knudsen and Katz[18] and Katz, *et al.*,[19] who give a method for calculating surface tension.

2.23 Review of gases as related to multiphase flow

2.231 Introduction

In every vertical multiphase flow condition a gas is present. This volume of free gas contributes more to the lightening of a pressure gradient than any other one factor. This is because gases have a low density compared to liquids. In vertical flow gas reduces pressure drop (at reasonably low G/L ratios); in horizontal flow, gas increases pressure drop.

Since the properties of other gases are compared to

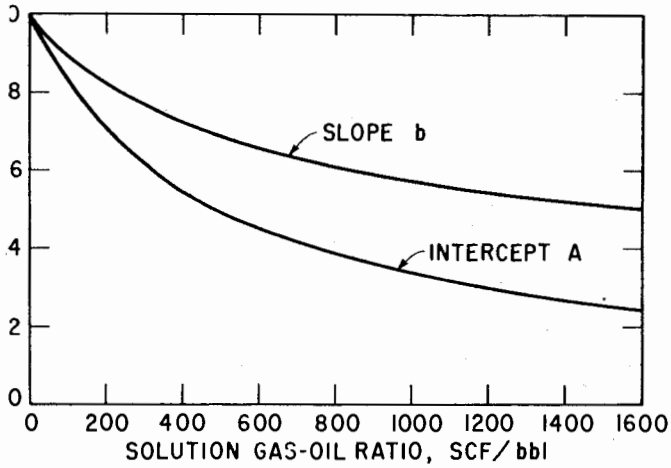


Fig. 2.8 A and b factors for use in Chew and Connally correlation (after Chew and Connally)

those of air in this discussion, the physical properties of air are given in Table 2.26.

In this book, one standard cubic foot (scf) of gas is defined as being one cu ft at 14.7 psia and 60°F. However, it is possible to define one standard cu ft at other conditions of pressure and temperature. For example, the States of Texas and Oklahoma define one scf of gas as being one cu ft at 14.65 psia and 60°F; while Louisiana defines one scf of gas as one cu ft at 15.025 psia and 60°F.

The specific gravity of air is 1.00. The specific gravity of a gas, γ_g, is defined as the ratio of the molecular weight of the gas to the molecular weight of air.

One lb-mole of a perfect gas at 14.7 psia and 60°F occupies 379 cu ft of space, and one lb-mole of any gas is one molecular weight of that gas, expressed in lbs.

For example, one lb-mole of methane (CH_4) is 16.04 lbs of methane since CH_4 has a molecular weight of 16.04. One lb-mole of methane (CH_4) and one lb-mole of ethane (C_2H_6) will both occupy the same volume at 14.7 psia and 60°F, although there will be 16.04 lbs of methane and 30.07 lbs of ethane.

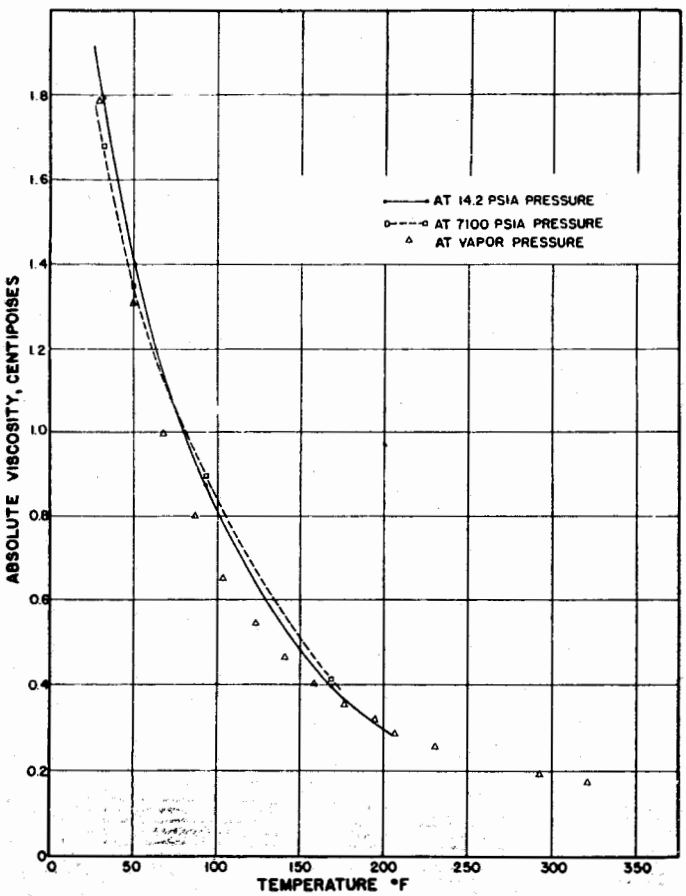


Fig. 2.9 Viscosity of water at oil field temperatures and pressure (after Beal, courtesy AIME).

2.232 Gas properties

2.2321 Density

The density of a gas is the one property that greatly affects the pressure gradient in vertical multiphase flow. Its effect is to lower the pressure gradient. The density of air is 0.0764 lb_m/cu ft as compared to 62.4 lb_m/cu ft for pure water.

2.2322 Viscosity

The viscosities of gases will increase with pressure and temperature. The pressure effect is the same as in liquids, but the temperature effect is opposite to that in liquids. Numerous correlations showing these relationships are found in Katz, *et al.*[20]

The two most widely used correlations for gas are those of Carr[21] and Lee[22]. Beggs and Brill[16] recommend the one by Lee and gave the following equations:

$$\mu = K \times 10^{-4} \exp(x\rho^{\gamma}) \qquad (2.246)$$

$$K = \frac{(9.4 + 0.02M)T^{1.5}}{209 + 19M + T}$$

$$X = 3.5 + \frac{986}{T} + 0.01M$$

$$\gamma = 2.4 - 0.2\,X$$

$T = °R$

$\mu = cp$

$\rho = gm/cm^3$

$M =$ molecular weight

Reference can be made to Figs. 2.11, 2.12, 2.13 and 2.14 for various correlations for gas viscosity.

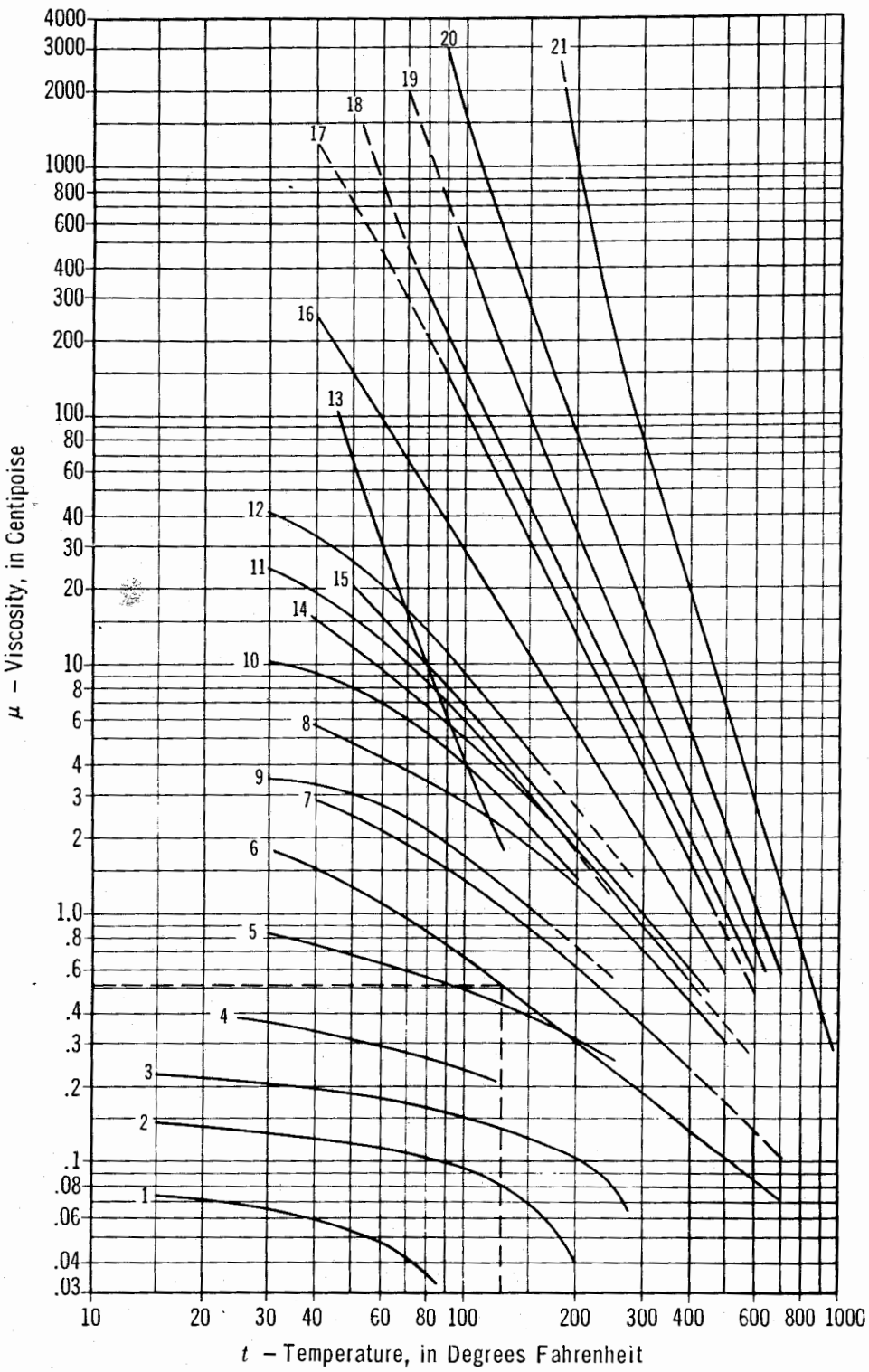
2.2323 Compressibility

Numerous correlations can be found for gas compressibility. Reference should be made to Fig. 2.15 for a simplified z factor chart based on specific gravity. Other commonly used correlations are shown in Figs. 2.16 and 2.17. Numerous other figures for determining z can be found in Appendix A.

2.233 Gas problems related to multiphase flow

2.2331 Introduction

Three types of problems related to gases are involved in most multiphase flow considerations. These are: (1) the calculation of gas density at a particular pres-



1. Ethane (C_2H_6)
2. Propane (C_3H_8)
3. Butane (C_4H_{10})
4. Natural Gasoline
5. Gasoline
6. Water
7. Kerosene
8. Distillate
9. 48 Deg. API Crude
10. 40 Deg. API Crude
11. 35.6 Deg. API Crude
12. 32.6 Deg. API Crude
13. Salt Creek Crude
14. Fuel 3 (Max.)
15. Fuel 5 (Min.)
16. SAE 10 Lube (100 V.I.)
17. SAE 30 Lube (100 V.I.)
18. Fuel 5 (Max.) or Fuel 6 (Min.)
19. SAE 70 Lube (100 V.I.)
20. Bunker C Fuel (Max.) and M.C. Residuum
21. Asphalt

Data extracted in part by permission from the Oil and Gas Journal.

Example: The viscosity of water at 125 F is 0.52 centipoise (Curve No. 6).

Fig. 2.10 Viscosity of water and liquid petroleum products (courtesy Crane Co.).

sure and temperature; (2) the determination of the actual volume that a certain quantity (scf) of gas will occupy under a set condition of pressure and temperature; and (3) the velocity of gas in a pipe at a particular condition of pressure and temperature. In order to understand these problems, a brief review of the real gas law is given.

The real gas law is $pV = znRT$, where z is the compressibility factor for gases and is defined as the ratio of the volume actually occupied by a gas at given con-

TABLE 2.26—PROPERTIES OF AIR

Specific gravity	1.00
Specific weight (60°F and 14.7 psia)	0.0764 $\frac{lb_f}{cu\ ft}$
Hydrostatic pressure gradient (60°F and 14.7 psia)	0.00053 psi/ft
Average molecular weight	28.96
Viscosity (60°F and 14.7 psia)	0.018 cp

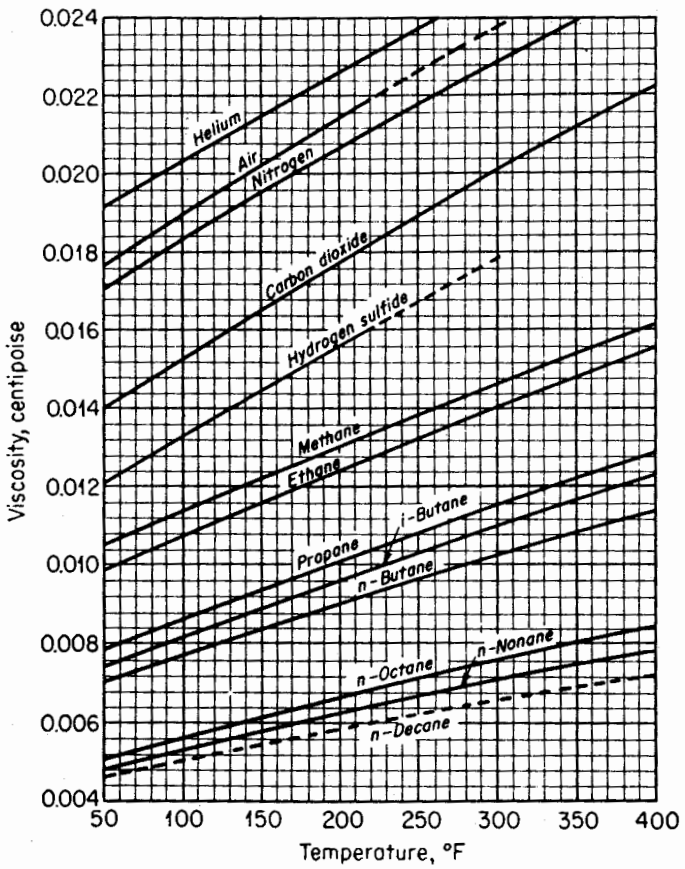


Fig. 2.11 Viscosity of natural gases at atmospheric pressure (after Carr, et al. courtesy AIME).

ditions of pressure and temperature to the volume it would occupy if it were assumed to behave as a perfect gas (ideally). This is expressed symbolically as:

$$z = \frac{V\ actual}{V\ ideal}. \qquad (2.247)$$

The factor R is the universal gas constant. It has a value of 10.72 for units of pressure in psia, volume in cu ft, temperature in degrees Rankine, and n in lb-moles. Values of R for other sets of units are found in many references, but here we will adopt a value of 10.72. The gas law is also used in combination with Boyle's and Charles' law, with z added, as follows:

$$\frac{p_1 V_1}{T_1 z_1} = \frac{p_2 V_2}{z_2 T_2} = \frac{pV}{zT} \qquad (2.248)$$

2.2332 Example problem on gas density

For calculating gas density, the following substitutions are made in the real gas law:

$\rho = \frac{lb_m}{V}$, and $n = \frac{lb_m}{M}$ are substituted into $pV = znRT$

to obtain:

$$\rho = \frac{p\ (M)}{zRT} = \frac{p(\gamma_g)(28.96)}{zRT} \qquad (2.249)$$

For R = 10.72: $\rho = 2.70\ \frac{p\ \gamma_g}{zT}$

The density at any condition of pressure and temperature can also be calculated from the equation:

$$\rho(p,T,z) = \gamma_g(0.0764)\left(\frac{p}{14.7}\right)\left(\frac{520}{T}\right)\left(\frac{1}{z}\right). \qquad (2.250)$$

Equation 2.250 simplifies to:

$$\rho = \frac{2.70\ p\ \gamma_g}{zT}.$$

These equations can be utilized to calculate the gas density for any set of conditions. For example, suppose we desire to obtain the density at 2,000 psia and a temperature of 200°F (660° abs) for a gas having a specific gravity of 0.65 (M = (0.65)(28.96) = 18.8). At those conditions of pressure and temperature, we find from Fig. 2.17 that the compressibility is 0.884. Substituting these values in the real gas law:

$$\rho = \frac{pM}{zRT}\ \frac{(2000)(18.8)}{(0.884)(10.72)(660)} = 6.02\ \frac{lb_m}{cu\ ft}$$

2.2333 Example problem on change in gas volume

For gas volume calculations, we use the relationship:

$$\frac{pV}{Tz} = \frac{p_1V_1}{T_1z_1} = \frac{p_2V_2}{T_2z_2} = \frac{p_{sc}\ V_{scf}}{T_{sc}\ z_{sc}} = \frac{14.7\ V_{scf}}{520}. \qquad (2.251)$$

Equation 2.251 can be rearranged into the form:

$$V_{(p,T,z)} = V_{scf}\left(\frac{14.7}{p}\right)\left(\frac{T}{520}\right)\left(\frac{z}{1}\right) \qquad (2.252)$$

The volume occupied by 700 scf of free gas ($\gamma_g = 0.65$) at 2,000 psia and 200°F is:

$$V_{(2000\ psia,\ 200°F)} = (700)\left(\frac{14.7}{2000}\right)\left(\frac{660}{520}\right)\left(\frac{0.884}{1}\right) = 5.77\ cu\ ft$$

2.2334 Example calculation, gas velocity in a pipe

Calculate the gas rate of flow in a pipe for a given free gas flow rate.

Data

Pipe size = 2.0 in. I.D.
p = 2,000 psia
T = 200°F (660° absolute)
$\gamma_g = 0.65$,
Free gas flow rate = 100,000 scf/day
Portion of pipe occupied by liquid = 60%
z = 0.884 at 2,000 psia and 200°F

Calculation

Step 1: Determine gas flow rate at standard conditions in cu ft/sec:

$$q = \frac{100{,}000\ scf/day}{86{,}400\ sec/day} = 1.158\ scf/sec$$

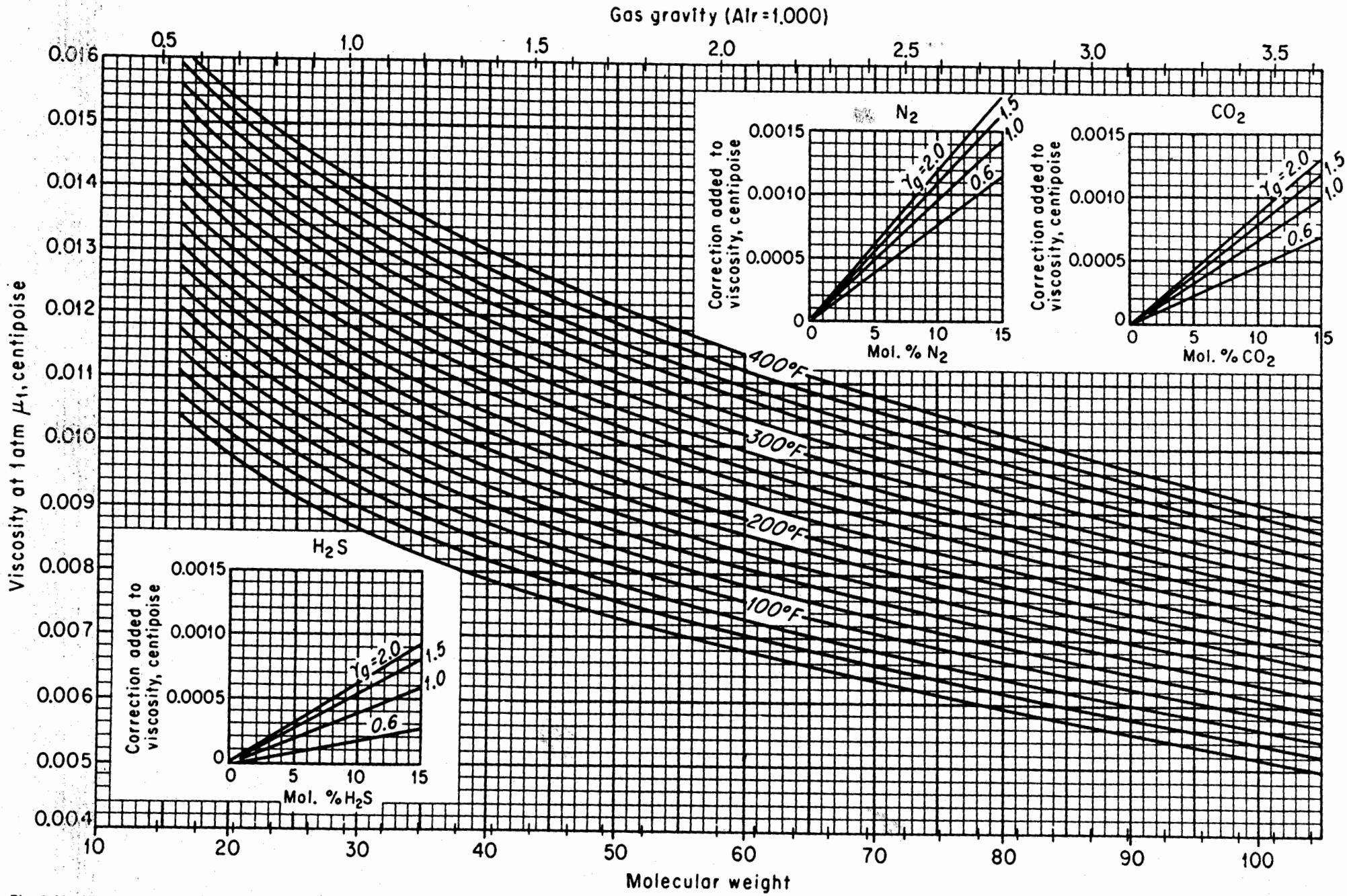


Fig. 2.12 Viscosity of paraffin hydrocarbon gases at 1 atm (after Carr et al. courtesy AIME).

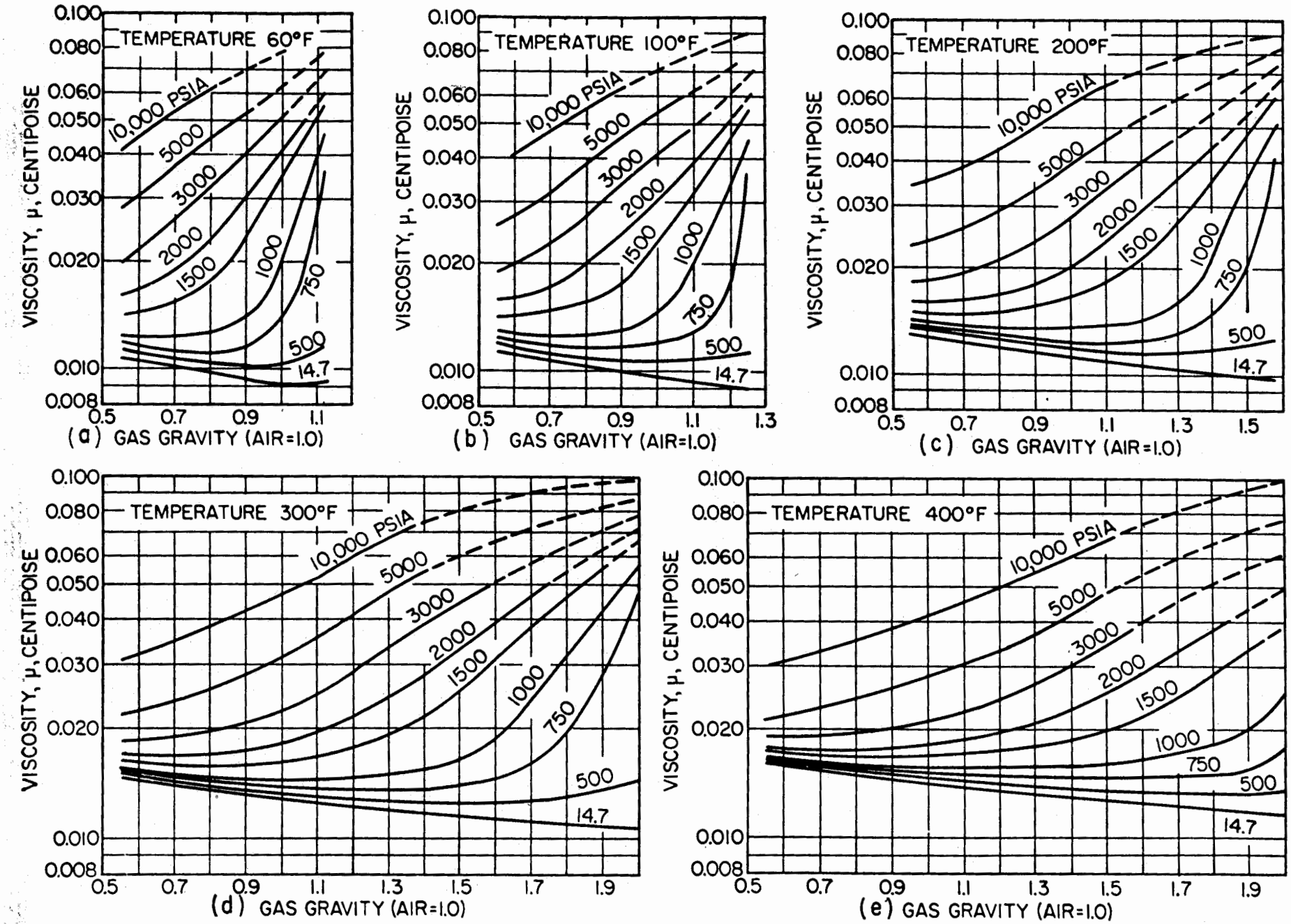


Fig. 2.13 Viscosity of natural gases (by permission from handbook of natural gas engineering by Donald L. Katz et. al., McGraw-Hill Book Co., Inc.).

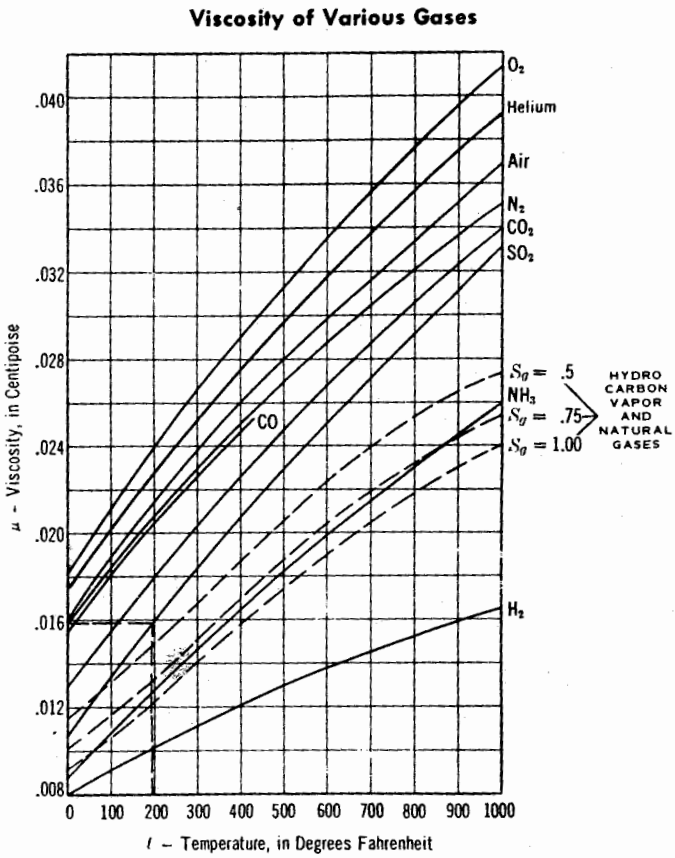


Fig. 2.14 Viscosity of various gases (courtesy Crane Co.).

Step 2: Determine gas flow rate in cu ft/sec at given pressure and temperature. The required flow rate can be calculated by substituting the standard rate from Step 1 into Eq. 2.251 in lieu of V_{scf} and determining the rate at the specified conditions instead of $V_{(p,T)}$.

$$q_{(2000\ psia,\ 200°F)} = 1.158\left(\frac{14.7}{2000}\right)\left(\frac{660}{520}\right)\left(\frac{0.884}{1}\right) = 0.00955 \text{ cu ft/sec}$$

Step 3: Determine the velocity

$$A_p = \pi\frac{d^2}{4} \qquad d = 2 \text{ in.} = \frac{2}{12} \text{ ft}$$

and the effective area for gas flow is 0.40 A_p, since for this problem the pipe is partially filled with liquid.

$$v = \frac{q}{A_p} = \frac{0.00955}{\frac{\pi}{4}\left(\frac{2}{12}\right)^2(0.40)} = \frac{(0.00955)(144)}{\pi\ 0.40} = 1.094 \text{ ft/sec}$$

2.24 Discussion of variables affecting pressure loss in multiphase flow

There are numerous variables that affect the pressure loss in multiphase flow. A brief review of these variables and their effect on the multiphase flow problem is necessary in order to understand the various correlations presented.

For example, Sections 2.241 and 2.242 discuss the volume factor for oil and gas in solution. The flow rates of both liquid and gas increase as the pressure decreases. The volume of oil decreases because the gas is being released from solution, and this increase in free gas volume causes the flow rates of both liquid and gas to increase. This change in flow rate could also change the flow regime, and this could result in different calculation procedures for some correlations.

2.241 Volume factor for oil

We use the oil formation volume factor to denote the volume at reservoir conditions occupied by one stock tank barrel (stk bbl) of oil plus gas in solution. In multiphase flow problems, we are concerned with the volume factor for oil at specified conditions of pressure and temperature encountered in a vertical or horizontal pipe.

Assuming a flowing bottom-hole pressure of 3,000 psi, we could have an oil volume factor of 1.29. At a point in the tubing string where the pressure is 2,000 psi, the oil volume factor could be 1.22. At a 500-psi pressure point the oil volume factor could change to 1.10; and at atmospheric pressure the volume factor is 1.00. This means that one stk bbl of oil changes in volume by 0.29 bbl in going from 3,000 psi to atmospheric pressure. This has a definite effect in multiphase flow calculations because of the changing volume, but of particular importance is the release of additional free gas.

Frick[23] has used the following formulas for calculating oil volume factors:

$$B_o = 0.972 + 0.000147\ F^{1.175} \qquad (2.253)$$

$$F = R_s\left(\frac{\gamma_g}{\gamma_o}\right)^{0.5} + 1.25\ T, \qquad (2.254)$$

where T is in °F and R_s is gas in solution, scf/stk bbl.

These formulas are based on Standing's work,[24] as noted in Fig. 2.18. Although this is a generalized correlation, it has been found to be quite applicable to black oils and can be used where volume factor correlations are not available for the specific crude under consideration. Most of the vertical and horizontal flow working curves and most computer solutions have utilized this correlation.

2.242 Gas in solution

2.2421 Crude

Under conditions of increasing pressure, a crude oil will continue to absorb more gas into solution. This becomes extremely important in multiphase flow problems because the amount of free gas affects the density of the mixture of gas and liquid. For example, let us assume that we have an oil well producing with a gas-oil ratio of 1,000 scf/bbl. At 3,000 psi there may be approximately 500 scf/bbl of gas in solution, leaving 500 scf/bbl of free gas. At a higher point in the tubing string, where the pressure is reduced to 2,000 psi, there may be 350 scf/bbl in solution and 650 scf/bbl of free gas. As the pressure is further reduced to 500 psi, there may be only 75 scf/bbl in solution and 925 scf/bbl of free gas.

Although the same stk bbl exists at each pressure point, the additional free gas together with the de-

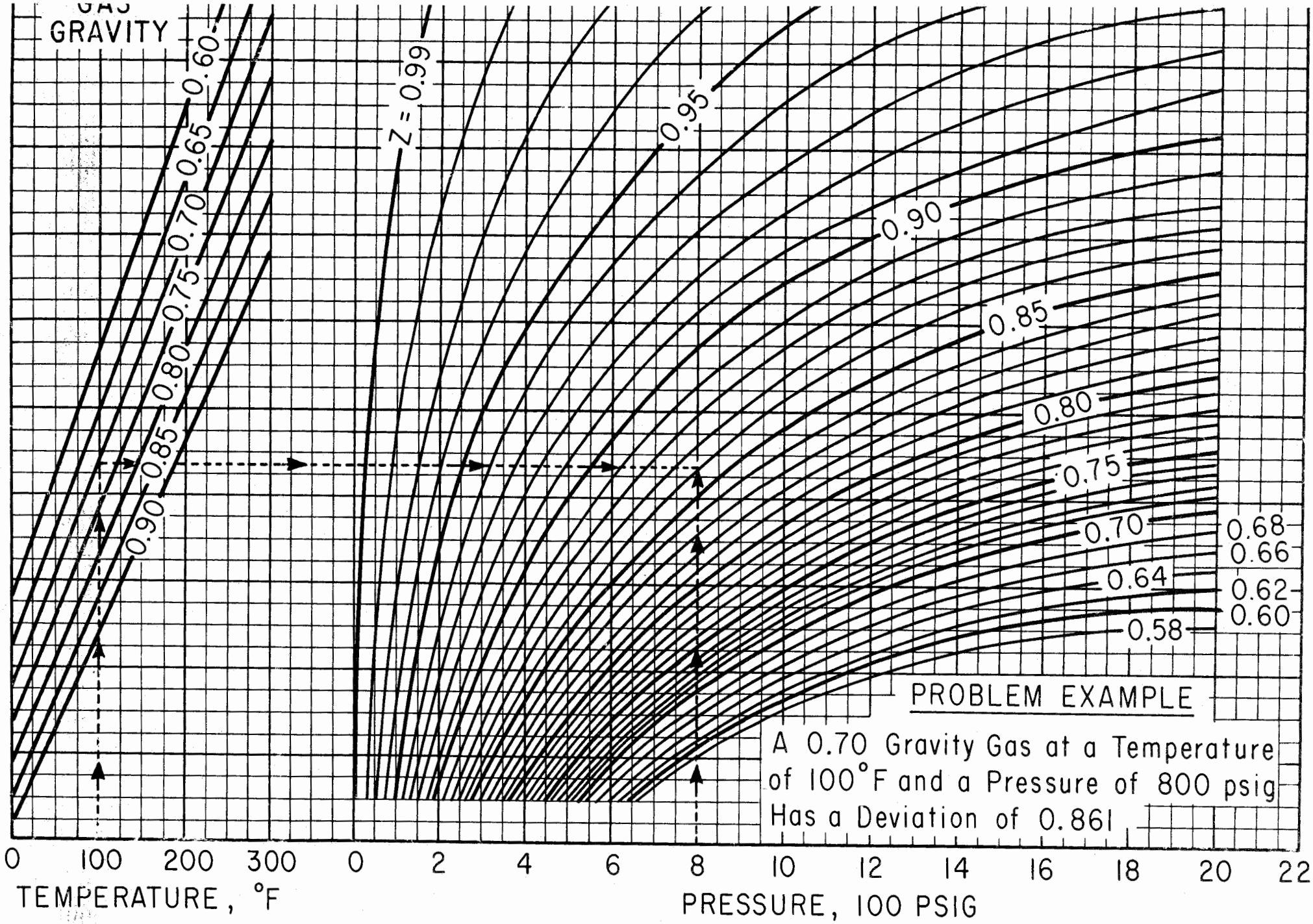


Fig. 2.15 Simplified Z factor correlation.

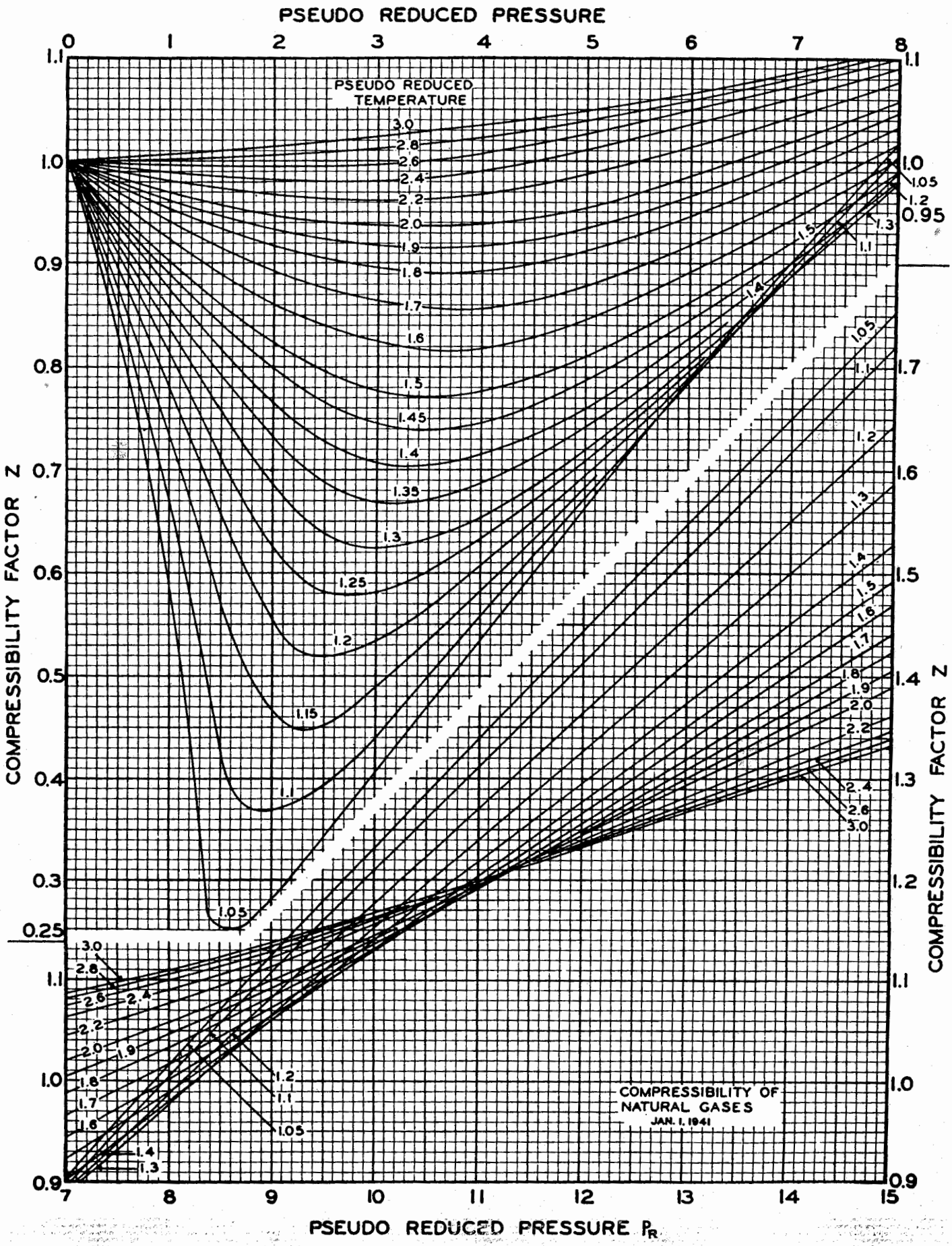


Fig. 2.16 Compressibility of natural gases as a function of reduced pressure and temperature (after Standing and Katz, courtesy AIME).

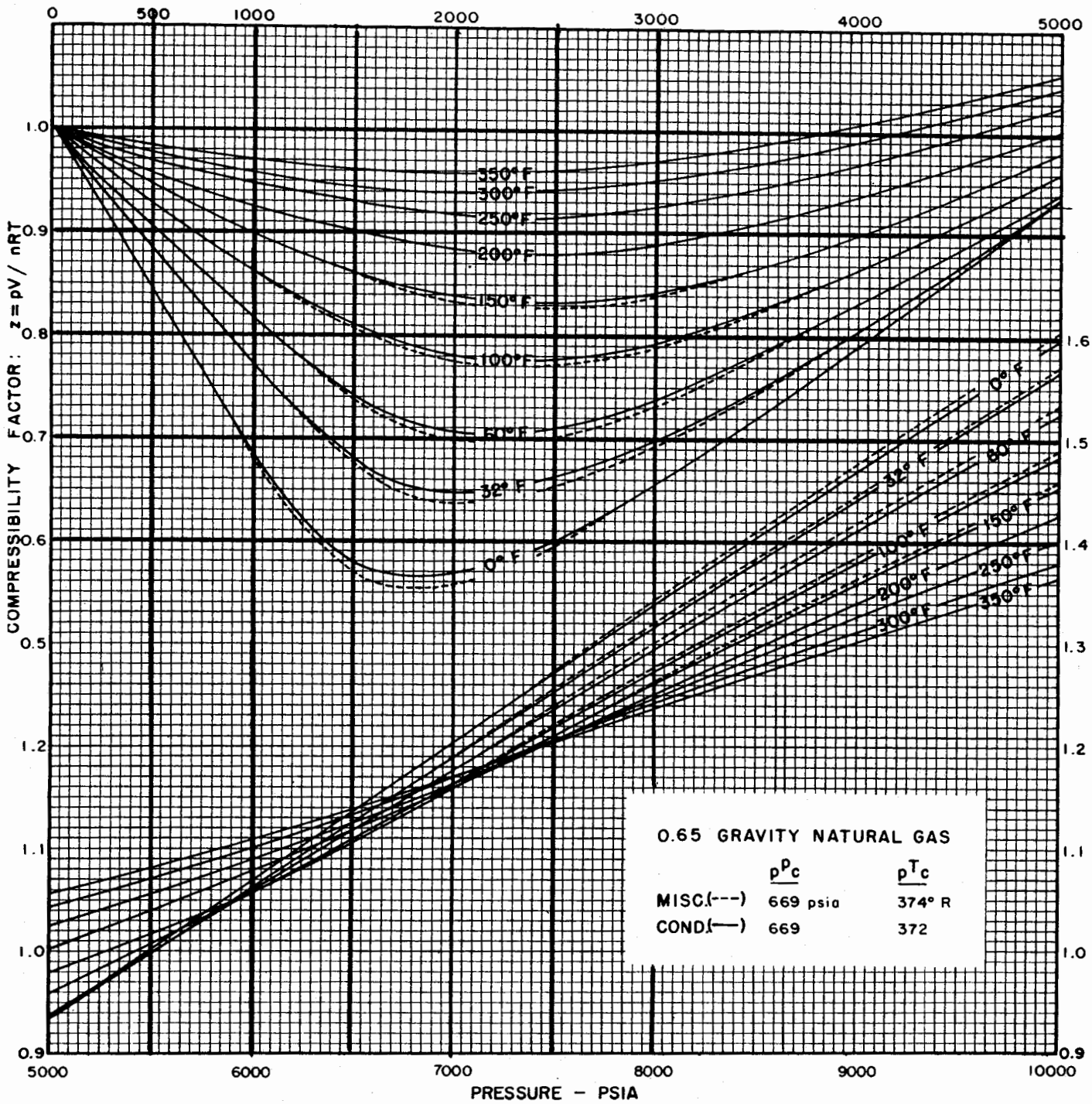


Fig. 2.17 *Compressibility factors for A 0.65 gravity natural gas.*

creased pressure has a definite effect on the density of the mixture of oil and gas.

Although the amount of solution gas is primarily controlled by pressure, it is also affected by temperature, density, gas specific gravity, and the composition of both the crude oil and the gas.

Frick[23] utilizes the following formula for calculating the gas in solution for a particular set of conditions:

$$R_s = \gamma_g \left[\frac{p}{18} \times \frac{10^{0.0125\ (°API)}}{10^{0.00091(T)}}\right]^{1/0.83} \qquad (2.255)$$

where: T = °F, p = psi

This formula was originally suggested by Standing[24]. It is presented in nomograph form in Fig. 2.19. This is a generalized correlation and should not be used when actual solution gas curves are available.

In addition, Lasater[25] presented a correlation for gas in solution as follows:

$$R_s = \left[\frac{(379.3)\ (350)\ \gamma_o}{M_o}\right]\left(\frac{Y_g}{1 - Y_g}\right) C \qquad (2.256)$$

C = factor to fit field data

where Y_g and M_o were obtained from appropriate correlations, as noted in Figs. 2.20 and 2.21A. A nomograph solution is noted in Fig. 2.21B. Chierici, *et al.*[26] has suggested that the Lasater correlation be used

EXAMPLE

REQUIRED:

Formation volume at 200°F of a bubble point liquid having a gas-oil ratio of 350 CFB, a gas gravity of 0.75, and a tank oil gravity of 30 °API.

PROCEDURE:

Starting at the left side of the chart, proceed horizontally along the 350 CFB line to a gas gravity of 0.75. From this point drop vertically to the 30°API line. Proceed horizontally from the tank oil gravity scale to the 200°F line. The required formation volume is found to be 1.22 barrel per barrel of tank oil.

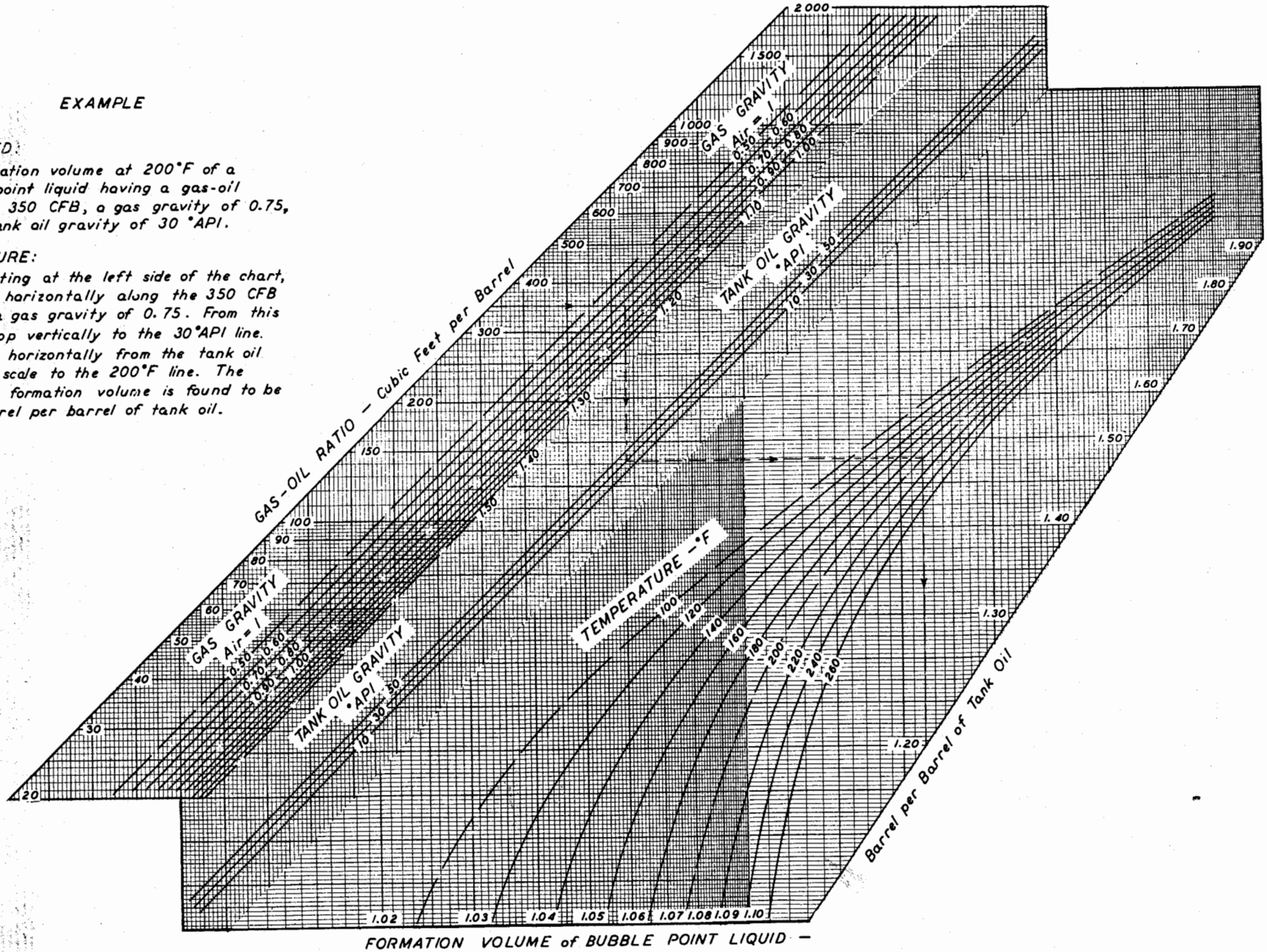


Fig. 2.18 Properties of natural hydrocarbon mixtures formation volume of bubble point liquid. (Courtesy Chevron Research Company).

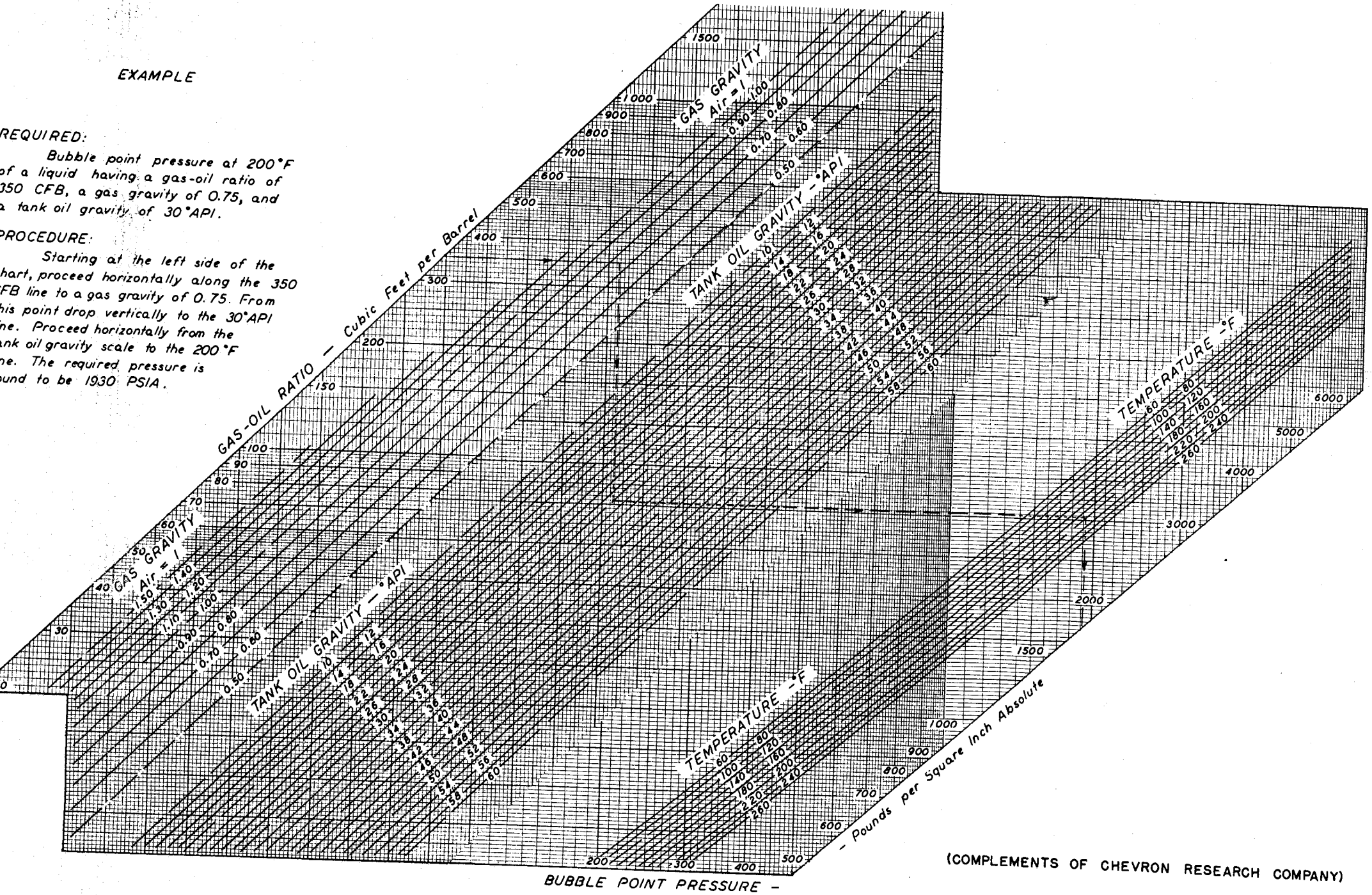
EXAMPLE

REQUIRED:

Bubble point pressure at 200°F of a liquid having a gas-oil ratio of 350 CFB, a gas gravity of 0.75, and a tank oil gravity of 30°API.

PROCEDURE:

Starting at the left side of the chart, proceed horizontally along the 350 CFB line to a gas gravity of 0.75. From this point drop vertically to the 30°API line. Proceed horizontally from the tank oil gravity scale to the 200°F line. The required pressure is found to be 1930 PSIA.



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2.19 Properties of natural hydrocarbon mixtures of gas and liquid bubble point pressure.

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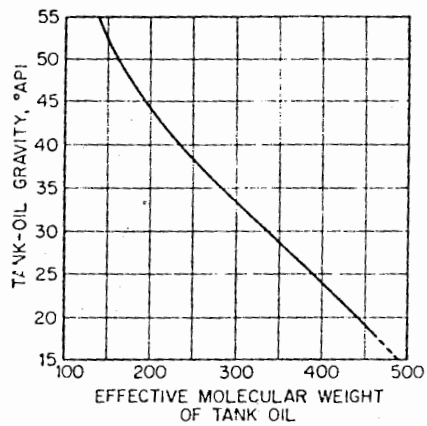


Fig. 2.20 Molecular weight vs. °API (after Lasater).

for °API > 15 and the Standing correlation for °API < 15.

2.2422 Water

Gas is slightly soluble in water, as reported in Refs. 27 and 28. From Ref. 27, it is noted that only 19.5 scf of gas is in solution in one bbl of water, at 5,000 psia and 180°F. The solubility decreases with increasing salinity. For all practical purposes, the amount of gas in solution in water can be ignored in multiphase flow problems.

Table 2.27 shows solubility vs. pressure at 180°F for fresh water:

TABLE 2.27

Pressure	Solubility, scf/B
500	4
1000	7.5
2000	12
3500	16.3
5000	19.5

In addition the solubility decreases as salinity increases. For example with 30,000 ppm, a correction factor of 0.85 is used in Table 2.27.

2.243 Surface tension

Very little research work has been done to determine the influence of surface tension in multiphase flow problems. Both Ros[1] and Hagedorn[2] have developed dimensionless correlation groups which included surface tension. Brill, *et al.*[29] showed that, in the calculation of a vertical pressure traverse by the method of Hagedorn, an increase in surface tension gave an increase in pressure gradient. Waldy[30] found the opposite to be true in his laboratory study.

The true effect of surface tension on multiphase flow is somewhat debatable. It is well known that liquid surface tension may vary several-fold, but many persons consider the effect to be a minor one. Baker[31] indicated that crude oil surface tensions varied from 40 to 20 dynes/cm in a range of densities from 10 to 75° API. He also showed that the effect of solution gas produced surface tension values that were less than 10 dynes/cm at pressures in excess of 2,500 psia (Figs. 2.21 and 2.22).

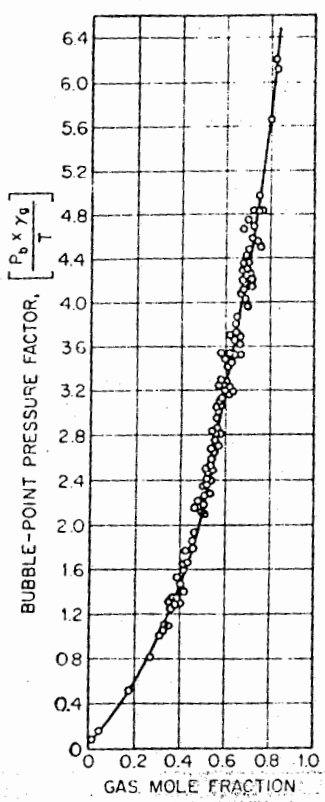


Fig. 2.21A Bubble-point pressure factor vs. gas mole fraction (after Lasater).

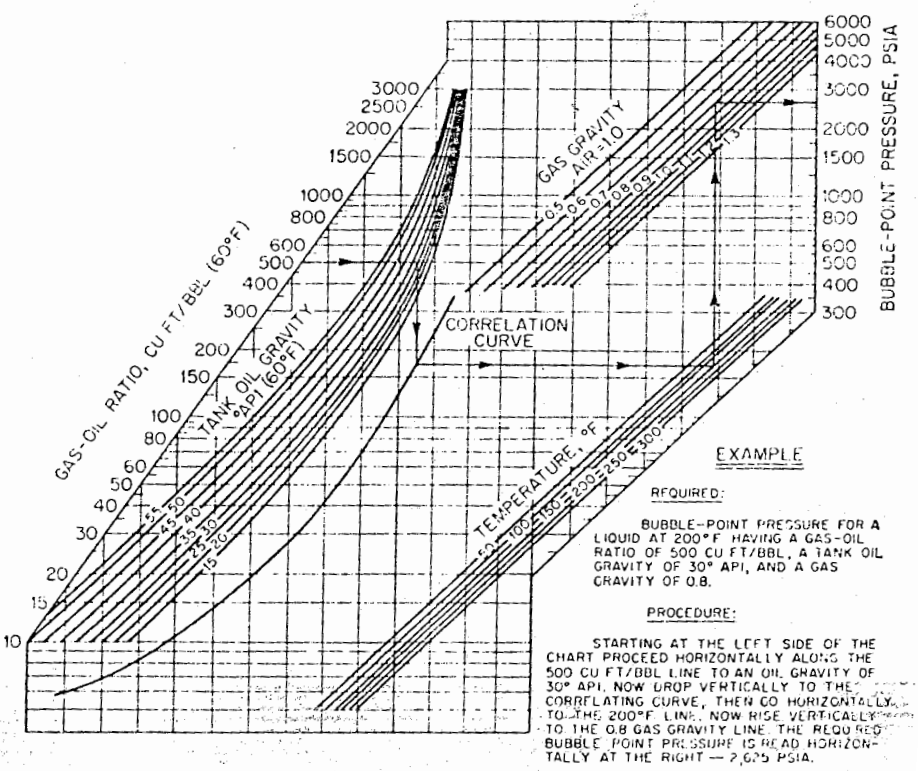


Fig. 2.21B Bubble point pressure by Lasater.

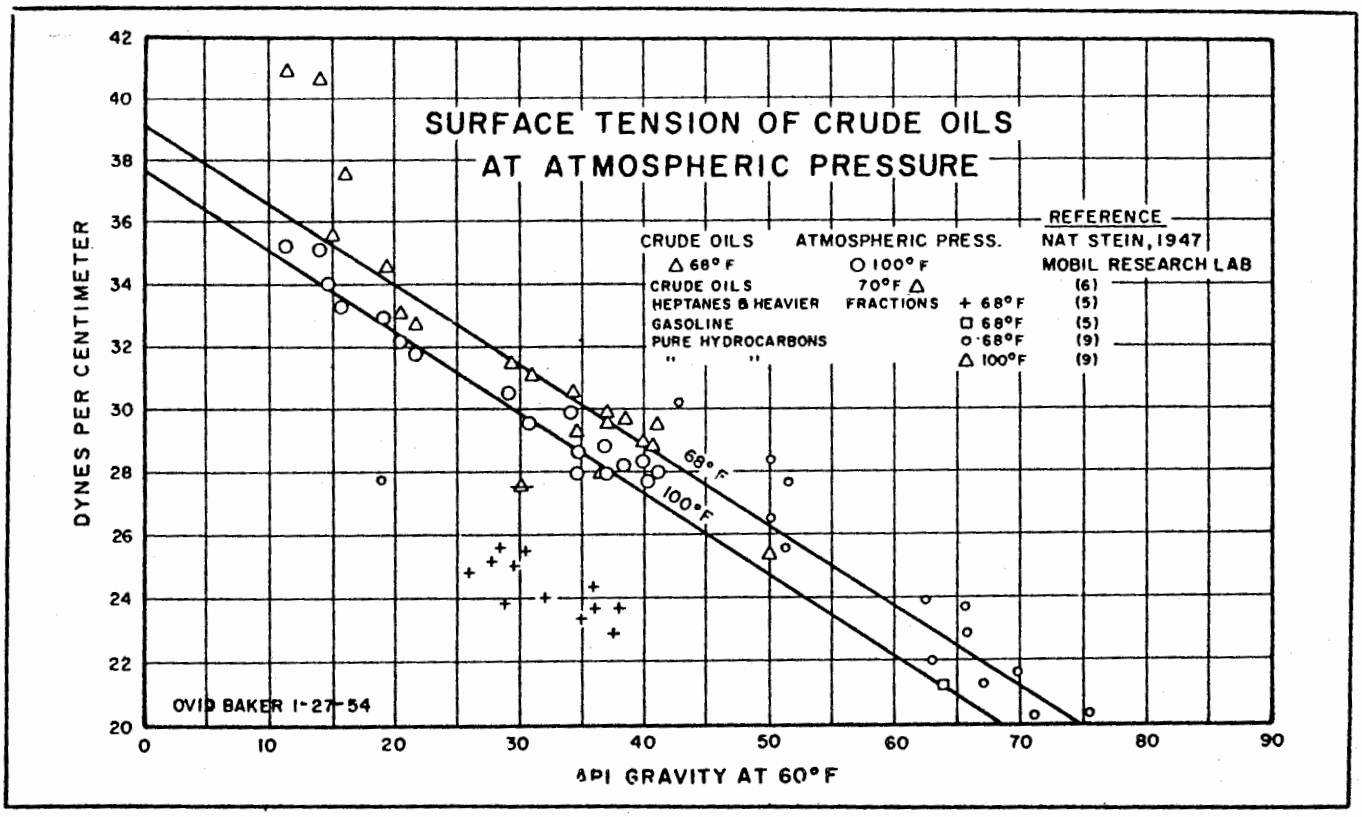


Fig. 2.22A Surface tension of crude oils (by Baker).

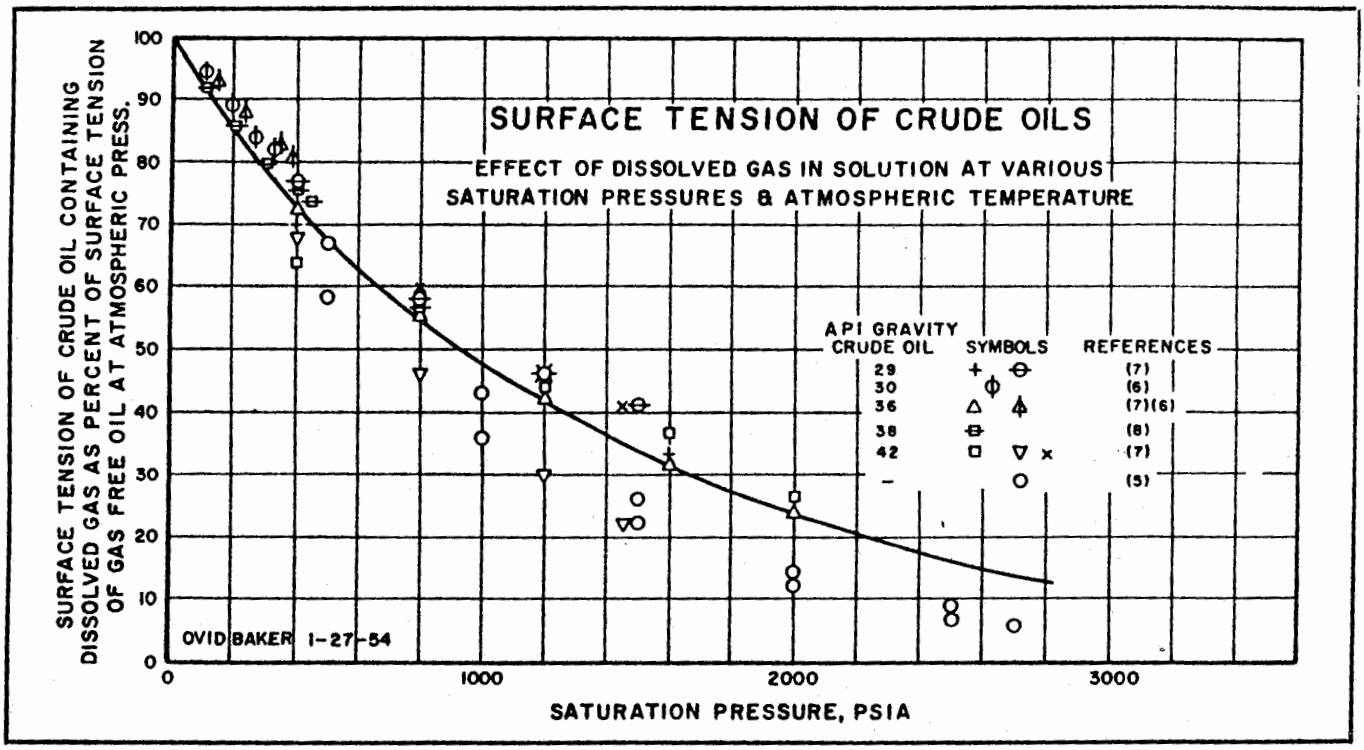


Fig. 2.22B Surface tension of crude oils (by Baker).

2.244 Wall contact angle

The contact angle of a fluid in a pipe is the measure of the relative wetting of the pipe wall by the fluid. A contact angle of zero indicates complete wetting by the denser phase; an angle of 90° indicates that neither phase preferentially wets the solid; and a contact angle of 180° indicates complete wetting by the least dense phase.

All visual observations have shown that the liquid wets the wall; therefore, the wall contact angle has no influence on multiphase flow loss calculations.

2.245 Viscosity of a multiphase flow mixture

The viscosity of a multiphase mixture cannot be accurately predicted. Each phase has some effect on the mixture viscosity.

One method for approximating the viscosity of a mixture is to take an arithmetic average, as described by the following equation:

$$\mu_m = \mu_L H_L + \mu_g (1 - H_L) \quad (2.257)$$

where: μ_m = viscosity of the mixture
μ_L = viscosity of the liquid
μ_g = viscosity of the gas
H_L = liquid holdup factor (fractional portion of pipe occupied by liquid)

Arrhenius[32] proposed an equation based on empirical data, as follows:

$$\mu_m = (\mu_L)^{H_L} (\mu_g)^{1 - H_L} \quad (2.258)$$

A comparison of these two equations is shown in Fig. 2.23. Of these two equations, Eq. 2.258 is considered to be the better. It should be emphasized that Eq. 2.258 is a method of handling the viscosity term in the two-phase Reynold's number. Its use, however, has no physical meaning in terms of *the* viscosity of *something* (anything).

There have been recent investigations of the effect of liquid viscosity on pressure loss in multiphase flow. Although no attempt was made to measure the mixture viscosity, the effect of liquid viscosity and its relationship to the mixture viscosity has been successfully handled. Ros[1] varied the viscosity of the liquids in his experimental work and found that, in general, viscosities of crudes greater than six centistokes had the effect of increasing pressure loss in multiphase flow as viscosity increased. Hagedorn and Brown[33] varied the crude viscosity in an experimental test well and found, in most cases, that all crude oils with viscosities greater than 10 cp exhibited increased pressure loss with increased viscosity.

Additional studies on the viscosities of mixtures are found in Refs. 32, 34, 35, and 36.

Some controversy has existed in handling water production in both vertical and horizontal flow. For example, water affects the viscosity, density, and surface tension of the liquids. It is recommended that the weighted average liquid properties be obtained and then used as if one liquid existed. For example, the liquid viscosity of a 50% water-oil mixture will be $0.50\ \mu_o + 0.50\ \mu_w$ with one solution for the viscosity of the gas liquid mixture being $\mu_m = \mu_L^{H_L} \times \mu_g^{(1 - H_L)}$.

This manner of handling water-cut excludes the effect of emulsions. Depending upon water percentage and crude properties we may have emulsions formed which cause the pressure losses to be much higher than predicted if handled strictly on the weighted average properties. Woelflin[37] did some experimental work on emulsions as shown in Figs. 2.24, 2.25, 2.26, and 2.27. He calibrated a marsh funnel in order to determine the Saybolt Universal Viscosity in seconds as noted in Fig. 2.24. Fig. 2.25 shows the variation in viscosity ratio with % water cut and for percentages between 50 and 60% the viscosity increases abruptly.

Fig. 2.26 shows the general curve for viscosity change with brine percentage with an abrupt increase occurring at about 65% brine. Woeflin commented that the abrupt increase in viscosity due to the formation of an emulsion will occur at different water percentages depending upon the crude. Fig. 2.27 shows the effect of temperature on the viscosity ratio.

Sobocinski and Huntington[38] noted that the most pronounced water-in-oil emulsion formation took place with a water-to-oil mass flow ratio of 4.

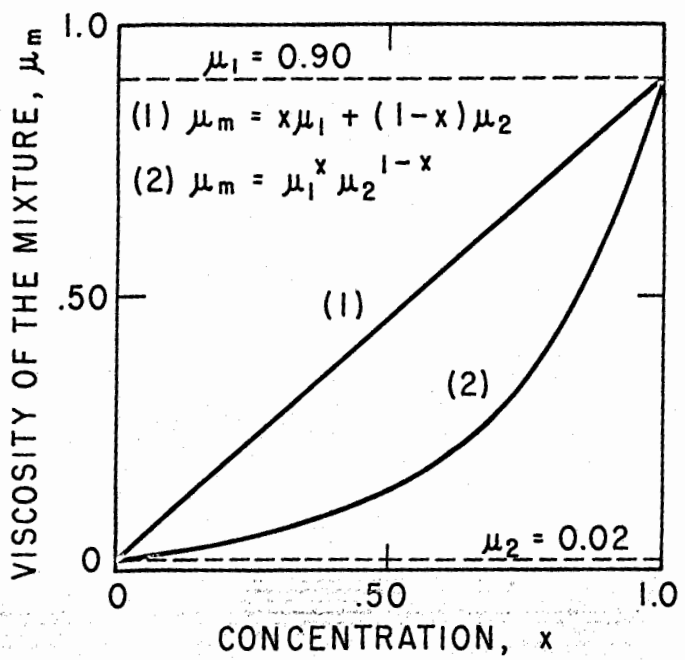


Fig. 2.23 Comparison of relationships predicting viscosities of mixtures.

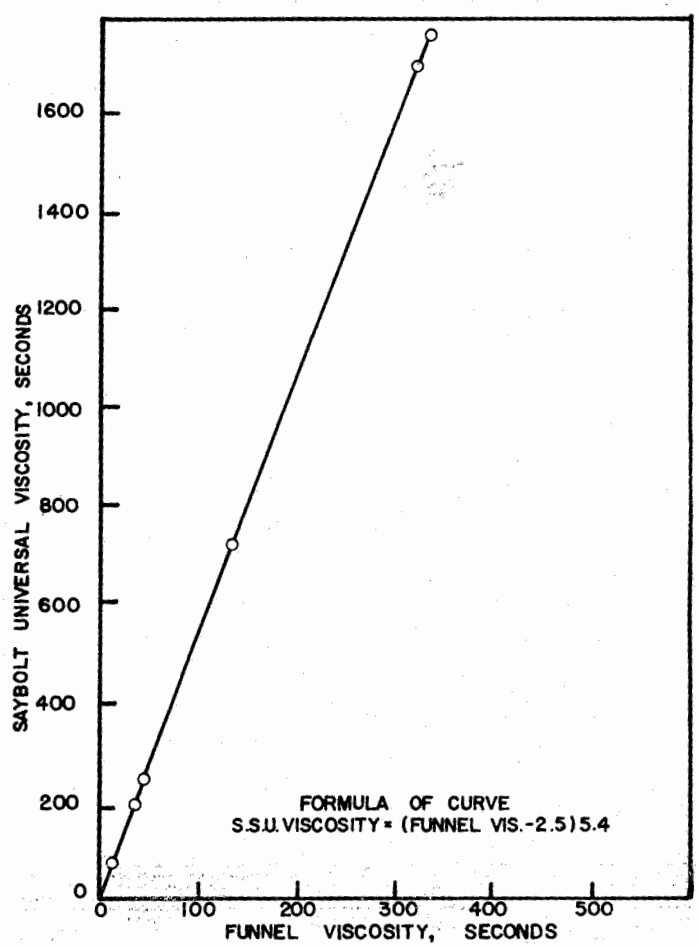


Fig. 2.24 Calibration of viscosimeter to saybolt universal viscosimeter (after Woelflin).

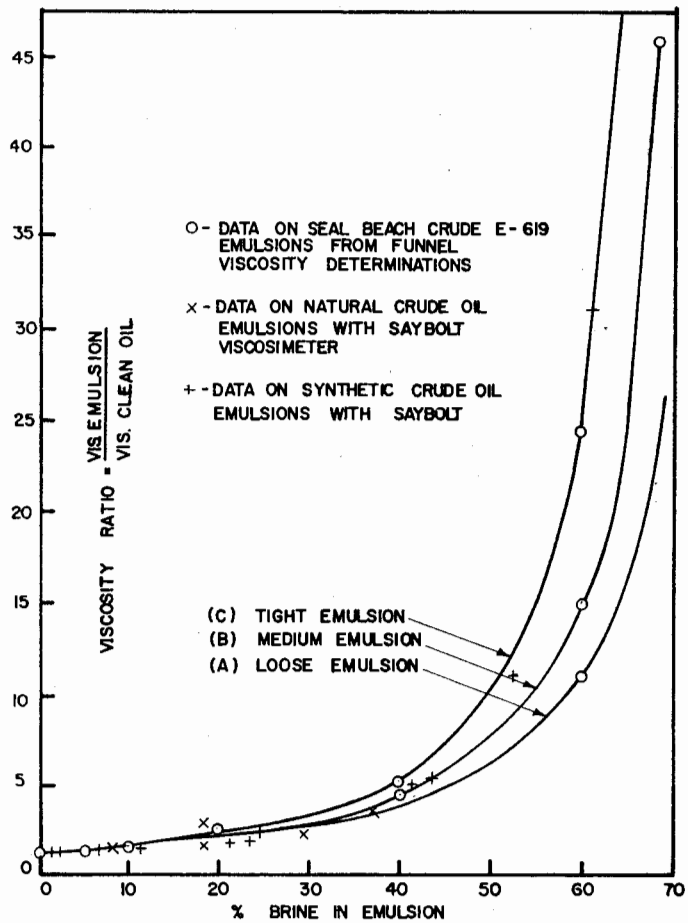


Fig. 2.25 Variation in viscosity of crude oil and brine with brine content (after Woelflin).

The physical properties of the water and oil used in the Sobocinski and Huntington experiment are given in the following table:

Component	*Density*	*Viscosity*	*Surface tension*
Water	62.3 lb_m/cu ft at 70°F	0.68 cs at 100°F	72.9 dynes/cm at 70°F
Gas-oil	36.8°API 60/60	3.38 cs at 100°F	23.5 dynes/cm at 95°F

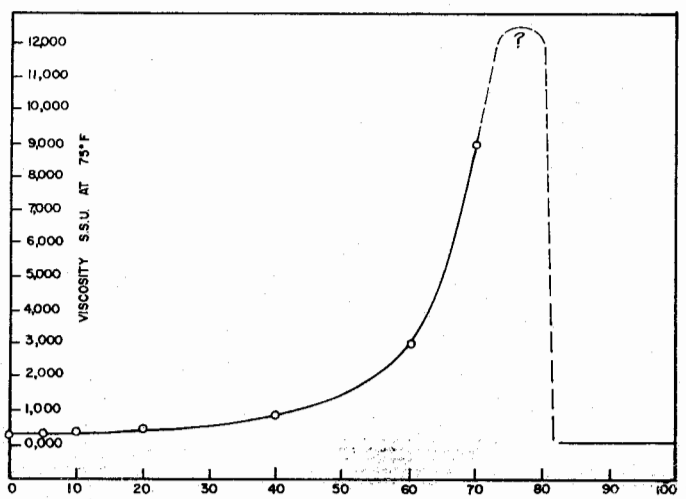


Fig. 2.26 General curve for viscosity of an emulsion with 0 to 100 percent brine (medium emulsion) (after Woelflin).

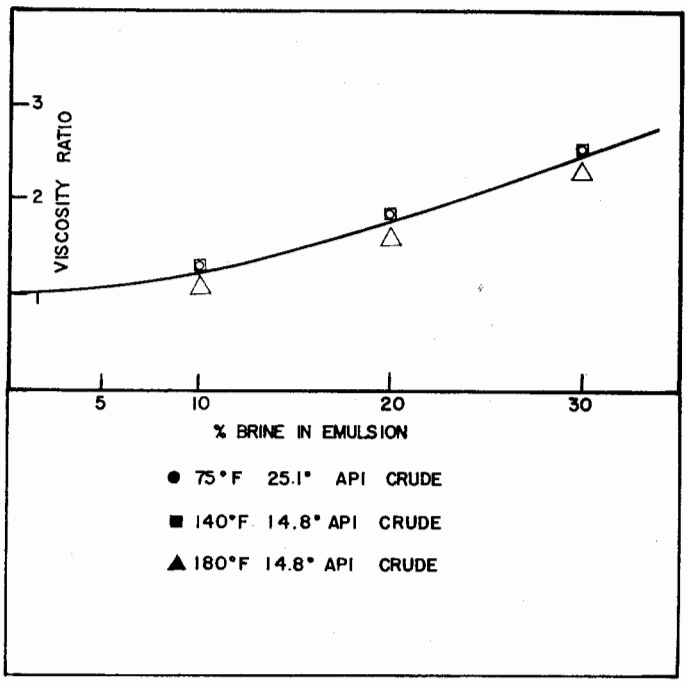


Fig. 2.27 Effect of temperature on viscosity ratio (after Woelflin).

In order to obtain at least a qualitative idea as to the behavior of the flow of two liquids and gas, Sobocinski and Huntington set a water/oil ratio of 0.66 with a total mass liquid rate of 34,100 lb/hr/sq ft. This liquid rate was then flowed with increasing gas rates. The types of flow patterns and the formation of emulsions is summarized in the following table:[38]

Air rates, lb_m/hr-sq ft	*Description*
2,000	The oil flowed on top of the water at 150% of the linear velocity of the water. There was no disturbance between the air, oil, and water boundaries. (Stratified flow)
2,000–3,000	Gentle ripples appeared on the oil surface but the surface between the oil and water remained smooth.
3,000–4,000	Linear velocity of oil increased to 227% of the water and deep ripples appeared on the oil surface with minor irregularities on the water-oil surface.
4,600	Waves appeared on the oil surface and also on the water surface causing incipient emulsification.
5,000–12,000	Emulsion became very pronounced. It gradually encroached up the pipe walls, forming a continuous film around the entire inside periphery of the tube. A greater portion of the emulsion remained on the bottom of the tube.
14,000	The in-place ratio of the two liquids approached the flowing ratio, indicating that there was no slippage of the liquids.

The pressure losses encountered by emulsified flow were found to be much greater than those under the

same liquid and gas flow rate for non-emulsified flow. For example, it was found that for a total liquid rate of 35,300 lb_m/hr-sq ft, a water-oil ratio of 4, and a gas rate of 13,000 lb_m/hr-sq ft, the pressure gradient was 1.4 lb_f/sq ft/ft. Under the same conditions of total liquid flow rate of 35,300 and a gas rate of 13,000, the pressure gradient was 0.28 lb_f/sq ft/ft for water and air and 0.29 lb_f/sq ft/ft for oil and air. As pointed out by Tek[39] in his discussion of this work, the pressure drop for emulsified flow was five times that of flowing only air and water or air and oil.

2.25 Development of the general energy equation

2.251 Introduction

Many of the investigators of multiphase flow have used the general energy equation in some form. Poettmann and Carpenter[40] and Hagedorn and Brown[2] used it in an energy balance form. Ros[1] and Duns and Ros[41] used it in a pressure balance form. Others have suggested a mechanical energy balance, but all forms are essentially the same.

The general energy equation expresses an energy balance between two points in a fluid flow system. It follows the law of conservation of energy, which states that the energy of a fluid entering Section 1 of a pipe, plus any additional work done on the fluid between Sections 1 and 2, minus any energy losses by the system between Sections 1 and 2, is equal to the energy of the fluid leaving Section 2.

The general energy equation is used to solve many problems involving multiphase flow in a vertical or horizontal direction. A brief development of the equation is made in the following section. This is essentially the same presentation that appears in other publications.[4,42]

2.252 Discussion of variables in the equation

Fig. 2.28 shows a typical flowing system. A steady state energy balance can be written with reference to elements used in this system.

If we choose the force-length-time (FLt) system and, more specifically, the units of ft-lb_f for each term, the following equation results:

$$U_1 + \frac{mv_1^2}{2\,g_c} + \frac{mg\,z_1}{g_c} + p_1V_1 + q - W = U_2 + \frac{mv_2^2}{2\,g_c} + \frac{mg\,z_2}{g_c} + p_2V_2 \qquad (2.259)$$

Each term of the equation is described briefly, as follows:

(1) Internal Energy (U) designates the internal energy carried with the fluid. Internal energy may be thought of as stored energy of some type. It may include rotational, translational, and vibrational energy of the molecules. We cannot measure internal energy as we do pressure and temperature nor can we calculate the absolute value of internal energy. A relative value is established by assigning zero to some particular set of conditions.

(2) Kinetic Energy $\left(\frac{mv^2}{2g_c}\right)$. This term represents the energy that the fluid possesses due to its velocity. In many multiphase flow problems this term is considered to be negligible. For vertical multiphase flow, it can become significant near the surface in the low density region. This is particularly true for high gas rates and small tubing sizes.

(3) Potential Energy $\left(\frac{mgz}{g_c}\right)$. The energy of position is designated by the potential energy term where z is the vertical distance above some reference point. For horizontal flow this term drops out, but must be included for any multiphase flow system that deviates from the horizontal. For vertical flow, it becomes a predominant term.

(4) Pressure Volume (pV). The energy of expansion or compression is accounted for by this term. It represents flow work and may be called the energy of pressure. It may be thought of as a compound property of the substance, or potential energy due to pressure.

(5) Transferred Heat (q). This represents heat energy that may be entering or leaving the system. In Eq. 2.259, q has a positive sign, indicating that heat is being added to the system.

(6) Work (W). This may carry either a positive or negative sign, depending on whether work is done on the flowing fluid (pump and negative sign), or the flowing fluid is doing work on its surroundings (turbine and positive sign).

(7) Enthalpy (H). Often, in thermodynamic equations, we find the terms U and pV together. Because of this common occurrence, the term enthalpy (H) has been defined, as follows:

$$H = U + pV \qquad (2.260)$$

Notice that Eq. 2.259 has $U_1 + p_1V_1$ and $U_2 + p_2V_2$. All flowing energy balances have this combination.

Because the absolute value of internal energy, U, cannot be measured, it is not possible to measure the absolute enthalpy of a substance. In actual practice, we are primarily interested in the changes in enthalpy from point to point.

(8) Entropy (S). The entropy of a system, like all state properties, is determined by the state of the system. Therefore, the entropy change during a process depends only on the initial and final states and not on the path.

Entropy is defined by:

$$S_2 - S_1 = \int_1^2 \frac{d\,q}{T} = \int_1^2 m\,Cp\,\frac{dT}{T} \qquad (2.261)$$

where: dq = heat transferred in a reversible process
= m Cp dT for the particular case of sensible heat transfer at constant pressure
m = lb mass
Cp = specific heat
T = temperature

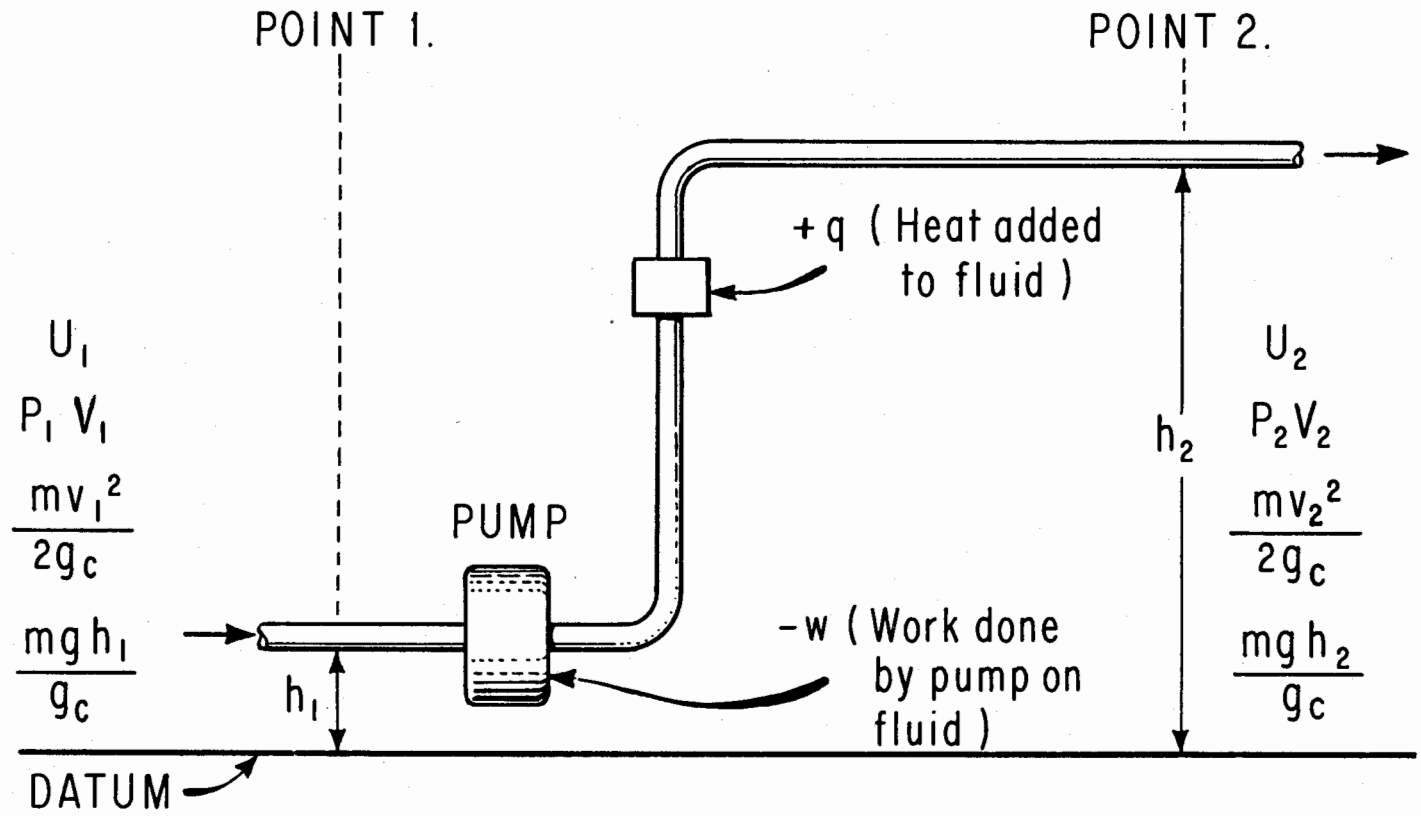


Fig. 2.28 Flow diagram.

Notice that while the formal definition of entropy entails a reversible process, the computed entropy change is valid for any process, reversible or irreversible, between the selected initial and final states.

Entropy is related to internal energy by the following equation:

$$\Delta U = \int_{S_1}^{S_2} Tds + \int_{V_1}^{V_2} p(-dV) + \int_{\text{effects}}^{\text{chem}} + \int_{\text{tension effects}}^{\text{surface}} + \int \text{etc.}, \tag{2.262}$$

where: $\int_{S_1}^{S_2} Tds$ = heat effects,

and $\int_{V_1}^{V_2} p(-dV)$ = compression effects.

Entropy may be expressed symbolically as

$$\int_{S_1}^{S_2} Tds = q + lw. \tag{2.263}$$

(9) Lost Work (lw). This term may be used as a "catch-all" in multiphase flow problems. It is generally thought of as lost work due to irreversibilities of the fluid. It may include losses such as wall friction, slippage, frictional effects between phases, viscosity effects, surface tension effects, etc.

2.253 Derivation of the equation

Starting with Eq. 2.259:

$$U_1 + \frac{mv_1^2}{2g_c} + \frac{mgz_1}{g_c} + p_1V_1 + q - W = U_2 + \frac{mv_2^2}{2g_c} + \frac{mgz_2}{g_c} + p_2V_2.$$

This reduces to:

$$\Delta U + \Delta\left(\frac{mv^2}{2g_c}\right) + \Delta\left(\frac{mgz}{g_c}\right) + \Delta(pV) + W - q = 0. \tag{2.264}$$

In terms of enthalpy, this equation may be written as

$$\Delta H + \Delta\left(\frac{mv^2}{2g_c}\right) + \Delta\left(\frac{mgz}{g_c}\right) + W - q = 0.$$

We may also define ΔU in terms of entropy as:

$$\Delta U = \int_{S_1}^{S_2} Tds + \int_{V_1}^{V_2} p(-dV) + \int \text{all other effects.}$$

Assuming $\int$ all other effects = 0, then:

$$\Delta U = \int_{S_1}^{S_2} Tds + \int_{V_1}^{V_2} p(-dV). \tag{2.265}$$

By substituting Eq. 2.265 into Eq. 2.264 and noting that Δ (pV) is a complete differential, we have:

$$\int_{S_1}^{S_2} Tds - \int_{V_1}^{V_2} pdV + \Delta\left(\frac{mv^2}{2g_c}\right) + \Delta\left(\frac{mgz}{g_c}\right) + \int_{V_1}^{V_2} pdV + \int_{P_1}^{P_2} Vdp + W - q = 0. \tag{2.266}$$

It is known from thermodynamics that:

$$\int_{S_1}^{S_2} Tds = q + lw. \tag{2.263}$$

By substitution of Eq. 2.263 into Eq. 2.266, we obtain:

$$\int_{P_1}^{P_2} Vdp + \Delta\left(\frac{mv^2}{2g_c}\right) + \Delta\left(\frac{mgz}{g_c}\right) + W + lw = 0. \tag{2.278}$$

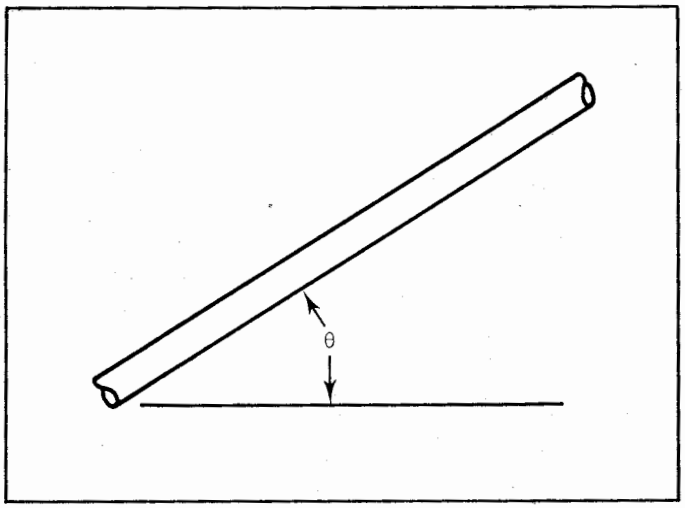


Fig. 2.29 Flow configuration.

The only term that has not been previously discussed is $\int V dp$—nothing more, nothing less. It is the algebraic sum of three terms: p_1V_1 (work to introduce fluids), p_2V_2 (work to expel fluids), and $\int p dV$. The term $\int V dp$ is obtained as noted in the previous derivation of the general energy equation.

2.254 Discussion of the general energy equation

Eq. 2.278 represents the general energy equation from which many flow problems may be solved, and it is the basis upon which many of our multiphase flow correlations are developed. This equation is sometimes presented in a slightly different form in which different units are used. Eq. 2.278 may be written for one lb-mass as:

$$\int \overline{V} dp + \Delta\left(\frac{v^2}{2g_c}\right) + \frac{g}{g_c}(\Delta z) + \overline{W} + l\overline{w} = 0 \tag{2.279}$$

which gives each term units of $\frac{\text{ft lb}_f}{\text{lb}_m}$.

The equation may be written in symbolic differential form based on one lb_m of the flowing fluid and with units of feet for each term as:

$$144\frac{g_c}{g} V dp + \frac{g}{g_c} dz + \frac{v dv}{g_c} + dW + d(lw) = 0. \tag{2.280}$$

The conversion factor 144 assumes Δp is in lb/sq in. rather than lb/sq ft.
This equation may also be written in the following form:

$$\frac{dp}{\rho} + \frac{v dv}{g_c} + \frac{g}{g_c} dz + d(lw) + dW = 0 \tag{2.281}$$

Assuming no work done on or by the fluid we have:

$$\frac{dp}{dz} + \frac{\rho v dv}{g_c dz} + \frac{g}{g_c}\rho + \rho\frac{d(lw)}{dz} = 0 \tag{2.282}$$

Solving for pressure gradient we have:

$$\frac{dp}{dz} = -\left[\frac{g}{g_c}\rho + \frac{\rho v dv}{g_c dz} + \rho\frac{d(lw)}{dz}\right] \tag{2.283}$$

In order to take care of any angle of flow and defining $\rho\frac{d(lw)}{dz}$ to be $\left(\frac{dp}{dz}\right)_F$ (friction) we have:

$$\frac{dp}{dz} = \frac{g}{g_c}\rho\sin\theta + \frac{f\rho v^2}{2g_c d} + \frac{\rho v dv}{g_c dz} \tag{2.284}$$

where: $\left(\frac{dp}{dz}\right)_F \equiv \frac{\text{"f"}\rho v^2}{2g_c d}$

Solving for the pressure gradient $\frac{dp}{dz}$:

$$\frac{dp}{dz} = \frac{g}{g_c}\rho\sin\theta + \frac{\rho v dv}{g_c dz} + \left(\frac{dp}{dz}\right)_F \tag{2.285}$$

where: $g/g_c\,\rho\sin\theta$ = elevation term
$\frac{\rho v dv}{g_c dz}$ = acceleration term
z = axial distance of flow
Δp = positive in direction of flow
θ = angle from horizontal

If we define $\left(\frac{dp}{dz}\right)_F$ to be $\frac{f\rho v^2}{2g_c d}$, we then have:

$$\frac{dp}{dz} = \frac{g}{g_c}\rho\sin\theta + \frac{f\rho v^2}{2g_c d} + \frac{\rho v dv}{g_c dz} \tag{2.284}$$

or we can write the equation in terms of the angle ϕ from the vertical as shown in Fig. 2.30.

$$\frac{dp}{dz} = \frac{g}{g_c}\rho\cos\phi + \frac{f\rho v^2}{2g_c d} + \frac{\rho v dv}{g_c dz} \tag{2.286}$$

Equations 2.284 and 2.286 are good for any fluid and for any pipe inclination where:

$f = f(N_{Re}, K)$
N_{Re} = Reynold's number
K = roughness

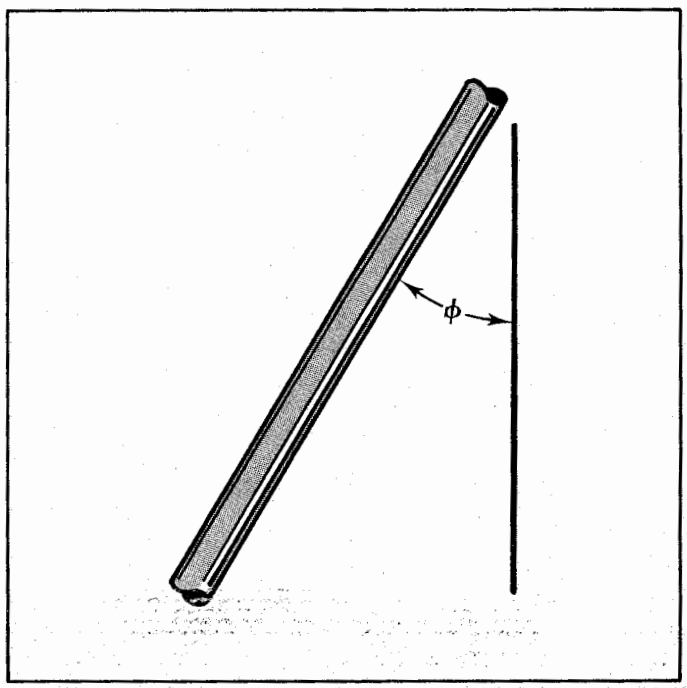


Fig. 2.30 Flow configuration.

Ve can state the preceding equation 2.284 as follows:

For units of psi/ft (gradient):
Total gradient = Elevation gradient
+ Friction gradient + Acceleration gradient

r:

$$\left(\frac{dp}{dz}\right)_{total} = \left(\frac{dp}{dz}\right)_{elevation} + \left(\frac{dp}{dz}\right)_{friction} + \left(\frac{dp}{dz}\right)_{acceleration} \quad (2.287)$$

Vhere:

$$\text{Elevation gradient} = \frac{dp}{dz}_{elevation} = \frac{g}{g_c}\rho \sin\theta$$

$$\text{Friction gradient} = \frac{dp}{dz}_{friction} = \frac{fv^2\rho}{2g_c d}$$

$$\text{Acceleration gradient} = \frac{dp}{dz}_{acceleration} = \frac{\rho v dv}{g_c dz}$$

Equation 2.284 is good for any fluid and for any ngles of flow. For vertical flow, $\theta = 90°$ and $\sin 90° = 0$. The equation then reduces to:

$$\frac{dp}{dh} = \frac{g}{g_c}\rho + \frac{f\rho v^2}{2g_c d} + \frac{\rho v dv}{g_c(dh)} \quad (2.288)$$

or horizontal flow $\theta = 0°$ and $\sin\theta = 0$, the equation ›duces to:

$$\frac{dp}{dx} = \frac{f\rho v^2}{2g_c d} + \frac{\rho v dv}{g_c(dh)} \quad (2.289)$$

ıd for those numerous cases where acceleration is ›gligible then:

$$\frac{dp}{dx} = \frac{f\rho v^2}{2g_c d} \quad (2.290)$$

?55 Application of equations to multiphase flow

For multiphase flow it is necessary that the terms of uation 2.284 be modified to take care of two phases as and liquid) flowing with the added possibility of o liquids (oil and water) flowing.
In particular the following variables must take into count the mixture of gas and liquid:

$\rho = \rho_m$ (density of mixture of gas + liquid)
$v = v_m$ (velocity of mixture)
$f = f_m$ (friction factor for flow of gas and liquid)

r single phase flow we have:

$N_{Re} = \frac{dv\rho}{\mu}$ = Reynold's number normally correlated th friction factor

r the flow of a mixture we have:

$N_{Re})_m = \frac{dv_m\rho_m}{\mu_m}$ where m denotes mixture, the equa-

n can then be modified to read:

$$\frac{dp}{dz} = \frac{g}{g_c}\rho_m \sin\theta + \frac{f_m\rho_m v_m^2}{2g_c d} + \frac{\rho_m v_m dv_m}{g_c(dz)} \quad (2.291)$$

2.2551 Hold-up

In order to determine ρ_m it is necessary that we know that part of the pipe occupied by liquid which we will define as H_L. In the flow of gas and liquids the gas has a tendency to flow faster than the liquids and we have slippage, that is, gas slipping past the liquid. Another way to visualize this is that the liquid tends to accumulate and will occupy more of the pipe space than if no slippage was occurring. This is noted quite often in vertical flow where a pipe is too large for a low flow rate of liquid and gas. A rather vivid example is the loading up of gas wells with liquids which materially affect the flow rate of gas.

Therefore, we find ρ_m from the following equation:

$$\rho_m = \rho_L(H_L) + \rho_g(1 - H_L) \quad (2.292)$$

In order to properly determine ρ_m we must have a valid hold-up correlation. Hold-up depends upon many variables including the angle of inclination. Some correlations choose to use a no-slip hold-up (λ) for correlating purposes. A no-slip density can be calculated:

$$\rho_{m(N.S.)} = \rho_L\lambda + \rho_g(1 - \lambda) \quad (2.293)$$

2.2552 Liquid mixture properties

In order to properly evaluate such variables as liquid density, viscosity, surface tension, etc., we normally use one of the following procedures. This will differ depending upon the investigator.

(a) Liquid density:

$$\rho_L = \rho_o F_o + \rho_w F_w \quad (2.294)$$
F_o = fraction oil, F_w = fraction water

(b) Liquid viscosity:

$$\mu_L = \mu_o F_o + \mu_w F_w \quad (2.295)$$

(c) Surface tension:

$$\sigma_L = \sigma_o F_o + \sigma_w F_w \quad (2.296)$$

2.2553 Two-phase mixture properties

(a) Viscosity:

(1) $\mu_m = \mu_L H_L + \mu_g(1 - H_L)$ (2.297)
(2) $\mu_m = \mu_L\lambda + \mu_g(1 - \lambda)$ (2.298)
(3) $\mu_m = \mu_L^{H_L} \cdot \mu_g^{(1-H_L)}$ (2.299)

As can be seen, viscosity can be determined in several ways and will vary depending upon the investigator.

(b) Density

(1) $\rho_m = \rho_L H_L + \rho_g(1 - H_L)$ (2.2100)
(2) $\rho_m = \rho_L\lambda + \rho_g(1 - \lambda)$ (2.2101)
(3) $\rho_m = \frac{\rho_L\lambda^2}{H_L} + \frac{\rho_g(1-\lambda)^2}{1 - H_L}$ (2.2102)

(c) Flow parameters:

(1) $v_m = v_{sL} + v_{sg}$ (2.2103)
(2) $g_m = g_L + g_g$

2.2554 Friction factors

The friction factors or energy loss factors for multiphase flow are found in several different ways depending upon the investigator. In some cases a standard Moody, Darcy-Weisbach, or Fanning friction factor correlation is used.

The group of numbers formed by the ratio of the wall shear stress and kinetic energy per unit volume defines the friction factor.

$$f = \frac{\tau_w}{\rho v^2/2g_c} \tag{2.2105}$$

It can be shown by a force balance between wall shear stress and pressure forces that:

$$\left(\frac{dp}{dz}\right)_F = \frac{f\rho v^2}{2g_c d} \tag{2.2106}$$

which is the well known Moody friction factor.

In determining a friction factor valid for multiphase flow numerous correlations have been proposed. Many use the same correlation as Darcy-Weisbach or Fanning to obtain a two-phase friction factor. However, the Reynold's number is modified for multiphase flow. For example, many investigators determine a two-phase Reynold's number and then use a standard Darcy-Weisbach chart. The two-phase N_{Re} is defined as:

$$N_{Re} = \frac{dv_m\rho_m}{\mu_m}$$

The subscript m denotes mixture properties, and ρ_m and μ_m may be defined by any of the previous equations.

Some investigators prepare their own correlating parameter to determine an energy loss factor which may be entirely different than the Reynold's number.

2.2555 Calculation of pressure traverses

In equation 2.291 we see that for a particular flow condition we must be able to determine ρ_m and f_m. Therefore, in order to calculate a pressure traverse we must have fluid property correlations that will permit us to determine such properties as R, B_o, μ_o, μ_g, σ_o, σ_g, etc. Once we have this information we know everything necessary to use equation 2.291 except H_L which is necessary to calculate ρ_m and f_m. Therefore if we have correlations to obtain H_L and f_m we can then solve the equation for $\frac{dp}{dz}$ over any pressure or length increment.

Beggs and Brill[16] gave the following procedures for calculating pressure traverses:

(A) Iterating on length increment:
- (1) Knowing p_1, select Δp.
- (2) Estimate Δh or ΔL or Δz to obtain T_2.
- (3) Calculate $\bar{T}$, $\bar{p}$, and all fluid properties (R_s, B_o, μ_o, μ_g, μ_m, etc.).
- (4) Calculate dp/dz.
- (5) Determine $\Delta z = \frac{\Delta p}{dp/dz}$.
- (6) Compare estimated and calculated values.
- (7) Repeat procedure until estimated and calculated values are close.

(B) Iterating on pressure increment:
- (1) Knowing p_1, select Δz.
- (2) Estimate Δp for Δz.
- (3) Calculate $\bar{p}$, $\bar{t}$, and all fluid properties.
- (4) Calculate dp/dz.
- (5) Calculate $\Delta p = \Delta z\left(\frac{dp}{dz}\right)$.
- (6) Compare estimated and calculated values of Δp.
- (7) Repeat until estimated and calculated values are close.
- (8) Set $z = z_1 + \Sigma\Delta z$ and $p = p_1 + \Sigma\Delta z$.
- (9) Continue for total length of pipe.

2.26 Single phase liquid flow

Although this book is primarily concerned with multiphase flow problems, a brief consideration of single phase liquid flow will help in understanding multiphase flow.

For constant area horizontal flow lines, the equation used for liquid flow is that of Darcy-Weisbach; or, it is the same as that of Moody[43] and is expressed as:

$$h_L = f\frac{L}{d}\frac{v^2}{2g}, \tag{2.2107}$$

where h_L is the head loss in feet of the liquid flowing. For single phase liquid flow, the frictional loss depends to a great extent on pipe diameter and pipe roughness. The pressure drop varies inversely with the fifth power of the diameter.

Fanning's equation has also been used by many investigators, and differs from the Darcy-Weisbach equation by a factor of 4 in the friction factor. Fanning's equation is:

$$h_L = \frac{4fLv^2}{d2g} \tag{2.2108}$$

In this book we will use the Moody or Darcy-Weisbach equation.

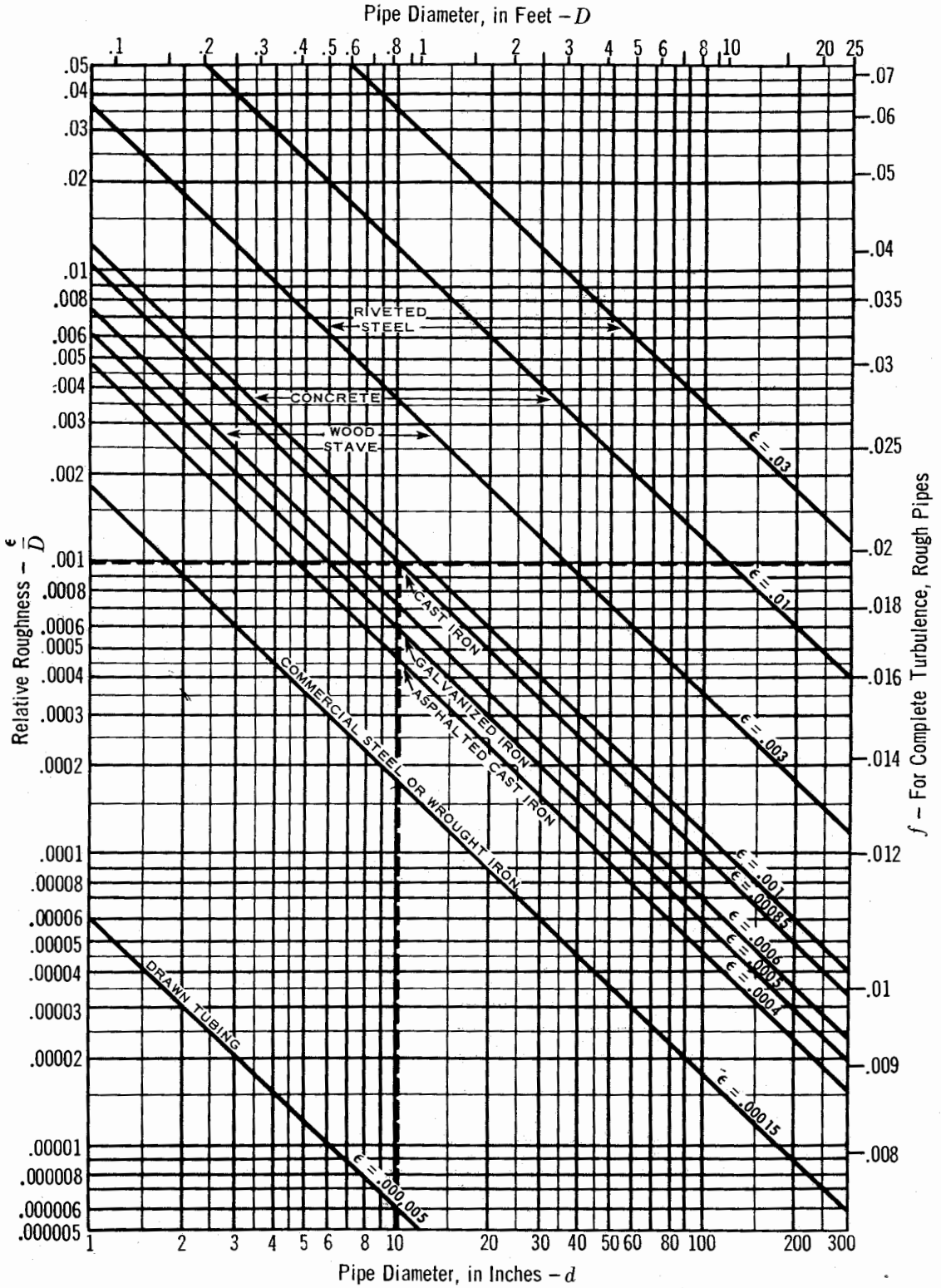
Eq. 2.2107 is normally written to express pressure drop in psi as:

$$\Delta p = 0.001294\,\frac{f\,L\rho v^2}{d} \tag{2.2109}$$

where Δp = psi.

It should be noted that the h_L in Eq. 2.2107 and 2.2108 and the Δp in Eq. 2.2109 reflect the energy loss of the lw term in Eq. 2.279. Fig. 2.31 shows friction factors for various types of pipes.

The friction factor is a function of two dimensionless quantities, ϵ/d and $\frac{dv\rho}{\mu}$, which are the relative roughness of the pipe and the Reynolds number (N_{Re}), respectively. For Reynolds numbers less than 2,000, laminar flow exists with velocity varying from zero at the pipe wall to a maximum at the center of the pipe in a parabolic profile. For Reynolds numbers greater than 4,000, the flow is turbulent and individual particles have no definite path or velocity. The velocity at the wall is zero, but the velocity profile changes



Data extracted from *Friction Factors for Pipe Flow* by L.F. Moody, with permission of the publisher, The American Society of Mechanical Engineers, 29 West 39th Street, New York.

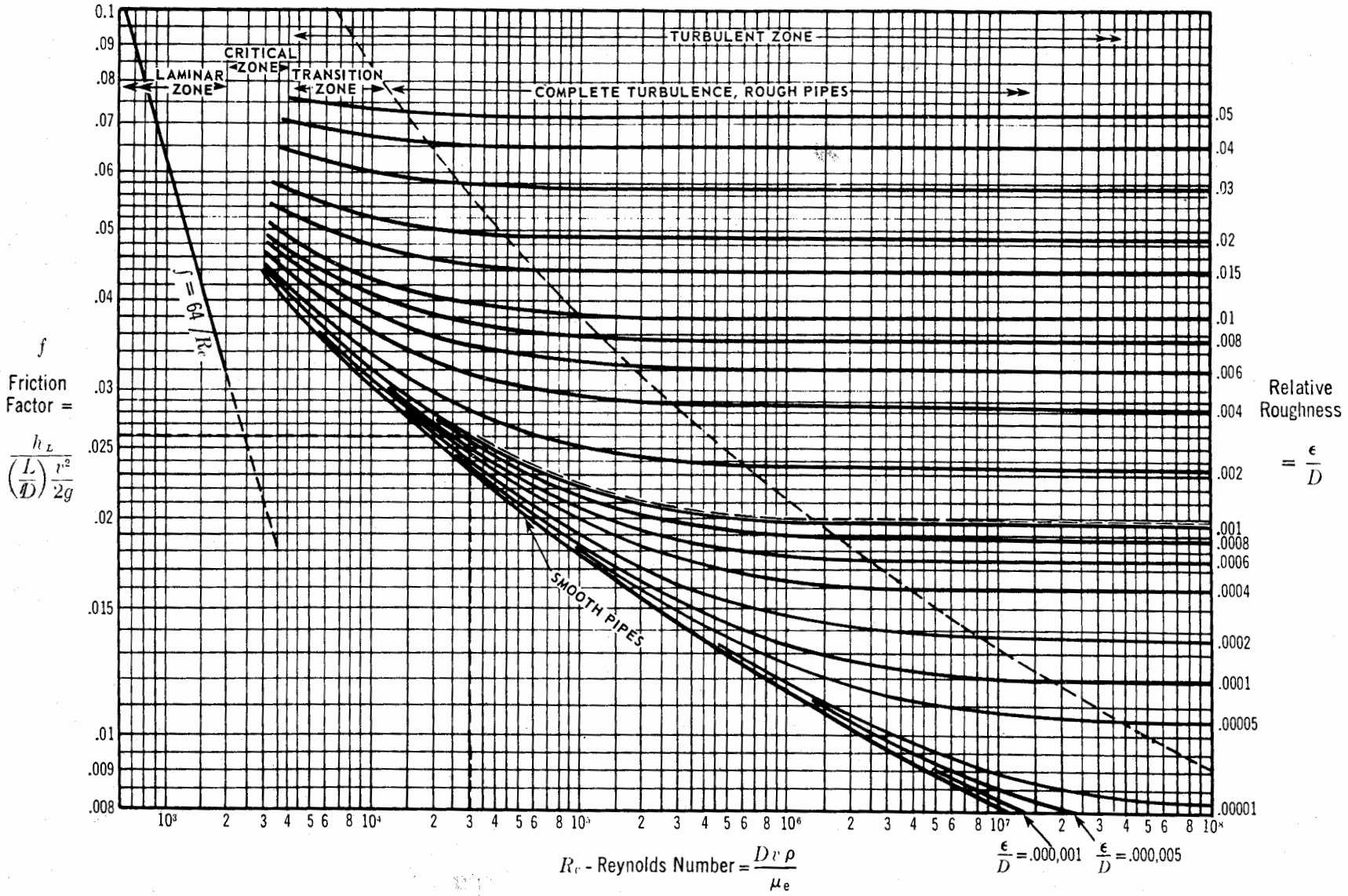
Problem: Determine absolute and relative roughness, and friction factor, for fully turbulent flow in 10-inch cast iron pipe (I.D. = 10.16″).

Solution: Absolute roughness (ϵ) = 0.00085 . . . . . Relative roughness (ϵ/D) = 0.001 . . . . . Friction factor at fully turbulent flow (f) = 0.0196.

Fig. 2.31 Relative roughness of pipe materials and friction factors for complete turbulence (after Crane Co.).

pidly near the wall. Between Reynolds numbers of 000 and 4,000 there is a transition region between minar and turbulent flow. (See Fig. 2.32, after oody[32])

For Reynolds numbers up to 2,000, the friction factor, f, can be expressed as $f = 64/N_{Re}$. It should also be noted from Fig. 2.32 that f is independent of pipe roughness in the laminar flow region. The film at zero



Problem:
Determine the friction factor for 10-inch cast iron pipe (10.16" I.D.) at a Reynolds number flow of 30,000.

Solution: The relative roughness (see page 98) is 0.001. Then, the friction factor (f) equals 0.026.

Data extracted from *Friction Factors for Pipe Flow* by L. F. Moody, with permission of the publisher, The American Society of Mechanical Engineers, 29 West 39th Street, New York 18, N. Y.

Fig. 2.32 Friction factors for any type of commercial pipe (after Crane Co.).

velocity at the pipe wall minimizes the roughness effect. The pressure loss is then due primarily to the viscosity of the liquid.

2.27 Single phase gas flow

If we were to start with the general energy equation:

$$\int_1^2 \bar{V}dp + \Delta\left(\frac{v^2}{2g_c}\right) + \left(\frac{g}{g_c}\right)\Delta z + l\bar{w} + \bar{W} = 0, \quad (2.279)$$

By neglecting the kinetic energy term, we have:

$$\int_1^2 \bar{V}dp + \left(\frac{g}{g_c}\right)\Delta z + l\bar{w} + \bar{W} = 0. \quad (2.2110)$$

If we assume that the flow is steady state, horizontal, and that no work is done by the gas in flow, the equation can then be written as:

$$\int_1^2 \bar{V}dp + l\bar{w} = 0. \quad (2.2111)$$

The losses $l\bar{w}$ can be defined by the same equation used in single phase liquid flow; i.e.,

$$l\bar{w} = \frac{f\,v^2\,L}{2\,g_c\,d}.$$

We then have:

$$\int_1^2 \bar{V}dp + \int_1^2 \frac{f^1 v^2\,L}{2\,g_c\,d}\,dL = 0. \quad (2.2112)$$

This equation can be used as the starting point for the derivation of horizontal flow equations for gases.

2.271 Horizontal gas flow

One of the first equations for horizontal gas flow was that of Weymouth, which can be derived by integrating Eq. 2.2112 to obtain:

$$q_g = 3.22\,\frac{T_{sc}}{p_{sc}}\left[\frac{(p_1^2 - p_2^2)\,d^5}{\gamma_g\,TL\,z_a\,f}\right]^{0.5}. \quad (2.2113)$$

This equation can be further reduced by assuming that:

$$f = \frac{0.032}{d^{1/3}},$$

which is a form of the Blasius equation,[44] to obtain:

$$q_g = 18.062\,\frac{T_{sc}}{p_{sc}}\left[\frac{(p_1^2 - p_2^2)\,d^{16/3}}{\gamma_g\,TL\,z_a}\right]^{0.5}. \quad (2.2114)$$

Another development assumes that f varies according to a relationship given by:

$$\frac{1}{f} = 52\left(\frac{\gamma_g\,q}{d}\right)^{0.1461}.$$

Solving for f and substituting in Eq. 2.2113 we obtain the expression for flow rate known as the Panhandle formula:

$$q_g = 435.87\,E\left(\frac{T_{sc}}{p_{sc}}\right)^{1.07881}\left(\frac{p_1^2 - p_2^2}{L}\right)^{0.5394}\left(\frac{1}{T}\right)^{0.5394}\left(\frac{1}{\gamma_g}\right)^{0.4606} d^{2.6182}, \quad (2.2115)$$

where E = efficiency factor, and z_a = gas compressibility factor at p and T.

Experience has shown that a value of 0.92 for E is suitable in most practical applications of the equation.

There are numerous other formulas for handling single phase gas flow. One of these is an equation by Clinedinst which includes a more rigorous consideration of compressibility[45] and is given by:

$$q_g = \frac{z_{SL}\,T_{sc}\,p_c}{p_{sc}}\left[\frac{d^5}{\gamma_g TLf}\left(\int_0^{Pr.1}\frac{P_r}{z}\,d\,P_r - \int_0^{Pr.2}\frac{P_r}{z}\,dP_r\right)\right]^{0.5} \quad (2.2116)$$

Values of the integral functions $\int_0^{Pr}\frac{P_r}{z}\,dP_r$ are readily available.[20]

2.272 Vertical gas flow

There have been numerous publications in the area of vertical single phase gas flow. Again, we may start with the general energy equation. By assuming the acceleration term to be negligible, the flow to be steady and isothermal, and that no work is done by the gas in flow, we can settle upon a suitable equation.

Smith,[46] Poettmann,[47] Cullender and Brinkley,[48] and others have offered equations for these calculations. The multiphase flow correlation of Hagedorn and Brown[2] can be used for calculating the pressure loss in a vertical flowing gas well. This correlation and its development are presented in Sec. 2.33 on multiphase vertical flow. This development lends itself to either multiphase flow problems or to dry gas wells. Also, it can be used for those borderline wells between a dry gas well and a definite multiphase flow well.

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2.3. VERTICAL FLOW

2.31 Introduction

Vertical multiphase flow is found in practically every tubing string used in the production of oil. It is necessary to be able to predict a vertical multiphase flow pressure traverse in order to correctly select completion strings, predict flow rates, and design artificial lift installations. Most of the progress towards a solution of the problem has been made since the publication of Poettmann and Carpenter's paper in 1952.[1] Most of the approaches use some form of the general energy equation.

The pressure gradient (or rate of change in pressure with respect to unit of flow length) for vertical multiphase flow is the sum of three contributing factors: hydrostatic pressure gradient, friction pressure gradient, and acceleration pressure gradient. The effects of chemical reactions between phases are neglected; however, such factors as viscosity, surface tension, density, etc. are included.

This section will introduce the vertical multiphase flow problem, offer a brief historical review, and discuss the best correlations available. It will also offer recommendations and conclusions in making use of these correlations.

2.32 Historical development of vertical multiphase flow

The first experimental work of practical significance in multiphase vertical flow was that of Davis and Weidner.[2] This work along with many other early contributions, as well as those that are considered the best correlations today, are shown in Table 2.31.

2.33 Development and utilization of the best correlations in predicting pressure loss

2.331 Introduction

Table 2.31 presents those correlations that have contributed either significantly or slightly to the vertical multiphase flow problem. The most important correlations are those of Duns and Ros,[14] Orkiszewski,[22] Hagedorn and Brown,[21] Beggs and Brill,[25] Govier and Aziz.[23] Most of these will be developed in complete detail.

Others that have contributed to a lesser extent are those of Poettmann and Carpenter,[1] Gilbert,[9] Fancher and Brown,[18] Cornish[27] and some earlier work by Hagedorn and Brown[20] on viscous effects. These methods will be discussed under limited correlations with the exception of the work by Gilbert.[9] His main contribution was a method of preparing working curves for the practicing engineer.

In this section, methods of calculating pressure loss

TABLE 2.31
HISTORICAL DEVELOPMENT

Reference no.	Date	Author(s)	Type of work	Pipe sizes	Fluids	Comments
2	1914	Davis & Weidner	Laboratory experimental	1¼"	Air-water	Holdup and friction had to be separated. Obtained minimum pressure traverse by injecting gas. Showed method of air entry of no consequence. Showed pipe roughness to be a factor.
3	1932	Versluys	Mathematical analysis			No practical value.
4	1931	Versluys	Theoretical			Discussed flow patterns. No practical value.
5	1929	Donoghue	Field experimental	5", 3", 2½", 2", 1½"	Oil	Showed that a minimum velocity of 5 ft/sec was necessary to keep a well flowing.
6	1947	Shaw	Laboratory experimental	1", 1½", 2", 2½"	Air-water	Showed effect of diameter, length of pipe, and submergence on flow rates and gas requirements.
1	1952	Poettmann & Carpenter	Semi-empirical method using field data	2", 2½", 3"	Oil, water, gas	Developed practical solution for 2", 2½", and 3" tubing. For G/L ratios less than 1,500 scf/bbl and for rates > 420 bpd.
7	1962	Winkler & Smith	Practical	1" through 3½"	Oil, water, gas	Prepared working curves from Poettmann and Carpenter correlation.
8	1960	U.S. Industries	Practical	1" through 4½"	Oil, water, gas	Prepared working curves from Poettmann and Carpenter correlation.
9	1954	Gilbert	Field data for practical use	2", 2½", 3"	Oil, water, gas	Presented set of vertical multiphase flow traverses.
10	1958	Govier & Short	Laboratory experimental	Small pipe	Air-water	Presented a correlation for calculating pressure losses but has not been extended to practical use.
11	1961	Tek	Semi-empirical	2", 2½", 3"	Oil, water, gas	Used Poettmann and Carpenter data to prepare another correlation. Has not been used in practice.
12	1961	Baxendall	Field data by Poettmann & Carpenter method	2½", 3½"	Oil, gas	Used Lake Maracaibo field data and prepared correlation similar to Poettmann & Carpenter (good correlation for that area).
13	1961	Ros	Laboratory experimental plus field data	All	All	Good correlation for all ranges of flow.
14	1961	Duns & Ros	Laboratory experimental plus field data	All	All	Good correlation for all ranges of flow. Easier to understand than original work of Ros.
15	1961	Griffith & Wallis	Laboratory experimental	Small	Air, water	Good results in slug flow region. Used by other investigators to improve their correlations.
16	1962	Griffith	Laboratory experimental	Small	Air, water	Used to improve other correlation in plug flow regime.
17	1961	Hughmark & Pressburg	Laboratory experimental	Small	Air, water	Presented holdup correlation used by Dukler in horizontal flow.
18	1963	Fancher & Brown	Field experimental	2"	Gas, water	Collected data to extend correlation of Poettmann and Carpenter to accurately predict pressure losses at low flow rates and high G/L's.
19	1963	Gaither, Winkler & Kirkpatrick	Field experimental (1,000' pipe)	1", 1¼"	Gas, water	Developed correlation for pipe sizes used in test. Has not been extended to practical field use.
20	1963	Hagedorn & Brown	Field experimental (1,500' tube)	1¼"	Air, crude	Developed correlation specifically to handle viscosity effects in 1¼" tubing.

TABLE 2.31 *Cont.*
HISTORICAL DEVELOPMENT

Reference no.	Date	Author(s)	Type of work	Pipe sizes	Fluids	Comments
21	1965	Hagedorn & Brown	Field experimental	1" through 4"	Oil, water, gas	Developed generalized correlation to handle all ranges of multiphase flow.
22	1967	Orkiszewski	Review of all methods plus own correlation	All	Oil, water, gas	Utilized work by Ros, and Griffith & Wallis to prepare own general correlation to predict pressure losses for all ranges of flow.
23	1972	Aziz & Govier	Laboratory & field data	All	All	Presented correlations developed mechanistically, tested against field data.
24	1972	Sanchez	Field data	Annular Flow	All	Checked present day correlations for ability to handle annular flow.
25	1973	Beggs & Brill	Laboratory	1", 1½"	Air, water	Developed generalized correlation to handle all ranges of multiphase flow and for any pipe angle.
26	1973	Chierici, Civcci & Scrocchi		All		Presented modification of Orkiszewski method for slug flow pattern, tested with field data.
27	1973	Cornish	Field data	Annular Flow	Oil & gas	Field correlation for very high flow rate wells in one area.

using the best correlations are presented. Example problems are worked using each method and comparisons are made. The correlations of Duns and Ros, Orkiszewski, Hagedorn and Brown, and Beggs and Brill are general and may be used for all pipe sizes and for any fluid. Other correlations are limited to only one pipe size, and some are best for particular fluid properties such as liquid viscosity.

The following example problem is worked by all methods.

Given data:

$d = 1.995$ in. or 1.380 in.
$p_1 = 500$ psig
$p_2 = 1{,}000$ psig
$p_a = 14.7$ psia
$T_1 = 120°F$
$T_2 = 150°F$
$\gamma_g = 0.65$
$\gamma_w = 1.07$
$\rho_o = 22°API$
$q_o = 400$ stk bbl/day
$q_w = 600$ stk blb/day
$\mu_g = 0.018$ cp (assume constant)
$\sigma_o = 30$ dynes/cm (assume constant)
$\sigma_w = 70$ dynes/cm (assume constant)
Gas-liquid ratio = 500 scf/stk bbl.

This problem is to be solved by determining the distance between the 500 and 1,000 psig pressure points. Both tubing sizes are used in most of the methods. Although in practice we would take the pressure increments much closer than 500 psi, it was felt that a wide separation of the pressures for an example problem would eliminate confusion. The recommended pressure increments are given for each method.

Calculation procedures and example problems are found in the Appendix. No actual field data was available for this example. The purpose of this example is to show the method of calculation only. The decision as to the best methods was based on hundreds of actual field wells. The following discussion will assist in a general understanding of what is needed to calculate a pressure traverse.

In Fig. 2.33, a portion of the pipe is occupied by gas and the rest by liquid. That portion of the pipe occupied by liquid is called "hold-up," and the symbol H_L is used. In order to correctly determine the "static gradient" it is necessary to know H_L. Some investigators, including Poettmann and Carpenter chose to use a no-slip H_L. Although this is not correct it does not prevent the correct calculation of $\frac{\Delta P}{\Delta H_{total}}$ since the slippage is included in $\frac{\Delta P}{\Delta H_{friction}}$

Other investigators such as Hagedorn and Brown[21] and Ros[13] have prepared hold-up correlations to determine H_L (Fig. 2.34).

As noted, many variables enter into the determination of hold-up. Once the hold-up has been obtained the density of the mixture of gas and liquid can be calculated by the following equation:

$$\rho_m = \rho_L (H_L) + \rho_g (1 - H_L) \qquad (2.31)$$

where:

ρ_m = density of mixture, lb/ft³
ρ_L = density of liquid, lb/ft³
ρ_g = density of gas, lb/ft³
H_L = hold-up (that part of pipe occupied by liquid)
$1 - H_L$ = that part of pipe occupied by gas

If the units of ρ_m are lb_m/ft^3 the static gradient, in psi/ft, can be determined by dividing ρ_m by 144 in^2/ft^2. The $\frac{\Delta p}{\Delta h_{friction}}$ is normally determined from

VERTICAL MULTIPHASE FLOW

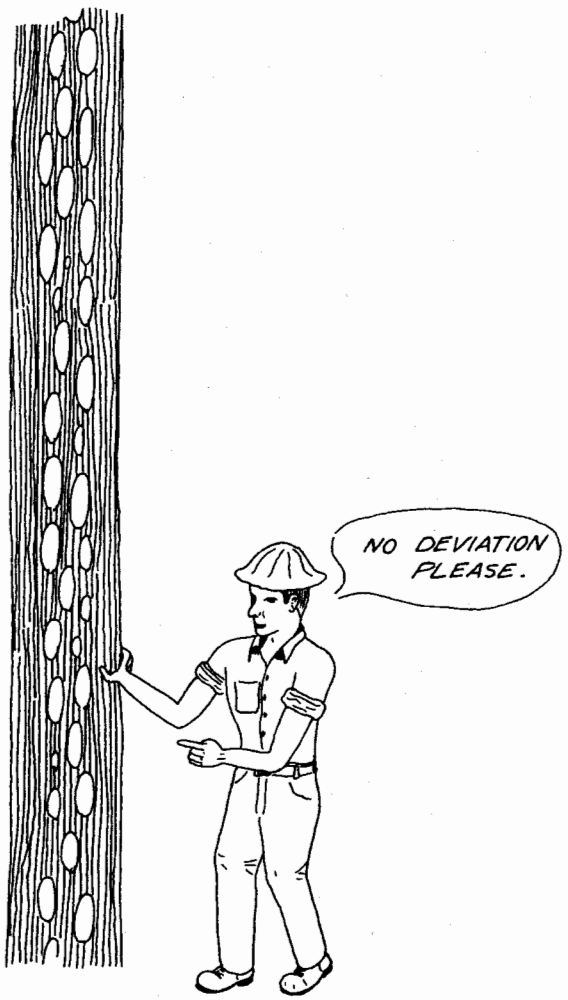
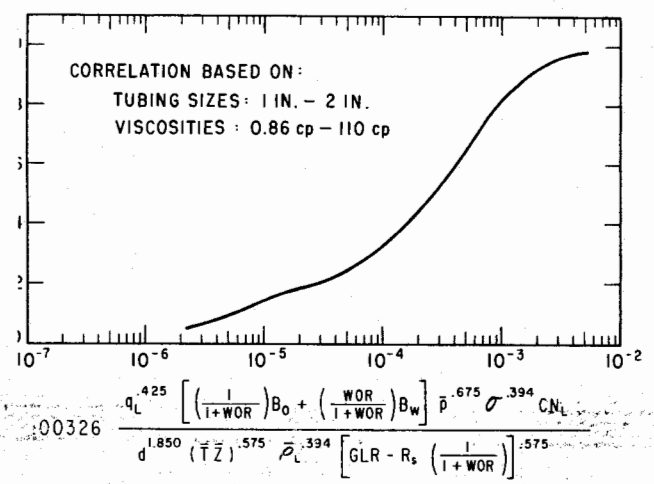


Fig. 2.33 Vertical multiphase flow.

TOTAL GRADIENTS COMPOSED OF:

STATIC GRADIENT
FRICTION GRADIENT
ACCELERATION GRADIENT



2.34 Holdup-factor correlation (after Hagedorn).

some form of the Fanning or Darcy-Weisbach equation and will take the form of:

$$\frac{f\,q_m^2}{K\,\rho_m}$$

where:

f = two-phase energy loss factor
q_m = mass flow rate
ρ_m = density of mixture
K = constant including diameter, etc.

"f" is determined from a plot similar to Fig. 2.35 as used by Hagedorn and Brown.[13]

The $\frac{\Delta p}{\Delta h}_{acceleration}$ can be neglected in most cases but offers no problem in computer calculations. Therefore it is best to include the term rather than worry about those ranges of flow where it is a significant contribution to the total $\frac{\Delta p}{\Delta h}$. For low pressure and hence low

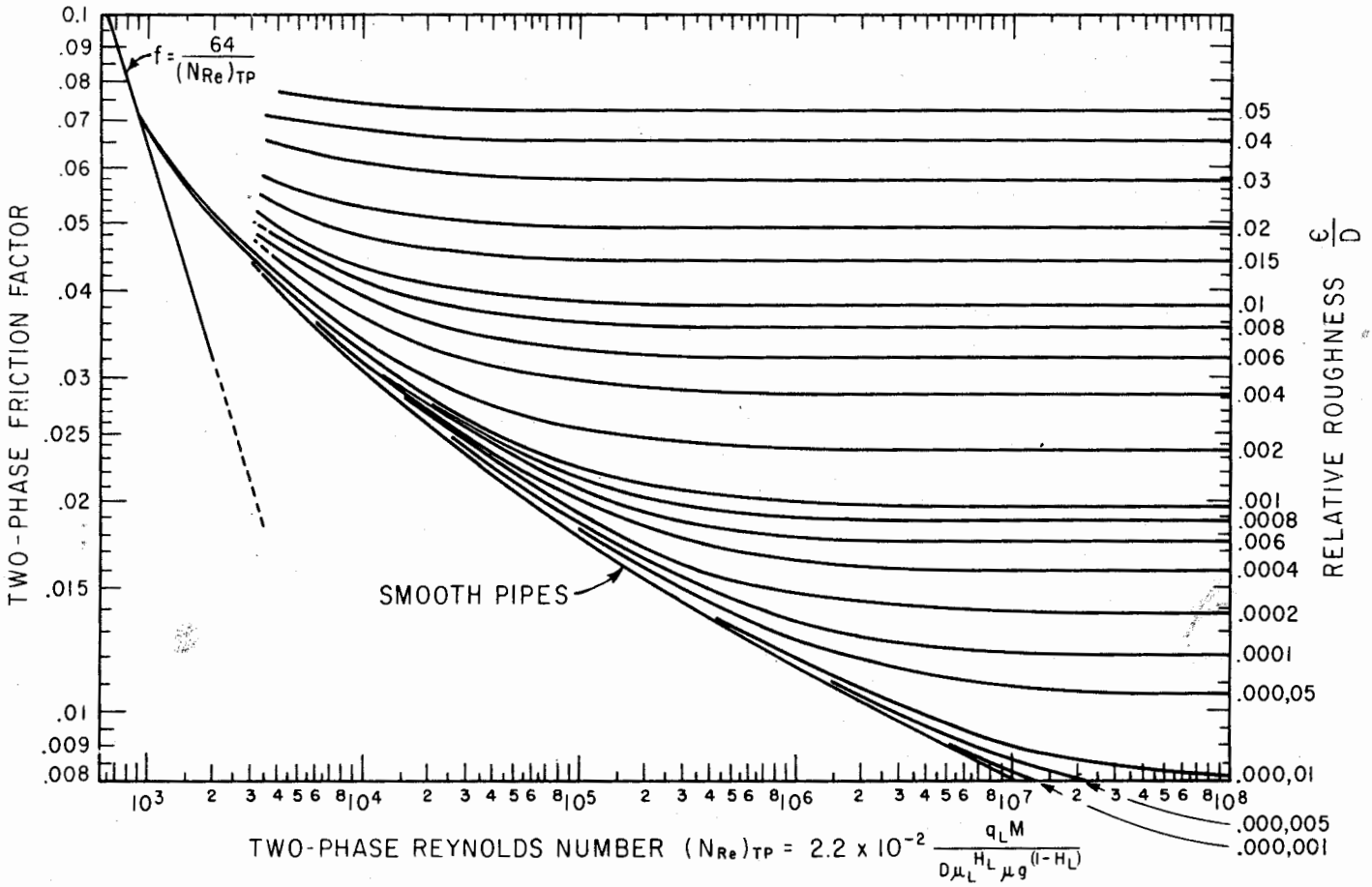


Fig. 2.35 Friction factor correlation (after Hagedorn).

density, the velocity increases rapidly and the acceleration term can become significant.

All correlations require that some type of friction factor or energy loss factor be determined to calculate $\frac{\Delta p}{\Delta h_{friction}}$. These vary, but in general some form of Fanning's, Moody's or Darcy-Weisbach's correlations have been used—perhaps modified slightly for multiphase flow.

Procedure to calculate pressure traverse for vertical multiphase flow is:

(1) From the general equation:

$$\frac{dp}{dh_{total}} = \frac{dp}{dh_{elevation}} + \frac{dp}{dh_{friction}} + \frac{dp}{dh_{acceleration}} \qquad (2.187)$$

(2) Determine $\frac{dp}{dh_{elevation}} = \bar{\rho}_m$

From the appropriate correlation find the hold-up (H_L) unless a no-slip hold-up is used:

(a) May require flow pattern determination
(b) May be independent of flow pattern

Calculate $\bar{\rho}_m = H_L\rho_L + (1 - H_L)\rho_g$

(3) Calculate $\frac{dp}{dh_{friction}} = \frac{f_m\rho_m v_m^2}{2g_c d}$

From the appropriate correlation find "f_m":

(a) May require flow pattern
(b) May be independent of flow pattern

(4) Calculate $\frac{dp}{dh_{acceleration}} = \frac{\rho_m\Delta(v_m^2)}{2g_c dh}$

Acceleration may be handled in a different manner depending upon the investigator.

(5) Solve for $\frac{dp}{dh_{total}}$

$$= \frac{dp}{dh_{elevation}} + \frac{dp}{dh_{friction}} + \frac{dp}{dh_{acceleration}} \qquad (2.187)$$

$$= \frac{g}{g_c}\rho_m + \frac{f_m\rho_m v_m^2}{2g_c d} + \frac{\rho_m\Delta(v_m^2)}{2g_c dh} \qquad (2.33)$$

or some form of this equation depending upon the investigator.

2.332 Limited correlations

2.3321 Introduction

In this section we will discuss some of the limited correlations including those of Poettmann and Carpenter, Fancher and Brown, and the initial work of Hagedorn and Brown. The Poettmann and Carpenter correlation will be discussed in more detail because of its impact on industry, and the development of their work will help in understanding the later work of others.

Later investigators used a similar approach but

ended the work of Poettmann and Carpenter to compass ranges of flow and pipe sizes outside of ose used in the original data. Their work has been e basis for at least two known publications of work- g curves, one by Winkler and Smith[7] and the other U.S. Industries.[8]

322 Poettmann and Carpenter method

Poettmann and Carpenter[1] developed a semi- ipirical method which incorporated the general ergy equation and data from 34 flowing oil wells d 15 gas-lift wells with tubing diameters of 2, 2½, d 3 in. The oil, gas, and water were considered to be single phase, and no attempt was made to determine holdup correlation. All energy losses, including ppage and liquid fallback, were combined into a nning-type "energy loss factor," which was assumed be constant over the entire length of the tube. It as assumed that the multiphase vertical flow of oil, ater, and gas was entirely turbulent. The energy ss term was correlated with the numerator of the ynolds number (Fig. 2.36).

Several points about the method and the assump- ons used should be noted:

(1) The correlation is applicable to the pipe sizes ed in the field study (2, 2½ and 3 in.) and extension the correlation to other sizes should be done with ution.

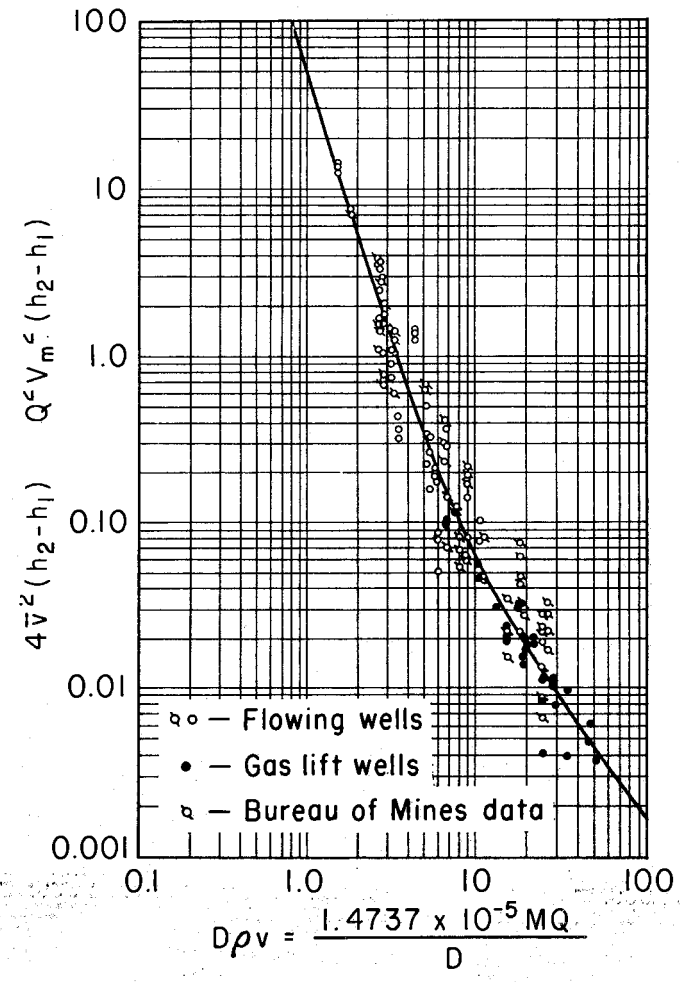


Fig. 2.36 Correlation of field data on flowing and gas lift wells.

(2) The total output flow rate was used to calculate the density at any one pressure point.

(3) Flow patterns were ignored.

(4) The effects of viscosity were assumed to be negligible, although work by Ros and Hagedorn and Brown have shown that viscous effects for crude oils with viscosities greater than 6 cp (or 10 cp) are very significant.

(5) The acceleration term of the general energy equation was considered to be negligible. This will be true for most cases but the term is significant at very high velocities, and was included in the friction factor.

(6) Although the energy loss factor (or friction loss factor) is assumed to be an average value over the length of vertical tube, it was actually found to vary from the bottom to the top of the tubing string by other investigators.[13]

Poettmann and Carpenter started with the general energy equation:

$$\int_{p_1}^{p_2} \bar{V}\, dp + \Delta h + \Delta\left(\frac{v^2}{2g_c}\right) + W + lw = 0, \quad (2.179)$$

where it is understood that Δh is multiplied by $g/g_c = 1.00\ lb_f/lb_m$. Assuming that the work, W, done by the fluid is zero; that the kinetic energy term is negligible; that the total energy losses can be correlated by Fanning's equation; and that field units are applicable; the final equation is:

$$\frac{dp}{dh} = \frac{1}{144}\left[\rho + \frac{(f)\,(w^2)}{7.413 \times 10^{10}\,\rho d^5}\right]. \quad (2.34)$$

The derivation of this equation, starting with the general energy equation, is found in Appendix C.11.

Appendix C.12 and C.13 include the detailed procedure for calculating a pressure traverse by this method and an example problem.

The relationship between f (energy loss term) and the numerator of the Reynolds Number for the experimental data is shown in Fig. 2.36. A value of f is selected for substitution in Eq. 2.34 to obtain a value of dp/dh.

Solution of the problem of calculating the distance between the 1,000 and 500 psig pressure points appears in Appendix C.13. The distance is calculated to be 2,425 ft for the 2⅜ O.D. tubing (2 in, nominal) and 1,650 ft for the 1¼ in. nominal tubing. The distance, based on density and assuming no frictional loss, is 2,950 ft and this value is the same for both tubing sizes. This means that the Poettmann and Carpenter method gives a loss in 2 in. tubing of 2,950–2,425 or 525 ft because of friction and slippage.

The Poettmann and Carpenter method should give more accurate results in the low density region if the densities are first averaged and then the pressure gradient obtained from Fig. 2.37. Similar figures are available for other tubing sizes.

For the Poettmann and Carpenter method it is recommended that pressure increments of 10 psi be used between 0 and 100 psig; 25 psi between 100 and 500 psig; 100 psi between 500 and 1,000 psig; and 200 psi between 1,000 and 5,000 psig.

The curves developed by Winkler and Smith and by U.S. Industries are a result of modifications of the

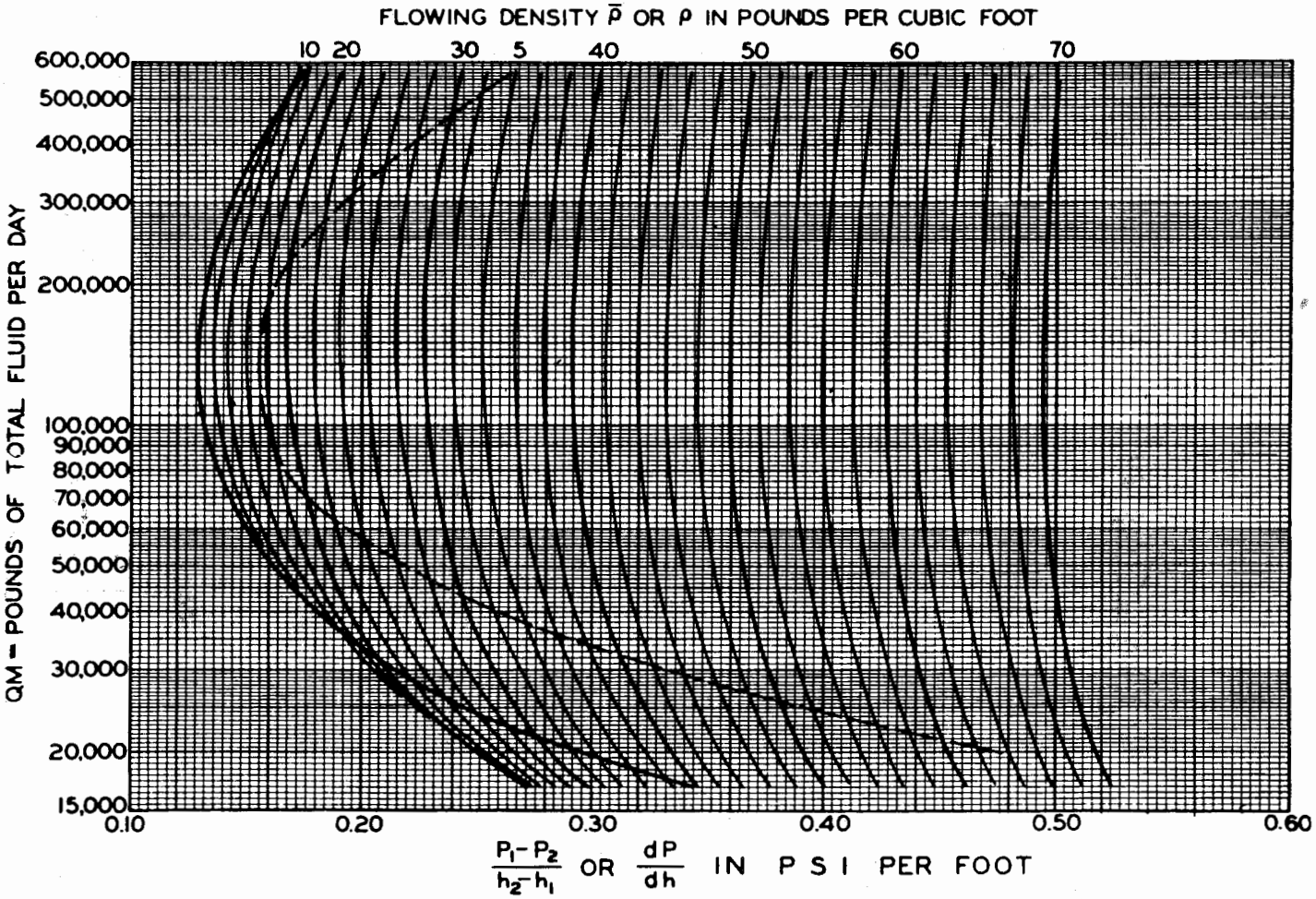


Fig. 2.37 Flowing density correlation (after Poettmann, courtesy API).

Poettmann and Carpenter correlation wherein the so-called "turnover," or reversal of curvature, has been eliminated. Since these curves are used by many practicing engineers, an explanation of this modification is given here.

This turnover can be noted in Fig. 2.37. The density curve representing 5 lb_m/cu ft will give a larger gradient value than does the 10 lb_m/cu ft curve for the same value of QM (pounds of total fluid per day). To remove this reversal, a calculation is started from 0 psig and small pressure increments (10–25 psi) are taken. The pressure gradients are calculated and corresponding depths are tabulated. In the low-density region the pressure gradients may be high, but they will eventually start to decrease.

For example, they may be 0.30, 0.25, 0.20, 0.15, and 0.10 psi/ft. This is in the turnover region. The gradients, after reaching a minimum of 0.10 psi/ft, may then start to increase and give values such as 0.12, 0.15, 0.18, and 0.20 psi/ft. It is then assumed that the gradient of 0.20 psi/ft remains constant over the interval from the pressure point at which it occurs to the surface. For example, if the minimum was attained a 150 psig, then the corresponding distance would be $\frac{150}{0.10} =$ 1,500 ft, and it would be assumed that a gradient of 0.10 psi/ft was constant from the surface to 1,500 ft. This kind of modification gives more accurate results than the original Poettmann and Carpenter method. A comparison between including and eliminating this reversal is shown in Figs. 2.38 and 2.39.

The Poettmann and Carpenter correlation is still used quite extensively in the field. It can be used with confidence for the following conditions:

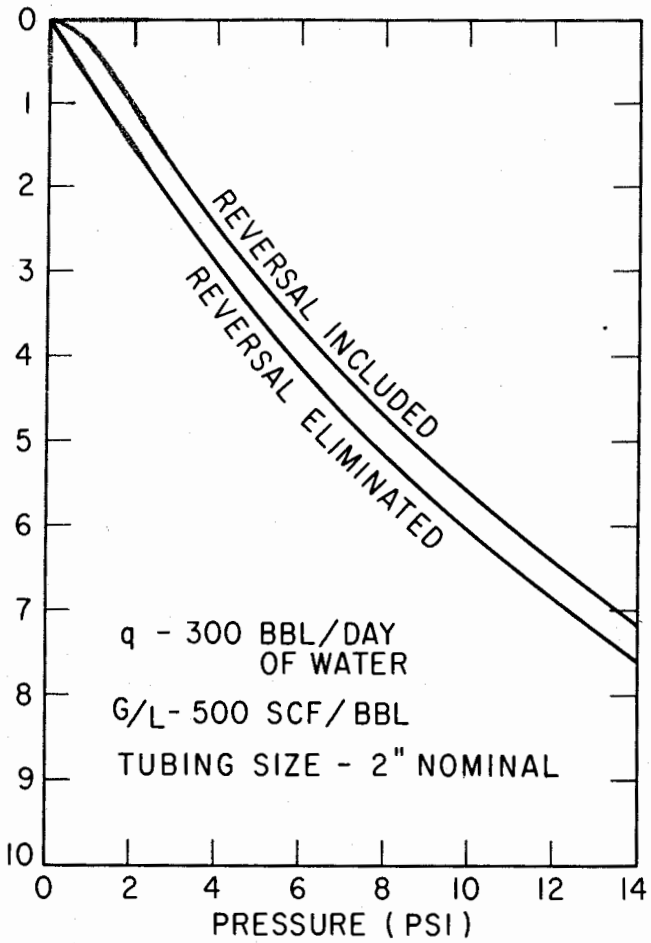
(1) Tubing sizes, 2 in., 2½ in., 3 in.
(2) For viscosities less than 5 cp
(3) For G/L's less than 1,500 scf/bbl
(4) For rates greater than 400 bpd.

The following derivations, procedures, and problems concerning the Poettmann and Carpenter method can be found in Appendix C.1.

(1) C.11—Derivation of the Poettmann and Carpenter equation for multiphase vertical flow.
(2) C.12—Detailed procedure for the calculation of a vertical pressure traverse by Poettmann and Carpenter method.
(3) C.13—Example problem worked by Poettmann and Carpenter method.

2.3323 Fancher and Brown method (extension of Poettmann and Carpenter method)

The Fancher and Brown[18] correlation is one of the best for 2 in. nominal tubing only. Field data were taken in 2⅜ in. O.D. (1.995 in. I.D.) plastic-coated tubing in an 8,000 ft test well. This correlation is an adjustment to the Poettmann and Carpenter method and is more accurate for lower flow rates and higher gas-



. 2.38 *Reversal in Poettmann and Carpenter method.*

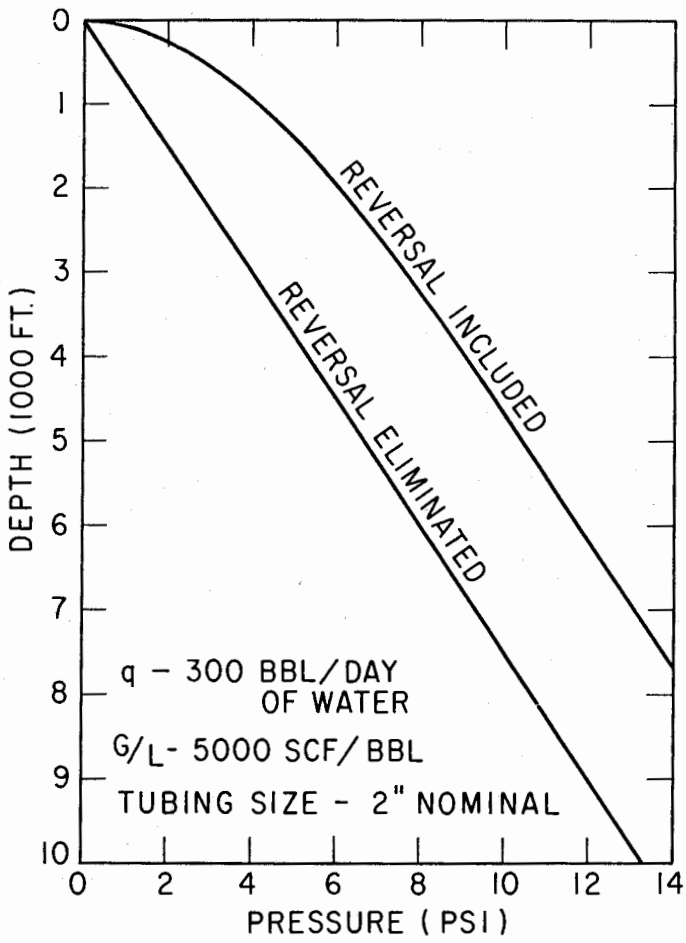


Fig. 2.39 Reversal in Poettmann and Carpenter method.

uid ratios at any flow rate. It may be used with good sults for gas-liquid ratios less than 5,000 scf/bbl and flow rates less than 400 bpd, and it may be extended to $2\frac{7}{8}$ in. tubing with a considerable degree of nfidence. However, it is not recommended for tubing es less than $2\frac{3}{8}$ in. O.D. and greater than $2\frac{7}{8}$ in. D. It does not account for fluid properties such as rface tension and viscosity.

The first attempt to correlate the data used the same rameters employed by Poettmann and Carpenter d, as shown in Fig. 2.40, it was found that an additional correlating parameter was needed to establish useful relationship. Fig. 2.41 shows the final correlation after including gas-liquid ratio as a parameter. ie friction factors for gas-liquid ratios other than own in Fig. 2.41 may be determined by interpolating tween the curves or extrapolating outside the 1,500 3,000 range.

Fancher and Brown[18] recorded pressures during the ading of a well. Fig. 2.42 shows this heading phenomenon. The dashed lines indicate the maximum and inimum pressures recorded. The solid line is the corded field data. A difference in pressure of as much 200 psi will be noted, and this is classified as severe ading. This phenomenon is discussed further in c. 2.35.

Fancher and Brown also recorded the reversal in rvature shown by Poettmann and Carpenter; however, it was not nearly as pronounced. Figs. 2.43 and 44 show the reversal recorded by Fancher and Brown and that calculated by the Poettmann and Carpenter method. This reversal is expected to occur in the low density regions, particularly for small pipe sizes and high velocities, and is more pronounced for the lower liquid rates and higher gas-liquid ratios.

Because this correlation is considered to be an excellent one for 2 in. tubing, a stepwise procedure for its use and a solution of the problem example worked by the Poettmann and Carpenter method are given in Appendices C.21 and C.22. The distance between the 500 and 1,000 psi pressure points was 2,350 ft in 2 in. tubing compared with 2,425 ft by the Poettmann and Carpenter method. The distance found by including the viscosity effect was essentially the same (2,350 ft in 2 in. and 1,380 ft in $1\frac{1}{4}$ in.).

The following procedures and example problems solved with the Fancher and Brown method can be found in Appendix C.2.

(1) C.21—Procedure for the calculation of a pressure traverse by Fancher and Brown method.
(2) C.22—Example problem by Fancher and Brown method.

2.3324 Method of Hagedorn and Brown to account for viscous effects ($1\frac{1}{4}$ in. tubing) (extension of Poettmann and Carpenter method)

Although the generalized method of Hagedorn and Brown is presented in the next section (2.3332), their original work to study viscous effects in $1\frac{1}{4}$ in. tubing

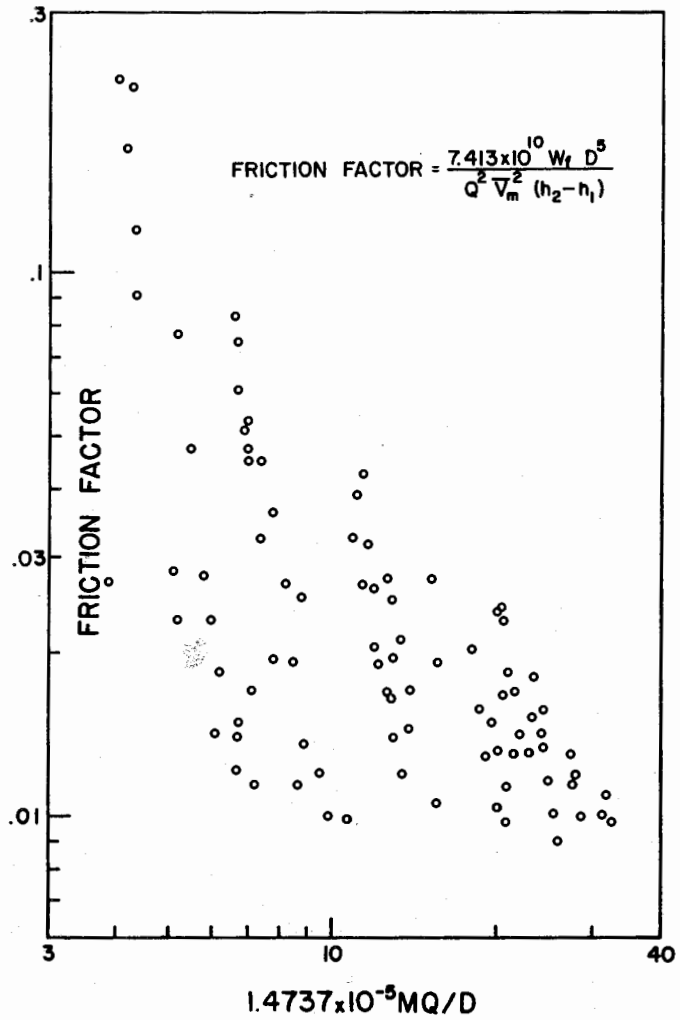


Fig. 2.40 Back-calculated friction factor (after Fancher and Brown, courtesy SPE).

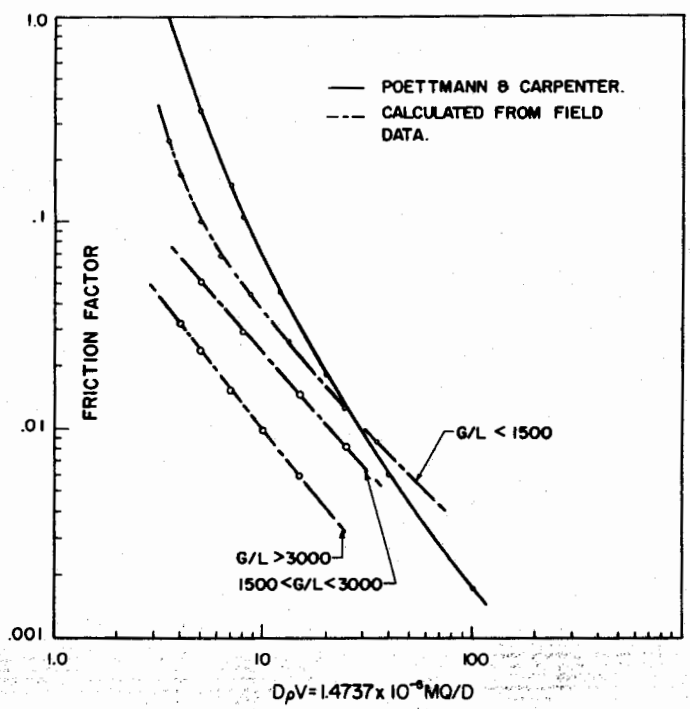


Fig. 2.41 Comparison of friction factor correlations (after Fancher and Brown, courtesy SPE).

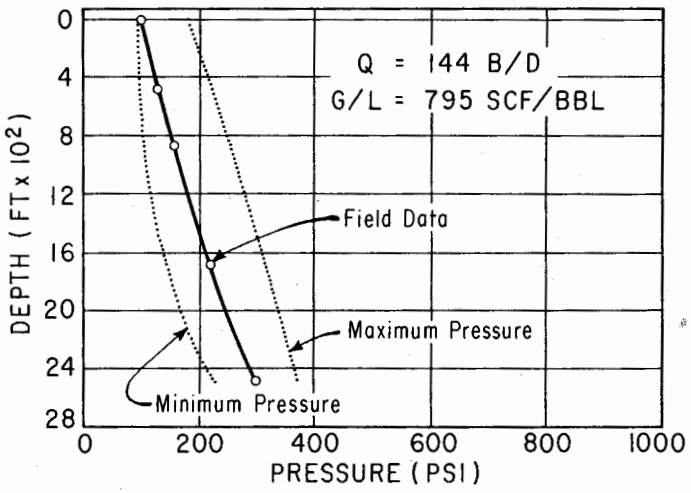


Fig. 2.42 Heading phenomen (after Fancher and Brown, courtesy SPE).

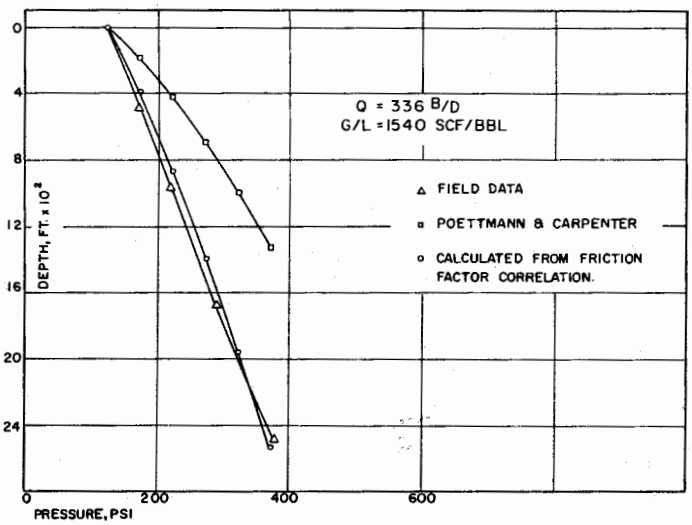


Fig. 2.43 Comparison of pressure traverses.

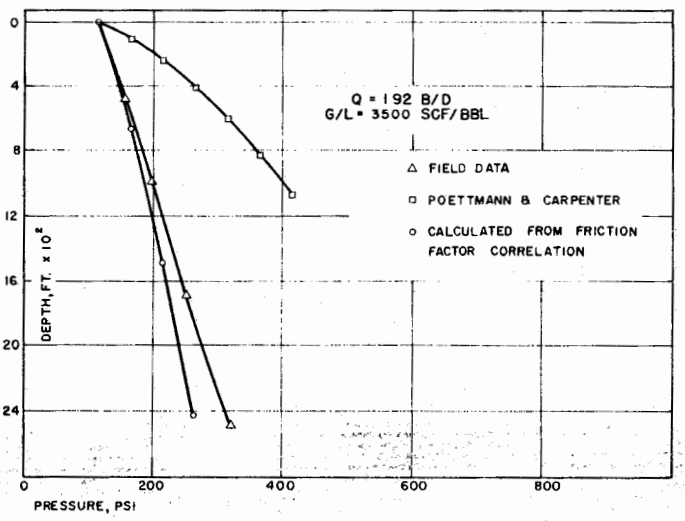


Fig. 2.44 Comparison of pressure traverses.

discussed briefly in this section, and in effect repre-ents an extension of Poettmann and Carpenter's ork.[20] Since viscous effects are extremely important, ·veral figures which show these effects are noted in ıis section.

Four different liquids, each with a different viscosity, ere tested so that viscous effects could be introduced ıto the correlation. Water with a viscosity of 0.86 cp, heavy Corsicana, Tex. crude oil of 110 cp, a mixture ' Corsicana crude oil and Varsol with a viscosity of 5 cp, and a mixture of the Corsicana crude oil and arsol with a viscosity of 10 cp were tested at various quid rates. Figs. 2.45, 2.46, and 2.47 show the effects ' liquid viscosity.

For example (Fig. 2.46), it takes almost 200 psig ıore pressure to produce the 110 cp crude oil than it ɔes for water at a rate of 206 bpd with a gas-liquid ıtio of 194 scf/bbl in 1¼ in. tubing at a depth of 1,000 . Because the oil is less dense than the water, this ·essure difference is believed to be due to liquid vis-ɔsity.

Fig. 2.48 shows a plot of liquid rate versus liquid ıscosity at a constant gas rate of 115 Mcfd in 1¼ in. ıbing. The liquid rates vary from 980 to 220 bpd with decided change in rate occurring between 10 and 30 ɔ. Hagedorn and Brown[20] selected 12 cp as the lower mit of viscosity at which viscous effects became im-ɔrtant. They proceeded to show how the 10 cp crude ıssed through the transition region between laminar nd turbulent flow, depending on its velocity in the ¼ in. tubing. This is shown in Figs. 2.49, 2.50, 2.51, nd 2.52. Fig. 2.53 shows that the G/L was not causing ıe change directly by reducing the viscosity. The 10 ɔ crude oil suffered increased pressure loss at the ɔwer rates because of viscous effects. They concluded,

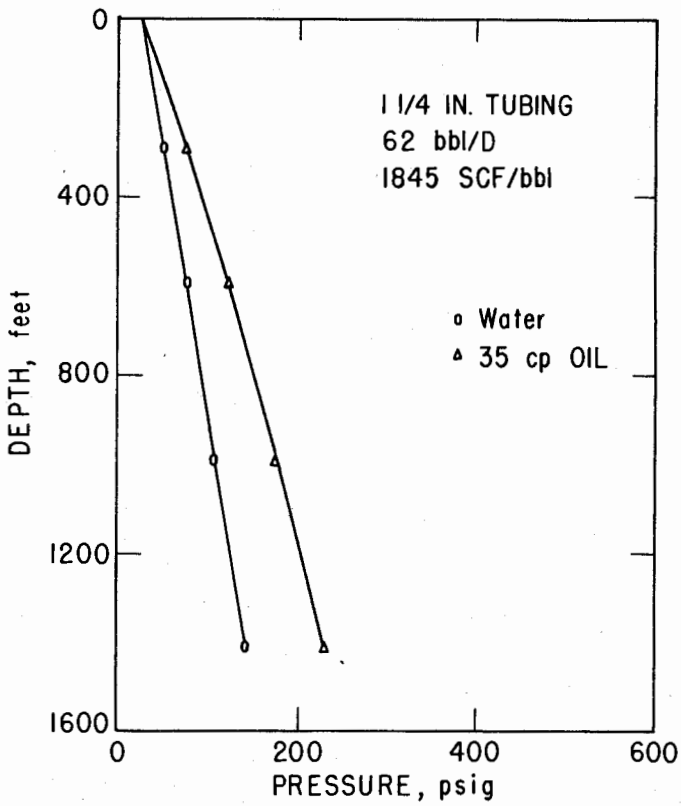
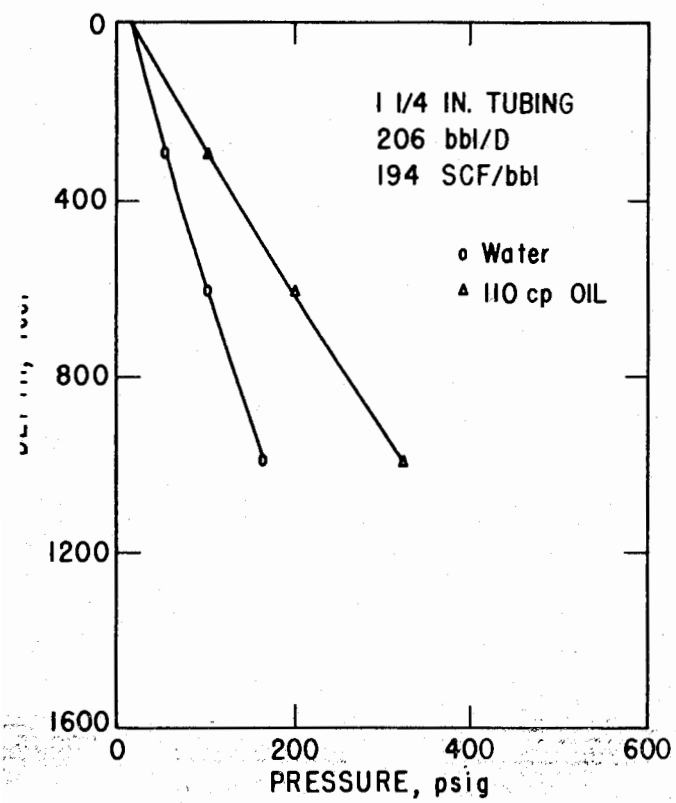


Fig. 2.46 Effect of viscosity.

as did other investigators, that any attempt to determine the viscosity of the gas-liquid mixture was impractical.

Figs. 2.54 and 2.55 show the correlations of Hagedorn and Brown from which friction factors should be se-



ig. 2.45 Effect of viscosity.

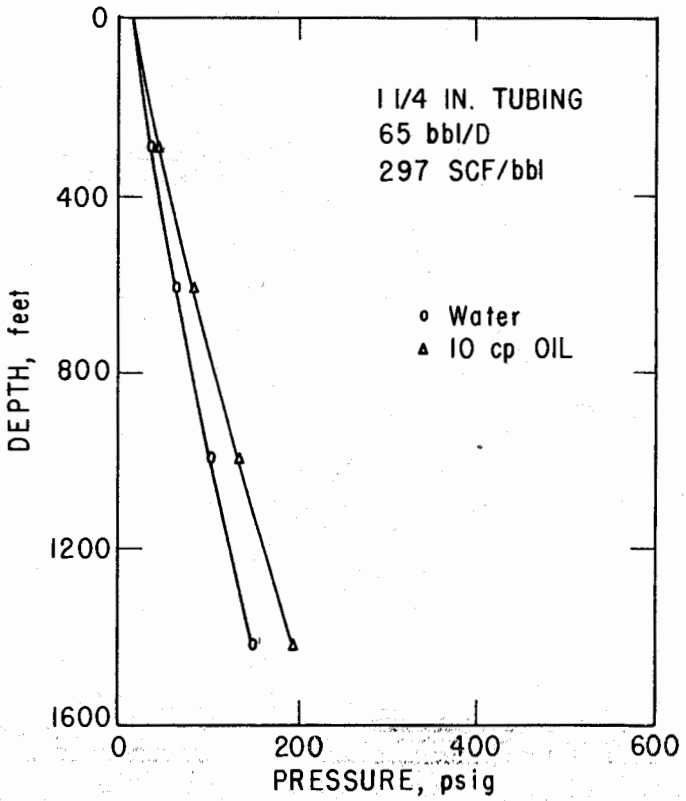


Fig. 2.47 Effect of viscosity.

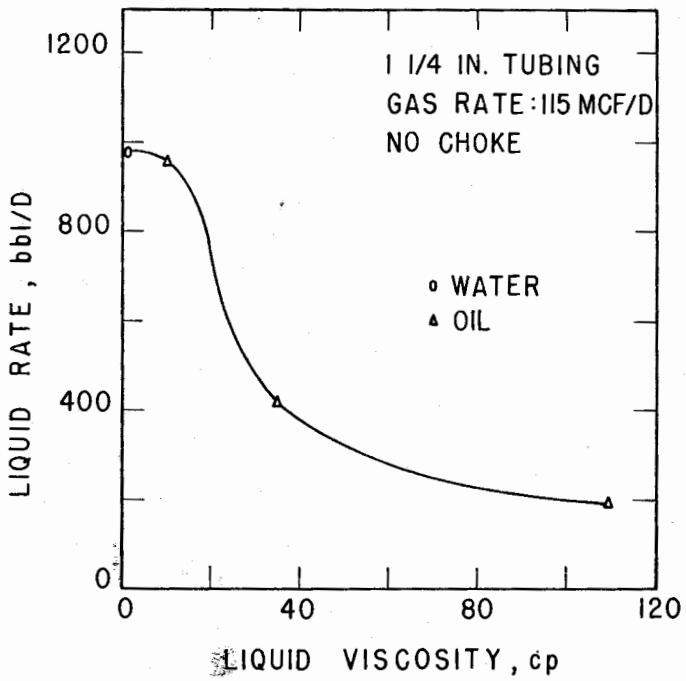


Fig. 2.48 Effect of liquid viscosity on liquid rate (after Hagedorn and Brown, courtesy SPE).

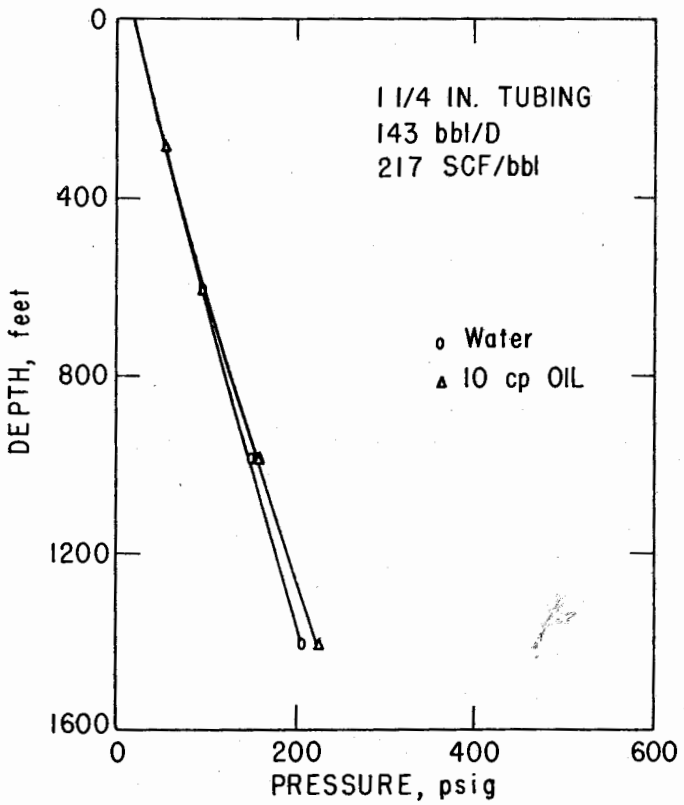


Fig. 2.50 Effect of viscosity.

lected. By trial and error, it was found that a value of 0.25 for the exponent of liquid viscosity resulted in the best correlation for viscosities greater than 12 cp. It is interesting to note that this value of 0.25 is the same exponent which appears in the Blasius formula used to determine friction factors for single phase turbulent flow through a tube. The gas-liquid ratio was selected as a parameter in the friction-factor correlations. Some of the significance of this parameter is lost since the in-situ gas-liquid ratio was not used. It is also emphasized that this correlation should be used only for 1¼ in. tubing.

Appendix C.3 contains a detailed procedure, along

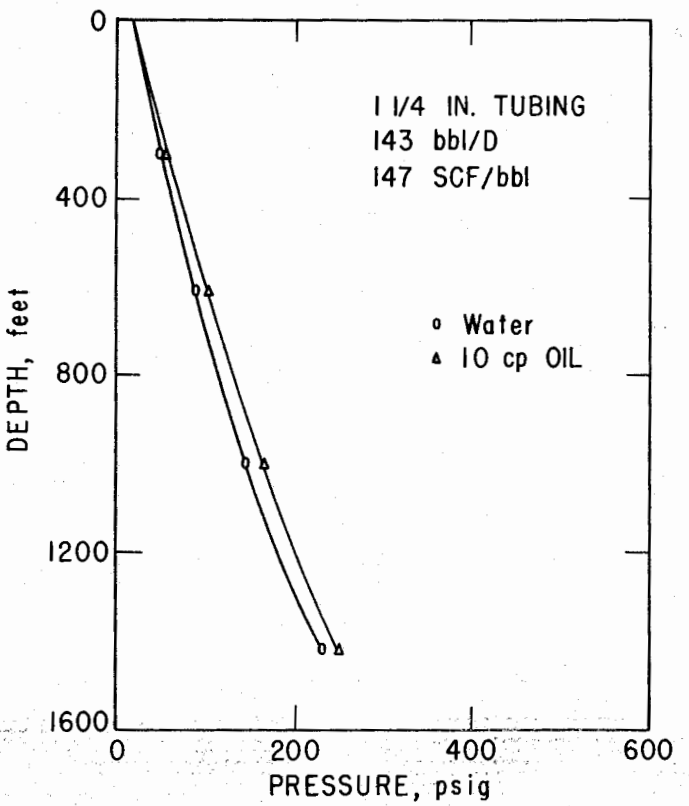


Fig. 2.49 Effect of viscosity.

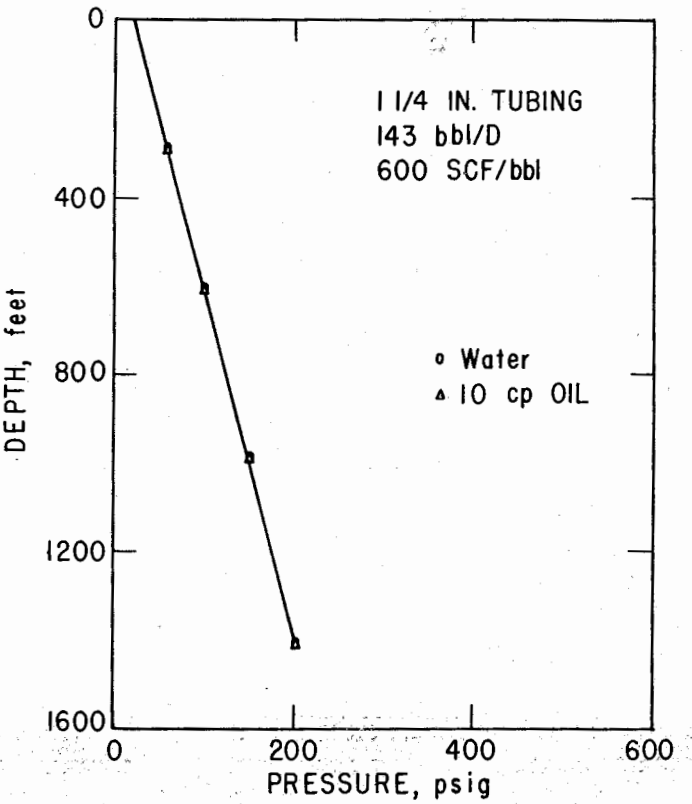


Fig. 2.51 Effect of viscosity.

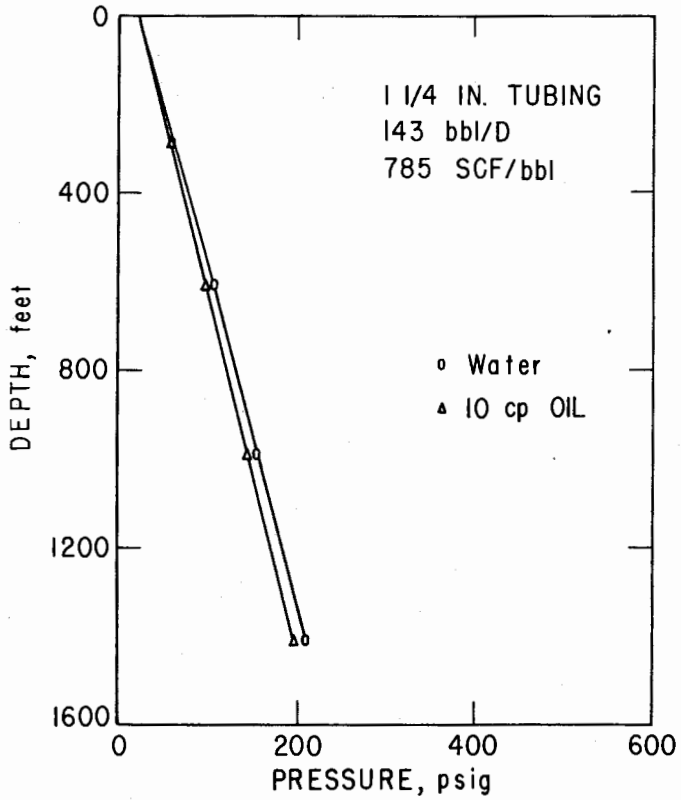


Fig. 2.52 Effect of viscosity.

with an example problem, which shows the applicability of the method. Distance between the 500 to 1,000 psig pressure points was determined to be 1,335 ft, compared to 1,650 ft, and 1,465 ft, determined by the Poettmann and Carpenter and Fancher and Brown methods respectively.

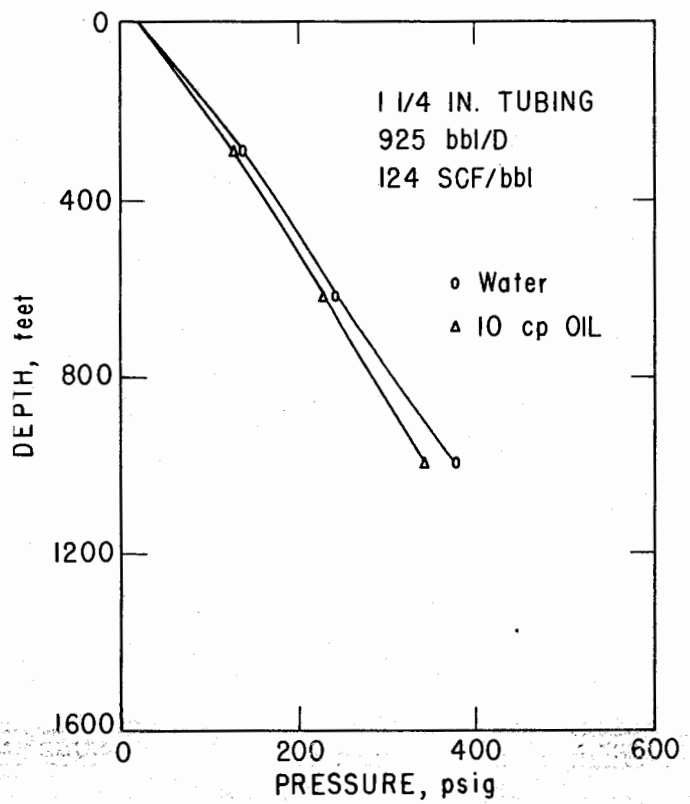


Fig. 2.53 Effect of viscosity.

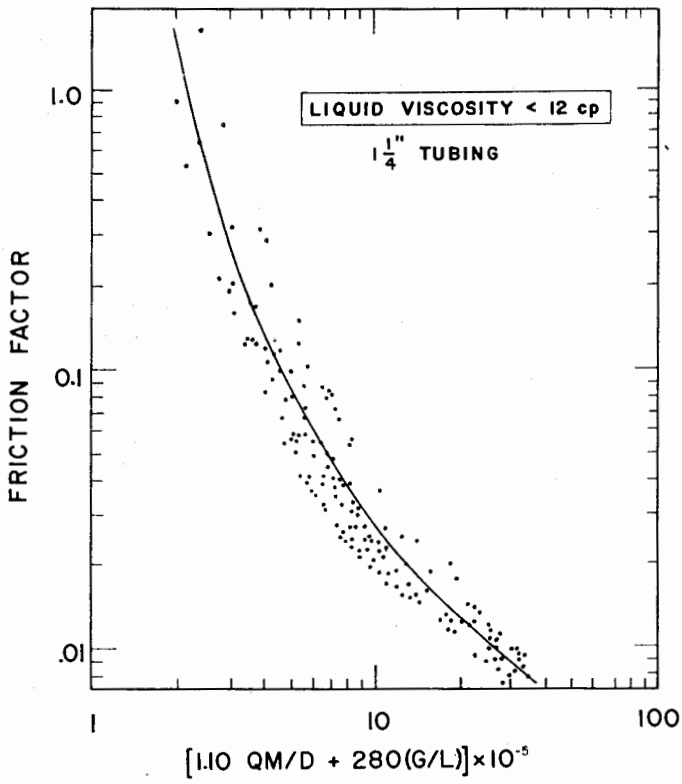


Fig. 2.54 Friction factor correlation.

Application of the relationships established by the Hagedorn and Brown method to account for viscous effects gives excellent results for $1^1/_4$ in. tubing.

The following procedure and example problem using the Hagedorn and Brown Method for $1^1/_4$ in. tubing can be found in Appendix C.3.

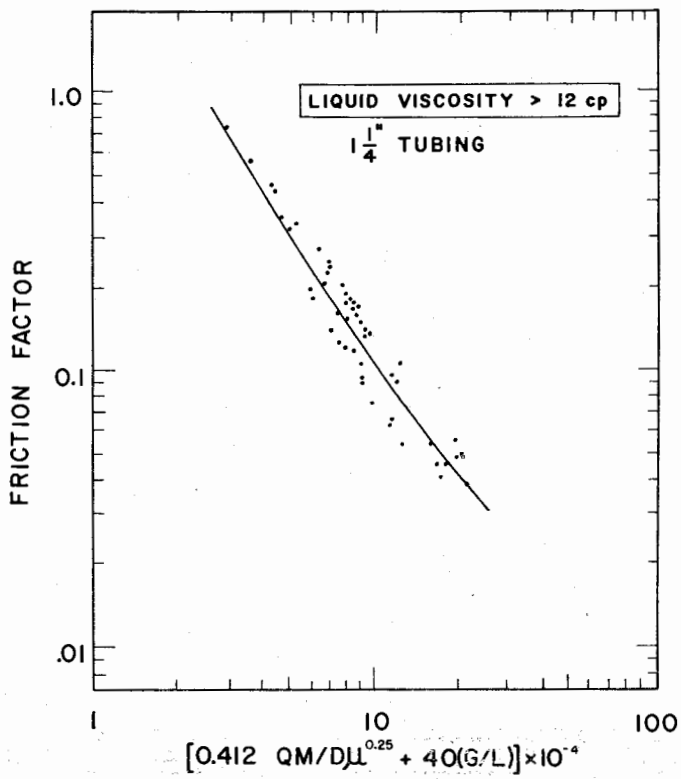


Fig. 2.55 Friction factor correlation (after Hagedorn and Brown, courtesy SPE, JPT).

(1) C.31 – Procedure for the calculation of a pressure traverse by the Hagedorn and Brown method (1¼ in. tubing only).
(2) C.32 – Example problem by Hagedorn and Brown method for 1¼ in. tubing only.

2.333 The best correlations for vertical multiphase flow

2.3331 Introduction

There are numerous correlations that give excellent results depending upon the ranges of flow conditions. Based on comparisons made by Lawson and Brill, the following methods and the order in which they will be discussed are those of Hagedorn and Brown,[21] Duns and Ros,[14] Orkiszewski[22] and Beggs and Brill.[25] All four of these methods represent generalized correlations to take care of all pipe sizes, fluid properties and flow rates. Also of significance is the correlation of Govier and Aziz.[23]

2.3332 Generalized correlation of Hagedorn and Brown

An effort was made by Hagedorn and Brown[21] to determine a generalized correlation which would include all practical ranges of flow rates, a wide range of gas-liquid ratios, all ordinarily used tubing sizes and the effects of fluid properties. Data were taken for pipe sizes ranging from 1 in. nominal to 2½ in. nominal tubing. The study included all of the prior work done by this team of investigators on the effects of liquid viscosity as discussed under "Limited Correlations" in Sec. 2.3324. A kinetic energy term was incorporated in the energy equation because it was considered to be very significant in small diameter pipes in the region near the surface where the fluid has a low density. Two adjustments were found necessary to improve this correlation. The Griffith correlation was used when bubble flow existed and the holdup was checked to make sure that it exceeded the holdup for no slippage and if not, the holdup for no slippage was used. Reference to Fig. 2.60-A shows a generalized flow diagram which should help in the following discussions.

These investigators also started with the general energy equation to obtain the pressure loss equation:

$$144\frac{\Delta p}{\Delta h} = \bar{\rho}_m + \frac{f\,w^2}{2.9652\times 10^{11} d^5\,\rho_m} + \bar{\rho}_m\frac{\Delta\left(\frac{v_m^2}{2g_c}\right)}{\Delta h} \qquad (2.35)$$

where

$$\bar{\rho}_m = \bar{\rho}_L\,H_L + \bar{\rho}_g(1-H_L) \qquad (2.31)$$

The steps used to derive these equations are shown in Appendix C.41.

The mixture viscosity was represented in the manner suggested by Arrhenius[28] and a Reynolds number for the two-phase mixture was defined by the equation:

$$(N_{R_e})_{T.P.} = 2.2\times 10^{-2}\frac{w}{d\mu_L^{H_L}\mu_g^{1-H_L}}. \qquad (2.36)$$

The relationship between this Reynolds number and the friction factor is shown in Fig. 2.35. Referring to Eq. 2.36, if the limit is taken of the Reynolds number for the mixture as $H_L \to 0$, $q_L \to 0$, it reduces to a single-phase gas-flow equation, and if the limit is taken as $H_L \to 1$, $q_g \to 0$, it reduces to the single-phase liquid-flow equation. The respective Reynolds numbers for all gas and all liquid reduce to

$$(N_{R_e})_g = C_1\frac{v_g\,\rho_g\,d}{\mu_g} \qquad (2.37)$$

and:

$$(N_{R_e})_L = C_1\frac{v_L\,\rho_L\,d}{\mu_L} \qquad (2.38)$$

Using methods similar to those of Duns and Ros[14] and Ros,[13] Brown and Hagedorn showed that the liquid holdup (H_L) is principally related to four dimensionless parameters:

$$N_{LV} = v_{sL}\left(\frac{\rho_L}{g\sigma}\right)^{1/4} \qquad (2.39)$$

$$N_{gV} = v_{sg}\left(\frac{\rho_L}{g\sigma}\right)^{1/4} \qquad (2.310)$$

$$N_d = d\left(\frac{\rho_L\,g}{\sigma}\right)^{1/2} \qquad (2.311)$$

and:

$$N_L = \mu_L\left(\frac{g}{\rho_L\sigma^3}\right)^{1/4} \qquad (2.312)$$

Converting to common oilfield units, these relations become:

$$N_{LV} = 1.938\quad v_{sL}\left(\frac{\rho_L}{\sigma}\right)^{1/4} \qquad (2.313)$$

$$N_{gV} = 1.938\quad v_{sg}\left(\frac{\rho_L}{\sigma}\right)^{1/4} \qquad (2.314)$$

$$N_d = 120.872\,d\left(\frac{\rho_L}{\sigma}\right)^{1/2} \qquad (2.315)$$

and:

$$N_L = 0.15726\,\mu_L\left(\frac{1}{\rho_L\sigma^3}\right)^{1/4} \qquad (2.316)$$

where:

d = ft
ρ = lb_m/cu ft
v_{sL} = ft/sec
v_{sg} = ft/sec
μ_L = cp
σ = dynes/cm.

A regression analysis technique was used to relate the four dimensionless groups, as well as a pressure term, and the result is shown as Fig. 2.34.

It should be noted at this point that the Hagedorn and Brown holdup correlation is in fact a pseudo-holdup correlation. Holdup was not actually measured but back-calculated from knowing the total pressure loss and using a friction factor obtained from a two-phase Reynolds number.

To account for the viscosity of the liquid, the term CN_L was included in the numerator of the correlating function used for the abscissa of Fig. 2.34. A plot of N_L versus CN_L is noted in Fig. 2.56. Water was chosen as

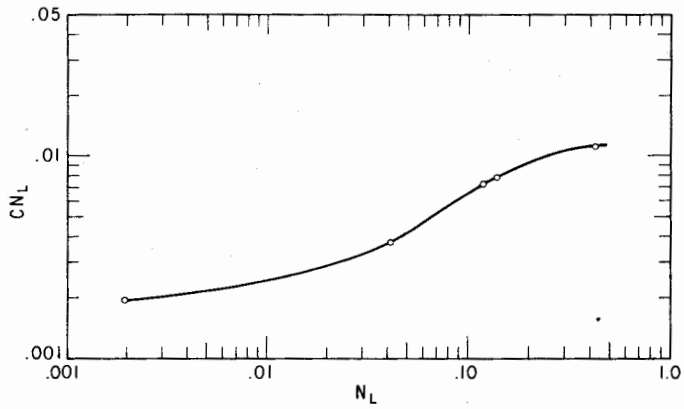


Fig. 2.56 Correlation for viscosity number coefficient C (after Hagedorn).

the base curve; C was taken as 1.00 for water. This plot shows that for low values of the liquid viscosity, the viscosity has very little effect.

An additional factor was needed to properly account for the holdup because it was impossible to obtain one curve to account for the deviation at high gas rates in high viscosity crude oils. This secondary correction factor (Ψ) was plotted against values of the group of terms $N_{gv}N_L^{0.38}/N_d^{2.14}$ (Fig. 2.57). In most cases it will be found that $\Psi = 1.00$.

The correlating function for liquid holdup can be expressed in field units as follows:

$$[(N_{LV})/(N_{gv})^{0.575}]\ (P/P_a)^{0.10}\ (CN_L/N_d) =$$

$$0.00326\ \frac{q_L^{0.425}\left[\left(\frac{1}{1+WOR}\right)B_o+\left(\frac{WOR}{1+WOR}\right)B_w\right]\overline{p}^{0.675}\,\sigma^{0.394}\,CN_L}{d^{1.850}\,(\overline{T}\overline{Z})^{0.575}\,\rho_L^{0.394}\left[GLR-R_s\left(\frac{1}{1+WOR}\right)\right]^{0.575}}, \qquad (2.317)$$

where: q_L = liquid production rate, stb/d
d = pipe diameter, ft
$\overline{Z}$ = average gas compressibility factor, dimensionless
$\overline{T}$ = average temperature, °R
$\overline{P}$ = average pressure, psia
ρ_L = average liquid density, lb_m/cu ft
σ = liquid surface tension, dynes/cm
GLR = gas-liquid ratio, scf/stb
R_s = gas in solution at $\overline{T}$ and $\overline{P}$, scf/stb
WOR = water-oil ratio, stbw/stbo.

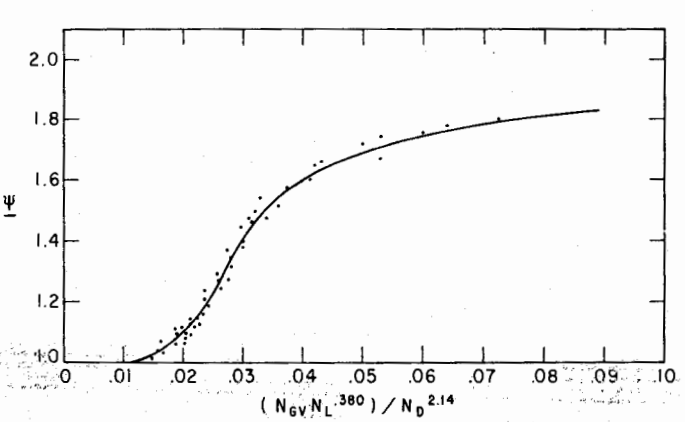


Fig. 2.57 Correlation for secondary correction factor (after Hagedorn).

Further work by Brill[29] and Hagedorn[30] has considerably improved this method. This work was performed in connection with better inclusion of the effects of holdup and slippage and the inclusion of the Griffith bubble flow correlation. Figs. 2.58 and 2.59 give an indication of the discrepancies that needed to be resolved. Fig. 2.58 presents a set of pressure traverses as calculated by Hagedorn and Brown for 100 bpd water in 3 in. tubing. There is a wide spread between the 0 and 50 scf/bbl gas-liquid ratio curve. It was thought that this was caused by an error in the original holdup correlation; but later it was found that the predicted holdup for low flow rates and low gas-liquid ratios was less than the holdup would be if there was no slippage. This discrepancy was more pronounced for large pipe sizes. Brill[29] and Hagedorn[30] suggested that a calculation be made for holdup from the Hagedorn and Brown correlation and that these results be compared to the holdup when no slippage is assumed. If the latter was found to be greater than the former, the H_L value excluding slippage was used. (Fig. 2.59 which requires using the H_L value for no slippage shows the same pressure traverse as found in Fig. 2.58.) The curves recently presented by Brown[31] were prepared in this manner. This discrepancy occurs only in larger tubing sizes at low flow rates and low gas-liquid ratios; therefore, except for these cases, this adjustment is not necessary.

At the suggestion of Brill and Hagedorn, the Griffith correlation[16] was added to the Hagedorn and Brown correlation. The Griffith correlation is used when flow is in the bubble flow regime. For a complete description of this correlation and its incorporation into the multiphase vertical flow problem, reference should be made to Orkiszewski.[22] The Griffith method is explained by Orkiszewski and an example problem is worked by him. The modification incorporates the Griffith correlation after the manner suggested by Orkiszewski but only for bubble flow.

In summary, the following two adjustments are made:

(1) The density of the mixture is calculated using the Hagedorn and Brown holdup correlation and this value is compared to the density of the mixture assuming that there is no slippage. The largest of these two values is then used.
(2) The flow regime is determined, and if it is bubble flow, the pressure traverse is calculated by the Griffith correlation.

It is recommended that these two adjustments always be used in the generalized Hagedorn and Brown method.

A procedure for the calculation of a pressure traverse by this method is given in Appendix C.42. It is recommended that Eq. 2.35 be solved for Δh and that a pressure traverse be calculated from the result. Rearranging Eq. 2.35, the equation becomes:

$$\Delta h = \frac{144\,\Delta p - \rho_m\,\Delta\left(\frac{v_m^2}{2g_c}\right)}{\rho_m + \frac{f\,w^2}{2.9652\times 10^{11}\,d^5\,\rho_m}}. \qquad (2.318)$$

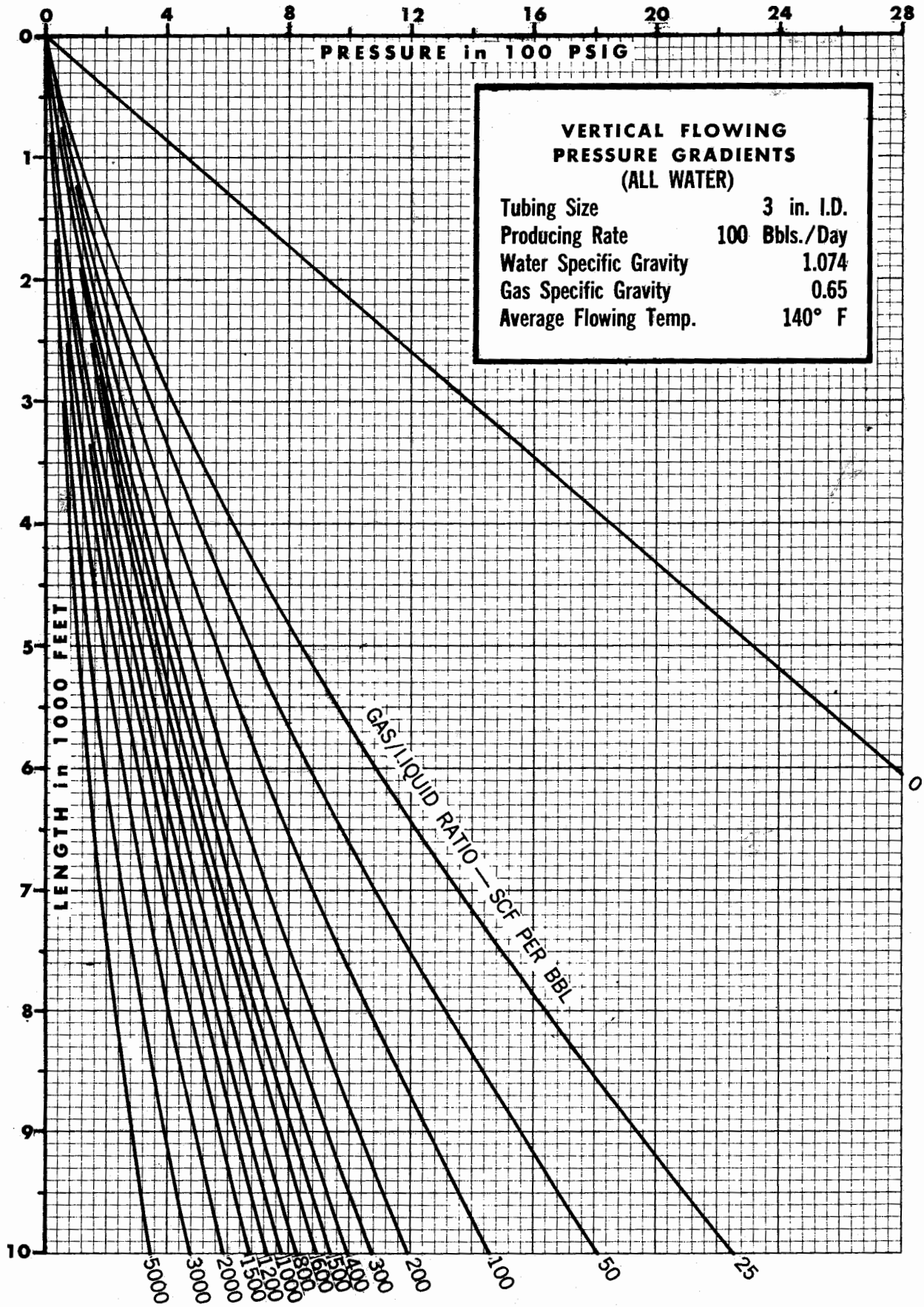


Fig. 2.58 Pressure traverses by Hagedorn original method.

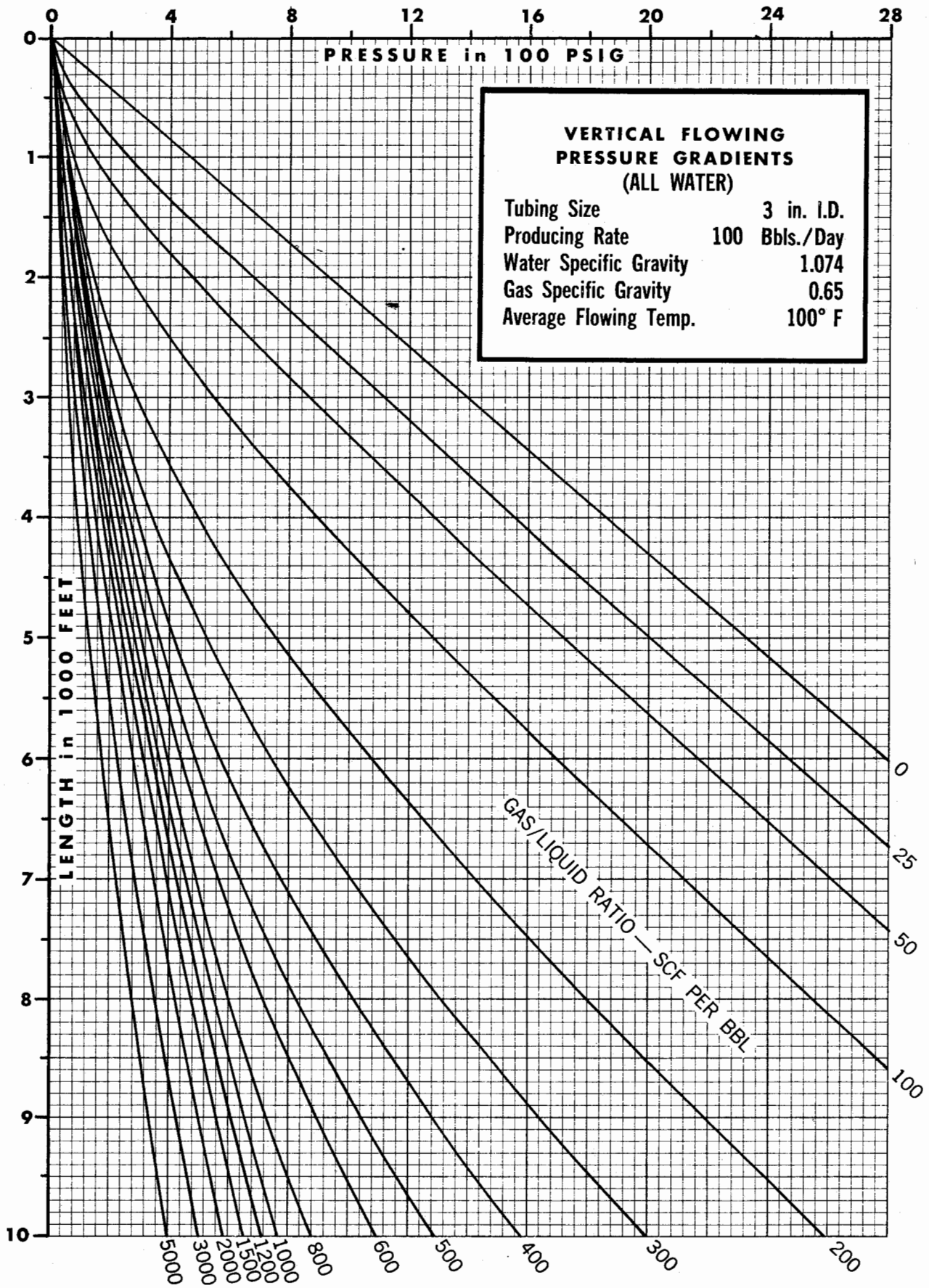


Fig. 2.59 Pressure traverses by modified Hagedorn method.

This form eliminates the necessity for a trial-and-error solution and permits solving for the distance between two pressure points. Fig. 2.60-B shows a detailed flow diagram of the Hagedorn and Brown method which shows how to calculate a pressure traverse by this method. This can be used in conjunction with Appendix C.42 and should also serve to clarify the previous discussions on this method.

A solution of the same example problem that was solved by other methods is given in Appendix C.43. For 2-in. nominal tubing the calculated distance between the 500 and 1,000 psig points was 1,870 ft. This compares to 2,425 ft obtained by the Poettmann and Carpenter method and 2,350 ft found by the Fancher and Brown method. The distance between the 500 and 1,000 psig points for 1¼ in. tubing was found to be 1,250 ft, as compared to: (a) 1,650 ft by Poettmann and Carpenter; (b) 1,465 ft by Fancher and Brown; and (c) 1,335 ft by Hagedorn and Brown Method for 1¼ in. tubing only.

As with some of the other correlations, the kinetic energy term can be neglected most of the time without inducing serious error. For example, a problem is worked in Appendix C.44 for a flow rate of 600 stbo/d with a GOR of 1,000 scf/bbl flowing in 2 in. tubing. The calculated distance between the 500 and 1,000 psig pressure points was 3,580 ft, excluding kinetic energy and 3,579 ft when the kinetic energy was included. However, for high gas-oil ratios and small tubing sizes the kinetic energy term can be significant. Therefore, in approaching a problem of this kind it is suggested that the kinetic energy term be included for gas-oil ratios in excess of 3,000 scf/bbl for all tubing sizes.

The following derivations, procedures and problem examples involving applications of the final correlation of Hagedorn and Brown are found in Appendix C.4:

(1) C.41—Derivation of Hagedorn and Brown's equation for multiphase flow.
(2) C.42—Detailed procedure for calculating a vertical pressure traverse.
(3) C.43—Example problem.
(4) C.44—Example problem showing the difference between including and excluding the kinetic energy term.
    (a) C.441—Excluding kinetic energy
    (b) C.442—Including kinetic energy

2.3333 The Duns and Ros method

The Duns and Ros method is the result of a large-scale laboratory investigation with modifications and adjustments using field data. A separate paper by Ros[13] contains information that is essentially the same as the work of Duns and Ros.[14]

In order to better understand the initial concept of the Duns and Ros method, Fig. 2.61 represents a generalized flow diagram. After assuming a pressure difference and calculating the various required properties, a flow region must be selected. The liquid holdup and friction factor differ depending upon the flow region after which the vertical length corresponding to the pressure difference assumed is determined. This method is described in detail in the following discussions and a calculation procedure with an example problem is found in Appendix C.5.

Duns and Ros chose an approach slightly different than that of most other investigators. They arbitrarily defined the static pressure gradient as the in-situ volume weighted density and developed correlations for wall friction from their extensive laboratory data for each of three broad flow regions. Although they made a specific point of using a pressure balance instead of an energy balance, their equation is a thermodynamic energy balance. The total gradient includes a static gradient, a wall friction gradient and an acceleration gradient. The effects of slip between the gas and liquid phase are incorporated in the static gradient and are kept separate from the effects due to friction. They separated the flow into three types of patterns and prepared separate correlations for slippage and friction in all three.

Typical examples of their results are shown in Fig. 2.62 which is for the flow of air and oil through an 8 cm pipe. The total pressure gradient and liquid holdup have been plotted for varying liquid rates.

The pressure gradient $\frac{dp}{dh}$ is expressed as a fraction of the hydrostatic liquid gradient $\rho_L g$, thus:

$$G = \left(\frac{1}{\rho_L g}\right)\left(\frac{dp}{dh}\right) \qquad (2.319)$$

where G = dimensionless pressure gradient. The throughputs are given in terms of superficial velocities v_{sg} and v_{sL}. As did other authors, Duns and Ros chose to use superficial velocities which implies that each phase is flowing alone in the pipe.

Holdup and pressure gradient depend to a large degree on the gas flow rate (Fig. 2.62). Duns and Ros showed that bubble flow prevailed at low gas rates. The liquid was the continuous phase and the gas existed in the form of bubbles. In this kind of a flow pattern the pressure gradient is almost equal to the hydrostatic gradient of the liquid. Wall friction increases the gradient whenever there is a high liquid flow rate.

For a low liquid flow rate ($v_{sL} < 40$ cm/sec), an increase in the gas rate causes the bubbles to increase in number and enlarge in size until they unite and form bullet-shaped gas plugs. These plugs become unstable and collapse at still higher gas flow rates. The pattern then changes to one of alternating liquid and gas slugs, or slug flow. Throughout these changing flow patterns the wall friction remains essentially negligible.

At higher gas flow rates ($v_{sg} > 1{,}500$ cm/sec and $v_{sL} < 40$ cm/sec), the flow pattern changed from slug flow to mist flow. When this change takes place, the gas then becomes the continuous phase and the liquid is dispersed and entrained in the gas as droplets. Wall friction is very significant for this type of flow and friction increases sharply as the flowing rate of the gas increases (Fig. 2.62). However, one important point to note is that the pressure gradient passes a minimum and then increases sharply. The lowest gradient is the minimum gradient attainable by the increased injection of gas into a gas-lift well.

If the liquid flow rate is increased to an amount where $v_{sL} > 160$ cm/sec, it becomes most difficult to observe the various flow patterns. Plug flow is non-

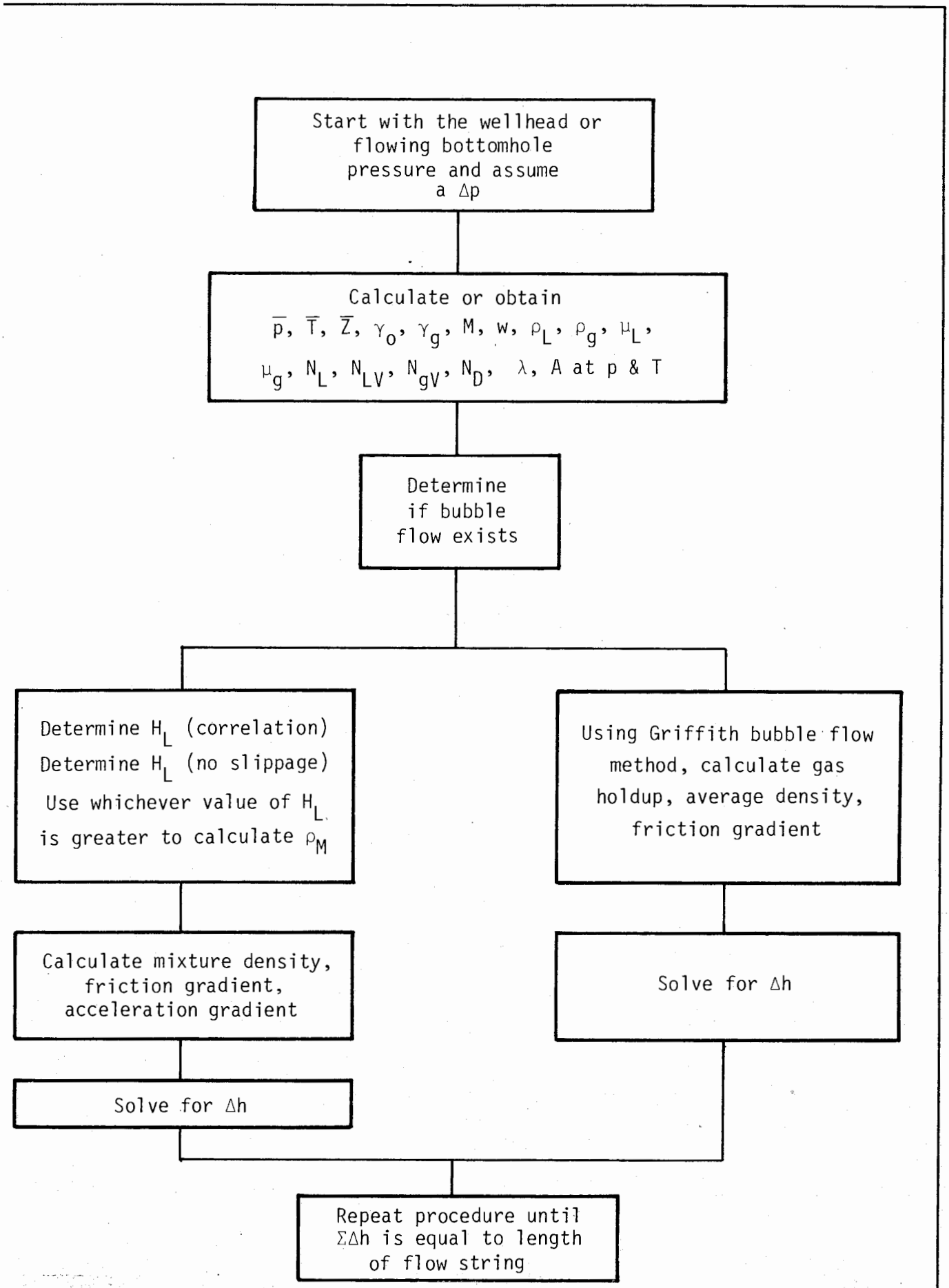


Fig. 2.60A Flow diagram for generalized method of Hagedorn and Brown.

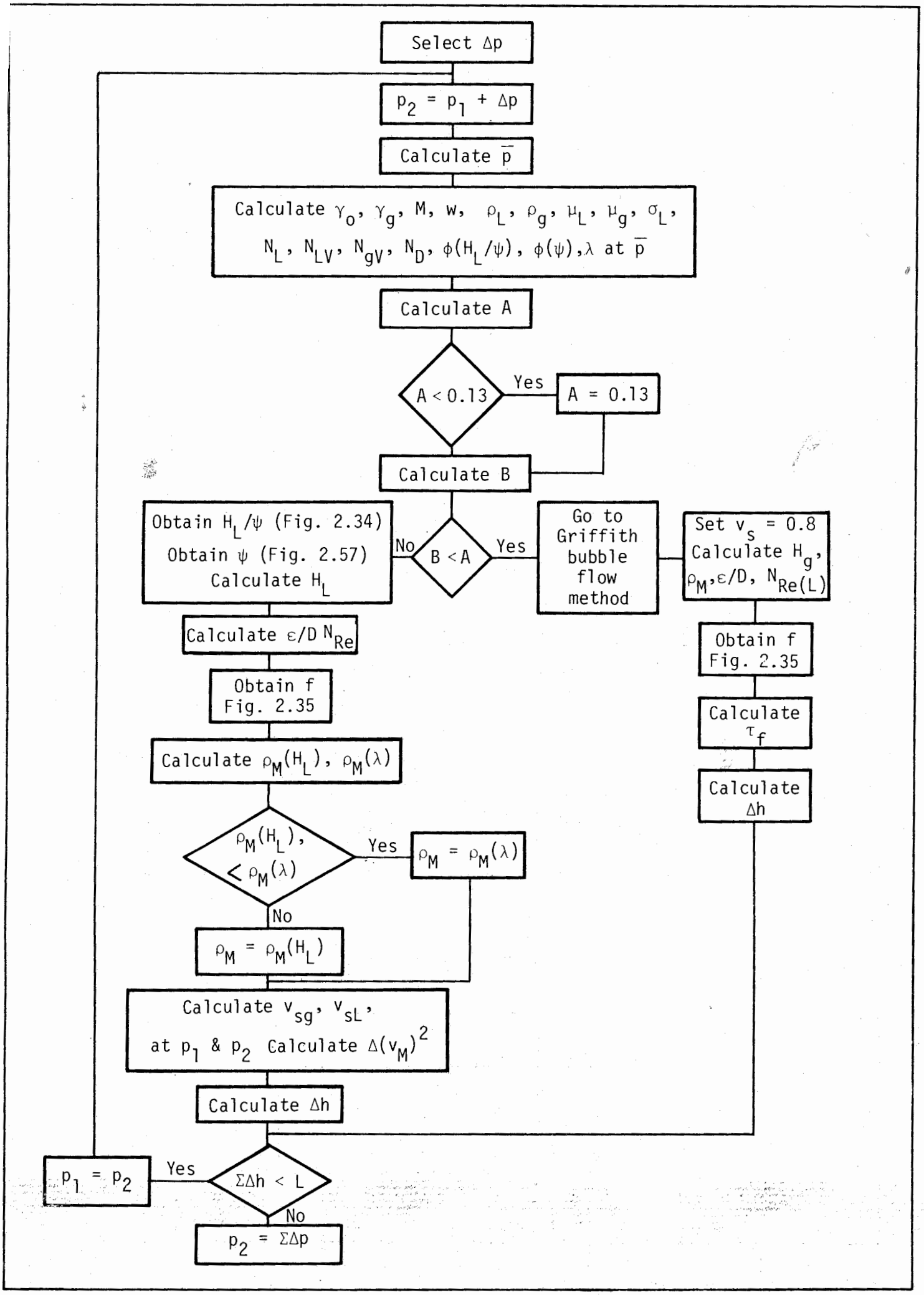


Fig. 2.60B Flow diagram for generalized method of Hagedorn and Brown.

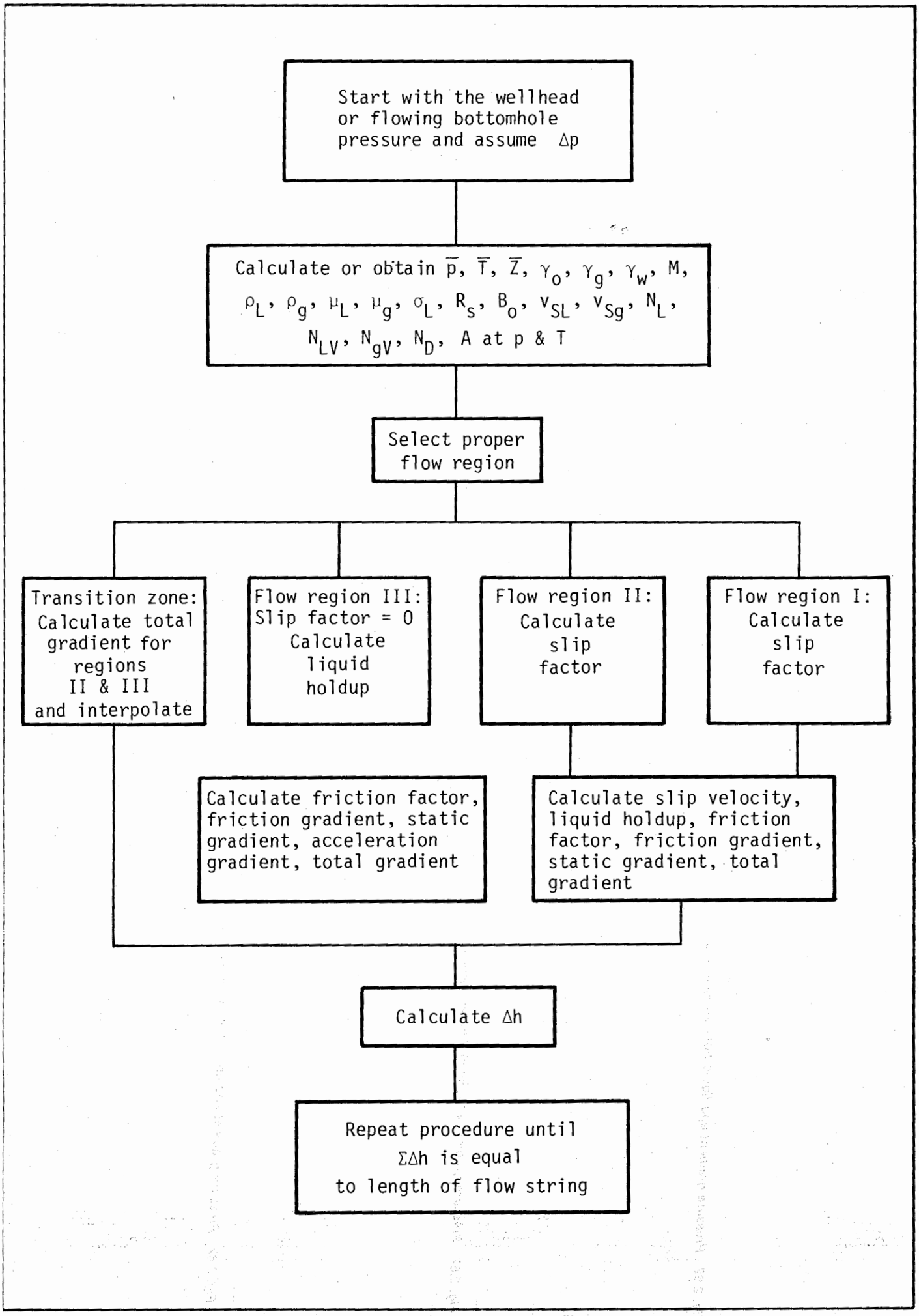
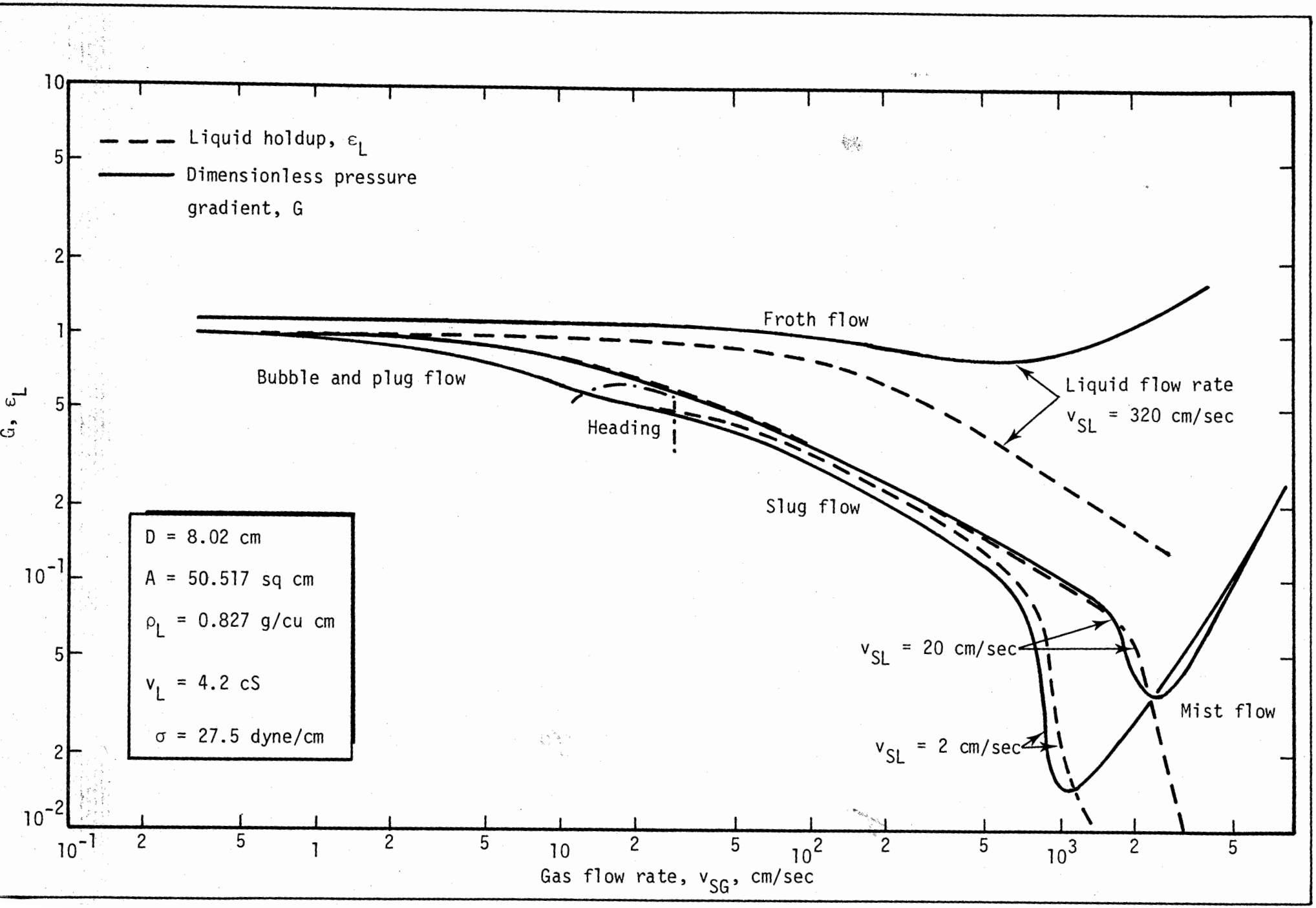


Fig. 2.61 Flow diagram for Duns and Ros method.



...against gas flow and liquid flow (after Ros)

istent, the pattern becomes turbulent, and the liquid frothy with dispersed gas bubbles. As the gas flow ıte is increased even more, some segregation takes ace and causes a rapidly repeating slug flow. This ıally changes to mist flow where $v_{sg} > 5{,}000$ cm/sec. uns and Ros developed four dimensionless groups hich were used extensively in their correlations. hese are:

$$N_{gv} = \text{gas-velocity number} = v_{sg}\left(\frac{\rho_L}{g\sigma}\right)^{1/4} \quad (2.39)$$

$$N_{Lv} = \text{liquid-velocity number} = v_{sL}\left(\frac{\rho_L}{g\sigma}\right)^{1/4} \quad (2.310)$$

$$N_d = \text{diameter number} = d\left(\frac{\rho_L g}{\sigma}\right)^{1/2} \quad (2.311)$$

ıd:

$$N_L = \text{liquid-viscosity number} = \mu_L\left(\frac{g}{\rho_L \sigma^3}\right)^{1/4} \quad (2.312)$$

Ros[13] made a correlation of slip velocity in dimenonless form with dimensionless groups. A different ip velocity correlation was used for each of two oad flow regimes, and a slip velocity of zero was dicated when the holdup is equal to λ for mist flow. ıese friction correlations, in conjunction with those eviously mentioned for each of the three types of ıtterns, give us a way to calculate the total pressure adient.

The pressure gradient given by Ros is:

)/dh = static gradient + friction gradient + acceleration gradient

ıe static gradient is equal to:

$$H_L\rho_L g + (1 - H_L)\,\rho_g g \quad (2.320)$$

ıere H_L is the liquid holdup. The acceleration gradit is generally considered to be negligible and the uation becomes:

$$dp/dh = H_L\rho_L g + (1 - H_L)\,\rho_g g + \text{friction gradient} \quad (2.321)$$

the pressure gradient dp/dh is expressed as a fracın of the hydrostatic liquid gradient, $\rho_L g$, the equaın becomes:

$$= (1/\rho_L g)\, dp/dh = H_L + (1 - H_L)\,\rho_g/\rho_L + \text{friction term} \quad (2.322)$$

ıere G = dimensionless pressure gradient.

Ros showed that for low liquid flow rates the pressre gradient was practically independent of the gas w rates in the range studied, but at high liquid tes the pressure gradient varied significantly with e gas rate.

The various flow regions were divided by Duns and ıs into three main regions depending on the amount gas present:

Region I. The liquid phase is continuous and bubble-flow, plug-flow and part of the froth-flow regime exists.

Region II. In this region the phases of liquid and gas alternate. The region thus covers slug-flow and the remainder of the froth-flow regime.

Region III. The gas is in a continuous phase and the mist-flow regime exists.

The different nature of these three main regions necessitates separate correlations for friction and holdup for each region; therefore in principle, six different correlations are to be expected. The identification of the flow region is a function of N_{Lv}, N_{gv}, L_1, L_2 and N_d. The regions of validity of the correlations are plotted and presented in Fig. 2.63 as a function of the liquid-velocity number, N_{Lv}, and gas-velocity number, N_{gv}. Because N_{Lv} and N_{gv} are directly related to liquid flow rate and gas flow rate, respectively, it can be seen from Fig. 2.63 that a change in one or both of these rates affects the region of flow.

Duns and Ros suggested the following limits for various flow regions:

Region I: $0 \le N_{gv} \le (L_1 + L_2\, N_{Lv})$
Region II: $(L_1 + L_2 N_{Lv} < N_{gv} < (50 + 36\, N_{Lv})$
Region III: $N_{gv} > (75 + 84\, N_{Lv}^{0.75})$

L_1 and L_2 are functions of N_d, and their relationships are presented in Fig. 2.64.

It was also found that the liquid holdup is related to the slip velocity, v_s, as follows:

$$v_s = v_{sg}/(1 - H_L) - v_{sL}/H_L \quad (2.323)$$

where v_{sg}, v_{sL} are average gas and average liquid superficial velocities, respectively.

The slip velocity was expressed in dimensionless form as:

$$S = v_s \sqrt[4]{\rho_L/g\sigma} \quad (2.324)$$

As soon as S has been determined, v_s, H_L and finally $(dp/dh)_{st}$ can be determined.

Different formulas are used for calculating S in each of the three flow regions. These formulas, which are functions of the four dimensionless numbers N_{Lv}, N_{gv}, N_d, N_L, are found in Appendix C.5, and make use of Figs. 2.65 and 2.66.

Duns and Ros[14] state that there is a gap between slug-flow and mist-flow and that the pressure gradient in this region is best obtained by interpolation.

The gradient resulting from wall friction is calculated with the following equations. For Regions I and II the following equation is applicable:

$$(dp/dh)_{fr} = \frac{4f_w\rho_L v_{sL}^2}{2d}\,(1 + v_{sg}/v_{sL}) \quad (2.325)$$

Expressed as a fraction of the static liquid gradient, Eq. 2.325 becomes:

$$G_{fr} = \frac{1}{\rho_L g}\,(dp/dh)_{fr} = 4f_w\,\frac{v_{sL}^2}{2gd}\,(1 + v_{sg}/v_{sL}) \quad (2.326)$$

From experimental data Duns and Ros formulated the following equation:

$$f_w = f_1 f_2/f_3 \quad (2.327)$$

A value for f_1 is obtained from Fig. 2.67 and is a function of the Reynolds number of the liquid, expressed as:

$$(N_{Re})_L = \rho_L v_{sL} d_{hy}/\mu_L, \quad (2.328)$$

where d_{hy} is the hydraulic diameter of the flow string. For annuli, $d_{hy} = d_{ci} - d_{ot}$ and:

$$N_d = (d_{ci} - d_{ot})\,\sqrt{\rho_L g/\sigma} \quad (2.329)$$

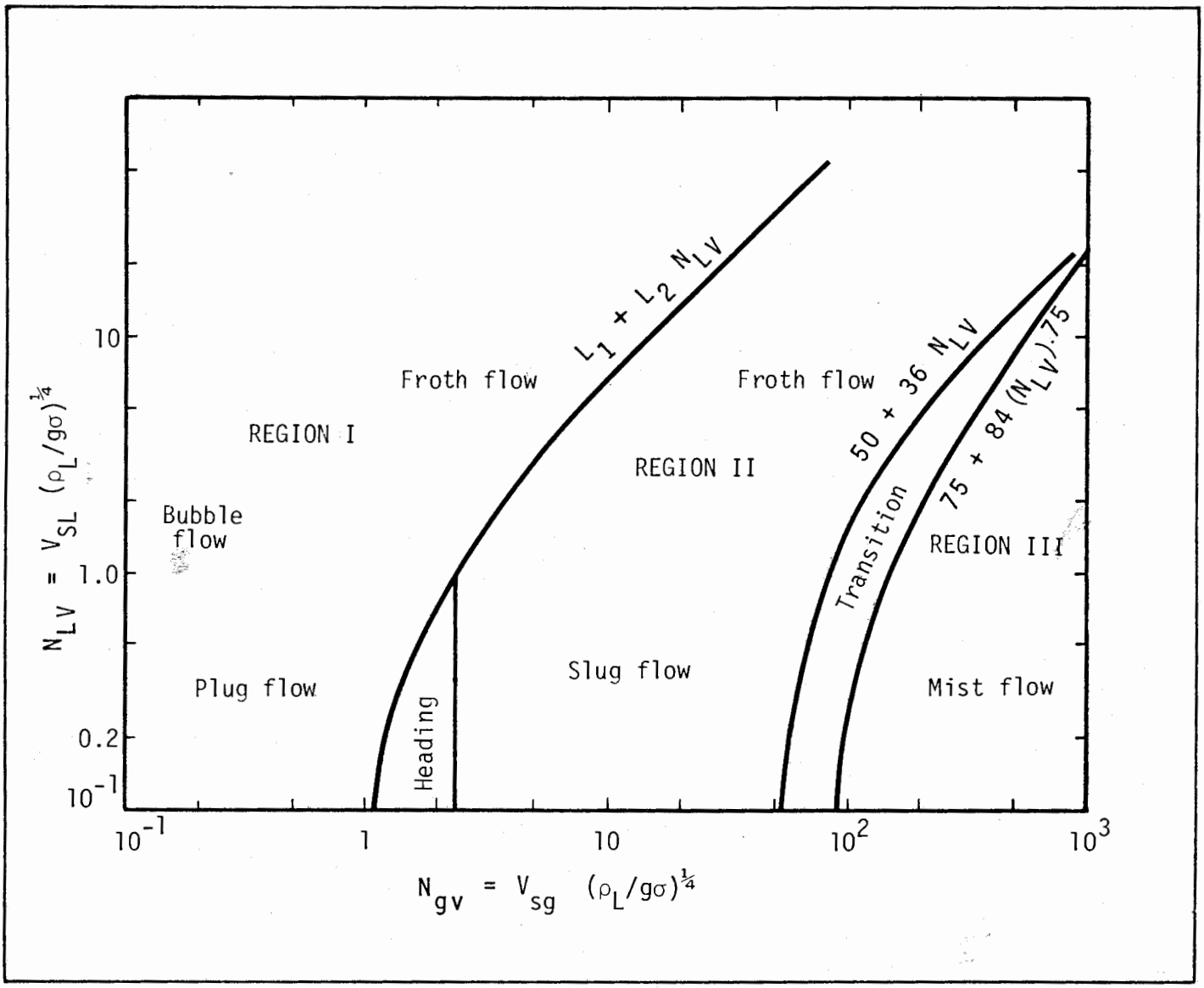


Fig. 2.63 Regions of validity of correlations (from Duns and Ros).

Fig. 2.67 resembles the well-known Moody diagram for use in single-phase flow, except for that transition range lying between laminar and turbulent flow.

The factor f_2 is a correction for the in situ gas-liquid ratio R. Values of this factor are given in Fig. 2.68 as a function of the group of terms $f_1 R N_d^{2/3}$. This factor is essentially 1.00 when R is small, but decreases very rapidly for high R values. The factor f_3 is considered by Duns and Ros to be an additional correction factor for liquid viscosity and in situ gas-liquid ratio. It is equal to $1 + f_1\sqrt{R/50}$ and becomes significant for viscosities greater than 50 cs.

The friction correlation for Regions I and II also includes the heading range. It is valid for conditions in the range from $N_{Lv} = 0$, $N_{gv} = 0$ to $N_{gv} = 50 + 36N_{Lv}$. For the friction correlation in Region III, the friction gradient is based on the gas phase and is obtained from the equations:

$$(dp/dh)_{fr} = 4f_w\rho_g v_{sg}^2/2d \qquad (2.330)$$

and:

$$G_{fr} = \frac{1}{\rho_L g}(dp/dh)_{fr} = 4f_w\frac{\rho_g}{\rho_L}\frac{v_{sg}^2}{2gd}, \qquad (2.331)$$

or:

$$G_{fr} = 2f_w N_\rho \frac{N_{gv}^2}{N_d}, \qquad (2.332)$$

where $N_\rho = \rho_g/\rho_L$ is a density influence number. Since there is no slip, the friction factor is that given in the Fanning diagram, but as a function of the Reynolds number of the gas:

$$(N_{Re})_g = \rho_g v_{sg} d_{hy}/\mu_g \qquad (2.333)$$

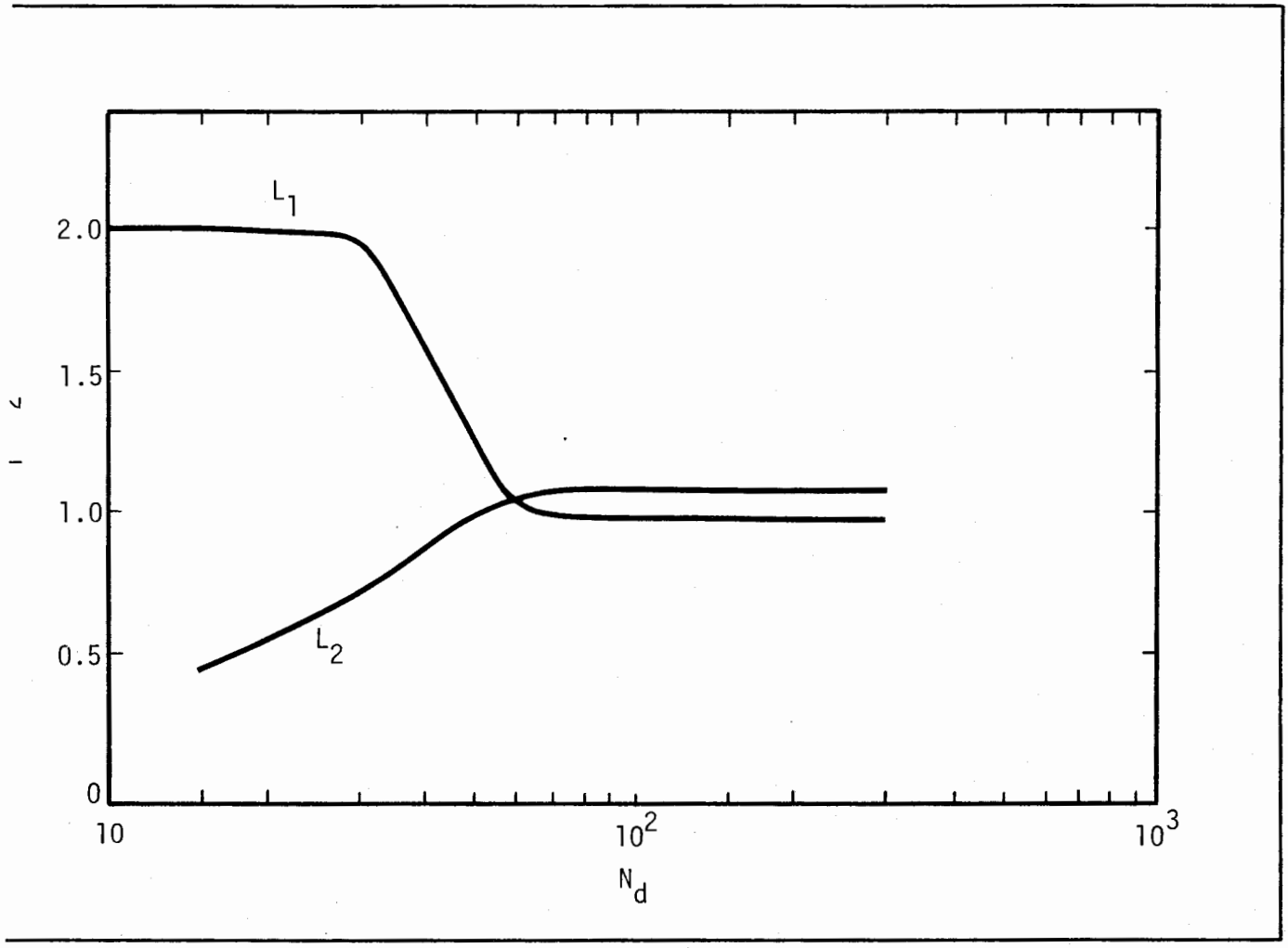
Therefore, $f_w = f_1$

In this expression, d_{hy} is the hydraulic diameter of the flow string.

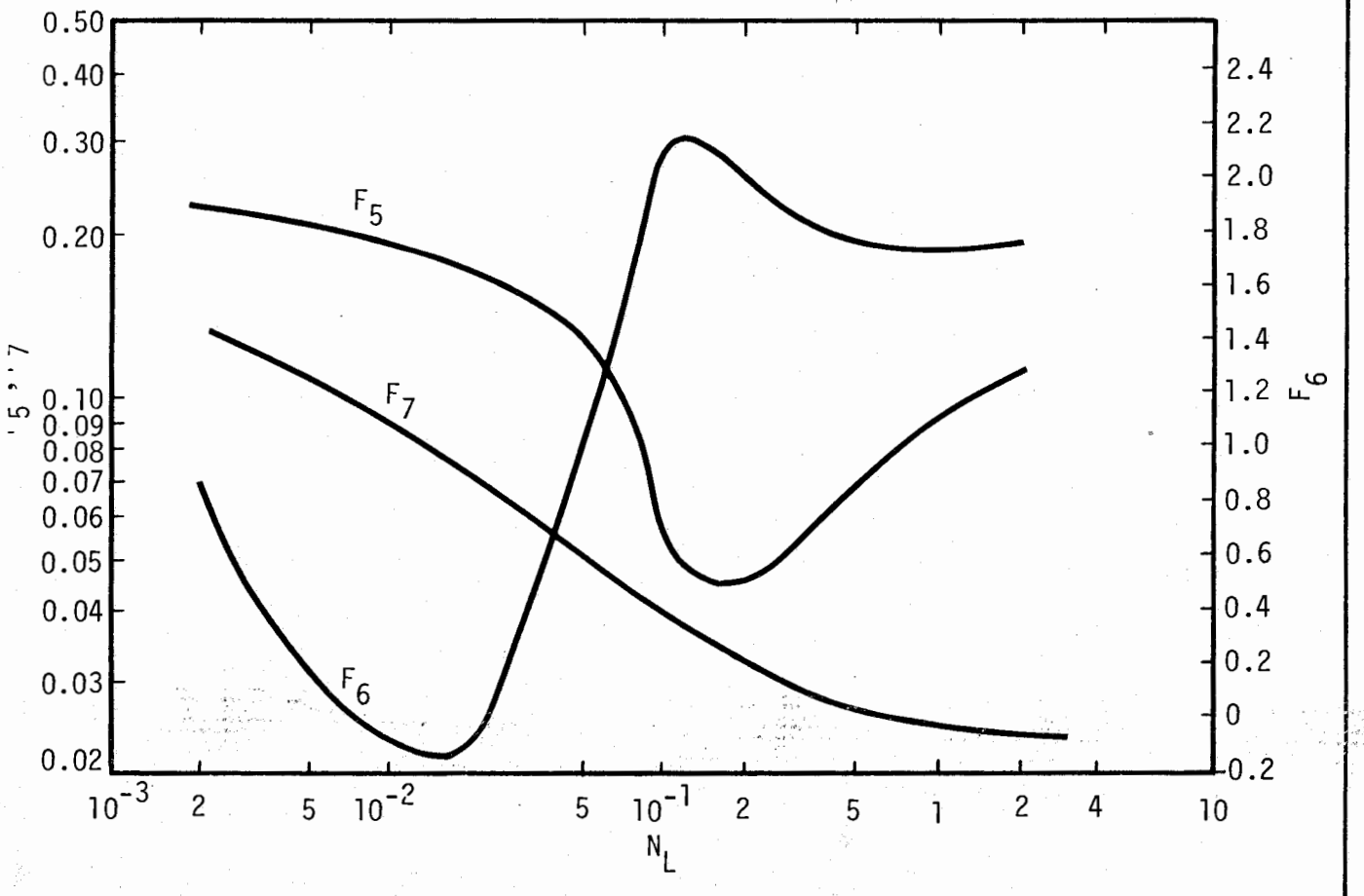
The wall roughness for mist flow is affected by the film of liquid on the wall of the pipe. The ripples of the wall film cause a drag on the gas. This process is governed by the dimensionless group

$$N_{We} = \rho_g v_{sg}^2 \epsilon/\sigma \qquad (2.334)$$

This Weber number is a function of the operating variables. It was found that the liquid viscosity affects its magnitude. To handle the influence of viscosity,



1. 2.64 L-factors against diameter number N_d (after Ros).



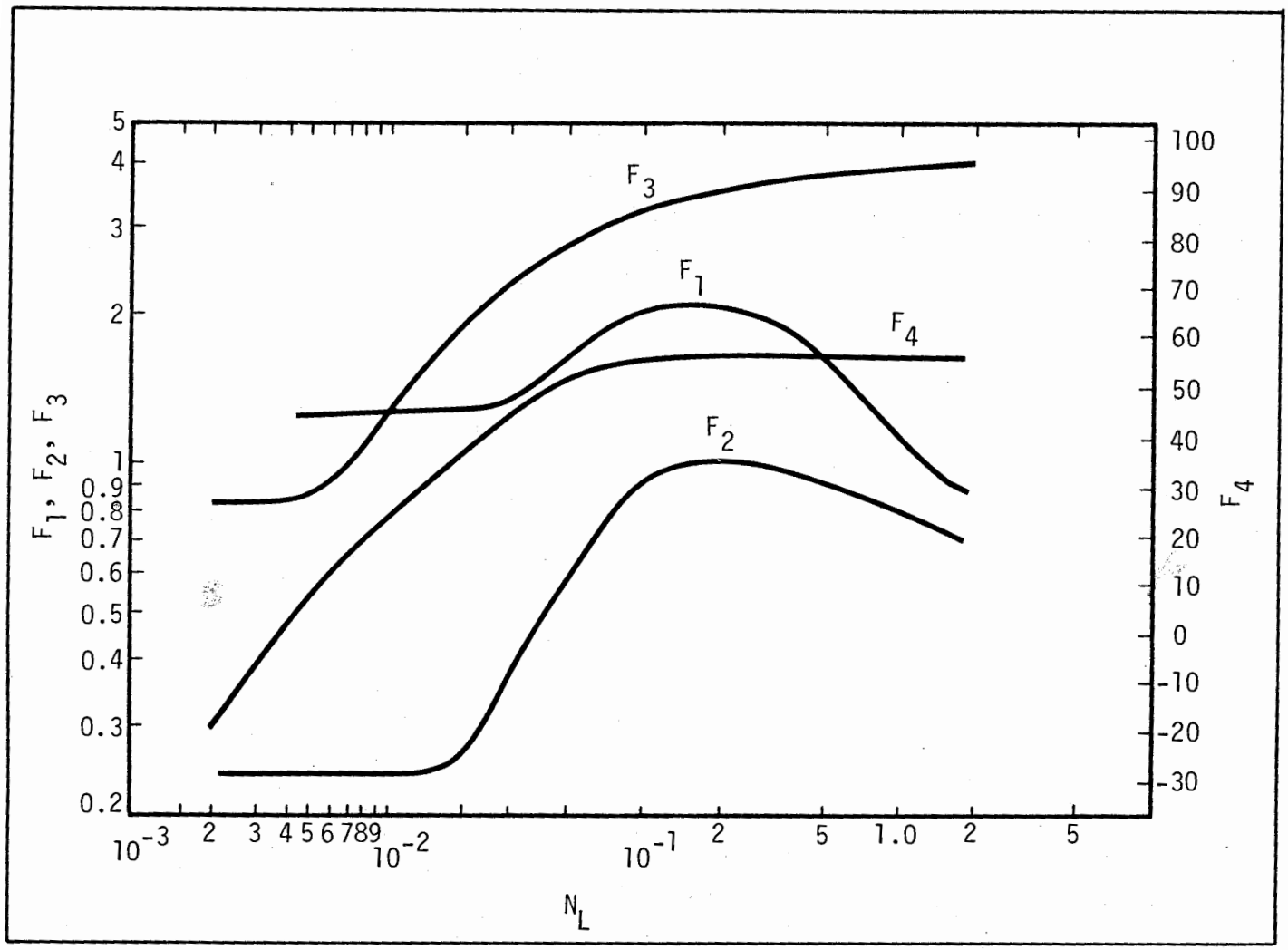


Fig. 2.66 F_1, F_2, F_3 and F_4 against viscosity number N_1 (after Ros).

N_{We} was related to the group of terms which included viscosity, as follows:

$$N_\mu = \mu_L^2/\rho_L\sigma\epsilon \qquad (2.335)$$

Eq. 2.335 reflects the interaction between viscosity and surface tension. The relationship is shown in Fig. 2.69.

The value of roughness can be very small but it is never less than that of the pipe itself, or approximately $d \times 10^{-3}$. However, at the transition zone to slug-flow, ϵ/d approaches 0.5. Values of f_1 are found by extrapolating the Moody diagram for $\epsilon > 0.05$ d by:

$$f_1 = \frac{1}{[4 \log_{10} (0.27\ \epsilon/d)]^2} + 0.067\ (\epsilon/d)^{1.73} \qquad (2.336)$$

This expression is subject to refinement by substituting $(d - \epsilon)$ for d in the calculation of the friction gradient, and $v_{sg}d^2/(d - \epsilon)^2$ for v_{sg}. When this is done, ϵ follows from an interation process.

The acceleration term can be neglected in all but the mist-flow region. The acceleration gradient is:

$$(dp/dh)_{ac} = -(\rho_L v_{sL} + \rho_g v_{sg})(dv_{sg}/dh) \qquad (2.337)$$

Gas expansion will be isothermal and pv_{sg} will be constant. Therefore,

$$\frac{d}{dh}(pv_{sg}) = v_{sg}(dp/dh) + p(dv_{sg}/dh) = 0 \qquad (2.338)$$

Using these equations with the relationship:

$$(1/\rho_L g)(dp/dh) = G$$

the reduced gradient becomes:

$$G_{ac} = \frac{1}{\rho_L g}(dp/dh)_{ac} = (\rho_L v_{sL} + \rho_g v_{sg})\ v_{sg}\ G/p \qquad (2.339)$$

The final expression for total gradient is:

$$G = G_{st} + G_{fr} + G_{ac} \qquad (2.340)$$

By substituting Eq. 2.339 into 2.340:

$$G = \frac{G_{st} + G_{fr}}{1 - (\rho_L v_{sL} + \rho_g v_{sg})(v_{sg}/p)} \qquad (2.341)$$

By the Duns and Ros method a determination of a pressure gradient in Regions I, II and III can be made in a straightforward manner, but numerous calculations are involved. These calculations are more easily visualized by noting the flow diagram of Fig. 2.70. The total pressure gradient is the sum of the static gradient and the friction gradient with a correction for acceleration by means of Eq. 2.339, when necessary. For N_{gv}

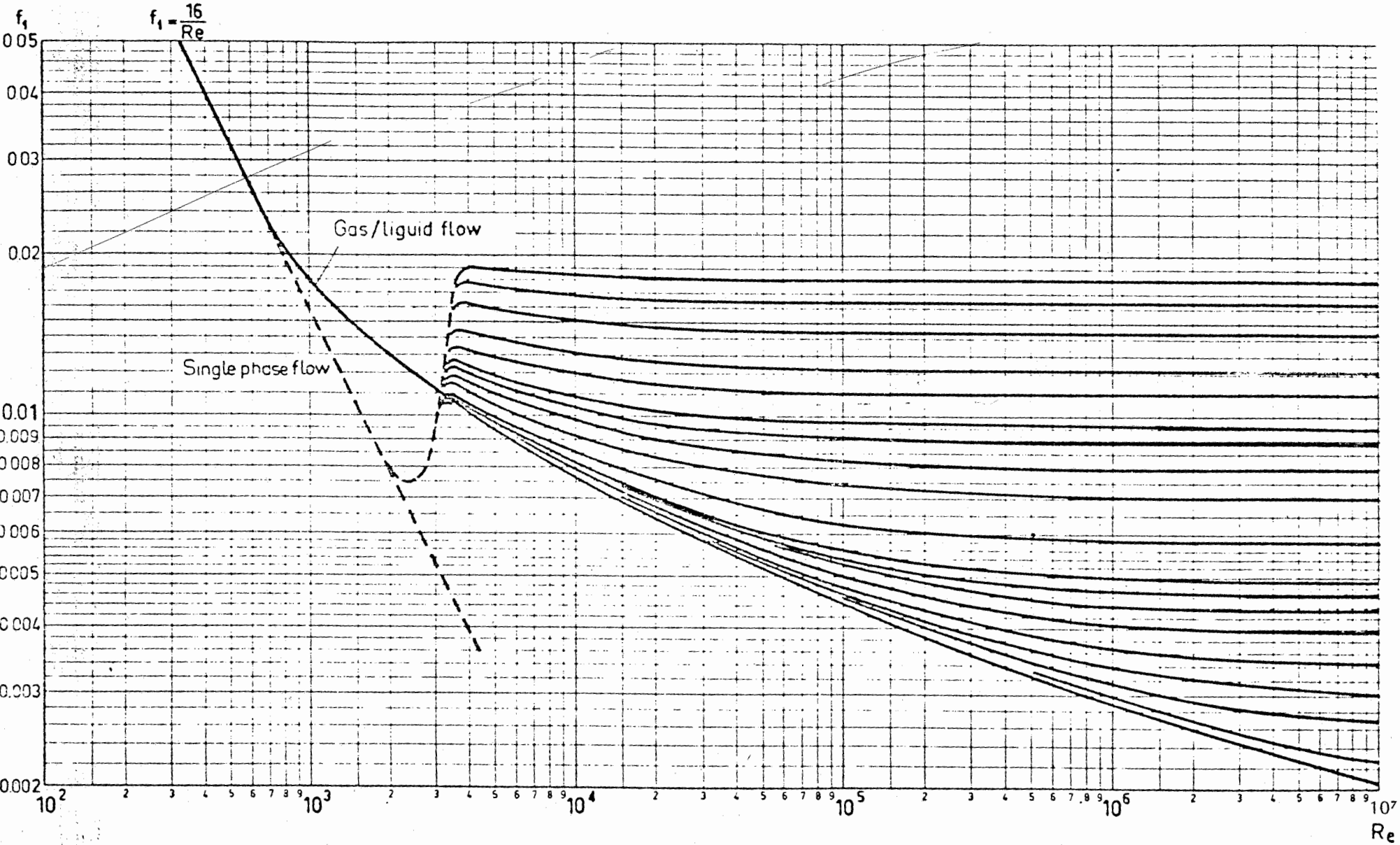


Fig. 2.67 F_1 against Reynolds N_{Re} (after Ros).

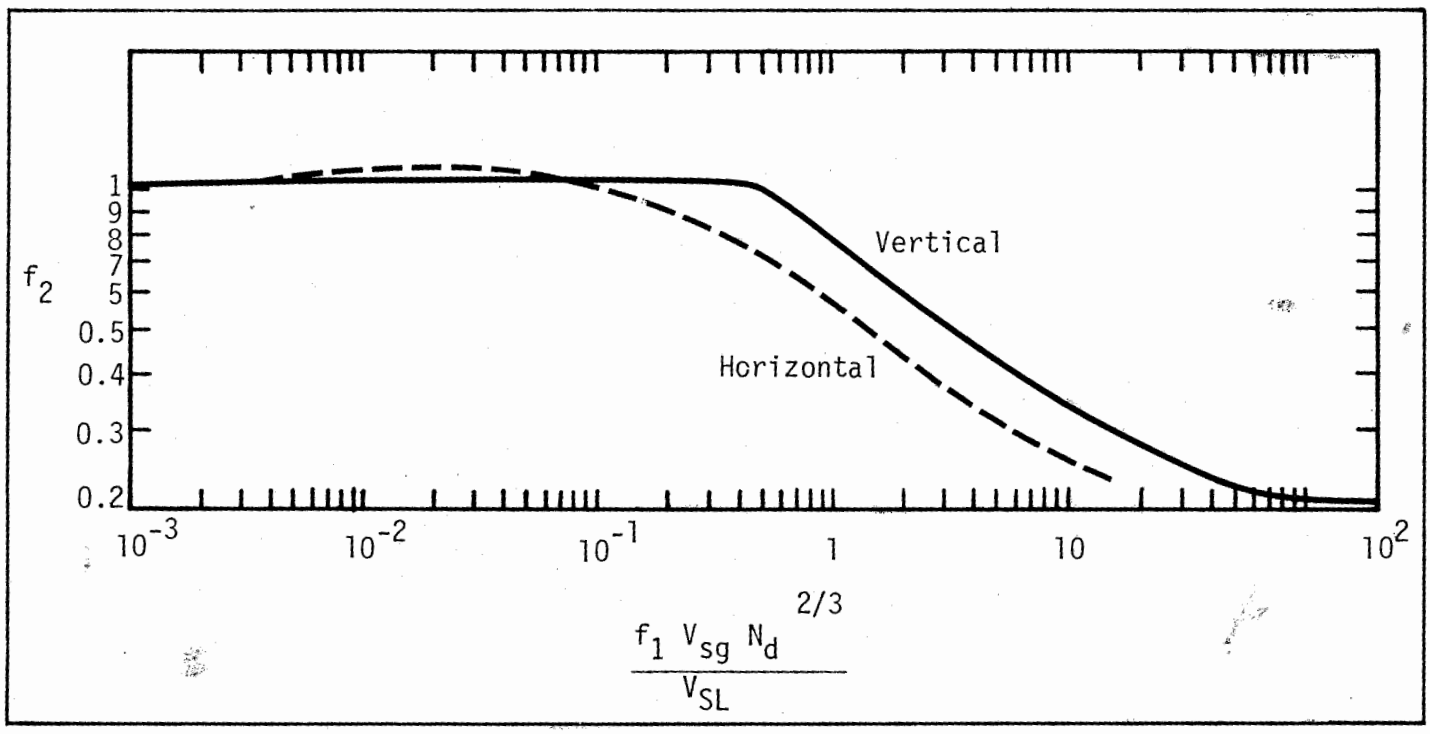


Fig. 2.68 Bubble friction correction (after Ros).

values in the transition region between slug flow and mist flow, a linear interpolation is required. Duns and Ros pointed out that their correlation is intended to be used for oil-gas mixtures only, but should also be accurate for water-gas mixtures. The correlations will not be valid for stable emulsions.

It is suggested that the pressure gradient for an oil-water-gas mixture be obtained by interpolation between the pressure gradient for an oil-gas mixture and a water-gas mixture. The pressure gradient for a wet mixture may be found from an experimentally determined relation given by:

$$\left(\frac{dp}{dh}\right)_{ow} = (1 - 7.3\ C_w)\left(\frac{dp}{dh}\right)_o + 7.3\ C_w\left(\frac{dp}{dh}\right)_w \tag{2.342}$$

where C_w = fractional part water. Eq. 2.342 is valid for Regions I and II and for water contents less than 10%. In correspondence, Ros has indicated that the

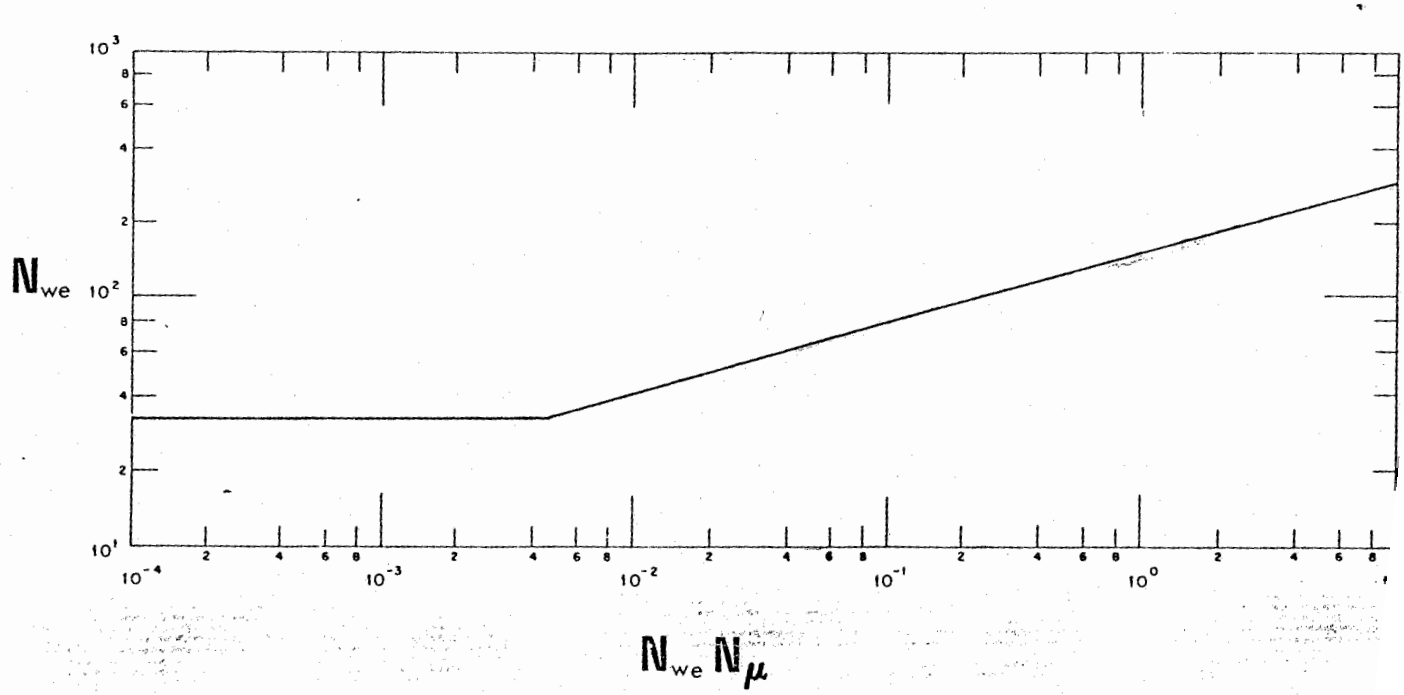
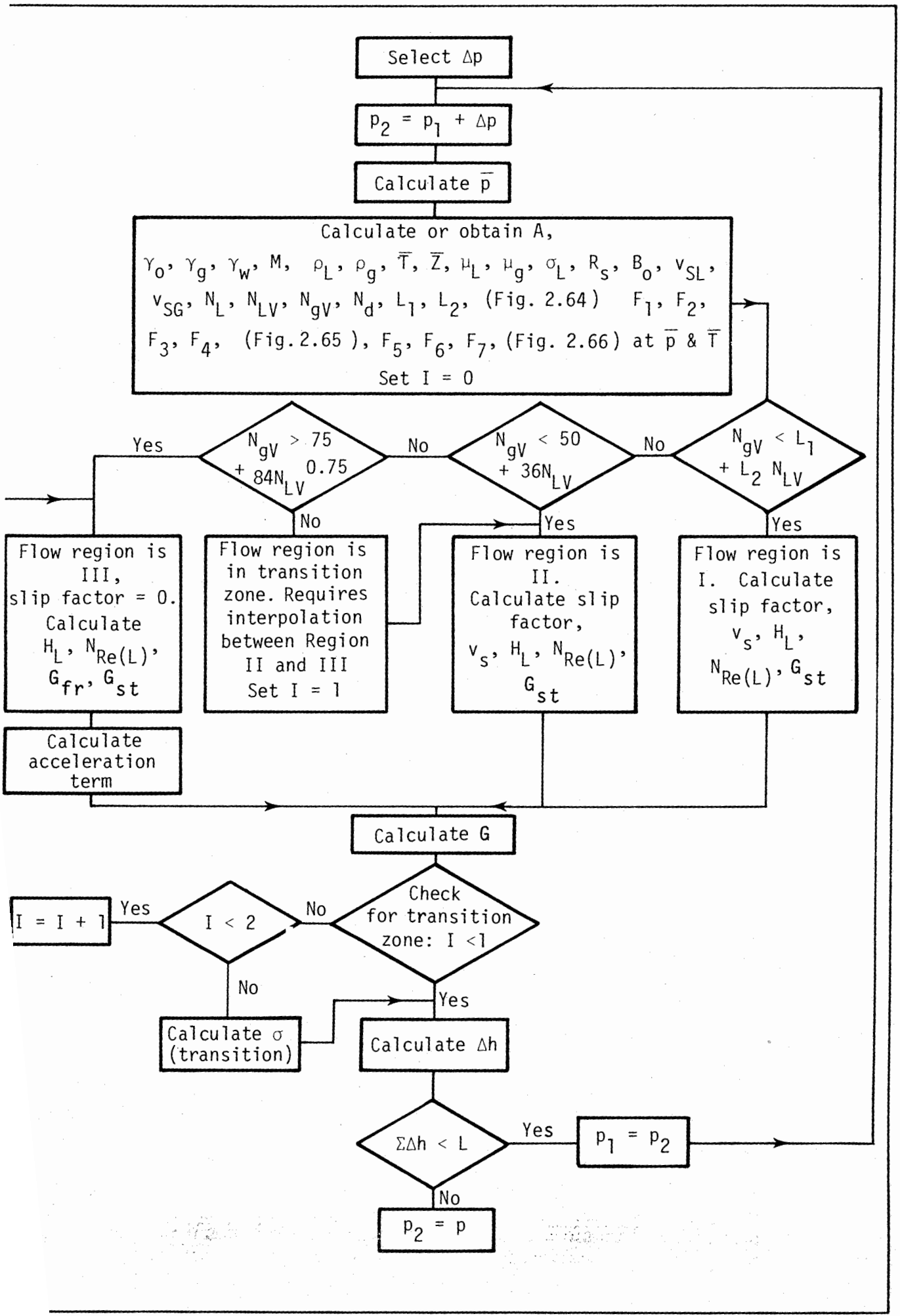


Fig. 2.69 Mist flow film thickness (after Ros).


Select Δp
$p_2 = p_1 + \Delta p$
Calculate $\bar{p}$
Calculate or obtain A,
γ_o, γ_g, γ_w, M, ρ_L, ρ_g, $\bar{T}$, $\bar{Z}$, μ_L, μ_g, σ_L, R_s, B_o, v_{SL}, v_{SG}, N_L, N_{LV}, N_{gV}, N_d, L_1, L_2, (Fig. 2.64) F_1, F_2, F_3, F_4, (Fig. 2.65), F_5, F_6, F_7, (Fig. 2.66) at $\bar{p}$ & $\bar{T}$
Set I = 0
$N_{gV} > 75 + 84N_{LV}^{0.75}$
Yes
No
$N_{gV} < 50 + 36N_{LV}$
No
$N_{gV} < L_1 + L_2 N_{LV}$
Yes
Yes
No
Flow region is III, slip factor = 0. Calculate H_L, $N_{Re(L)}$, G_{fr}, G_{st}
Flow region is in transition zone. Requires interpolation between Region II and III
Set I = 1
Flow region is II. Calculate slip factor, v_s, H_L, $N_{Re(L)}$, G_{st}
Flow region is I. Calculate slip factor, v_s, H_L, $N_{Re(L)}$, G_{st}
Calculate acceleration term
Calculate G
Check for transition zone: I <1
I < 2
Yes
No
I = I + 1
No
Yes
Calculate σ (transition)
Calculate Δh
ΣΔh < L
Yes
$p_1 = p_2$
No
$p_2 = p$

correlation should be valid for all water percentages, except where emulsions occur.

A detailed procedure for the calculation of a pressure traverse by the Duns and Ros method is given in Appendix C.51. The same example problem which was worked by other methods, has also been worked by the Duns and Ros method (Appendix C.52).

For 2 in. tubing the distance between the 500 and 1,000 psig pressure points was found to be 2,100 ft and for 1¼ in. tubing the distance was found to be 1,125 ft. These numbers compare to 2,425, 2,350, 1,870 and 1,870 ft in 2 in. tubing by the method of Poettman and Carpenter, Fancher and Brown, Hagedorn and Brown, and the modified method of Hagedorn and Brown, respectively.

The Duns and Ros correlation is one of the best for multiphase flow calculations that cover all ranges of flow. The range of their laboratory data is the most complete taken for this field of work. It is recommended that this Duns and Ros correlation be used with the assurance that a reasonable degree of accuracy is obtainable for all flow conditions.

The following procedure and example problems can be found in Appendix C.5.

(1) C.51 – Procedure for the calculation of a pressure traverse by the Duns and Ros method.
(2) C.52 – Example problem by the Duns and Ros method.

2.3334 Orkiszewski correlation

The Orkiszewski correlation[22] resulted from an analysis of the many published methods to determine if any one of them is broad enough to accurately predict pressure drops for a wide range of well conditions. The various methods were first categorized, based upon similarity in theoretical concepts. Certain methods were selected from each of the three categories that were established. These selections were based on whether the method was original or unique, and whether or not they were developed from a broad base of data. The discriminatory feature of each category was based on the way that liquid holdup was considered in the computation of density; how friction losses were handled; and whether a distinction was made in flow regimes.

In one category, the liquid holdup was not considered in the computation of density; the liquid holdup and wall friction losses were expressed by using an empirically correlated friction factor; and no distinction was made between flow regimes. Another category used holdup in the density computation; friction losses were based on the composite properties of the liquid and gas; and no distinction between flow regimes was made. The third category considered liquid holdup in the calculated density term; liquid holdup was determined from some concept of slip velocity; friction losses were determined by the properties of the continuous phase; and four flow regimes were recognized.

Orkiszewski emphasized that liquid holdup was derived from observed physical phenomena and that the pressure gradient was related to the geometrical distribution of the liquid and gas phase. He recognized four types of flow patterns and prepared separate correlations to establish the slippage velocity and friction for each. The four types of flow were bubble, slug, transition, and mist.

Because slug flow was present in 95% of the cases that he studied, Orkiszewski considered his correlation to be a modification of the work done by Griffith and Wallis.[15] Where transition and mist flow existed, he used the Duns and Ros correlation.[14]

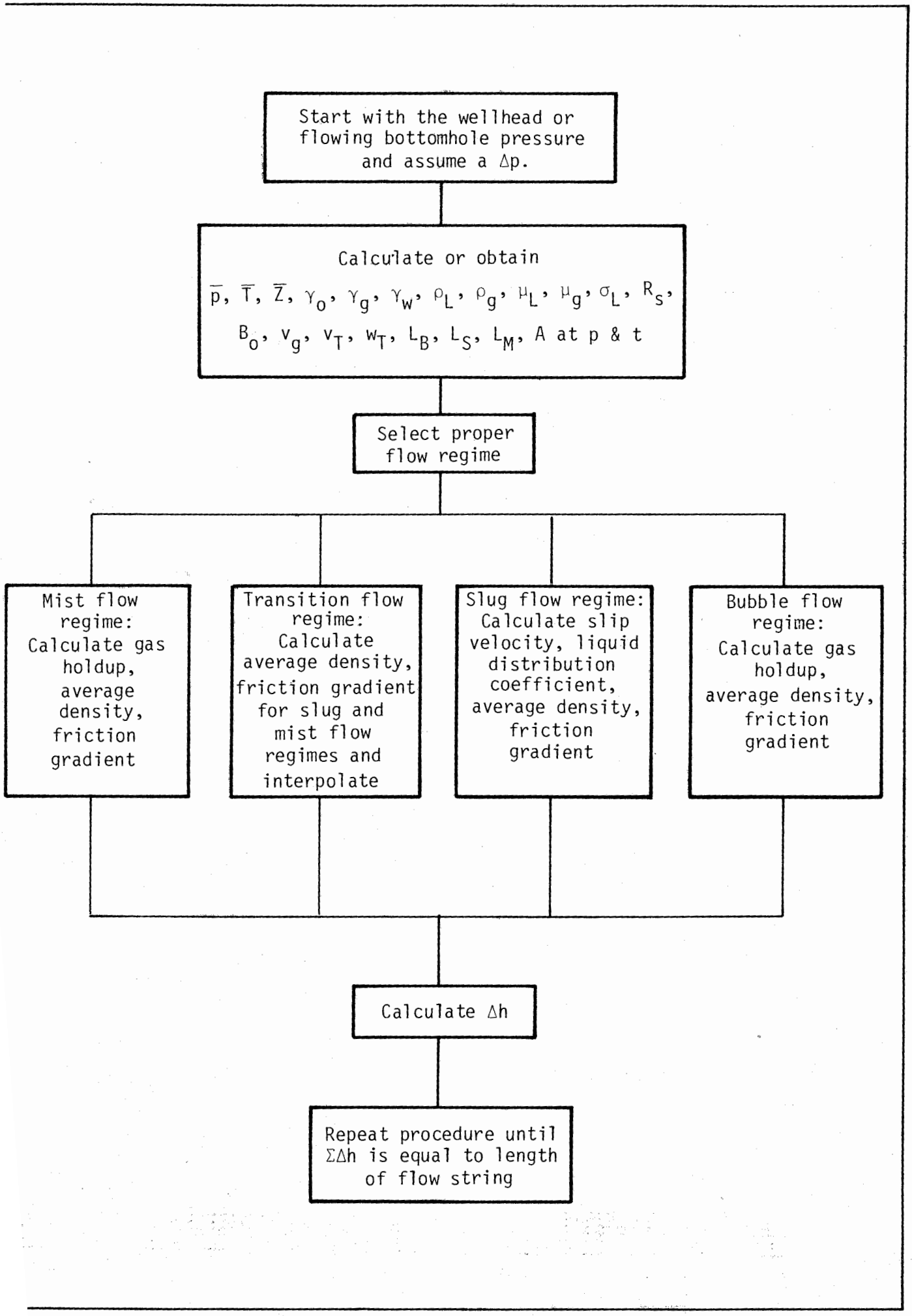
Considering similarity in theoretical concepts and different categories, Orkiszewski compared five selected methods and determined the amount of deviation between predicted and measured pressure drops. Two of the methods, that of Duns and Ros and that of Griffith and Wallis, showed the greatest accuracy. These two methods were then programmed for computer calculations and were further tested with data from 148 wells.

Neither method proved to be accurate over the entire range of conditions used in the test. The Griffith and Wallis correlation was reliable in the lower flow-rate range of slug flow, but it was inaccurate for the higher range of flow rates. The Duns and Ros method gave similar findings and also indicated an inaccuracy for high-viscosity oils in the low flow-rate range. The Griffith and Wallis correlation seemed to provide a better basis for an improved general solution, although its predicted values showed more error than the correlation of Duns and Ros. Therefore, the Griffith and Wallis correlation was selected by Orkiszewski because the prediction of slip velocity is derived from physical observation and the friction drop is negligible. The Duns and Ros method in this range is presented as a complex set of interrelated parameters and equations and is therefore difficult to relate to what physically occurs inside the pipe.

Orkiszewski extended the Griffith and Wallis[15] work to include the high-velocity flow range. A parameter was developed to account for (1) liquid distribution among the liquid slug, the liquid film, and entrained liquid in the gas bubble and (2) the liquid holdup at the higher flow velocities. This parameter was used to calculate wall friction losses and flowing density. It was correlated chiefly from the published data of Hagedorn and Brown.[21]

Orkiszewski summarized the results of the detailed study in Table 2.19.[22] He concluded that the modified Griffith and Wallis method is the only one of the three methods that predicts pressure drops with sufficient accuracy and precision over the entire range of the conditions used in the study. Also, the precision might be further improved if the liquid phase distribution could be more rigorously analyzed. Because the Griffith and Wallis method is expressed in terms of flow regime and liquid distribution, and because there are limitations in the other methods in this respect, Orkiszewski additionally concluded that for general engineering work this method is accurate over a broader range than other correlations.

Fig. 2.71 shows a generalized flow diagram of the method. After assuming a pressure difference and calculating the various required properties, a flow region is selected. Depending upon the flow region, pressure loss calculations – which in general include friction and holdup – are made. The vertical l



1 Flow diagram for the Orkiszewski method.

corresponding to the pressure difference is then determined.

The Orkiszewski equation is:

$$\Delta p_k = \frac{1}{144}\left|\frac{\rho + \tau_f}{1 - w_t q_g/4637\, A_p^2\, \bar{p}}\right|_k \Delta h_k, \quad (2.343)$$

where: $\bar{\rho}$ = average fluid density, lb/cu ft
Δp = pressure drop, psi
$\bar{p}$ = average pressure, psia
w_t = total mass flow rate, lb_m/sec
τ_f = friction loss gradient, psi/ft
q_g = gas volumetric flow rate, cu ft/sec
Δh = depth change, ft

The flow regime may be determined by testing $q_g/q_t = 1 - \lambda$, or v_g in accordance with the following limits:

Flow regime	Limits
Bubble flow	$q_g/q_t < (L)_B$
Slug	$q_g/q_t > (L)_B$, $\bar{\bar{v}}_g < (L)_S$
Transition	$(L)_M > \bar{\bar{v}}_g > (L)_S$
Mist	$\bar{\bar{v}}_g > (L)_M$

The above variables are expressed as:

$$\bar{\bar{v}}_g = \frac{q_g}{A_p}\left(\frac{\rho_L}{g\sigma}\right)^{1/4} \equiv N_{gv} \quad (2.344)$$

$$(L)_B = 1.071 - (0.2218\, v_t^2/d_{hy})$$

with the limit $(L)_B \geq 0.13$ (2.345)

$$(L)_S = 50 + 36\, \bar{\bar{v}}_g q_l/q_g \quad (2.346)$$

$$(L)_M = 75 + 84\, (\bar{\bar{v}}_g q_l/q_g)^{0.75} \quad (2.347)$$

where:

$\bar{\bar{v}}_g$ = dimensionless gas velocity
v_t = total fluid velocity (q_t/A_p), ft/sec
ρ_L = liquid density, lb_m/cu ft
σ = liquid surface tension, lb_m/sec^2
d_{hy} = hydraulic diameter, ft
q_L, q_g, q_t = volumetric flow rate of liquid, gas, and total flow rate respectively, cu ft/sec
$(L)_B$, $(L)_S$, $(L)_M$ = bubble-slug, slug-transition, transition-mist boundaries, respectively, dimensionless

The average density and the friction loss gradient are calculated by the following equations:

A) *Bubble flow:*

The void fraction of gas (H_g) can be expressed as:

$$H_g = \tfrac{1}{2}\left[1 + \frac{q_t}{v_s A_p} - \sqrt{(1 + q_t/v_s A_p)^2 - \frac{4q_g}{v_s A_p}}\right] \quad (2.348)$$

where v_s = slip velocity, ft/sec.

Griffith[15] stated that a good approximation of an average v_s is 0.8 ft/sec which is very important in that other equations fall out easily. Thus, the average flowing density can be computed as:

$$\bar{\rho} = H_L\, \rho_L + H_g\, \rho_g \quad (2.349)$$

The friction gradient is:

$$\tau_f = f\, \rho_L v_L^2/2g_c d_{hy} = \left(\frac{dp}{dh}\right)_f \quad (2.350)$$

where: $v_L = q_l/A_p H_L$ (2.351)
A_p = pipe area, sq ft
H_L = liquid holdup

The friction factor "f" is obtained from Fig. 2.72 by using a Moody[32] relative roughness factor obtained from Fig. 2.73. The Reynolds number is calculated from:

$$N_{Re} = 1{,}488\, \rho_L d_{hy} v_l/\mu_L \quad (2.352)$$

where: ρ_L = density of liquid, lb_m/cu ft
μ_L = liquid viscosity, cp

B) *Slug Flow:*

The average density term is:

$$\bar{\rho} = \frac{w_t + \rho_L v_s A_p}{q_t + v_s A_p} + \delta\rho_L \quad (2.353)$$

where $v_s = C_1 C_2 \sqrt{g d_{hy}}$ (2.354)
δ = liquid distribution coefficient.

The heart of the problem is to find v_s and δ. To find v_s, C_1 and C_2 must be determined as noted in Eq. 2.353. C_1 is expressed in Fig. 2.74 as a function of the bubble Reynolds number, (N_b), and C_2 is expressed in Fig. 2.75 as a function of both N_b and Reynolds number (N_{Re}). These Reynolds number factors are given by:

$$N_b = 1{,}488\, v_s d_{hy}\, \rho_l/\mu_L \quad (2.355)$$

$$N_{Re} = 1{,}488\, \rho_L d_{hy}\, v_t/\mu_L \quad (2.356)$$

Because v_s is a nonlinear correlation, it must be obtained by iteration. A value of v_s is assumed, N_b is calculated, and C_1 and C_2 are determined. If the value of v_s calculated from Eq. 2.354 does not agree with the assumed value, the procedure is repeated until a sufficiently close approximation is obtained.

The curves of Fig. 2.75 were extrapolated by Nicklin, Wilkes and Davidson.[33] This extrapolation was incorporated into the Orkiszewski correlation, and v_s also may be calculated by the following equations depending on the value of N_b.

When $N_b \leq 3{,}000$:

$$v_s = (0.546 + 8.74 \times 10^{-6}\, N_{Re})\sqrt{g d_{hy}} \quad (2.357)$$

When $N_b \geq 8{,}000$:

$$v_s = (0.35 + 8.74 \times 10^{-6}\, N_{Re})\sqrt{g d_{hy}} \quad (2.358)$$

When $3{,}000 < N_b < 8{,}000$:

$$v_{si} = (0.251 + 8.74 \times 10^{-6}\, N_{Re})\sqrt{g d_{hy}} \quad (2.359)$$

$$v_s = \tfrac{1}{2}\left[v_{si} + \sqrt{v_{si}^2 + \frac{13.59\, \mu_L}{\rho_L \sqrt{d_{hy}}}}\right] \quad (2.360)$$

The value of δ is dependent on the continuous liquid phase and whether v_s is less than or greater than 10; therefore, δ is calculated by the equation which meets the following conditions.

If $v_t < 10$:

a) Continuous liquid phase—water;

$$\delta = 0.013 \log \mu_L/d_{hy}^{1.38} - 0.681 + 0.232 \log v_t - 0.428 \log d_{hy} \quad (2.361)$$

b) Continuous liquid phase—oil;

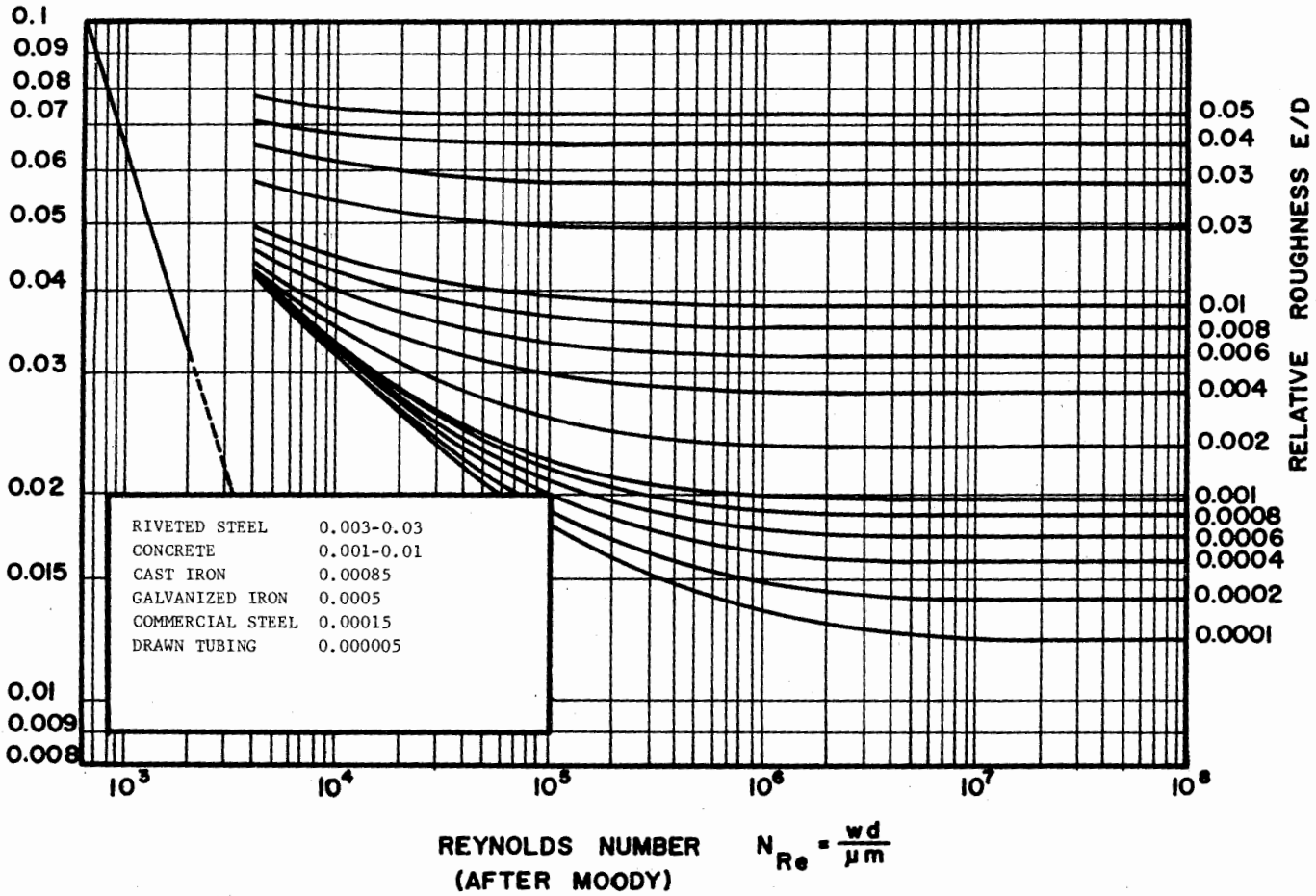


Fig. 2.72 *Friction factor curve.*

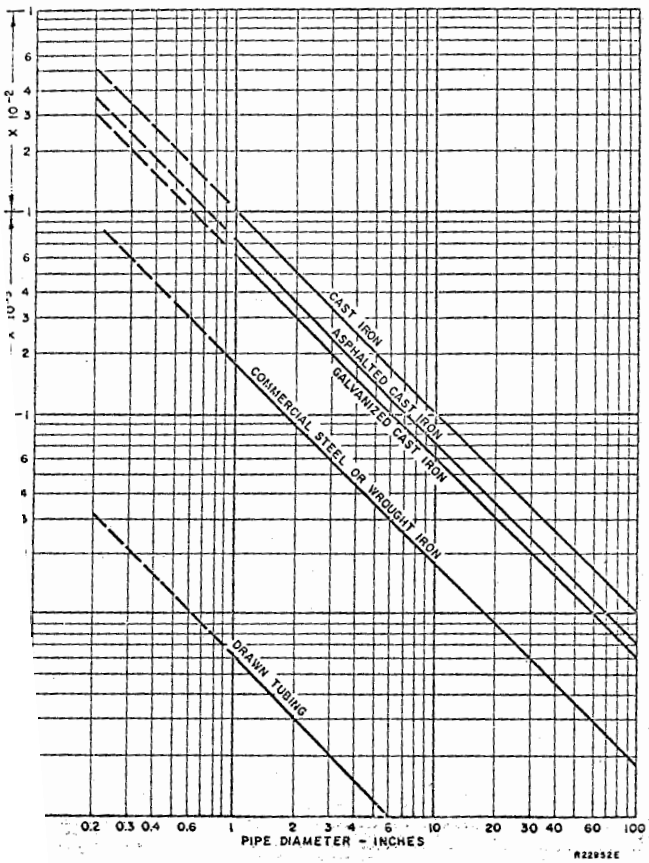


Fig. 2.73 *Effect of pipe diameter and material on relative rough-*

$$\delta = 0.0127 \log(\mu_L + 1)/d_{hy}^{1.415} - 0.284 + 0.167 \log v_t + 0.113 \log d_{hy} \quad (2.362)$$

If $v_t > 10$:

a) Continuous liquid phase—water;

$$\delta = (0.045 \log \mu_L)/d_{hy}^{0.799} - 0.709 - 0.162 \log v_t - 0.888 \log d_{hy} \quad (2.363)$$

b) Continuous liquid phase—oil;

$$\delta = 0.0274 \log(\mu_L + 1)/d_{hy}^{1.371} + 0.161 + 0.569 \log d_{hy} + X \quad (2.364)$$

where $X = -\log v_t(0.01 \log(\mu_L + 1)/d_{hy}^{1.571} + 0.397 + 0.63 \log d_{hy}$

The value of δ is constrained by the following limits:

(a) $v_t < 10$:

$$\delta \geq -0.065\, v_t \quad (2.365)$$

(b) $v_t > 10$:

$$\delta \geq \frac{-v_s A_p}{q_t + v_s A_p}(1 - \bar{\rho}/\rho_L) \quad (2.366)$$

Reed[34] points out that Eqs. 2.361 and 2.363 for calculating the liquid distribution coefficient do not give the same value at $v_t = 10$ for all flow conditions. This could cause a discontinuity in the liquid distribution coefficient and the pressure gradient at $v_t = 10$. An example of this situation is given in which the calculated pressure gradient decreased by one-half as v_t increased past 10. Therefore, when using the Orkiszewski cor-

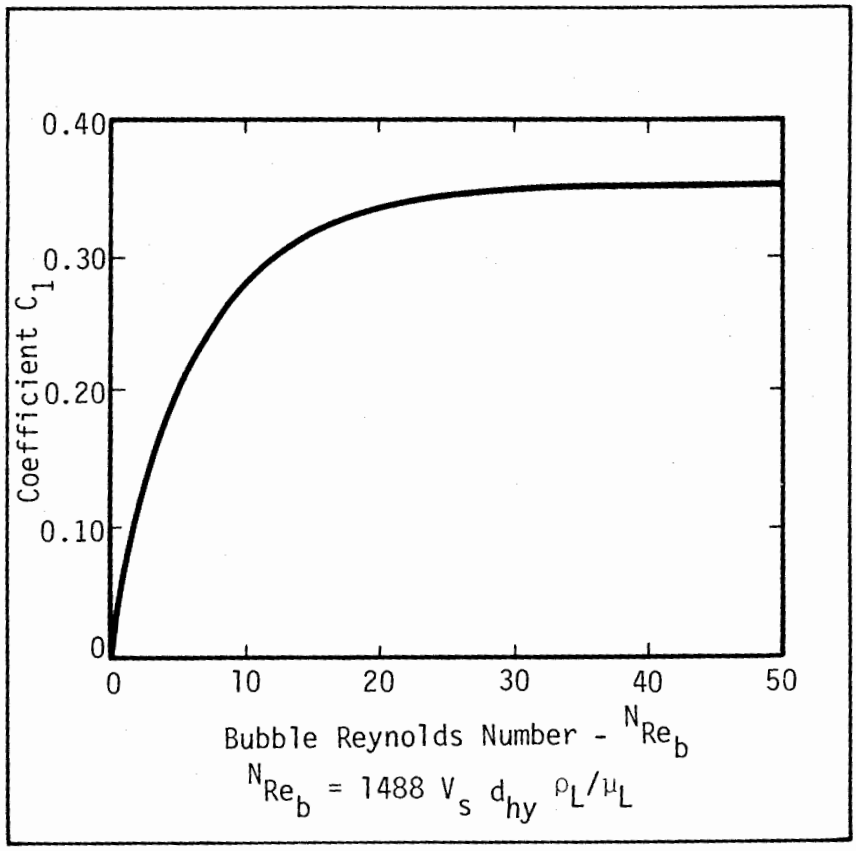


Fig. 2.74 Griffith and Wallis' C_1 vs. Bubble Reynolds number.

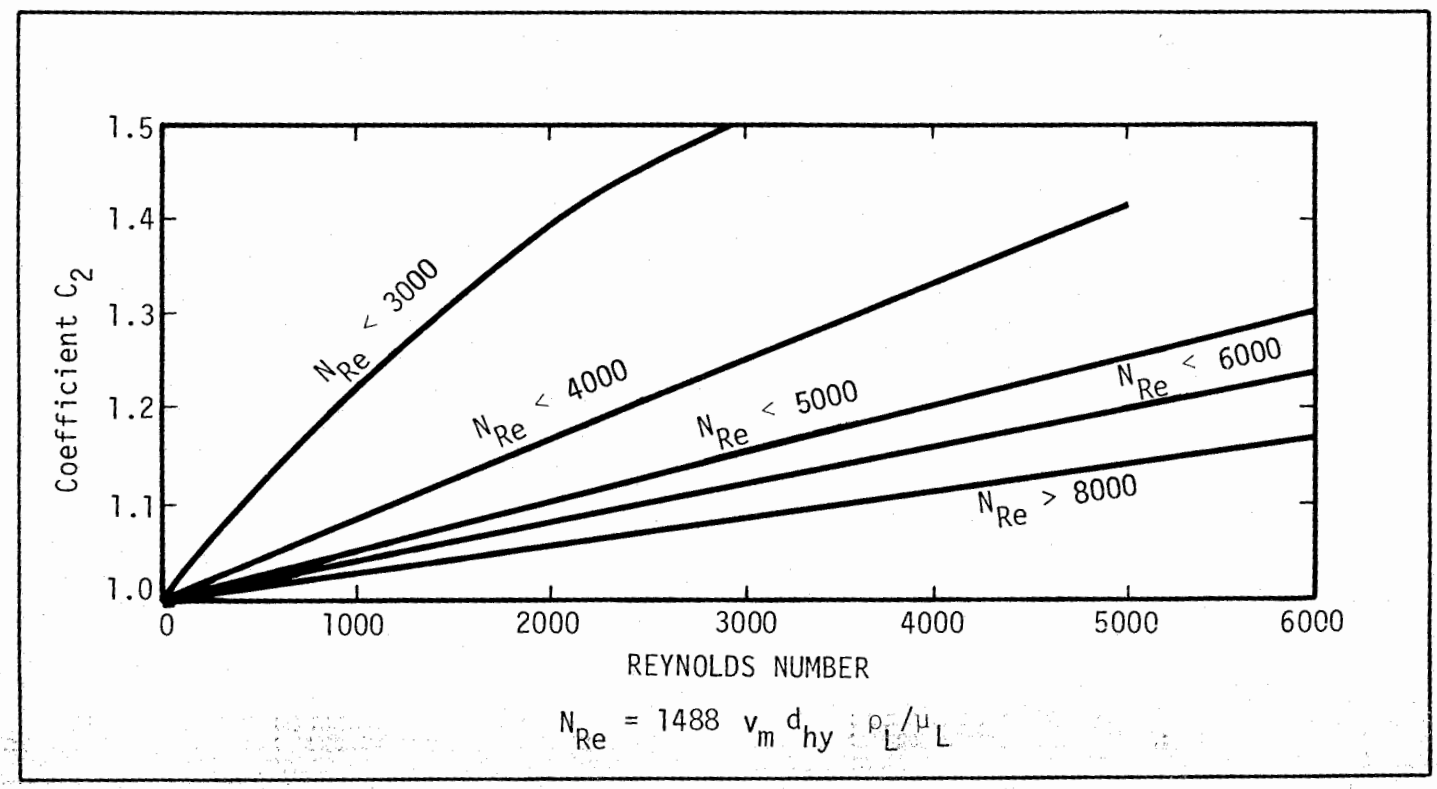


Fig. 2.75 Griffith and Wallis' C_2 vs. Bubble Reynolds number.

elation in the slug flow regime, it is suggested that nese equations be checked for continuity for this spe- ial value of v_t.

The value of the wall friction-loss term in the slug ow regime is obtained by:

$$\tau_f = \frac{f_L v_t^2}{2g_c\, d_{hy}} \left[\frac{q_L + v_s}{q_t + v_s A_p} + \delta \right] \quad (2.367)$$

) *Transition flow:*

The average density term calculated by the Duns nd Ros method is:

$$\bar{\rho} = \frac{(L)_M - \bar{\bar{v}}_g}{(L)_M - (L)_S} \bar{\rho}_{slug} + \frac{\bar{\bar{v}}_g - (L)_S}{(L)_M - (L)_S} \bar{\rho}_{mist} \quad (2.368)$$

nd the friction gradient which is weighted in the same anner is calculated by:

$$\tau_f = \frac{(L)_M - \bar{\bar{v}}_g}{(L)_M - (L)_S} (\tau_f)_{slug} + \frac{\bar{\bar{v}}_g - (L)_S}{(L)_M - (L)_S} (\tau_f)_{mist} \quad (2.369)$$

lore accuracy is attained in the friction loss prediction the gas volumetric flow rate for mist flow is used. his flow rate is given by:

$$q_g = A_p\, (L)_M (\rho_L / g\sigma)^{-1/4} \quad (2.370)$$

) *Mist flow:*

$\bar{\rho}$ is calculated by:

$$\bar{\rho} = (H_L)\, \rho_L + H_g\, \rho_g \quad (2.371)$$

ecause there is virtually no slip between the phases, $_g$ is:

$$H_g = 1/(1 + q_L/q_g) = q_g/q_T = V_{sg}/v_m \quad (2.372)$$

he friction loss gradient is expressed by Duns and os as:

$$\tau_f = f\, \rho_g v_{sg} / 2g_c\, d_{hy} \quad (2.373)$$

here v_{sg} is the superficial gas velocity and f is ob- ined from Fig. 2.72 as a function of the gas Reynolds mber:

$$N_{Re_g} = 1{,}488\, \rho_g\, d_{hy}\, v_{sg}/\mu_g \quad (2.374)$$

correlated form of the Moody relative roughness or ϵ/d_{hy} was developed by Duns and Ros. This fac- was also used in their correlation with specified tations that the factor be no smaller than 0.001 no greater than 0.5. Within these prescribed limits factor is determined by one of the following equa- , depending on the value of N_w:

< 0.005:

$$\epsilon/d_{hy} = 34\, \sigma/(\rho_g\, v_{sg}^2\, d_{hy}) \quad (2.375)$$

> 0.005:

$$\epsilon/d_{hy} = 174.8\, \sigma (N_w)^{0.302}/(\rho_g\, v_{sg}^2\, d_{hy}) \quad (2.376)$$

$$N_w = 4.52 \times 10^{-7} (v_{sg}\, \mu_l/\sigma)^2\, \rho_g/\rho_l \quad (2.377)$$

ence should be made to Fig. 2.76 which shows led flow diagram for the calculation of a pres- averse and summarizes the preceding discus-

ailed procedure for the calculation of a pressure traverse by the Orkiszewski method is given in Appendix C.6. The same example problem which was worked by other methods has also been worked by the Orkiszewski method (Appendix C.63).

For 2 in. tubing the distance between the 500 and 1,000 psig pressure points was found to be 2,178 ft and for 1¼ in. tubing the distance was found to be 1,080 ft. These numbers compare to 1,870 and 2,100 ft in 2 in. tubing by the generalized method of Hagedorn and Brown and the Duns and Ros method, respectively.

The following procedure and example problems can be found in Appendix C.6.

(1) C.61—Procedure for the calculation of a pressure traverse by the Orkiszewski method.
(2) C.62—Example problem for the calculation of a pressure traverse by Orkiszewski method (same example presented by Orkiszewski[22]).
(3) C.63—Example problem for the calculation of a pressure traverse by the Orkiszewski method (same problem as used in other methods).

2.3335 Beggs and Brill correlation

The Beggs and Brill correlation[25] was developed from experimental data obtained in a small scale test facility. The facility consisted of 1-in. and 1.5-in. sections of acrylic pipe 90 ft long. The pipe could be inclined at any angle. The parameters studied and their range of variation were (1) gas flow rate, 0 to 300 mscfd; (2) liquid flow rate, 0 to 30 gal/min; (3) average system pressure, 35 to 95 psia; (4) pipe diameter, 1 and 1.5-in.; (5) liquid holdup, 0 to 0.870; (6) pressure gradient, 0 to 0.8 psi/ft; (7) inclination angle, −90° to +90°; and (8) horizontal flow pattern. Fluids used were air and water.

For each pipe size, liquid and gas rates were varied so that all flow patterns were observed when the pipe was horizontal. After a particular set of flow rates was set, the angle of the pipe was varied through the range of angles so that the effect of angle on holdup and pressure gradient could be observed. Liquid holdup and pressure gradient were measured at angles from horizontal of 0, plus and minus 5, 10, 15, 20, 35, 55, 75 and 90 degrees. The correlations were developed from 584 measured tests.

Different correlations for liquid holdup are presented for each of three *horizontal* flow regimes. The liquid holdup which would exist *if the pipe were horizontal* is first calculated and then corrected for the actual pipe inclination angle. The horizontal flow patterns are illustrated in Fig. 2.77. The variation of liquid holdup with pipe inclination is shown in Fig. 2.78 for three of the tests. The holdup was found to be a maximum at approximately +50 degrees from horizontal and a minimum at approximately −50 degrees. The original flow pattern map has been slightly modified to include a transition zone between the segregated and intermittent flow regimes. The modified flow pattern map is superimposed on the original in Fig. 2.79. A two-phase friction factor is calculated using equations which are independent of flow regime but depend on holdup. A graph of a normalized friction factor has a function of liquid holdup and input liquid content is shown in Fig. 2.80.

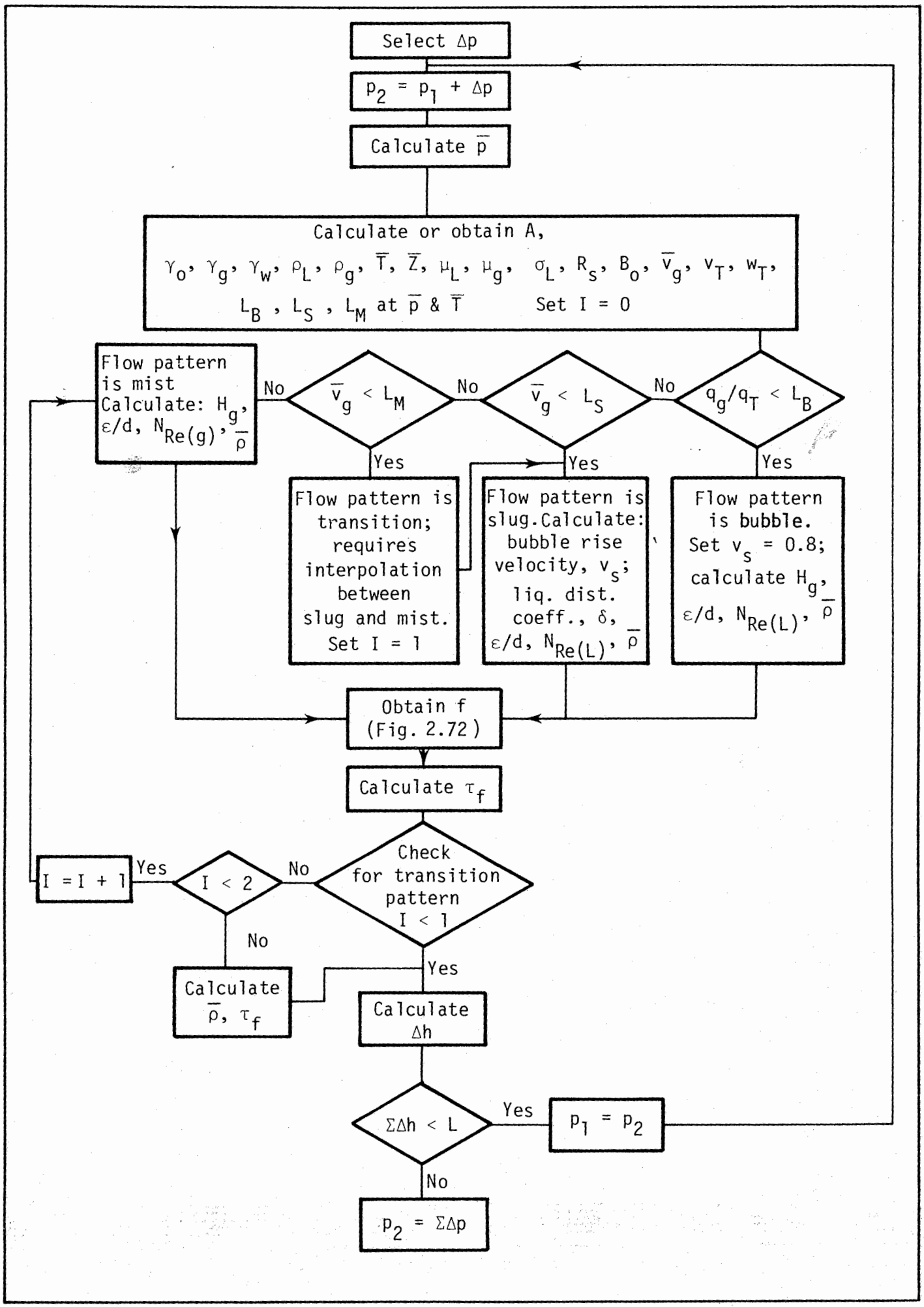
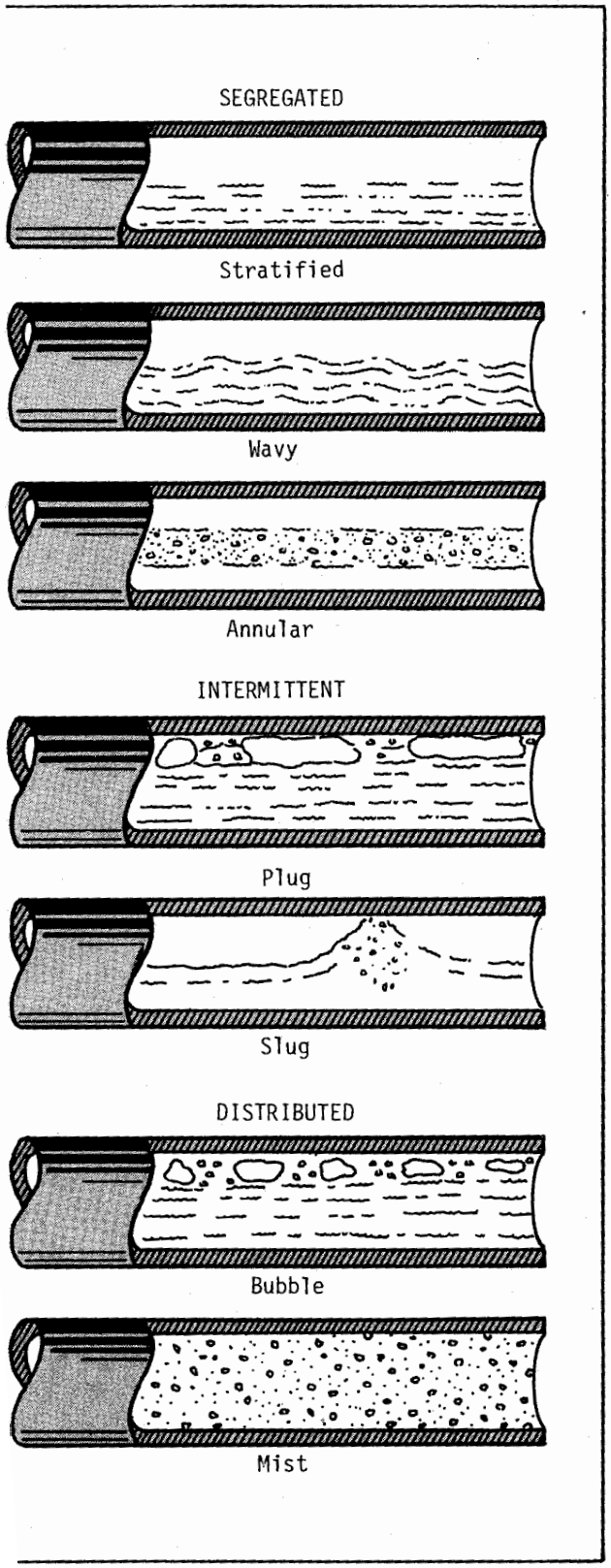


Fig. 2.76 Detailed flow diagram for the Orkiszewski method.



77 *Horizontal flow patterns.*

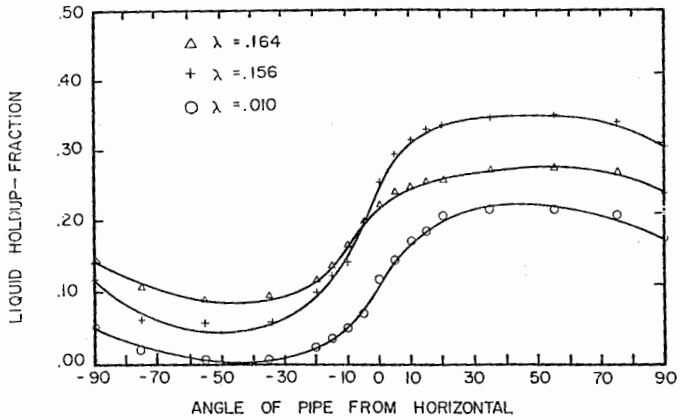


Fig. 2.78 Liquid holdup vs. angle.

$$N_{FR} = \frac{v_m^2}{g\,d}$$

$$\lambda_L = \frac{v_{sL}}{v_m}$$

$$L_1 = 316\,\lambda_L^{0.302}$$
$$L_2 = 0.0009252\,\lambda_L^{-2.4684}$$
$$L_3 = 0.10\,\lambda_L^{-1.4516}$$
$$L_4 = 0.5\,\lambda_L^{-6.738}$$

The horizontal flow regime limits are:

SEGREGATED:

Limits: $\lambda_L < 0.01$ and $N_{FR} < L_1$

or $\lambda_L \geq 0.01$ and $N_{FR} < L_2$

TRANSITION:

Limits: $\lambda_L \geq 0.01$ and $L_2 < N_{FR} \leq L_3$

INTERMITTENT:

Limits: $0.01 \leq \lambda_L < 0.4$ and $L_3 < N_{FR} \leq L_1$

or $\lambda_L \geq 0.4$ and $L_3 < N_{FR} \leq L_4$

DISTRIBUTED:

Limits: $\lambda_L < 0.4$ and $N_{FR} \geq L_1$

or $\lambda_L \geq 0.4$ and $N_{FR} > L_4$

When the flow falls in the transition regime the liquid holdup must be calculated using both the segregated and intermittent equations and interpolated using the following weighting factors.

H_L (transition) = A × H_L (segregated) + B × H_L intermittent)

where: $$A = \frac{L_3 - N_{FR}}{L_3 - L_2}$$

$$B = 1 - A$$

The same equations are used to calculate liquid holdup for all flow regimes. The coefficients and exponents used in the equations are different for each flow regime.

Two-phase density:

$$H_{L(\phi)} = H_{L(0)}\,\psi \qquad (2.378)$$

egime determination:
following variables are used to determine which gime would exist if the pipe were in a *horizontal* n. This flow regime is a correlating parameter ves no information about the actual flow regime the pipe is horizontal.

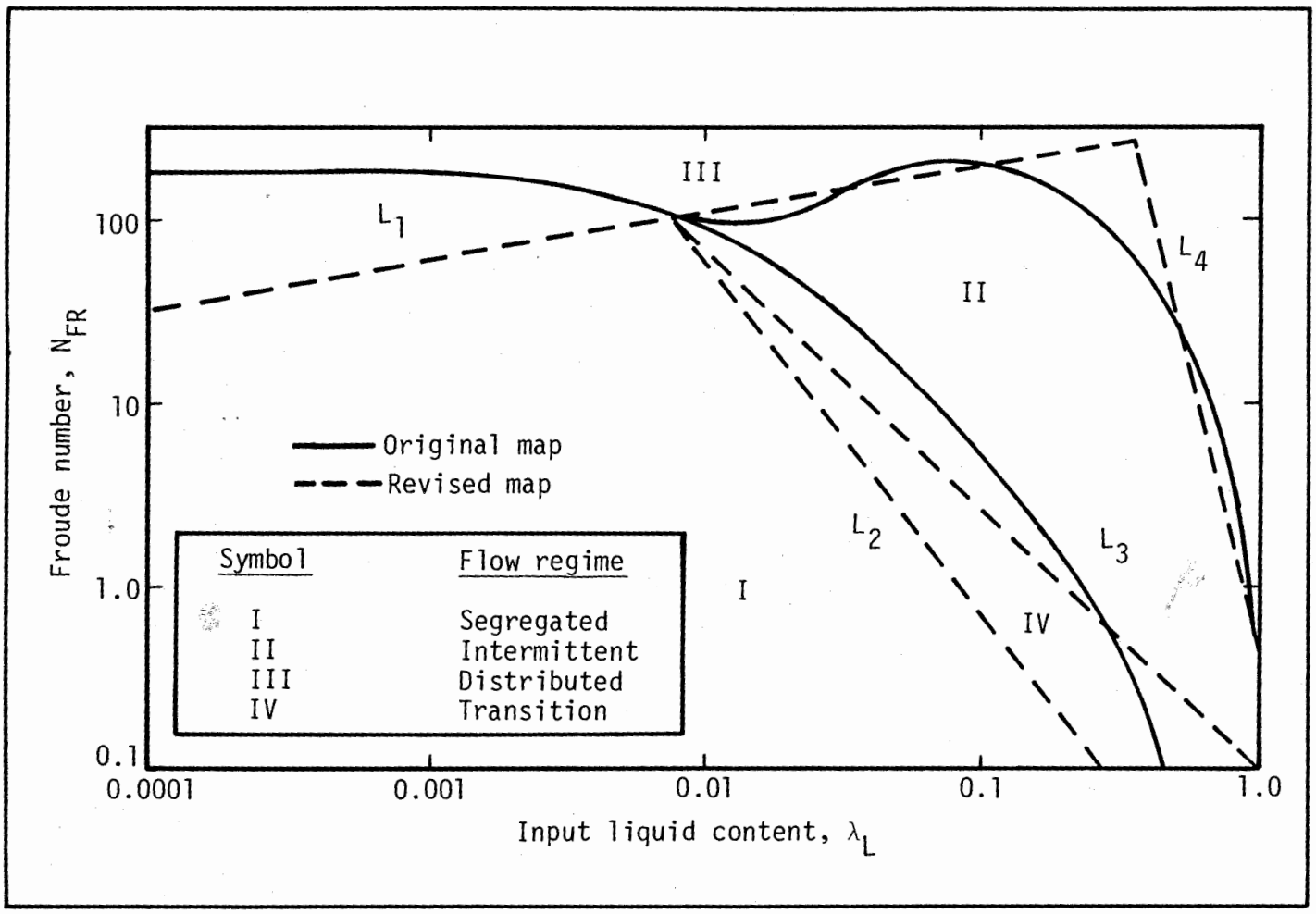


Fig. 2.79 Horizontal flow pattern map.

where $H_{L(0)}$ is the holdup which would exist at the same conditions in a horizontal pipe. It is calculated from:

$$H_{L(0)} = \frac{a\,\lambda_L{}^b}{N_{FR}{}^c} \qquad (2.379)$$

where a, b, and c are determined for each flow pattern from table (2.32-(a))

TABLE 2.32-(a)

Flow pattern	a	b	c
Segregated	0.98	0.4846	0.0868
Intermittent	0.845	0.5351	0.0173
Distributed	1.065	0.5824	0.0609

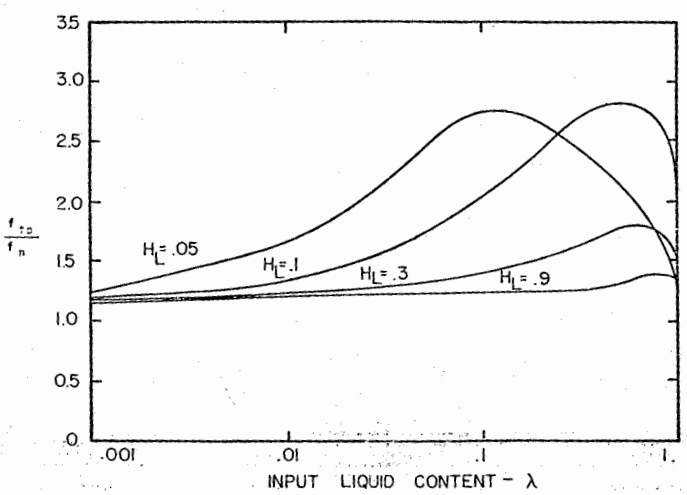


Fig. 2.80 Two-phase friction factor.

with the restriction that $H_{L(0)} \geq \lambda_L$

The factor for correcting the holdup for the effect of pipe inclination is given by:

$$\psi = 1 + C\,[\sin(1.8\phi) - 0.333\,\sin^3(1.8\phi)] \qquad (2.380)$$

where ϕ is the *actual* angle of the pipe from horizontal.

For vertical upward flow, $\phi = 90°$ and ψ becomes

$$\psi = 1 + 0.3\,C \qquad (2.381)$$

where:

$$C = (1 - \lambda_L)\,\ln(d\,\lambda_L{}^e\,N_{LV}{}^f\,N_{FR}{}^g) \qquad (2.382)$$

where d, e, f, and g are determined for each flow condition from Table 2.32(b).

TABLE 2.32-(b)

low pattern	d	e	f	g
regated uphill	0.011	−3.768	3.539	−1.614
rmittent uphill	2.96	0.305	−0.4473	0.0978
ributed uphill	No correction		$C = 0, \psi = 1, H_L \neq f(\phi)$	
low patterns ownhill	4.70	−0.3692	0.1244	−0.5056

h the restriction that $C \geq 0$.

ction factor:

$$\left(\frac{dp}{dz}\right)_f = \frac{f_{tp}\,\rho_n\,v_m^2}{2\,g_c\,d} \tag{2.383}$$

ere: $\rho_n = \rho_L\,\lambda_L + \rho_g\,\lambda_g$

$$f_{tp} = f_n\,\frac{f_{tp}}{f_n} \tag{2.384}$$

he no-slip friction factor is determined from the ooth pipe curve on a Moody diagram or from

$$_1 = 1/[2\log(N_{Re}/4.5223\log N_{Re} - 3.8215)]^2 \cdots \tag{2.385}$$

ng the following Reynolds number:

$$N_{Re} = \frac{\rho_n\,v_m\,d}{\mu_n} \tag{2.386}$$

ere: $\mu_n = \mu_L\,\lambda_L + \mu_g\,\lambda_g$

he ratio of the two-phase to no-slip friction factor is culated from:

$$\frac{f_{tp}}{f_n} = e^S \tag{2.387}$$

ere: $S = [\ln(y)]/\{-0.0523 + 3.182\ln(y) - 0.8725[\ln(y)]^2 + 0.01853[\ln(y)]^4\} \cdots$ (2.388)

l: $y = \dfrac{\lambda_L}{[H_{L(\phi)}]^2}$ (2.389)

e value of S becomes unbounded at a point in the erval $1 < y < 1.2$; and for y in this interval, the func- n S is calculated from:

$$S = \ln(2.2y - 1.2) \tag{2.390}$$

eleration term:

lthough the acceleration pressure gradient is very all except for high velocity flow, it should be in- ded if possible.

$$\left(\frac{dp}{dz}\right)_{acc} = \frac{\rho_s\,v_m\,v_{sg}}{g_c p}\,\frac{dp}{dz}$$

we define an acceleration term as

$$E_k = \frac{\rho_s v_m v_{sg}}{g_c p} \tag{2.391}$$

otal pressure gradient can be calculated from:

$$\frac{dp}{dz} = \frac{\left(\frac{dp}{dz}\right)_{el} + \left(\frac{dp}{dz}\right)_f}{1 - E_k} \tag{2.392}$$

where: $\rho_s = \rho_L H_L + \rho_g H_g$,

$$\left(\frac{dp}{dz}\right)_{el} = \frac{g}{g_c}\,\rho_s \tag{2.393}$$

A flow diagram for calculating a pressure traverse in a vertical well is shown in Fig. 2.81. A detailed procedure for the calculation of a pressure traverse by the Beggs and Brill method is given in Appendix C.71. The same example problem which was worked by other methods has also been worked by the Beggs and Brill method (Appendix C.72).

For 2 in. tubing the distance between the 500 and 1,000 psig pressure points was found to be 1,901 ft. This compares to 1,870, 2,100 and 2,178 ft found by the methods of Hagedorn and Brown, Duns and Ros, and Orkiszewski respectively.

The following procedure and example problem can be found in Appendix C.7.

(1) C.71—Procedure for the calculation of a pressure traverse by the Beggs and Brill method.

(2) C.72—Example problem for the calculation of a pressure traverse by the Beggs and Brill method (same problem as used in other methods).

In 1972 Aziz, Govier, and Fogarasi[23] presented a correlation which was flow regime dependent. They also proposed a new vertical two-phase flow regime map. New methods were given for the bubble and slug flow regimes but the Duns and Ros method was used for mist flow. In the comparison study by Vohra, et al[35] the Aziz correlation performed with accuracy comparable to the Orkiszewski method.

Chierici, Ciucci, and Sclocchi[26] in 1973 proposed a new method to be used in the slug flow regime only. They suggested using the Griffith and Wallis correlation in bubble flow and the Duns and Ros method in mist flow. Chierici, et al also specified which fluid property correlations were to be used for calculating two-phase flowing pressure gradients by their method. The Vohra, et al study showed that this method calculates pressure drops too large in most cases.

2.34 Casing annular flow

2.341 Introduction

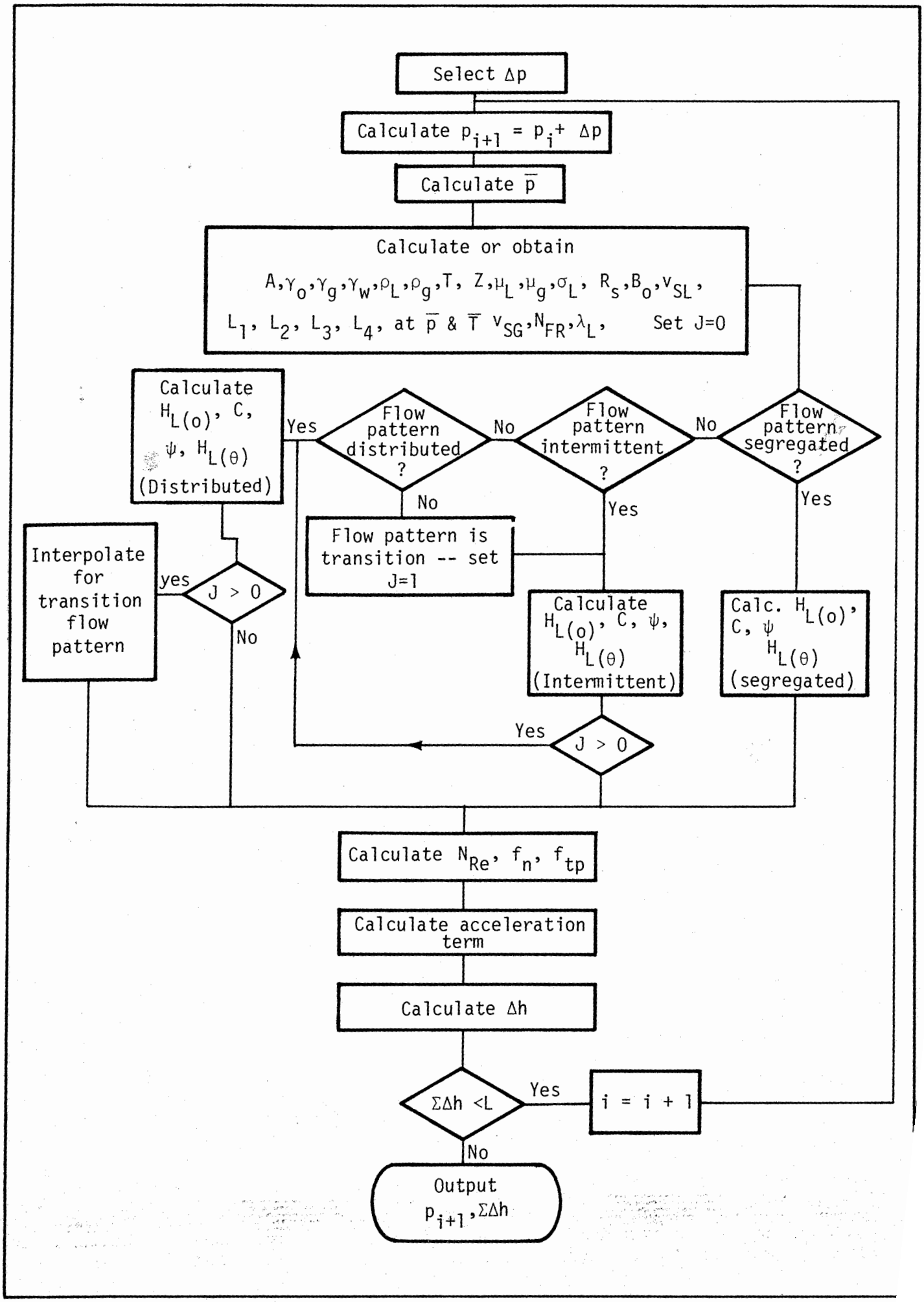
There are many wells in the world today that produce at high flow rates. Many of these produce through the annulus. High rate wells, although few in number when compared with the total number of producing wells world-wide, account for a considerable portion of the world's production. Many of the previously discussed correlations can be adapted to annular flow with the following general substitution for hydraulic diameter. We recall that the definition for hydraulic radius is:

$$r_h = \frac{\text{cross-sectional area}}{\text{wetted perimeter}} \tag{2.394}$$

For annular flow:

$$r_h = \frac{\pi(d_o^2 - d_i^2)/4}{\pi(d_o + d_i)} = \frac{d_o - d_i}{4} \tag{2.395}$$

where: d_o = outside diameter (external)
d_i = inside diameter (internal)


Select Δp
Calculate $p_{i+1} = p_i + \Delta p$
Calculate $\bar{p}$
Calculate or obtain
$A, \gamma_o, \gamma_g, \gamma_w, \rho_L, \rho_g, T, Z, \mu_L, \mu_g, \sigma_L, R_s, B_o, v_{SL}$,
L_1, L_2, L_3, L_4, at $\bar{p}$ & $\bar{T}$ $v_{SG}, N_{FR}, \lambda_L$, Set J=0
Calculate $H_{L(o)}$, C, ψ, $H_{L(\theta)}$ (Distributed)
Yes
Flow pattern distributed ?
No
Flow pattern intermittent ?
No
Flow pattern segregated ?
Yes
No
Yes
Flow pattern is transition -- set J=1
Interpolate for transition flow pattern
yes
J > 0
No
Calculate $H_{L(o)}$, C, ψ, $H_{L(\theta)}$ (Intermittent)
Calc. $H_{L(o)}$, C, ψ $H_{L(\theta)}$ (segregated)
Yes
J > 0
Calculate N_{Re}, f_n, f_{tp}
Calculate acceleration term
Calculate Δh
$\Sigma\Delta h < L$
Yes
i = i + 1
No
Output $p_{i+1}, \Sigma\Delta h$

ır a circle $r_h = \frac{d_o}{4}$ it is implied that the equivalent ameter = 4 r_h. Therefore

$$dia_{eqv} = d_h = d_o - d_i \qquad (2.396)$$

ıerefore for annular flow we can use the following bstitutions into equations commonly used for tubing ıw. When the diameter appears to the first power we ake the direct substitution of $d_o - d_i$ for diameter ch as the Reynolds number where $N_{Re} = \frac{dv\rho}{\mu}$ or the ughness factor of $\frac{K}{d}$. In many correlations the diam-er appears to the 5th power and then we can substi-te as follows: For d^5 we substitute $(d_o + d_i)^2(d_o - d_i)^3$ hich is arrived at as follows: $d^5 = d \times d^4$ where d^4 ›pears in (area)^2 we have:

$$\begin{aligned}&(d_o - d_i)(d_o^2 - d_i^2)^2\\ &= (d_o - d_i)\,[(d_o - d_i)(d_o + d_i)]^2\\ &= (d_o + d_i)^2\,(d_o - d_i)^3\end{aligned}$$

Most of the several correlations discussed previously ın be used for annular flow. Only a very few correla-ons such as that of Baxendell[12] have been proposed ıly for annular flow.

Sanchez[24] calculated the bottom-hole flowing pres-ıre for many wells, using the methods of Baxendell, 'uns and Ros, Orkiszewski, and Hagedorn and Brown. He found the results shown in Table 2.33.

TABLE 2.33

	Baxendell	Duns & Ros	Orkiszewski	Hagedorn & Brown
⁄erage error	−5.04	+0.215	+1.75	−8.89
tandard eviation	2.95	6.94	4.40	5.82

.342 Cornish method

Cornish[27] presented a rather simple correlation in /hich he calculated the same bottom-hole flowing ressures as Sanchez and found the average percent rror to be +1.31 and the standard deviation to be .94. In using his method it is necessary to have lab-ratory derived PVT data in order to calculate density, ıd that the wells make only small volumes of water. his correlation places no upper limit on the oil flow te, but unfortunately no quantitative lower limit as found. Other correlations such as Duns and Ros,[14] agedorn and Brown,[21] and Orkiszewski[22] may be ed at the lower rates.

However, Cornish noted from personal experience ıt accurate traverses can be predicted, using PVT :a to calculate the mixture density, for rates as low 5000 bpd in a 9⅝-in. casing by 2⅞-in. tubing ıulus. This is equivalent to a minimum two-phase nolds number of 1.78×10^5, and it is tentatively gested that a flow rate equivalent to a two-phase nolds number of about 10^5 is the lower limit to the rate range of applicability for this method.

e used the same form of the gradient equation as other investigators and included the acceleration term. Although these are high rate wells the velocity is normally low due to large flow areas.

Since this correlation does not appear in the literature, it is discussed in some detail in the following section.

The pressure traverse equation used by Cornish is as follows:

$$\Delta h = \frac{144\Delta p}{\bar{\rho}(1 + K)} \qquad (2.397)$$

where: $$K = \left(\frac{w}{\rho}\right)\frac{1}{2g}\left\{\frac{f}{d} + \frac{2\ln \rho_1/\rho_2}{\Delta h}\right\} \qquad (2.398)$$

Δp = pressure drop, lb/in^2
Δh = incremental height, ft
$\bar{\rho}$ = average fluid density, lb/ft^3
w = mass flow rate per unit area, lb/sec ft^2
f = friction factor
d = pipe diameter, ft
g = gravitational acceleration, ft/sec^2
ρ_1, ρ_2 = fluid density at limits of pressure increment p, lb/ft^3

The second term of equation 2.398 accounts for acceleration.

This equation is similar in many ways to that of Poettmann and Carpenter[1] and Hagedorn and Brown[21] and can be derived from the general energy equation. The fundamental difference between the work of Cornish and others is that no general correlations are made from masses of field and/or experimental data. The pressure traverse is evaluated using PVT data for the crude oil being considered, and friction factors obtained from a standard Moody Diagram.

Cornish[27] assumed a "no-slip" holdup value to calculate ρ_m. Since most rates are very high, this should be satisfactory. Cornish[27] suggested using fluid density as derived from PVT data as noted in Fig. 2.82.

If PVT data is not available a "no-slip" density must be calculated. For example if Standing's correlation is used for B_o and R_s, some error may be introduced as shown in Fig. 2.83.

In order to obtain a value for "f" note Fig. 2.72.

We calculate $$N_{Re} = \frac{wd}{\mu_m} \qquad (2.399)$$

where: w = mass flow rate per unit area
d = diameter, ft
μ_m = mixture viscosity, lb/ft-sec

As for other flow problems f is a function of N_{Re}, and e/d where e = characteristic roughness height and $\mu_m = (\mu_L{}^{H_L})(\mu_g{}^{(1-H_L)})$

For tubing flow, the roughness presents no problem, and common values are given in Fig. 2.72. However, for casing annulus flow, typical of the majority of high rate wells, some means of accounting for the different roughness of the inside of the casing and the outside of the tubing must be devised. It is proposed that the effective roughness height be determined by the relationship.

$$e' = e_c\left(\frac{d_c}{d_c + d_t}\right) + e_t\left(\frac{d_t}{d_c + d_t}\right) \qquad (2.3100)$$

where: e′ = effective roughness, ft
d = diameter, ft

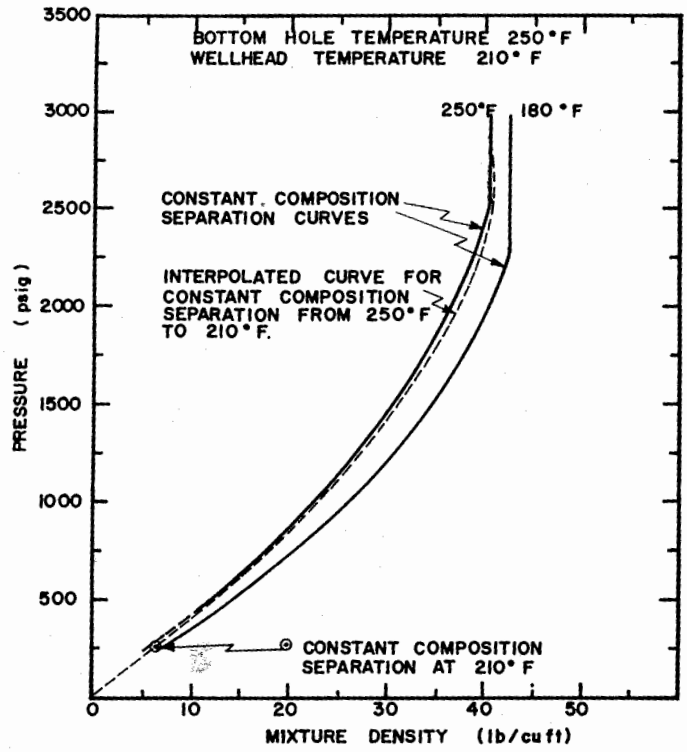


Fig. 2.82 Mixture density curve.

and subscripts:

c = casing
t = tubing

For annulus flow, the pipe diameter, d, is replaced by a hydraulic diameter, d_e, defined as:

$$d_e = \frac{4 \times \text{cross-sectional area}}{\text{wetted perimeter}} \qquad (2.3101)$$

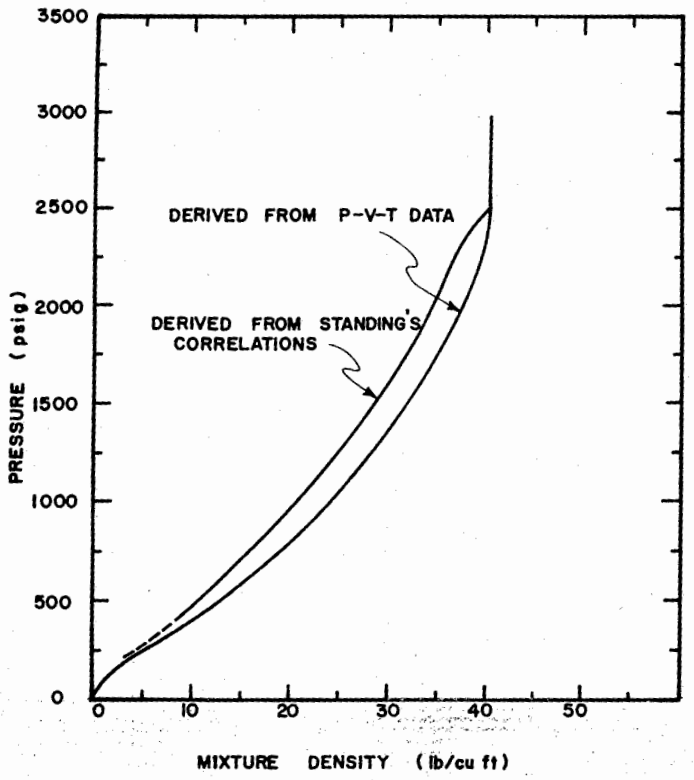


Fig. 2.83 Comparison of mixture density curves.

which, for a circular annulus, reduces to:

$$d_e = d_c - d_t \qquad (2.396)$$

This diameter is used in the pressure traverse equation, the Reynold's number, and the roughness ratio e'/d_e. In classical fluid mechanics, this substitution is considered effective only if the ratio of inner to outer diameter does not exceed 0.3. However, in the case of the small annulus ($6\frac{5}{8}$ in. × $2\frac{7}{8}$ in.) data available, the substitution is satisfactory.

The effects of couplings or upsets in the annulus have not been studied, but some allowance for their presence can be included in the effective roughness height, e'.

Because of the lack of good published information on annular flow the procedure for the calculation of a traverse by Cornish is given here.[27]

procedure for calculating pressure traverse

1. Construct pressure-density curve for required bottom hole and wellhead temperatures. If this is not available standard procedures are used for calculating density assuming no-slip.
2. Calculate pipe area and diameter (use hydraulic diameter for annulus flow).
3. Select flowrate, q_o, in stb/d and calculate m, and N_{Re}.

$$m = \frac{\rho_{sat} B_o q_o (5.615)}{86{,}400} \quad \text{(lb/sec)}$$

$$w = \frac{m}{A} \qquad \text{(lb/sec ft}^2\text{)}$$

$$N_{Re} = \frac{wd}{\mu_m} \quad \text{(Note: use } d_e \text{ for annulus flow)}$$

4. Calculate e' and $\frac{e'}{d_e}$ (annulus flow only):

$$e' = e_c\left(\frac{d_c}{d_c + d_t}\right) + e_t\left(\frac{d_t}{d_c + d_t}\right)$$

$$d_e = d_c - d_t$$

5. With N_{Re} and $\frac{e'}{d}$ read friction factor, f, from Fig. 2.72.
6. Select pressure increments.
7. Calculate $\bar{p} = \frac{p_1 + p_2}{2}$ and read ρ_1, ρ_2 and $\bar{\rho}$ from the pressure density curve (similar to Fig. 2.82).
8. Evaluate K:

$$K = \frac{1}{\bar{\rho}^2} \frac{w^2}{2g} \left(\frac{f}{d} + \frac{2 \ln \rho_1/\rho_2}{\Delta h}\right)$$

Note: If $P_1 \geq 500$ psig, neglect the ln ρ_1/ρ_2 ter in hand calculations.

9. Solve for Δh

$$\Delta h = \frac{144 \Delta p}{\bar{\rho}(1 + K)}$$

10. Calculate cumulative Δh:

$$\sum \Delta h = \sum_{i=i}^{n} \Delta h_i + \Delta h$$

Repeat steps 6 to 10 for required range of pressure. Plot pressure versus arbitrary depth scale.

(a) To determine wellhead flowing pressure:
  i) enter plot determined in Step 12 at bottom hole flowing pressure. Read depth on arbitrary scale (H_1, ft)
  ii) subtract from this depth an amount equal to the depth of the perforations (or casing shoe) from surface, i.e. $H_2 = H_1 - h$.
  iii) read pressure at depth H_2. This pressure is the wellhead flowing pressure.

(b) To determine bottom hole flowing pressure:
  i) enter plot at p_{wn} (wellhead flowing pressure) and reverse procedure of 12(a).

An example problem follows.

AMPLE PROBLEM

The following data are given:

Casing: 9⅝" J55, 40 lb/ft, LTC/N80, 435 lb/ft XL
Tubing: 2⅞" J55, 6.5-lb/ft EUE regular
Perforations: 8,105 ft BGL

Construct a pressure-density curve as Fig. 2.82.
Average casing I.D.: 8.800 in.
tubing O.D.: 2.875 in.

$$A = \frac{\pi}{4}(8.8^2 - 2.875^2)\frac{1}{144} = 0.375 \text{ ft}^2$$

$$d_e = (8.800 - 2.875)\frac{1}{12} = 0.494 \text{ ft}$$

$q_o = 32{,}400$ bpd at 180°F
$\rho_{sat} = 40.3$ lb/ft³
$B_o = 1.54$

$$m = \frac{\rho_{sat} B_o q_o (5.615)}{86{,}400} = \frac{(40.3)(1.54)(32{,}400)(5.615)}{86{,}400}$$

$$= 131.0 \text{ lb/sec}$$

$$w = \frac{m}{A} = \frac{131.0}{0.375} = 348.8 \text{ lb/sec ft}^2$$

$$N_{Re} = \frac{w d_e}{\mu_m}$$

Note: Often the Reynold's number will be so high that any increase in its value from saturation pressure to wellhead pressure will not affect the value of the friction factor. However, it is advisable to calculate the Reynold's number at two or more pressures, say, saturation pressure and an assumed wellhead pressure.

At saturation pressure $\mu_m = \mu_o$, and so:

$$N_{Re} = \frac{w d_e}{\mu_o} = \frac{(348.8)(0.494)}{(0.222)(6.72)(10^{-4})} = 1.16 \times 10^6$$

μ_o is found from fluid properties:

At 1,150 psig, and 210°F:

$\mu_o = 0.386$ cp (given)
$\mu_g = 0.0145$ cp (given)

$$\rho_g = \frac{(0.94)(0.0764)(1164.7)(520)}{(14.7)(670)(0.958)} = 4.63 \text{ lb/ft}^3$$

$\rho_m = 25.0$ lb/cu ft (from Fig. 2.82)

$$H_L = \frac{\rho_m - \rho_g}{\rho_o - \rho_g} = \frac{25.00 - 4.63}{42.40 - 4.63} = 0.534$$

$$(\mu_m = \mu_o^{H_L})(\mu_g^{1-H_L}) = (0.386)^{0.534}(0.0145)^{0.466}$$
$$= (0.602)(0.1485)$$
$$= 0.0894 \text{ cp.}$$

$$(N_{Re})_{1,150 \text{ psig}} = \frac{(348.8)^{0.494}}{(0.0894)(6.72)(10^{-4})} = 2.865 \times 10^6$$

4. $e_c = 0.00015$ ft $e_t = 0.0005$ ft

$$e' = e_c \frac{d_c}{d_c + d_e} + e_t \frac{d_c}{d_c + d_e}$$

$$e' = (0.00015)\frac{(8.800)}{(11.675)} + (0.0005)\frac{(2.875)}{(11.675)}$$

$$= 0.000113 + 0.000123 = 0.000236.$$

$$\frac{e'}{d_e} = \frac{0.000236}{0.494} = 0.00048$$

Use $\frac{e'}{d_e} = 0.0005$.

5. From Fig. 2.72 friction factor, $f = 0.018$, for all values of pressure.
6. Let $\Delta p = 200$ psi
7. $p_1 = 0$ psig $p_2 = 200$ psig
   From Fig. 2.82

$$\rho_1 = 0.34 \text{ lb/ft}^3, \rho_2 = 3.4 \text{ lb/ft}^3, \bar{\rho} = 1.4 \text{ lb/ft}^3$$

$$\bar{p} = \frac{p_1 + p_2}{2} = \frac{0 + 200}{2} = 100 \text{ psig}$$

8. $K = \frac{1}{\bar{\rho}^2}\frac{w^2}{2g}\left\{\frac{f}{d_e} + \frac{2 \ln \rho_1/\rho_2}{\Delta h}\right\}$ Assume $\Delta h = 1{,}000$ ft

$$= \frac{1}{(1.4)^2}\frac{(348.8)^2}{(2)(32.2)}\left\{\frac{0.018}{0.494} + \frac{4.604}{1000}\right\} = 39.59$$

9. $\Delta h = \frac{144\,\Delta p}{\bar{\rho}\,(K+1)} = \frac{(144)\,(200)}{(1.4)\,(39.59 + 1.0)} = 506$ ft.

10. $\Sigma\Delta h = 506$ ft, since this is the first pressure increment.

For the calculation of a complete traverse by hand a tabulation could be set up as follows in Table 2.34.

The complete pressure traverse is plotted in Figure 2.84. To obtain the wellhead flowing pressure, enter at point A.

$$p = 2{,}670 \text{ psig}, h_1 = 12{,}650$$

At $h_2 = 12{,}700 - 8{,}105 = 4{,}595$ ft (point B) read $p = 750$ psig.

2.35 Heading phenomenon

2.351 Introduction

The phenomenon termed "heading" normally occurs during the final stages of the flowing life of an oil well, but it has also been known to occur at various stages of continuous liquid production rates and gas-liquid ratios ranging from 150 to 5,000 scf/bbl. Marshall[36] defined heading as a flow regime characterized by

TABLE 2.34

p	ρ	$\bar{p}$	$\bar{\rho}^*$	$2 \ln \frac{\rho_{n+1}}{p_n}$	$\left\{\frac{f}{d} + \frac{2 \ln \frac{\rho_{n+1}}{p_n}}{\Delta h}\right\}$	$\frac{w^2}{\bar{p}^2 2g}$	(k+1)	Δh	$\Sigma \Delta h$
0	0.34								
200	3.4	100	1.4	4.604	0.04104	964.77	40.59	506	506
400	11.0	300	6.1	2.348	0.03875	50.82	2.97	1590	2096
600	16.5	500	14.0	0.810	0.03725	9.65	1.36	1514	3810
800	20.8	700	18.8	0.472	0.036909	5.35	1.20	1279	4889
1000	25.0	900	23.0	0.368	0.036805	3.57	1.13	1107	5996
1200	27.9	1100	26.2	0.220	0.036657	2.75	1.10	998	6994
1400	30.8	1300	29.2	0.198	0.036635	2.22	1.08	912	7906
1600	33.2	1500	32.1	0.150	0.036587	1.83	1.07	841	8747
1800	35.1	1700	34.4	0.112	0.036549	1.60	1.06	791	9538
2000	37.0	1900	36.2	0.106	0.036543	1.44	1.05	756	10294
2200	38.6	2100	38.0	0.084	0.036521	1.31	1.05	723	11017
2400	39.4	2300	39.0	0.046	0.036483	1.24	1.05	706	11723
2600	40.3	2500	40.0	0.040	0.036477	1.18	1.04	691	12414
2800	40.3	2700	40.3	0.000	0.036400	1.16	1.04	685	13099
3000	40.3	2900	40.3	0.000	0.036400	1.16	1.04	685	13784

* Note: $\bar{\rho}$ is read at $\bar{p}$, from Fig. 2.82.

regular and perhaps irregular cyclic changes in pressure at any point in the tubing string. There is also evidence of a lack of homogeneity of the flowing mixture.

This phenomenon is evident at the surface because intermittent slugs of liquid and gas are produced and there is usually a difference in the sound produced by these slugs as they pass through the surface fittings. There has been no attempt to describe heading analytically; consequently, there is no correlation of factors that will predict the prevalence of this condition or that will predict the range of pressure that occurs during a heading cycle. However, it is possible to determine the average pressure gradient that exists. Because there is this lack of information on this aspect of multiphase flow, this subject will be treated separately.

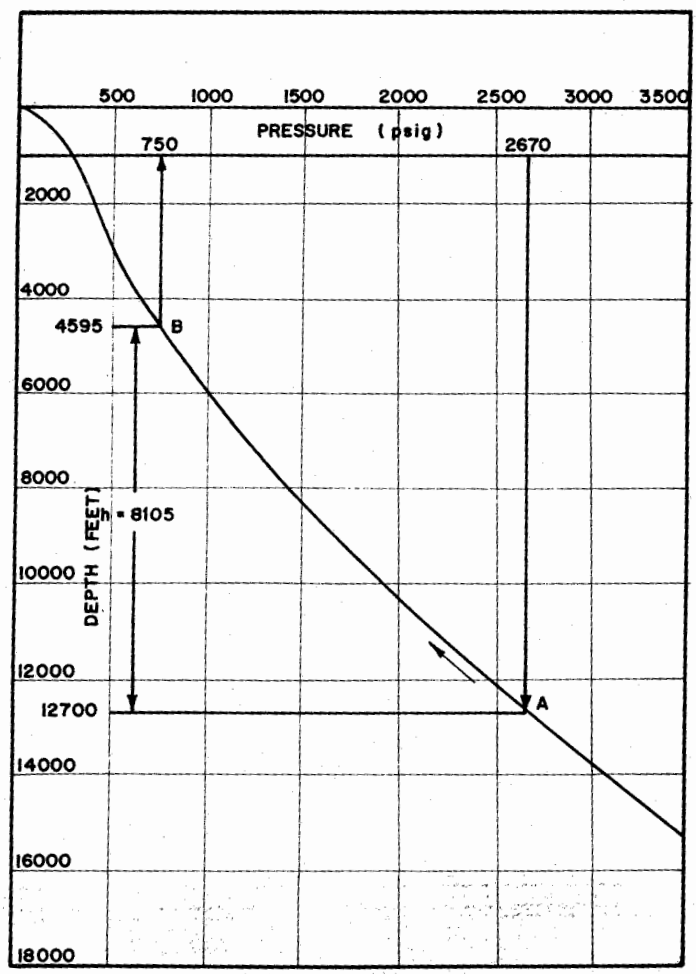


Fig. 2.84 Pressure traverse annular flow.

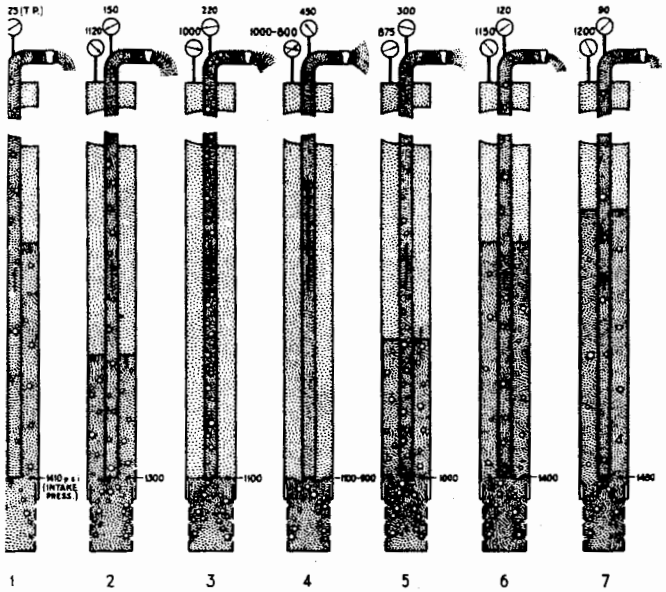
2.352 Literature review

A review of the literature reveals that the first comprehensive discussion of the phenomenon of heading was the one given by Gilbert.[9] Fig. 2.85 shows his pictorial representation and a description of the heading cycle that occurs during the latter stages of a flowing well. This flowing condition usually occurs whenever there is no packer in the well or when there is communication between the tubing and casing.

Fancher and Brown[18] did some of the first experimental field work where pressure traverses were recorded during a heading cycle. Figs. 2.86 and 2.87 show the recorded maximum and minimum pressures and the average pressure during a cycle in 2 in. tubing. Also shown are the predicted traverses of Poettmann and Carpenter,[1] Fancher and Brown,[18] and Ros.[13] Pressures were recorded at many points between the maximum and minimum line.

Fancher and Brown found that there was no correlation between heading, flow rate, and gas-liquid ratio, but it was observed that no heading occurred when the liquid rate was greater than 192 bpd.

Ros[13] and later Duns and Ros[14] also mentioned the heading phenomenon as occurring in the transition zone between various flow patterns and prepared a correlation to predict pressure loss in this flow region. During their experimental testing, they observed that flow occurred in a cyclic manner for a very limited range of flow rates and gas-liquid ratios. This occurred for superficial velocities of less than 10 cm/sec for the



ig. 2.85 Latter stages of a flowing well (after Gilbert, courtesy API).

'eading cycle:

(1) Because of the bypassing of gas into the annulus, the liquid level is lowly being lowered; annulus oil is being displaced into the tubing.

(2) The well is still producing at a low rate and the tubing column is heavy ecause gas is being diverted into annulus and oil from the annulus is being liverted into the tubing.

(3) The weight of the tubing column is being reduced because no further gas an be stored in the annulus. This further reduces the intake pressure and...

(4) Allows gas from the annulus to "blow around." For a short time the well s gas lifted with annulus gas at a high rate, and this reduces the intake ressure to its lowest value. The well gas is NOT being used to best advantage.

(5) The extra annulus gas has been dissipated and fluid, because of the low ntake pressure, is flowing at a high rate into both the tubing and the annulus. he tubing column is becoming heavier and the outflow rate is diminishing.

(6) Fluid is still flowing into the well at a faster rate than it will flow out f the well with the existing intake pressure.

(7) The rate of outflow is again in balance with the rate of inflow, and ypassing of gas into the annulus starts repetition of the cycle.

Observations:

This is not an efficient type of flow because it produces a large proportion f the oil with a deficient supply of gas and a small proportion of the oil with an excess supply of gas.

The usual practice of beaning a well back to a low rate of flow to avoid his type of surging is not efficient, and in many instances results in avoidable eduction of income.

This type of flow is characteristic of the latter part of the flowing life of wells in most areas, but is also characteristic of many relatively new low GLR wells.

Heading action of this kind can be minimized by use of tubing-casing packers, ut where packers are not already installed, use of casing-actuated intermitters may be preferable.

liquid and between 10 and 30 cm/sec for the gas. They found that heading was more severe in smaller pipes for lower viscosity liquids and for the lower liquid velocities.

Marshall,[36] in a study of heading phenomena based upon Hagedorn and Brown data, noted that minor and severe heading occurred; and he based these categories on pressure variations. Minor heading was said to exist when pressure fluctuations of less than 10 to 15 psi existed at any point in the tubing string. When the pressure within the tubing string fluctuated in excess of 15 to 20 psi, this was classified as severe heading. Fluctuations as high as 200 psi were recorded by Fancher and Brown, and this heading condition could be definitely recognized at the surface during their flowing tests.

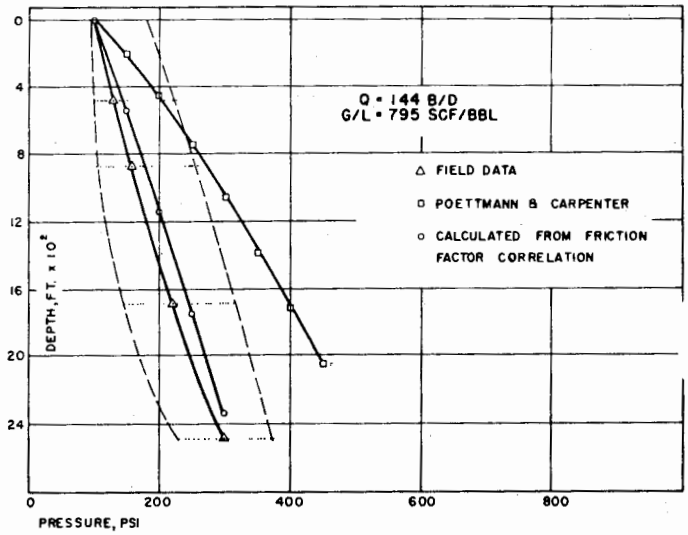


Fig. 2.86 Comparison of pressure traverses (after Fancher and Brown).

Figs. 2.88 through 2.98 were presented by Marshall to show the pressures at various points within the tubing string. They indicate conditions of smooth or continuous flow, minor heading, and severe heading. They also show fluctuations for wells that are loading up and dying. There is no ready explanation for conditions represented by Fig. 2.90, which represents a liquid rate and gas-liquid ratio of 34 bpd and 582 scf/bbl, respectively.

A team of investigators consisting of Zarrinal, Brown, and Tanaka conducted experimental laboratory work under the sponsorship of the API.[37] This work provided some data on the flow regime that exists just before a well loads up and dies.

Numerous tests were made in relatively short test strings of plastic pipe that varied in diameter from 1 in. to 2 in. The bottom hole pressure was held constant, and the liquid feed-in (or PI) was varied by adjusting a needle valve to control entry to the production string. Varying gas injection rates were used, and the rates

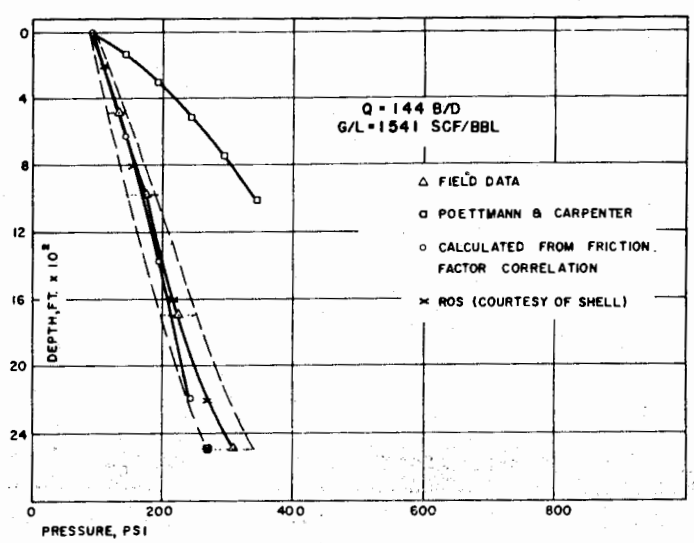


Fig. 2.87 Comparison of pressure traverses (after Fancher and Brown).

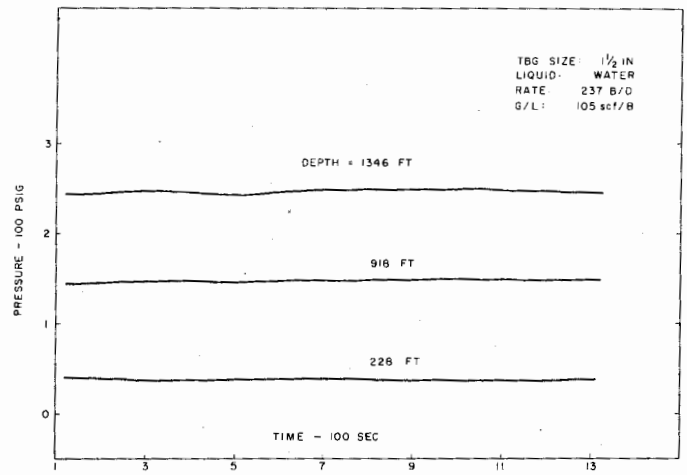


Fig. 2.88 Continuous flowing well no heading.

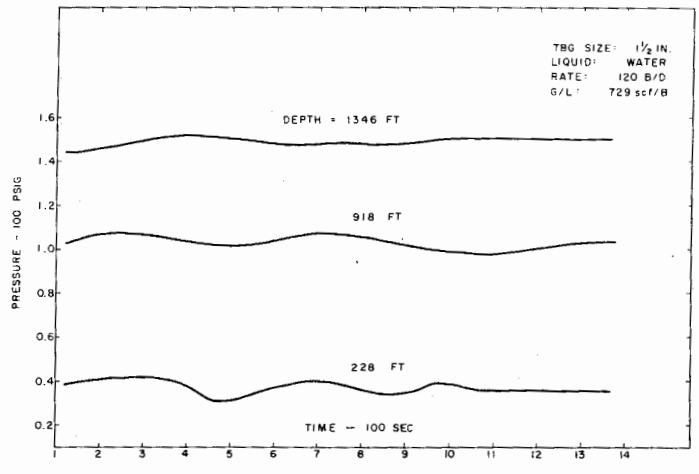


Fig. 2.91 Minor heading (after Marshall).

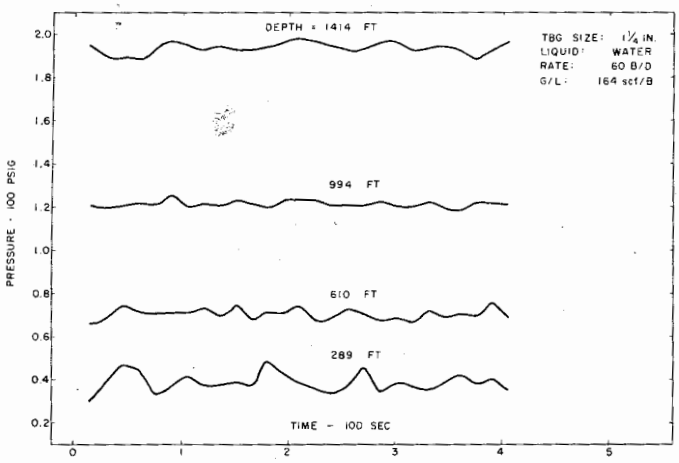


Fig. 2.89 Minor heading: well not loading up (after Marshall).

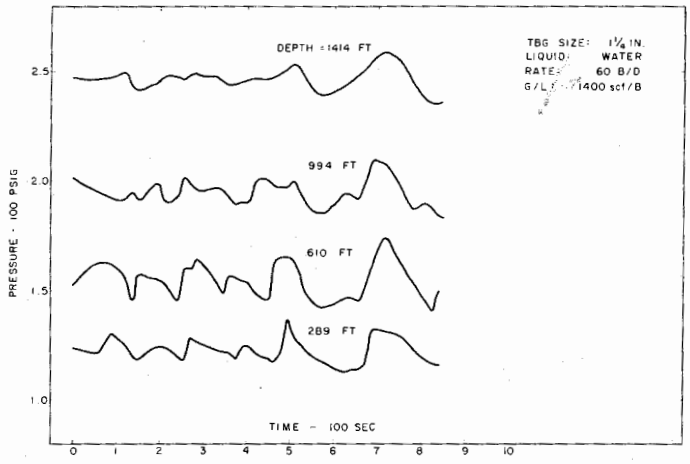


Fig. 2.92 Minor heading (after Marshall).

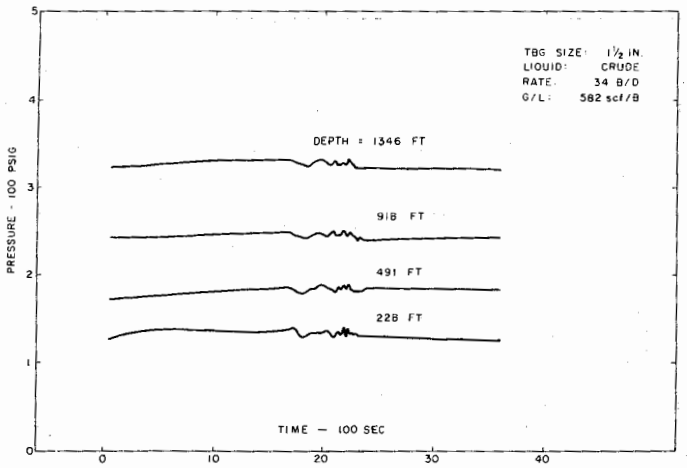


Fig. 2.90 Minor heading (after Marshall).

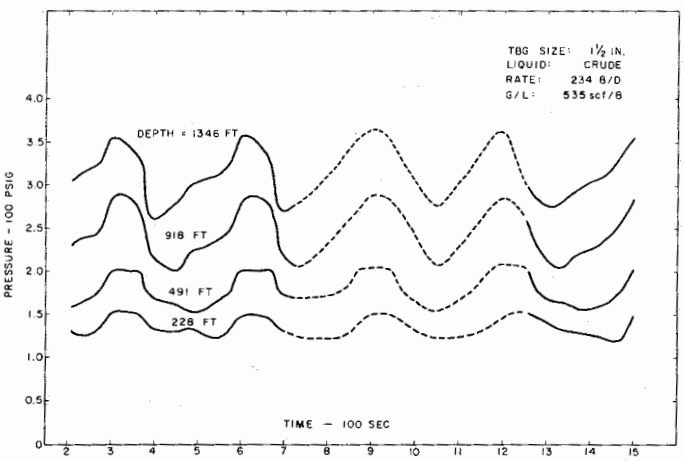


Fig. 2.93 Severe heading (after Marshall).

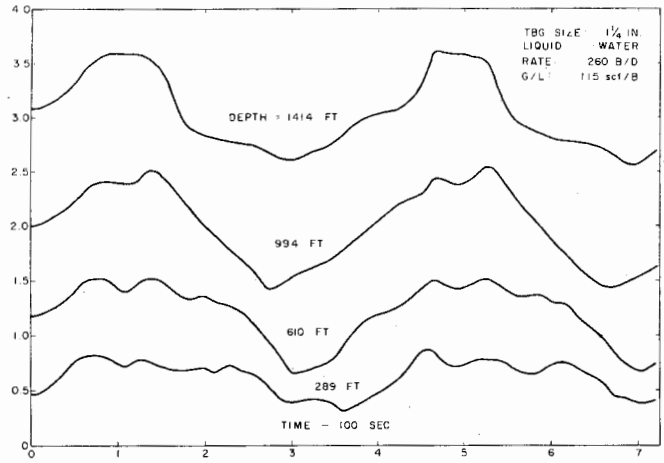
were increased until the well began to flow liquid over the top of the string. This rate was considered to be the gas rate at which a particular well would begin to flow. The gas rates were then increased to observe the flow patterns for wells considered to be definitely flowing. Many heading patterns were noted. For almost all pipe sizes and values of submergence, a type of heading occurred when the amount of gas being injected was just sufficient to just keep the well flowing.

Zarrinal, et al showed that the liquid holdup in this region of heading was substantially greater than that predicted by the holdup curve of Hagedorn and Brown.

2.353 Conclusions

Two principal conclusions can be drawn concerning the phenomenon of heading.

(1) Heading cannot always be predicted with any degree of accuracy.



ig. 2.94 *Severe heading (after Marshall).*

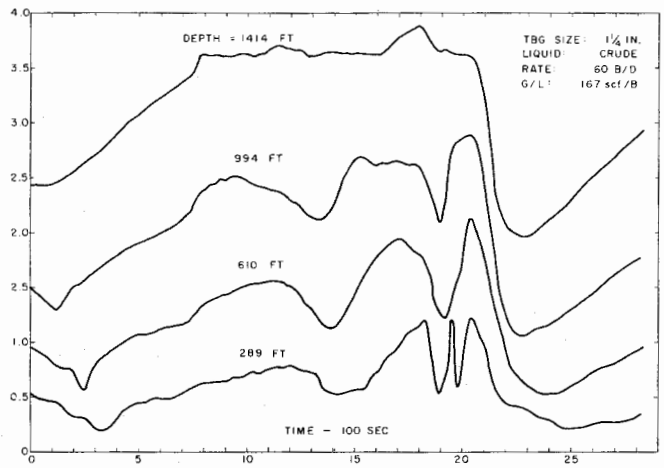


Fig. 2.95 Severe heading (after Marshall).

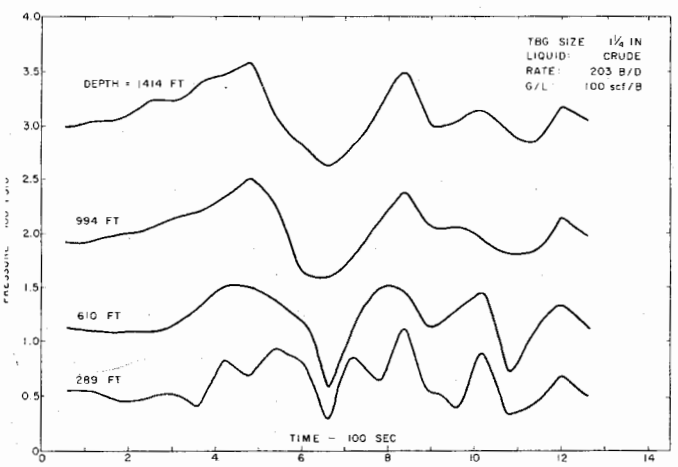


Fig. 2.96 Severe heading (after Marshall).

(2) The work done by Duns and Ros is the only contribution which predicts this type of flow; therefore, the correlation developed by this team of investigators should be used for wells in which flow is determined to be in the heading region.

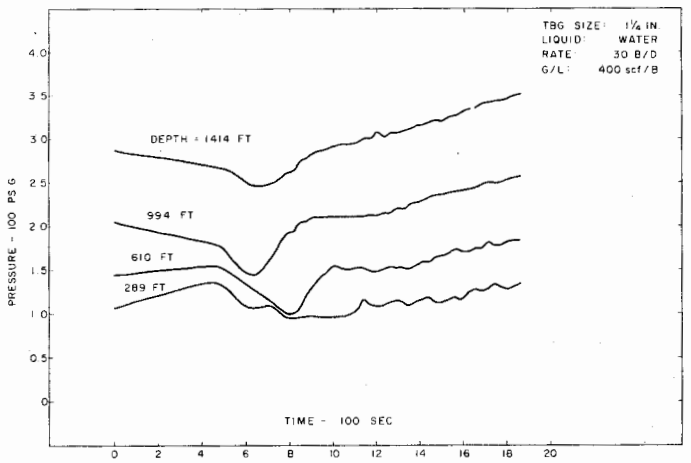


Fig. 2.97 Heading: well loaded (after Marshall).

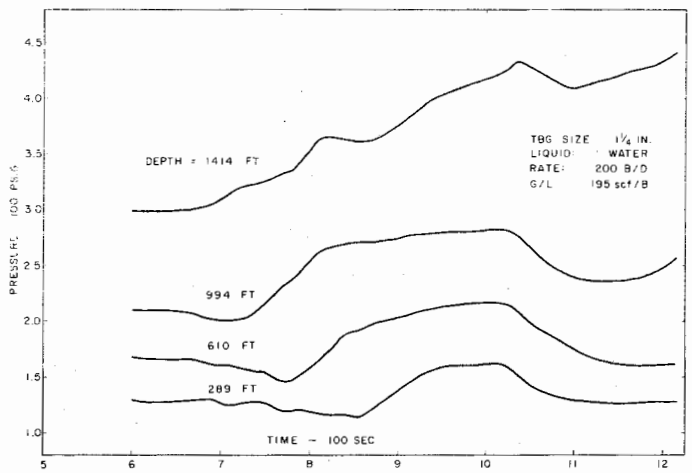


Fig. 2.98 Well loading (after Marshall).

2.36 Summary and evaluation of methods and their range of application

2.361 Introduction

Lawson and Brill[38] presented a study to ascertain the accuracy of several pressure-loss prediction methods. The results of their study will assist the petroleum production engineer in selecting the most accurate method for his problem and indicates the general accuracy to be expected from the methods.

The correlations included in their study were those of Poettmann and Carpenter,[1] Baxendell and Thomas,[12] Duns and Ros,[14] Fancher and Brown,[18] Hagedorn and Brown,[20] and Orkiszewski.[22] Each of these correlations was proposed specifically for predicting pressure losses in vertical oilwell tubing for the upward flow of multiphase well fluids.

A later study by Vohra, et al[35] presented three more good correlations—those of Beggs and Brill, Aziz, et al, and Chierici, et al. The purpose of their study was to extend the work of Lawson and Brill to cover the new correlations.

They used the same flowing pressure surveys and related data for 726 field and experimental wells as

TABLE 2.35
MINIMUM, MAXIMUM, AND AVERAGE VALUES OF WELL DATA
(After Lawson & Brill)

	Minimum	Maximum	Arithmetic average
Internal diameter, in.	1.049	8.760	1.878
Producing liquid rate, stb/d	1.0	5,082.0	395.2
Producing GLR, scf/stb	21	788,000	–
Well depth, ft	918	12,458	3,993
Measured bottom-hole pressure, psia	104	5,140	1,252
Measured wellhead pressure, psia	20	2,856	518
Wellhead temperature, °F	50.0	180.0	93.9
Bottom-hole temperature, °F	80.0	330.0	126.0
Produced oil API gravity	9.5	56.2	34.4
Produced water gravity	1.00	1.15	1.03
Produced gas gravity	0.600	1.400	0.850
Viscosity of produced oil at 100°F, cp	0.90	20,000.00	–
Viscosity of produced oil at 210°F, cp	0.26	140.00	–

were reported by Lawson and Brill. Fluid physical properties were estimated using the same correlations as employed by Lawson and Brill.

Table 2.35 shows the values of well data used by Lawson and Brill.[38]

The source and description of well test data was given by Lawson and Brill,[38] as noted in Table 2.36.

Lawson and Brill used the following fluid property correlations as noted in Table 2.37.

2.362 Discussion of results

For statistical evaluation of each pressure-loss correlation they used three numerical measures of reliability. These were the percent difference, (PD), average of percent differences (APD), and standard deviation of percent difference values from the average percent difference (SD).

$$PD = \frac{\Delta p_m - \Delta p_c}{\Delta p_m} \times 100 \text{ percent} \qquad (2.3101)$$

$$APD = \frac{\sum_{i=1}^{n} PD_i}{n} \qquad (2.3102)$$

TABLE 2.36
DESCRIPTION OF WELL TEST DATA
(After Lawson & Brill)

Data source	Number of tests	General description	Comments on estimated quantities
Poettmann and Carpenter	49	Field data; mostly 2-3/8 and 2-7/8 in. OD tubing; medium flow rates; gas-oil and gas-oil-water; low to medium pressures.	Surface temperature assumed to be 80°F with 1.0°F/100 ft temperature gradient. Viscosities from Beal correlation.
Fancher	83	One field well with gas injection; 2-3/8-in. OD tubing; medium-high flow rates; 95% water cut; low pressures.	Produced gas gravity was taken to be 0.6 as an average between solution gas gravity of 0.65 and injected gas gravity of 0.57. Measured pressure at lowest depth above gas injection depth was used. Measured pressures were read from plots in Appendix to MS thesis.
Hagedorn	346	Experimental well with air injection; 1, 1¼, and 1½-in. nominal tubing; air-water and air-refined oil flow; oil viscosities from 10 to 110 cp; broad range of flow rates; low pressures.	Temperature was set constant at 80°F. Pressure measurement at lowest depth above gas injection depth was used. Air viscosity was set at 0.0185 cp; air compressibility factor at 1.0; solution GLR at 0. Surface tension and liquid phase viscosity were set at values given in PhD dissertation.
Baxendell and Thomas	25	Field data from 2-7/8 and 3½-in. OD tubing; high flow rates; gas-oil flow low to medium pressures.	Temperature was set constant at 180°F. Oil viscosities estimated from data given at 160° and 200°F. Deepest pressure measurement was used.
Orkiszewski	22	Field data from 3½-in. OD tubing with two tests on 8.76-in. ID tubing; medium to high flow rates; heavy oils moderate pressures.	Fluids from all wells were assumed to have same properties as those given for Well 22 except for oil API gravities given for each well. Well 22 temperature gradient was used for other wells.
Espanol Herrera	44	Field data on 2-3/8 and 2-7/8-in. OD tubing; mostly low flow rates; light oils; high pressures.	Inside diameters of 1.996 and 2.376 in. were used.
U. of Tulsa Data Group 1	56	Field data on 2-3/8 and 2-7/8-in. OD tubing; mostly low rates; low to medium pressures.	Viscosities were estimated using Beal correlation.
Camacho	101	Field data on 2-3/8 and 2-7/8-in. OD tubing; gas wells making low to medium amounts of water; medium pressures.	

TABLE 2.37
FLUID PHYSICAL PROPERTY CORRELATIONS
(After Lawson & Brill[38])

Fluid physical property	Correlation used	Comments
Pseudocritical temperature and pressure of hydrocarbon gases	Katz	The curve for miscellaneous gases was used.
Hydrocarbon gas compressibility factor	Standing and Katz	The correlation becomes more in error as the nonhydrocarbon content of the gases is increased, but may still be accurate to about 2% when only small amounts of nonhydrocarbons are present. Z-factors for reduced temperatures less than 2.05 were calculated for reduced temperatures of 1.05.
Solution GOR	Lasater	Predicted values of gas mol fraction greater than 0.85 were taken as 0.85. The °API vs effective molecular weight of tank oil correlation was extrapolated for oil API lower than 18 and greater than 55. This correlation predicts the bubble-point pressures of the mixtures used to generate the correlation to an average of 3.8%. Accuracy for predicting solution GOR's would be expected to be about 5% also for the original data.
Oil formation volume factor	Standing	The correlation has an accuracy of 1.17% when applied to mixtures used to develop the correlation.
Hydrocarbon gas viscosity	Carr et al	The correction factors for H_2S, CO_2 and N_2 were not used since gas analyses were not available. Fortunately, corrections are small for nonhydrocarbon content less than about 5%. Effect of water vapor on viscosity was not considered.
Oil viscosity	Beal and Chew and Connally	Beal's correlation was used to estimate "dead oil" viscosity at 100° and 210°F as a function of API gravity. These values may be in error several-fold, since oil composition also affects oil viscosity. Temperature dependency of viscosity was assumed to follow the Beal correlation. When the range of the Chew and Connally correlation was exceeded, values at 1,600 scf/bbl were used.
Water viscosity	Beal	Data are for a specific gravity of 1.0. The curve was extrapolated to higher temperatures. No attempt was made to account for the effects of salt content, dissolved gas content, or pressure.
Oil surface tension	Baker	Values at 68°F were used for temperatures below 68°F, and values at 100°F were used for temperatures above 100°F. The fractional change of surface tension with pressure was taken as 0.1 for pressures over 3,900 psia.
Water surface tension against air	Data of Ref. 36	Data for pure water against air were used. Surface tension was assumed to vary linearly with temperature and have values of 69.56 dyne/cm at 104°F and 58.9 dyne/cm at 212°F.
Water surface tension against hydrocarbon gas	Data of Hough et al; also Katz	Surface tension values were found by linear interpolation between data for 74° and 280°F in the methane-water system. Values for temperatures below 74°F and above 280°F were taken as the values at 74° and 280°, respectively.

$$SD = \frac{\sum_{i=1}^{n} (PD_i - APD)^2}{n-1}, \qquad (2.3103)$$

where

Δp_c = calculated pressure loss
Δp_m = measured pressure loss
n = number of cases

Statistical results for all correlations are presented in Table 2.38. The following statistical results were obtained:

TABLE 2.38
STATISTICAL RESULTS FOR ALL WELL TESTS

Method	Average percent difference	Standard deviation
Poettmann and Carpenter	−107.3	195.7
Baxendell and Thomas	−108.3	195.1
Fancher and Brown	− 5.5	36.1
Duns and Ros	− 15.4*	50.2*
Hagedorn and Brown	− 1.3	26.1
Orkiszewski	− 8.6	35.7
Beggs and Brill	− 17.8	27.6
Aziz et al	+ 8.2	34.7
Chierici et al	− 42.8	43.9
"No-slip" static head	+ 53.5	33.0

*Results for the 427 well tests in the range of the correlation.

Lawson and Brill noted that Table 2.38 included statistical results for comparing measured pressure drop with that resulting from a static head of the fluids if no gas slippage occurs. As expected, this prediction method underpredicts the pressure drop, since friction, acceleration, and greater values of liquid holdup resulting from gas slippage are not considered.

The statistical results (Table 2.38) show the best two correlations to be those of Hagedorn and Brown and Beggs and Brill for the 726 wells tested. It is quite obvious that additional data from field wells is needed to properly evaluate each correlation.

Lawson and Brill and Vohra et al prepared Figs. 2.99 through 2.103 which give further information about the performance of the pressure-loss prediction methods. The figures illustrate the effects that internal tubing diameter, produced water/oil ratio (WOR), produced GLR, produced oil API gravity, and superficial liquid velocity at tubinghead conditions exert on the performance of each of the prediction methods. Since these flow variables have a major effect on the pressure gradients in producing wells, an analysis of prediction errors based on ranges of these variables should point out some of the strengths and weaknesses of the individual correlations.

The following discussion of these figures is taken directly from Lawson and Brill and Vohra, et al.

Internal tubing diameter:

Fig. 2.99a and 2.99b shows a tendency for some of the correlations to overpredict pressure losses for the 2.992-in. tubing. Note that the Hagedorn and Brown

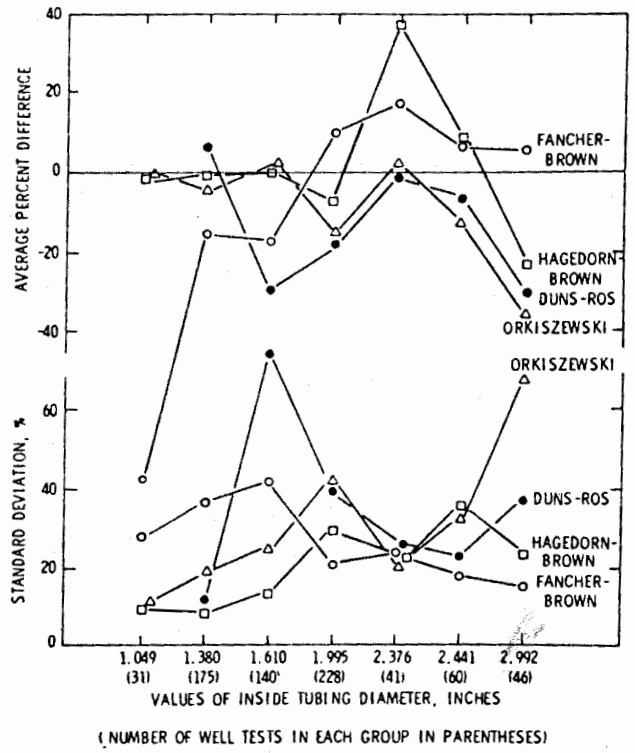


Fig. 2.99a Statistical results for well data grouped by internal tubing diameter (after Lawson and Brill).

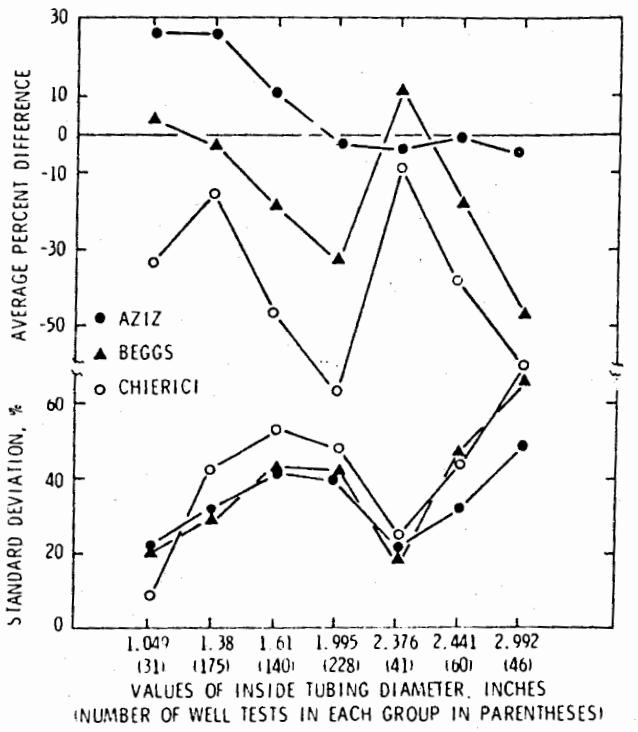


Fig. 2.99b Statistical results for well data grouped by internal tubing diameter (after Lawson and Brill).

and the Orkiszewski correlations do quite well for 1 to 2-in. tubing.

Only five test cases had tubing diameters above 3.2 in. and therefore no reliable estimates on performance could be determined for these larger pipe sizes.

API Gravity:

Fig. 2.100a indicates that the Hagedorn and Brown

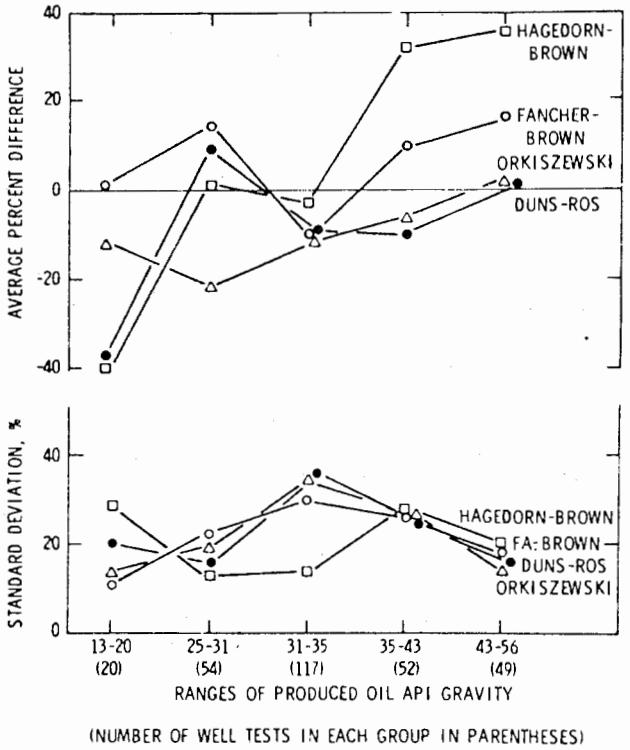


Fig. 2.100a Statistical results for well data grouped by produced oil API gravity (after Lawson and Brill).

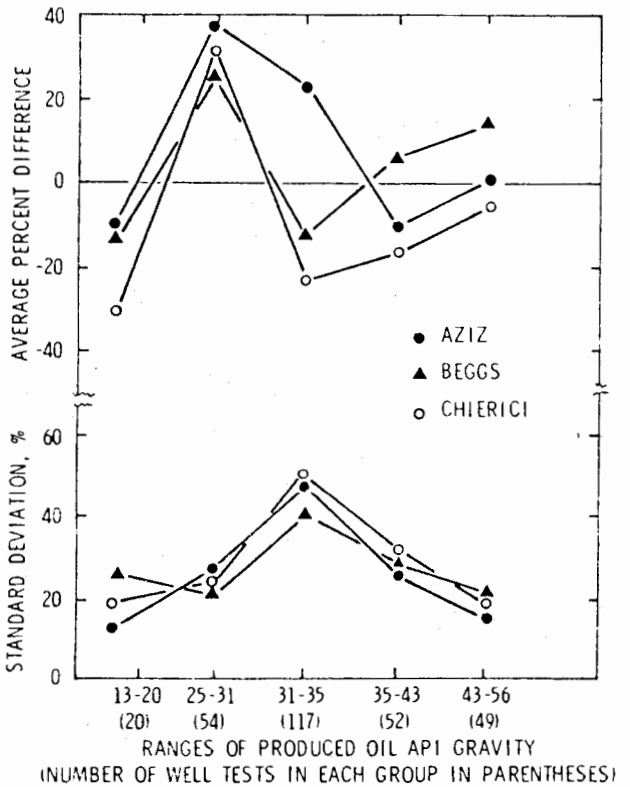


Fig. 2.100b Statistical results for well data grouped by produced-oil API gravity (after Lawson and Brill).

method overpredicts pressure loss for heavier oils and underpredicts pressure loss for lighter oils. The Orkiszewski method seems to improve as oil API gravity increases. No trend is indicated by the standard deviation plot.

Fig. 2.100b shows that for oils in the range of 20° to 31° API gravity, all three correlations underpredicted pressure losses. The Aziz, et al method performed better for the heaviest (below 20° API) and lightest (above 43° API) oils, whereas the Beggs and Brill method yielded better results for oils of intermediate API gravity.

Producing GLR:

Fig. 2.101a shows that the Hagedorn and Brown and the Orkiszewski correlations generally tend to show more scatter in prediction accuracy and also to overpredict pressure loss for higher producing GLRs. Also, the Duns and Ros correlation overpredicts as GLR is increased. The Fancher and Brown correlation tends to underpredict pressure losses, yet out-performs other correlations at high producing GLRs.

Fig. 2.101b shows that both the Beggs and Brill and the Chierici, et al schemes overestimated calculated down-hole pressures in all ranges of gas/liquid ratio. However, the Aziz, et al correlation underpredicted pressure drops for gas/liquid ratios below 5,000 scf/stb; above 5,000 scf/stb, this correlation overestimated flowing bottom-hole pressures and exhibited the least scatter.

Producing WOR:

Figs. 2.102a and 2.102b show that the correlations do a better job of predicting pressure losses when little or no water is present. Where the produced liquid is predominantly water, the correlations seem to overpredict pressure loss and exhibit more scatter in the prediction errors. Duns and Ros did not recommend using their prediction method with averaged physical properties for the liquid phase, as was done in this

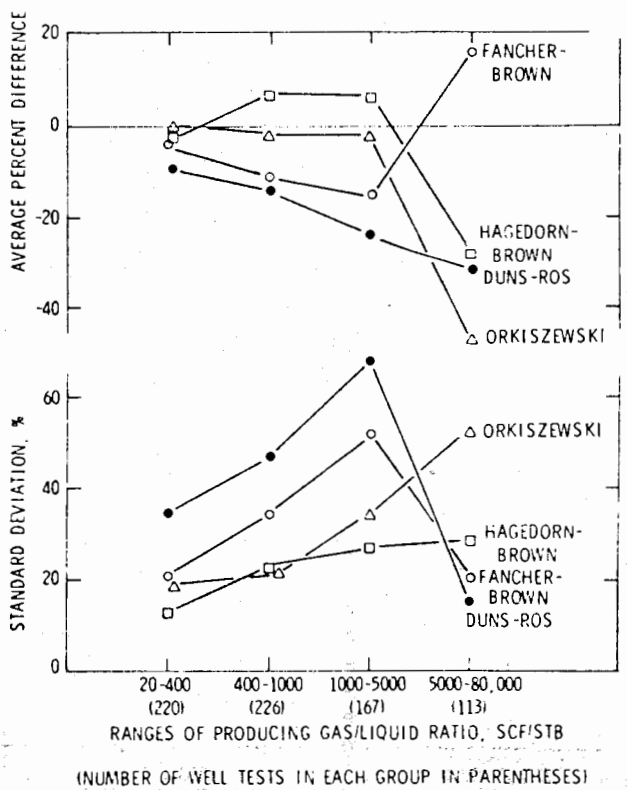


Fig. 2.101a Statistical results for well data grouped by producing gas/liquid ratio (after Lawson and Brill).

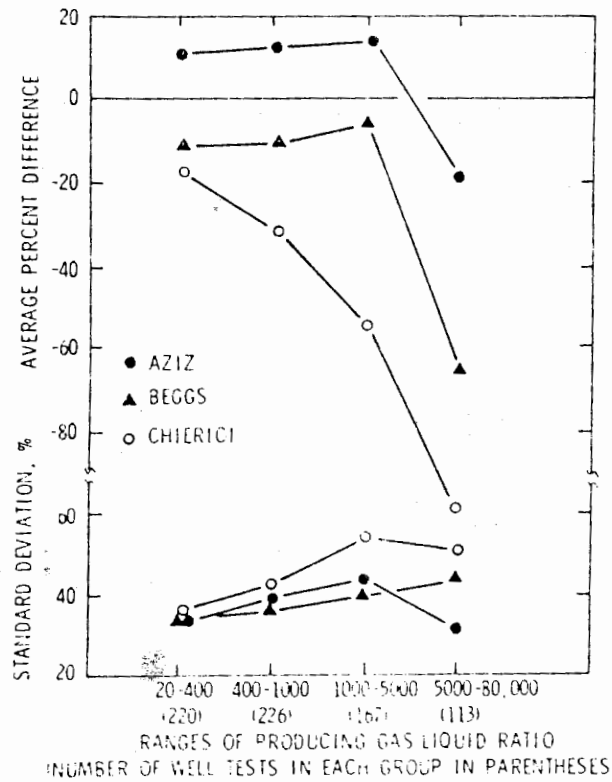


Fig. 2.101b Statistical results for well data grouped by producing gas/liquid ratio (after Lawson & Brill).

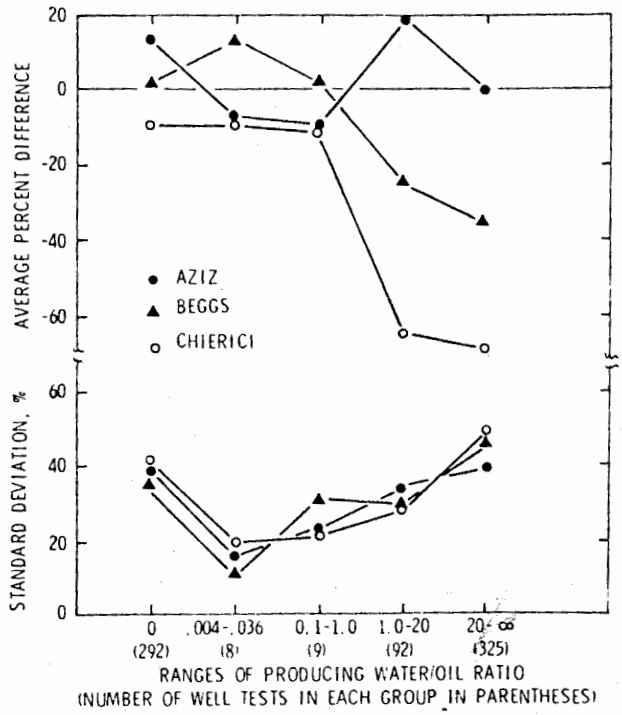


Fig. 2.102b Statistical results for well data grouped by producing water/oil ratio.

work. Instead they recommended that two calculations be performed, one with oil and one with water, each at the same total liquid production rate. The gradients were then to be averaged to get an estimate for the combined water and oil gradient. The lack of consistency in the results of all three correlations could be attributed to the possible formation of oil-water emulsions. For such mixtures, averaging separate oil and water properties does not yield appropriate values for the various liquid properties required in each correlation.

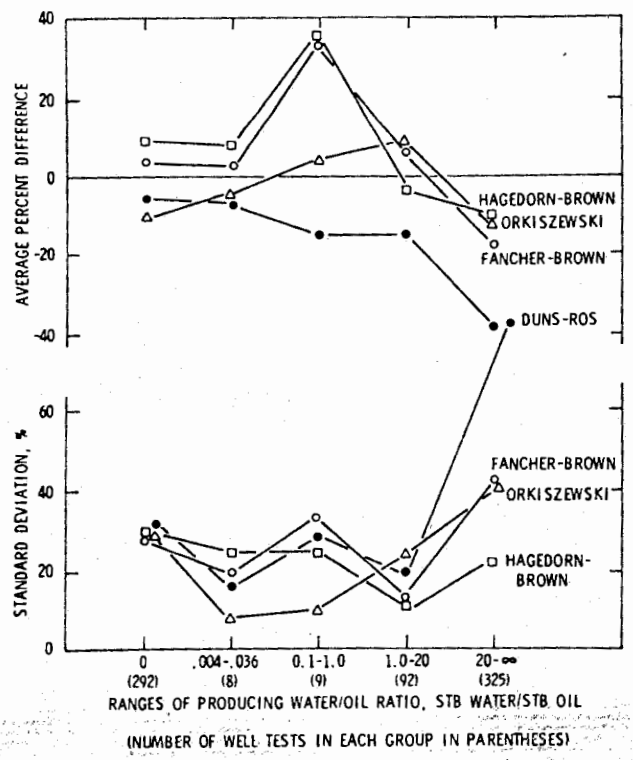


Fig. 2.102a Statistical results for well data grouped by producing water/oil ratio (after Lawson & Brill).

Superficial velocity of produced liquids:

Figs. 2.103a and 2.103b show the steady improvement in standard deviation with increasing superficial liquid velocities at the well-head. The poorer predictions at lower liquid superficial velocities could be an indication of a more cyclical or noncontinuous type of flow, which would be expected to be harder to predict by correlating techniques based on continuous flow.

The preceding figures should be used with caution in selecting a method for a particular set of conditions. Some of the trends may be caused by a change in another variable. As an example, what effect does a changing diameter have on a high or low gas liquid ratio.

2.363 Conclusions

Lawson and Brill gave the following conclusions:

"1. A comparison study such as this one depends a great deal on the quality and range of basic well data. Different well data may result in different conclusions as to the method having the best over-all performance.

2. Inaccuracies of fluid physical property correlations for predicting volumes and properties of down-hole fluids have an unknown effect on the statistical results. Each pressure loss prediction method, which combines a pressure loss correlation and fluid physical property correlations, must be considered as a unit when tested against measured pressure losses.

3. No single pressure-loss prediction method was found superior to all others considered for all ranges of producir

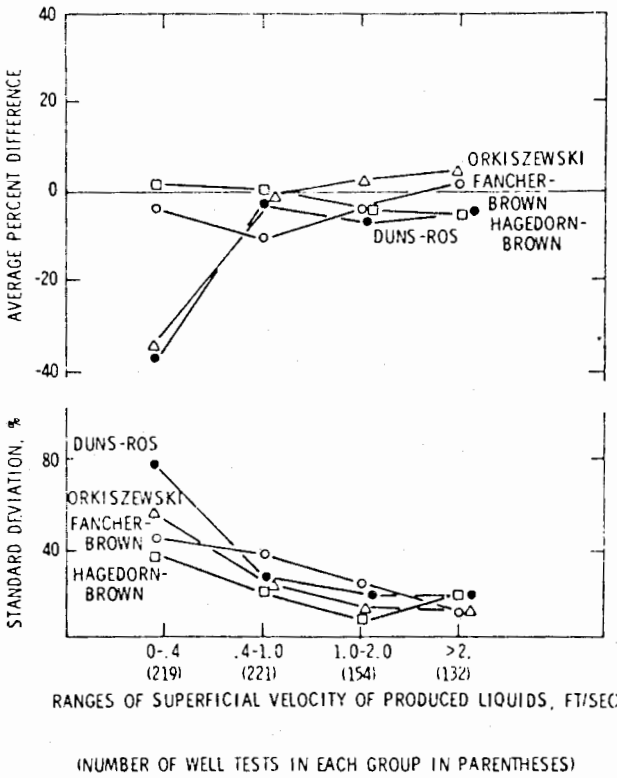


Fig. 2.103a Statistical results for well data grouped by superficial velocity of produced liquids (after Lawson and Brill).

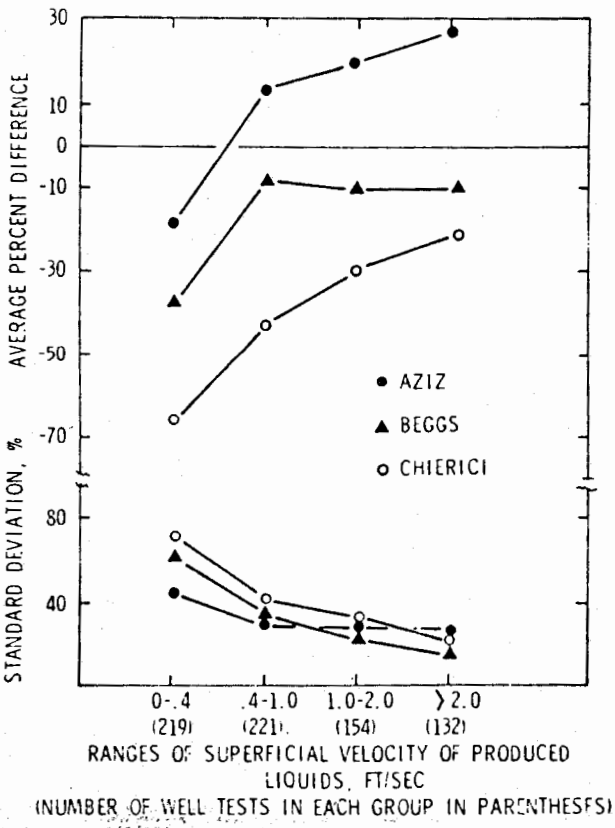


Fig. 2.103b Statistical results for well data grouped by superficial velocity of produced liquids (after Lawson and Brill).

well flow variables. Over all, the Hagedorn and Brown method, when tested against all 726 well tests, performed best.

4. Figs. 2.99 through 2.103 may be used with caution to eliminate or select methods for particular ranges of producing well flow variables.

5. The inability of the methods to consistently predict producing well pressure gradients with accuracy indicates the need for more research in the area of vertical multiphase flow."

6. The method of Beggs and Brill is the only method of the preceding group that will handle flow at any angle therefore making it the most universal of all methods.

7. In regard to the discussion on limited correlations, the following points should be made:

(a) The Poettmann and Carpenter method is accurate for 2, 2½, and 3 in. tubing sizes for flow rates above 400 bpd and gas-liquid ratios less than 1,500 scf/bbl provided no viscous effects are present.

(b) The Fancher and Brown method is accurate for 2 in. tubing in all ranges of flow rates and gas-liquid ratios provided no viscous effects are present.

(c) The initial work of Hagedorn and Brown to study viscous effects should be used in 1¼ in. tubing only, and will handle viscous effects quite accurately for this pipe size.

2.37 Practical application of vertical multiphase flow

2.371 Introduction

The accuracy of vertical multiphase flow correlations is good enough that the following are some of the uses that can be made of them.

(1) Select correct tubing sizes (tubing or annular flow).

(2) Predict when a well will quit flowing and hence predict time for artificial lift.

(3) Design artificial lift installations.

(4) Determine flowing bottom hole pressures.

(5) Determine PIs of wells.

(6) Predict maximum flow rates.

An understanding of the effect of various variables such as tubing diameters, flow rates, gas-liquid ratios, viscosity, density, etc., is necessary for good well completion and production practices. Some extreme examples represent the use of 4 in. tubing in Alaska, 9⅝ in. × 2⅞ in. annular flow in the Middle East and 1¼ in. tubing in South Texas. These selections of completion strings must be made prior to the drilling of the well.

We have two choices in making use of vertical multiphase flow correlations. The calculation can be made by computer or "working curves" can be used. Most companies have available at least one program for vertical multiphase flow, and there are numerous sets of working curves available either by widely published sets of curves or through restricted company published curves. If time permits the computer calculations are recommended, but there are numerous occasions where the production engineer must make use of working curves.

2.372 Effect of variables

3.3721 Introduction

In showing the effect of various variables, reference should be made to Brill, et al[39] and Lopez.[40] Many of

the figures are selected from their work in which the Hagedorn and Brown correlation was used. Other correlations would show the same general trends. The effect of such variables as tubing size, flow rate, viscosity, etc., are given in the following sections. Typical flowing wells have been selected to show these effects of variables.

3.3722 Effect of tubing size

Fig. 2.104 shows the effect of tubing sizes on the flowing bottom hole pressure for a 10,000 ft well producing 200 stb/d against 150 psig wellhead pressure. The pressure varies from 3,175 psig for 1 in. tubing to 1,150 psig for 3 in. tubing. The tubing size is extremely important, because the decision about the size should be made prior to drilling the well.

For example, if it was known that the well would have a flowing pressure of 1,500 psi to produce 200 stb/d, then we would use 2½ in. tubing (Fig. 2.104). This same procedure was used to determine that 4 in. tubing was necessary in some of the Cook Inlet wells in Alaska, and that 1¼ in. tubing was sufficient for many wells in South Texas.

Lopez[40] presented Figs. 2.105, 2.106 and 2.107 showing effect of tubing sizes for rates of 500, 3,000, and 8,000 bpd. The following data was selected by Lopez for each case where applicable (Table 2.39).

The results of Figs. 2.105 through 2.107 of Lopez are tabulated in Table 2.310.

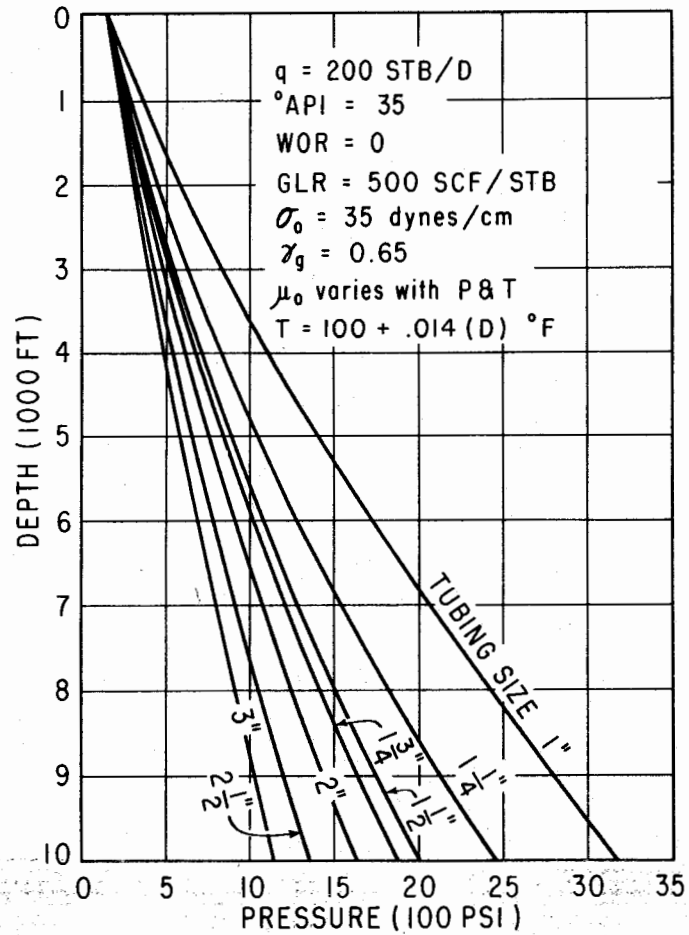


Fig. 2.104 Effect of tubing size.

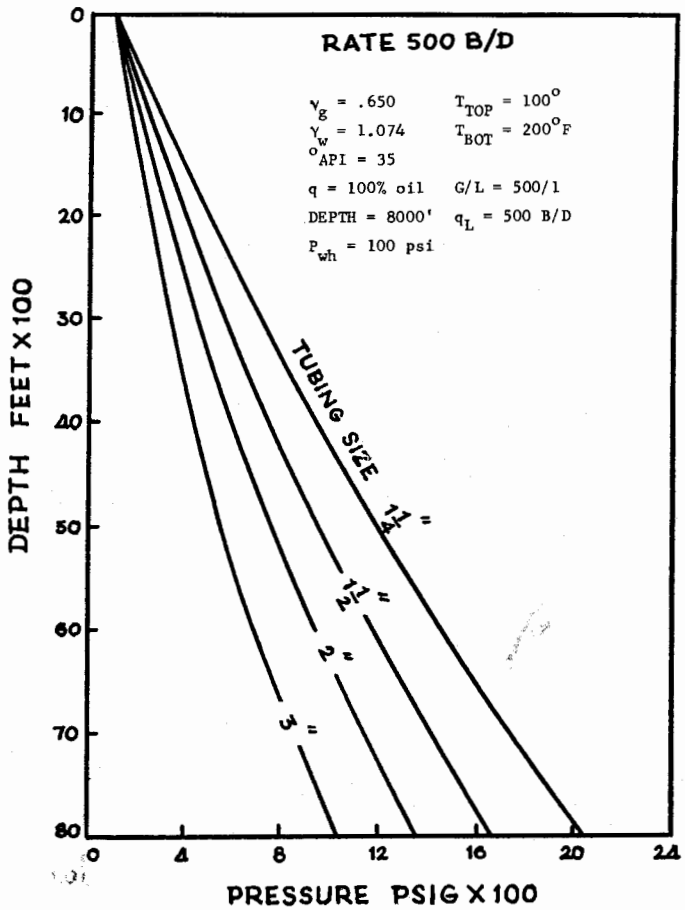


Fig. 2.105 Effect of tubing size.

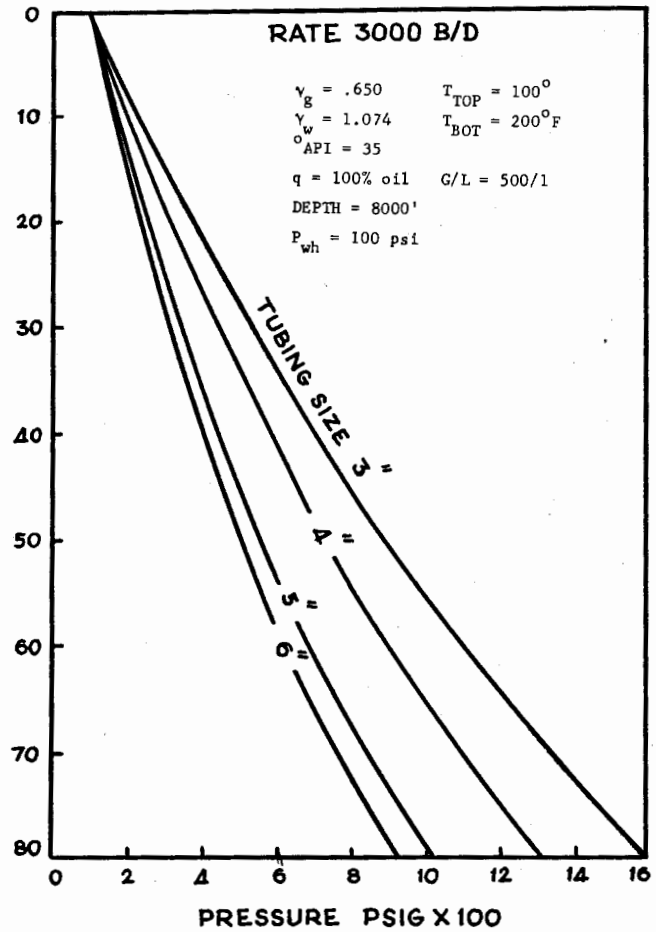
Fig. 2.108 shows the effect of gradient reversal in the low density region (near the top of the well). This reversal is very pronounced for 1 in., 1¼ in., and 1½ in. tubing sizes but is practically non-existent in 2½ in. and 3 in. tubing sizes. In most field wells it is not too important since a wellhead pressure greater than 100 psi normally exists.

2.3723 Effect of flow rate

The effect of surface rate is shown in Figs. 2.109, 2.110 and Table 2.311. Fig. 2.109 shows how the surface flow rate through 2 in. tubing affects flowing pressure gradients. Results are tabulated in Table 2.311. The flow rate is varied from 4,000 to 50 stb/d and the wellhead pressure is held constant at 100 psig. A pressure of 2,336 psig is needed to produce 2,000 stb/d, whereas 986 psig is required to produce 50 stb/d. The same general trend would exist for any tubing size, except that the approximate maximum and minimum flow rates possible would change. Fig. 2.110 shows the same effect for a tubing size of 4 in.

The surface flow rate establishes the required flowing bottom-hole pressure which, in turn, influences the selection of the tubing size.

Fig. 2.111 shows the effect of production rate in 1¼ in. tubing for a wellhead pressure of zero causing a reversal effect near the surface.



. 2.106 *Effect of tubing size.*

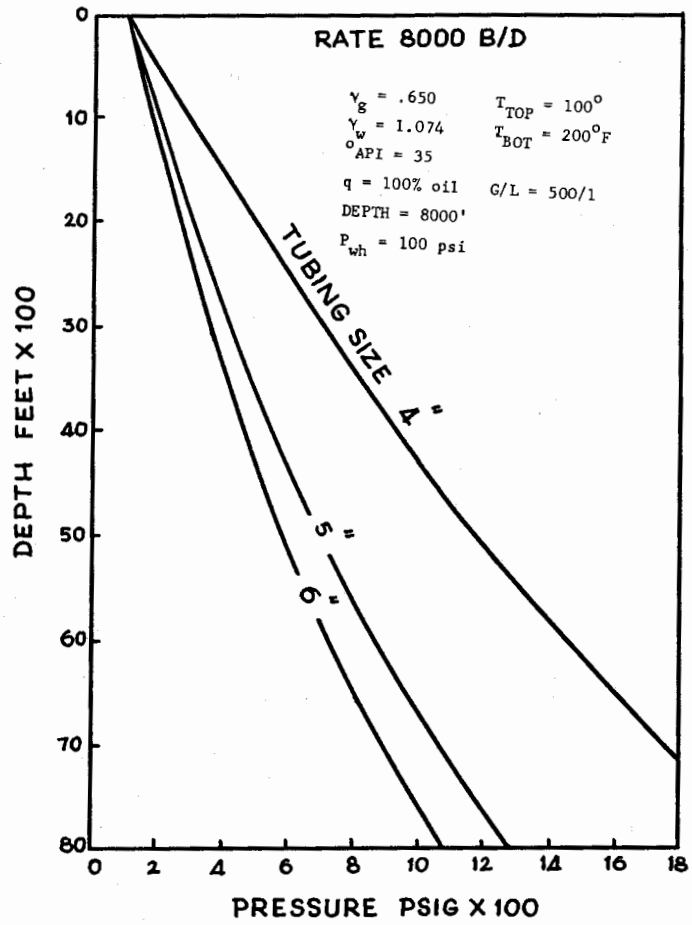


Fig. 2.107 Effect of tubing size.

724 Effect of gas-liquid ratio

The effect of gas-liquid ratio is indicated in Figs. .12, 2.113 and 2.114. Figs. 2.112 and 2.113 show the ect of gas-liquid ratio in two common tubing sizes 2 in. and 2½ in. respectively.

The effect of gas-liquid ratio on flowing pressure adients for a flow rate of 500 stb/d of 35° API oil rough 2½ in. tubing is shown in Fig. 2.113. An in- ease in the gas-liquid ratio causes a decrease in the quired flowing bottom-hole pressure. A point is ached where any increase in the gas-liquid ratio tually increases the flowing bottom-hole pressure. his is a result of both the gradient reversal near the rface and the increased friction losses over the entire bing length. This is explained by the fact that a ntinued increase in the amount of gas results in creased velocities which, in turn, increases the fric- n. Table 2.312 shows the results.

In this particular example, for a gas-liquid ratio of 00 scf/stb a flowing bottom-hole pressure of 752 ig is needed, and for a gas-liquid ratio of 5,000 scf/), a flowing bottom-hole pressure of 768 psig is re- ired. For a set of conditions used, there exists a gas- uid ratio between 5,000 and 3,000 scf/stb which will sult in a minimum flowing bottom-hole pressure. A s-liquid ratio of 10,000 scf/stb shows a flowing pres- re of 915 psi.

TABLE 2.39
TYPICAL WELL DATA

SGG	=	0.650	
SGW	=	1.074	
API	=	35.000	
CUT	=	0.000%	
Depth	=	8,000	ft
PWH	=	100	psig
THT	=	100	°F
BHT	=	200	°F
DI	=	2.441	in.
GLR	=	500	scf/stb
QL	=	500	stb

TABLE 2.310
TUBING SIZE EFFECT
(showing flowing bottom hole pressures)

	Rate			
Dia., in.	500	3,000	4,000	8,000
1¼	2,042	–	–	–
1½	1,680	–	–	–
2	1,371	–	–	–
3	1,042	1,592	1,819	–
4	–	1,319	1,459	2,068
5	–	1,025	1,072	1,285
6	–	950	972	1,092

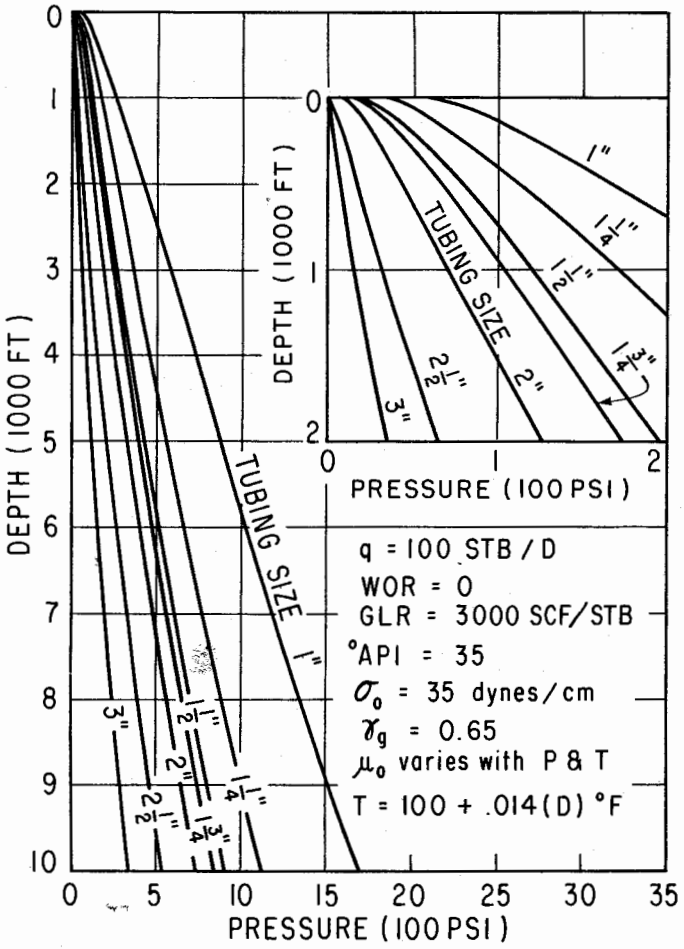


Fig. 2.108 Effect of tubing size on gradient reversal.

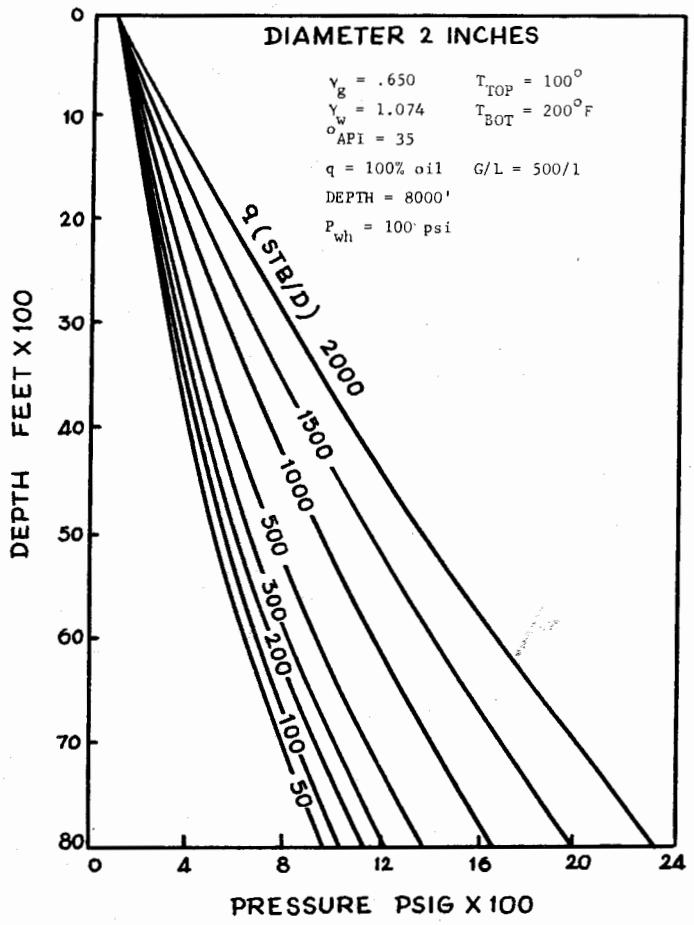


Fig. 2.109 Effect of rate.

The operator who continues to increase his injection gas volume to gain more production should note that a point is reached where additional gas will actually increase the flowing bottom-hole pressure, thus decreasing the production rate. Note Table 2.312.

A tubing size of 1¼ in. and a liquid rate of 200 bpd are noted in Fig. 2.114. In Fig. 2.114 3,400 psig is required for a gas-liquid ratio of 200 scf/bbl, compared to 1,600 psig for a gas-liquid ratio of 3,000 scf/bbl.

2.3725 Effect of density

The effect of a change in density in terms of API gravity when the viscosity is held constant at 1 cp can be seen in Fig. 2.115A. Although density and viscosity are related, it is necessary to fix the viscosity to isolate the effect of liquid density. As the API gravity increases, the flowing pressure at any depth is seen to decrease. A heavier oil is more difficult to produce than a light 50°API oil. Results are shown in Table 2.313. We note a flowing pressure of 1,460 psi for a 10°API crude and 1,275 psi for an API gravity of 50.

If the change in viscosity is also included with the effect of change in density then a pronounced difference occurs as noted in Fig. 2.115B.

2.3726 Effect of water-oil ratio

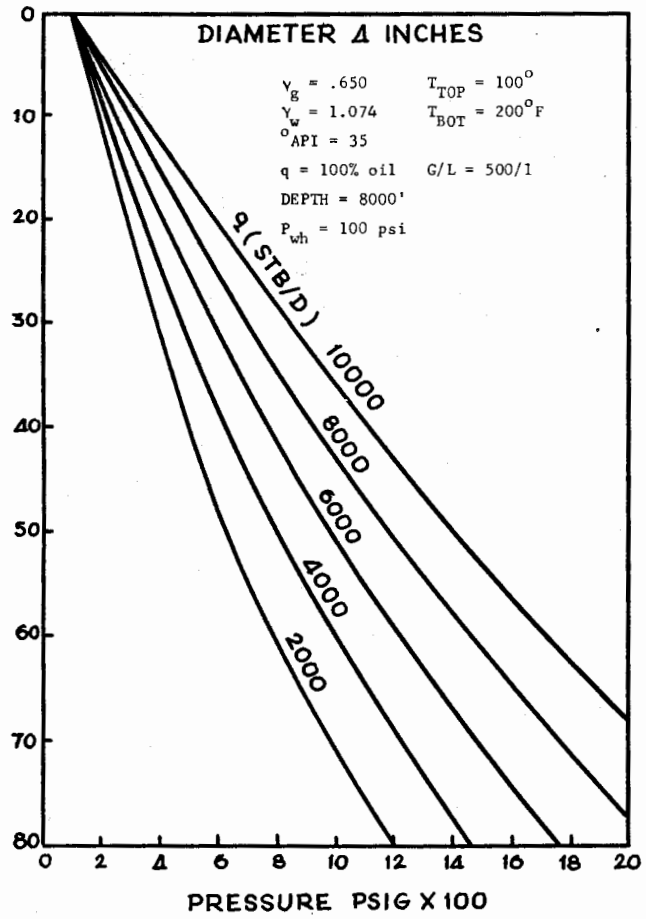
The effect of increased water production is shown in Fig. 2.116. It will be seen that the 100% 35° API oil can be produced with a flowing pressure of 1,750 psig, whereas the flowing pressure for 100% water is almost 2,250 psig. The gas-liquid ratio is assumed to be constant in this case. The effect of changing G/L can be found in Chapter 3.

2.3727 Effect of viscosity

Figs. 2.117 and 2.118 show the effect of viscosity. The API gravity has been shown along with the viscosity of the crude.

Equations for calculating gas-liquid mixture viscosities are all highly suspect and are approximations which are based on necessity rather than fact.

When using the Chew and Connally correlation it is impossible to separate the effect of density and solution gas from the effect of viscosity, since the live oil viscosity is a function of API gravity, temperature, and solution gas. Actual practice attests to the difficulty in producing highly viscous crudes. Results of Fig. 2.116 are tabulated in Table 2.314. For example, a viscosity of 10 cp requires a flowing pressure of 1,462 psi compared to a flowing pressure of 2,612 psi for a 500 cp crude.



2.110 *Effect of rate.*

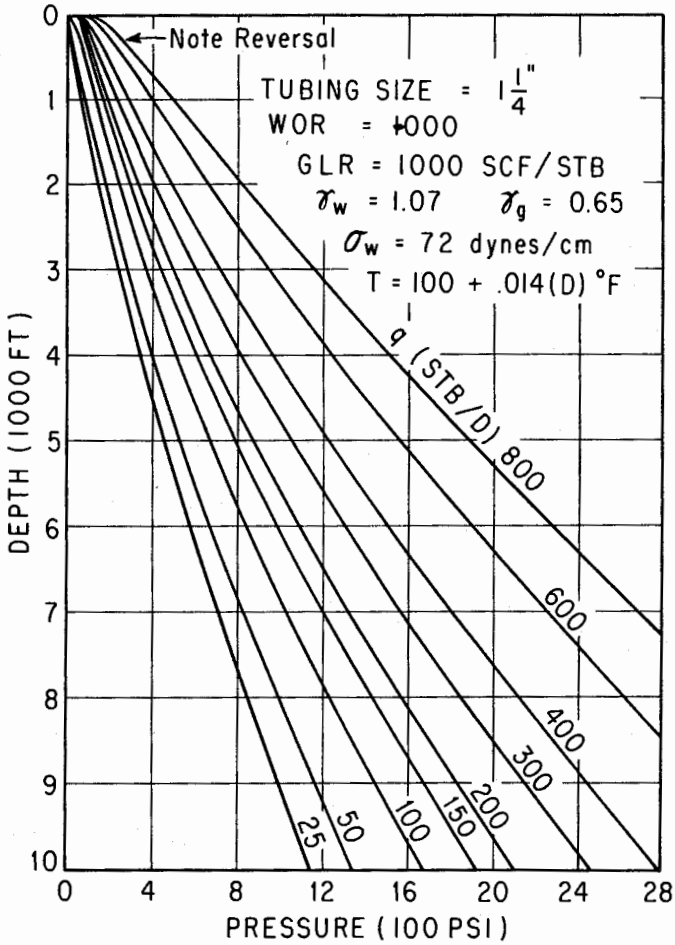


Fig. 2.111 Effect of production rate on pressure gradients.

TABLE 2.311
SURFACE FLOW RATE EFFECT SHOWING FLOWING BOTTOM HOLE PRESSURES

	Diameter	
Rate, bpd	2 in.	4 in.
50	986	–
100	1,049	–
200	1,152	–
300	1,235	–
500	1,371	976
1,000	1,678	1,046
1,500	1,995	1,112
2,000	2,336	1,180
4,000	3,777	1,459
6,000	–	1,753
8,000	–	2,068
10,000	–	2,398

728 Effect of slippage

The relationship between flow rate and bottom-hole ɛssure is shown in Fig. 2.119 for 2½ in. tubing in a)00 ft well with 100 psig well-head pressure. It will be seen that if the gas-liquid ratio is held conɪnt, the flowing rate can be decreased to a point ıere the required flowing pressure begins to increase. For example, for 400 scf/bbl this occurs between 100 and 150 stk/bpd. This means that the liquid rate is so low that excessive fallback begins to develop. This is because the velocity of the gas becomes higher than the velocity of the liquid, hence liquid hold-up increases (approaches 1), mixture density increases, and pressure required to lift the fluids increases. For flow rates below this point increased efficiencies might be obtained with smaller tubing sizes. However, the smaller tubing should be checked because it may still require a higher pressure. Results are shown in Table 2.315.

For example, for a gas-liquid ratio of 400 scf/bbl the minimum pressure required occurs at 150 bpd and is 1,531 psi. For rates lower than 150 bpd such as 50 bpd the pressure increases to 1,762 psi. It should be noted that for the higher G/L's such as 1,500 psi this effect does not occur.

In checking with various companies it has been found that the increase in pressure due to slippage begins to occur at less than 3,000 bpd for 7 in. × 2⅞ in. annular flow, less than 1,500 bpd in 4 in. tubing, and less than 7000–8000 bpd for casing sizes of 9⅝ in. or greater.

2.3729 Effect of surface tension

The effect of surface tension is somewhat questionable. Brill, et al,[25] showed by calculation that an in-

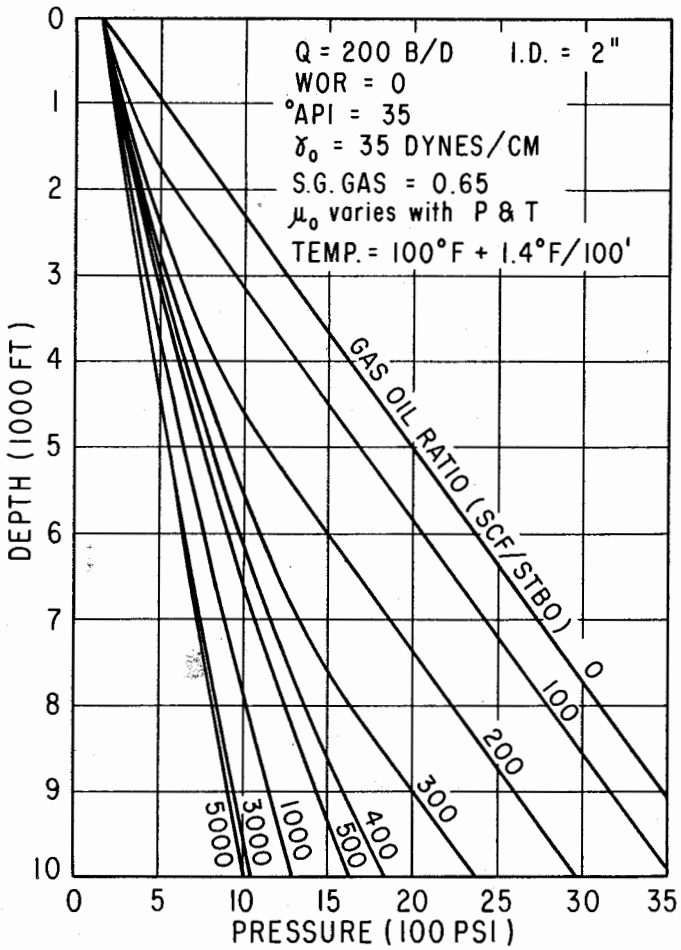


Fig. 2.112 Effect of gas-oil ratio (2" tubing).

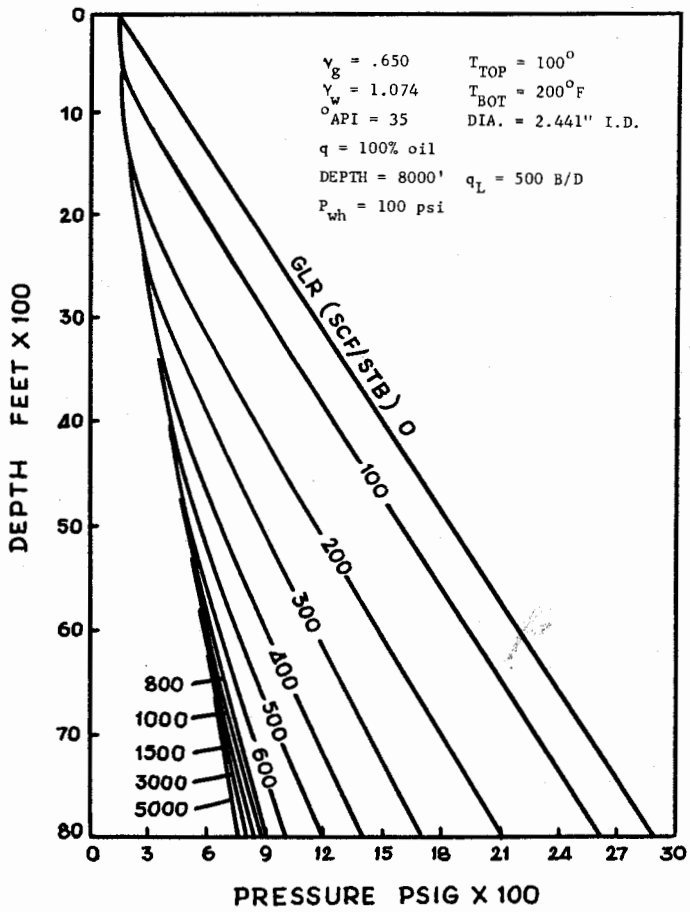


Fig. 2.113 Effect of GLR.

crease in surface tension showed an increase in pressure gradient holding all other variables constant (Fig. 2.120). Waldy[41] conducted a laboratory study in which the opposite effect was noted (Fig. 2.121).

He showed that lower surface tension liquids resulted in higher pressure traverses. Although surface tension enters into the calculation of most pressure traverses, and its value may change by a factor of 5 to 10 (say from 30 to 3), its overall effect on the total pressure traverse is not thought to be too significant. Additional research in this area will be beneficial.

2.37210 Effect of kinetic energy

The effect of kinetic energy due to acceleration can be neglected in most cases. Reference can be made to Fig. 2.122 which shows a well with fluids reaching a high velocity.

As noted, there is not much change when the acceleration term is included. However, in computer calculations it is suggested that the acceleration term be included, and then there is no problem when the pressure loss due to acceleration is appreciable such as very high velocities in the low density region.

2.373 Preparation of working curves

2.373 Introduction

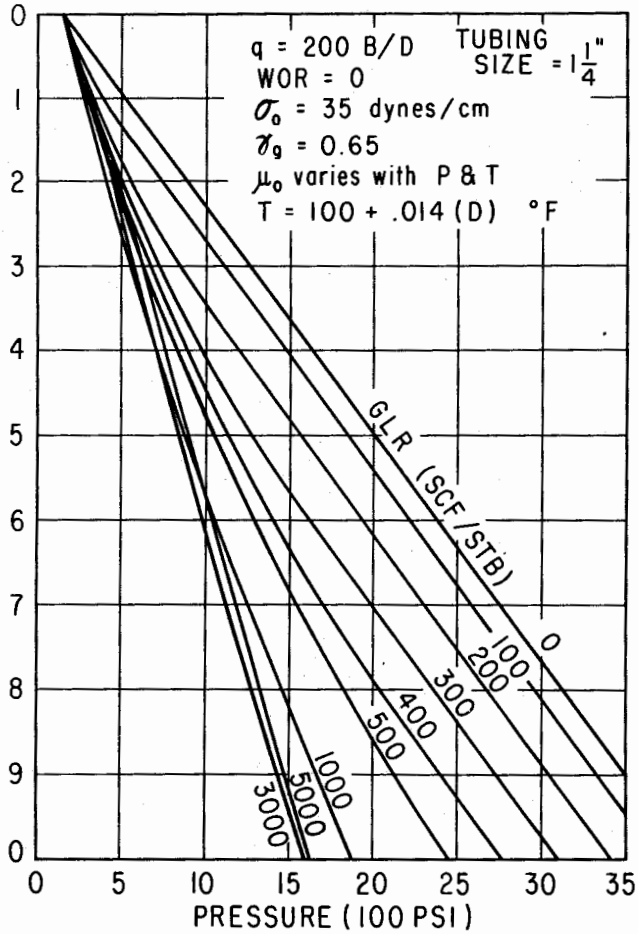
In order for the practicing engineer to have immediate access to use of multiphase flow correlations, a set of working curves becomes a necessity. Although computer solutions are more accurate, a computer is not always readily available. The field engineer can apply working curves to problems needing immediate attention.

There are three separate publications available which present multiphase vertical flowing pressure traverses. These are, in order of publication: (1) *Handbook of Gas Lift*, by U.S. Industries, Inc.;[8] (2) *Gas Lift Manual* by H. W. Winkler and Sid Smith, Camco, Inc.;[7] and (3) *Gas Lift Theory and Practice*, by Kermit E. Brown, Prentice Hall, Inc., 1967.[31] Others have been prepared by individual companies such as those of Shell prepared from the Duns and Ros correlation.[14]

Curves in the U.S. Industries and Camco publications were prepared from a modification of Poettmann and Carpenter's data of 1954.[1] Curves in the book by Brown were prepared from the generalized Hagedorn and Brown procedure.[21]

All three sets of these curves were calculated using generalized and average conditions. Average temperature conditions and Standing's correlations for gas in solution and formation volume factor were used. Also the curves were prepared for liquid phases of 100% oil, 100% water, and 50% oil–50% water.

The curves prepared from the Poettmann and Carpenter correlation are not accurate for tubing sizes smaller than 1½ in. The curves prepared from the Generalized Hagedorn and Brown correlation can be



2.114 *Effect of gas—liquid ratio.*

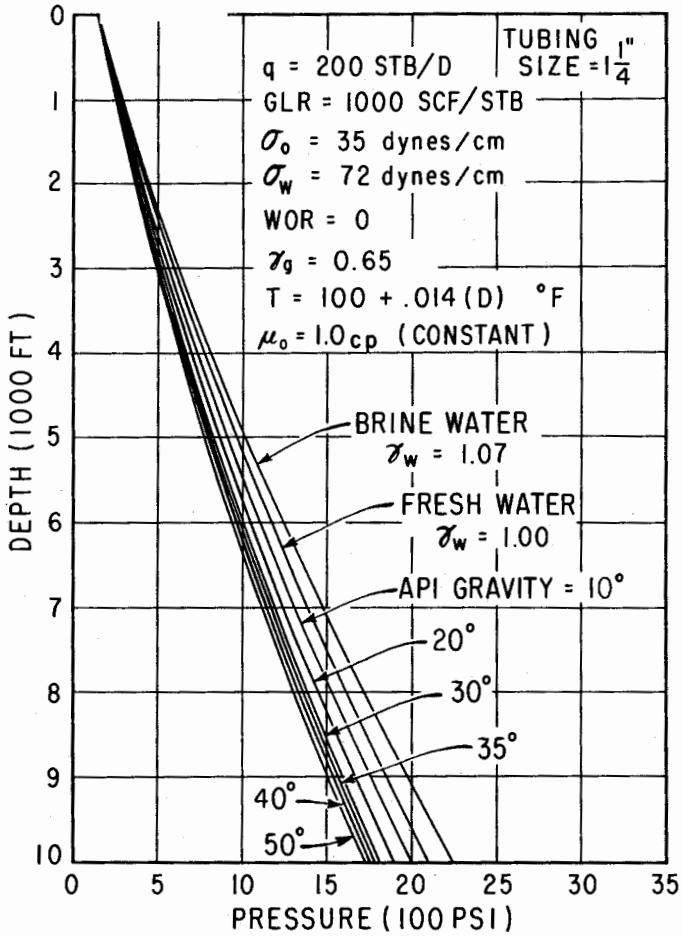


Fig. 2.115a Effect of density (°API) (after Brill, et al., courtesy SPE).

TABLE 2.312
GAS-LIQUID RATIO EFFECT

GLR	FBHP
0	2,938
100	2,669
200	2,234
300	1,783
400	1,398
500	1,175
600	1,042
800	913
1,000	862
1,500	801
3,000	752
5,000	768
10,000	915

TABLE 2.313
DENSITY EFFECT

API	FBHP
10	1,460
20	1,392
30	1,313
35	1,285
40	1,281
50	1,272

:d for pipe sizes from 1 in. through 4 in. Typical rking curves are shown in Fig. 2.123.

'igs. 2.124 and 2.125 show working curves where reversal in gradient is eliminated and included pectively. Some working curves such as those of own have eliminated the reversal where others such those of Ros have included reversal (Fig. 2.126). s also shifted the curves downward so appropriate ition must be taken in using the curves.

'he curves of Brown all start with the well-head ssure of zero and start from an arbitrary depth of o. Those of Ros start with a well-head pressure greater than zero and from depths greater than zero so that the reversal can be included without any crossing over of the curves.

2.374 Example problems

Since we have completed the discussion on vertical multiphase flow and effect of various variables we will now work example problems utilizing a series of "working curves" found in Appendix "C." A typical curve is noted in Fig. 2.123. A computer solution would be worked in the same manner except equivalent depths would not have to be determined. Equivalent depths are explained in the first example. Standard SPE symbols will be used where possible:

$\bar{p}_R$ = static reservoir pressure
p_{wf} = flowing reservoir pressure

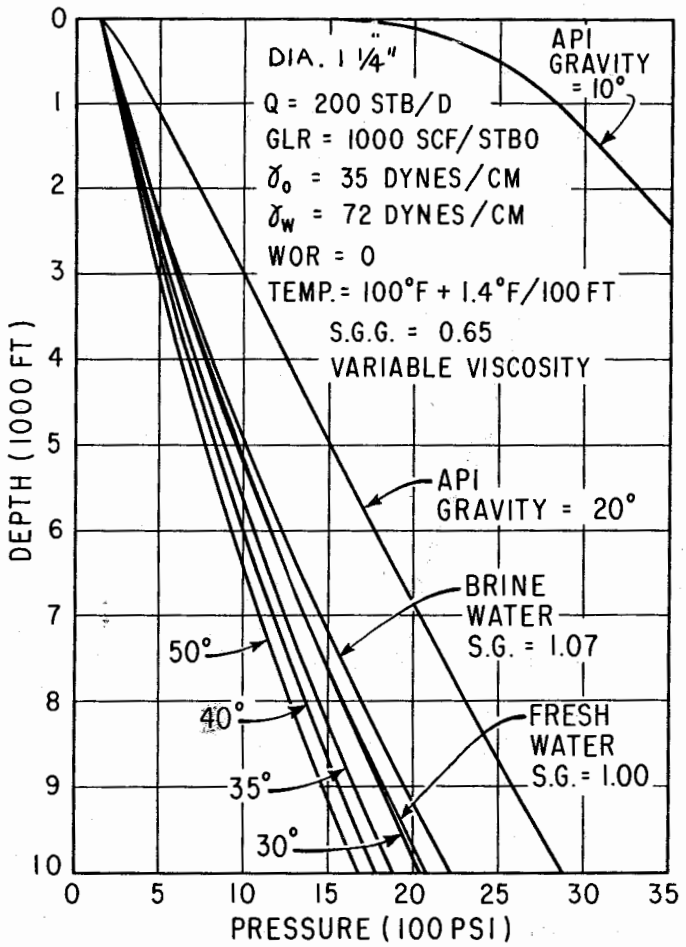


Fig. 2.115b Effect of API gravity (variable viscosity).

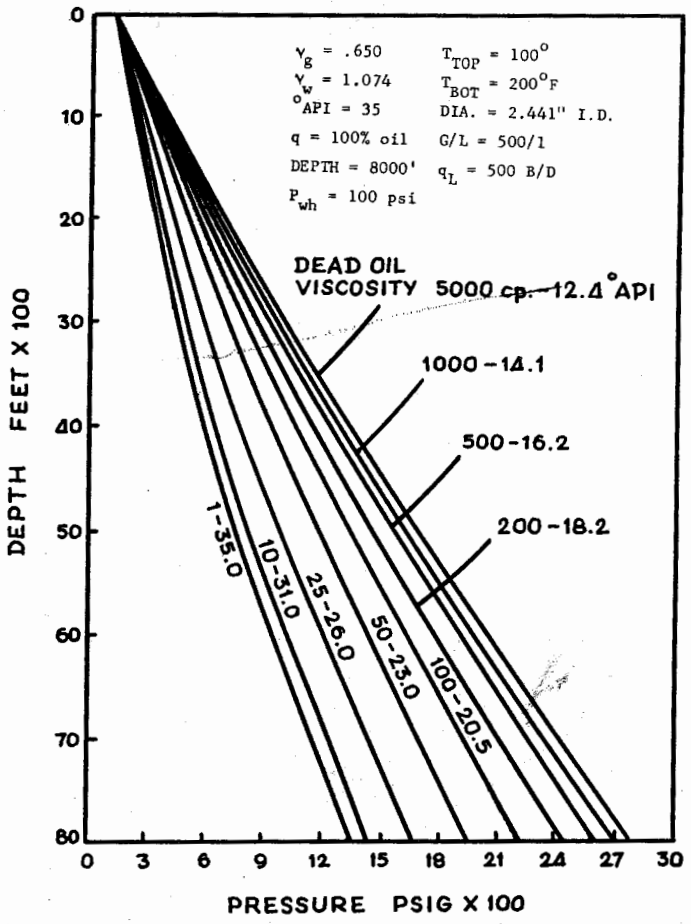


Fig. 2.117 Effect of viscosity.

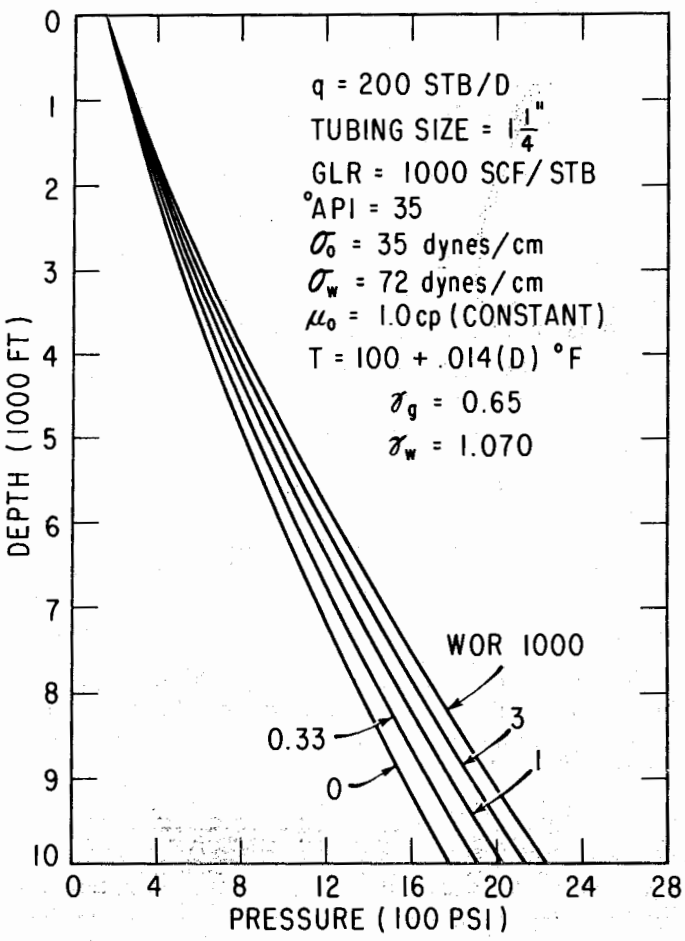


Fig. 2.116 Effect of WOR (after Brill, et al., courtesy SPE).

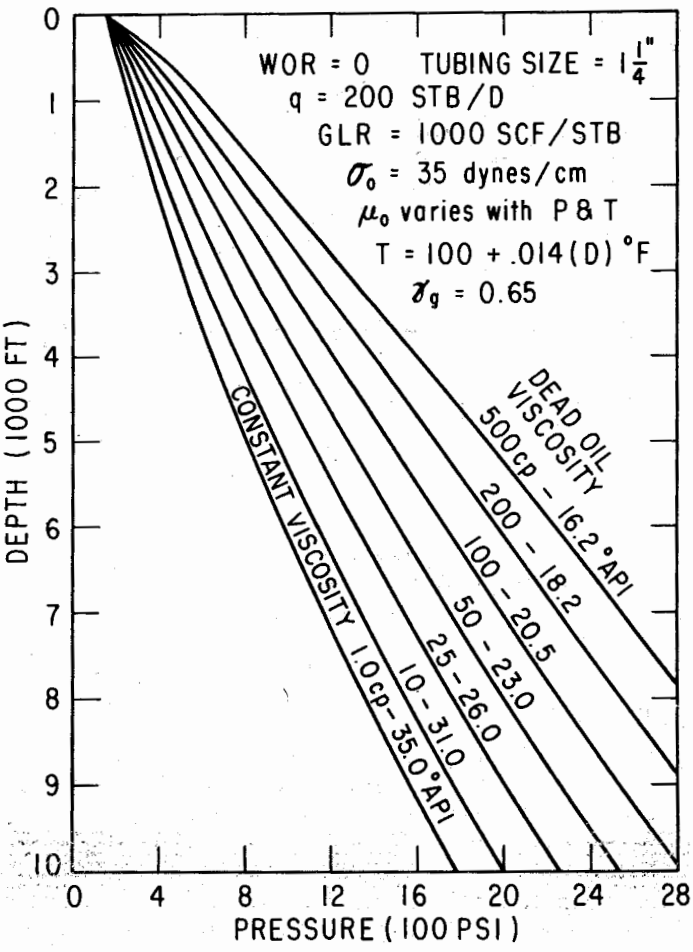
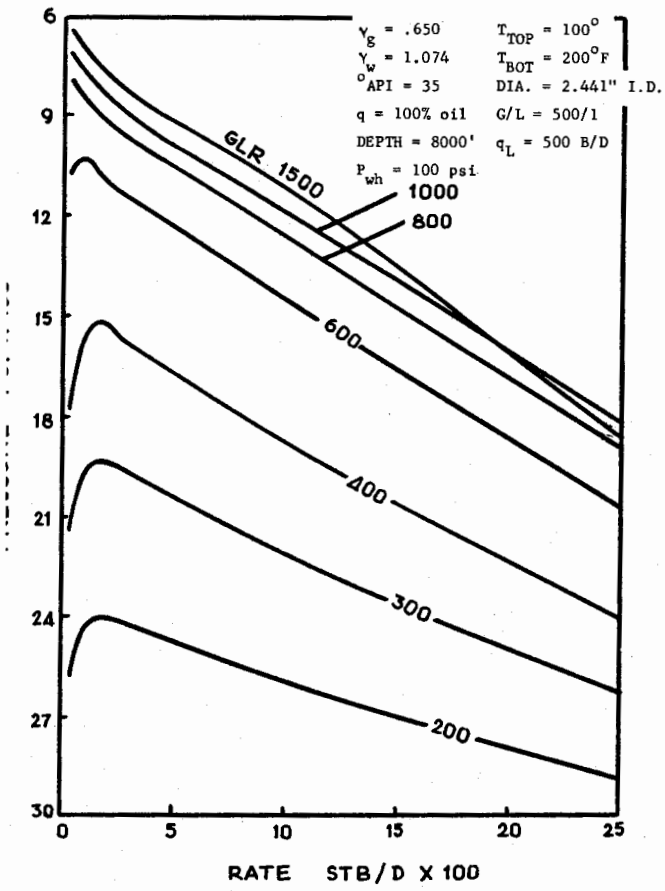


Fig. 2.118 Effect of viscosity (after Brill, et al., courtesy SPE).

TABLE 2.314
VISCOSITY EFFECT

Viscosity	FBHP
1	1,371
10	1,462
25	1,711
50	1,979
100	2,236
200	2,457
500	2,612
1,000	2,704
5,000	2,771



ig. 2.119 Effect of slippage.

TABLE 2.315
SLIPPAGE EFFECT SHOWING FLOWING BOTTOM HOLE PRESSURES

	FLOW RATE						
LR	200	300	400	600	800	1,000	1,500
50	2,593	2,163	1,762	1,081	782	704	631
100	2,469	2,007	1,588	1,028	852	774	706
150	2,426	1,952	1,531	1,066	892	815	742
200	2,412	1,944	1,535	1,096	922	846	778
300	2,437	1,975	1,586	1,145	971	897	826
000	2,572	2,190	1,861	1,439	1,261	1,184	1,110
500	2,881	2,619	2,395	2,069	1,917	1,860	1,876

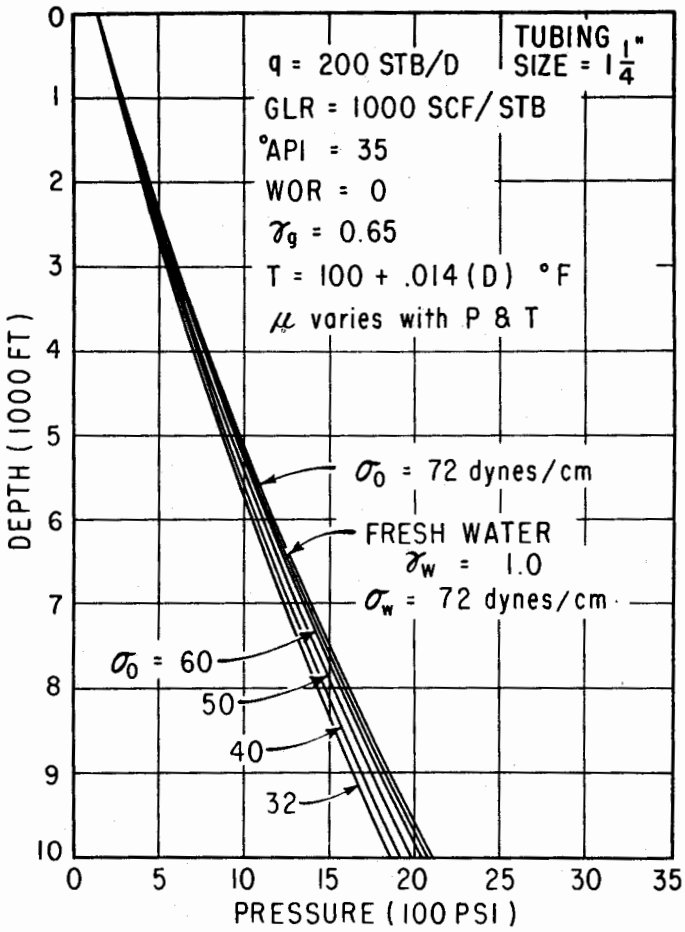


Fig. 2.120 Effect of surface tension (after Brill, et al., 17, courtesy SPE).

p_{wh} = well-head pressure
q_o = oil flow rate
q_w = water flow rate
q_L = liquid flow rate
G/O = Gas-oil ratio
G/L = gas-liquid ratio
γ_g = gas gravity

EXAMPLE PROBLEM #1: HOW TO DETERMINE THE FLOWING BOTTOM HOLE PRESSURE OF A WELL

Given: Depth = 8000 ft
Tubing = 2 in.
$\bar{p}_R$ = 3,000 psi
p_{wh} = 200 psi
G/O = 200 scf/bbl
γ_g = 0.65
q = 400 bpd (all oil)

Find p_{wf}

Solution:

The method for determining equivalent depth will be given in this example, but will not be repeated in other examples since the same general procedure applies.

Step-wise procedure

Reference should be made to Fig. C.88.

(1) Select a depth equivalent to the tubing pressure of 200 psig by proceeding vertically downward from

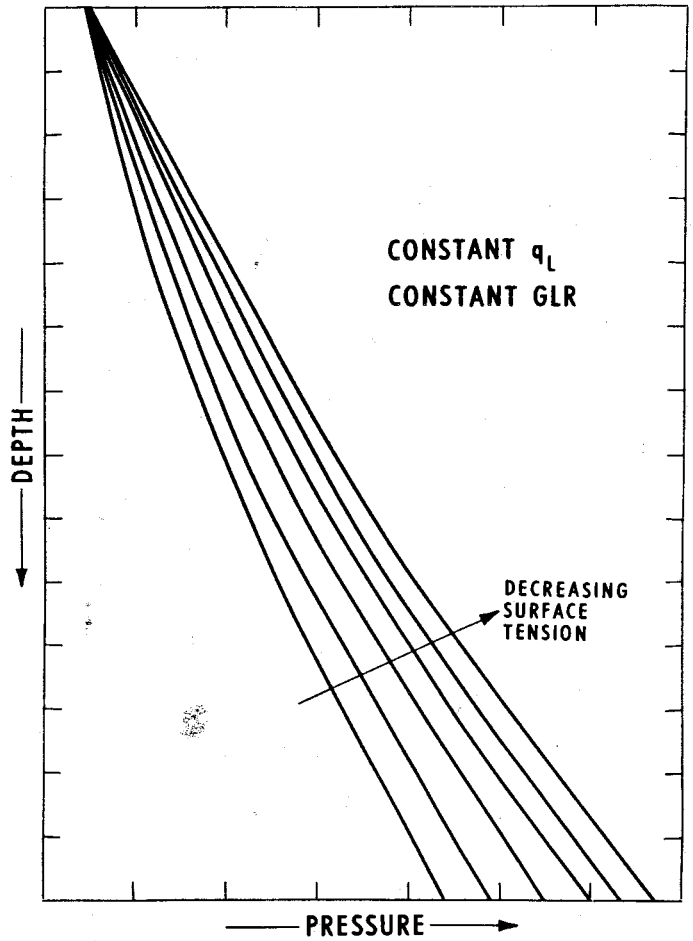


Fig. 2.121 Predicted effect of surface tension on flowing bottom-hole pressure.

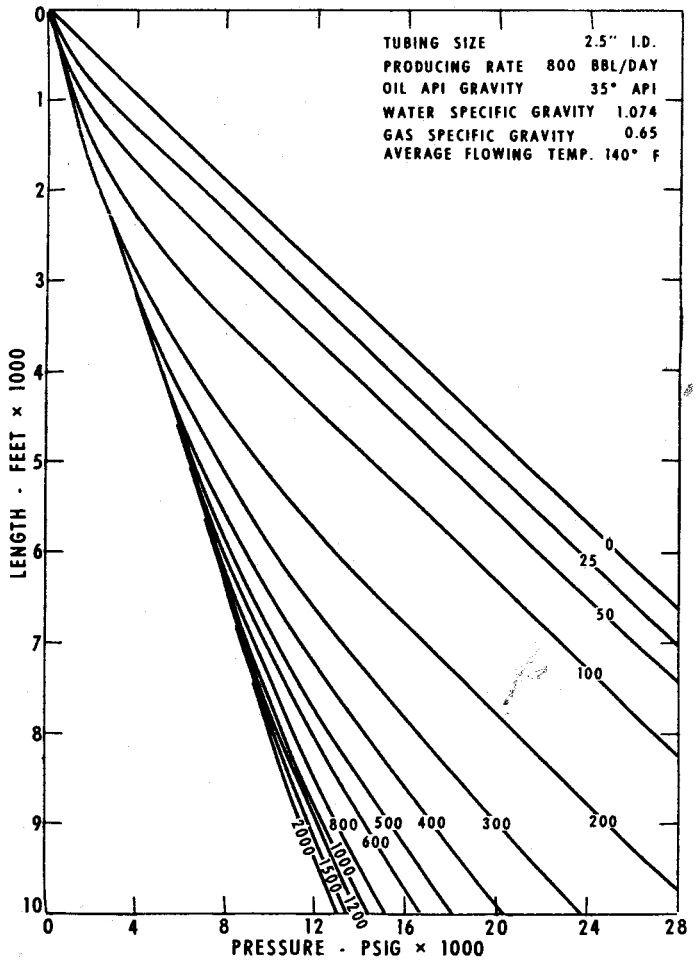


Fig. 2.123 Vertical flowing pressure gradients (50% oil—50% water).

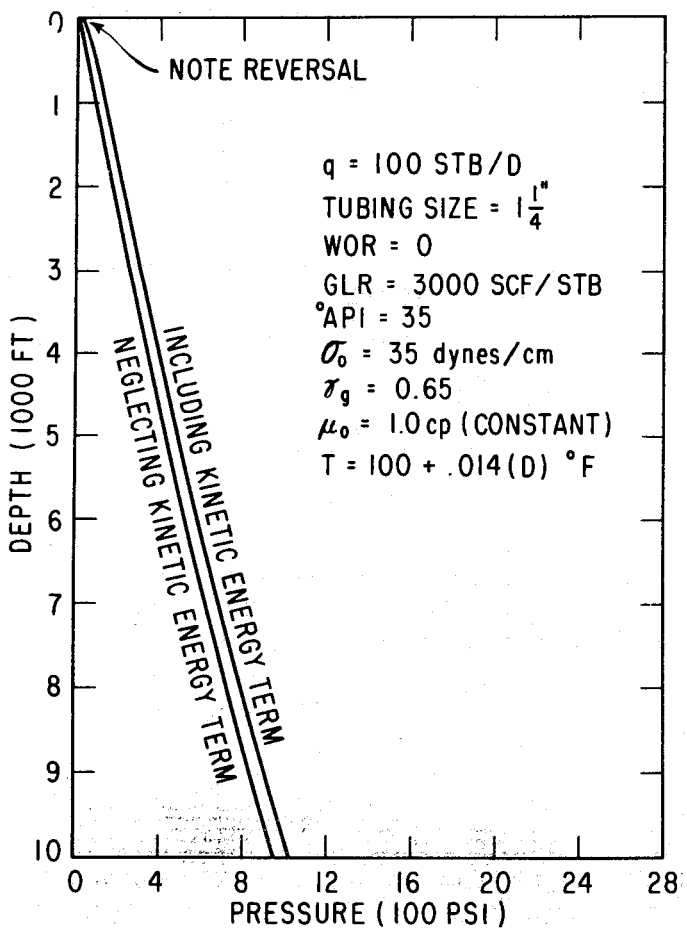


Fig. 2.122 Effect of kinetic energy (after Brill, et al., courtesy SPE).

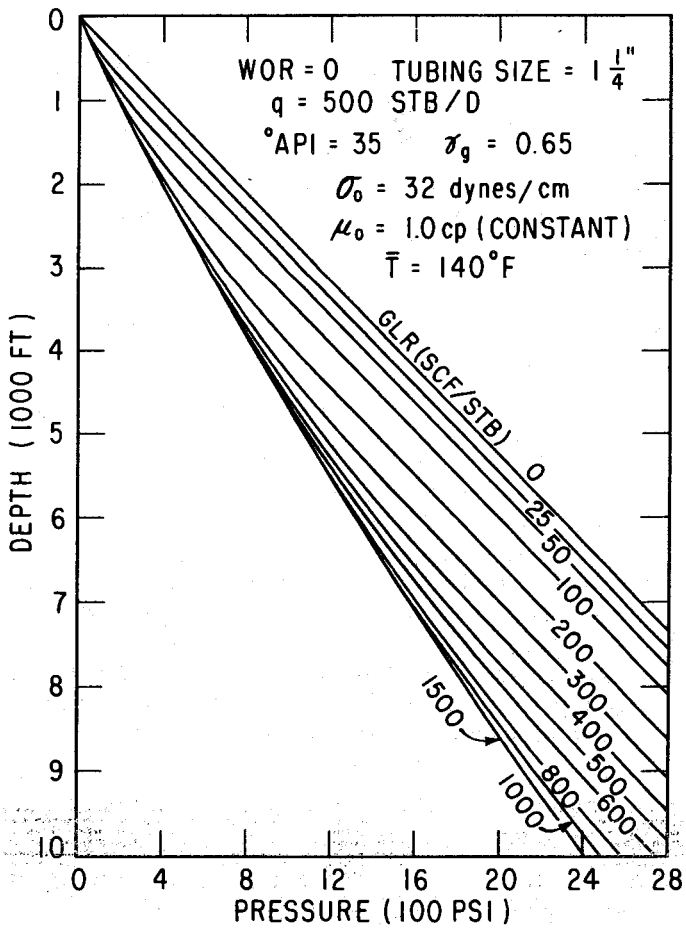


Fig. 2.124 Typical vertical pressure traverse curves (reversal removed).

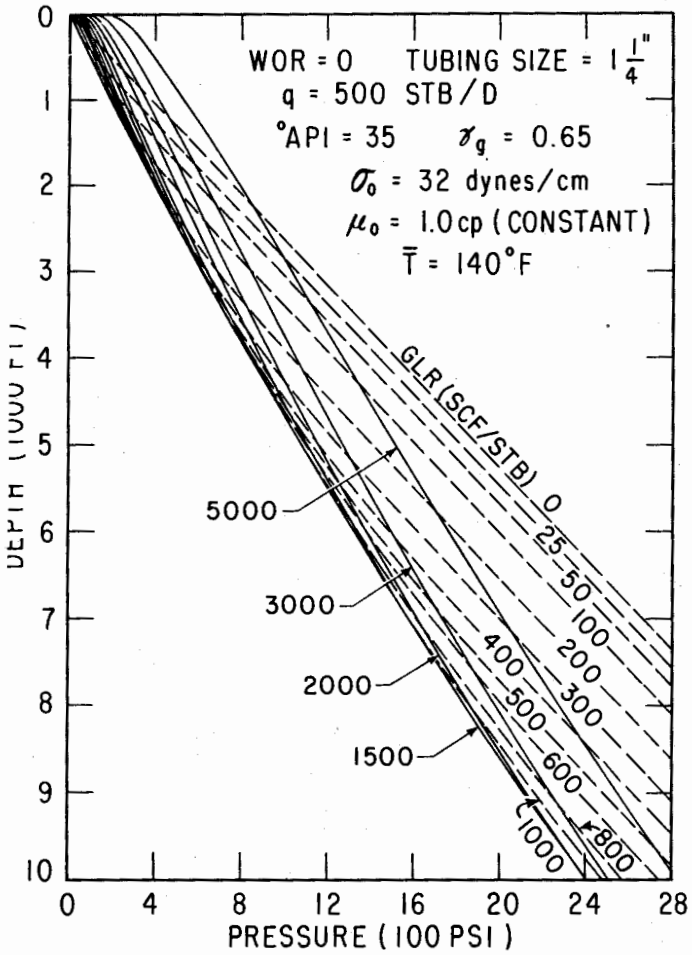


Fig. 2.125 Vertical pressure traverse curves (reversal included).

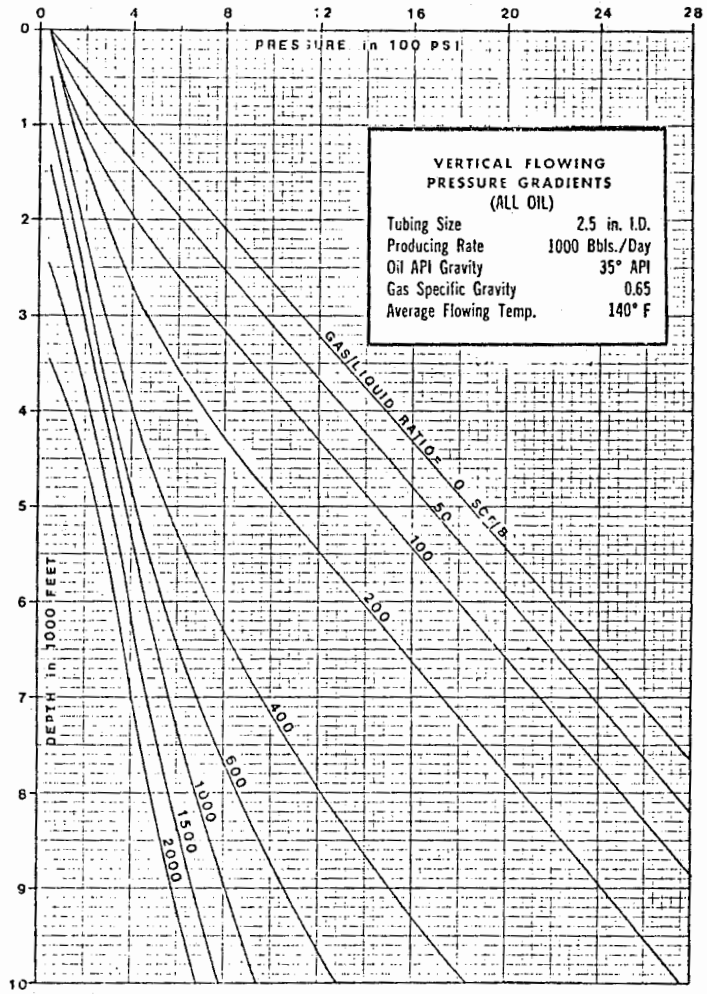


Fig. 2.126 Typical vertical pressure traverse curves (after Ros).

200 psig at zero depth until intersecting the 200 scf/bbl gas-oil ratio line. This is found to be 2,000 ft.

(2) Add the equivalent depth of 2,000 ft to the well depth of 8,000 ft, giving a graph depth of 10,000 ft.

(3) At the intersection of the 10,000 ft depth line and the 200 scf/bbl GOR line, read the pressure to be 2,700 psig.

EXAMPLE #1-B: HOW TO DETERMINE p_{wf} AND p_{wf} MINIMUM

Given: q = 200 bpd (all oil)
2 in. = tubing
d = 7000 ft
G/O = 500 scf/bbl
p_{wh} = 120 psi
Refer to Fig. C.86.

(a) Find p_{wf} for G/O = 500. After finding the equivalent depth of 1800 ft for 120 psi we find p_{wf} = 1,060 psi

(b) Find p_{wf} for G/O = maximum = 3,000 scf/bbl. This gives the minimum flowing gradient. Following the same procedure we find p_{wf} = 680 psi.

CLASS PROBLEM #1-A: TO FIND THE FLOWING BOTTOM HOLE PRESSURE

Given: Tubing = 3 in.
q_o = 1000 bpd (all oil)
p_{wh} = 80 psi
d = 8000 ft
G/O = 400 scf/bbl

Find p_{wf}

CLASS PROBLEM #1-B: TO FIND THE FLOWING BOTTOM HOLE PRESSURE

Given: q = 100 bpd (all oil)
p_{wh} = 100 psi
Tubing = 2 in.
d = 8000 ft
G/O = 300 scf/bbl

Find p_{wf}

EXAMPLE PROBLEM #2-A: HOW TO FIND THE AVERAGE PRODUCTIVITY INDEX FOR A WELL (Flowing pressure above bubble point)

Given: 2½ in. tubing
q_L = 1000 bpd (95% water)
p_{wh} = 80 psi
d = 9000 ft
G/L = 400 scf/bbl
$\bar{p}_R$ = 2,600 psi

A new problem arises here in that the well is making 95% water. In this case use the all water curve of

Figure C.110. When the well produces in excess of 85–90% water it is appropriate to use all water curves. Curves are available for all oil, 50% oil and 100% water. Interpolations can be performed, but will generally give results very near the answer obtained when using the curve nearest the percentage of 100% oil; 100% water or 50–50. For example when water production is greater than 75%, use all water curves, but interpolate when in doubt. A computer solution to account for exact percentages of water and oil is much better.

Following our same solution procedure we first find $p_{wf} = 2{,}175$ psi.

$$\text{Average PI} = \frac{q}{\bar{p}_R - p_{wf}} = \frac{1{,}000}{2{,}600 - 2{,}175} = 2.35$$

This represents an average PI between flowing pressures of 2,600 and 2,175 psi and assumes a linear relationship.

CLASS PROBLEM #2-A: TO FIND PI

Given 3 in. tubing (all oil)

$q = 1000$ bpd
$\bar{p}_R = 3{,}000$ psi
$p_{wh} = 120$ psi
$G/O = 600$ scf/bbl
$d = 8000$ ft

(a) Find PI assuming a linear relationship, and find $(q_o)_{max}$.
(b) Construct a complete IPR curve according to Vogel solution and find $(q_o)_{max}$.

CLASS PROBLEM #2-B: TO FIND PI FROM FLOW TEST

Given: 2 in. tubing
$q = 600$ bpd (all water)
$p_{wh} = 120$ psi
$d = 8000$ ft
$\bar{p}_R = 2{,}800$ psi
$G/L = 300$ scf/bbl

Find the average PI assuming a linear IPR.

EXAMPLE PROBLEM 2B: HOW TO FIND THE AVERAGE PI FOR A WELL WITH PRESSURE BELOW THE BUBBLE POINT AND HOW TO CONSTRUCT THE IPR CURVE

Given: 2½ in. tubing
$q_o = 800$ bpd
$\bar{p}_R = 2{,}000$ psi
$d = 8{,}500$ ft
$p_{wh} = 100$ psi
Bubble point pressure $= 2{,}400$ psig
$G/O = 600$ scf/bbl

Solution:

For this problem it must be assumed that the IPR curve is not linear. An average PI can be found between one static and flowing pressure for the one test. Using the one test an IPR curve can be constructed according to the method of Vogel as outlined in Chapter I by assuming various pressures and calculating the flow rate. The procedure is the same as set out in Chapter I except that we find p_{wf} from a multiphase flow correlation.

(1) Find p_{wf} for 800 bpd in 2½ in. tubing to be 1,350 psi (Fig. C.104).
(2) Using Vogel's reference curve of Fig. 1.30 we find $(q_o)_{max}$ as follows:

$$\frac{p_{wf}}{\bar{p}_R} = \frac{1{,}350}{2{,}000} = 0.675 \quad \text{Reading from Fig. 1.30}$$

we find $q_o/q_{max} = 0.50$

$$\frac{800}{(q_o)_{max}} = 0.50 \qquad (q_o)_{max} = 1{,}600 \text{ bpd}$$

A table of q_o vs. p_{wf} is then prepared by assuming values of p_{wf} (See Table 2.316)

TABLE 2.316

q_o	p_{wf}
0	2,000
1,600	0
800	1,350
1,120	1,000
640	1,500
1,440	500

A graph is then prepared.

CLASS PROBLEM 2B-1: TO FIND AVERAGE PI

Given: $d = 8000$ ft
2 in. tubing
$p_{wh} = 100$ psi
$G/O = 400$ scf/bbl
$\bar{p}_R = 1{,}800$ psi
$q_o = 200$ bpd (all oil)

Find the average PI:

(a) Assume linear
(b) Find $(q_o)_{max}$ – method of Vogel

CLASS PROBLEM 2B-2: TO FIND AVERAGE PI

Given: $d = 8000$ ft
2½ in. tubing
$q_L = 600$ bpd (85% water)
$p_{wh} = 120$ psi
$G/L = 500$ scf/bbl
$\bar{p}_R = 2{,}400$ psi

Find average PI (assume linear)

EXAMPLE PROBLEM #3A: HOW TO DETERMINE AVERAGE PI FROM 2 FLOW TESTS (LINEAR PI)

To off-set any error in multiphase flow correlations, it is recommended that the PI be determined by taking two flow tests on the well.

iven: d = 7000 ft
G/O = 500 scf/bbl
2 in. tubing
Producing all oil

TEST I	TEST II
$q = 600$ bpd	$q = 1{,}000$ bpd
$p_{wh} = 360$ psi	$p_{wh} = 160$ psi

he manner prescribed previously we find:

$$p_{wf_1}(\text{Test I}) = 1{,}840 \text{ psi}$$
$$p_{wf_2}(\text{Test II}) = 1{,}640 \text{ psi}$$

ce we are assuming the inflow curve to be a straight :, then PI can be determined from the value of where $PI = -\frac{dq}{dp_{wf}}$

$$= -\left(\frac{q_2 - q_1}{p_{wf_2} - p_{wf_1}}\right) = -\left(\frac{1{,}000 - 600}{1{,}640 - 1{,}840}\right) = -\left(\frac{400}{-200}\right)$$
$$= 2 = \text{positive value}$$

: static pressure can also be obtained from these) tests by making a plot of q vs. p_{wf} and extending to o flow rate, or can be calculated from knowing the and using the data of one test.

$$PI = \frac{q}{\Delta P} = \frac{600}{\bar{p}_R - 1{,}840} = 2$$

$\bar{p}_R = 2{,}140$ psi.

$\bar{p}_R$ can be found using the following two equations:

$$PI = \frac{q_1}{\bar{p}_R - p_{wf_1}} = \frac{q_2}{\bar{p}_R - p_{wf_2}}$$

m which:

$$\bar{p}_R = \frac{q_1 p_{wf_2} - q_2 p_{wf_1}}{q_1 - q_2}$$
$$= \frac{(600)(1{,}640) - (1{,}000)(1{,}840)}{600 - 1{,}000} = 2{,}140 \text{ psi}$$

AMPLE PROBLEM 3B: NON-LINEAR PI—BEHAVIOR CORDING TO VOGEL'S INFLOW REFERENCE CURVE :e Chapter 1)

Given two flow tests as noted in Example 3-A:

Test I	Test II
$q = 600$ bpd	$q = 1{,}000$ bpd
$p_{wh} = 360$ psi	$p_{wh} = 160$ psi
$p_{wf_1} = 1{,}840$	$p_{wf_2} = 1{,}640$ psi

nd $\bar{p}_R$ and establish the inflow performance curve. om Vogel's equation:

$$q_o/(q_o)_{max} = 1 - 0.2\left(\frac{p_{wf}}{\bar{p}_R}\right) - 0.8\left(\frac{p_{wf}}{\bar{p}_R}\right)^2$$

st I

$$\frac{600}{(q_o)_{max}} = 1 - 0.2\left(\frac{1{,}840}{\bar{p}_R}\right) - 0.8\left(\frac{1{,}840}{\bar{p}_R}\right)^2$$

Test II

$$\frac{1{,}000}{(q_o)_{max}} = 1 - 0.2\left(\frac{1{,}640}{\bar{p}_R}\right) - 0.8\left(\frac{1{,}640}{\bar{p}_R}\right)^2$$

By solving the preceding two equations we find $\bar{p}_R = 2{,}109$ psi and $(q_o)_{max} = 2{,}774$ bpd. A complete IPR curve can be prepared with this information.

In this solution we have two equations and two unknowns and the following quadratic equation is found:

$$4\,\bar{p}_R^2 - 1{,}712\,\bar{p}_R - 3{,}764.93^2 = 0$$
$$\bar{p}_R = \frac{1{,}712 \pm \sqrt{1{,}712^2 + (16)(3{,}764.93^2)}}{8}$$
$$\bar{p}_R = \frac{1{,}712 \pm 15{,}156}{8} = 2{,}108.6 \text{ psi}$$

Once we have solved for $\bar{p}_R$ then $(q_o)_{max}$ can be solved from either equation and is found to be 2,774 bpd.

CLASS PROBLEM #3A-1: TO FIND THE PI FROM 2 FLOW TESTS

Given: 2½ in. tubing
Production—all oil
d = 6,000 ft
G/O = 600 scf/bbl

Test I	Test II
$q_o = 1{,}000$ bpd	$q_o = 2{,}000$ bpd
$p_{wh} = 420$ psi	$p_{wh} = 120$ psi

(a) Find $\bar{p}_R$ and PI for linear relationship
(b) For Vogel reference curve—construct IPR curve

CLASS PROBLEM #3A-2: TO FIND THE PI FROM 2

Given: d = 7000 ft
Production—95% water
2 in. tubing
G/L = 500 scf/bbl

Test I	Test II
$q_L = 1{,}000$ bpd	$q_L = 600$ bpd
$p_{wh} = 200$ psi	$p_{wh} = 400$ psi

Find PI and $\bar{p}_R$:

(a) Based on linear relationship

EXAMPLE PROBLEM #4: NEGATIVE PI

A flowing well of 7000 ft depth is known to be producing 800 bpd of 35° API crude from 2 in. tubing and the flowing surface well head pressure is 80 psi. The flowing G/O is 500 scf/bbl. A pressure bomb is run in this well for a PI test and a restriction does not allow the bomb to be run deeper than 6,000 ft. The static pressure recorded at this point was 1,090 psi and the flowing pressure recorded at this point was 1,160 psi. Find the PI at the depth of the bomb and explain why this is negative.

Do you feel that these two pressures could be recorded such that the static was greater than the flowing pressure? How would you explain this?

Determine the true PI of the well by extending both pressures to the bottom of the well.

Solution:

It is necessary that both pressures be extended downwards or corrected to 7,000 ft.

(1) 35° API = 0.368 psi/ft gradient under static conditions. Therefore, $\bar{p}_R$ at 7,000 ft = 1,090 + (1,000) (0.368) = 1,458 psi.

(2) p_{wf} at 7000 ft is found by extending the flowing pressure of 1,160 psi to the bottom of the well, that is, from 6000 ft to 7000 ft. This can be done by tracing the appropriate gradient curve (Fig. C.90). However, in checking the 2 in. curve for 800 bpd, we note a flowing pressure of 1,120 psi at 6000 ft or a 40 psi discrepancy between the measured pressure and the pressure predicted by the curves. This may be caused by any number of reasons such as errors, paraffin, scale, or corrosion in the tubing string causing a higher measured value than calculated value. The measured value should be used and the pressure point of 1,160 psi should be shifted directly (vertically down) over the 500 scf/bbl line and a flowing pressure of 1,400 psi is noted.

$$(3)\ PI = \frac{q}{\Delta p} = \frac{800}{1{,}458 - 1{,}400} = \frac{800}{58} = 13.8$$

CLASS PROBLEM #4-A: NEGATIVE PI

Given: d = 8000 ft
2 in. tubing
q_o = 300 bpd (all oil), 35°API oil
G/O = 500 scf/bbl
p_{wh} = 120 psi

Restriction in tubing at 6,000 ft required that "pressure recording gage" be stopped at 6,000 ft.

$\bar{p}_R$ at 6,000 ft = 660 psi
p_{wf} at 6,000 ft = 740 psi

This indicates a negative PI at 6000 ft because $p_{wf} > \bar{p}_R$. Find the correct PI by extending both $\bar{p}_R$ and p_{wf} to 8,000 ft.

EXAMPLE PROBLEM #5: HOW TO FIND THE TUBING SIZE NECESSARY

Given: $\bar{p}_R$ = 3,000 psi
PI = 1.5 (linear)
q = 1000 bpd (95% water)
d = 8000 ft
G/L = 400 scf/bbl
p_{wh} = 120 psi

Find minimum tubing size necessary for the conditions given.

Solution:

(1) Find necessary p_{wf} for well to "give up" 1,000 bpd

$$\text{Drawdown} = \frac{1{,}000}{1.5} = 667 \text{ psi.}$$

p_{wf} = 3,000 − 667 = 2,333 psi required.

(2) By trial and error solution find the minimum tubing size to give a flowing pressure equal to or less than 2333 psi.

(3) Assume a tubing size and determine p_{wf} for 1,000 bpd in same manner as in previous problems.

Tubing size, in.	p_{wf}, psi
2	2,380
2½	1,980

We note that 2½ in. tubing is needed in that 2 in. requires 2,380 psi which is greater than 2,333 psi. However, 2 in. tubing is very close to being sufficient and might be selected based on costs and completion practices.

CLASS PROBLEM #5-A: TO FIND TUBING SIZE NECESSARY

Given: d = 8000 ft
q_o = 1,000 bpd (all oil)
p_{wh} = 80 psi
G/O = 600 scf/bbl
$\bar{p}_R$ = 2,300 psi
PI = 1.0

What minimum tubing size can be used?

CLASS PROBLEM #5-B: TO FIND TUBING SIZE NECESSARY

Given: q_o = 3,000 bpd (all oil)
p_{wh} = 120 psi
d = 8000 ft
$\bar{p}_R$ = 2,500 psi
PI = 3
G/L = 350 scf/bbl

Find minimum tubing size.

CLASS PROBLEM #5-C: TO FIND TUBING SIZE NECESSARY

Given: d = 8,000 ft
q_w = 1500 bpd
q_o = 100 bpd
p_{wh} = 100 psi
$\bar{p}_R$ = 2,400 psi
PI = 10
GOR = 3,000 scf/bbl

Find minimum tubing size.

Note: You must find G/L from G/O.

EXAMPLE PROBLEM #6: HOW TO FIND THE FLOWING WELLHEAD PRESSURE

Given: 2 in. tubing
q_o = 600 bpd (all oil)
GOR = 400 scf/bbl
d = 8000 ft
$\bar{p}_R$ = 2,400 psi
PI = 2 (assume linear)

ind the maximum wellhead pressure against which his well will produce.

olution:
(1) Determine p_{wf} for 600 bpd

$$p_{wf} = \bar{p}_R - \frac{q}{PI} = 2{,}400 - \frac{600}{2} = 2{,}100 \text{ psi}$$

(2) From Fig. C.89 locate 2,100 psi on the 400 scf/bbl G/O line and note this arbitrary depth of 10,025 ft.
(3) Subtract the depth of the well (8000 ft) from 0,025 = 10,025 − 8,000 = 2,025 ft.
(4) Read p_{wh} on the 400 scf/bbl G/O line at 2,025 ft = 20 psi.

LASS PROBLEM #6-A: TO DETERMING FLOWING /ELLHEAD PRESSURE

Given: d = 8,000 ft
q_o = 800 bpd (all oil)
2½ in. tubing
G/O = 300 scf/bbl
PI = 1.0
$\bar{p}_R$ = 2,800 psi

Find p_{wh}

CLASS PROBLEM #6-B: TO DETERMINE FLOWING VELLHEAD PRESSURE

Given: d = 7,000 ft
2 in. tubing
q = 1,000 bpd (90% water)
G/L = 300 scf/bbl
$\bar{p}_R$ = 2,500 psi
PI = 10 (Work also for PI = 2)

Find p_{wh} (well head flowing pressure).

CLASS PROBLEM #6-C: TO DETERMINE FLOWING WELLHEAD PRESSURE

Given: d = 8,000 ft
2 in. tubing
q_L = 800 bpd (50% oil)
G/O = 600 scf/bbl
$\bar{p}_R$ = 1,900 psi
PI = 2

Find p_{wh}

Caution: $G/L = \frac{600}{2} = 300$ scf/bbl

EXAMPLE PROBLEM #7: HOW TO FIND THE NECESSARY GAS-LIQUID RATIO

Given: d = 8,000 ft
2 in. tubing
p_{wh} = 100 psi
$\bar{p}_R$ = 2,800 psi
PI = 2.2 (assume linear)
q_w = 1,000 bpd (all water)

Find the required G/L for this well to flow (Fig. C.95).

Solution:

(1) Determine required p_{wf} for 1,000 bpd

$$p_{wf} = \bar{p}_R - \frac{q}{PI}$$

$$p_{wf} = 2{,}800 - \frac{1{,}000}{2.2} = 2{,}345 \text{psi}$$

(2) Find depth equivalent for p_{wh} = 100 psi. To do this we must assume a G/L. For the first assumption assume G/L to fall on the minimum gradient line. This is found to be 580 ft.
(3) Add 580 ft to depth of 8,000 ft = 8,580 ft
(4) At depth of 8,580 ft on the curve move horizontally until you find the flowing pressure of 2,345 psi. Read the gas-liquid ratio at this pressure to be 385 scf/bbl. By checking, we see that the G/L of 385 scf/bbl falls on the minimum gradient line for p_{wh} = 100 psi as mentioned in Step 2. Therefore no further assumptions are necessary.

CLASS PROBLEM #7-A: TO FIND THE REQUIRED G/L

Given: d = 8,500 ft
2 in. tubing
q_L = 600 bpd (90% water)
p_{wh} = 200 psi
$\bar{p}_R$ = 2,200 psi
PI = 2

Find the required G/L.

CLASS PROBLEM #7-B: TO FIND THE REQUIRED G/L

Given: d = 7,500 ft
2½ in. tubing
p_{wh} = 100 psi
q_o = 600 bpd (all oil)
$\bar{p}_R$ = 1,680 psi
PI = 1.0

Fing the required G/L.

CLASS PROBLEM #7-C: TO FIND TOTAL G/L

Given: d = 8,500 ft
q_L = 4,000 bpd (50% oil)
4 in. tubing
PI = 4
$\bar{p}_R$ = 2,500 psi
p_{wh} = 80 psi
G/O = 400 scf/bbl

Find:

(a) Total G/L necessary for well to flow.
(b) If gas can be injected around the bottom of the tubing string, how much injection gas is needed per day.

EXAMPLE PROBLEM #8: HOW TO DETERMINE THAT FLOW RATE AT WHICH THE TUBING SIZE IS TOO LARGE (slippage becomes excessive).

Given: $d = 9000$ ft
$2\frac{1}{2}$ in. tubing
$p_{wh} = 100$ psig
$G/O = 400$ scf/bbl
q_o = variable (all oil)

Solution:

This problem is worked independently of the reservoir factors such as PI and static bottom hole pressure although the IPR curve may be included on the same plot as shown in Fig. 2.119.

Solution:

(1) Assume flow rates making sure that several flow rates below 500 bpd are in the group.

(2) From the appropriate gradient curve determine the necessary flowing bottom hole pressure for each assumed rate.

(3) Plot flow rate vs. pressure as noted in Fig. 2.119, and following the curve towards decreasing rates, select that rate at which the pressure begins to increase. This represents that point at which excess slippage begins to occur in the tubing string. For this example this begins to occur at about 250 bpd.

Note: This does not necessarily mean that we should change to the next smaller tubing size at this point, but the required pressure increases very rapidly as we continue to lower the rate. Smaller tubing sizes should be checked in the same manner before a decision is made.

CLASS PROBLEM #8-A: TO DETERMINE THAT FLOW RATE AT WHICH THE TUBING SIZE MAY BE TOO LARGE

Given: $d = 7000$ ft
$p_{wh} = 120$ psi
$G/O = 200$ scf/bbl
Tubing = $2\frac{1}{2}$ in.
q_o = variable (all oil)

Find that flow rate at which the flowing bottom hole pressure begins to increase due to slippage. For the flow rate determined, find p_{wf} for tubing diameters of 2 in., $1\frac{3}{4}$ in., $1\frac{1}{2}$ in., and $1\frac{1}{4}$ in. Which size gives the minimum flowing pressure?

CLASS PROBLEM #8-B: TO DETERMINE THAT FLOW RATE AT WHICH THE TUBING SIZE MAY BE TOO LARGE

Given: $d = 8000$ ft
$p_{wh} = 80$ psi
4 in. tubing
$G/O = 200$ scf/bbl
q_o = variable (all oil)

Find that flow rate at which the flowing bottom hole pressure begins to increase due to slippage.

CLASS PROBLEM #8-C: TO FIND OPTIMUM TUBING SIZE

Given: $p_{wh} = 120$ psi
$d = 7000$ ft
$G/O = 200$ scf/bbl
$q_o = 200$ bpd
$2\frac{1}{2}$ in. tubing

Find the flowing bottom hole pressure necessary for $2\frac{1}{2}$ in., 2 in., $1\frac{3}{4}$ in., $1\frac{1}{2}$ in. and $1\frac{1}{4}$ in. tubing. Select the optimum tubing size.

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2.4 HORIZONTAL FLOW

2.41 Introduction

The prediction of pressure loss in horizontal pipes for multiphase flow concerns two principal operational phases in the petroleum industry: (1) the sizing of transmission lines and long offshore lines for the transportation of oil and gas mixtures and (2) the sizing of surface flow lines from the wellhead to the battery, and the prediction of pressure loss in existing flow lines to determine well production rates.

The formulas and techniques used for predicting pressure losses in horizontal multiphase flow also apply in sizing the lines for: (1) glycol injection into wet gases to prevent or control the forming of hydrates, (2) piping systems in industrial plants, including refineries, and (3) transportation systems carrying gas with water or condensate, or those carrying gas with a multicomponent mixture of gas, condensate, and water.

The basic factors involved in horizontal flow of multiphase fluids in pipes are essentially the same ones that apply for vertical flow. In both systems, the total pressure loss is the sum of the frictional and kinetic energy losses. The principal difference lies in the energy balances of the two systems due to differences in the positional considerations between the vertical and horizontal flow processes.

Horizontal flow, as described in this chapter, does not include inclined flow; however, many correlations have used data from pipes that undoubtedly varied a few degrees from the horizontal. (Refer to Fig. 2.127). This could account for some of the discrepancy in correlations. If a pipe is elevated a few degrees from the horizontal, the holdup may change from 50% to as much as 90%, depending on the liquid and gas rate. Inclined flow is further discussed in Section 2.5.

There has been a tremendous amount of theoretical and experimental research in the field of horizontal multiphase flow. Fig. 2.128 shows the number of publications for all gas-liquid flow studies through 1964, as given by Gouse.[1] It has been estimated that the total cost for all of the work represented by these publications is about $23 million. Most of these technical contributions have been made since 1950.

Many attempts have been made to differentiate between the various flow patterns. Several correlations require that the flow pattern be first determined before they can be used. Eaton, Andrews, Knowles, Silberberg and Brown[2] showed that a flow-pattern determination is not necessary. This aspect is discussed in Sec. 2.42.

The main objective of horizontal multiphase-flow studies has been to develop techniques by which a pipeline may be properly sized, or by which the pressure loss may be calculated for a known pipe size. Pressure losses for multiphase flow differ greatly from those encountered in single-phase flow. Losses may be from five to ten times as much as those expected for a comparable flow rate in single-phase flow. They may be even greater for emulsified flow, as shown by Sobocinski and Huntington.[3]

2.42 Flow patterns

Many investigators of horizontal multiphase flow have chosen to separate their experimental data into groups that fit the various flow patterns or regimes. These investigators who have studied flow patterns have offered correlations for their predictions; however, there is a great deal of discrepancy in all of this work, and most investigators have failed to discover the degree of influence of flow pattern on pressure loss. Some investigators have shown abrupt changes in flow patterns. Personal observations of the authors have shown that the changes are somewhat gradual and that there is a transition region when moving from one pattern to another. This necessitates approaching the problem of flow pattern from a qualitative standpoint. Any dip or change in the flow line from a horizontal position will change the flow pattern. A slight upgrade will cause the pipe to fill with liquid and, therefore, will create a change in flow pattern.

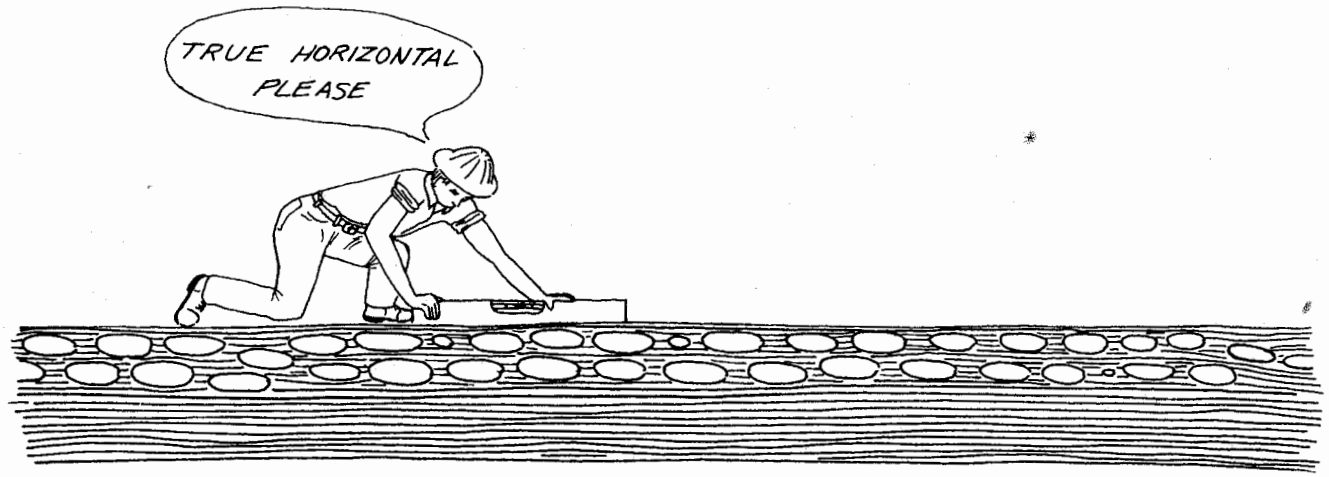


Fig. 2.127 Horizontal multiphase flow.

Although some controversy exists about the value of flow patterns or flow regimes, a brief discussion is given here because all of the aspects of horizontal multiphase flow have not been satisfactorily solved for all ranges of flow rates. Eventually, it may be necessary for the best solution to the problem to at least partially account for flow patterns in horizontal flow.

In the horizontal multiphase flow literature, prior to the article by Knowles, et al,[4] flow patterns were classified into eight categories. These were described by Alves,[5] and are listed as follows in the order of increasing gas velocity. (Refer to Fig. 2.129)

1. Bubble flow
2. Plug flow
3. Stratified flow (layered, separated)
4. Wavy flow (ripple flow, cresting)
5. Slug flow
6. Semi-annular flow
7. Annular flow (ringed)
8. Spray flow (mist, froth, dispersed)

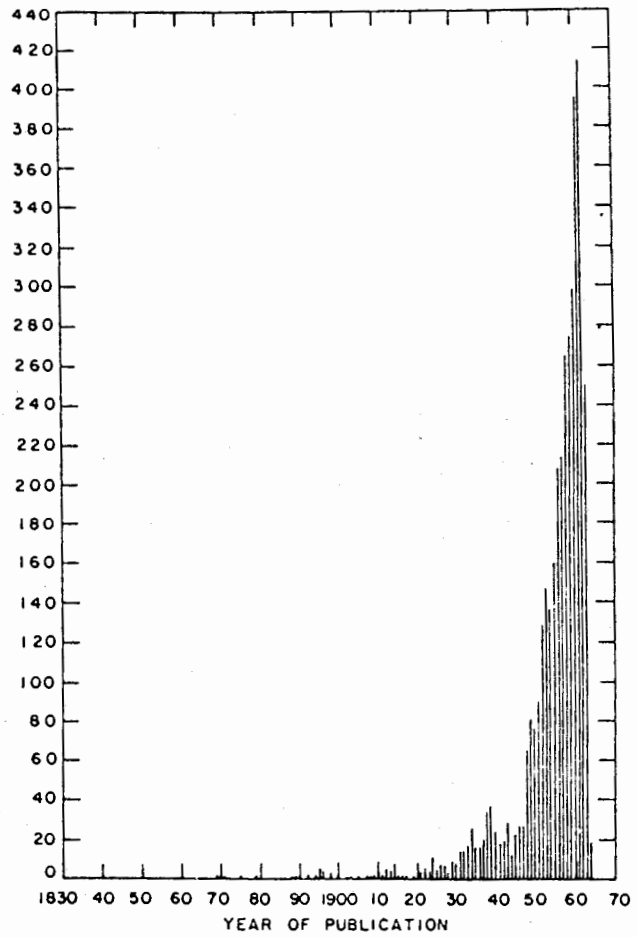
Table 2.41 lists those authors who have made the greatest contributions in determining either flow patterns or flow regimes. Although most of the final selections of methods to predict pressure losses do not require flow patterns, these references are given to show the concern of numerous authors for the necessity of being able to determine flow patterns or regimes.

The ultimate solution to the horizontal multiphase flow problem may, in fact, require some knowledge of flow patterns. Lockhart and Martinelli[6] require that the flow regime be known in order to predict pressure loss. However, when it is noted that most practical flow problems place both gas and liquid in turbulent flow, then Lockhart and Martinelli really use only one of the four flow regimes. Baker[10] really offers the only significant work whereby flow patterns are required to calculate pressure loss, and his solution is probably best when the flow falls in the slug flow region as predicted by his flow pattern map. A procedure for the determination of flow patterns by Baker is given in Appendix D.11 with an example for annular flow in Appendix D.12 and slug flow in Appendix D.13. His flow pattern map is noted in Fig. 2.130. It is very likely that the flow pattern as observed will not be the same as calculated in using Baker's correlation.

From Table 2.41 the best qualitative information available for the prediction of flow patterns is the data obtained by Knowles, et al,[4] and a detailed description of these can be found in Ref. 16. However, Knowles offered no solution for calculating pressure losses based on flow patterns. Quantitative data recorded during these tests have been discussed by Andrews, Silberberg, and Brown.[18]

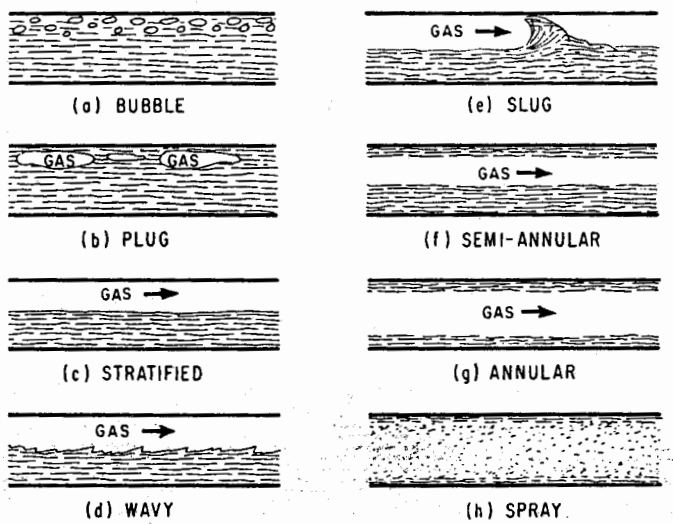
The final correlation of Knowles, et al[4] for flow pattern is noted in Fig. 2.131. From the values of two dimensionless parameters, flow may be predicted to be in one of the following regimes: (1) laminar stratified and wave, (2) ripple, (3) slug one, (4) slug three, (5) froth, or (6) mist.

A procedure for the determination of flow pattern by



Number of predictions on two-phase gas-liquid flow phenomena appearing per year vs. the year of publication (excluding atomization, cavitation, and condensation) (after Gouse[38])

ig. 2.128 Number of predictions on two-phase gas-liquid 'ow phenomena appearing per year vs. the year of publication excluding atomization, cavitation, and condensation) (after Gouse).



g. 2.129 Flow patterns.

the method of Knowles, et al is given in Appendix D.2, Sec. D.21. An example problem is given in Appendix D.2, Sec. D.22.

Although Knowles, et al did an extensive amount of work to determine flow patterns, they concluded that it is not necessary to know the flow pattern to determine pressure loss. In Fig. 2.132, a plot of Δp versus gas rate for the 2 in. line, the horizontal part of the curve includes flow patterns which encompass laminar, stratified, wave, ripple, and slug one regimes, as defined by Knowles, et al. After the curve has turned upward, it essentially has a constant slope for slug three, mist, annular, and froth regimes. There is a significant difference in flow patterns for a broad span in pressure loss. It is the author's opinion that an abrupt change occurs whenever it is necessary to cram the fluids through rather than having them flow easily. Fig. 2.132 shows that this change occurs for lower and lower gas rates as the liquid rates increase. For example, the change occurs at a gas rate of 1,000 Mscf/d for a liquid rate of 50 bpd; and also for a gas rate of 100 Mscf/d for a liquid rate of 2,500 bpd. In each case, the line capacity was being approached by the quantity of the combined fluids involved.

A further verification that flow pattern can be neglected in determining pressure loss can be noted in Fig. 2.133. This is a frictional loss plot of Andrews, et al.[18] These curves show that flow pattern changes cause no discontinuity in the relationship between pressure loss and the controlling variables. A noticeable change in Fig. 2.133 occurs where the abscissa is equal to 3×10^5. This is the point where froth flow starts to form.

The problem with any flow pattern correlation is the possibility of pattern change over the length of the pipeline under consideration. There is a good chance that there may be two or more different flow patterns if a high pressure drop occurs over a given pipe length. Many of the pipe flow correlations suggest the use of an average pressure for the length of the pipe. If a correlation requires flow pattern determinations, a serious error may result because the flow pattern for this average pressure could be considerably different from the pattern which exists at the high and low pressure. To eliminate this error, smaller pressure decrements should be chosen; with computer computations, this presents no problem. Beggs and Brill[19] presented a flow pattern correlation for horizontal flow, and corrected hold-up for any angle of flow. Govier, et al[20,21] discussed all aspects of flow patterns.

2.43 Liquid holdup

Since we have no static gradient term for horizontal flow it may be thought that a determination of holdup is not necessary. This is not true in that the holdup is necessary in order to calculate true velocities for the acceleration term and holdup is also involved in determining the friction factor for Dukler's correlation.[23] In numerous cases the acceleration term can be neglected and the horizontal multiphase flow calculation becomes one of merely obtaining a good friction factor.

Few good holdup correlations are available for horizontal flow. Dukler, et al[22,23] checked several correlations and found that the one of Hughmark[24] for

TABLE 2.41
FLOW PATTERNS OR REGIMES

Reference no.	Date	Author	Comments
6	1949	Lockhart & Martinelli	Determined four broad flow regimes: (1) gas and liquid both in laminar flow, (2) gas and liquid both in turbulent flow, (3) gas in turbulent flow with the liquid in laminar flow, and (4) gas in laminar flow with the liquid in turbulent flow.
7	1949	Bergelin & Gazley	Discussed flow patterns with particular emphasis on wavy flow.
8	1949	Kosterin	Developed flow pattern correlation.
9	1952	Johnson & Abou-Sabe	Presented flow pattern map.
5	1954	Alves	Classified eight separate flow patterns.
10	1954	Baker	Classified seven flow patterns and offered equations to calculate pressure loss for each pattern. (Fig. 2.130)
11	1955	White & Huntington	Presented flow pattern map.
12	1957	Brigham, Holstein & Huntington	Studied flow patterns. Showed stratified flow to be difficult to achieve.
13	1958	Chisholm & Laird	Reported semi-annular flow.
3	1958	Sobocinski & Huntington	Observed liquid gradually climbing walls of pipe forming semiannular flow.
14	1959	Hoogendoorn	Gave detailed description of flow patterns and presented equations for calculating pressure loss.
15	1963	Al-Sheikh	Made a computer study of flow patterns and derived twenty-five correlations. No practical value found for his work to date.
16	1966	Knowles	Presented complete thesis study on flow patterns. (Fig. 2.130)
4	1966	Knowles, et al	Classified eight flow categories.
17	1968	Calderon	Studied reliability of Baker's flow pattern map. Found it not to be reliable.
	1973	Beggs & Brill	Presented complete range of flow patterns from laboratory study.
	1972	Govier & Aziz	Complete book with detailed discussions.

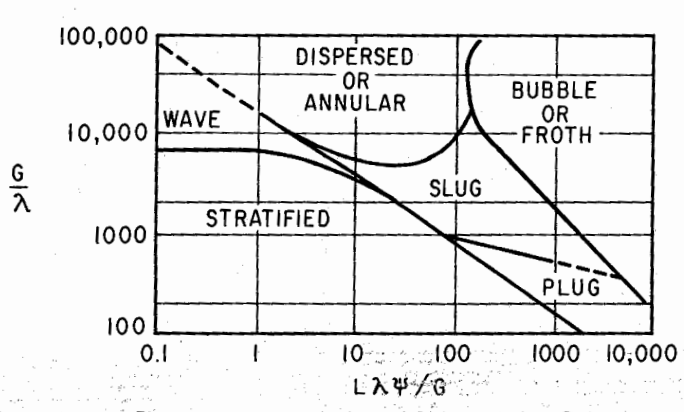


Fig. 2.130 Flow pattern correlation of Baker's (after Baker, courtesy Oil and Gas Journal).

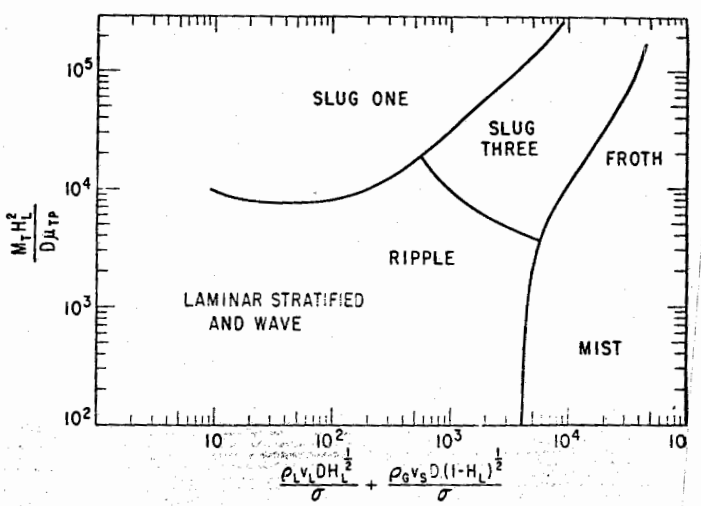
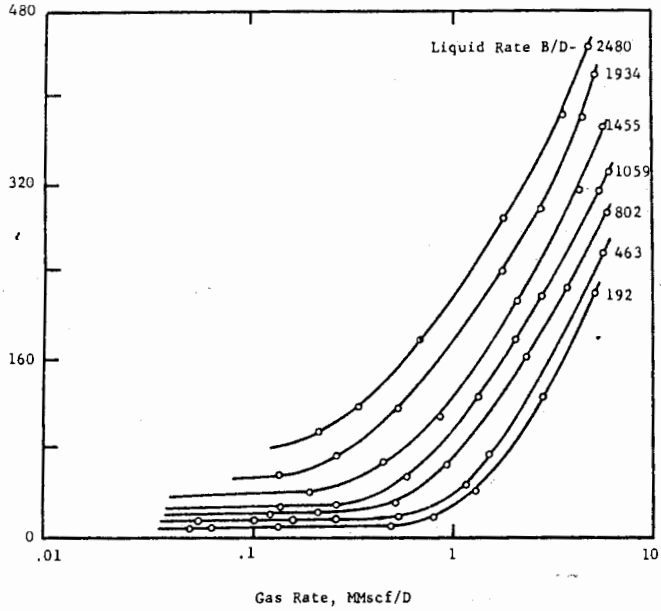


Fig. 2.131 Flow pattern correlation (after Knowles).



ig. 2.132 Pressure loss versus gas rate for high pressure water ests.

ertical flow was the most reliable for horizontal flow. lowever, a later hold-up correlation was presented y Baker, et al[25] to use with the Dukler method.

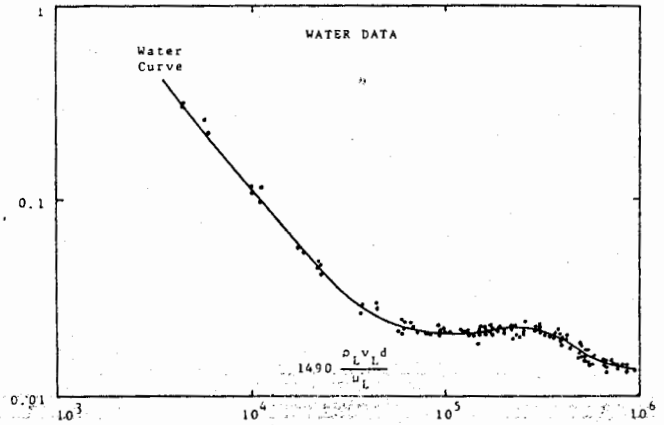
One of the best hold-up correlations was prepared y Eaton, et al.[2] They performed experiments in an ,800 ft field line on both 2 in. and 4 in. pipe where the oldup was measured by trapping the liquid between uick-closing valves.

Beggs and Brill[19] presented a correlation to deter- ıine holdup for horizontal flow which could be cor- ected for any angle of flow. Other evaluations of oldup have been made.[26,27]

44 Historical development of horizontal multiphase flow

441 Introduction

A large amount of literature is available on multi- hase horizontal flow. This makes it very difficult to etermine which of these many publications has con- ributed most to the development of correlative in- ormation on this type of flow. The first known work in



ig. 2.133 Energy-loss factor correlation for water with the mass atio included.

this subject was published in 1830. Time and space precludes the possibility of reviewing all of these publications; therefore, five correlations that offered the best results were selected for analysis.

The five best general correlations are those of Lockhart and Martinelli,[6] Baker,[10] Dukler, et al,[20] Eaton, et al,[2] and Beggs and Brill.[19] The best of these five are those of Dukler, et al and Eaton, et al with the one of Eaton limited to viscosities less than 12 cp. In addition, the correlation of Beggs and Brill can be used for any angle as well as horizontal flow and appears to be good. Example problems illustrating their use are worked. However, many other investigators have made significant contributions, and a brief review of some of these follows.

2.442 Historical review of correlations

The first publication on horizontal multiphase flow of real significance was that of Lockhart and Martinelli[6] in 1949. As mentioned, this is one of the five methods selected for final analysis and a discussion of this correlation can be found in Sec. 2.4521.

Table 2.42 shows those authors contributing most to horizontal flow. Some of these are a repetition of Table 2.41 but are listed because of work they did in addition to flow patterns or regimes.

2.45 Utilization of best correlations in predicting pressure losses and determining line sizes

2.451 Introduction

Because of the many correlations developed for multiphase horizontal flow calculations, it is difficult to select those considered to be the best. The best correlations for all ranges of fluid flow rates and pipe sizes are those of Dukler, et al,[20] Eaton, et al,[2] and Beggs and Brill.[19] Therefore, these three methods will be discussed in complete detail.

The methods of Lockhart and Martinelli,[6] Baker,[10] and Andrews, et al[18] are limited but are very good for some ranges of flow and pipe sizes. Therefore, these three will be discussed under Limited Correlations. Numerous other correlations are very good in various ranges but space does not permit their presentation.

Most practical horizontal multiphase flow conditions are in the turbulent flow region. This critical Reynolds number for single-phase flow is approximately 2,100. This depends to some extent on the amplitude of the disturbance which could be caused by bends, turns, or pipe roughness. Flow will be laminar for N_{Re} less than 2,000. Above $N_{Re} = 4{,}000$, flow is most likely to be turbulent although laminar flow has been noted at much higher Reynolds numbers. In order for flow rates in practical units to have some physical significance for varying Reynolds numbers, the following example is shown.

Given:

Line sizes = 4 in. I.D. and 18 in. I.D.,

$\bar{T} = 100°F$	$\mu_g = 0.02$ cp
$\gamma_g = 0.65$	$\mu_o = 1.0$ cp
$\rho_o = 42°API$	$\bar{p} = 14.7$ psia

TABLE 2.42

Reference no.	Date	Author(s)	Type work	Comments
6	1949	Lockhart & Martinelli	Laboratory experimental	Considered one of the first classical approaches to horizontal flow. Method used in field today. Good in certain ranges of flow. Presented four flow regimes.
7	1949	Bergelin & Gazely	Laboratory experimental	Observed five flow patterns. Offered solution.
8	1949	Kosterin	Theoretical	Gave solution and presented flow pattern diagram.
9	1952	Johnson & Abou Sabe	Laboratory experimental	Constructed plot to predict flow patterns. Work conducted at or near atmospheric pressure.
30	1953	Schneider	Laboratory experimental	Developed two-phase friction factor correlation.
10	1954	Baker	Practical from field data	Described seven flow patterns. Offered an equation to solve for pressure loss for each equation. Good in slug flow.
31	1954	Schneider, White & Huntington	Laboratory experimental	Published work of Schneider in Ref. 24. Defined friction losses similar to Kosterin. Results not good.
5		Alves	Laboratory experimental	Observed seven flow patterns and presented flow pattern map. Gave no general solution.
32	1955	Chenoweth & Martin	Laboratory experimental to 100 psi	Attempted to improve work of Lockhart and Martinelli. Used pressures up to 100 psia and larger pipe sizes. Not a successful solution.
11	1955	White & Huntington	Laboratory experimental	Presented flow pattern map for experimental work conducted at atmospheric pressure.
33	1956	Bertuzzi, Tek & Poettmann	Semi-empirical from experimental data	Adopted a semi-empirical approach from experimental data. Used energy balance. Correlated two-phase friction factor with N_{Re}. Has not been found to be a good correlation.
34, 35 36, 37 38, 39	1957 through 1967	Baker	Practical	Additional articles by Baker which have contributed to the understanding of the horizontal flow problem.
13	1958	Chisholm & Laird	Laboratory experimental	Used same approach as Lockhart and Martinelli. Extended to use with pipes of varying degrees of roughness.
3	1958	Sobocinski & Huntington	Laboratory experimental	Used 3 in pipe. Proposed solution to the flow of water-oil-air mixtures. Baker noted that they failed to account for slug flow.
14	1959	Hoogendoorn	Laboratory experimental	Presented flow pattern map. Measured holdup. Acceleration pressure loss determined. Presented pressure loss calculations.
28	1959	Yocum	Field	Field study—Middle East Lines.
40	1960	Bankoff	Laboratory experimental	Presented a holdup correlation and method for calculation of pressure drop. Not good for field use.
29	1961	Baker, W. J. & Keep	Field	Presented some good correlations.
41	1963	Beadle, Harlan & Brown	Simplified Fanning equation	Used value of friction of 0.01 which gave good results in limited range.
35	1963	Ciafaloni	Mathematical	Thesis study on four methods.
2	1966	Eaton	Field experimental	Developed solution to problem with holdup and friction factor correlation. One of the better correlations.

TABLE 2.42—*Cont.*

Reference no.	Date	Author(s)	Type work	Comments
18	1967	Andrews	Field experimental	Developed good correlation for 2 in. line only.
22	1964	Dukler	Collected data similarity analysis	Developed good correlation for all ranges of flow and pipe sizes.
23	1969	Dukler	Presented monograph on similarity analysis	Gave complete detailed discussion of Houston data bank and Dukler's methods.
25	1970	Baker, et al	Design manual	Gave design procedure and examples for Dukler's methods.
47	1970	Degance & Atherton	Analyzed some methods	Presented seven different papers on all aspects of multiphase flow. Excellent discussion of limitations and accuracy.
19	1973	Beggs & Brill	Laboratory	Presented correlation to account for any angle of flow.
20	1973	Govier & Aziz	All	Presented book covering subject in detail.

For horizontal flow we can refer to the general flow quation for any angle:

$$\frac{dp}{dz} = \frac{g}{g_c}\rho_m \sin\theta + \frac{f\rho v_m^2}{2g_c d} + \frac{\rho_m v_m dv_m}{g_c dz} \quad (2.284)$$

For horizontal flow $\theta = 0$, $\sin\theta = 0$ and $dz = dx$ 'hen:

$$\frac{dp}{dx} = \frac{f\rho_m v_m^2}{2g_c d} + \frac{\rho_m v_m dv_m}{g_c dz} \quad (2.41)$$

r:

'OTAL GRADIENT = FRICTION GRADIENT + ACCELERATION GRADIENT (2.42)

r:

$$\frac{\Delta p}{\Delta h_{total}} = \frac{\Delta p}{\Delta h_{friction}} + \frac{\Delta p}{\Delta h_{acceleration}} \quad (2.43)$$

r in terms of pressure loss over some distance:

$$\Delta p_{total} = \Delta p_{friction} + \Delta p_{acceleration} \quad (2.44)$$

The acceleration term may be handled differently lepending upon the investigator.

For most cases we have negligible acceleration ffects and then:

$$\frac{dp}{dx} = \frac{f\rho_m v_m^2}{2g_c d} \quad (2.45)$$

where "f" is a two-pnase energy loss factor.

As noted, equation 2.45 is the same as for vertical low except for the elimination of the static gradient erm.

The preceding equations are good for any conduit ize, gas-liquid ratio, flow rate, fluid property, etc.

Table 2.43 shows the necessary flow rates of gas and il to give Reynolds numbers of 200, 1,000, 2,500, and 5,000, assuming each fluid to be flowing in the pipe lone.

A flow rate of 125 bpd places oil in the transition region between laminar and turbulent flow (N_{Re} = 2,500) in 4 in. pipe and the same is accomplished for a 563 bpd rate in 18 in. pipe. These rates are calculated assuming that the oil is flowing alone in the pipe. Also, gas rates of 15,250 scf/d in 4 in. and 68,700 scf/d in 18 in. place the gas in turbulent flow. Most field conditions are in the turbulent flow region for multiphase flow; however, very viscous crudes may be in laminar flow.

TABLE 2.43

FOR 4 IN. ID PIPE

N_{Re}	v_{sl}, ft/sec	q_o, bpd	v_{sg}, ft/sec	q_g, scf/d
200	0.00792	10	0.004	1,220
1,000	0.0396	50	0.02	6,100
2,500	0.099	125	0.05	15,250
5,000	0.198	250	0.1	30,500

FOR 18 IN. I.D. PIPE

N_{Re}	v_{sl}, ft/sec	q_o, bpd	v_{sg}, ft/sec	q_g, scf/d
200	0.00176	45	0.000888	5,500
1,000	0.0088	225	0.00444	27,500
2,500	0.022	563	0.01332	68,700
5,000	0.044	1,125	0.02664	136,500

The sizing of a flow line or transmission line generally involves a first assumption of pipe diameter. In order to make a better first guess, it was suggested by Baker[10] that the liquid-alone diameter and the gas-alone diameter be calculated and the two added together. The sum of these two diameters should be the first assumed diameter. The next diameter guess is then improved by using the (press)$^{1/5}$ ratio as follows:

$$\text{Diameter guess} = (\text{diameter assumed})\left(\frac{\Delta p \text{ calculated}}{\Delta p \text{ actual}}\right)^{1/5}.$$

Although an original assumption must be made for the diameter guess, the second diameter guess should be made by the above equation, and this procedure

continued until the assumed value is equal to the calculated value. When the problem is to determine the pressure traverse or length between pressure points, the following procedure can eliminate the trial and error involved. On a graph, plot pressure versus $\frac{1}{\frac{dp}{dz}}$. Graphical integration between the two pressure points will give the correct distance. This procedure applies to all methods including horizontal, inclined and vertical flow.

Baker made a comparison of several methods for a 12 in. test line, as shown in Table 2.44. The following conditions were given:

Diameter = 12 in.
Length = 134,370 ft
Pressure upstream = 424.7 psia
Pressure downstream = 319.7 psia
Liquid flow rate = 6912 bpd of 37°API oil
Gas flow rate = 3,191 Mscf/d of 0.7 sp gr gas
Temperature = 60°F.

At pipeline conditions:

Oil volume = 7,140 bpd
Gas volume = 2,569 Mscf/d
Oil density = 6.869 lb/gal
Oil viscosity = 4.2 cp
Surface tension = 22.3 dyne/cm
Gas density = 1.45 lb_m/cu ft
Gas viscosity = 0.0105 cp

The methods of Beadle, et al, of Eaton, et al,[2] Dukler, et al[22,23] and Beggs & Brill[19] have been added to this list. Many of these correlations were for small pipe sizes; consequently the large deviations in predicted pressure loss for a 12 in. line are understandable. Also, Baker indicated that the line was not exactly horizontal.

TABLE 2.44
(AFTER BAKER)

Author	Experimental pipe size, in.	Two-phase pressure drop, psi	Deviation, %
Test line	12	105	0
Baker	1–10	78	−26
Martinelli	1	59	−44
Schneider, White, Huntington	1–2	53	−50
White and Huntington	1–2	53	−50
Capenegro	4–8	33	−69
Flanigan	4–10	34	−68
Reid	4–6	37	−64
Baxendall	4–6	23	−78
Chenoweth and Martin	1.5–3	31	−70
Bertuzzi, et al	0.5–2	14	−86
Chisholm and Laird	1	65	−38
Beadle, et al	2–4	53	−50
Eaton, et al	2–4	109	+4
Dukler, et al (Case I)		22	−79
Dukler, et al (Case II)		30.7	−70.8
Beggs & Brill	1–1.5	29.9	−71.5

2.452 Limited correlations

2.4521 The Lockhart and Martinelli correlation

Lockhart and Martinelli[6] presented an excellent piece of work that has been widely used by industry. More recent publications have proven to be better. However, it is still considered to be very good for low gas and liquid rates and good in general for small pipe sizes. They presented the results of experimental work conducted in pipeline sizes from 0.0586 in. to 1.017 in. They proposed that four types of flow mechanisms can exist during the simultaneous flow of a liquid and gas or vapor.

As presented earlier by Martinelli, et al[44,45] the pressure drop per unit length during two-phase flow was:

$$\left(\frac{\Delta p}{\Delta L}\right)_{TP} = \phi_g^2 \left(\frac{\Delta p}{\Delta L}\right)_g \tag{2.46}$$

where $\left(\frac{\Delta p}{\Delta L}\right)_g$ is the pseudogradient for the gaseous phase (neglecting effects of liquid flow) and ϕ_g is the parameter which is a function of a dimensionless variable X. The variable X is a function of the ratio of weight rates of the liquid and gas, the ratio of the densities of the liquid and gas, the ratio of viscosities of the liquid and gas, and the pipe diameter.

Each type of flow mechanism was associated with a different form of X. However, Lockhart and Martinelli[6] presented a more generalized correlation, and the data for all four mechanisms of flow can be compared directly (Fig. 2.134).

The main correlating variables are ϕ_g, ϕ_L and several derived variables. All of these are correlated with a parameter called X.

Three correlating parameters were defined as follows:

$$X^2 = \frac{N_{Reg} m_{C_L}}{N_{ReL} n_{C_g}} \left(\frac{w_L}{w_g}\right)^2 \frac{\rho_g}{\rho_L} = \left(\frac{\Delta p}{\Delta L}\right)_L \Big/ \left(\frac{\Delta p}{\Delta L}\right)_g \tag{2.47}$$

$$\phi_L^2 = \left(\frac{\Delta p}{\Delta L}\right)_{TP} \Big/ \left(\frac{\Delta p}{\Delta L}\right)_L \tag{2.48}$$

$$\phi_g^2 = \left(\frac{\Delta p}{\Delta L}\right)_{TP} \Big/ \left(\frac{\Delta p}{\Delta L}\right)_L \tag{2.49}$$

where $\left(\frac{\Delta p}{\Delta L}\right)_L$ is the pressure gradient that would exist if only liquid were flowing in the conduit, $\left(\frac{\Delta p}{\Delta L}\right)_g$ is the pressure gradient that would exist if only gas were flowing in the conduit and $\left(\frac{\Delta p}{\Delta L}\right)_{TP}$ is the actual two-phase pressure gradient.

Lockhart and Martinelli concluded that ϕ_L, and ϕ_g, and the liquid holdup are functions of X. (The relationship between these parameters and X for various types of flow are given in Fig. 2.134.) The correct form of X for each flow type was evaluated by substituting the appropriate exponents n and m and the proper constants C_L and C_g into Eq. 2.47.

A derivation of their equations can be found in Appendix D.3, Sec. D.31. A stepwise procedure is given in Sec. D.32 and example problems for the calculation

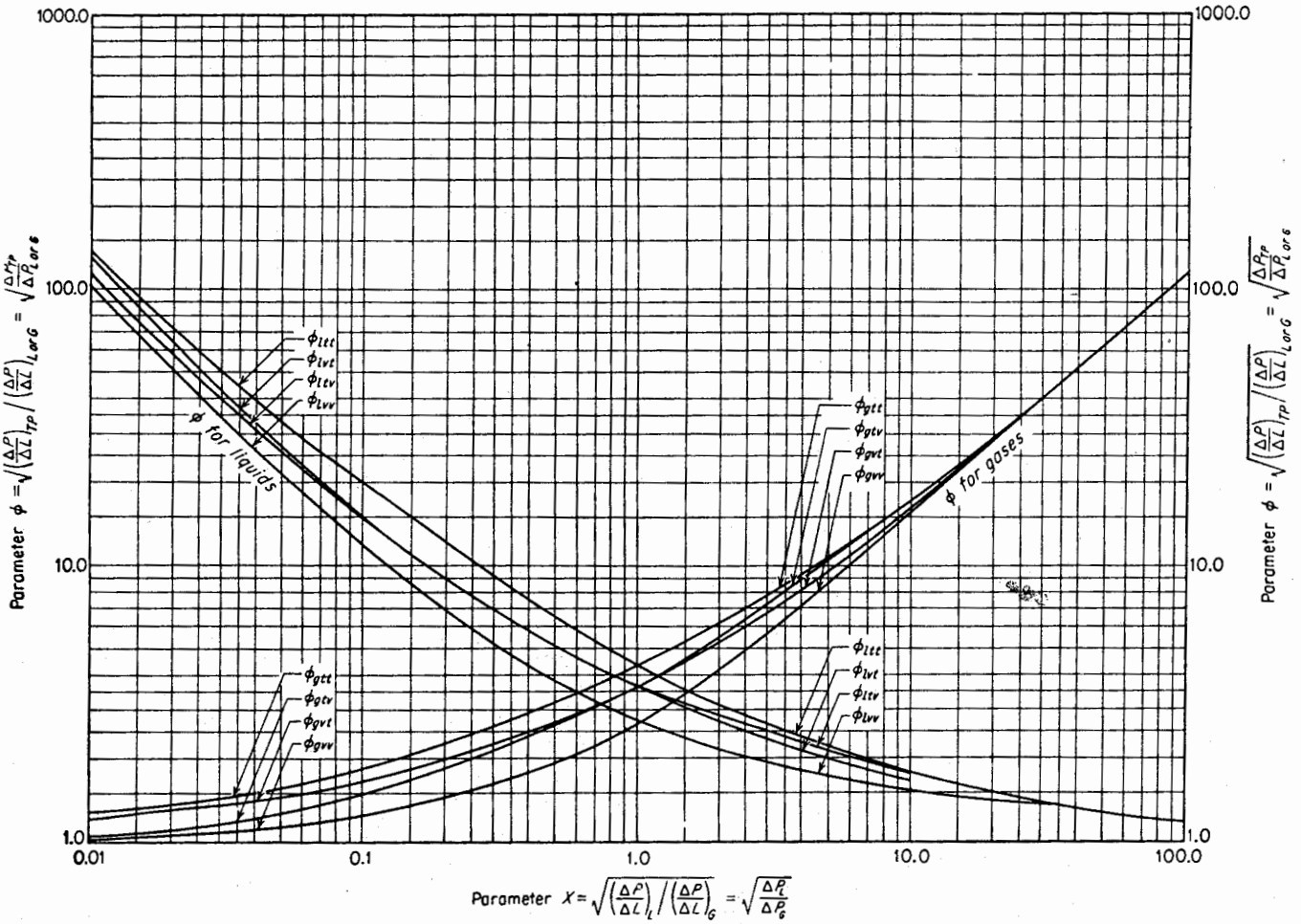


Fig. 2.134 Correlation for multiphase flow (Alves, courtesy Chem. Eng. Progr.).

a pressure traverse and determining line sizes are ted in Sec. D.33 and D.34.

Dukler, et al[22,23] compared the Lockhart and Mar-ıelli[6] correlation with correlations of Baker,[10] Ban-ff,[40] Chenoweth and Martin,[32] Yagi,[46] and one of their n. They reported that the Lockhart and Martinelli :relation becomes less accurate as the pipe diameter :reases. This conclusion was not reached in the study Calderon.[17] In general, the Lockhart and Martinelli :relation over-predicts pressure losses.

The following procedures and examples can be found Appendix D.3:

- D.31—Derivation of Lockhart and Martinelli's final equation.
- D.32—Procedure for calculation of a horizontal pressure traverse by the Lockhart and Martinelli method.
- D.33—Example problem by the Lockhart and Martinelli method to determine the downstream pressure for a known upstream pressure.
- D.34—Example problem to determine the necessary line size by the method of Lockhart and Martinelli (trial and error procedure).

522 Baker's correlation

Baker's[10,35,36,37,38,39] series of paper deals with the bject of multiphase flow in horizontal pipes and in hilly terrain. In his initial work, Baker described seven different flow patterns and presented a method for predicting flow patterns. In using his method, the two most accurate flow regions are slug and annular. His method is better in general for pipe sizes greater than 6 in. and most of his data was selected from 8 in. and 10 in. pipes. A rather abrupt discontinuity occurs from one pattern to the next. If by any chance the flow pattern point falls near one of the borders of the flow pattern map, then the problem should be worked using both patterns and the one giving the safest design should be used. He postulated an equation for each different flow pattern. Although Baker presented an excellent piece of work that has been widely used in industry, more recent correlations have proven to be better. However, his work is good in large pipe sizes and for slug flow.

Baker stated that "the borders of the flow-regions (Fig. 2.130) are shown as lines, but in reality these borders are rather broad transition zones." Basically, he presented an approach similar to that of Lockhart and Martinelli.[6] The main difference between the two was that Baker used the concept of flow patterns and also presented different equations for each pattern. A calculation procedure and example problem are given in Appendix D.4. The equations for all flow patterns can be found in Appendix D.41. Fig. 2.135 shows the friction factor chart as adjusted by Baker. The calcu-

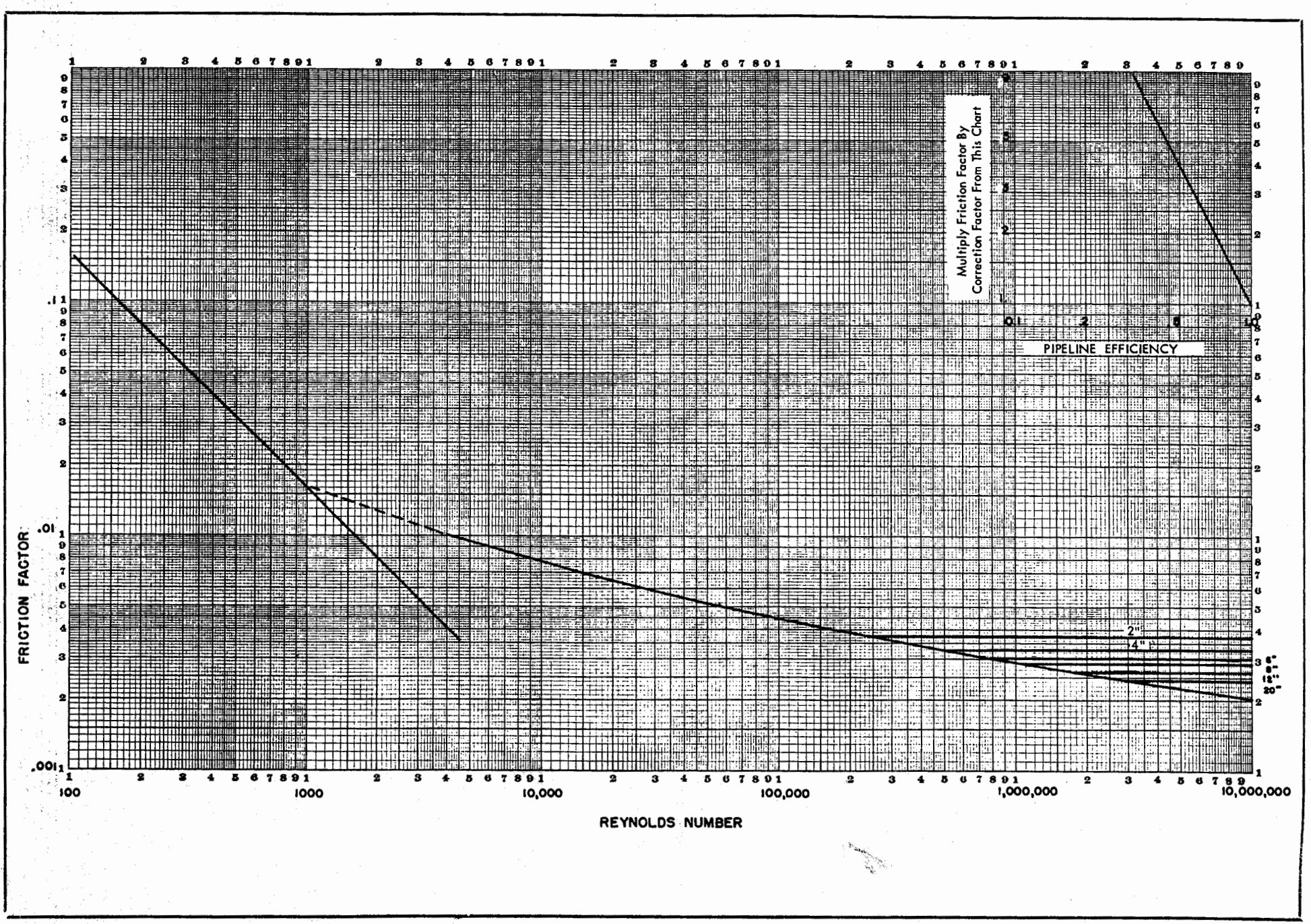


Fig. 2.135 Friction factor chart (after Baker).

ated pressure gradient may change by a factor of 5 when a different flow pattern is selected under the same conditions of flow. This shows that at conditions near the boundary of a flow pattern, the correlation is very sensitive to the choice of flow pattern and corresponding flow equation to use.

The following procedures and example problems can be found in Appendix D.4:

1) D.41—Procedure to determine pressure loss in horizontal lines by the Baker method.
2) D.42—Example problem to determine pressure loss in 4 in. and 16 in. lines by Baker method.
3) D.43—Example problem to determine line size needed by the Baker method.

.4523 The correlation of Andrews, et al

The publication of Andrews, et al[18] presented a correlation to determine the two-phase pressure loss in 1.065 in. ID steel pipe at field conditions. Tests were made with water, distillate, crude oil, and natural gas.

An energy balance was used to describe the flow movement. Assuming that no work is done by or on the system, the equation is:

$$144\,\bar{V}\,dp + \frac{v\,dv}{g_c} + dW_F = 0 \qquad (2.410)$$

where W_F is used to represent the irreversible losses due mainly to friction, and which can be described by the Darcy-Weisbach equation.

For one lb_m of flowing two-phase mixture, the total irreversible energy losses become:

$$(W_F)_t = \frac{f_{TP}\,v_T^2}{2\,g_c\,d}\,\Delta L, \qquad (2.411)$$

where f_{TP} is the energy-loss factor.

For the total mass of liquid and gas flowing per second, a balance may be written as:

$$w_T(W_F)_t = w_L(W_F)_L + w_g(W_F)_g \qquad (2.412)$$

Combining Eq. 2.411 and 2.412 with Eq. 2.410, the pressure loss equation becomes:

$$-\Delta p = \frac{w_t}{144(\bar{q}_L + \bar{q}_g)}\left[\frac{f_{TP}\,\bar{v}_t \Delta L}{2\,g_c\,d} + \frac{w_L}{w_t}\frac{(\Delta v_L^2)}{2\,g_c} + \frac{w_g}{w_t}\frac{(\Delta v_g^2)}{2\,g_c}\right] \qquad (2.413)$$

The solution of Eq. 2.413 is accomplished by a trial-and-error procedure; but, by rearranging the equation, ΔL can be obtained directly after assuming a Δp value.

In the high pressure range, the kinetic energy term can be neglected and Eq. 2.413 reduces to:

$$-\Delta p = \frac{w_t\,f_{TP}\,\bar{v}_t\,\Delta L}{144(q_L + \bar{q}_g)\,2g_c\,d} \qquad (2.414)$$

The liquid holdup was measured in the field and, working with four dimensionless groups, Eaton, et al[2] developed a holdup correlation independent of flow pattern and this method was applied to these data. The plot given in Fig. 2.136 is for water and gas, and corrections should be made for viscosity and surface tension when using the curve for hydrocarbons.

Liquid holdup, H_L, must be determined from Fig. 2.136 in order to calculate the phase velocities, v_L, v_g. The phase velocities must be used in evaluating the kinetic energy term and also in the friction loss correlation (Fig. 2.137).

Andrews, et al[18] found that the water data could be correlated very well with the liquid Reynolds number and a parameter $K = w_l/w_t$ (Fig. 2.137).

In the application of this correlation to oil and distillate, they found that the distillate data lay close to the water curve but that the oil curve diverged at the higher Reynolds numbers. For this reason, for hydrocarbons the exponent of K is 0.1, and the liquid viscosity appears in the abscissa only to the 0.1 power. They also found that in the case of turbulent flow the frictional losses appear abnormally high at the lower Reynolds numbers.

From the results of their study, it seems that calculations using this method should give better results than for any other method in use for 2 in. pipe except for viscosities greater than 10 to 15 cp.

A stepwise procedure for applying this method, with example problems, can be found in the Appendix D.5.

2.453 Best correlations for horizontal multiphase flow

2.4531 Introduction

The best correlations are those of Dukler[22,23] (Case I and II), Eaton, et al,[2] and Beggs and Brill.[19] For all ranges of flow the best of these three methods is believed to be the Dukler Case II method. The main reasons for this is that Eaton failed to properly account for highly viscous crudes and that his method deteriorates at low GORs. However, as is true for vertical flow, most horizontal flow takes place in a highly turbulent manner. The Eaton method is very good for 2 in. and 4 in. lines since most of this data was taken from these two line sizes. The method of Beggs and Brill[19] requires additional checks against field data but appears very good.

2.4532 The correlation of Dukler, et al.

2.45321 Introduction

Work on horizontal two-phase fluid flow was published in 1964 by Dukler, et al[22] and in a later manual in 1969.[23] They accumulated all the published data on horizontal two-phase flow and formed what they call the data bank. The data consisted of short tube laboratory data and long tube oil field data. More than 20,000 experimental measurements have been taken, half of them since 1959. The data have been culled and approximately 2,600 data points remain consistent. They report that some of the data contain errors, and they have discarded many short-tube data where end effects were known to be large.

The work presented by Dukler, et al[22,23] consists essentially of two parts. The first part is a comparison among the correlations of Baker,[10] Bankoff,[40] Chenoweth and Martin,[32] Lockhart and Martinelli,[6] and Yagi.[46] The second part is a new correlation developed through the concept of similarity analysis. This is discussed in complete detail in the AGA-API publication by Dukler.[23] In Ref. 23 Dukler first discusses this concept for single phase flow and then for two-phase flow.

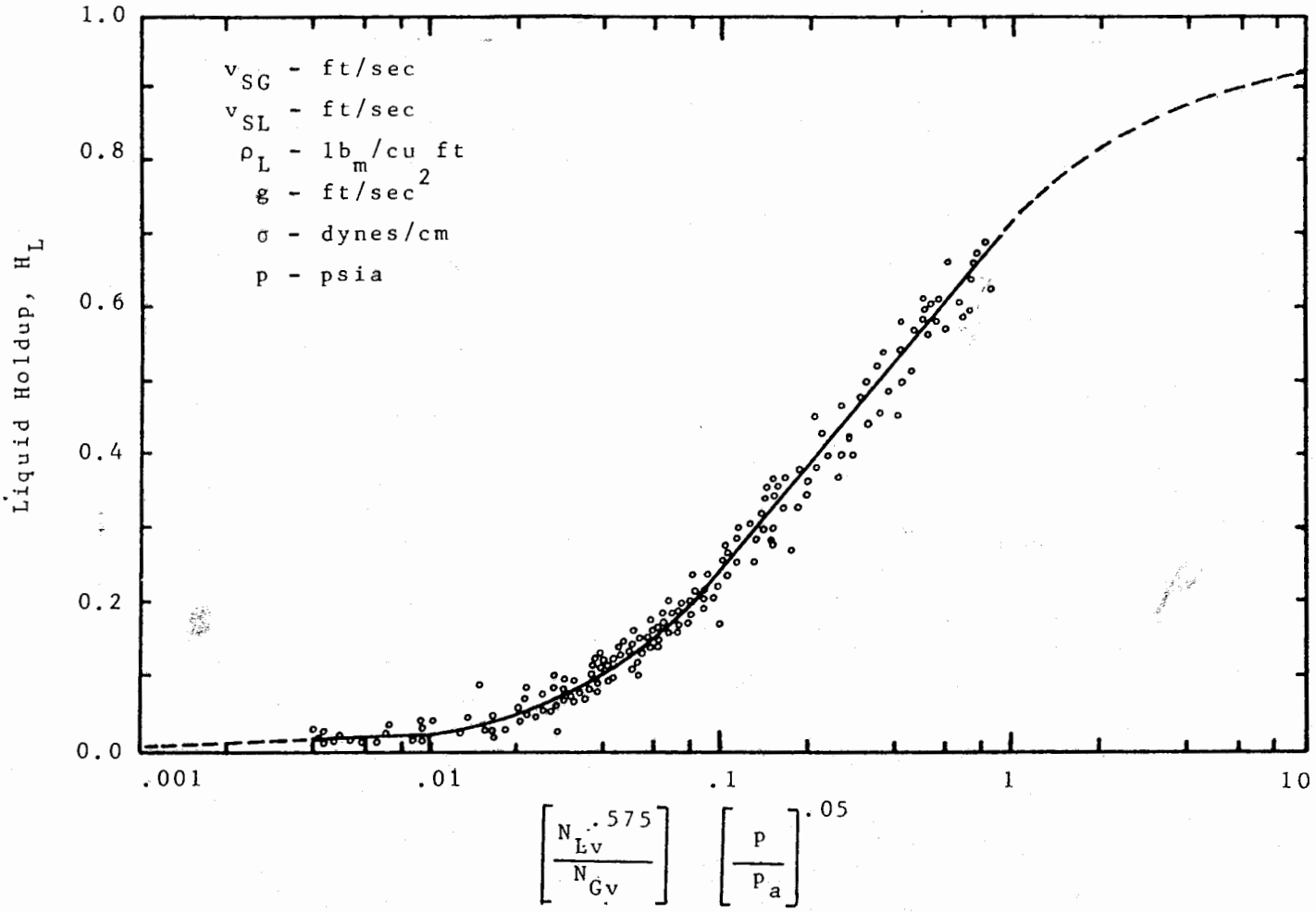


Fig. 2.136 Liquid holdup correlation (after Andrews).

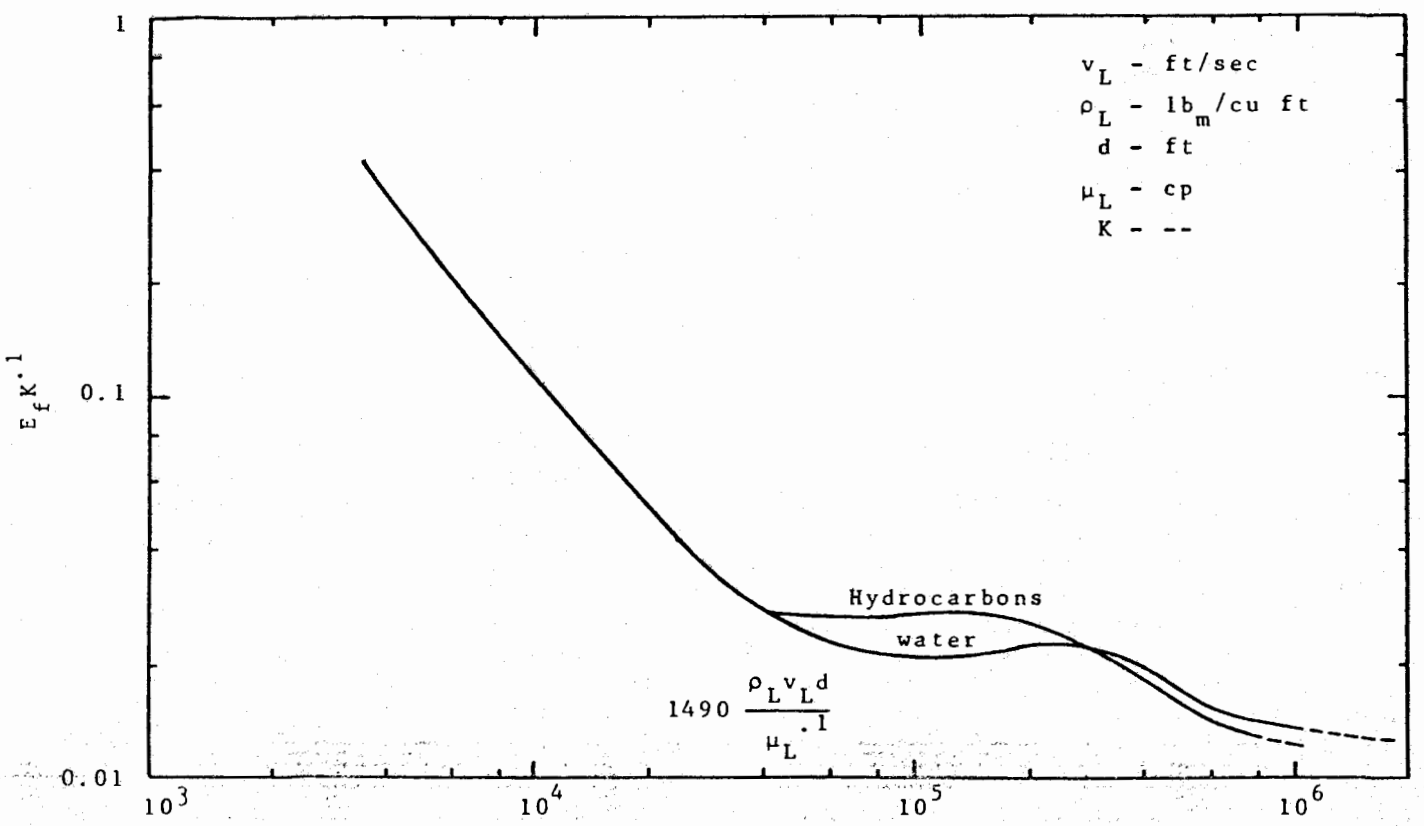


Fig. 2.137 Final energy-loss factor correlation curves (after Andrews).

.e identifies four different forces which can be identified as acting on the fluid. These are identified as forces ue to pressure, viscous shear forces, forces due to ravity, and forces due to inertia or acceleration of ıe fluid. For horizontal flow there is no force due to ravity; thus only three forces remain. Dukler's assumption was to assume the existence of dynamic and inematic similarity and to arrive at appropriate reıtionships. It is suggested that pages 79 through 145 ı Ref. 23 be studied for a thorough explanation of his concept.

The conclusions presented in the first part of their ork can be summarized as follows: From the correlaions tested, those presented by Bankoff[40] and Yagi[46] ere found to be completely inadequate. In the Chenoeth and Martin[32] and the Lockhart and Martinelli[6] orrelations, an almost uniform trend was observed nd deviations became larger as the pipe diameter ncreased. From an overall comparison of the correlaions they tested, these investigators reported that he one presented by Lockhart and Martinelli[6] was ıetter than the other four tested.

They also tested several holdup correlations includng those of Hoogendoorn,[14] Hughmark,[24] and Lockıart and Martinelli.[6] Hughmark's holdup correlation vas found to be the best of the three tested. This seems ınusual in that his holdup was for vertical tubes. He nade no mention of testing the Eaton holdup correlaion which was measured in 2 in. and 4 in. pipe under ield conditions.

In the second part of their study, correlations were ɔresented for two cases. Case I was applied when no slip between phases and a homogeneous flow were assumed to exist. In Case II it was assumed that slip ɔccurred but that the ratio of each phase velocity to the average velocity was constant. Both of Dukler's methɔds will be discussed. His methods are based on a simple correlation which does not require determination ɔf flow patterns. One first calculates the properties of the liquid and gas at conditions of flow. Using the equations prescribed by Dukler, a two-phase Reynolds number is calculated from which a two-phase friction factor is determined, and hence the pressure loss determined. Case I requires no trial and error procedures whereas Case II does require an original assumption of holdup.

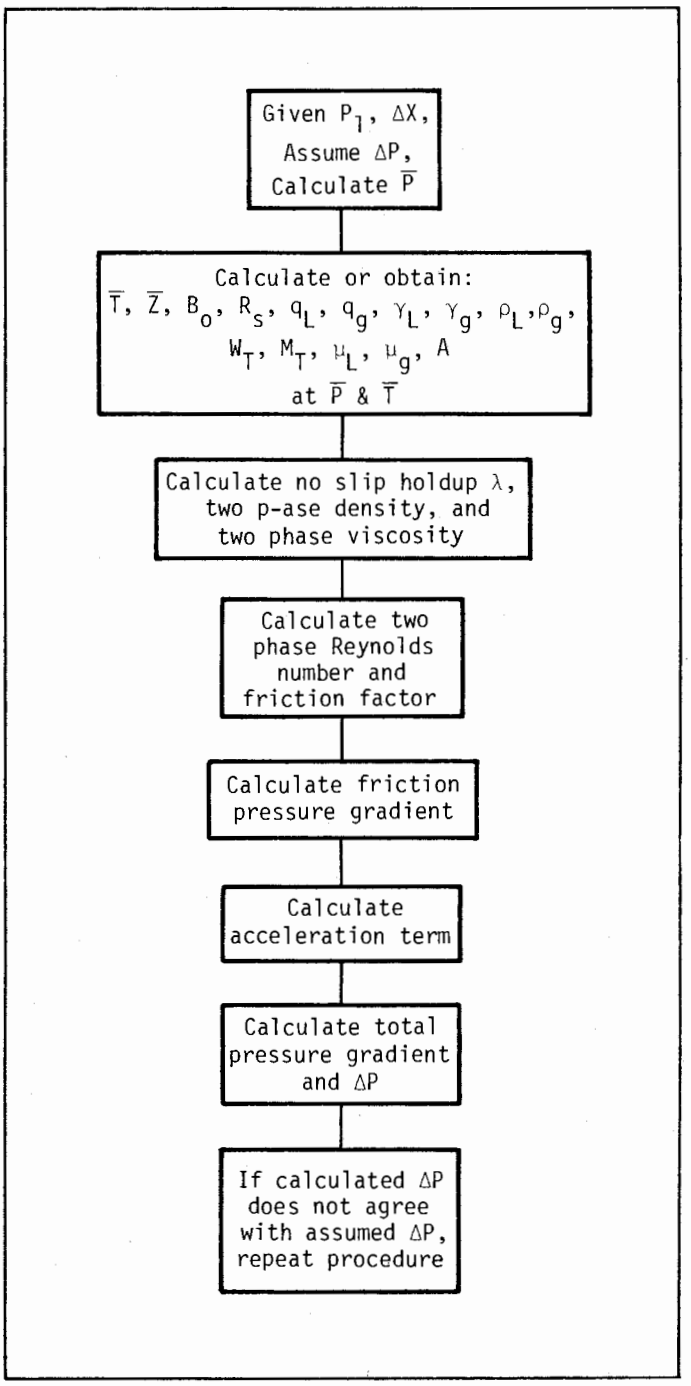


Fig. 2.138 Flow diagram for Dukler's case I method for horizontal multiphase flow.

2.45322 Case I—Dukler

The Case I method of Dukler is for no slip and homogeneous flow. Although most horizontal flow is highly unsteady, the assumption of steady-state can be very useful. The key concept of Dukler's Case I is that the holdup is defined as the liquid volumetric flow rate to the total volumetric flow rate or as the ratio of the liquid superficial velocity to the total superficial velocity. This is a very simple method to use and requires no flow pattern calculation. It is essentially as simple as a single-phase pressure-drop calculation. Reference should be made to Fig. 2.138 which shows a generalized flow diagram for calculations involving this procedure. As noted, if we are given p_1 and Δx, then we assume Δp and go through his procedure to calculate Δp. This procedure is repeated until the two Δps coincide. Once the fluid and other properties are calculated, the no slip holdup, the two-phase density, and the two-phase viscosity are calculated. From these values a two-phase Reynolds number is calculated and a value for the friction factor is obtained. The value of the friction gradient is calculated and added to the acceleration gradient if necessary. This gives the total Δp and the procedure is repeated if necessary.

The trial and error procedure can be eliminated by assuming a Δp and solving for Δx directly until the $\Sigma \Delta x$ = the length of the line. For Case I the two-phase mixture was treated as an equivalent single fluid. The equation relating friction factor and Reynolds number is:

$$f_{TP} = 0.00140 + \frac{0.125}{(N_{Re})_{TP}^{0.32}} \qquad (2.415)$$

No correlations or charts are necessary to obtain f_{TP}. As a matter of fact no figures at all are needed for calculation by Case I. The equation for the Reynolds number is:

$$(N_{Re})_{TP} = \frac{4\ w_t}{\pi\ d\ \mu_{TP}} \qquad (2.416)$$

where: $$\mu_{TP} = \mu_L\ \lambda + \mu_g(1 - \lambda) \qquad (2.417)$$

and $$\lambda = H_L(\text{no slip}) = \frac{q_L}{q_L + q_g} \qquad (2.418)$$

The frictional gradient can be obtained from:

$$\left(\frac{\Delta p}{\Delta L}\right)_f = \frac{2\ M_t^2\ f_{TP}}{g_c\ \rho_{TP}\ d} \qquad (2.419)$$

The total calculated pressure drop is obtained by adding the pressure drop due to friction, from Eq. 2.419, to that due to the acceleration of the gas and liquid.

The final equation is then:

$$\left(\frac{\Delta p}{\Delta L}\right)_t = \left(\frac{\Delta p}{\Delta L}\right)_f \Big/ (1 - a) \qquad (2.420)$$

where the acceleration term is:

$$a = \frac{16\ w_t\ w_g\ \bar{p}}{\pi^2 g_c d^4 p_1 p_2 \rho_g} \qquad (2.421)$$

A procedure for the calculation of a pressure traverse along with example problems for Case I are given in Appendix D.6:

(1) D.61 – Procedure for Dukler's Case I Method.
(2) D.62 – Example Problem to Calculate Pressure Loss – Dukler's Case I.
(3) D.63 – Example Problem to Determine Size of Offshore Line – Dukler's Case I.

2.45323 Case II – Dukler

Dukler's Case II or so called constant slip method is probably the most widely used method today for a wide range of conditions. Dukler's Case II method does require a valid holdup correlation. Degance and Atherton[47] mention that Hughmark's holdup correlation as recommended by Dukler[22] fails for values less than 0.2. In this range, they recommend the use of Eaton's holdup correlation. In noting a table prepared by Dukler, they point out that Hughmark's correlation is best in the middle range, Eaton's best for low values less than 0.2, and Hoogendoorn's best for values close to 1.0. Degance and Atherton provided an extension of the Eaton correlation for values of 0.7 to 1.0 which compares favorably with Hughmark's and Lockhart and Martinelli's correlations. In the 1970 AGA-API publication of the Dukler method by Baker, et al,[25] a new holdup correlation is given. This was originally suggested by Dukler in his reference.[23] (Refer to Fig. 2.138.)

For Dukler's Case II, some confusion may exist in noting that a no slip holdup (λ) must still be calculated the same as in Case I, as well as making a determination of the true holdup (H_L). This is easily understood in noting Fig. 2.140 which requires the no slip holdup to determine the friction factor and Fig. 2.139 which requires the no slip holdup to determine the true holdup.

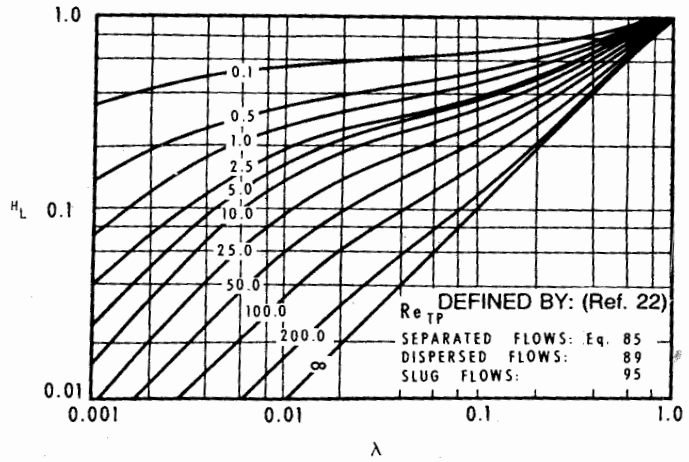


Fig. 2.139 Dukler holdup correlation (after Dukler).

Fig. 2.141 is a generalized flow diagram showing the necessary steps to follow in calculating a pressure traverse by Dukler's Case II method. It is noted that if the upstream pressure (p_1) is given a value, p_2 is assumed and all fluid and other properties calculated for the average conditions at average pressure. The no slip holdup is then calculated from which the no slip viscosity is calculated. A value for f_{TP}/f_n is read directly from Fig. 2.140. A value of true holdup is then assumed from which the two-phase density and the approximate value of the two-phase Reynolds number is calculated. With the value of no slip holdup and approximate Reynolds number, a value of H_L is read from Fig. 2.139. If the assumed value of H_L and value from Fig. 2.139 agree within 5%, the accuracy is sufficient. If not, the procedure should be repeated with the new value of H_L. With the correct value of H_L a new value for density and hence Reynolds number is calculated. With the new value of $(N_{Re})_{TP}$ the true friction factor is determined from which the distance (Δx) for assumed Δp is then determined. The acceleration pressure drop can generally be neglected for transmission and production lines. In process piping it may, however, be significant and is handled as noted by the equation in Appendix D.71, Step 18, and added to the frictional pressure loss. This procedure is continued until the $\Sigma\Delta x$ = length of line.

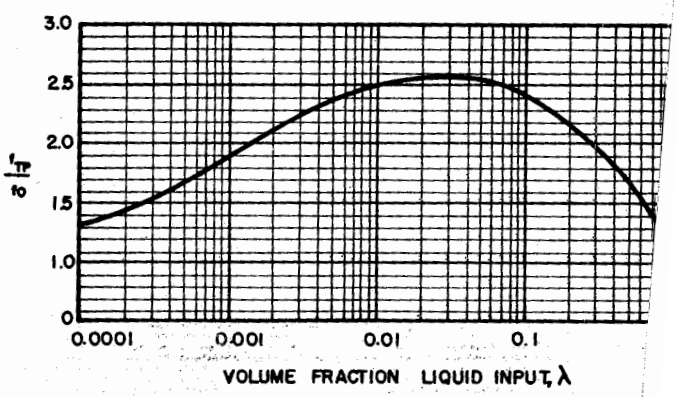


Fig. 2.140 Two phase flow friction factors (after Dukler).

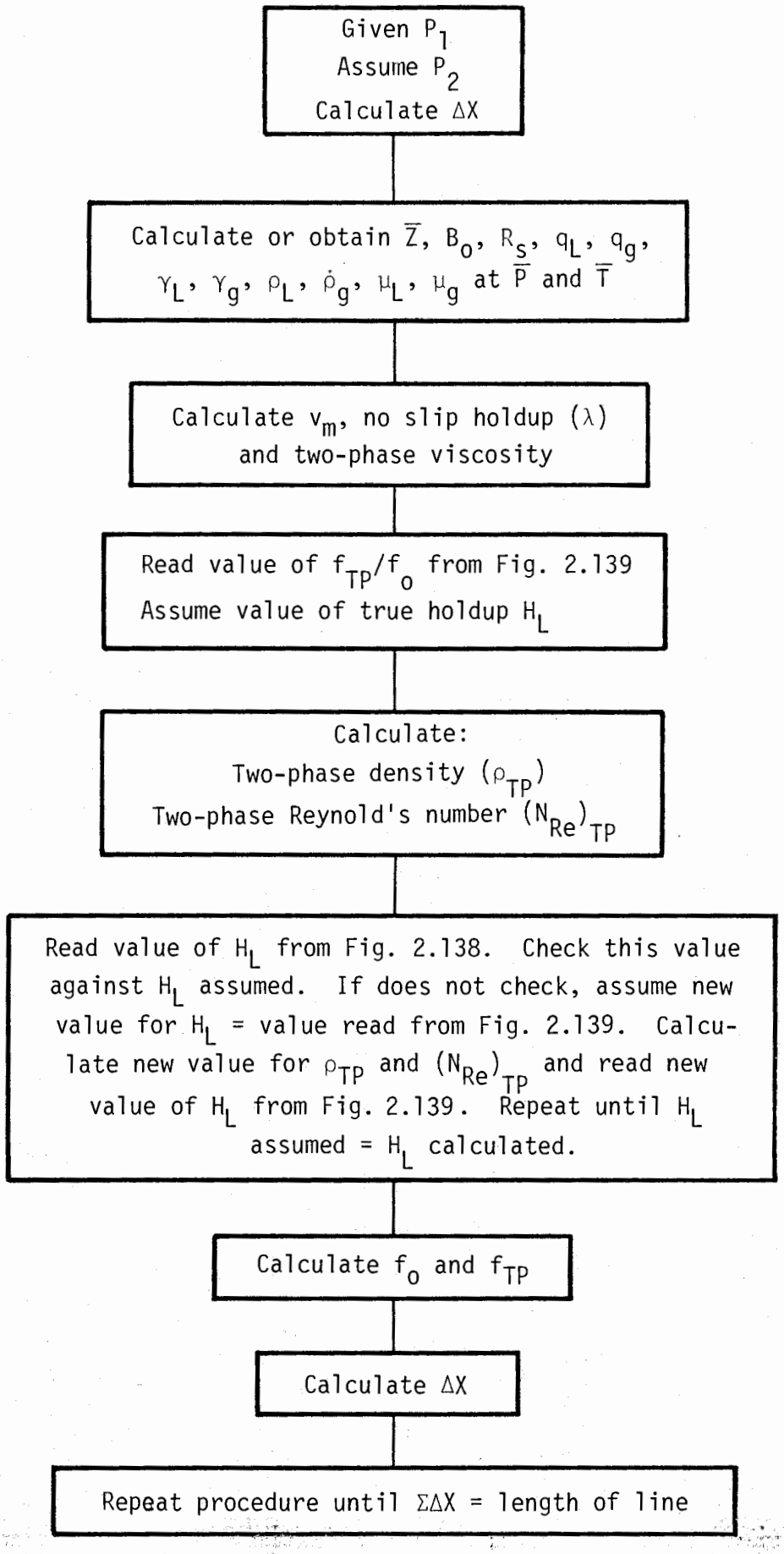


Fig. 2.141 Flow diagram for Dukler's case II method.

The exact procedure and a problem example are found in Appendix D.7. The AGA-API Design Manual II by Baker, et al[25] also gives a procedure with examples. However, their procedure in solving for Δp over the entire length of the line should be used with caution where large pressure differences occur. Such factors as H_L, $(N_{Re})_{TP}$, and f_{TP} change over the entire length. Their final equation in solving for Δp can be easily changed to solve for ΔL for a small assumed value of Δp and then summing the ΔLs until the length of the line is reached. This permits averaging fluid properties and determining holdup over a smaller Δp and shorter distance. It should improve the accuracy of the correlation.

In Case II it was assumed that slip occurred but that the ratio of each phase velocity to the average velocity was constant. The friction factor is correlated empirically with the Reynolds number and a method for predicting liquid holdup in place is required. Dukler originally suggested that the holdup correlation of Hughmark[24] be used but later presented another holdup correlation in Ref. 20 and 22 which is recommended for his calculation procedure (See Fig. 2.139). Eaton's holdup correlation may also be used with Dukler's procedure. The Reynolds number for the mixture is given by:

$$(N_{Re})_{TP} = \frac{d\, v_m\, \rho_{TP}}{\mu_{TP}} \qquad (2.422)$$

where:

$$\mu_{TP} = \mu_L\, \lambda + \mu_g (1 - \lambda) \qquad (2.417)$$

$$\rho_{TP} = \rho_L \left(\frac{\lambda^2}{H_L}\right) + \rho_g \frac{(1-\lambda)^2}{(1 - H_L)} \qquad (2.423)$$

H_L = liquid holdup, and:

$$\lambda = \frac{q_L}{q_L + q_g} \qquad (2.418)$$

The frictional pressure gradient is then given by:

$$\Delta p_{friction} = \frac{2\, f_{TP}\, \Delta L\, \bar{v}_m^2\, \rho_{TP}}{1{,}728\, g_c\, d} \qquad (2.424)$$

or the equation may be solved for ΔL eliminating some iteration.

The total pressure drop is the sum of the frictional pressure drop plus an acceleration term which is normally negligible.

An iterative procedure is again required to evaluate the increment terms.

A stepwise procedure for the calculation of a pressure traverse by these methods is given in Appendix D.7, with an example problem.

Because these data covered a wide range of pipe sizes, flow rates and conditions, this correlation is good for a wide range of conditions. Very good results were also obtained when the correlation was applied to the Eaton data.

It appears that the pipe diameter is adequately accounted for with results being good for both large and small pipe sizes. However, slightly better results were obtained for the large pipes.

At low liquid rates and increasing gas rates, calculated pressure drops are less than those measured. For high liquid rates, calculated pressure drops are greater than those measured, particularly when combined with high gas rates. For low total mass flow rates the acceleration term has little influence, but at high mass rates it becomes more important, especially for small pipe sizes.

The correlation of Dukler, et al seems to handle viscous effects adequately. Calculated pressure drops show no error trends over the range of liquid viscosities studied.

The following procedure and example problem can be found in Appendix D.7.

(1) D.71 – Procedure to calculate the pressure loss in a horizontal flow line from a known upstream pressure, Case II.
(2) D.72 – Example problem to calculate pressure loss in Baker's experimental line, Case II.

No additional examples are shown here but attention is called to the AGA-API Design Manual II by Baker, et al[10] for additional examples.

2.4533 The correlation of Eaton, et al.

2.45331 Introduction

In 1964, an extensive field study by Eaton, et al[2] was conducted in Delcambre, Louisiana. Controlled tests covering various gas and liquid rates were conducted in long tubes.

The data were taken in a horizontal multiphase test set-up located at the Union Oil Company of California Tigre Lagoon Field, near Delcambre, La. The test unit consists of two 1,700 ft test lines, the diameters of which were 2 and 4 in., respectively. Long lines were selected to more nearly approach oil field conditions.

The parameters studied were: (1) variation of the gas rate (0–10 MMscf/d); (2) variation of the liquid rate (50–5,500 bpd); (3) variation of liquid viscosity (1–13.5 cp); (4) variation of the mean system pressure (70–950 psig); (5) variation of pipe diameter; and (6) liquid holdup (0–1).

Three liquids were tested in each flowline. The liquid flow rates were varied from 50–2,500 bpd in the 2 in. line and 50–5,500 bpd in the 4 in. line, and for each liquid rate the volumetric gas/liquid ratio was varied from zero to the maximum allowed by the system.

They proposed an energy balance equation in differential form based on 1 lb_m of flowing fluid and assuming horizontal flow in which no external work was done by the fluid, which is:

$$144\, V dp + \frac{v dv}{g_c} + \frac{g}{g_c} dW_f = 0. \qquad (2.425)$$

When gas and liquid are flowing through a horizontal conduit, a similar equation can be written for each phase.

This was the approach taken by Eaton, et al[2] and the derivation of their equations can be found in Appendix D.8 with a stepwise procedure for solving problem examples. Reference should be made to Fig. 2.142 which shows a generalized flow diagram for Eaton's method. His method is rather simple to use, in particular where kinetic energy effects are negligible. After

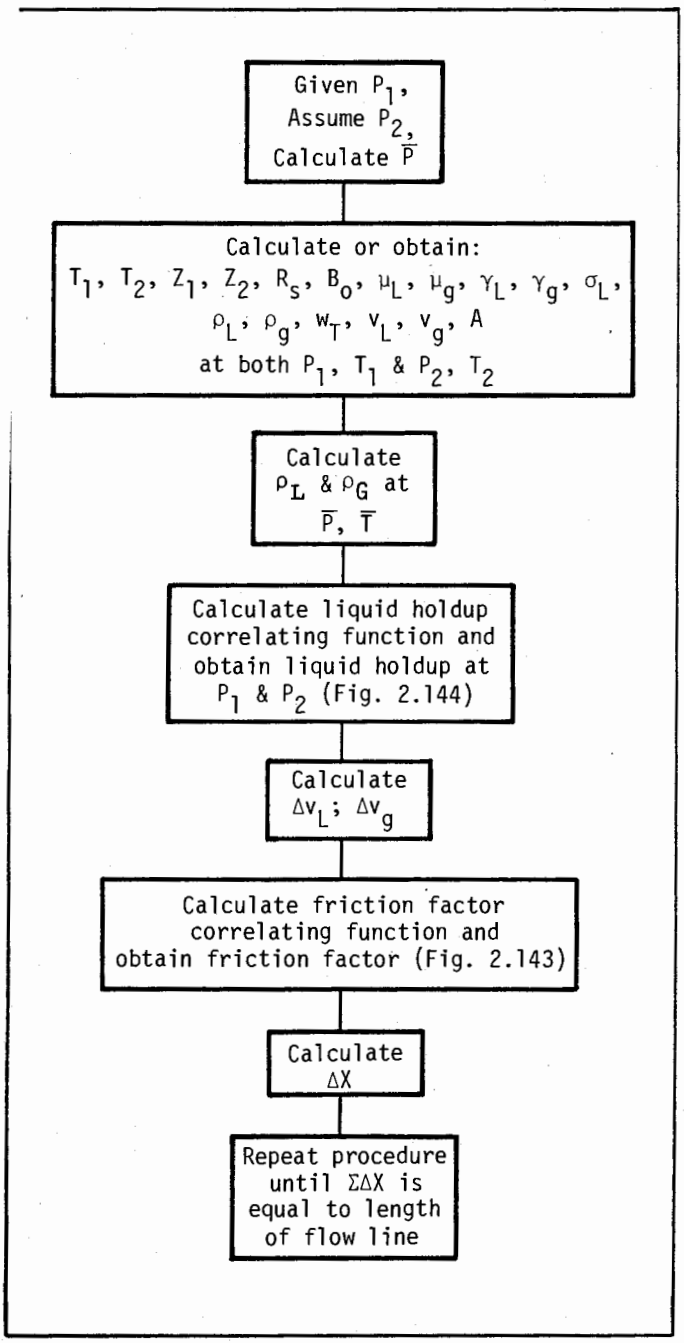


Fig. 2.142 Flow diagram for the calculation of pressure loss by method of Eaton.

assuming a p_2 from knowing p_1, all fluid and other properties are calculated. A friction factor correlating function is then calculated and the friction factor determined from Fig. 2.143. His equation then permits solving for distance directly corresponding to Δp. If kinetic energy is included, holdup is selected from Fig. 2.135 after calculating the holdup correlating function. His equation including kinetic energy is then solved for distance. In neither instance is a trial and error solution needed, except of course where the problem is to determine a line size.

The pressure loss equation of Eaton, et al is:

$$-144\left[\frac{w_L}{\bar{\rho}_L}+\frac{w_g}{\bar{\rho}_g}\right]\Delta p+\frac{w_L\,\Delta v_L^2+w_g\,\Delta v_g^2}{2g_c}$$

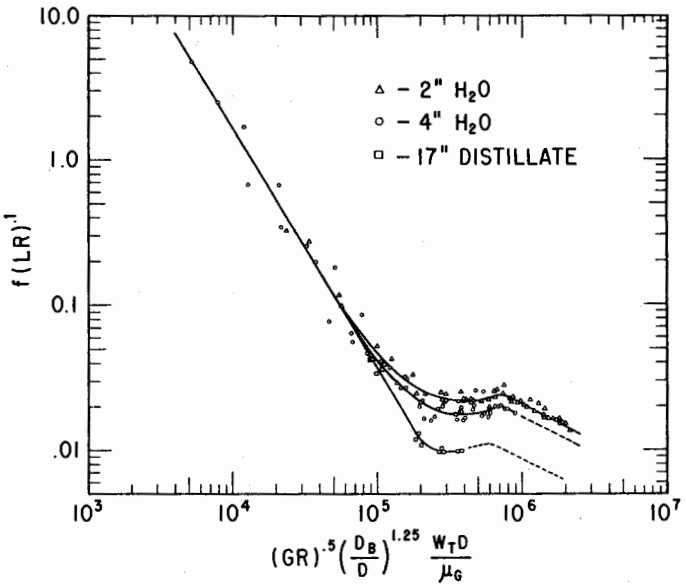


Fig. 2.143 Energy loss factor correlation with data (after Eaton).

$$+\frac{f\,w_t\,\bar{v}_m^2}{2\,g_c\,d}\,\Delta L=0 \quad (2.426)$$

In order to evaluate the kinetic energy terms, the holdup must be known. Eaton, et al developed a holdup correlation as well as an energy loss correlation; these are discussed in the following section.

2.45332 Energy-loss correlation of Eaton, et al.

The following list of variables having an effect on the energy loss factor was given by Eaton, et al:

1. Liquid Reynolds number:

$$(N_{Re})_L=\frac{\bar{\rho}_L\bar{v}_L d_L}{\mu_L} \quad (2.427)$$

2. Gas Reynolds number:

$$(N_{Re})_g=\frac{\bar{\rho}_g\bar{v}_g d_g}{\mu_g} \quad (2.428)$$

3. Liquid mass rate to total mass rate ratio:

$$LR=\frac{w_L}{w_t} \quad (2.429)$$

4. Gas mass rate to total mass rate ratio:

$$GR=\frac{w_g}{w_t} \quad (2.430)$$

It was found that the energy loss factor could be correlated as follows:

$$\text{Ordinate values}=f(LR)^{\alpha}$$

and:

$$\text{Abscissa values}=(GR)^{\beta}\frac{M_t}{\mu_g}$$

where

$$M_t=\text{total mass flux}=w_t/A_p$$

It was determined that $\alpha=0.10$ and $\beta=0.50$ by trial-and-error plotting. This resulted in good correla-

tions for each individual pipe size. The pipe diameter influence was then considered.

An approach was made to normalize the pipe diameter effect by using a base diameter and forming a ratio of the base diameter and any pipe diameter for use in the correlating function. Because 1 in. ID pipes are about the smallest used for oil field applications, this was selected as the base diameter, d_B. Actually, any size could have been used for this value. This has no particular significance except to make the parameter dimensionless.

The dimensionless group is d_B/d; it was included in the energy-loss correlation function. The following dimensionless groups were found best in correlating the energy-loss factors:

$$\text{Ordinate values} = f(LR)^{0.10}$$

and:

$$\text{Abscissa values} = (GR)^{0.50}\left(\frac{d_B}{d}\right)^{1.25}\left(\frac{M_t}{\mu_g}\right)$$

The final energy-loss correlation is shown in Fig. 2.143. The straight-line portion of the correlation represents laminar flow; only the bubble, stratified, and slow, undulating wave flow patterns fall in this regime. The region corresponding to the transition region for single-phase flow contains several different flow patterns. Slugs, rapid waves, and ripples with a great deal of overlap were observed to fall within this region. Mists and emulsion type flows fall consistently in the right-hand region of Fig. 2.143.

2.45333 Liquid holdup correlation of Eaton, et al.

In an approach similar to that of Ros[48] and Hagedorn and Brown[49] for vertical flow, Eaton, et al[2] applied dimensional analysis to the problem to obtain five significant groups in field units:

$$N_{Lv} = 1.938\, v_{sL}\sqrt[4]{\frac{\rho_L}{\sigma}} \qquad (2.431)$$

$$N_{gv} = 1.938\, v_{sg}\sqrt[4]{\frac{\rho_L}{\sigma}} \qquad (2.432)$$

$$N_d = 120.872 d\sqrt{\frac{\rho_L}{\sigma}} \qquad (2.433)$$

$$\frac{p}{p_a} = \frac{p}{14.65} \qquad (2.434)$$

$$N_L = 0.15726\,\mu_L\sqrt[4]{\frac{1}{\rho_L\,\sigma^3}} \qquad (2.435)$$

Initially, values for N_{Lv}, N_{gv}, p/p_a, and N_d were used as independent variables with experimental values of H_L as the dependent variable in a regression analysis. The analysis indicated that the water-gas holdup data were correlative with:

$$H_L = \psi\left[\frac{N_{Lv}^{0.575}}{N_{gv}\,N_d^{0.0277}}\left(\frac{p}{p_a}\right)^{0.05}\right]. \qquad (2.436)$$

Finally, the N_L number was included and the dimensionless group

$$(N_L/N_{LB})^{0.10}$$

was formed, which is the viscosity influence group. N_L is the viscosity influence number for any liquid under any set of conditions as developed by Ros. N_{LB} is the base value of the water viscosity number calculated at 60°F and 14.65 psia. N_{LB} is constant and equals 0.00226. The final correlating function for liquid holdup is then given by:

$$H_L = \psi\left[\frac{N_{Lv}^{0.575}}{N_{gv}\,N_d^{0.0277}}\left(\frac{p}{p_a}\right)^{0.05}\left(\frac{N_L}{N_{LB}}\right)^{0.10}\right]. \qquad (2.437)$$

Fig. 2.144 shows the variation of liquid holdup with this final dimensionless group.

The correlation for liquid holdup presented by Eaton, et al is intended for use in horizontal circular conduits. Care should be exercised in using the correlation for liquids the viscosity of which is greater than 10 cp. The limiting values on the other dimensionless groups are defined as

1. $0.0697 \le N_{Lv} \le 13.246$
2. $1.5506 \le N_{gv} \le 140.537$
3. $5.0 \le p/p_a \le 65.0$
4. $20.3395 \le N_d \le 39.6277$

Reference should be made to Appendix D.8 for a stepwise procedure for calculating pressure loss and/or determining line sizes, as well as problem examples for the Eaton, et al method.[2] It should be possible to apply this correlation over a wide range of pipe sizes. The energy-loss factor correlation for determining two-phase friction factors includes curves for 2 in., 4 in., and 17 in. pipes (Fig. 2.143). Friction factors for pipe sizes not included can be obtained by interpolation.

The correlation does not apply as the flow degenerates to single phase. In the range of low GORs some curious anomalies have been observed by other investigators. It may be that these anomalies were due to low abscissa values on the energy-loss correlation,

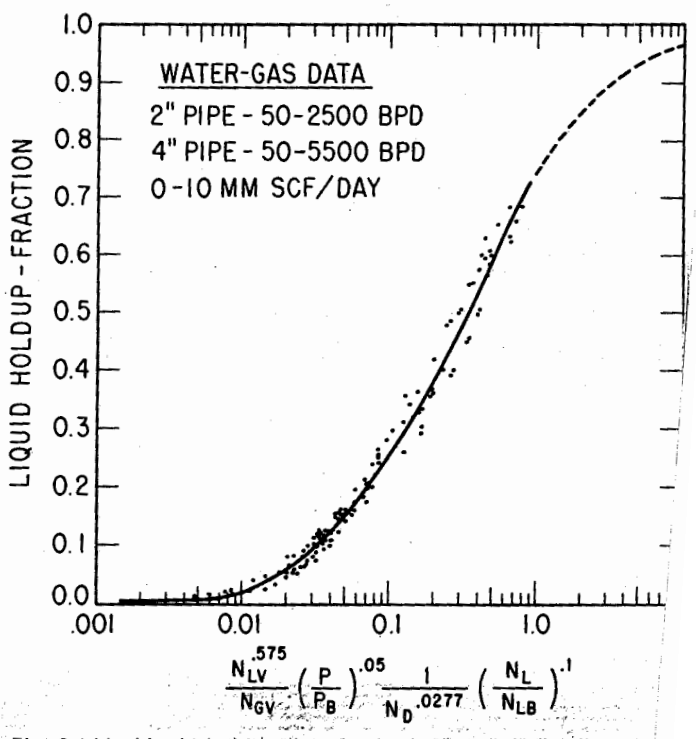


Fig. 2.144 Liquid holdup data for both 2" and 4" flowlines (a Eaton).

resulting in very large friction factors and large calculated pressure drops.

Liquid viscosities used to develop the correlation did not exceed 13.5 cp. Because liquid viscosity does not appear in the energy-loss correlation (Fig. 2.143), it is doubtful if reliable results will be obtained where viscosities exceed 10 to 15 cp or even lower for very low flow rates.

Perhaps the most important contributions of the work of Eaton, et al is the liquid holdup correlation, which relates holdup to fluid properties, flow rate, and system properties without regard to the flow pattern existing in the line. This correctly implies that flow-pattern changes are continuous and that they cause no abrupt discontinuity in the energy-loss correlation. The ability to predict liquid holdup at any place in the line also allows one to calculate real velocities of both the liquid and gas phases. These values are important and necessary in predicting pressure losses due to changes in kinetic energy in each phase. Changes in kinetic energy are negligible at low flow rates and can normally be neglected for pressures greater than 100 psia due to the decreased volume of the gas at higher pressures.

2.45334 Derivations, procedures, and example problems for method of Eaton, et al. (Appendix D)

(1) D.81—Mathematical development of the Eaton, et al correlation.
(2) D.82—Procedure for the calculation of a pressure traverse by the method of Eaton, et al.
(3) D.83—Example problem to determine pressure loss in a 2 in. line 1,500 ft long by the method of Eaton, et al (kinetic energy term neglected).
(4) D.84—Example problem to determine pressure loss by the method of Eaton, et al (kinetic energy term included), Same as Ex. D.33.
(5) D.85—Procedure for selection of line size by the method of Eaton, et al.
(6) D.86—Example problem to determine line sizes by the method of Eaton, et al.

2.4534 The correlation of Beggs and Brill

The development and application of this correlation was discussed in detail in Section 2.3335. For application to horizontal flow the inclination correction factor is zero, but otherwise the calculation procedure is the same as for vertical flow.

2.45341 Procedure and example problem by the method of Beggs and Brill (Appendix D)

(1) D.91—Procedure for calculating a pressure traverse in a horizontal line by the method of Beggs and Brill.
(2) D.92—Example problem to determine pressure loss in a 4 in. line by the method of Beggs and Brill.

2.46 The use of spheres in horizontal flow

One of the main problems in horizontal flow lines, in particular those with some incline, is the accumulation of liquids. Rubber spheres may be used periodically to sweep out the liquids from the line. McDonald and Baker[39] gave a detailed procedure for the operation of a multiphase flow pipeline with spheres. This was generally intended to be used in transmission lines of larger diameters. Baker prepared Fig. 2.145 which shows the pipeline efficiency obtained in utilizing spheres. The efficiency depends upon the liquid entering the pipeline and the frequency of passing the spheres through the line.

Baker also proposed using this efficiency from Fig. 2.145 to calculate the two-phase pressure drop in the line from the following gas flow equation:

$$\Delta p = \frac{L\, q_g\, \gamma_g\, \overline{T}\, \overline{Z}\, f_o}{34{,}722{,}000\, d^5\, \overline{p}\, E^2} \qquad (2.438)$$

where: E = efficiency from Fig. 2.145
d = diameter, in.
f_o = friction factor
L = length, ft
q_g = gas flow rate, scf/hour
$\overline{T}$ = average temperature, °R
$\overline{Z}$ = average compressibility factor
$\overline{p}$ = average pressure, psia

2.47 Summaries of the best correlations and their range of application

2.471 Introduction

Although a great deal of time and money has been spent on the multiphase horizontal flow problem, there still is much to be desired. In particular, the correlations for true horizontal flow must be tied in with the following section on inclined flow. Very few flow or transmission lines are truly horizontal. Additional studies are still necessary under closely controlled experimental conditions. Many holdup correlations for example are not representative because the holdup may have been measured one or two times only for a particular flow pattern. If this is measured between quick closing valves, then the holdup may differ by 100% depending on when the two valves are closed. Friction loss correlations need additional improvement.

Many practical realities must also be faced. For large lines, a change in either the gas or liquid rate may change pressures considerably as affected by holdup in the lines.

2.472 Summary

The following three best methods were chosen for final analysis on the basis of their overall accuracy:

1(a). Dukler, et al method—Case I
1(b). Dukler, et al method—Case II
2. Eaton, et al method
3. Beggs and Brill method

Three additional methods were discussed as limited correlations:

1. Lockhart and Martinelli method
2. Baker method
3. Andrews, et al method

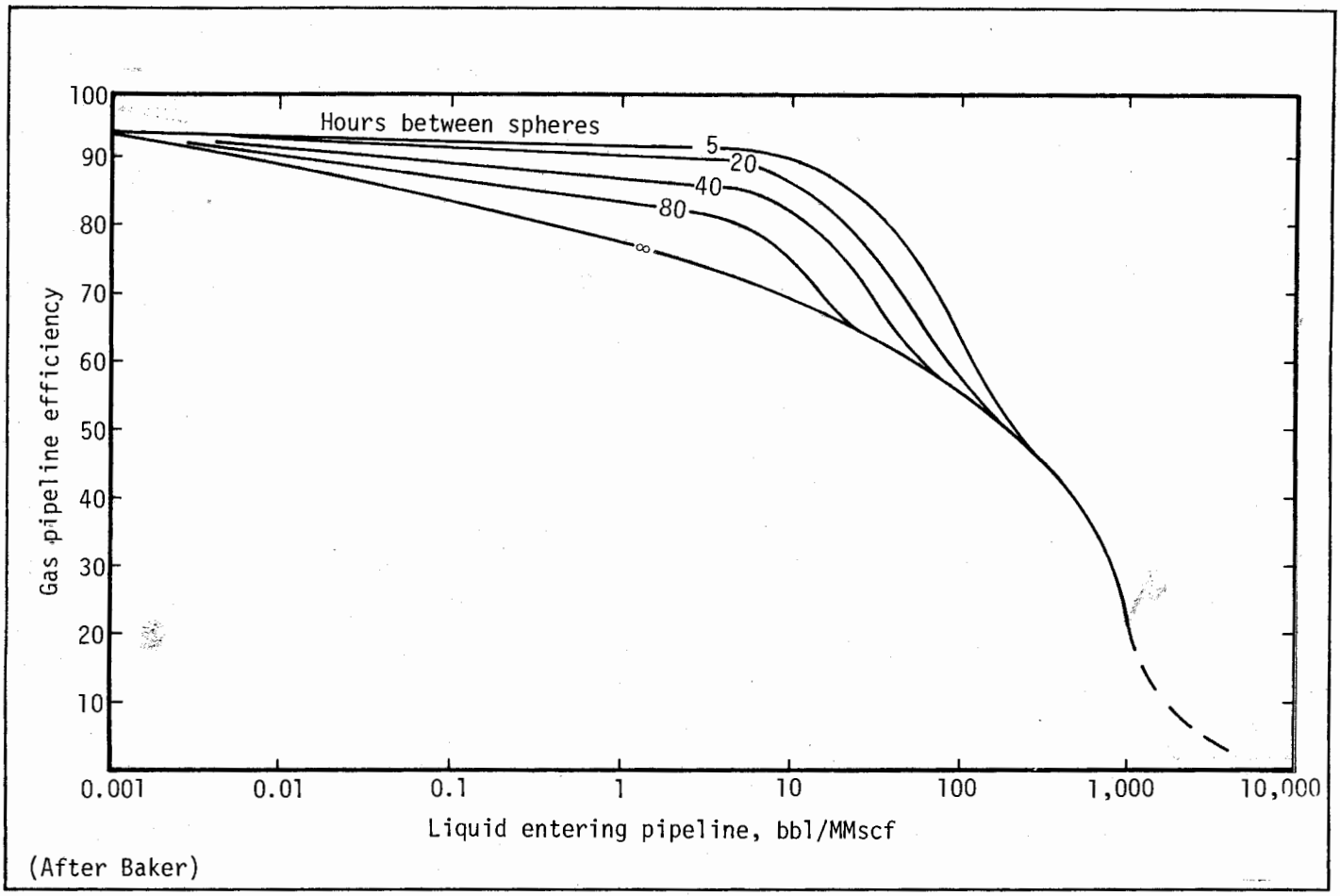


Fig. 2.145 Effect of spheres on the flow efficiency of two-phase lines (after Baker).

In addition, there have been Russian publications which should be noted.[50,51,52] In particular, the one by Gushov may have some merit but presently has not shown to be better than Dukler's or Eaton's work.

Govier, et al[20,21,53,54] has performed a great deal of work in this area and reference should be made to the book of Govier and Aziz for a detailed discussion on the subject of horizontal flow.

Final decisions on correlations to use in certain ranges are difficult. Some comparative papers have been presented including work by Hernandez and Brill[55] on friction factor correlations.

Table 2.45 summarizes the best correlations in detail, giving ranges of use in regard to flow rates, gas-liquid ratios, fluid properties, etc.

Reference should also be made to the book by Govier and Aziz[20] for more details on correlations.

Based on our present knowledge of multiphase horizontal flow, the following conclusions are noted.

TABLE 2.45

Correlation	Pipe sizes	Handles viscous effects	Liquid flow rates	Gas/liquid ratios	Flow patterns required
Andrews, et al	2 in. only	Below 10 cp only	All	All	No
Baker	All (better above 6 in.)	Yes	All	All	Yes (detailed)
Dukler, et al (Case I and Case II)	All	Yes	All	All	No
Eaton, et al	All (better below 6 in.)	Below 10 cp only	All	Better above 300 scf/bbl	No
Lockhart and Martinelli	All	Yes	All	All	Requires flow regime
Beggs & Brill	All	No	All	All	Yes

2.473 Conclusions

1. The Dukler Case II method is the best for a wide range of flow conditions.
2. The Eaton, et al method is as good as Dukler's Case II in certain ranges except it will not handle low GORs or high viscosity crudes and, therefore, does not cover as wide a range of flow conditions.
3. At low gas and liquid rates and in general for small diameters, the Lockhart and Martinelli correlation is best and is the simplest approach to the horizontal multiphase flow correlation.
4. The Andrew's correlation is good for 2 in. pipe except for highly viscous crudes.
5. The Beggs and Brill method is good for a wide range of conditions and may be used to account for deviations of the line from horizontal, thereby making it a more realistic approach.
6. The Baker correlation is good for slug and annular flow and gives comparable results to the better methods for these two flow regions.
7. Calculations should be made in pressure decrements of 25 psi or less especially where a high pressure drop occurs over a short line. Reference should be made to Appendix D.84 where a problem worked by the Eaton method shows a pressure drop of 270 psi to 200 psi compared to taking one pressure of 250 psi compared to 50 psi increments. However, for very low pressure drops in long lines, small pressure decrements are not necessary.

2.474 Recommendations

1. The Dukler, et al correlation and the Eaton, et al correlation should be tested and compared over a wider range of conditions.
2. The Eaton correlation should be tested for high liquid viscosities and a new energy-loss correlation should be developed to take care of viscosity and low GORs.
3. The Beggs and Brill correlation should be more extensively tested for large diameter pipes.
4. More experiments under field conditions should be conducted and used for further comparisons of the correlations of Lockhart and Martinelli, Dukler, et al, Eaton, et al, Baker, and Beggs and Brill.

2.48 Practical application of horizontal multiphase flow

2.481 Introduction

The principal application of horizontal multiphase flow correlations for the flowing or artificial lift wells is to determine the necessary flowing well-head pressure to move the fluids to the separator or to determine the optimum surface flow line size. One of the most serious problems for any flow system is having to produce a well into excessive well-head pressure. In many instances the surface flow line must, by necessity, be anywhere from 1500 ft to several miles long. If this line is too small, then a high wellhead pressure is necessary to move the fluids over to the separator. By using a good horizontal multiphase flow correlation the optimum line size can be selected. For example, the use of a 4 in. flow line in comparison to a 2 in. flow line can mean several hundred bpd more production.

As is true for vertical flow, we may have two choices in making use of horizontal multiphase flow correlations. The calculations can be made by computer or "working curves" can be used. Most companies have at least one horizontal flow line computer program available. It is recommended that either the Dukler Case II or the Eaton correlation be used. There is only one published set of working curves available for horizontal flow and those are found in Brown's book.[56] These are prepared from the Eaton correlation and have proven to be very good except in the very low pressure, low rate, and low gas-oil ratio range.

Figures 2.146 and 2.147 are typical curves. As noted, they are for all water but may be used for all oil if the free gas-oil ratio is used. The reason for this is that the static gradient is not necessary for horizontal flow and no fluids are having to be lifted vertically. Another very decided difference between this curve and a vertical curve is the difference in the effect of increased gas-oil ratios. For vertical flow, we recall that an increased gas-oil ratio causes a decrease in pressure for a certain set of conditions until the minimum gradient is reached. This is because the increased gas lightens the static gradient and causes a decreased pressure traverse. The opposite effect takes place for horizontal flow, that is, an increased gas-oil ratio causes an increase in pressure. The reason for this is that no fluids are being lifted vertically and therefore the gas merely

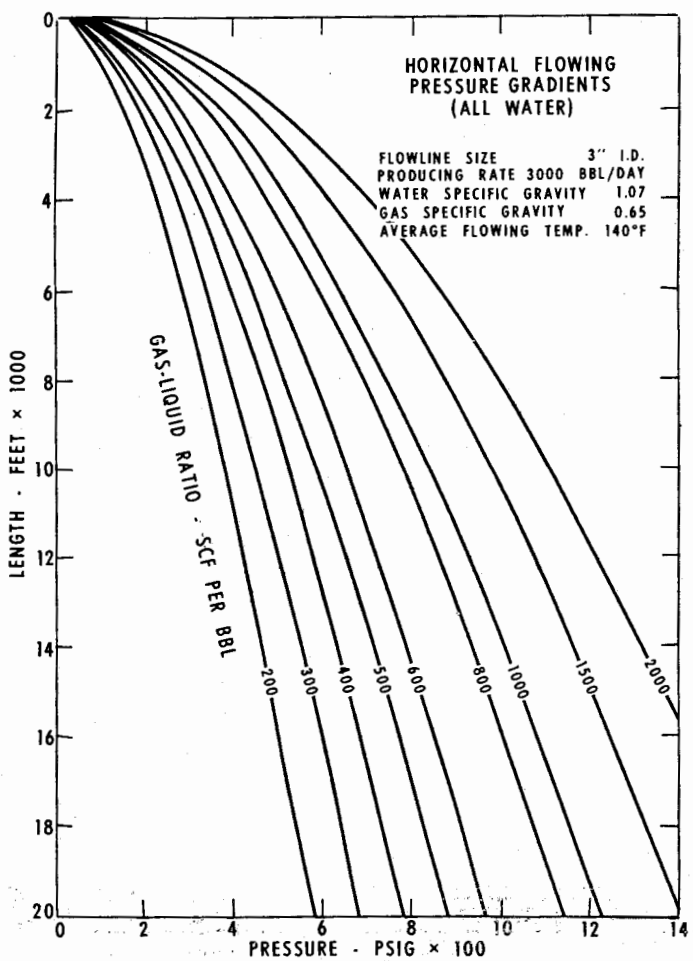


Fig. 2.146 Effect of gas-liquid ratio.

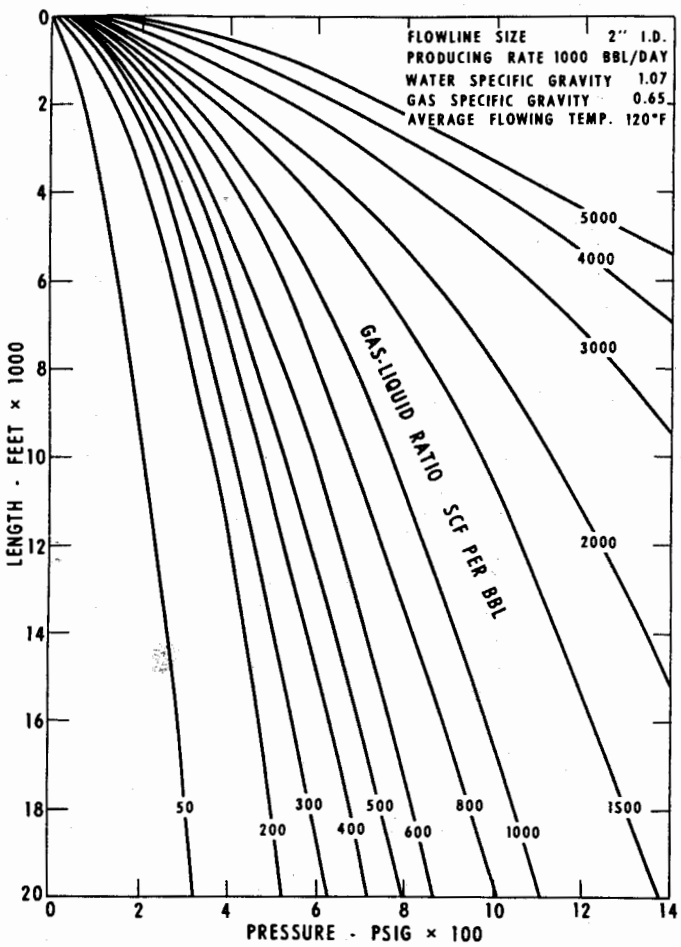


Fig. 2.147 Horizontal flowing pressure gradients (all water).

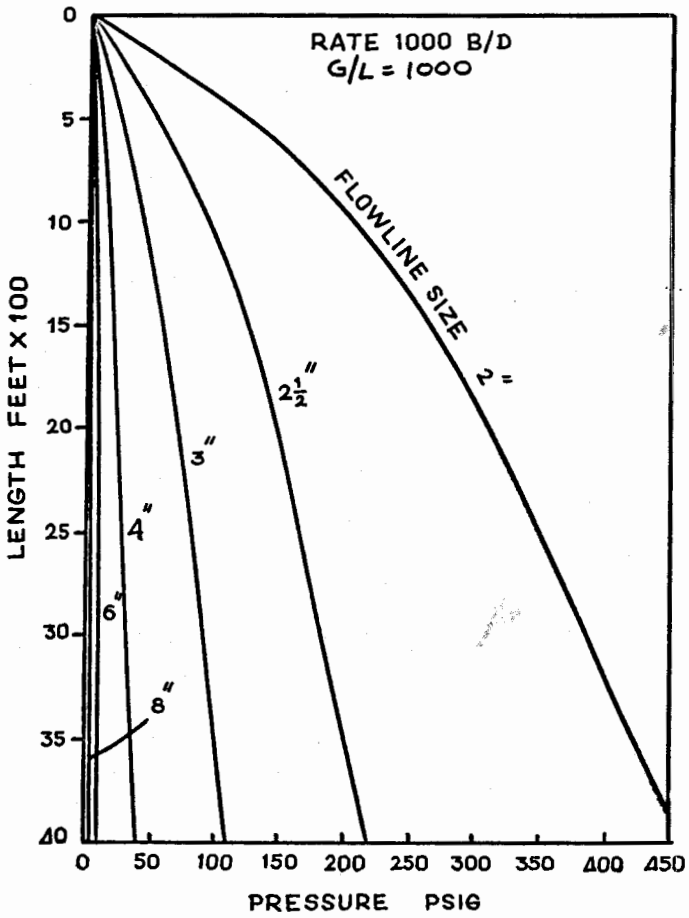


Fig. 2.148 Effect of line diameter.

represents additional fluids to be moved in the horizontal line.

A complete set of these horizontal working curves can be found in Appendix E of Brown's book. These were prepared from the correlation of Eaton and may be used with confidence for both oil and water production but the free gas-oil ratio (GOR-R_s) should always be used. The curves should not be used when the viscosity of the liquid is greater than 10 cp.

2.482 Effect of variables

2.4821 Introduction

In general, the effect of variables such as pipe size and viscosity have the same effect in horizontal multiphase flow as they did in vertical flow. The one variable that differs completely in its effect is the gas-liquid ratio. The following series of figures shows the effect of some of these variables. These effects are important in calculating pressure losses and determining line sizes. A study of these variables was made by Lopez[57] and his results are presented here. He used the Dukler Case II calculation procedure.

2.4822 Effect of line sizes

The effect of pipe diameter on horizontal pressure losses is shown in Figs. 2.148 and 2.149 for flow rates of 1,000, 3,000 and 8,000 stb/d respectively. As noted, the pressure loss for a given length of flow line decreases very rapidly with increasing diameter sizes. For example, a production rate of 1,000 stb/d in a 4,000 ft line flowing against atmospheric pressure would require a well-head pressure of 40 psig in a 4 in. line and 462 psig in a 2 in. line. Table 2.46 shows these results with 3,000 bpd added to the table.

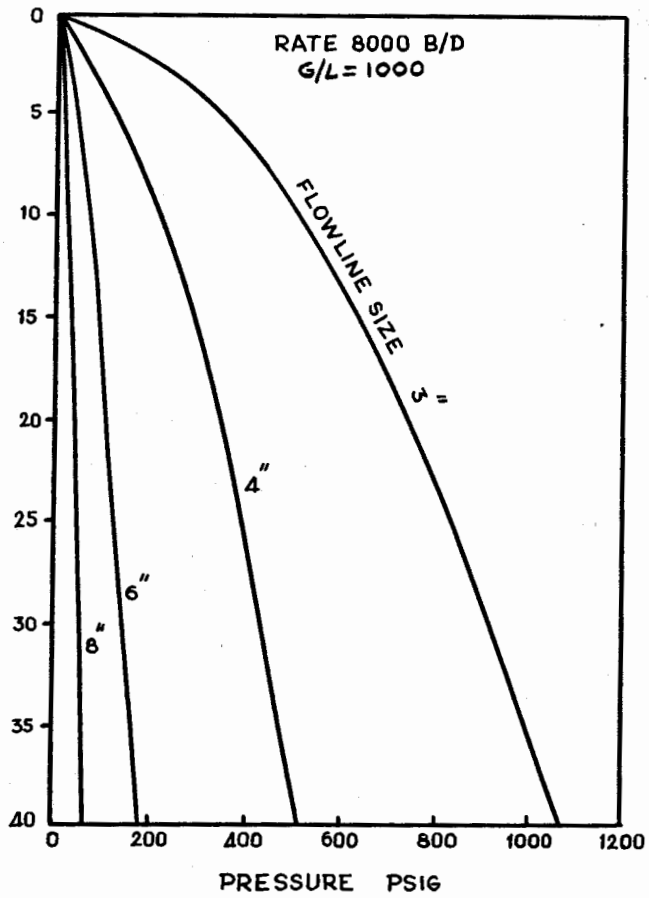
2.4823 Effect of flow rate

Figures 2.150, 2.151 and 2.152 show the effect of flow rate on the horizontal pressure losses, through flowlines of 2, 4, and 6 in., respectively. For the set of conditions listed in Table 2.47, a wellhead pressure of 209 psig is required to flow 500 stb/d through 2 in. pipe against atmospheric pressure, compared to 712 psig for 1,500 stb/d. Refer to Table 2.47 for a complete tabulation of results.

A common error in production flow lines is to place two or more wells into one long flow line, exceed its capacity, and thereby increase the wellhead pressure at each well. This in turn lowers the overall productive capacities of the individual wells and the field.

2.4824 Effect of gas-liquid ratio

The effect of gas-liquid ratio on horizontal pressure losses is shown in Fig. 2.153. For vertical flow we recall



g. 2.149 Effect of line diameter.

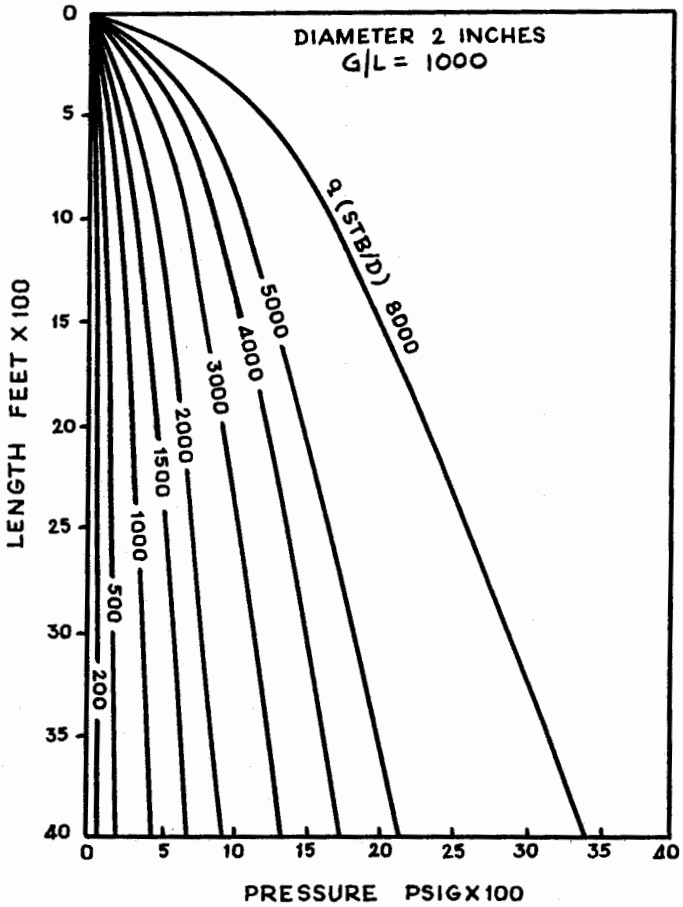


Fig. 2.150 Effect of flowrate.

TABLE 2.46
FLOWLINE EFFECT

a.	2	2½	3	4	6	8
L			Pressure Drop			
)00	462	223	110	40	9	3
)00	1,360	745	408	172	53	17
)00	–	–	1,084	526	184	72

iat an increased gas-oil ratio causes a decrease in ·essure for a certain set of conditions until the mini- um gradient is reached. This is because the increased as lightens the static gradient and causes a decreased ·essure traverse. The opposite effect takes place for)rizontal flow, where no fluids are being lifted verti- ılly and therefore the gas merely represents addi- onal fluids to be moved in the horizontal line.

These results can be noted in Table 2.48. For ex- nple, a G/L of 200 requires 116 psi compared to a /L of 1,500 requiring 330 psi. We see that as the /L increases the pressure increases.

1825 Effect of viscosity

Figure 2.154 shows the variation in horizontal pres- ıre gradient at a 4,000 ft pipe for various API gravi- es which in turn can be correlated with viscosities. iscous crudes offer more of a problem in horizontal)w than they do in vertical multiphase flow.

The reason for this is that generally the crudes are cooler in the surface flow line and hence more viscous. Results are shown in Table 2.49. We note that a pressure drop of 215 psi occurs for a viscosity of 1.0 cp, compared to 400 psi for a viscosity of 500 cp.

2.4826 Effect of water-oil ratio

The effect of water-oil ratio, and hence density, is not as pronounced as for vertical flow because liquids are not having to be lifted vertically. Therefore, except for viscous crudes, there is not much difference in producing crude as compared to water in a horizontal flow line. Fig. 2.155 shows the superposition of all curves when the water-oil ratio is changed for different values between 0.1 to 1,000 stbw/stbo. Results are shown in Table 2.410.

For example, a water-oil ratio of 0.1 requires a pressure drop of 223 psi compared to a pressure drop of 216 for a 1,000 water-oil ratio.

2.4827 Effect of other factors

The effect of surface tension is likely the same as for vertical flow and is not clearly defined. The effect of slippage probably enters into the pressure loss calculation but does not have the decided effect that it does in vertical flow because again no fluids are being lifted vertically.

The effect of kinetic energy can be neglected in most cases, but for high fluid rates and in the low pressure systems where the density is low and the velocity in-

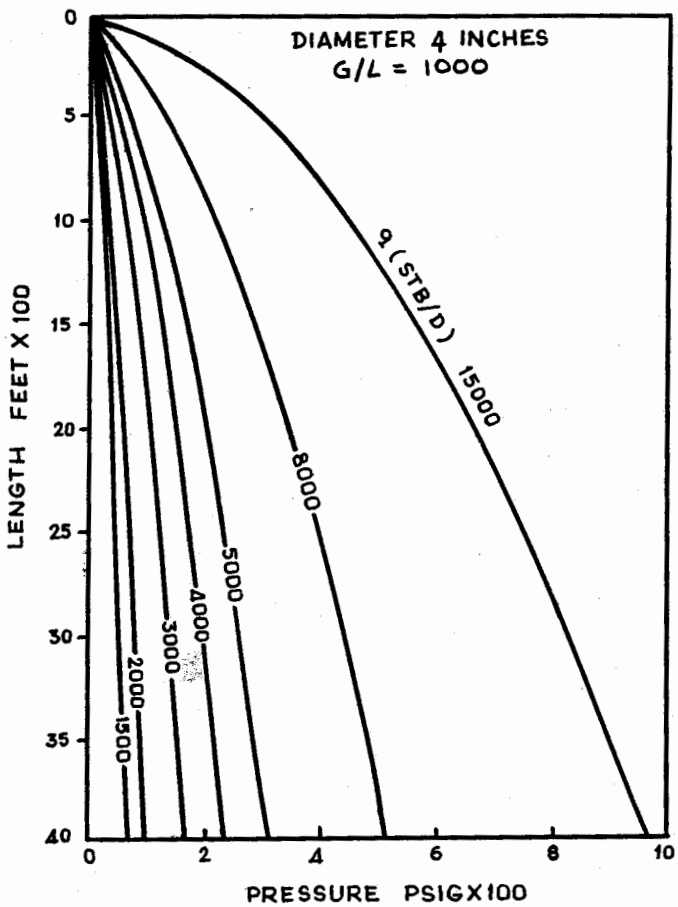


Fig. 2.151 Effect of flowrate.

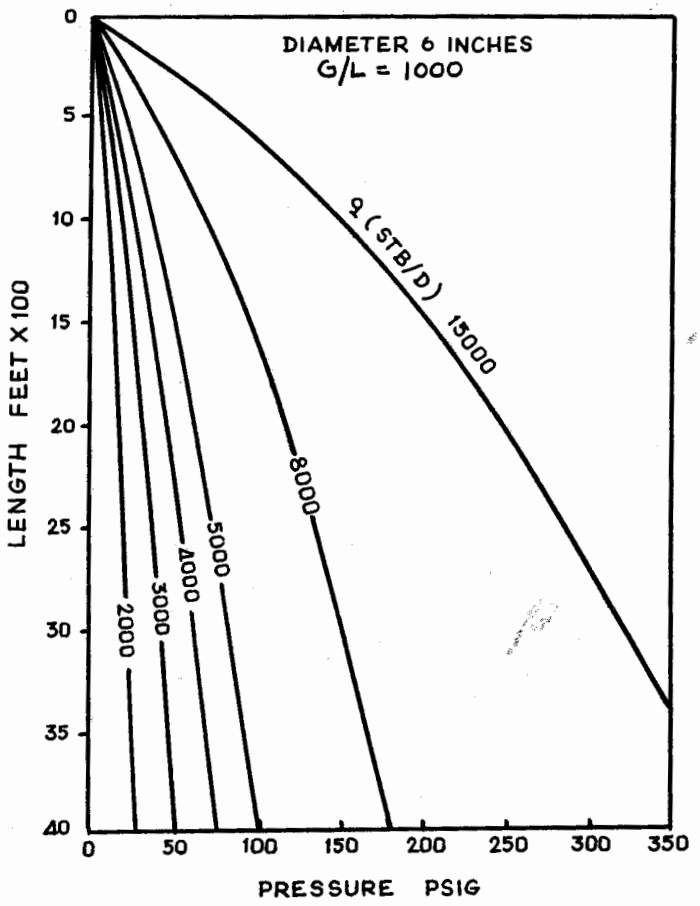


Fig. 2.152 Effect of flowrate.

creasing rapidly, the acceleration term can be significant. For computer calculations it is recommended that the acceleration term be included due to ease of calculation.

2.483 Example problems

Similar to the section on vertical multiphase flow, a series of problems will be worked illustrating the practical use of horizontal multiphase flow correlations. The series of working curves found in Appendix D will be utilized for these problems.

EXAMPLE PROBLEM #1: HOW TO DETERMINE THE NECESSARY WELLHEAD PRESSURE

For surface flow lines, it is likely that the separator pressure is the only known constant. This is generally a constant for a particular well or lease and is predetermined due to gas sales line pressure, required liquid dump pressure, or composition of the fluids with reference to a required back pressure to give the most economical separation. The necessary wellhead pressure to move a certain fluid rate through a set flow line size and length can then be determined. This can be illustrated through an example as follows:

Given: Flow line size = 2 in. I.D.
Separator pressure = 100 psig = P_{sep}
Flow line length = 5,000 ft = L
Gas-liquid ratio = 1,000 scf/bbl (γ_g = 0.65)
q_L = 1,000 bpd (95% water)

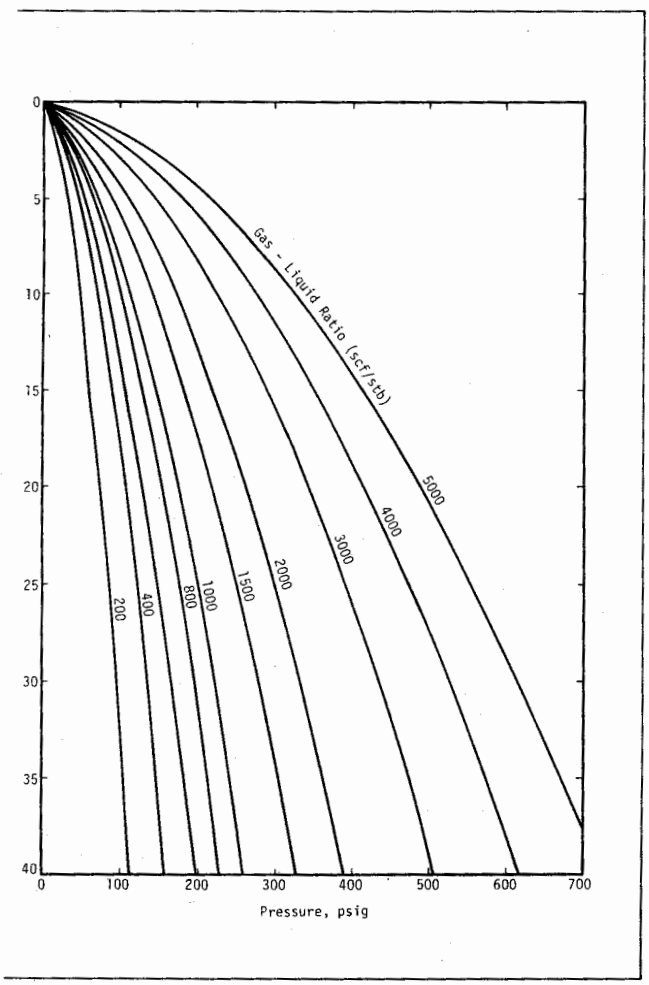
Determine the wellhead pressure necessary to move these fluids through the 5000 ft of horizontal flow line into a separator pressure of 100 psig.

The following solution is given:

1. The solution can be made by long hand, or, preferably, by machine computation by starting with the separator pressure. Pressure increments are assumed and the wellhead pressure determined.
2. The solution can be easily obtained by referring to a set of working curves such as those of Brown.[56] Refer to Fig. D.103 for the solution to this problem. Although these curves are for 100% water, they may be also used for 100% oil with fair accuracy since no vertical lift is involved in the horizontal flow problem.
   (a) Find the equivalent length due to separator pressure by following a line vertically downwards on Fig. D.103 to the 1,000 scf/bbl line from the separator pressure of 100 psig at zero length. This length is found to be 200 ft.
   (b) Add this equivalent length of 200 ft to the total length of 5,000 ft and find 200 + 5,000 = 5,200 ft.
   (c) From a length of 5,200 ft on the ordinate, proceed horizontally to the right until intersecting the 1,000 scf/bbl line. Read the pressure at this intersection to be approximately 545 psig. This is the required wellhead pressure.

TABLE 2.47
FLOW RATE EFFECT

q_L	200	500	1,000	1,500	2,000	3,000	4,000	5,000	8,000	15,000
Dia.	Pressure Drop									
2	65	209	462	712	938	1,360	1,761	2,160	3,464	–
4	–	–	41	72	104	172	243	314	526	970
6	–	–	–	18	29	53	77	103	184	384



. 2.153 *Effect of gas liquid ratio.*

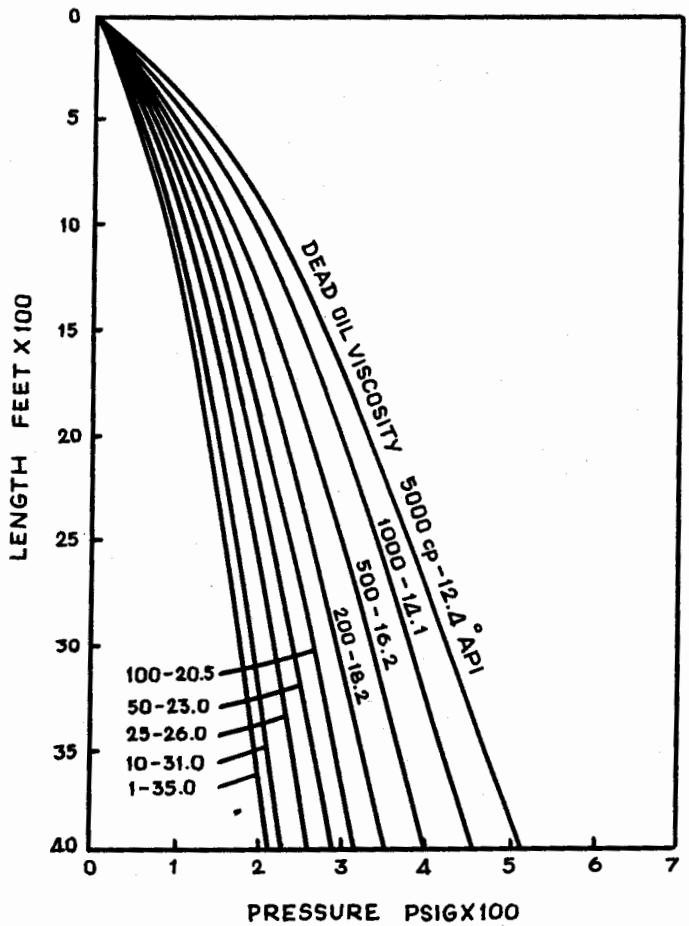


Fig. 2.154 Effect of viscosity.

TABLE 2.48
EFFECT OF GAS LIQUID RATIO

G/L	Pressure drop
200	116
400	163
600	198
800	231
1,000	262
1,500	330
2,000	393
3,000	509
4,000	618
5,000	722

TABLE 2.49
VISCOSITY EFFECT

Viscosity	Pressure drop
1	215
10	231
25	264
50	292
100	323
200	360
500	400
1,000	456
5,000	530

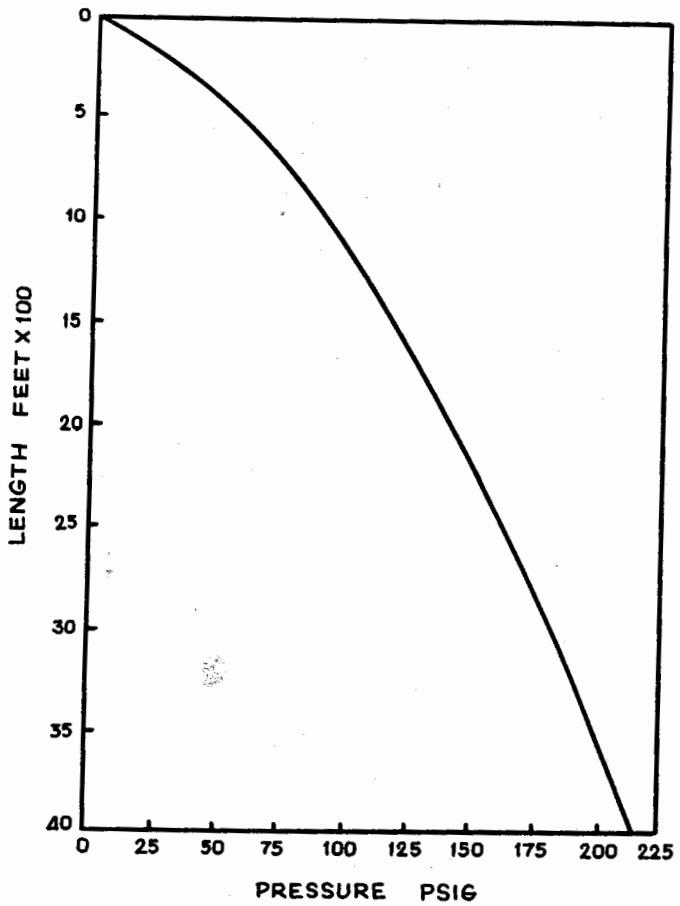


Fig. 2.155 Effect of WOR.

TABLE 2.410
WOR

WOR	Pressure drop
0.10	223
0.25	224
0.33	224
0.50	224
1.00	223
2.00	222
3.00	221
5.00	220
10.00	218
1,000.00	216

CLASS PROBLEM #1-A: TO DETERMINE p_{wh}

Given the data of the previous example problem where:

$q_L = 1{,}000$ bpd (95% water)
$G/L = 1{,}000$ scf/bbl
$p_{sep} = 100$ psig
$L = 5{,}000$ ft

By use of working curves, find the necessary wellhead pressure (p_{wh}) for a 3 in. and 4 in. flow line.

CLASS PROBLEM #1-B: TO DETERMINE p_{wh}

Given data: $q_L = 5{,}000$ bpd (90% water)
$G/L = 600$ scf/bbl
$p_{sep} = 200$ psig
$L = 10{,}000$ ft

Find the wellhead pressure (p_{wh}) for a 3 in. and 4 in. flow line.

EXAMPLE PROBLEM #2: HOW TO SELECT THE FLOW LINE SIZE

Given data: $p_{sep} = 80$ psig
$p_{wh} = 150$ psig
$q_L = 1{,}000$ bpd (95% water)
$G/L = 1{,}000$ scf/bbl
$L = 2{,}500$ ft

1. For a computer solution we would increase the estimated line size by nominal standard pipe sizes until we obtain a wellhead pressure of 150 psig or less.
2. If we use working curves such as Figs. D.101–D.1022, we work the problem in the manner already described. Several line sizes are assumed and the pressure loss tabulated for each line size until one is obtained giving a wellhead pressure of 150 psig or less.

Assumed line size	p_{wh}	
2 in.	380	Fig. D.103
2½ in.	220	Fig. D.108
3 in.	155	Fig. D.1013
3½ in.	125	--------
4 in.	110	Fig. D.1018

As can be seen, a 3½ in. flow line will be necessary to give a wellhead pressure less than 150 psig. The 3½ in. line would require a wellhead pressure of 125 psig to move the required production 2,500 ft into an 80 psig separator, and would be our size selection for this problem.

CLASS PROBLEM #2-A: TO SELECT THE FLOW LINE SIZE

Given data: $p_{sep} = 100$ psi
$p_{wh} = 180$ psi
$q_L = 2{,}000$ bpd (90% water)
$L = 4{,}800$ ft
$G/L = 750$ scf/bbl

Select the minimum flow line size to operate for these conditions.

CLASS PROBLEM #2-B: TO SELECT THE FLOW LINE SIZE

Given data: $q_L = 3000$ bpd
$G/L = 600$ scf/bbl
$p_{sep} = 100$ psi
$L = 6{,}000$ ft
$p_{wh} = 320$ psi

Find the minimum line size.

EXAMPLE PROBLEM #3: HOW TO FIND THE SEPARATOR PRESSURE

Given data: $L = 6000$ ft
$p_{wh} = 800$ psi

q_L = 4000 bpd
d = 3 in.
G/L = 600 scf/bbl

ind the pressure on the separator.

olution:

1. Disregarding the length of the line of 6000 ft find ıe arbitrary length where the pressure of 800 psi ‹ists on the 600 scf/bbl gas-liquid ratio line. This is ıund to be at 8400 ft (Fig. D.1016).
2. Subtract the length of the line of 6000 ft from 400 ft = 2400 ft.
3. At 2400 ft on the ordinate, read the pressure on ıe 600 scf/bbl line. This is found to be 415 psi which the separator pressure.

_ASS PROBLEM #3-A: TO FIND THE SEPARATOR RESSURE

Given data: L = 9000 ft
p_{wh} = 450 psi
q_L = 1,000 bpd
G/L = 800 scf/bbl
d = 2.5 in.

ind the separator pressure.

_ASS PROBLEM #3-B: TO FIND THE SEPARATOR RESSURE

Given data: q_L = 600 bpd
d = 2 in.
L = 10,000 ft
G/L = 1500 scf/bbl
p_{wh} = 600 psi

nd the separator pressure.

(AMPLE PROBLEM #4: TO DETERMINE THE AS-LIQUID RATIO

This problem has little significance because there is ı reason to be solving for the gas-liquid ratio in the rface flow line. However, it does illustrate further e of the working curves.

Given data: d = 2½ in.
L = 10,000 ft
p_{sep} = 200 psi
q_L = 2000 bpd
p_{wh} = 600 psi

nd the gas-liquid ratio.

lution:

1. This represents a trial and error solution, so first sume a gas-liquid ratio = 400 scf/bbl.
2. Find the equivalent length for the separator pres- re of 200 psi on the 400 scf/bbl line = 1300 ft (Fig. 1010).
3. Add this 1300 ft to L of 10,000 ft = 11,300 ft.
4. Find the pressure at 11,300 ft on the 400 scf/bbl ıe = 605 psi.
5. This is near enough to the 600 psi wellhead pres- re, therefore the assumption of another G/L is not cessary. If the pressure had been greater than 600 i a new assumption for the gas liquid ratio of less an 400 scf/bbl should be assumed. If the pressure had been less than 600 psi a new gas-liquid ratio greater than 400 scf/bbl should be assumed and the procedure repeated.

CLASS PROBLEM #4-A: TO FIND THE GAS-LIQUID RATIO

Given data: d = 3 in.
q_L = 3,000 bpd
p_{wh} = 500 psi
p_{sep} = 100 psi
L = 6,000 ft

Find the G/L.

CLASS PROBLEM #4-B: TO FIND THE GAS-LIQUID RATIO

Given data: d = 4 in.
q_L = 5,000 bpd
L = 12,000 ft
p_{sep} = 100 psi
p_{wh} = 700 psi

Find the G/L.

EXAMPLE PROBLEM #5: HOW TO DETERMINE THE FLOW RATE POSSIBLE

Given data: d = 3 in.
p_{sep} = 160 psi
p_{wh} = 600 psi
L = 4,500 ft
G/L = 500 scf/bbl

Find flow rate possible through this line.
The best way to solve this problem is to prepare a plot of q vs. p_{wh} similar to Fig. 2.156.

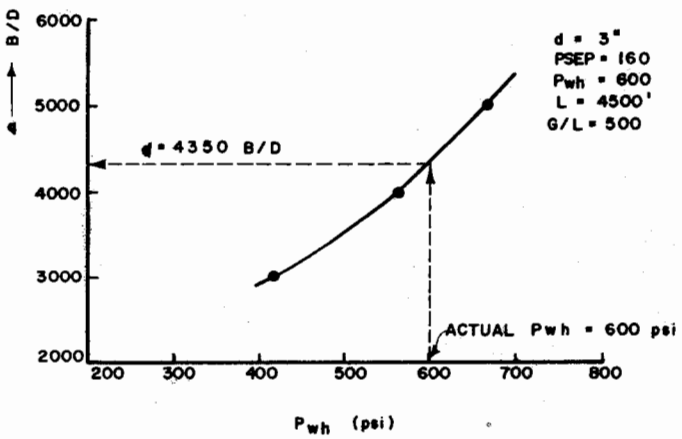


Fig. 2.156 Determination of possible flow rate.

It is necessary to assume a rate and determine the necessary wellhead pressure for this rate.

Solution:

1. Assume various rates.
2. Determine the necessary wellhead pressure for each rate making certain that the assumed rates require wellhead pressures which bracket the actual pressure of 600 psi. Prepare a table as follows:

Assumed rates	p_{wh} required
3,000	415
4,000	565
5,000	665

3. Make a plot of q vs. p_{wh} as noted in Fig. 2.156 constructing the curve as indicated.
4. Proceed vertically upwards from the wellhead pressure of 600 psi until intersecting the constructed curve.
5. Proceed horizontally to the left from the intersected point and read the actual production rate of 4,350 bpd.

CLASS PROBLEM #5-A: TO DETERMINE THE FLOW RATE POSSIBLE

Given data: $2\frac{1}{2}$ in.
$P_{sep} = 300$ psi
$p_{wh} = 800$ psi
G/L = 1,000 scf/bbl
L = 10,000 ft

Find the flow rate.

CLASS PROBLEM #5-B: TO DETERMINE THE FLOW RATE POSSIBLE

Given data: 4 in.
$p_{sep} = 80$ psi
$p_{wh} = 200$ psi
G/L = 600 scf/bbl
L = 4,000 ft

Find the flow rate.

EXAMPLE PROBLEM #6: HOW TO DETERMINE IF THE SURFACE FLOW LINE HAS RESTRICTIONS

There are cases where the flow line has become partially plugged due to paraffin, and/or scale deposits. This will cause an increase in surface wellhead pressure in order to move the produced fluids on to the separator. By checking the wellhead pressure required against the actual wellhead pressure this can be determined.

Given data: 3 in. flow line
$p_{wh} = 480$ psi
q = 2000 bpd (90% water)
G/L = 1,000 scf/bbl
$p_{sep} = 100$ psi
L = 4,000 ft

Solution:

Disregard the actual given wellhead pressure and determine the necessary wellhead pressure for other given conditions. Check the determined value against the actual p_{wh} and if the determined value is less, then some type of restriction exists. This assumes that we, of course, are measuring p_{wh} downstream from any choke or restriction in the Christmas tree.

The p_{wh} is found in the same manner as previously described in example problem #1 (Fig. D.1014)

For this problem, the p_{wh} required is found to be 350 psi and this is less than the actual value of 480 psi by 130 psi. There is indeed some restriction in the flow line which should be investigated.

CLASS PROBLEM #6-A: TO DETERMINE IF THERE ARE RESTRICTIONS IN THE FLOW LINE

Given data: 4 in. flow line
q = 4,000 bpd (90% water)
G/L = 800 scf/bbl
L = 5,000 ft
$p_{sep} = 80$ psi
$p_{wh} = 430$ psi

Determine if any restriction exists in the flow line.

CLASS PROBLEM #6-B: TO DETERMINE IF RESTRICTIONS EXIST IN THE FLOW LINE

Given data: 2 in. flow line
L = 3,000 ft
G/L = 500 scf/bbl
$p_{sep} = 120$ psi
$p_{wh} = 400$ psi
q = 1,000 bpd (95% water)

Determine whether or not there exist any restrictions in the flow line.

CLASS PROBLEM #6-C: TO DETERMINE IF RESTRICTIONS EXIST IN THE FLOW LINE

Given data: 2 in. Flow line
q = 600 bpd (95% water)
G/L = 1500 scf/bbl
$P_{sep} = 100$ psi
L = 6000 ft
$p_{wh} = 460$ psi

Determine whether or not there are any restrictions in the flow line.

EXAMPLE PROBLEM #7: HOW TO USE THE ALL-WATER CURVES FOR ALL OIL PRODUCTION

As noted, the horizontal working curves have been prepared for all water and since no vertical lift of the fluids is required, they can also be used for all oil production. However, the gas-oil ratio which in turn is the gas-liquid ratio must be the free gas-oil ratio and not include that gas in solution. In addition, we need to know the average flowing temperature and the gas gravity.

Given data: 2 in. flow line
L = 5,000 ft
$p_{sep} = 100$ psi
q = 1,000 bpd (all oil), 35 °API
Total produced GOR = 600 scf/bbl
Average temperature = 110°F
$\gamma_g = 0.65$

Find p_{wh}.

1. Assume a value of p_{wh} by using that value obtained from the curves for all water = 425 psi.

2. Average p_{sep} and $p_{wh} = \frac{100 + 425}{2} = 262$ psig = 77 psia.

3. From Fig. 2.19—determine gas in solution under nditions of $\bar{p} = 277$ psia. $\bar{T} = 110°F$, 35° API, and $\gamma_g = 65$.

$$R_s \cong 55 \text{ scf/bbl}$$

4. Subtract R_s of step 3 from producing GOR of 00 scf/bbl = 600 − 55 = 545 scf/bbl.

5. Find p_{wh} using 545 scf/bbl and we find p_{wh} = 10 psi.

6. Repeat steps 2 through 5 using the value of p_{wh} ound in step 5. However, in most cases the first answer 1 step 5 will be very nearly correct as is true in this roblem. Answer ≅ 410 psi.

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2.5 INCLINED OR HILLY TERRAIN MULTIPHASE FLOW

2.51 Introduction

Unfortunately, very few of our surface production lines as noted in Figure 2.157 are truly horizontal. Most of them vary a little, and many of them have to pass over rather steep hills (Fig. 2.158).

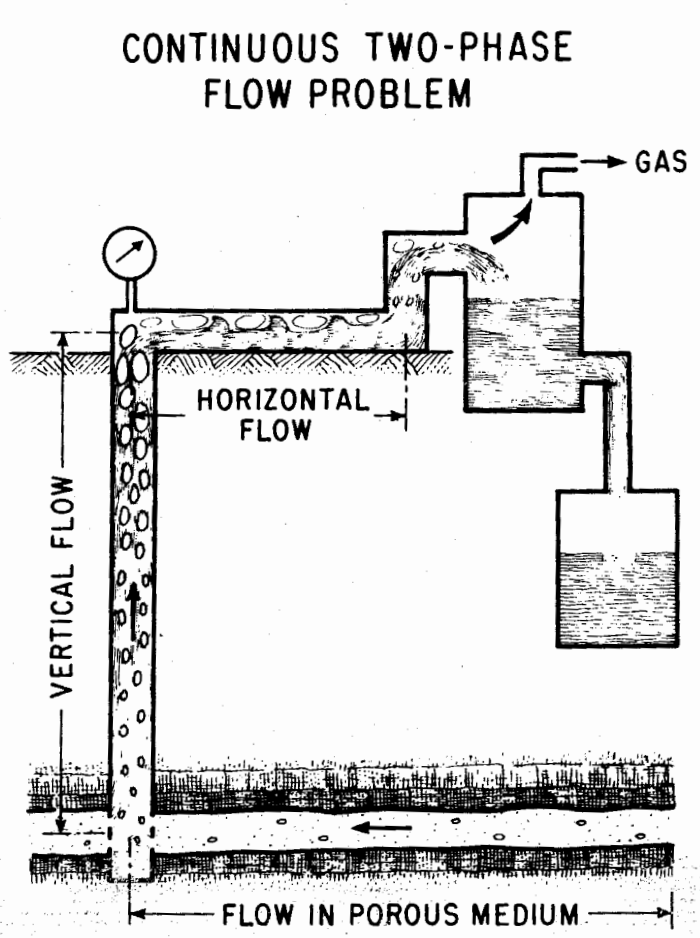


Fig. 2.157 Three stages of production.

Inclined flow is defined as flow through pipes that deviate from true horizontal, such as flow over hilly terrain. Directional flow is defined as flow through pipes (normally tubing) that deviate from true vertical, and is referred to in this book as flow through pipes in directionally-drilled wells. Both inclined and directional flow offer similar problems, but are discussed separately. They are indeed very complex problems and no completely satisfactory solution is available at the present time.

The total pressure loss for a hilly terrain pipeline system (inclined flow) is the sum of the frictional losses, the acceleration losses, and the elevation pressure losses necessary to lift the fluids through any vertical distance. Beggs and Brill[1] noted recovery on the downhill sides of the slopes, but most present day solutions assume no recovery on the downhill side.

The same general equation can be applied to inclined flow:

$$\frac{dp}{dz} = \frac{g}{g_c}\rho_m \sin\theta + \frac{f\rho_m v_m^2}{2g_c d} + \frac{\rho_m v_m dv_m}{g_c dz} \qquad (2.184)$$

where θ = angle from horizontal.

Beggs and Brill arrived at the following equation for inclined flow:

$$\frac{\Delta p}{\Delta z} = \frac{\frac{g}{g_c}\rho_{TP}\sin\theta + \frac{f_{TP}\, G_m\, v_m}{2g_c d}}{1 - \frac{\rho_{TP} v_m\, v_{sg}}{g_c p}} \qquad (2.51)$$

In solving this equation for the depth increment:

$$\Delta z = \frac{\Delta p\left(1 - \frac{\rho_{TP}\, v_m\, v_{sg}}{g_c p}\right)}{\frac{g}{g_c}\rho_{TP}\sin\theta + \frac{f_{TP}\, G_m\, v_m}{2g_c d}} \qquad (2.52)$$

Where $G_m = G_L + G_g$ = mass flux rate

$$\rho_{TP} = \rho_L H_L + \rho_g(1 - H_L)$$

and H_L is a function of the angle of inclination from the horizontal.

The most widely used solution to this problem to date has been offered by Flanigan.[2] However, recent work by Beggs and Brill[1] appears to be more accurate with slight modifications. It should indeed cover a wider range of conditions. It should also be more suitable for surface flow lines whereas the work of Flanigan was conducted in larger transmission lines. Other investigators which should be noted are Odishariya,[3] Guzhov,[4] and Mamayev.[5]

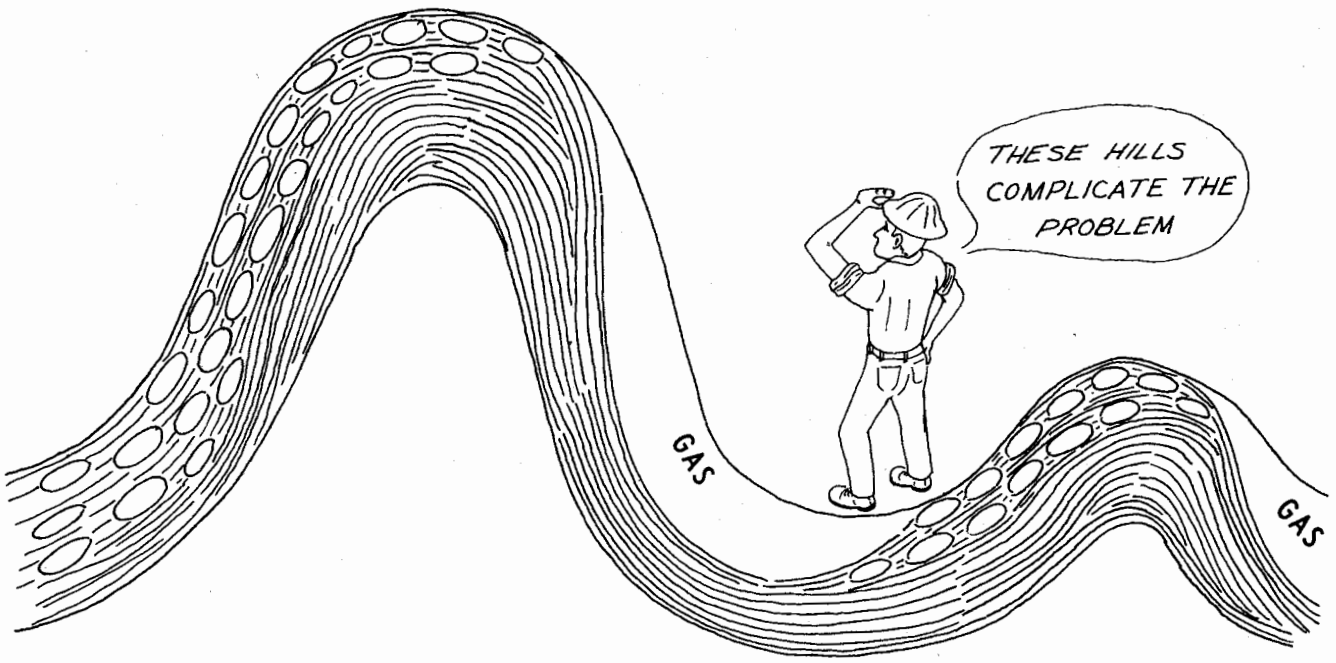
2.52 Best correlations for inclined flow

2.521 Flanigan correlation

2.5211 Introduction

Flanigan[2] conducted numerous field tests for inclined flow and noted the following:

1. Most of the pressure drop occurred in the uphill section of the line.
2. The pressure drop in the line decreased as the gas flow rate increased.



BEST CORRELATIONS:

FLANIGAN
BEGGS & BRILL

TOTAL GRADIENT COMPOSED OF:

FRICTIONAL GRADIENT
ACCELERATION GRADIENT
STATIC GRADIENT—UPSLOPE
RECOVERY GRADIENT—DOWNSLOPE

Fig. 2.158 Horizontal flowing pressure gradients (all water).

In regard to item 2, it has been observed that for two-phase flow in horizontal lines, the pressure drop increases as the gas flow rate increases.

Flanigan explains this apparent difference by analyzing the phenomena described in Fig. 2.159. In this figure he assumes that there are two main components of pressure drop in two-phase flow. The first is the component due to friction which is the predominant one existing in horizontal lines. The second component is the elevation effect due to the liquid head which becomes the principal component in inclined and vertical sections at low gas velocities. The sum of the two components is the total pressure drop, except for those cases of very high velocities when the acceleration component becomes important.

Flanigan separated these two components and presented a method to determine each of them. This explanation of the friction and elevation components is discussed in the next two sections.

2.5212 Friction pressure-drop component

In determining the frictional loss, Flanigan analyzed the correlation presented by Ovid Baker[6] on the line efficiency for horizontal two-phase flow as a function of the liquid-to-gas ratio and found that the scattering of the data was too great to assure accuracy. He reworked the data and developed the correlation shown in Fig. 2.160. As can be seen, the efficiency, E, is shown to be a function of gas velocity, v_g, as well as the liquid-to-gas ratio, R, raised to the 0.32 power. According to Flanigan,[2] this correlation gives a maximum deviation of plus or minus 9% pipeline efficiency.

The data for this correlation were obtained from pipelines of 4, 6, 8 and 10 in. diameter, while the gas velocities ranged from 1 to 12 ft per second, and the liquid-to-gas ratio ranged from 20 to 1,200 bbl/MMcf. The fluids used were natural gas and condensate. Once the line efficiency is available, a pressure drop equation similar to the Panhandle equation can be used to determine the friction pressure drop component.

2.5213 Elevation pressure-drop component

In examining the test data from the 16 in. line, Flanigan noted the following:

1. For relatively low gas velocities, the largest portion of the total pressure-drop occurred in the uphill sections of the line.
2. The elevation component pressure drop is directly proportion to the sum of the uphill rises in the pipeline.
3. The elevation difference between the terminals of the line is, by itself, of no particular significance.
4. The slope of the hill is of no importance other than

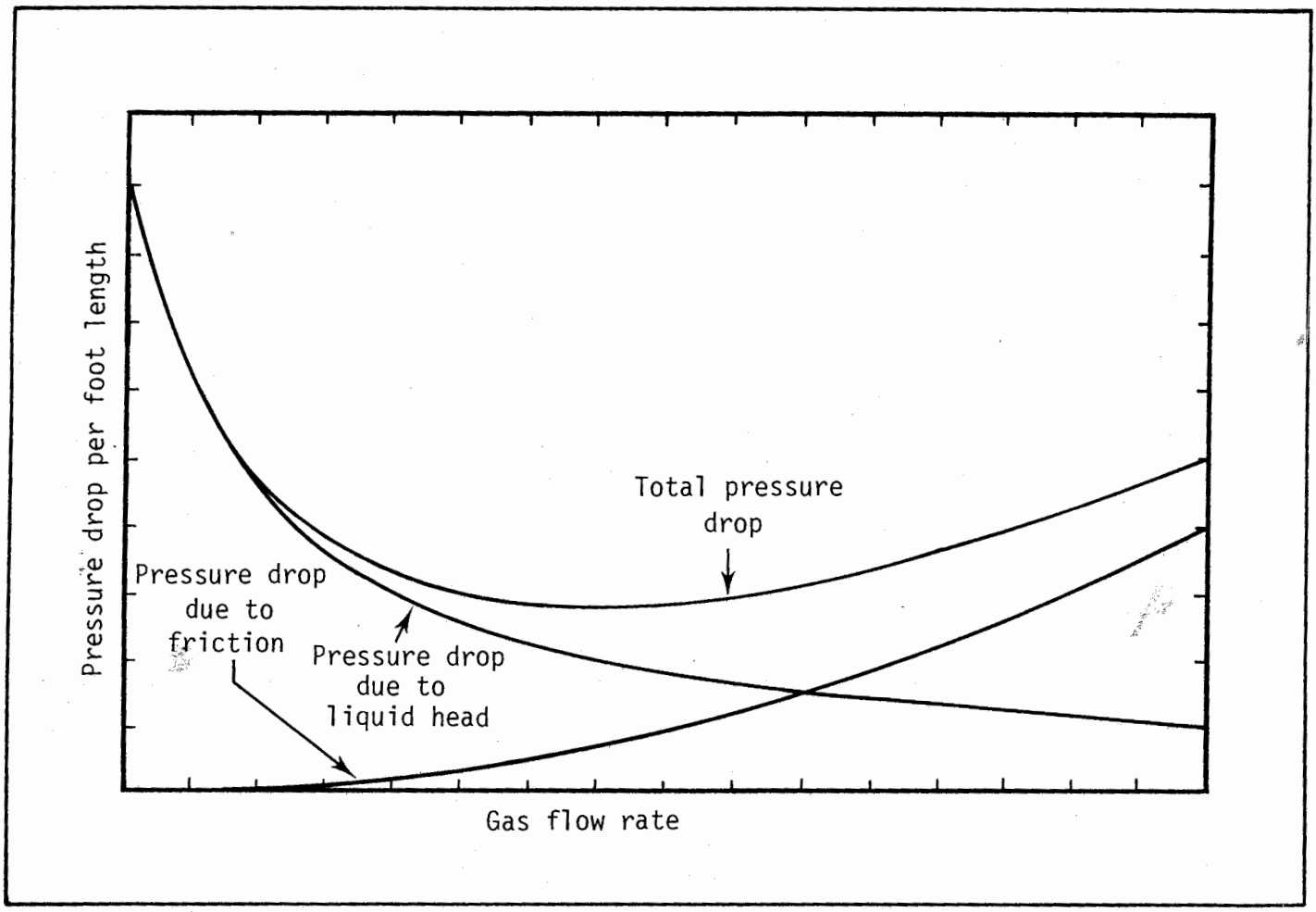


Fig. 2.159 Pressure-drop components in two-phase flow (after Flanigan).

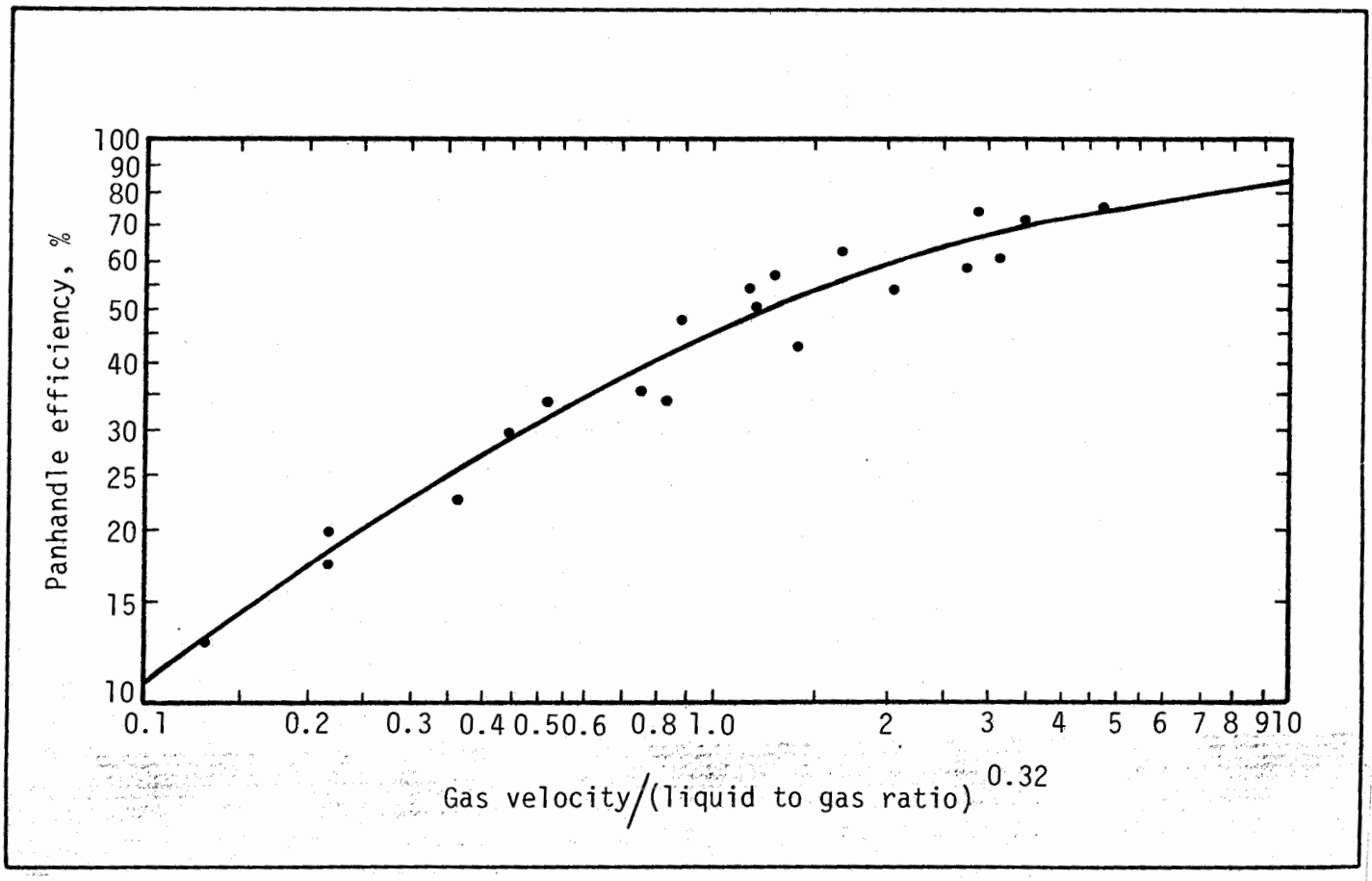


Fig. 2.160 Efficiency correlation (after Flanigan).

the way in which it affects the sum of the uphill rises.

5. The pressure drop in the upslope section varies inversely with the gas velocity.

In view of the above, he treated the uphill sections as though they affected the pressure drop in the same manner as would a vertical column containing an equivalent amount of liquid. Since a two-phase line is not completely filled with liquid, Flanigan used an H_L term in his equation to represent that fraction of the total static pressure drop which exists as the elevation component in two-phase flow. The following equation was used to calculate the pressure drop due to elevation:

$$\Delta p = \frac{\rho_L H_F \Sigma H}{144} \qquad (2.53)$$

where: Δp = pressure drop, psi
ρ_L = liquid density, lb_m/cu ft
H_F = elevation factor, dimensionless
ΣH = summation of the uphill rises in the direction of flow, ft

Using the data from the 16 in. line, as well as some other data available in the literature from tests carried out in vertical tubes, and making the corresponding corrections for the pressure drop due to friction, he covered a wider range of data and developed a correlation between H_F and the superficial gas velocity (Fig. 2.161). This figure reveals a considerable scattering of data which represents a wide variety of fluids over a wide range of flow conditions.

Later authors for inclined multiphase flow have used the Flanigan correlation for the pressure loss due to hills. Although other friction loss correlations have been developed, the Flanigan correlation is used today in conjunction with them. For example Baker, et al[7] have recommended using the Flanigan correlation and the holdup factor in conjunction with Duker's correlation and procedure for horizontal flow. It is the author's belief that there are friction loss correlations more accurate than those of Flanigan, and we recommend that the Flanigan correlation be used to determine pressure loss due to the inclines in conjunction with either Eaton or Dukler friction loss correlations.

The following detailed calculation procedure and example problems can be found in Appendix E:

(1) E.11—Flanigan's calculation procedure.
(2) E.12—Example problem for determining effect of inclines by method of Flanigan.
(3) E.13—Example problem for selecting the size of an offshore line to bring fluids to an onshore facility including hilly terrain by Flanigan.
(4) E.14—Example problem E.13 worked using Eaton method to determine friction loss and Flanigan method for hills.
(5) E.15—Example problem E.13 worked using Lockhart and Martinelli method for friction loss and Flanigan method for hills.
(6) E.16—Example problem to determine necessary upstream pressure for a given flow line size and downstream pressure including effect of hills.

2.522 Ovid Baker's correlation

Ovid Baker[8] also worked with the calculation of pressure traverses in lines which contain inclined sections. Baker suggests, as did Flanigan, a calculation of the two-phase pressure drop by summing the pressure drop due to friction in the entire length of the line and the pressure drop due to liquid head in the hills. Baker's procedure for the calculation of the pressure loss due to friction has been presented in Section 2.4. His formula to account for the hills and frictional loss is

$$\Delta p_{TPH} = \Delta p_{TP} \text{ (all line considered horizontal)} + \frac{N\ h\ H_F\ \rho_L}{144} \qquad (2.54)$$

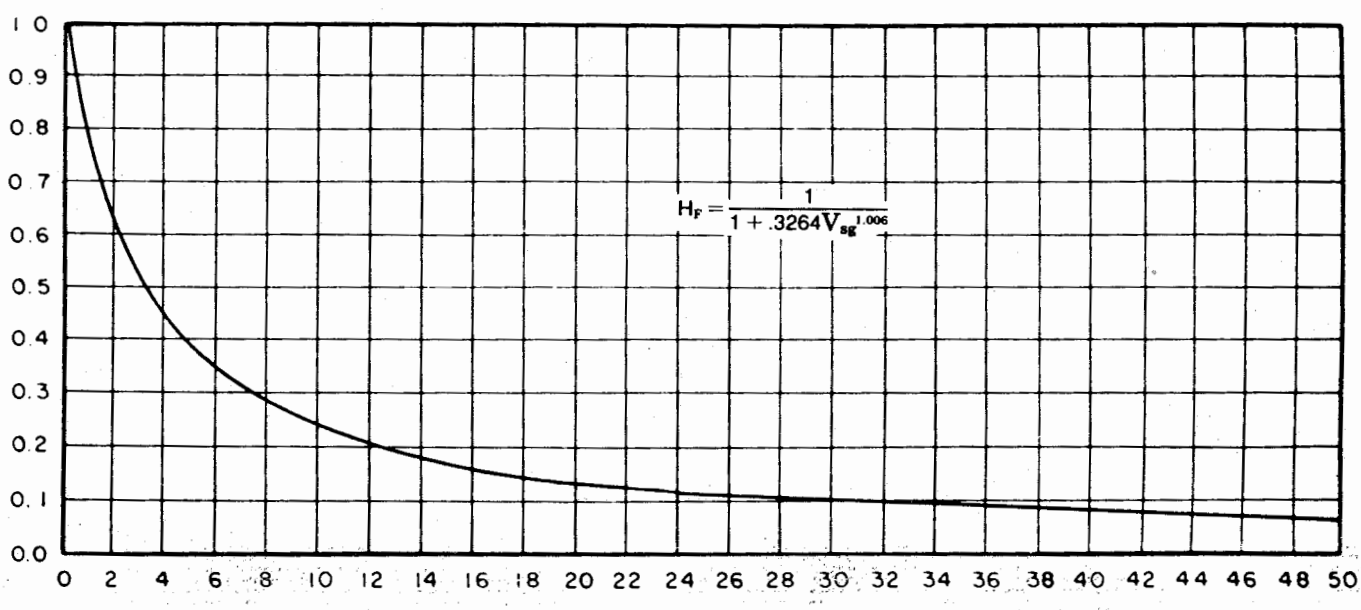


Fig. 2.161 Flanigan's correlation. 1.006 = exp of V_{sg}

where:

Δp_{TPH} = calculated two-phase pressure drop for pipelines in hilly terrain, psi
Δp_{TP} = calculated two-phase pressure drop, assuming total length of line to be horizontal, psi
N = number of hills
h = average height of hills, ft
H_F = elevation factor from Fig. 2.161 (Flanigan's correlation)
ρ_L = density of liquid flowing in line, lb_m/cu ft

Baker also noted that for $v_{sg} > 50$ the following formula applied:

$$H_F = \frac{0.00967\ (L)^{1/2}}{v_{sg}^{0.7}} \qquad (2.55)$$

Baker's main contribution was this formula which extended the work of Flanigan to take care of $v_{sg} > 50$ (Note Fig. 2.161). However the formula should not be used in lieu of the Flanigan correlation for elevation, except outside the range of Flanigan's correlation. His friction loss component, however, is more accurate than Flanigan's.

It should be noted that the correction factor for hills is the same used by Flanigan since:

$$Nh = \Sigma H$$

2.5221 Procedures and example problems

It should again be stressed that a diameter guess is improved by the formula:

$$\text{diameter guessed} = (\text{diameter assumed}) \left(\frac{\Delta p \text{ calculated}}{\Delta p \text{ actual}}\right)^{1/5}$$

This method of successive guessing should be used for inclined lines as well as horizontal lines as pointed out in Section 2.4.

A complete calculation procedure along with example problems can be found in Appendix E.2, including the following:

(1) E.21 – Baker's calculation procedure for inclined flow.
(2) E.22 – Example problem to determine line size by method of Baker including hilly terrain.

2.523 Beggs & Brill correlation

Beggs and Brill[1] conducted an experimental investigation of two-phase, gas-liquid flow in inclined pipes to determine the effect of inclination angle on liquid holdup and pressure loss.

The experimental study consisted of measuring liquid holdup and pressure drop in pipes of 1 in. and 1.5 in. ID. Fluids used were air and water and flow rates were varied from 0 to 4 cu ft/min liquid and from 0 to 300 Mscfd gas.

Data were taken at inclination angles of ± 90°, ± 85°, ± 75°, ± 55°, ± 35°, ± 20°, ± 15°, ± 10°, ± 5° and 0° from the horizontal.

Empirical correlations were developed for liquid holdup and friction factor as functions of flow properties and inclination angle. Liquid holdup is strongly influenced by inclination angle and the friction factor is influenced by liquid holdup. Different holdup correlations were developed for each of three flow regimes. The friction factor correlation was found to be independent of the flow regime, but requires a value for liquid holdup.

Beggs and Brill[1] noted the following conclusions:

1. The inclination angle of a pipe in which two-phase flow is occurring definitely affects liquid holdup and pressure drop for most flow conditions.
2. In inclined two-phase flow, the liquid holdup reaches a maximum at an angle of approximately +50° and a minimum at approximately −50° from horizontal. The fact that holdup is approximately equal at angles of +90° and +20° explains why vertical holdup correlations can be used with some degree of success for horizontal flow.
3. Pressure recovery in the downhill section of a two-phase pipeline in hilly terrain can definitely exist and should be considered in pipeline design.
4. The accuracy of a liquid holdup correlation for horizontal two-phase flow can be improved by consideration of flow pattern.

A detailed calculation procedure and example problem can be found in Appendix E.3.

2.53 Limited correlations

2.531 Introduction

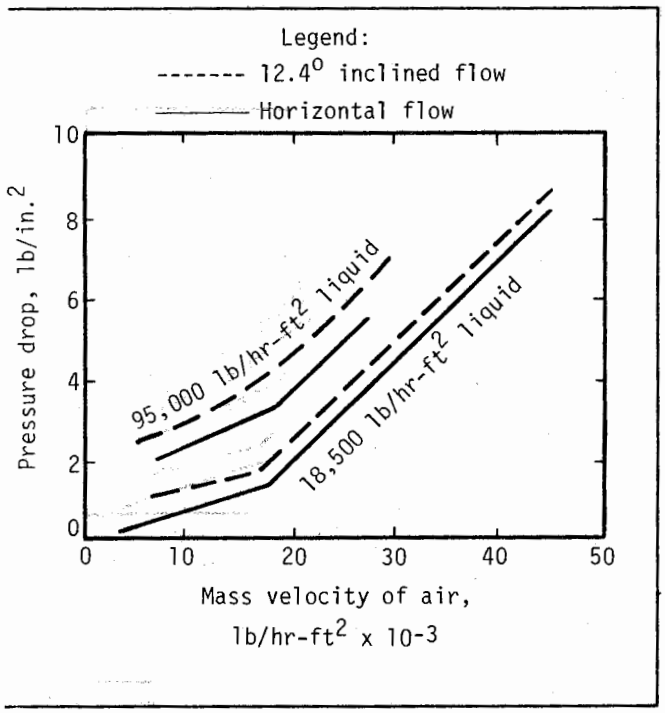
There are several authors who have contributed to the inclined flow problem in a limited manner. Many have done extensive experimental work which at the present time has not been placed in practical use. Others have merely observed flow patterns, etc. without offering any solution to the determination of line sizes or pressure losses. These publications are briefly discussed in the following sections, and are mentioned so that those making additional efforts to solve the inclined flow problem will be aware of these contributions.

2.532 Brigham, Holstein and Huntington's correlation

Brigham, et al[9] compared multiphase flow in horizontal pipes with that in inclined pipes with 12° and 4° slopes. Brigham showed that in the low velocity regions a 12.4° incline showed a much greater pressure drop than was noted for horizontal flow (Fig. 2.162). However, at the greater velocities where spray flow existed there was very little difference between the pressure drop for inclined and horizontal flow.

2.533 Rene Sevigny's correlation

Sevigny[10] conducted numerous laboratory tests of air and water mixtures flowing through a 0.8245 in. ID brass pipe which was inclined at angles of ± 90, ± 60, ± 30, ± 15, ± 10, ± 5 and 0° to the horizontal. Although his work has not contributed to the practical solution of the problem, it may eventually assist in the ultimate solution. A correlation curve was developed which includes the angle of inclination. The mathematical development of the correlation may be found in his work, "An Investigation of Isothermal, Cocurrent, Two-fluid, Two-phase Flow in an Inclined Tube."[10]



g. 2.162 "ΔP" comparisons for two-phase flow in horizontal and 2.4° inclined pipe flow (after Brighan Holstein and Huntington).

Sevigny started with an energy balance equation nd arrived at an equation to express the energy losses ue to friction. For correlation purposes, Sevigny as-ımed no slippage between phases. The object of the ɔrrelation effort was to evolve an equation or curve hich would express a friction factor in terms of mass ow rates, fluid properties, and system geometry.

Expressing the Reynolds numbers and friction factor ı terms of mass flow rates, fluid properties and system eometry, his final equation for f is:

$$\frac{\frac{w_g - w_L}{\mu_g - \mu_w}}{\frac{w_g}{\mu_g} + \frac{w_L}{\mu_w}} \text{EXP}\left[\frac{g_c\, d\, A^2}{2(q_g+q_L)^2 L}\left(\left\{\frac{q_g\, p_1 \ln\left(\frac{p_1}{p_2}\right) + q_L(p_1-p_2)}{w_g + w_L}\right\} - (h_2-h_1)\frac{g}{g_c}\right)\right]^{\sqrt{H_L}} \quad (2.56)^*$$

Sevigny used his data and the data of others to calulate a value for the friction loss factor. A correlation urve was developed for f as a function of superficial iquid holdup, H_L.

.5331 Sevigny's calculation procedure

For a problem in which it is desired to calculate a ownstream pressure, p_2, knowing the upstream pressure, p_1, the geometry of the line, the flow rates, and he viscosity of the gas, the procedure would be as fol-ɔws:

1. Calculate H_L.
2. Obtain the corresponding value of the ordinate r f, according to the curve of the figure in reference 10.
3. Assuming a value for downstream pressure, or $_2$, calculate the value of f from Eq. 2.56.

* This equation has been corrected from the original one of Ref. 10 y introducing the second open parenthesis and the minus sign be-ɔre $(h_2 - h_1)$.

4. Compare f value obtained from the curve and f value obtained from the equation.
5. If they are not sufficiently close, assume a new value of p_2 and repeat steps 3 and 4 until the two values of f can be considered equal.

No practical solution was found for this method and therefore no example problems are given.

2.534 Bonnecaze, Erskine, and Greskovich correlation

Bonnecaze, et al[11] using a momentum balance, derived an equation to calculate pressure drop in inclined pipes for slug flow only. Their equation is:

$$\frac{\Delta p}{\Delta L} = \alpha\left[\frac{g}{g_c}\rho_L \sin\theta + \frac{2\,\rho_L\, f\, v_m^2}{g_c\, d}\right] \quad (2.57)$$

where: $\frac{\Delta p}{\Delta L}$ = pressure gradient, lb_f/cu ft
θ = angle from horizontal
f = Fanning friction factor
d = pipe diameter, ft

$$\alpha = \frac{H_L - H_{LB}}{1 - H_{LB}}$$

H_L = in-situ liquid holdup
H_{LB} = liquid holdup in the gas bubble

In order to apply this equation, values for H_L and H_{LB} must be known. The equation developed for H_L was:

$$H_L = 1 - \frac{(1-\lambda)}{1.20 + 0.35\left(1 - \frac{\rho_g}{\rho_L}\right)\frac{\delta}{\sqrt{N_{FR}}}} \quad (2.58)$$

where: $\lambda = q_L/(q_L + q_g)$
q_L = liquid flow rate, cu ft/sec
q_g = gas flow rate, cu ft/sec
N_{FR} = Froude number = v_m^2/gd

$$\delta = \begin{Bmatrix} +1, \text{ uphill} \\ 0, \text{ horizontal} \\ -1, \text{ downhill} \end{Bmatrix}$$

An equation was developed for H_{LB} in terms of mixture velocity and bubble velocity. Bonnecaze, et al measured pressure drop in pipes at angles from horizontal of 0°, ± 2°, ± 6°, and ± 10°. Agreement of calculated and experimental data was poor, ranging from −1.9% at +10° to 220% at −2°. No procedure is presented for applying this correlation.

2.535 Singh and Griffith correlation

A recent study on slug flow in inclined pipes was performed by Singh and Griffith.[12] They developed a model in which the total pressure drop was the sum of the gravity and friction pressure drops. The gravity pressure drop was expressed as

$$\left(\frac{\Delta p}{\Delta L}\right)_g = \rho_m \frac{g}{g_c}\sin\theta \quad (2.59)$$

where: $\left(\frac{\Delta p}{\Delta L}\right)_g$ = pressure gradient, lb_f/cu ft
θ = angle from horizontal

The friction pressure drop is:

$$\left(\frac{\Delta p}{\Delta L}\right)_f = \frac{f \rho_L v_m^2}{2 g_c d} \qquad (2.510)$$

where: $\left(\frac{\Delta p}{\Delta L}\right)_f$ = pressure gradient, lb_f/sq ft

f = Fanning friction factor.

Liquid holdup was related to bubble rise velocity, v_b, as:

$$H_L = 1 - \frac{q_g}{v_b A_p} \qquad (2.511)$$

and bubble rise velocity was empirically related to mixture velocity by:

$$v_b = 0.95\ v_m + 1.15 \qquad (2.512)$$

As is evident from the above equation, v_b and thus H_L was found to be independent of inclination angle.

2.54 Conclusions and recommendations

Either the Beggs and Brill or Flanigan correlation should be used for inclined flow. The Flanigan procedure offers a safe solution because it assumes no recovery on the downhill side of an incline. However, the Beggs and Brill correlation still needs additional field verification and perhaps some adjustment for field data. It also gives safe results.

2.55 Practical application

The effect of variables is essentially the same as for vertical and horizontal flow and is not given here.

2.551 Example problems

The hilly terrain problems will be worked by the method of Flanigan for the elevation component and the method of Eaton for all other losses. The working curves of Appendix D will be used. These problems can also be worked by the Beggs and Brill procedure.

EXAMPLE PROBLEM #1: HOW TO DETERMINE THE EFFECT OF HILLS IN HORIZONTAL FLOW

The effect of hills only will be considered in this first problem.

Given data: A flow line passes over one 500 ft hill.

d = 2 in.
L = 5000 ft
q_L = 2,000 bpd (100% water)
G/L = 1,000 scf/bbl (free gas)
γ_g = 0.65
γ_w = 1.10
$\bar{T}$ = 120°F

Average pressure = 725 psia

Solution procedure:

1. Find the superficial gas velocity. In order to use the correlation of Flanigan, it is necessary to obtain the average gas velocity of the pipe on the hill. More accuracy would be obtained if the hill were taken in increments because the gas velocity changes. Also, when more than one hill is involved the gas velocity for each hill may be necessary depending upon the total pressure drop in the system. However, in Flanigan's original work, he used the average velocity of the entire system and obtains one factor for all the hills. If the total pressure drop is low, this offers no problem.

The superficial gas velocity assumes the gas to be flowing alone in the pipe and is calculated as follows:

$$v_{sg} = \frac{q_g}{A_p}$$

where: q_g = flow rate, cu ft/sec

A_p = area pipe, ft^2

$$q_g = \frac{(q_L \text{ in bpd})\ (G/L \text{ in scf/bbl})}{86{,}400 \text{ sec/day}}$$

$$= \frac{(1{,}000)\ (2{,}000)}{86{,}400} = 23.2 \text{ scf/sec}$$

We now need to convert scf/sec to cu ft/sec under average pipe line conditions.

Average Pressure = $\bar{p}$ = 725 psia (Given)
$\bar{T}$ = 120°F = 580°R
$\bar{z}$ = 0.905

Then q_g at 725 psia and 120°F = $(23.2)\left(\frac{14.7}{725}\right)\left(\frac{580}{520}\right)\left(\frac{0.905}{1}\right) = 0.474\ ft^3/sec$

$$v_{sg} = q/A = 0.474/0.0218 = 21.7 \text{ ft/sec}$$

This can be simplified by the following equation:

$$v_{sg} = 31{,}194 \frac{qz}{d^2\ \bar{p}} \frac{\bar{T}}{520} \qquad (2.513)$$

where: q = MMcfd
d = in.
$\bar{p}$ = psia
$\bar{T}$ = °R

2. Obtain elevation factor from Fig. 2.161. $H_F = 0.13$
3. Calculate effect of hills by formula:

$$\Delta p_{hills} = \frac{\rho_L H_F \Sigma H}{144}$$

$$\rho_L = (62.4)(1.10) = 68.6 \text{ lb/ft}^3$$

$$\Sigma H = 500 \text{ ft}$$

$$\Delta p_{hills} = \frac{(68.6)\ (0.13)\ (500)}{144} = 31 \text{ psi}$$

CLASS PROBLEM #1-A: TO DETERMINE THE PRESSURE LOSS BECAUSE OF HILLS

Given data: A flow line passes over a hill having a vertical rise of 700 ft.

3 in. flow line
5,000 ft long, total length
q_L = 3,000 bpd (95% water)
G/L = 600 scf/bbl (free gas)
γ_g = 0.70
γ_w = 1.07
γ_o = 0.82
Average press = 500 psia
Average temp = 120°F

Find the pressure loss due to the hill.

CLASS PROBLEM: TO DETERMINE THE PRESSURE LOSS BECAUSE OF HILLS

Given data: A flow line passes over 6 hills with the following vertical heights:

Hill no.	Vertical height, ft
1	120
2	80
3	220
4	40
5	70
6	180

4 in. flow line
2,000 ft long, total length
$q_L = 6{,}000$ bpd (95% water)
G/L = 200 scf/bbl (free gas)
$\gamma_g = 0.70$
$\gamma_w = 1.07$
$\gamma_o = 42°$ API
Average pressure in line = 300 psi
Average temperature = 110°F

Find the pressure loss because of the hills.
Work also for G/L = 1,500 scf/bbl.

EXAMPLE PROBLEM #2: HOW TO DETERMINE THE TOTAL PRESSURE LOSS IN A LINE INCLUDING FRICTION & HILLY TERRAIN

In this problem, the upstream pressure and the vertical height of the various hills in the line are known.

Additional data:

L, total = 10,000 ft
p, upstream = 850 psi = p_1
$q_L = 1{,}500$ bpd (all water)
G/L = 1,000 scf/bbl
Temperature average = 140°F
$\gamma_g = 0.65$
$\gamma_w = 1.07$
$2\frac{1}{2}$ in. line

We have 11 hills with vertical heights of 100, 50, 100, 50, 100, 200, 50, 100, 50, 100, 100 ft respectively.

Find the downstream pressure, p_2.

Solution procedure:

In order to obtain an average velocity of the gas, we need the average pressure and therefore, p_2. We then have a trial and error solution.

In order to assume p_2 we need to know the $\Delta p_{friction} + \Delta p_{hills}$. We can obtain $\Delta p_{friction}$ from the horizontal flow correlation (Fig. D.109) and then assume Δp_{hills}.

1. $\Delta p_{friction} = 850 - 520 = 330$ psi
2. Assume $\Delta p_{hills} = 200$ psi
3. $\Delta p_{total} = 330 + 200 = 530$ psi
4. Assumed value of $p_2 = 850 - 530 = 320$ psi
5. Calculate v_{sg}

$$\text{where } v_{sg} = \frac{31{,}194\,(q\,\bar{Z})}{d^2\,\bar{p}}\,\frac{\bar{T}}{520}$$

q = MM scfd
$\bar{p}$ = psi
d = in.

$$\bar{p} = \frac{850 + 320}{2} = 585 \text{ psi};\ \bar{z} = 0.92$$

$$q = \frac{(1{,}500)\,(1{,}000)}{1{,}000{,}000} = 1.5 \text{ MM scfd}$$

$$v_{sg} = \frac{(31{,}194)(1.5)(0.92)}{(2.5)^2\,(585)}\left(\frac{600}{520}\right) = 13.5 \text{ ft/sec}$$

6. Find elevation factor from Fig. 2.161.
   $H_F = 0.19$
7. Calculate $\Delta p_{hills} = \frac{\rho_L H_F \Sigma H}{144}$ — Where $\Sigma H = 1{,}000$

$$= \frac{(62.4)(1.07)(0.19)(1{,}000)}{144} = 88 \text{ psi}$$

8. Check Δp calculated against Δp assumed. We assumed 200 psi, but calculated 88 psi. Assume a new $\Delta p_{hills} = \Delta p$ calculated = 90 psi
9. Repeat the procedure for the new value of $\Delta p_{hills} = 90$ psi
   The new calculated value of $\Delta p_{hills} \cong 90$ psi
10. The total pressure loss = 330 + 90 = 420 psi
11. $p_2 = 850 - 420 = 430$ psi

As a further suggestion the first value of ΔP_{hills} could be taken as the value determined by assuming that p_2 = value from friction loss only. This calculates a value of $\Delta p_{hills} = 112$ psi which is much closer than the assumed value of 200 psi.

CLASS PROBLEM #2-A: TO DETERMINE THE DOWNSTREAM PRESSURE

Given data: L = 4,000 ft
d = 3 in.
q = 2,000 bpd (100% water); $\gamma_w = 1.10$
G/L = 800 scf/bbl (Work also for G/L = 400 scf/bbl)
Temp avg = 100°F
$\gamma_g = 0.70$
Pressure upstream = 600 psi

We have 6 hills with vertical rises of 180, 140, 220, 150, 70, and 100 ft respectively.

Find the downstream pressure, p_2.

CLASS PROBLEM #2-B: TO DETERMINE THE DOWNSTREAM PRESSURE

Given data: L = 6,000 ft
Pressure upstream = 1,000 psi
$q_L = 800$ bpd (100% water); $\gamma_w = 1.08$
G/L = 500 scf/bbl
Temp avg = 110°F
$\gamma_g = 0.65$
2 in. line

We have one hill of vertical height = 550 ft.
Find the downstream pressure, p_2.

EXAMPLE PROBLEM #3: HOW TO DETERMINE THE UPSTREAM PRESSURE INCLUDING HILLS FOR ALL OIL FLOW

This is the same procedure as the previous example except that we are flowing all oil and the gas liquid ratio must be that of free gas only.

Given data: L = 12,000 ft

We have 9 hills having vertical heights of 30, 80, 120, 250, 300, 140, 280, 180, and 350 ft respectively.

Line size = 4 in.
q = 4,000 bpd (all oil, 42° API)
Temp avg = 140°F
$\gamma_g = 0.65$
GOR = 1,600 scf/bbl
Pressure downstream = 300 psi

Find the upstream pressure p_1.

Solution:

1. Find Δp due to friction only assuming all water initially = 800 − 300 = 500 psi (Fig. D.1021)
2. Assume $\Delta p_{hills} = 100$ psi
3. $\Delta p_{total} = 500 + 100 = 600$ psi
4. Determine p upstream = 600 + 300 = 900 psi
5. Calculate v_{sg} with p_1 assumed = 900

$$p = \frac{900 + 300}{2} = 600 \text{ psia}$$

$$v_{sg} = \frac{31{,}194\ q\ \bar{Z}}{d^2 \bar{p}} \left(\frac{T}{520}\right)$$

$$\text{where: } q = \frac{(4{,}000)(G/O)}{1{,}000{,}000}$$

where: G/O must be the free gas at $\bar{P}$ and $\bar{T}$.
Refer to Fig. 2.19 and $R_s = 145$ scf/bbl
Free GOR = 1,600 − 145 = 1,455 scf/bbl

$$q = \frac{(4{,}000)(1{,}455)}{1{,}000{,}000} = 5.82 \text{ MMscfd}$$

$$v_{sg} = \frac{(31{,}194)(5.82)(0.93)}{(4)^2 \quad (600)} \times \left(\frac{600}{520}\right) = 20.4 \text{ ft/sec}$$

6. From Fig. 2.161 find $H_F = 0.13$.
7. Total vertical height = 1,730

$$\Delta p_{hills} = \frac{\rho_L H_F \Sigma H}{144}$$

$$\rho_L = 50.9 \text{ lb/ft}^3 = (0.816)\ (62.4)$$

$$\Delta p_{hills} = \frac{(50.9)(0.13)(1{,}730)}{144} = 79.5 \text{ psi}$$

8. Check the calculated value of 79.5 psi against the assumed value of 100 psi and we see that a new value must be assumed. Assume 75 psi and repeat the procedure:
   (1) $\Delta p_{friction} = 765 - 300 = 465$
   (2) Assume $\Delta p_{hills} = 75$ psi
   (3) Total $\Delta p = 465 + 75 = 540$
   (4) $p_1 = 540 + 300 = 840$ psi
   (5) $v_{sg} = \frac{31{,}194\ q\bar{Z}}{d^2\bar{p}} \left(\frac{T}{520}\right)$ where $\bar{p} = \frac{840 + 300}{2}$
   $= 570$ psia
   From Fig. 2.19, $R_s = 140$ scf/bbl
   Free gas = 1,600 − 140 = 1460 scf/bbl

$$q = \frac{(4{,}000)(1{,}460)}{1{,}000{,}000} = 5.85 \text{ MMscfd}$$

$$v_{sg} = \frac{(31{,}194)(5.85)(0.930)}{(4)^2 \quad 570} \left(\frac{600}{520}\right) = 21.6$$

(6) From Fig. 2.161 find $H_F = 0.12$

(7) $\Delta p_{hills} = \frac{(50.9)(0.12)(1{,}730)}{144} = 73.5$ psi

(8) This checks close enough to assume a value of 75 psi. $\Delta p_{hills} = 75$ psi
(9) $\Delta p_{total} = 465 + 75 = 540$ psi
(10) p_1 = upstream = 540 + 300 = 840 psi

CLASS PROBLEM #3-A: TO DETERMINE THE UPSTREAM PRESSURE FOR ALL OIL FLOW

Given data: $\Sigma H = 3{,}000$ ft of hills
L = 10,000 ft
Line size = 3 in.
$q_o = 3{,}000$ bpd (all oil) $\gamma_o = 35°$ API
Temp avg. = 140°F
Total GOR = 800 scf/bbl
$\gamma_g = 0.65$
Pressure downstream = 200 psi

Find the upstream pressure (p_1).

CLASS PROBLEM #3-B: TO DETERMINE THE UPSTREAM PRESSURE FOR ALL OIL FLOW

Given data: L = 5000 ft
Total vertical height of hills = 1500 ft
Line size = 2 in.
$q_o = 600$ bpd (all oil) $\gamma_o = 40°$ API
Temp avg. = 110°F
Total GOR = 600 scf/bbl
$\gamma_g = 0.70$
Pressure downstream = 100 psi

Find the upstream pressure (p_1).

EXAMPLE PROBLEM #4: HOW TO SELECT LINE SIZE FOR FLOW LINE PASSING OVER HILLY TERRAIN

This represents a trial and error procedure since the flow rate, upstream and downstream pressures, gas-liquid ratio and fluid properties are given.

A good first assumption is to find the line size necessary to take care of $\Delta p_{friction}$ and then assume the next largest size in order to take care of the hills.

Given data: L = 8,000 ft $\Sigma H = 2{,}000$ ft
$p_1 = 600$ psi
$p_2 = 200$ psi
$q_w = 1{,}500$ bpd (all water) $\gamma_w = 1.10$
G/L = 1,000 scf/bbl $\bar{T} = 120°$F
$\gamma_g = 0.65$

Find line size.

Solution procedure:

1. Determine the line size necessary to take care of friction only
2. Assume a line size and determine p_1 (upstream); check this against the actual value of 600 psi. Prepare a table of line size vs. p_1 required (Refer to horizontal working curves in Appendix D.10.)

Line size, in.	p_1 required, psi
2	1,100
2½	630
3	405
3½	310
4	260

We have available 600 psi at the well head and note that a 2½ in. line requires 630 psi and that a 3 in. line requires 405 psi. There should be less than 200 psi to overcome the hills; therefore, assume a 3 in. line and check the total pressure loss.

3. $\Delta p_{total} = 600 - 200 = 400$ psi
4. $\Delta p_{friction} = 405 - 200 = 205$ psi
5. $\Delta p_{hills} = 600 - 405 = 195$ psi
6. Calculate v_{sg}:

$$p = \frac{600 + 200}{2} = 400 \text{ psi}$$

$$v_{sg} = \frac{31194\, q\, \overline{Z}}{d^2 \overline{p}} \left(\frac{T}{520}\right)$$

$$q = \frac{(1{,}000)(1{,}500)}{1{,}000{,}000} = 1.5 \text{ MMscfd}$$

$$v_{sg} = \frac{(31{,}194)(1.5)(0.95)}{(3)^2\,(400)} = 12.3 \text{ ft/sec}$$

7. From Fig. 2.161 find $H_F = 0.20$.
8. $\Sigma H = 2000$ ft

$$\Delta p_{hills} = \frac{\rho_L H_F \Sigma H}{144} = \frac{(62.4)(1.10)(0.20)(2000)}{144}$$

$$= 190 \text{ psi}$$

9. Total $\Delta p = 205 + 190 = 395$ psi
10. Upstream required = 200 + 395 = 595 psi
    Upstream available = 600 psi
    Therefore, the 3 in. line is satisfactory.

CLASS PROBLEM #4-A: TO SELECT LINE SIZE FOR FLOW LINE PASSING OVER HILLY TERRAIN

Given data: L = 10,000 ft
p upstream = 200 psi
p downstream = 80 psi
q_w = 1000 bpd (water) $\gamma_w = 1.07$
G/L = 500 scf/bbl
$\overline{T}$ = 100°F
$\gamma_g = 0.70$

Find line size for: (a) $\Sigma H = 700$ ft
(b) $\Sigma H = 2000$ ft
(c) $\Sigma H = 4000$ ft

CLASS PROBLEM #4-B: TO SELECT LINE SIZE FOR FLOW LINE PASSING OVER HILLY TERRAIN

Given data: L = 4,000 ft
ΣHills = 500 ft
p upstream = 300 psi
p downstream = 100 psi
q_w = 3,000 bpd (water) $\gamma_w = 1.10$
G/L = 800 scf/bbl
Temp avg. = 120°F
$\gamma_g = 0.65$

Find necessary line size.

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2.6 DIRECTIONAL WELL MULTIPHASE FLOW

2.61 Introduction

The number of wells being drilled directionally are increasing and will continue to increase in the future. (Fig. 2.163). The reasons for this are: (1) wells will be drilled from offshore platforms, (2) more wells are being drilled from man-made islands such as offshore California, (3) Arctic drilling utilizes one location for numerous directional wells and (4) wells must be drilled under metropolitan areas. Some of these wells are highly deviated as noted in Fig. 2.164.

In the past we have used vertical multiphase flow correlations for directionally drilled wells by accounting only for the true vertical depth of the well. This has proved to be successful in some cases and in particular seems to be fairly good for deviations not exceeding 15–20°. However, for those wells exceeding 20° from the vertical, the standard vertical multiphase flow correlation will give erroneous results. Two factors change in a directional well as compared to a vertical well. The directional well has a greater length of tubing to reach the same depth which means that the frictional loss is greater. Although the vertical distance is the same the holdup differs considerably and, in fact, can be greater for certain angles of deviation than for the vertical.

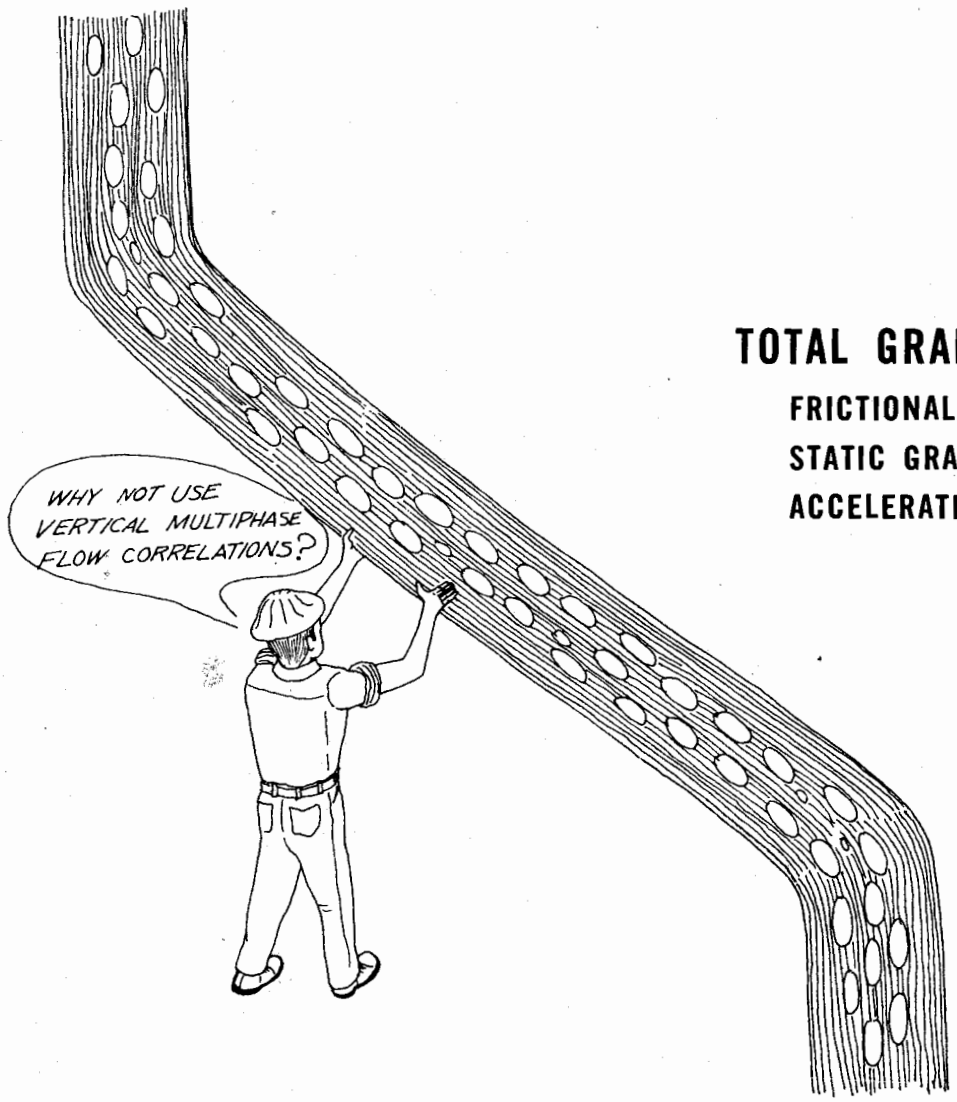

Fig. 2.163 Directional multiphase flow.

The same general equation can be used for directional multiphase flow:

$$\frac{dp}{dz} = \frac{g}{g_c}\rho_m \cos\phi + \frac{f_m \rho_m v_m^2}{2g_c d} + \frac{\rho_m v_m dv_m}{g_c dz} \quad (2.184)$$

($\sin\theta$ may be used if angle is taken from the horizontal) where ϕ = angle from vertical.

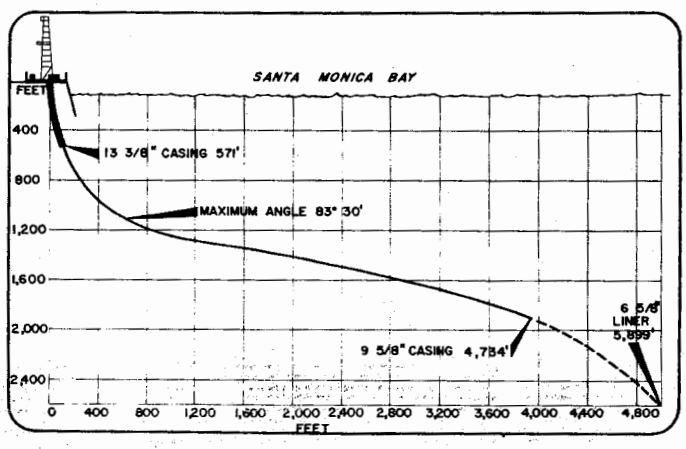

Fig. 2.164 83° slanted hole at Redondo Beach.

Beggs and Brill offered the following equation:

$$\frac{dp}{dz} = \frac{\frac{g}{g_c}\rho_{TP}\sin\theta + \frac{f_{TP}G_m v_m}{2g_c d}}{1 - \frac{\rho_{TP} v_m v_{sg}}{g_c p}} \quad \text{where } \theta \text{ is angle from horizontal} \quad (2.51)$$

where: $\rho_{TP} = \rho_L H_L + \rho_g(1 - H_L)$

2.62 Directional multiphase flow correlations

2.621 Introduction

The most recent work in the area of directional multiphase flow was performed by Beggs and Brill[1] at the University of Tulsa. However, their work has not yet been field tested. Practical solutions have been presented by Ney[2], Fuentes[3], and Cardozo[4].

The solutions of Ney and Fuentes, Cardozo, and Beggs and Brill will be presented here.

2.622 General solution to the problem

The directional well problem may be solved by starting with the general flow equation:

$$\frac{dp}{dz} = \frac{g}{g_c}\rho_m \sin\theta + \frac{f\rho v^2}{2g_c d} + \frac{\rho v dv}{g_c dz} \qquad (2.184)$$

or:

$$\frac{dp}{dz_{total}} = \frac{dp}{dz_{static}} + \frac{dp}{dz_{friction}} + \frac{dp}{dz_{acceleration}}$$

This equation permits the friction term and acceleration term to include the directional distance (measured depth) and the static term is corrected to the vertical distance only by the sine of the angle. Most standard vertical multiphase flow correlations such as Hagedorn and Brown, Duns and Ros, and Orkizewski can be adapted for directional multiphase flow merely by the addition of sin θ to the static gradient term. This permits friction and acceleration to be taken over the total tubing length and density to be taken over the vertical length only. As a matter of fact this appears to be an excellent solution to the problem.

For example the original Hagedorn and Brown equation for vertical flow is as follows:

$$144\frac{\Delta p}{\Delta h} = \bar{\rho}_m + \frac{f\,w^2}{2.9652\times 10^{11}\,d^5\bar{\rho}_m} + \bar{\rho}_m\frac{\Delta\left(\frac{v_m^2}{2g_c}\right)}{\Delta h} \qquad (2.35)$$

For directional flow over a measured distance of Z the equation can be modified as follows:

$$144\frac{\Delta p}{\Delta z} = \bar{\rho}_m \sin\theta + \frac{f\,w^2}{2.9652\times 10^{11}\,d^5\bar{\rho}_m} + \bar{\rho}_m\frac{\Delta\left(\frac{v_m^2}{2g_c}\right)}{\Delta z} \qquad (2.61)$$

The equations of Ros and Orkizewski can be modified in the same manner and offer a reasonable solution to the problem.

Inherent errors in this type of solution is that a hold-up correlation for vertical flow is used. Beggs and Brill showed that the holdup can change appreciably depending on the angle, and that at some angles the holdup will exceed that for vertical flow.

2.623 Beggs and Brill correlation

This correlation has been described previously and is found in Section 2.335. It is good for any angle of flow, and with slight modifications will undoubtedly result in one of the best solutions to the problem.

A procedure and example problem for directional flow can be found in Appendix F.2.

2.624 Solutions of Ney and Fuentes

Ney[2] presented two solutions and his work was extended by Fuentes[3].

Ney used the correlations of Hagedorn and Brown for friction factors and holdups and from the general energy equation and the angle of deviation, he arrived at equation 2.62:

$$144\frac{\Delta p}{\Delta L} = \frac{g}{g_c}\rho_m \cos\phi + f\frac{v_m^2\rho_m}{2g_c d} + \frac{\rho_m}{2g_c}\frac{\Delta(v_m^2)}{\Delta L} \qquad (2.62)$$

For horizontal flow where $\phi = 90°$, equation 2.62 involves only the friction and acceleration terms, and thus agrees with horizontal flow theory.

We note in Fig. 2.165 that $\Delta h = \Delta L \cos\phi$, where ΔL is the length of the pipe and ϕ is the angle which the pipe makes with the vertical.

Reference should be made to Appendix E.31 for an example problem illustrating this procedure. It is worked for angles of 0, 30 and 45 degrees.

2.625 Solution combining a vertical and horizontal multiphase flow correlation

Ney and Fuentes offered another solution which combines the use of a vertical flow correlation and a horizontal flow correlation. In general this solution is as follows: The pressure loss is first calculated taking into account only the vertical depth of the well. For example if the vertical depth is 8,000 ft and the deviated tubing length is 10,000 ft, then the pressure loss is determined from a vertical flow correlation for 8,000 ft. However, we have another 2,000 ft of tubing (10,000 − 8,000) in which frictional loss occurs. Therefore, a horizontal flow correlation is used to obtain the frictional loss for 2,000 ft of tubing and this additional loss is added to the loss determined for the 8,000 ft. The sum of these two losses then represent the total pressure loss for the deviated well. Ney and Fuentes checked several field tests by both these methods and they both gave fairly good results.

2.6251 Solution procedure

The following procedure is offered to more clearly explain this solution.

1. Starting with the surface flowing pressure, calculate that portion of the pressure drop necessary to lift the fluids the vertical height of the well. For example, if the true vertical depth was 1,000 ft and the actual length of tubing was 1,500 ft, then a pressure drop is calculated for the static gradient for 1,000 ft, using the holdup correlation of Hagedorn and Brown for the vertical depth.
2. The frictional portion of the pressure loss is then calculated for the entire length of the tubing string (1,500 ft).
3. The acceleration portion of the pressure loss is then calculated for the entire length of tubing (1,500 ft). This term can be neglected for most cases.
4. The three pressure components are then summed to

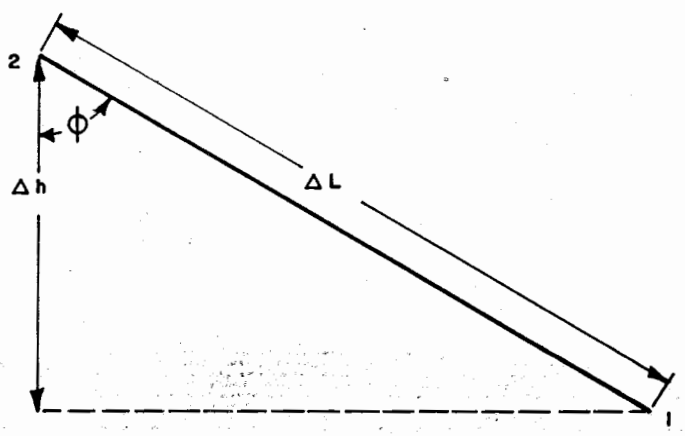

Fig. 2.165 Directional well geometry.

obtain the total loss for the true length of tubing (1,500 ft).

An additional problem arises in establishing a pressure traverse in that we generally start with a known tubing pressure at the top, assume a pressure increment, and calculate the corresponding depth. This works very well for a vertical well, but a different procedure is necessary for a directional well. For computer applications, the following procedure is suggested.

1. Start with a known surface pressure (for example, 300 psi).
2. Assume a pressure increment (say 100 psi).
3. Calculate the vertical depth to 400 psi from a reliable vertical flow correlation (Assume to be 1,000 ft.)
4. Determine the equivalent true length of tubing at this depth. (Assume = 1,500 ft at 1,000 ft vertical depth.)
5. Determine the frictional loss in the extra tubing length, that is, 1,500 − 1,000 = 500 ft. (For example, assume 10 psi.)
6. Add this to the pressure increment (100 + 10 = 110 psi total loss in 1,500 ft).
7. Now repeat the procedure, starting at 1,000 ft with 300 + 110 = 410 psi instead of the normal 400 psi that would be used for a vertical well.
8. Continue with the next pressure increment of 100 psi.

2.6252 Problem examples and procedures for the directional well

Reference should be made to Appendix E for a problem example, along with a detailed calculation procedure. The same example problem as worked in Sec. 2.624 is also worked by this method.

For the case of a well with an inclination angle of 45°, the results using both methods are the same. This can be noted by looking at the calculated pressure gradients. The solution of Ney and Fuentes as described in Sec. 2.624 gives:

$$\frac{\Delta p}{\Delta L} = \frac{500 \text{ psi}}{2{,}947 \text{ ft}} = 0.169 \text{ psi/ft.}$$

The combined method of Ney and Fuentes as described in Sec. 2.625 also gives:

$$\frac{\Delta p_t}{\Delta L} = \frac{515.5 \text{ psi}}{3{,}039 \text{ ft}} = 0.169 \text{ psi/ft.}$$

2.626 Correlation of Cardozo

In 1971 Cardozo[4] developed a new correlation based on flowing pressure data from 26 field wells. He showed that the Hagedorn and Brown correlation for holdup gave too high a value in directional wells. Therefore, he extended the Hagedorn and Brown holdup correlation to take care of directional wells.

In order to solve for the true H_L value for the directional wells the following procedure was used. A value of H_L was assumed and the friction factor f_m was determined from the following equation:

$$f_m = \frac{(2.9652)(10^{11})(d^5)}{q_L^2 M^2} \left\{144 \frac{\Delta p}{\Delta L} \bar{\rho}_m - \bar{\rho}_m^2 \left[\cos\phi + \frac{\Delta(v_m^2)}{2g_c L}\right]\right. \quad (2.63)$$

A value of friction factor f was taken from the normal friction factor diagram (Fig. 2.32) using the Reynolds number described by Hagedorn and Brown. These two friction factors were then plotted for various values of H_L as shown in Fig. 2.166 until they intersected. The intersection point represented the correct friction factor and hence the correct holdup. These values of H_L were plotted on Fig. 2.167 and compared to the original Hagedorn H_L curve. He noted that the computed values of holdup for wells with deviation angle greater than 30° fall below the original curve developed by Hagedorn and Brown for vertical wells.

Based on this observation, Cardozo found a factor to adjust the abscissa of the Hagedorn and Brown holdup correlation. This factor was $[0.223^{(1-\cos\phi)}]$, where ϕ is the deviation angle from the vertical and 0.223 is a constant obtained from field data. This was then introduced into the holdup correlation of Hagedorn and Brown as follows:

$$0.00326 \frac{B_o\, q^{0.425}\, P^{0.675}\, \sigma^{0.394}\, CN_L}{d^{1.850}\, (TZ)^{0.575}\, \rho_L^{0.394}\, GLR - R_s)^{0.575}} [0.223^{(1-\cos\phi)}] \quad (2.64)$$

Cardozo suggested that this function could be used for deviation angles up to 50°. He tested his correlation using Eq. 2.137 of the Hagedorn and Brown method and field data from 26 wells and found it to give good results.

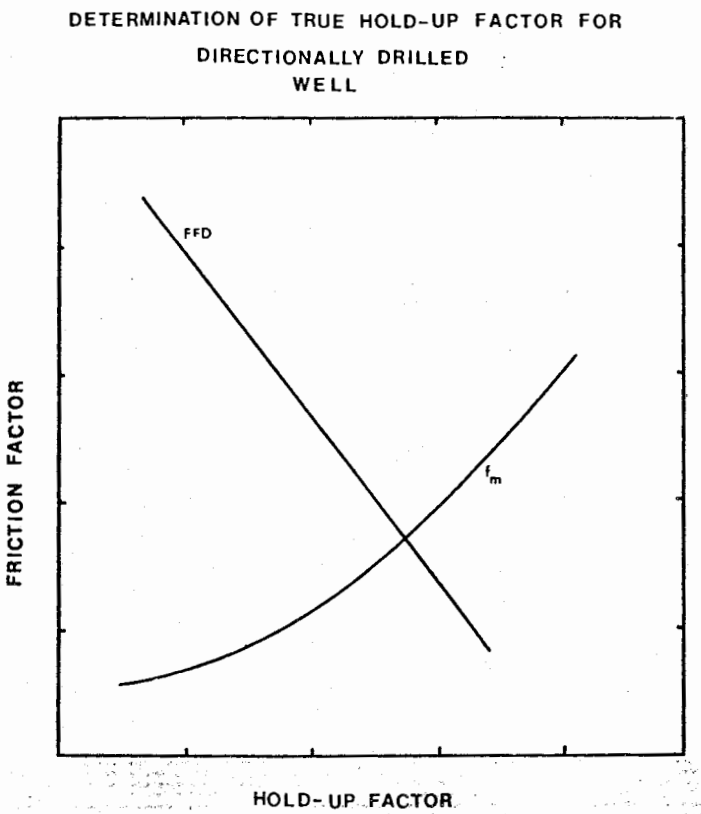

Fig. 2.166 Determination of true hold-up factor for directionally drilled well (after Cardozo).

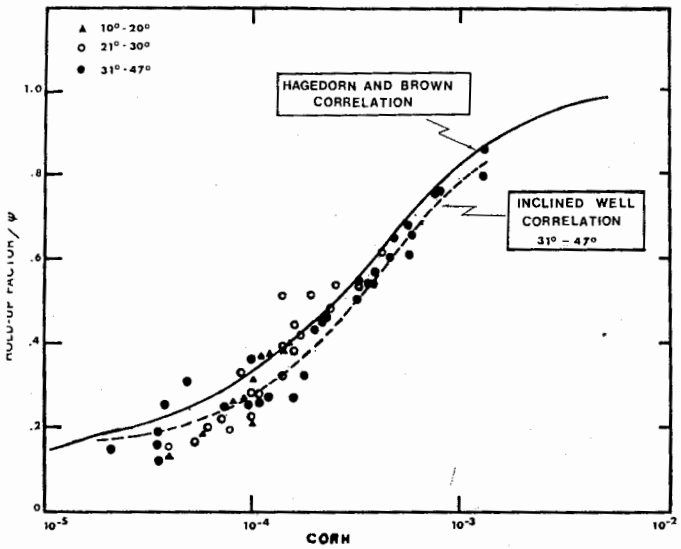

Fig. 2.167 Cardozo's holdup correlation (after Cordozo).

The procedure for the calculation of a pressure traverse would be identical to that of the generalized Hagedorn and Brown procedure. The only difference would be that a holdup value be selected from the adjusted correlation and the true length of the tubing must be used. Therefore, no example problem is given for this method in the Appendix.

2.627 Summary and recommendations

It is recommended that the Beggs and Brill correlation be used where enough field data has been obtained to assure its accuracy.

If field data is not available it is recommended that one of the general correlations be used but modified by multiplying the sine of the angle from the horizontal by the static gradient term. This permits friction and acceleration to be taken over the entire length of the tubing whereas the static gradient is taken over the vertical distance only.

2.628 Practical application of directional well multiphase flow correlations

The solution procedure of Beggs and Brill is given in Appendix F.2. The problems worked here will make use of the vertical and horizontal working curves to illustrate one procedure.

EXAMPLE PROBLEM #1: HOW TO DETERMINE THE FLOWING BOTTOM HOLE PRESSURE IN A DIRECTIONALLY DRILLED FLOWING WELL

This sample uses the vertical working curves of the Hagedorn and Brown correlation and the horizontal working curves of the Eaton method.

Given data: $p_{wh} = 120$ psi
2 in. tubing
$q = 1{,}500$ bpd (95% water)
$G/L = 600$ scf/bbl
Vertical depth = 6,000 ft
Tubing length = 8,000 ft

Find the flowing bottom hole pressure.

Solution procedure:

1. Find the flowing bottom hole pressure required for a 6,000 ft vertical well. From Figure C.96 of reference 5 this is found to be 1,900 psi.
2. We need the frictional loss for an additional 2,000 ft (8,000 − 6,000) of tubing. Since this additional frictional loss occurs over the 2,000 ft of tubing length we will use the average pressure in the tubing string to obtain this value.

$$\text{Assume } \bar{p} = \frac{120 + 2{,}000}{2} = 1{,}060 \text{ psi and check.}$$

3. Obtain the frictional loss for the 2,000 ft of tubing from Fig. D.104.

(a) Locate the average pressure of 1,060 psi on the 600 scf/bbl line. Note that the friction loss appears to be constant (straight line) at this position so we can take 2,000 ft and check the loss. Starting at bottom (12,200 ft and 1,060 psi) we proceed upwards 2,000 ft and read 960 psi at 10,200 ft. The friction loss in 2,000 ft is 1,060 − 960 = 100 psi.

4. Add the pressure obtained from the vertical flow correlation and the horizontal flow correlation to obtain the final answer (p_{wf}).

$$1{,}900 + 100 = 2{,}000 \text{ psi}$$

A computer solution would be more accurate in that the additional loss due to friction could be accounted for in each increment.

CLASS PROBLEM #1-A: TO DETERMINE FLOWING BOTTOM HOLE PRESSURE IN A DIRECTIONALLY DRILLED WELL

Vertical depth = 7,000 ft
Measured depth = 11,000 ft
$q_L = 1{,}000$ bpd (90% water)
$G/L = 800$
$P_{wh} = 100$ psi
Tubing size = 2 in.

Find p_{wf}.

CLASS PROBLEM #1-B: TO DETERMINE THE FLOWING BOTTOM HOLE PRESSURE IN A DIRECTIONALLY DRILLED WELL

Given data:

Vertical depth = 8,000 ft
Tubing length = 12,000 ft
3 in. tubing
$p_{wh} = 160$ psi
$q = 1{,}500$ bpd (oil)
GOR = 1,500 scf/bbl
Temp, surf = 110°F
Temp, bottom = 180°F
$\gamma_g = 0.65$

(a) Find the flowing bottom hole pressure.
(b) Find p_{wf} for all water with other conditions remaining the same.

CLASS PROBLEM #1-C: TO DETERMINE THE FLOWING BOTTOM HOLE PRESSURE IN A DIRECTIONALLY DRILLED WELL

Given data:

Vertical depth = 9,000 ft
Angle of deviation = 50° from vertical
q = 4,000 bpd (all oil)
4 in. tubing
p_{wh} = 200 psi
GOR = 600 scf/bbl
Avg. temp = 140°F
γ_g = 0.65

Find p_{wf}.

Other Solutions:

(A) The solution by computer can be taken directly from any of the good vertical flow solutions such as the Hagedorn and Brown equation

$$144\frac{dp}{dh} = \bar{\rho}_m + \frac{f\,q_L^2\,M^2}{2.965\times10^{11}\,d^5\,\bar{\rho}_m} + \frac{\bar{\rho}_m\,\Delta\left(\frac{v_m^2}{2g}\right)}{\Delta h} \quad (2.65)$$

The above equation can be solved directly for vertical distance for each pressure increment as prescribed for vertical multiphase flow and then the friction term only:

$$144\left(\frac{dp}{dh}\right) = \frac{f\,q_L^2\,M^2}{2.965\times10^{11}\,d^5\,\bar{\rho}_m} \quad (2.66)$$

is added for each additional length of pipe for each increment.

Solution procedure:

1. Knowing the well-head pressure, assume a second pressure (higher) which occurs at some vertical depth.
2. Find the vertical depth (Δh) for the pressure difference between the well-head pressure and pressure assumed in (1).
3. Knowing the deviation and using Δh calculate ΔL (true length of tubing).
4. Using the frictional term only of the vertical flow correlation, calculate the frictional loss for the additional length ($\Delta L - \Delta h$).
5. Add the frictional pressure found in step (4) to the assumed pressure of step (1).
6. Continue the iteration procedure using the pressure found in step (5) for the pressure at Δh, etc. until the total depth of the well is reached.

No example problem is given here since a computer solution to this problem is required.

(B) The methods of Cardozo can be used directly as described in previous sections.

A more accurate way of solving this problem is by changing the equation to:

$$144\frac{dp}{dz} = \bar{\rho}_m \sin\theta + \frac{f\,q^2\,M^2}{2.965\times10^{11}\,d^5\,\bar{\rho}_m} + \frac{\bar{\rho}_m\,\Delta\left(\frac{v_m^2}{2g}\right)}{\Delta z} \quad (2.67)$$

$\frac{dp}{dz}$ can now be determined directly where dZ = measured length of pipe and θ = angle from horizontal. Also the equation can be handled in terms of cos ϕ where ϕ = angle of deviation from the vertical.

$$144\frac{dp}{dz} = \bar{\rho}_m \cos\phi + \frac{f\,q^2\,M^2}{2.965\times10^{11}\,d^5\,\bar{\rho}_m} + \bar{\rho}_m\frac{\Delta\left(\frac{v_m^2}{2g}\right)}{\Delta z} \quad (2.68)$$

where $\frac{dp}{dz}$ = measured length of pipe, ϕ = angle from vertical. This same adjustment can be made to other correlations such as that of Orkiszewski and Duns and Ros.

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5. Brown, Kermit E., "Gas Lift Theory and Practice," The Petroleum Publishing Co., 1973, Tulsa, Oklahoma.

2.7 SUMMARY AND CONCLUSIONS

2.71 Introduction

The multiphase flow correlations available that offer a solution to all angles of flow are indeed limited. The correlation of Beggs and Brill appears to be the most promising with additional field data yet needed to permit certain modifications to improve its accuracy.

No one correlation for either vertical or horizontal flow is best for all ranges of flow. It appears that the best solution will be the combining of various correlations in one computer program that will first determine the most accurate correlation for a particular set of conditions and then use that correlation for calculation purposes. It is conceivable that correlations may be changed for any one tubing configuration or surface flow line, that is, several correlations may be used depending upon which one is most accurate for a section of the tube.

2.72 Areas for further investigation

2.721 Directional wells

The petroleum industry is faced with the necessity of drilling more and more directional wells as offshore activities continue to increase. Some directional wells have deviations of as much as 70° from the vertical, and wells with true vertical depths of 8,000 ft may have measured tubing lengths of 10,000 to 15,000 ft. At the present time no multiphase flow correlation is available which adequately solves this problem, although the Beggs and Brill solution appears most promising.

2.722 Inclined flow (hilly terrain)

The hilly terrain problem is one that warrants additional study. Since the design of the numerous offshore lines is now being required, it represents one of our most urgent problems. Again the solution of Beggs and Brill appears most promising.

2.723 Heading phenomenon

The phenomenon of heading that occurs in a well is not fully understood. The conditions under which heading can be expected are very unpredictable; sometimes it occurs at the lower liquid rates for a particular tubing size, other times it seems to occur at practically any gas-liquid ratio—regardless of tubing size. Data on heading were obtained by Fancher and Brown[18] and the condition is discussed in detail by Marshall,[19] who studied the data taken by Hagedorn and Brown.[5] This phenomenon was also discussed by Ros.[7]

2.724 Emulsified flow

There are numerous occasions when gas, water, and oil flow simultaneously in pipelines. This is very common in surface production flow lines in bringing oil from the wellhead to the separator and storage facilities. When the flow of these mixtures occurs, there is the possibility of an emulsion being formed, and if this happens, the pressure loss may increase drastically.

Emulsified flow in pipes is one area where very little research has been done, and some controversy exists as to how pressure loss in pipes is influenced by emulsions.

2.725 Viscosity effects

Viscosity effects are not completely understood in the multiphase flow problem. This is substantiated by the fact that correlations give widely different answers, depending on the viscosity of the liquid.

2.726 Slippage at low flow rates

No two correlations will give the same answer for decreasing flow rate for the same pipe size at which a tubing size should be changed out to the next smaller pipe size. Many of the $2\frac{7}{8}$-in. O.D. and $2\frac{3}{8}$-in. O.D. tubing sizes as run in the U.S. are too large for the flow rates produced through them. For example, a rate less than 200 B/D is inefficient through either size.

2.727 Conclusions

The multiphase flow correlations will never replace the pressure gage in accuracy for determining flowing pressures in vertical conduits. However, continued improvements will permit much greater accuracy for clean tubing strings. The practical situations of having scale deposits, paraffin, corrosion and other effects will always leave doubt when using vertical multiphase flow correlations where this is known to exist.

For the best use of these correlations several flowing pressure traverses should be conducted to verify the use of any one correlation in a particular field. A set of working curves should then be prepared for the practicing engineer to use for that field since he may not have immediate access to a computer.

The ultimate solution will encompass a computer program calling for the particular correlation that is most accurate for a particular set of conditions.

These correlations can also be used to greatly improve all facets of drilling and completing a well. It should dictate the tubing sizes, casing sizes, and hence the drilling program.

Horizontal flow lines and transmission flow lines fall in the same category. The hilly terrain problem including downhill recovery will be greatly improved and reliability should be in the range of $\pm$ 5%.

Finally, it is urged that the industry become more aware of the benefits that can be directly obtained from proper use of multiphase flow correlations.

Chapter 3

The flowing well including choke bean performance

3.1 INTRODUCTION

A thorough understanding of the flowing well is necessary prior to placing it on artificial lift. There are two surface conditions under which a flowing well is produced, that is, it may be produced with a choke at the surface or it may be produced with no choke at the surface. The majority of all flowing wells utilize surface chokes. Some of the reasons for this are (1) safety; (2) to maintain production allowable; (3) to maintain an upper flow rate limit to prevent sand entry; (4) to produce the reservoir at the most efficient rate; (5) to prevent water or gas coning; and (6) others.

In particular, flowing wells utilize a choke in their early stages of production. As time progresses, the choke size may have to be increased and eventually removed completely in order to try to optimize production.

The second condition that we are concerned with is producing the flowing well with no restrictions at the surface except normal Christmas tree turns, bends, etc. Even these may be streamlined in order to obtain the maximum flowing rate possible.

As discussed in Chapter 2 on Multiphase Flow, we use multiphase flow correlations to predict when a well will quit flowing, select a tubing size for a new well, determine flowing pressures, PIs, etc. We will also show how the surface facilities including the length and size of the horizontal production flow line affects the flowing well. Numerous problem examples will be worked including problems combining the vertical and horizontal multiphase flow correlations, and wells utilizing and not utilizing surface chokes.

3.2 THE OVERALL FLOWING SYSTEM

Several factors must be considered simultaneously in order to determine the production rate and/or properly diagnose a flowing well. These are:

Inflow performance
Vertical flow performance
The surface system
Surface facilities
Subsurface facilities

All these items are connected in some way. Together they govern the flow of oil, gas, and water coming from the reservoir to the surface facilities. We have discussed inflow performance in Chapter 1 and Vertical Flow Performance in Chapter 2.

We shall also consider the facilities existing in the field to provide the necessary continuity of the system and means to exercise control upon each one. Fonseca gave the following discussion.[1]

1. Between the reservoir rock and the oil well, we have the subsurface equipment which consists of casing, tubing, packers, bridge plugs, bottomhole chokes, valves, seating nipples, safety devices, etc. All this equipment and the manner in which it is installed in the well constitutes what is called the mechanical conditions of the well and it is designed in a variety of ways to provide adequate communication between the reservoir and the well; and to allow an effective control of the producing formation, including the possibility of special workover, stimulation, and recompletion operations in order to offset production problems.
2. Between the oil well and the flow line system, we have the surface equipment to control the well, including safety facilities and facilities to allow special operations related to the performance of the producing well. For our purpose, the main component of this system is the flow line choke which permits the control of the flowing pressure at the surface (tubing and casing), and at the bottom of the hole.
3. Between the flow lines and the surface facilities, we have equipment to permit the separation of the different phases (gas, oil, and water).

Fig. 3.1 is a scheme of the four items under consideration, the ways used to connect them, and an indication of the types of flow to consider when studying the overall system.

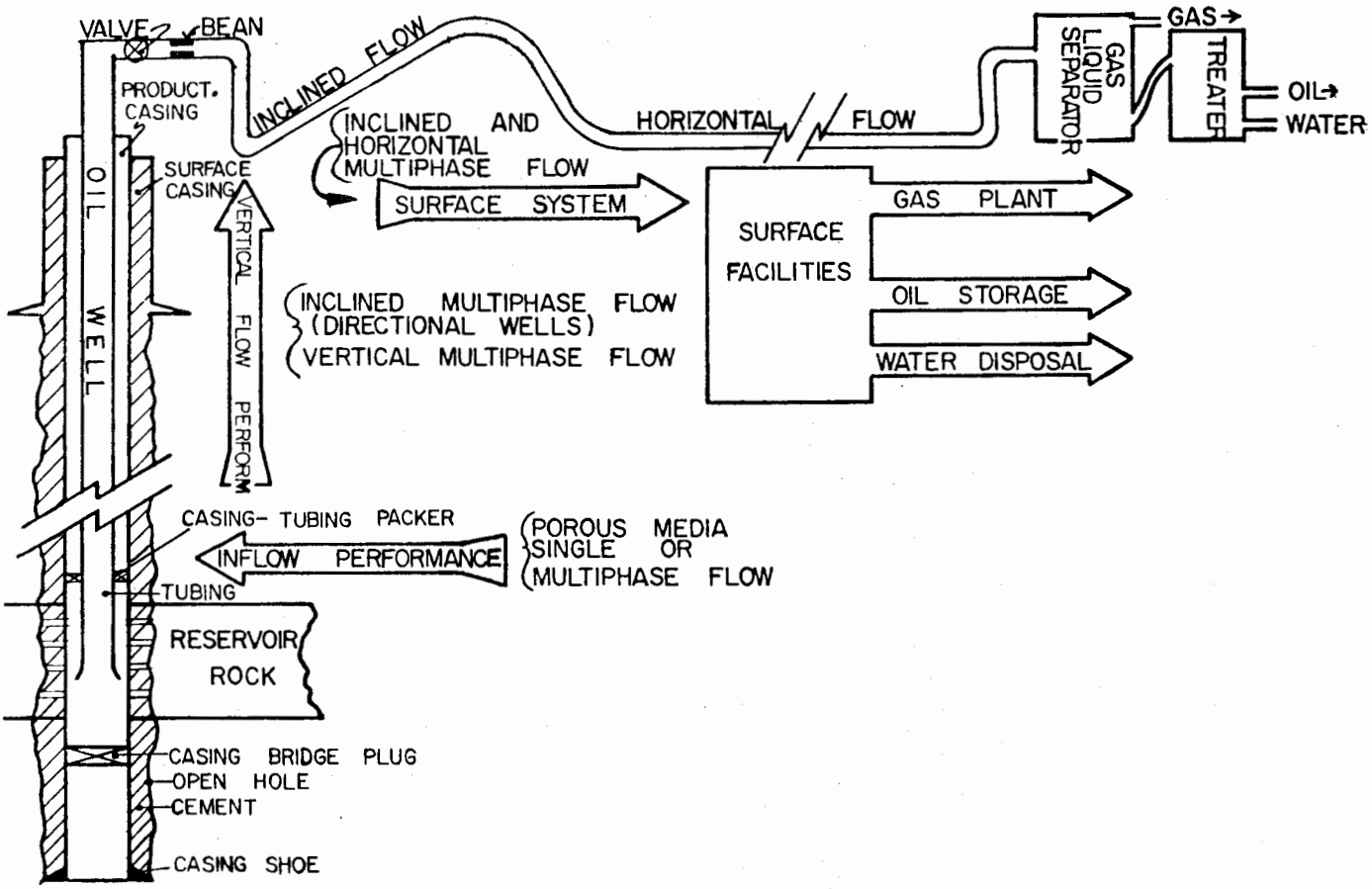


Fig. 3.1 The overall flow system (after Fonseca).

After the separators, since the phases are already separated, we have only single-phase flow problems which are simple to solve. For our purpose it is only necessary to consider the flow to the separator, because it is the last possible flow restriction to interfere in the well performance.

Although all problems are not yet satisfactorily solved, from a practical point of view a skilled engineer has enough information to put all the available knowledge together in order to determine the most efficient well flowing performance. Although much has been done in each individual system, attempts have not yet been made to use all the information available and take full advantage of it in well performance studies.

This chapter will show how to select and use all available information. It was stimulated by our belief in the fact that perfect well performance cannot be pursued if we do not study the overall system as a unit. A change in any one factor influences the others, many times to a much greater degree than is believed.

For example, the simple act of changing the surface choke size of a flowing well may cause the well to die, resulting in loss of money, both in lost production and expenses to bring the well in again.

In analyzing a well, problems are easily detected and solved when they occur at the surface. Subsurface problems, on the other hand, are difficult to detect and analyze and may lead to false conclusions in a well diagnosis.

Before attempting any modification in arriving at a solution for sudden alterations in a well performance, a thorough investigation of the mechanical conditions is highly recommended.

The investigation of the mechanical conditions of a well should start with the well completion procedure. In so doing we may suspect the effectiveness of the cement job, for instance, which may cause intercommunication between geologically isolated formations. The conditions of the hole before the casing was cemented may furnish clues for the well diagnosis, so we should analyze the caliper log and other logs that were run in the open or cased hole. Also the nature of the fluids used to drill and complete the wells should be known.

Leaks in casing, tubing, packers, and bridge plugs must always be looked at as a possible cause of trouble. Sometimes it is easy to inspect anomalies in the well based on these possibilities. For instance, if a deep well starts making water and we suspect a casing leak near the surface, the simple determination of the salinity of the water may save expensive, difficult, and complicated analyses.

If the well performance is closely observed, any anomaly can be immediately detected, and the combination of the data from the well performance and the knowledge of the mechanical conditions of the well is sufficient to diagnose most production problems.

Flow through perforations represents a study in itself. Flow through various sand control devices and methods also has an effect on well capability.

3.3 METHODS OF ANALYSIS

The multiphase flow path of oil and gas from the reservoir into the storage tanks is noted in Fig. 3.1, and is divided into three stages—inflow, vertical flow, and horizontal flow. This is shown further in Fig. 3.2 and the pressure losses are:

(1) ΔP_1—flow through the porous medium (inflow performance). ΔP_1 will vary from 10–50% of the total loss.
(2) ΔP_2—multiphase flow from the bottom of the well to the surface. ΔP_2 will vary from 30–80% of the total pressure loss.
(3) ΔP_3—multiphase flow from the well head into the storage tanks (surface flow performance). ΔP_3 will vary from 5–30% of the total pressure loss depending upon many factors, and in particular on whether or not the well needs to be choked.

Stages 2 and 3 require their own multiphase flow correlation in order to determine ΔP_2 and ΔP_3. These ΔPs are shown graphically in Fig. 3.2. This type of plot can be the engineer's worksheet in determining the behavior of the flowing well. The vertical line of intersection of the three stages of production gives the equilibrium flow rate.

Similar to prior plots, we first plot the inflow performance curve shown in Fig. 3.2. This is a curved line here, indicating two-phase flow in the porous medium.

The following stepwise procedure will help to understand this plot.

(1) Plot the inflow curve.
(2) From the depth of the well, G/L, and diameter find the values of the well-head pressures corresponding to different rates and plot.
(3) Plot the Surface Choke Performance Curve. Several of these may be plotted or it can be selected for the desired rate.
(4) Draw in a vertical line at the rate desired. This will be the equilibrium condition for the well giving q, p_{wf}, p_{wh}, and bean size necessary.

There are many ways in which correlations and the effects of parameter changes can be shown. One of these has already been described in Fig. 3.2. Another is the pressure depth diagram of Fig. 3.3. In this diagram we note ΔP_1 (loss in porous medium) and ΔP_2 (loss in vertical flow traverse). The loss in the surface facilities is $(p_{wh} - P_{sep})$ and finally $P_{sep} - 0$ to the storage tank.

A third diagram is shown in Fig. 3.4 which is the flowrate diagram for unrestricted flow. We can use this type of diagram to illustrate or study the effects of changes in the G/L, $\overline{P_R}$, and PI. This does assume a constant well head pressure and no restrictions at the surface. The vertical flow performance curve represents the flowing bottom hole pressure required for

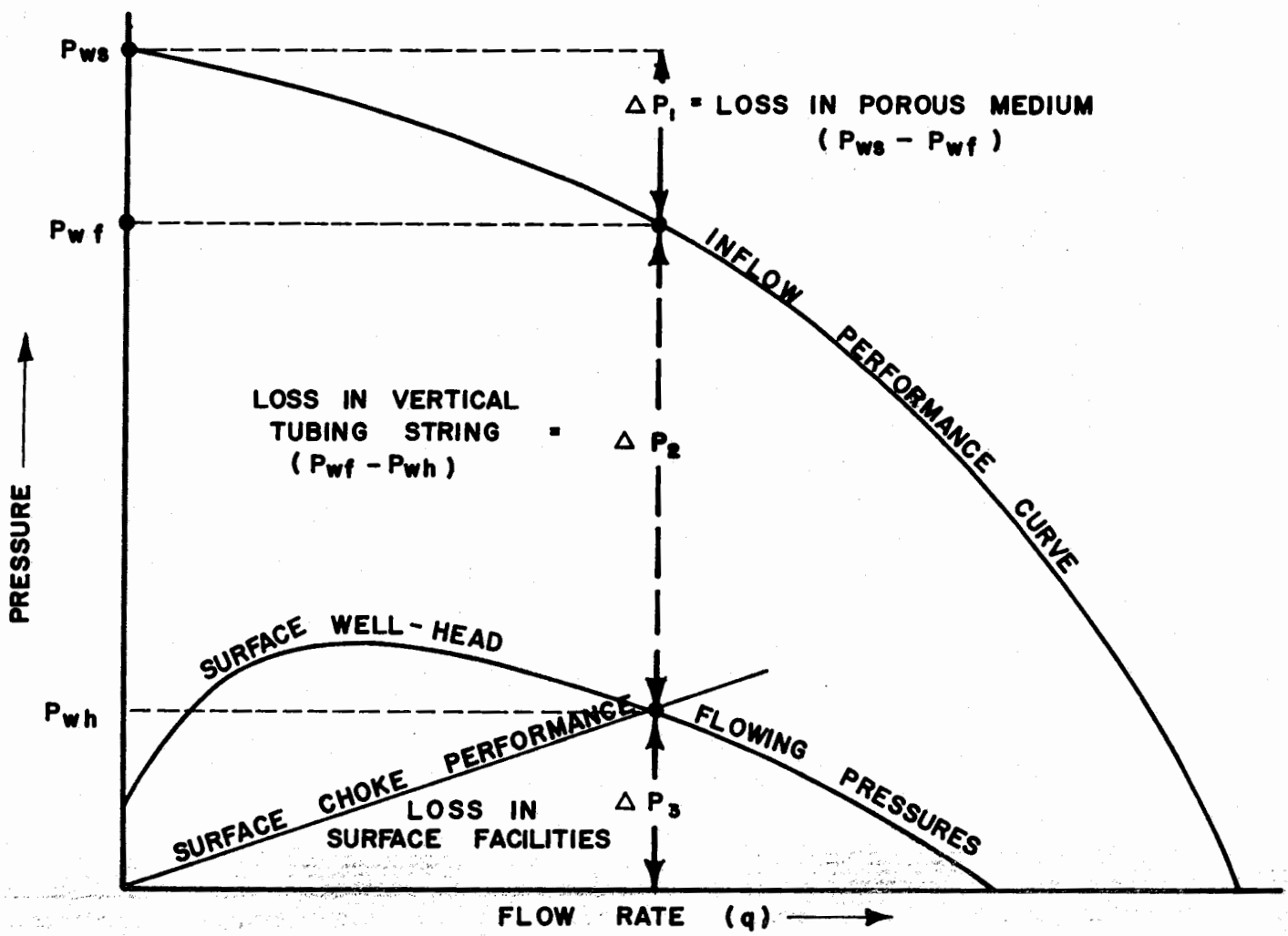
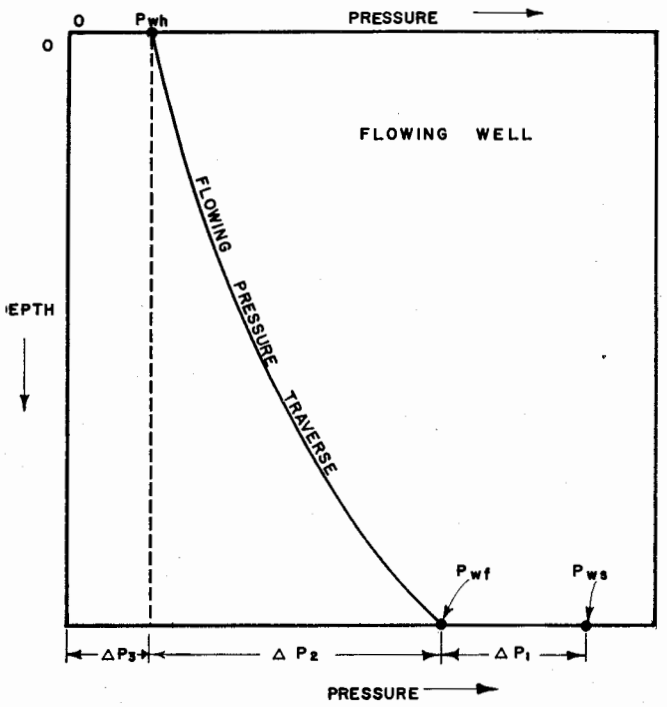


Fig. 3.2 Pressure—flowrate diagram. (After Juch).

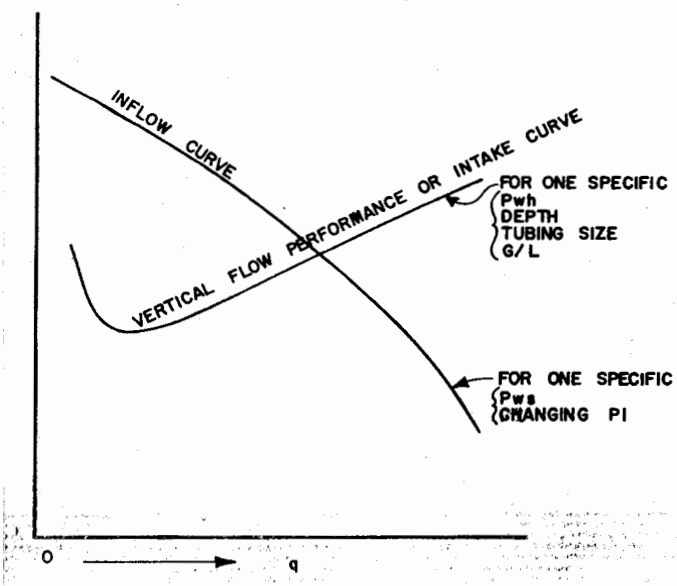


ig. 3.3 Pressure depth diagram.

arying flow rates. The equilibrium condition is shown y the intersection of the two curves.

The vertical flow performance curve can be changed o include the surface flow performance, and thus he entire multiphase flow performance can be shown n Fig. 3.4, that is, from the bottom of the well to the eparator.

Gilbert[2] chose to show a 3-dimensional model (Fig. .5) which shows the influence of the production rate nd G/L on the flowing bottom hole pressure. This amily of curves is for one tubing size, depth, and ellhead pressure. In checking Fig. 3.5 we see two eavy-lined curves, #1 and #2. Curve #1 shows that s the G/L decreases the rate of flow for minimum ressure increases and the minimum pressure itself ncreases.



ig. 3.4 Flowrate diagram for unrestricted flow.

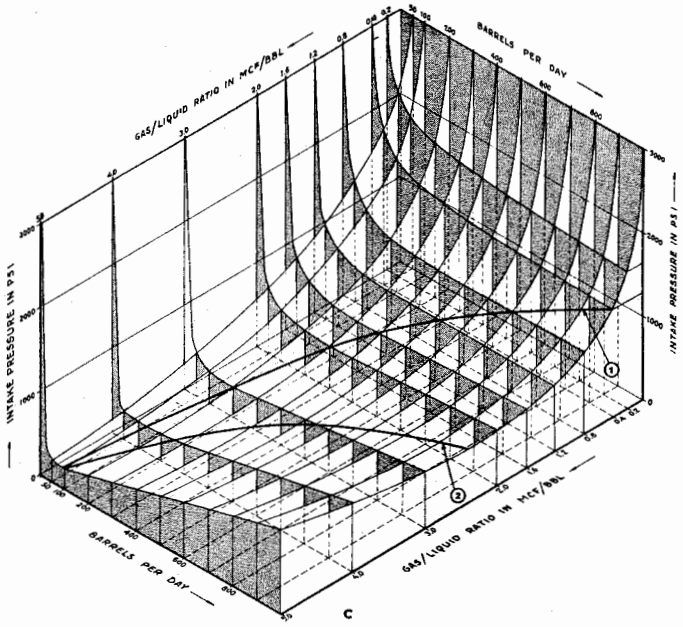


Fig. 3.5 The two-phase vertical-lift function for 2.875-in. tubing set at 8,000 ft. (after Gilbert).

The following table, taken from Fig. 3.5, shows this:

G/L	q	p_{wf}
3,000	200	low value
800	700	300
400	1,000	900

Curve #2 shows that for any q = constant, there is a G/L which requires a minimum p_{wf}. For example, 800 b/d has a minimum intake pressure (p_{wf}) at a G/L of approximately 2,000/1.

3.4 IRREGULAR PRODUCTION

3.41 Heading phenomena

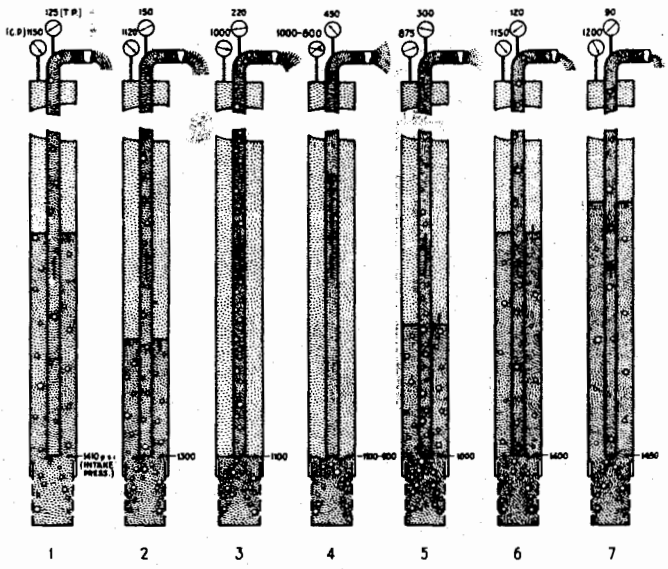
Numerous wells fall into the category of irregular behavior with heading being the principal cause. This has been described in some detail in Chapter 2, Section 2.24 on multiphase flow. However, it should be discussed here in regard to its effect on the flowing well. As a general rule the heading phenomena can be expected to occur only during the latter stages of the flowing life of a well and then at relatively low flow rates. It will differ considerably depending upon whether or not a packer is installed in the well, or whether communication exists between the tubing and casing by any other means such as a sliding side-door. Wells with no communication may also flow in a heading manner, but with a lower variation in pressure and rate.

Unless a well is a high gas oil ratio well (greater than 300 scf/bbl per 1,000 ft of depth) it cannot be expected to flow in a heading manner for very long, and is in its latter stages of flowing life. If the well has a G/L greater than 300 scf/bbl per 1,000 ft of depth it can be assisted to flow in a cyclic manner for an extended period of time.

Gilbert[2] described this phenomena and the heading cycle in a well with no packer as noted in Fig. 3.6. For a well having reached this stage of production, it will probably flow longer with communication between the tubing and casing than if a packer were installed in the well.

This type of situation generally brings up the question of whether a well will flow longer naturally with or without communication between tubing and casing. If a packer is installed all the gas is immediately routed through the tubing string and the well will flow longer in a continuous or semi-continuous manner. Heading can also occur in a well without a packer.

However, the well without a packer will flow longer in a cyclic manner than the well with a packer and will continue to produce in this manner even though it would die if a packer were installed. This is described fully in Fig. 3.6.



Heading cycle:

(1) Because of the bypassing of gas into the annulus, the liquid level is slowly being lowered; annulus oil is being displaced into the tubing.

(2) The well is still producing at a low rate and the tubing column is heavy because gas is being diverted into annulus and oil from the annulus is being diverted into the tubing.

(3) The weight of the tubing column is being reduced because no further gas can be stored in the annulus. This further reduces the intake pressure and...

(4) Allows gas from the annulus to "blow around." For a short time the well is gas lifted with annulus gas at a high rate, and this reduces the intake pressure to its lowest value. The well gas is NOT being used to best advantage.

(5) The extra annulus gas has been dissipated and fluid, because of the low intake pressure, is flowing at a high rate into both the tubing and the annulus. The tubing column is becoming heavier and the outflow rate is diminishing.

(6) Fluid is still flowing into the well at a faster rate than it will flow out of the well with the existing intake pressure.

(7) The rate of outflow is again in balance with the rate of inflow, and bypassing of gas into the annulus starts repetition of the cycle.

Observations:

This is not an efficient type of flow because it produces a large proportion of the oil with a deficient supply of gas and a small proportion of the oil with an excess supply of gas.

The usual practice of beaning a well back to a low rate of flow to avoid this type of surging is not efficient, and in many instances results in avoidable reduction of income.

This type of flow is characteristic of the latter part of the flowing life of wells in most areas, but is also characteristic of many relatively new low GLR wells.

Heading action of this kind can be minimized by use of tubing-casing packers, but where packers are not already installed, use of casing-actuated intermitters may be preferable.

Fig. 3.6 Latter stages of a flowing well (after Gilbert, courtesy API).

In order to further prolong the flowing life without making any downhole installation of artificial lift, the installation of a regulator (stop-cock) device on the tubing wing will keep the well flowing under its own source of power for a much longer period of time (Fig. 3.7).

For this type of installation to be successful, we must have communication between the tubing and casing. Even if a packer is installed, this communication must be provided by opening a sliding side-door or punching a hole in the tubing in order for the stop-cock to work successfully. In general this is a surface means of more fully utilizing the energy of the well to assure it will be able to flow in cycles. This type of flow is fully described in Fig. 3.7.

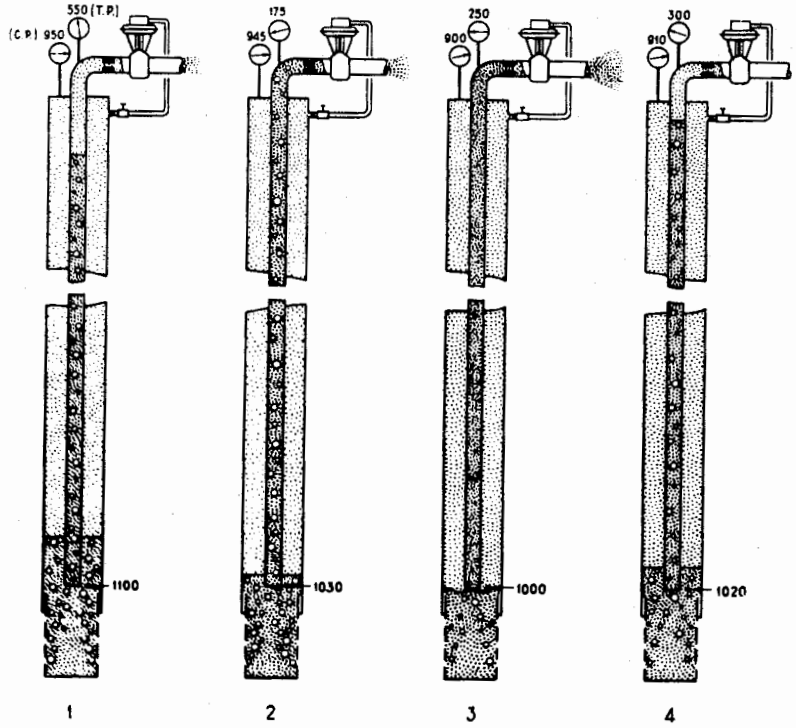
This type of flow may be improved even more by the installation of a "plunger" or "free piston" in the tubing string. The piston eliminates a great portion of the slippage or fallback of liquids that occurs in the tubing string.

Although for many wells, this represents only a temporary means of prolonging the installation of lift equipment, there are numerous wells where this method of lift may suffice for several years. In particular, as long as the gas liquid ratio remains above 300 scf/bbl per 1,000 ft of lift, the system will function satisfactorily. As a matter of fact in many cases it represents the best method to produce the well. This is because the high gas liquid ratio may cause pump-lock problems or gas chamber escape problems for chamber gas lift. It also offers the decided advantage of supplying its own source of power for production.

Juch[3] discussed heading in a different manner and explained it by constructing flow rate pressure diagrams as noted in Figs. 3.8 and 3.9. He noted that heading can occur due to the peculiar vertical multiphase flow characteristics whereby that under certain conditions the pressure loss decreases (respectively increases) when the G/L and or q increases (respectively decreases).

In symbolic form $\frac{d(\Delta P)}{d(G/L)}$ can be negative and/or $\frac{d(\Delta P)}{dq}$ can be negative. Both these effects are more pronounced at low G/Ls and low flow rates and therefore heading is a good indication that the well may die in the near future.

Juch[3] gave two examples (Figs. 3.8 and 3.9) where heading may cause a well to die. Both of these examples show wells that were good producers in the past but are presently in trouble. In Fig. 3.8 we note that $\frac{d(\Delta P)}{dq}$ is negative. It is in a range whereby a decrease in rate causes an increase in ΔP (loss in vertical string). In other words a decrease in the surface choke opening results in an increased ΔP due to extra slippage or fallback of liquids, whereas normally a decreased rate causes a decreased ΔP. A possible solution to this problem would be to increase the surface choke size (bean size) sufficiently to move past that minimum rate where slippage is excessive.



Heading Cycle:

1. At the start of the flowing period, the tubing is opened by the rising casing pressure which actuates the motor valve. The column of gas which has collected in the upper part of the tubing is produced, and the consequent reduction of pressure ensures flow of the fluid mixture in the tubing below this gas column.
2. The tubing pressure trends downward while fluid is being displaced out of the annulus, and then
3. Rises as annulus gas starts to break around the foot of the tubing.
4. When the casing pressure reaches the predetermined minimum, the motor valve closes the tubing outlet, but flow into the well continues with very little decrease in rate, both gas and liquid flowing into the annulus. Tubing pressures continue to rise. The casing pressure, which is directly related to the amount of gas stored in the annulus, also increases in response to gas and liquid entering the well. When the casing pressure reaches the predetermined maximum, repetition of the cycle is started by opening of the motor valve.

Observations:

By regularizing flow, casing-actuated intermitters can be used to increase the rate of flow and extend the flowing life of wells when they reach the heading stage.

Intermitters are not to be recommended for wells which will produce more on the pump, but they are recommendable:

a. To increase the rate of flow of wells which have been beaned back to avoid heading.
b. To regularize and increase the flow of new low-ratio wells until increase of gas ratios may permit steady flow at desired rates.

Intermitters have been used to flow wells which would not flow without an intermitter, wells with water cuts exceeding 50 percent and wells at rates exceeding 500 bbl per day. Intermitters have been misapplied, and mostly by inexpert selection of bean size and casing-pressure ranges. The bean should preferably be as large as necessary to ensure a continuous drop of casing pressure during the on-period (1 to end of 3) and no larger. The casing-pressure range must be large enough to ensure a flow of gas around the foot of the tubing, signalized by a steady buildup in tubing pressure. A longer range is unnecessary and often undesirable.

Fig. 3.7 Casing control of a flowing well (after Gilbert).

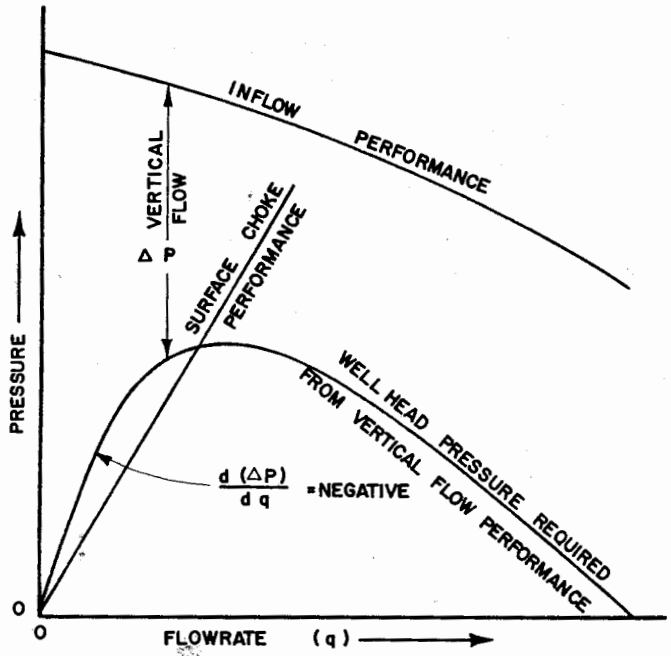


Fig. 3.8 Heading explanation (After Juch).

For example, note the following Table 3.1:

TABLE 3.1

q	p_{wf}	p_{wh}	ΔP vertical
1,000	1,800	500	1,300
2,000	1,600	100	1,500

Under these conditions we have a lower flowing bottom hole pressure of 1,600 psi for q = 2,000 bpd as compared to 1,800 psi for 1,000 bpd due to the high well-head pressure of 500 psi for q = 1,000 bpd. However, the loss in the vertical tubing string of 1,300 psi for 1,000 bpd is less than the 1,500 psi for 2,000 bpd.

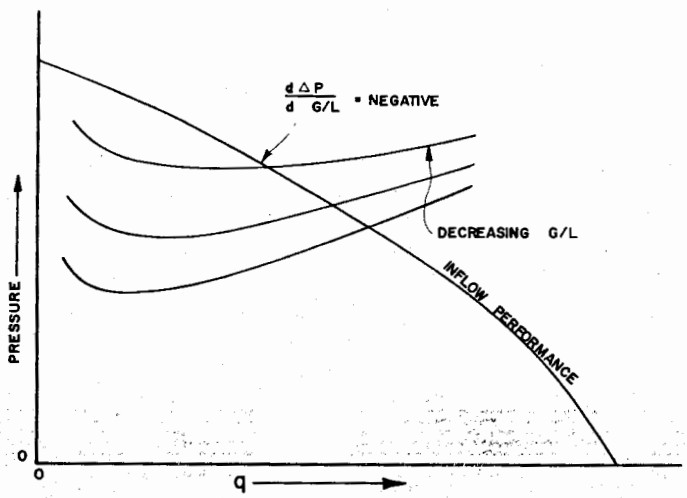


Fig. 3.9 Heading explanation by changing G/I. (After Juch).

If we plot p_{wf} vs. q then $\frac{dp}{dq}$ is negative. This is normal since we expect $\frac{dp}{dq}$ to be negative.

If we plot ΔP vs. q then $\frac{d(\Delta P)}{dq}$ is positive in a normal manner, and is what we expect.

However, at the lower flow rates $\frac{d(\Delta P)}{dq}$ can become negative, indicating a range of unstable flow. For example, note the following Table 3.2:

TABLE 3.2

q	p_{wf}	p_{wh}	Vertical ΔP
100	1,500	100	1,300
400	1,300	300	1,000

Again if we plot p_{wf} vs. q we find $\frac{dp}{dq}$ to be negative in a normal manner. However, if we plot ΔP vs. q we find that $\frac{d(\Delta P)}{dq}$ is also negative. This is not normally what we expect or desire. We are then in an unstable flow regime and can expect irregular flow behavior.

Fig. 3.9 shows an example whereby $\frac{d(\Delta P)}{d(G/L)}$ is negative. Normally the change in the vertical pressure loss ΔP decreases as the gas liquid ratio increases, that is we expect $\frac{dp}{d(G/L)}$ to be negative in a normal sense, but we do not expect $\frac{d(\Delta P)}{d(G/L)}$ to be negative since it is positive in a normal sense. Reference to Fig. 3.9 shows $\frac{d(\Delta P)}{d(G/L)}$ to become negative at that point at which the vertical performance curve for a constant G/L line starts swinging upwards towards the low rates (becomes negative). And when this occurs at an intersection with the inflow performance curve, then the heading phenomena is likely to occur.

Assume for example:

G/L	ΔP (vertical)	p_{wf}	p_{wh}
200	2,000	2,100	100
500	1,500	1,900	400

If we plot G/L vs. q, we obtain $\frac{d(\Delta P)}{d(G/L)}$ a negative value indicating a region of unstable flow.

Nind[4] discussed in some detail the effect of a small change in gas liquid ratios, and provided the following mathematical analysis.

The flowing well head pressure can be found from the equation:

$p_{wh} = p_{wf} - \Delta P$, where ΔP = pressure loss in vertical flow string.

From Gilbert's equation for surface chokes:

$$p_{wh} = \frac{CR^{0.5}\, q}{S^2} \qquad (3.1)$$

S = dia. bean (64th of in.)
q = rate, (bpd)
R = G/L, Mcf/bbl
C = constant

y substitution we note that:

$$p_{wf} - \Delta P = \frac{CR^{0.5}q}{S^2} \tag{3.2}$$

can also be defined in terms of PI and $\bar{p}_R - p_{wf}$

here: $q = J(\bar{p}_R - p_{wf})$ (3.3)

hen: $p_{wf} - \Delta P = \frac{CR^{0.5}}{S^2} J(\bar{p}_R - p_{wf})$ (3.4)

hich can be put in the form:

$$p_{wf}\left(1 + \frac{CR^{0.5}J}{S^2}\right) = \frac{CR^{0.5}}{S^2} J\bar{p}_R + \Delta P \tag{3.5}$$

By differentiating equation 3.5 we can find the manner in which p_{wf} varies with small changes in the gas/quid ratio.

In differentiating we have:

$$\frac{p_{wf}}{R}\left(1 + \frac{CR^{0.5}}{S^2}J\right) + \frac{1}{2R}\frac{CR^{0.5}}{S^2}Jp_{wf} = \frac{1}{2R}\frac{CR^{0.5}}{S^2}J\bar{p}_R + \frac{d}{dR}(\Delta P)$$

$$\therefore \frac{dp_{wf}}{dR}\left(1 + \frac{CR^{0.5}}{S^2}J\right) = \frac{1}{2R}\frac{CR^{0.5}}{S^2}q + \frac{d}{dR}(\Delta P)$$

finally: $$\frac{dp_{wf}}{dR}\left(1 + \frac{CR^{0.5}}{S^2}J\right) = \frac{1}{2R}p_{wh} + \frac{d}{dR}(\Delta P) \tag{3.6}$$

By way of explanation, if a well is producing with a is liquid ratio less than the optimum, an increase in e G/L will decrease ΔP in the tubing.

Then $\frac{d(\Delta P)}{dR}$ is negative.

From the previous equation we note that if: $\frac{p_{wh}}{2R}$ is ss than $\frac{-d(\Delta P)}{dR}$, then $\frac{d(p_{wf})}{dR}$ will be negative and the oduction rate should increase normally for an ineased G/L since p_{wf} will also decrease permitting ldditional production.

However, if $\frac{p_{wh}}{2R}$ is greater than $\frac{-d(\Delta P)}{dR}$, then $\frac{d(p_{wf})}{dR}$ ll be positive and p_{wf} at the bottom of the tubing ill rise and production will decrease with an ineased gas liquid ratio.

The exact point of balance between these two cases pends on the shape of the distribution curves for o-phase vertical flow. However, in general, the naller the bean at the surface, the greater will be h, and the greater the probability that p_{wh} will inease as the G/L increases. This appears physically und in that the additional gas must pass through e small choke.

When the G/L is near the minimum (critical) the lue of $\frac{d(\Delta P)}{dR}$ will be small so that the right hand side of equation 3.6 will be positive and p_{wf} will increase with increased G/L.

In general, however, we expect that p_{wf} will decrease as G/L increases until a minimum flowing gradient is reached in the tubing string.

Nind noted two examples illustrating this behavior, by utilizing Gilbert's pressure traverses.

EXAMPLE PROBLEM #1

Given data: Depth = 2,000 ft
3½ in. tubing
q = 600 bpd
G/L = 1,000 scf/bbl = 1.0 Mcf/bbl
p_{wh} = 600 psi
ΔP in tubing string = 300 psi
For G/L = 1,200 scf/bbl, then ΔP = 260 psi

Therefore $\frac{d(\Delta P)}{dR} = \frac{260 - 300}{1{,}200 - 1{,}000}$
$= -200$ psi/Mcf

$$\frac{p_{wh}}{2R} = \frac{600}{2} = 300 \text{ psi/Mcf}$$

Therefore $\frac{d}{dR}(\Delta P) + \frac{1}{2R}p_{wh} = -200 + 300 = 100$ psi/Mcf.

Then $\frac{d(p_{wf})}{dR}$ is positive.

EXAMPLE PROBLEM #2

Given data: Depth = 5,000 ft
2⅜ in. tubing
q = 200 bpd
G/L = 1.0 Mcf/bbl
p_{wh} = 200
ΔP in tubing string = 470 psi
For G/L = 1.2 Mcf/bbl then ΔP = 430 psi

Thus: $\frac{d}{dR}(\Delta P) = \frac{-40}{0.2} = -200$ psi/Mcf

$$\frac{p_{wh}}{2R} = \frac{200}{2} = 100 \text{ psi/Mcf}$$

$$\frac{d}{dR}(\Delta P)\frac{+1}{2R}p_{wh} = -200 + 100 = -100 \text{ psi/Mcf}$$

and therefore: $\frac{d(p_{wf})}{dR}$ is negative

The above discussions shed some light on the phenomenon of heading.

Most of the heading or irregular behavior occurs in a well that does not have a packer. Some causes for temporary changes in the G/L may be (1) slugs of accumulated water; (2) gas blowing around the annulus for wells with no packer; and (3) possible tubing leaks or other communication between the tubing casing.

Nind[4] gave an example problem which illustrates the inefficiency of a heading well. The following data was given:

EXAMPLE PROBLEM #3

Depth = 4000 ft
2 in. tubing
$\bar{p}_R$ = 1,500 psi
PI = 0.4 (assume linear)
Surface choke = $^{24}/_{64}$ in.

The well produces 22 hours with a gas liquid ratio of 100 scf/bbl and for 2 hours with a G/L = 2000 scf/bbl. Find the production rate of the well produced steadily at the same daily gas liquid ratio.

To solve the problem assume various rates and find the corresponding well-head pressures for G/Ls of 100 scf/bbl and 2,000 scf/bbl. See Table 3.3.

This information is plotted in Fig. 3.10. Bean performance curves are also included for the $^{24}/_{64}$ in. bean for gas liquid ratios of 100 scf/bbl and 2,000 scf/bbl. Also included on this plot is the IPR curve and the well-head pressures for G/Ls of 100 and 2,000 scf/bbl respectively. The production rates of 230 bpd for a G.L of 100 scf/bbl and 295 bpd for 2000 scf/bbl are taken from the plot. The average daily production rate is found to be:

$$q = (230)\frac{22}{24} + 295\frac{2}{24} = 236 \text{ bpd}$$

TABLE 3.3

Assumed rate	p_{wf}	p_{wh} G/L = 100	p_{wh} G/L = 2,000	p_{wh} G/L = 300
50	1,370	300	940	490
100	1,250	260	890	470
200	1,000	120	680	320
400	500		170	
600	0			

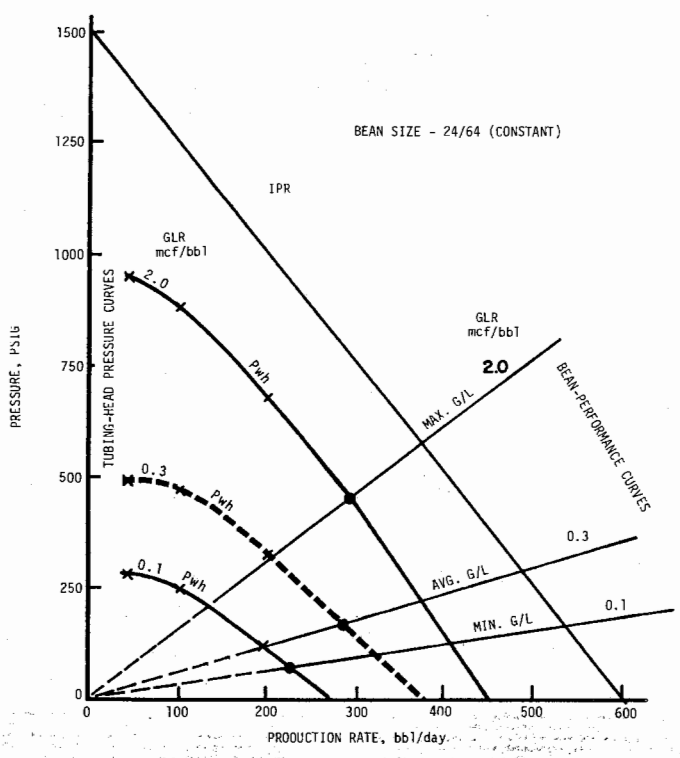


Fig. 3.10 Performance of a heading well (from Principles of Oil Well Production *by Nind. Copyright McGraw-Hill. Used with permission of McGraw-Hill Book Company).*

The total gas produced per day is:

$$q_g = (230)(100)\frac{22}{24} + (295)(2000)\frac{2}{24} = 70{,}300 \text{ scfd}$$

The average G/L $= \frac{70{,}300}{236} = 298$ scf/bbl. This G/L for a $\frac{24}{64}$ bean is then plotted on Fig. 3.10. Use 300 scf/bbl along with the corresponding surface well head pressure for 300 scf/bbl and their intersection show a steady flow rate of 290 bpd for a G/L of 300 scf/bbl, and P_{wh} = 160 psi. This shows that the well will produce at a higher flow rate under steady conditions of flow.

Nind comments that a greater production rate will be achieved for the same G/L for a well on steady production as compared to a heading well.

The type of heading in the preceding example can only occur for wells completed without a packer or wells with communication between casing and tubing. In present day practice, practically no wells are completed without a packer, and therefore Nind's discussions must be considered with this in mind.

In summary, a well in the heading stages should be checked closely. If the G/L is at least 300 scf/bbl per 1000 ft of depth it is a good candidate for flowing intermittently. For this type of installation a definite must is communication between the tubing and casing so that gas storage in the annulus provides a source of energy to bring the liquid out in cycles. The unloading of water that has accumulated in gas wells can be accomplished in the same manner, and is given an additional chance of success by the installation of a plunger.

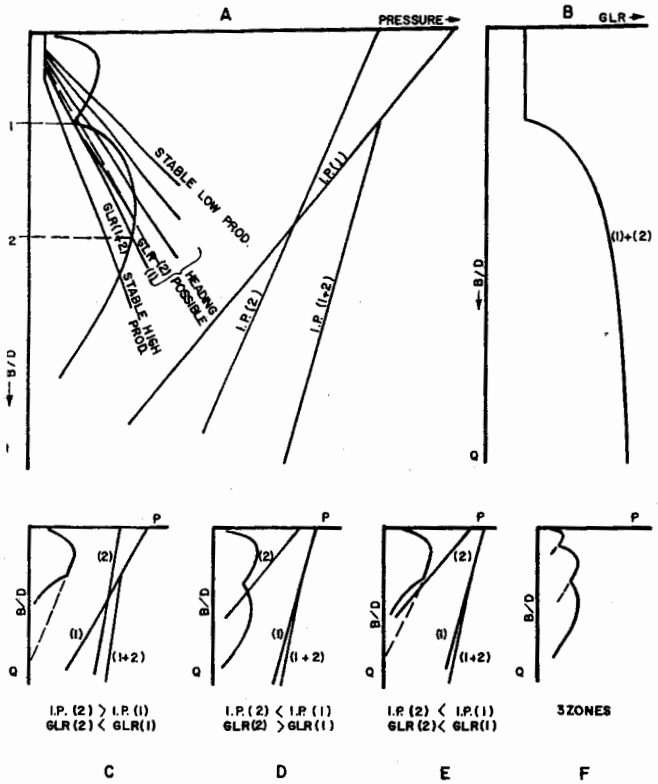
3.42 Irregular behavior of wells completed in stratified formations

Many wells are completed in stratified formations whereby each zone may produce with different G/Ls, PIs, and static pressures. This type of well may show all types of unstable effects and any predictions from multiphase flow correlations, surface choke beans, etc. are difficult to make. Therefore, flowing tests are necessary in order to obtain a better idea as to the well's producing behavior.

It is suggested that a pressure flow rate diagram be constructed and that the zone characteristics be superimposed on this diagram. Juch[3] gave a typical example as noted in Figs. 3.11A and 3.11B. For example, the production rate may start to increase from q_1 to q_2. The G/L ratio of the fluid entering the tubing string starts to increase and the ΔP of the vertical flow string starts to decrease until this higher G/L fluid reaches the surface choke. At this time the surface well head pressure begins to rise immediately. This in turn reduces the fluid entry rate into the well-bore. This again reduces the G/L which permits the flow rate to drop from q_2 to q_1 again. This well head pressure also reduces and the cycle will repeat itself. Figs. 3.11 A, C, D, E, and F show in general the configuration for varying PIs and G/Ls.

3.43 Purging of wells

Under the category of irregular behavior, we find wells that have a tendency to load up with liquids; in



g. 3.11 *Flow analysis of multiple zone production (after Juch).*

articular if they are making some water. A means of rolonging and supplementing natural flow is sometimes used in the following manner.

The well is equipped with a by-pass arrangement at ıe surface whereby a standard intermitter is used in arallel with the normal choke size used on the well, ig. 3.12. For example, a well may be flowing steadily ı a $^{12}/_{64}$ in. choke but over a period of several hours it ıay have a tendency to load up and die. The reason ır this is that the well is making some water and the ow rate is such that the velocity will not permit the ntinual removal of the water, thereby permitting it accumulate in the tubing string. By allowing the ıtermitter to open on a by-pass (perhaps every 4 ours) with a larger choke (perhaps $^3/_8$ in.), the velocity ı the flow string increases and the accumulated liquids are removed. This represents a so-called "purging" of the well.

The control may be either by pressure or a clock. The increase in flowing well-head pressure after the well is unloaded may be used to close in the regulator and in turn it will open when the tubing pressure decreases to a certain low which signals a loading up of the well. The cycles by clock are determined by trial and error.

3.44 Summary

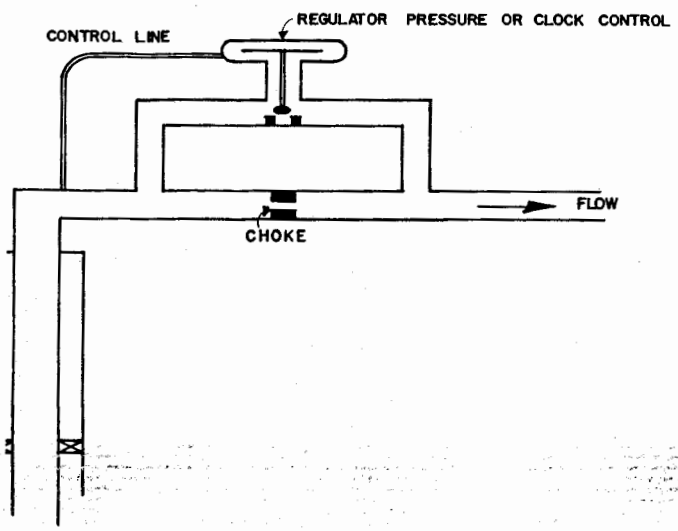
In summary, a production engineer should be able to forecast and design future production operations (Refer to Fig. 3.13). Juch[3] noted that the future production potential (unrestricted production) can be predicted by constructing a series of inflow curves and multiphase flow curves for successive time intervals in one p-q diagram (Fig. 3.14). If, for instance, the static bottom hole pressure declines from p_{R_1} to p_{R_2} and the well's G/L remains constant, the equilibrium point will move from point A to B and the production rate will decline from q_1 to q_2.

If, however, the G/L declines during this interval, point A may move to say point C. If the G/L increases or if gaslift is applied the production rate may decline to a lesser degree, say from point A to D.

To construct future inflow and multiphase flow curves, future trends in static reservoir pressure, PI, GOR, and water cut should be available. In most cases a reasonable estimate of these reservoir characteristics versus cumulative withdrawal can be made available. With the advent of more recent modeling work, many of these parameters can be predicted with good accuracy provided good inflow information is available.

An analysis of the pressure flowrate diagram can show:

(1) Future production rates
(2) Prediction of when the well will die
(3) Production gains by artificial lift or pressure maintenance



g. 3.12 *Well purging arrangement.*

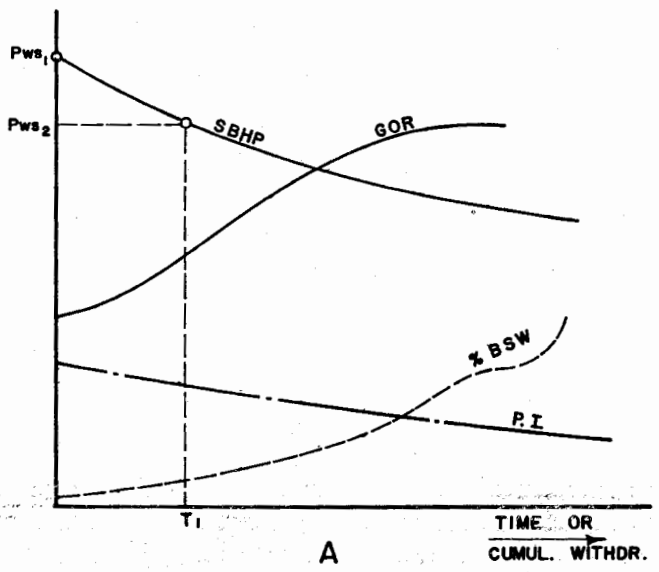


Fig. 3.13 Performance forecast for well (after Juch).

(4) Prediction of results of stimulation, repair, change of tubing size, etc.

Finally, the economic measures of value can be applied to determine the cash return on the project.

3.5 FLOW OF FLUIDS THROUGH SURFACE CHOKES, RESTRICTIONS, AND FITTINGS

3.51 Introduction

In order to properly interpret flowing well behavior, we must have a good idea of the effect of surface chokes, hereafter called "beans." Practically all flowing wells utilize some surface restriction in order to regulate the flowing rate. Only a very few are produced with absolutely no restrictions in order to obtain maximum production rate. The control at the well-head may be necessary for any one of the following reasons:

(1) Maintain correct well allowable
(2) Maintain sufficient back pressure to prevent sand entry
(3) Protect surface equipment
(4) Prevent gas coning
(5) Prevent water coning
(6) Produce reservoir at most efficient rate

Unfortunately, the solution to the problem of multiphase flow through chokes has not been satisfactorily solved for all cases. Most solutions are offered only for the case of critical flow, that is, when the downstream pressure is approximately less than one-half the upstream pressure. There exists some controversy even here since we are flowing a mixture of gas and liquids through the choke and not one phase. Gilbert suggested using his correlation for values of $P_{downstream}/P_{upstream} \leq 0.70$. Pure analytical solutions are doubtful, and most investigators have offered empirical correlations based on field and laboratory data.

Fig. 3.15 gives an idea of how the bean size influences the rate of flow in a flowing well. The graph shows that for the same G/L the rate of flow normally increases when the bean size increases (upper part of Fig. 3.15). The tubing head pressure, however, first increases with the decreasing bean size and then decreases after a maximum is reached for a bean size of $^{10}/_{64}$ in. This is due to the fact that when the bean size is decreasing, the flowing bottom-hole pressure is building up by virtue of the flow reduction. But, for lower rates of flow, gas slippage occurs, reducing the flow efficiency to such an extent that the ever increasing flowing bottom-hole pressure effect is overcome by the vertical pressure loss in the tubing, causing the tubing head pressure to decrease.

Quoting directly from Nind: "Suppose the well is producing at position 1 and that, for some reason, the production rate increases slightly (Fig. 3.16). Then the pressure imposed by the bean increases to A and the tubing-head pressure based on the combined inflow and vertical-lift performance decreases to B. A back pressure of amount AB is imposed on the well, and this is transmitted to the formation, thus reducing the production rate.

Similarly, if for some reason the production rate dropped slightly, there would be a release of back pressure and the production rate would be restored to its original value. Thus position 1 is a position of stable equilibrium, and the well will flow steadily at the corresponding production rate."

"On the other hand, suppose the well's producing position is as defined by point 2. Assume that, for some reason, the production rate decreases slightly (Fig. 3.17). The pressure imposed by the bean decreases to C, but the tubing-head pressure based on combined inflow and vertical-lift performance decreases to D. The effect is to increase the back pressure on the well, to increase the bottom-hole flowing pressure, and to cut the production rate down still further. The net effect will be that the well dies.

Alternatively, if a small production increase takes place,

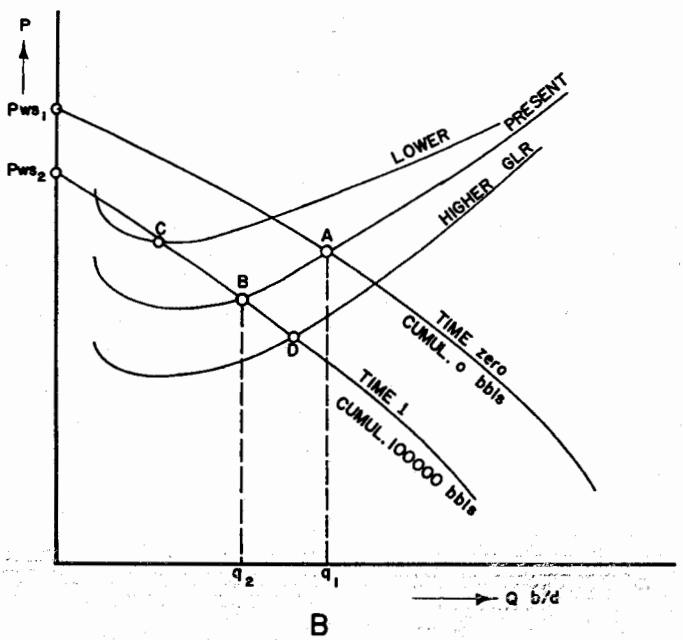


Fig. 3.14 Performance forecast for well (after Juch).

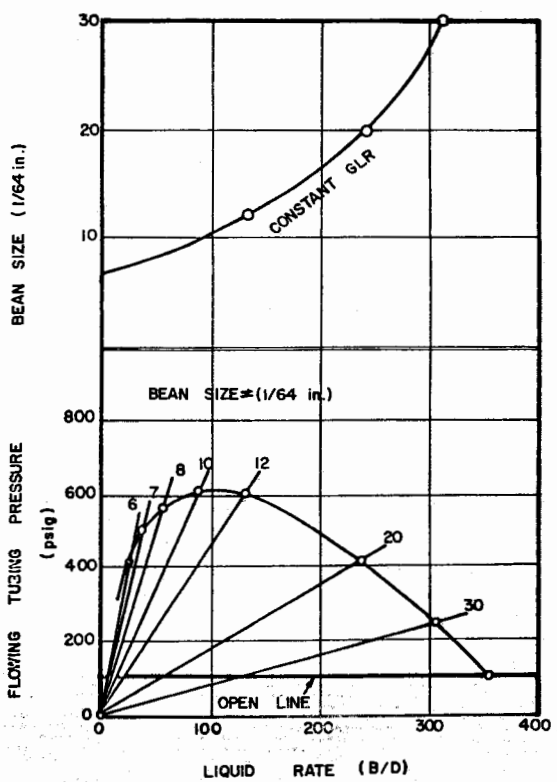


Fig. 3.15 Effect of bean size on the liquid rate.

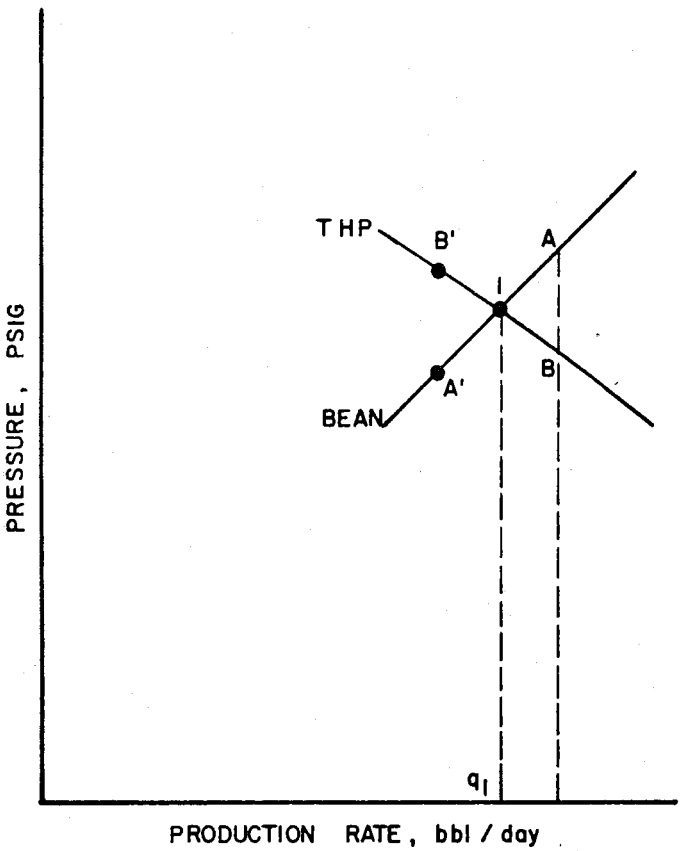


Fig. 3.16 Stability of flow: higher possible flow rate (From Principles of Oil Well Production *by Nind. Copyright 1964, McGraw-Hill. Used with permission of McGraw-Hill Book Company).*

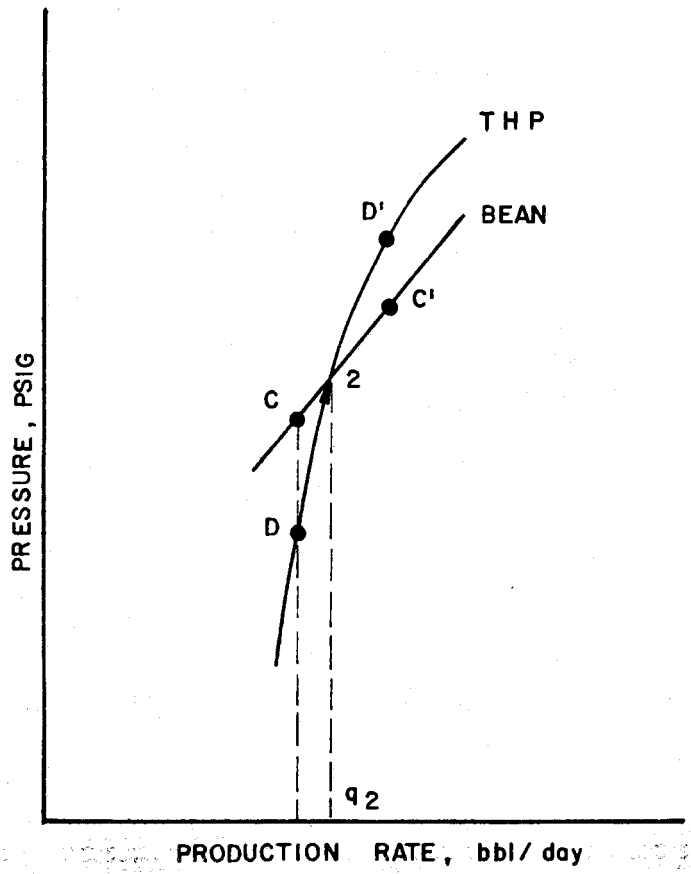


Fig. 3.17 Stability of flow: lower possible flow rate (From Principles of Oil Well Production *by Nind. Copyright 1964, McGraw-Hill. Used with permission of McGraw-Hill Book Company).*

the back pressure on the well is reduced, a further production increase takes place, and this continues until the production position 1 is reached. So 2 is a position of unstable equilibrium, and the well cannot be flowed steadily at the corresponding production rate through the particular bean under consideration."

"Analysis of the above type indicates that theoretically, the higher flow-rate position is stable at all times, but, in practice, since the left-hand arm of the tubing-head pressure curve is so steep, a position such as that shown at 3 in Fig. 3.18 is usually unstable.

The reason is that the correcting moment that results from a small decrease in production rate through the tubing is extremely small and the resultant dead oil accumulation in the annulus will frequently be sufficient to kill the well. Thus it may be said that it is not possible to maintain steady flow conditions at rates below that giving the maximum tubing-head pressure."

The application of this analysis should be used with caution for wells completed with a packer.

3.52 Correlations for choke flow

3.521 Introduction

Most present day correlations for multiphase flow across chokes are valid only for critical flow across the choke. We have good correlations for single phase flow of either liquids or gases, but good correlations for multiphase flow are scarce. This is especially true for flow in the sub-sonic flow region.

By way of review, remember that critical flow of a fluid is defined as fluid flow at a velocity equivalent to the velocity (frictionless) of propagation of a pressure (sound) wave in the fluid medium. This is consistent with hydrodynamic terms, and is not to be confused with (1) "critical pressure" associated with gas-liquid phase equilibrium, or (2) the "critical flow region"

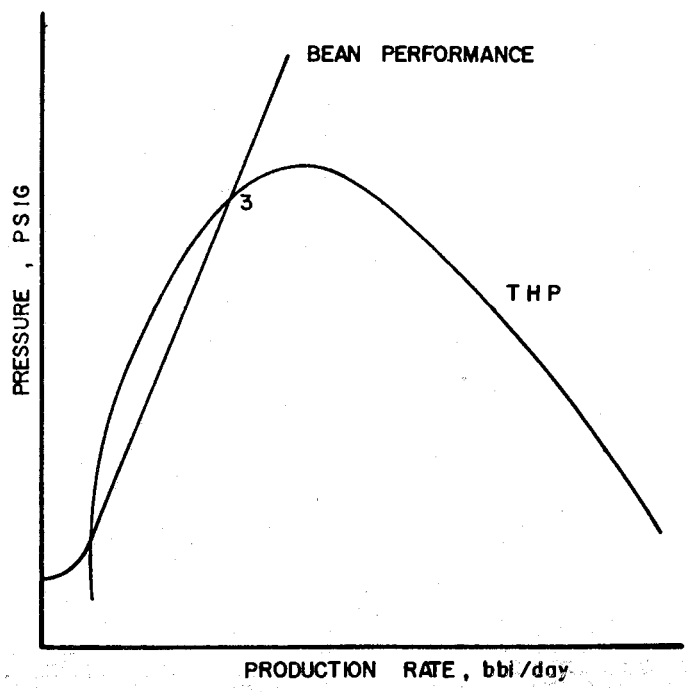


Fig. 3.18 Well beaned back too far (From Principles of Oil Well Production *by Nind. Copyright 1964, McGraw-Hill. Used with permission of McGraw-Hill Book Company).*

TABLE 3.4

Substance	V_p (ft/sec)
Oil	3,990
Brine	5,600
Distilled water (at 25°C)	4,912
Drilling mud	4,870
Berea sandstone (21.1% porosity, 20% oil and 80% water)	7,900
Nichols Buff sandstone (21.7% porosity, $S_o = 20\%$, $S_w = 80\%$)	7,100

associated with the friction factor Reynolds Number correlation.

For critical flow to occur the relative velocity of a fluid at any point (V_f-ft/sec) must be equivalent to the elastic wave velocity (V_p-ft/sec) at that point. That is V_f/V_p = Mach Number = 1.00.[5]

The velocities of elastic wave propagation (V_p) of various fluids and fluid-rock media as determined by Wyllie, et al[6] are listed in Table 3.4.

Critical flow for gases occurs at approximately the same conditions where the ratio of downstream to upstream pressure is $\cong$ 0.528. Mach 1 or sonic flow for gases is different than for liquids (1,100 ft/sec for air). Then for a mixture of oil and gas we ask ourselves the question—what is the critical flow for a mixture. Some have noted that the sonic velocity for gas liquid mixtures is less than that for either phase alone.

Recent work at the University of Tulsa presents a solution to the choke problem in the sub-sonic flow region and that report is obtainable through the API.

3.522 Single phase choke flow

Crane[7] gave the following discussion of single phase flow through orifices and chokes.

The rate of flow of any fluid through an orifice or nozzle may be expressed by:

$$q = Cd\ A\sqrt{2g\ h_L} \tag{3.7}$$

if the velocity of approach is neglected

The velocity of approach may have considerable effect on the quantity discharged through a nozzle or orifice. The factor correcting for velocity of approach is:

$$\frac{1}{\sqrt{1-\left(\frac{d_o}{d_1}\right)^4}} \tag{3.8}$$

This may be incorporated in Equation 3.7 as follows:

$$q = \frac{Cd\ A}{\sqrt{1-\left(\frac{d_o}{d_1}\right)^4}}\sqrt{2g\ h_L} \tag{3.9}$$

The quantity:

$$\frac{Cd}{\sqrt{1-\left(\frac{d_o}{d_1}\right)^4}} \tag{3.10}$$

is defined as the flow coefficient C. Values of C for nozzles and orifices are shown on Fig. 3.19. Use of the flow coefficient C eliminates the necessity for calculating the velocity of approach, and Equation 3.9 may now be written:

$$q = CA\sqrt{2g\ h_L} = CA\sqrt{\frac{2g(144)\Delta P}{\rho}} \tag{3.11}$$

The flow coefficient C is plotted for Reynolds numbers based on the internal diameter of the upstream pipe. (see Fig. 3.19)

Flow of liquids: For nozzles and orifices discharging incompressible fluids to atmosphere, C values may be taken from Fig. 3.19 if h_L or ΔP in Equation 3.11 is taken as the upstream pressure. For low viscosities, i.e., water, gasoline, etc., the Reynolds number need not be calculated since it will fall in the range of the values on Fig. 3.19 where the flow coefficient C is a constant.

Flow of gases and vapors: The flow of compressible fluids through nozzles and orifices can be expressed by the same equation used for liquids except the net expansion factor Y must be included.

$$q = YCA\sqrt{\frac{2g(144)\Delta P}{\rho}} \tag{3.12}$$

The expansion factor Y is a function of:

1. The specific heat ratio, k.
2. The ratio of orifice or throat diameter to inlet diameter.
3. Ratio of downstream to upstream absolute pressures.

This factor[8,9] has been experimentally determined on the basis of air, which has a specific heat ratio of 1.4, and steam having specific heat ratios of approximately 1.3. The data is plotted on Fig. 3.20 and values of other specific heat ratios have been included to extend the use of the data.

Equation 3.12 may be used for orifices discharging compressible fluids to atmosphere by using:

1. Flow coefficient C given on Fig. 3.19 in the Reynolds number range where C is a constant for the given diameter ratio.
2. Expansion factor Y per Fig. 3.20.
3. Differential pressure ΔP, equal to the inlet gauge pressure.

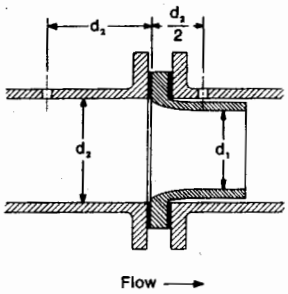
This also applies to nozzles discharging compressible fluids to atmosphere only if the absolute inlet pressure is less than the absolute atmospheric pressure divided by the critical pressure ratio, r_c.

3.523 Multiphase flow choke correlations

Fonseca[1] made a study of the choke correlations and offered the following comments.

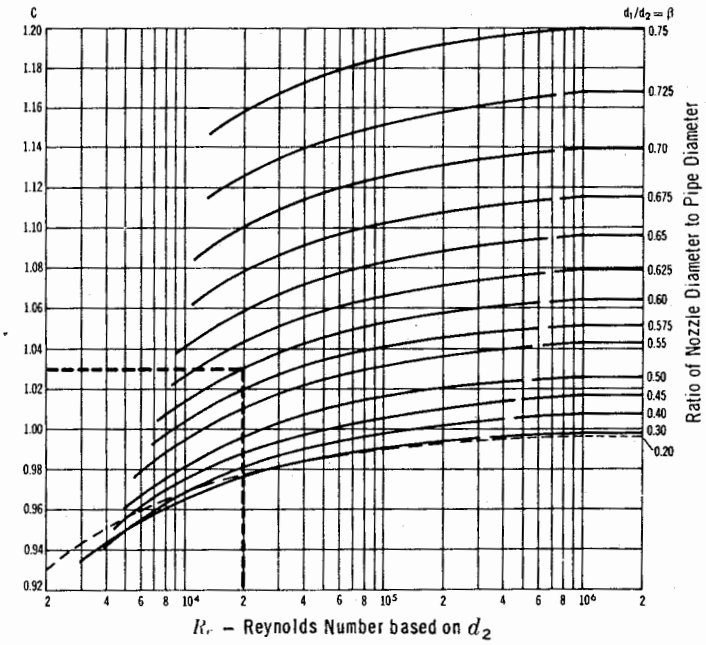
The approach used by most investigators may be classified into one of the following:

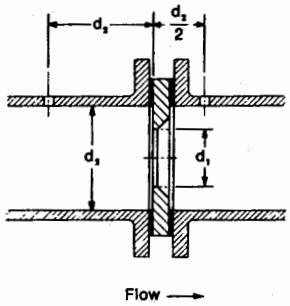
1. Empirical correlations from field or laboratory data
2. Empirical correlations using dimensional analysis to select and group the most important variables
3. Theoretical approaches, applying mathematical analysis to a simplified physical model with development of equations



$$C = \frac{C_d}{\sqrt{1 - \beta^4}}$$

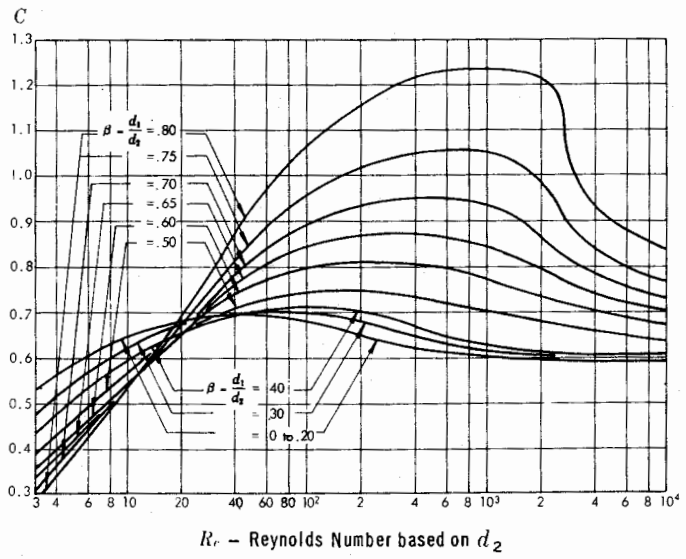
Example: The flow coefficient C for a diameter ratio β of 0.60 at a Reynolds number of 20,000 (2×10^4) equals 1.03.





$$C = \frac{C_d}{\sqrt{1 - \beta^4}}$$

$$K_{\text{orifice}} \simeq \frac{1 - \beta^2}{C^2\beta^4}$$



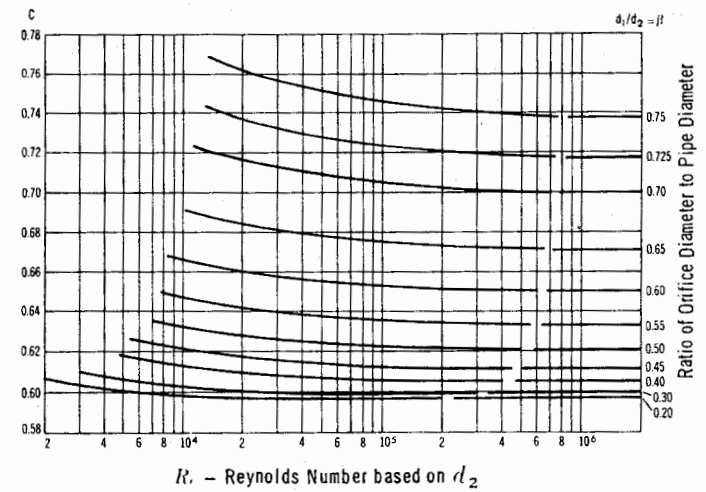
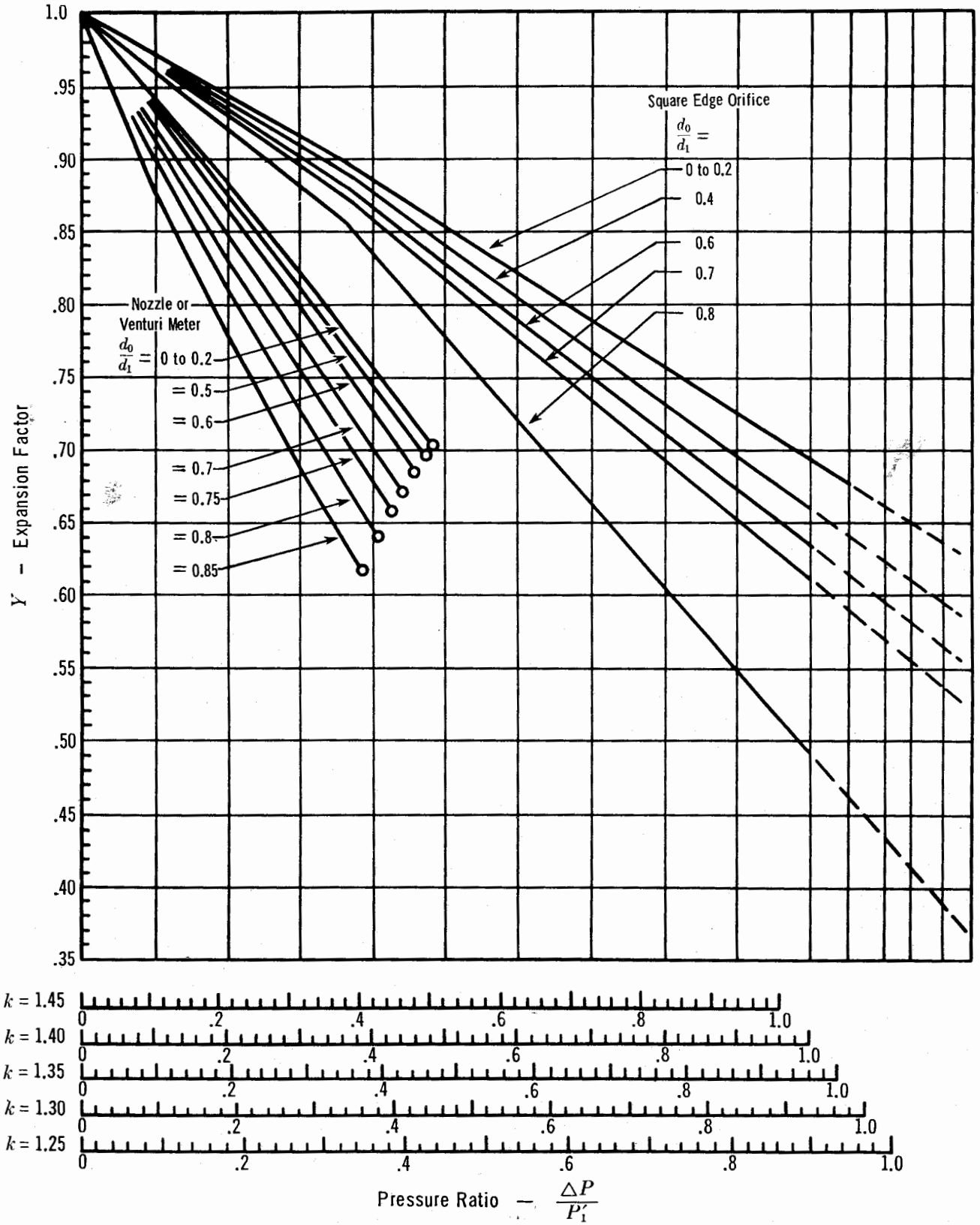


Fig. 3.19 Flow coefficient for nozzles and orifices (after Crane).



Data extracted from *Fluid Meters, Their Theory and Application*, Fourth Edition, 1937, and *Orifice Meters with Supercritical Flow* by R. G. Cunningham, with permission of the publisher, The American Society of Mechanical Engineers, 29 West 39th Street, New York 18.

Fig. 3.20 Net expansion factor for compressible flow through nozzles and orifices (after Crane).

5231 Tangren, et al:[10]

In 1949 R. F. Tangren, C. H. Dodge, and H. S. Seifret eveloped an equation of state and one of motion for as-water mixtures flowing through a "de Laval" ozzle.[10] The significance of the Tangren approach was) show that when gas bubbles are added to an in-ompressible liquid, the mixture becomes compressible nd, above the critical flow velocity, the medium be-omes incapable of transmitting pressure changes pstream against the flow.

This conclusion was important in oil fields where it ecame a common practice to select a choke to be in-talled on a flowing well in such a way that critical ow velocity is attained in order to avoid the fact that mall variations in the downstream pressure (flow ine and separators) affect the tubing head pressure nd therefore the capacity of the well to produce.

.5232 Gilbert's approach[2]

Assuming a knife-edge choke and making several implifying assumptions with regard to the pressure-olume characteristics of the oil and gas, it can be hown theoretically that:

$$p_{wh} = \frac{C\,R^{0.5}\,q}{S^2} \qquad (3.1)$$

where: p_{wh} = flowing well tubing head pressure, psia
R = gas-liquid ratio, Mcf/stk bbl (Note, Mcf/bbl)
q = gross liquid rate, stk bpd
S = bean size, 64ths of an in.
C = Constant approximately equal to 600 in the above units.

Using daily individual well production data from the Ten Section Field in California, Gilbert,[2] in 1954, published the following formula to be used as a guide for determining the first guess in the trial and error procedure in selecting a suitable bean size:

$$p_{wh} = \frac{435\,R^{0.546}q}{S^{1.89}} \qquad (3.13)$$

The units are the same as given previously, except that p_{wh} is in psig. Therefore, the value of a constant is:

$$K = \frac{435\,R^{0.546}}{S^{1.89}}$$ is a constant for each bean size and gas-liquid ratio.

It may be written as $p_{wh} = Kq$, which gives a straight line on rectangular coordinate paper. If the G/L is known, it is possible to draw straight lines on a graph where P_{wh} is already plotted against q for each choke size. The intersections of these lines with the curve p_{wh} vs. q resulting from the combination of IPR and multiphase vertical flow show the rates possible for different choke sizes.

Gilbert prepared charts as noted in Fig. G.1 (Appendix) that give approximate solutions for any one of the four variables when the others are known. The procedure for use of these charts is given on the page prior to Fig. G.1 (Appendix).

Gilbert noted that an error of 1/128 in. in bean size can give an error of 5 to 20% in pressure estimates. Gilbert's formula assumes that the actual mixture velocities through the bean exceed the speed of sound, therefore the downstream or flow line pressure has no effect on the rate of upstream pressure. The speed of sound is known to occur when the upstream pressure is at least twice the downstream pressure, (100% greater). However, Gilbert noted that his formula was good when the downstream pressure was less than 0.70 of the upstream pressure.

Example problems are found at the end of this section.

3.5233 Ros' formula (Poettmann and Beck adaptation)[11,12]

Based on the analysis of the energy balance equation, in 1960, Ros[11] developed a flow-meter formula for critical gas-liquid flow through a restriction. Later on, in 1963, Poettmann and Beck[12] converted the Ros equation to oil field units and reduced it to graphical form, resulting in the following expression:

$$q_o = \frac{86{,}400\,CA}{5.61\rho_{LS} + .0765\gamma_g R_p} \sqrt{\frac{9273.6p}{V_L(1 + 0.5m_L)}} \left[\frac{0.4513\sqrt{R + 0.7660}}{R + 0.5663}\right] \qquad (3.14)$$

In simplified form: $P_{wh} = 17.4q\,R^{0.5}/S^2$

where:

$$R = \frac{0.00504\,T\,Z\,(R_p - R_s)}{B_o P}, \quad p = p_i/4636.8$$

$$m_L = 1/\left(1 + R\,\frac{\rho_g}{\rho_L}\right), \quad V_L = m_L/\rho_L$$

q_o = barrels of stock tank oil per day
C = discharge coefficient (1.03)
A = cross sectional area of throat in sq. in. (throat is the minimum cross sectional area of a choke).
ρ_{LS} = density of crude in lb/cu ft at 60°F. and 14.7 psia
γ_g = specific gravity of gas referred to air at 60°F. and 14.7 psia
R_p = gas-oil ratio in scf of gas per bbl of stock tank oil
p_{wh} = tubing pressure in psi
p_i = tubing pressure in lb (force) per sq ft
V_L = volume of liquid per unit mass of total fluid, cu ft/lb
m_L = mass of liquid per unit mass of total fluid (dimensionless)
T = tubing temperature (absolute) assumed to be 85°F. (or 545°R)
z = compressibility factor of gas at tubing pressure and 85°F
R_s = solubility of gas in crude at tubing pressure and 85°F
B_o = formation volume factor of crude at tubing pressure and 85°F
ρ_L = density of crude at pressure, p, and 85°F. in lb mass/cu ft
ρ_g = density of gas at pressure, p, and 85°F. in lb mass/cu ft

Using the above equations and the Borden and Rzasa correlations for crude oil gravities of 20, 30, and 40° API Poettmann and Beck constructed graphs (Figs. G-2, G-3, G-4 in Appendix), assuming a gas gravity of

0.6, a tubing temperature of 85°F., and downstream pressure less than 0.55 of the upstream pressure. The charts are for 20°, 30°, and 40° API respectively. The following problems can be solved by the use of the previously mentioned graphs:

1. Design of chokes or beans for new wells.
2. Estimates of gas-oil ratios and gas production rates from existing wells, knowing tubing pressures and oil flow rates.
3. Prediction of performance of a given choke or bean, knowing the producing gas-oil ratio.
4. Check for paraffin obstruction or choke cut by gas or sand.

Good results are obtained from the charts if there is no water production and if the flow is two-phase and at critical flow conditions.

3.5234 Sheldon/Schuder approach[13]

Some manufacturers of valves tried to solve the multiphase flow problem through orifices by adding the two sizing coefficients used for single-phase flow of liquid and of gas. The results obtained were not accurate because of additional pressure loss introduced when the actual gas and liquid velocities tend to equalize at the orifice. Scheldon and Schuder in 1965 made a series of tests in order to determine the form and magnitude of required corrections. They arrived at some relationships which made possible the selection of proper choke sizes, based on calculated sizing coefficients for two-phase flow.

3.5235 Omana's correlation[14]

In 1968 Omana used controlled field data taken at the facilities of Union Oil Company of California's Tigre Lagon Field in Louisiana to check the existing correlations and to develop his own. Field experiments were conducted with water and natural gas, and although his correlation gave better results than those tested with the data obtained, his correlation is not widely accepted today for the following reasons:

1. Limitations of choke size (4 to 14/64 in.)
2. Limitation in flow rate (800 bpd maximum)
3. Limitation in pressure (400 to 1,000 psig)
4. Use of water instead of oil or water-oil mixture in the field experiments.

However, for choke sizes up to 14/64 in. this correlation is considered to be very accurate.

3.5236 Achong's correlation[15]

Achong[15] derived a choke correlation very similar to that of Gilbert[2] for use in the Lake Maracaibo Field. He started with the formula for predicting pressure loss across a choke in its simplest form as follows:

$$\Delta P = \frac{C\, q \sqrt{R}}{A} \tag{3.15}$$

q = liquid flow rate
R = gas liquid ratio
A = cross-sectional area of bean

C depends on units of q, R, P, A, and shape of bean. A more general formula and one with less error is:

$$\Delta P = \frac{C\, q\, R^n}{A^m} \tag{3.16}$$

The coefficients n and m are determined from field data, and they account for the influence of temperature, gas gravity, etc.

For example, Achong derived accurate constants for the condition and type of beans (Cameron-positive type) used in Lake Maracaibo, Venezuela and found the formula to be as follows:

$$p_{wh} = \frac{3.82\, q\, R^{0.65}}{S^{1.88}} \tag{3.17}$$

where: p_{wh} = psig
q = bpd
R = scf/bbl
S = 64th of an inch

or in the metric system:

$$p_{wh} = \frac{0.904\, q\, R^{0.65}}{S^{1.88}} \tag{3.18}$$

where: p_{wh} = Kg/cm²
q = cu m/day
R = cu m/cu m
S = diameter in mm

A nomograph for this solution can be found in the Appendix (Fig. G.5)

3.5237 Conclusions and recommendation

For the purpose of beaning back a flowing well at the well head, it is concluded that the Poettmann and Beck approximation of the Ros formula should be used when no water is produced and sufficient data is available. Gilbert's formula can be used in the other cases, including those in which the well is making only oil. Omana's correlation may be used with confidence for choke sizes up to 14/64 in. for water or oil.

It is recommended that additional data be taken for multiphase flow through chokes in order to check existing correlations and prepare new ones where applicable, and in particular for sub-sonic flow.

We will show numerous examples using beans and some problems which utilize no restriction at the surface. In order to use bean correlations correctly we must assume that critical flow exists across the bean (that is, the upstream pressure is approximately twice the downstream pressure).

EXAMPLE PROBLEM #4: HOW TO DETERMINE THE FLOW RATE THROUGH A BEAN

Given data: °API = 40
Wellhead pressure = 400 psia
Choke size = 14/64 in.
G/L = 1000 scf/bbl
Downstream pressure = 100 psi

Find the flow rate in bpd by:

(a) Poettmann and Beck method (Ros)
(b) Gilbert method

Solution:

(a) Poettmann and Beck
  (1) Check ratio of pressure downstream to pressure upstream

$$\text{Ratio} = \frac{100}{400} = 0.25$$

If value is less than 0.50, assume critical flow and use Fig. G.4 (Appendix)

(2) Note G/L on abscissa = 1,000 scf/bbl
(3) Proceed vertically upwards to intersection with well-head pressure on diagonal line of 400 psi.
(4) From this intersection move horizontally to choke size of $^{14}/_{64}$ in.
(5) From this intersection proceed vertically upwards to the top of the nomograph and find the flow rate to be 152 bpd.

(b) Method of Gilbert

(1) Use formula of Gilbert:

$$\frac{p_{wh}}{1} = \frac{435\ R^{0.546} q}{S^{1.89}}$$

$$q = p_{wh}\left(\frac{S^{1.89}}{435\ R^{0.546}}\right)$$

$$q = (400)\left(\frac{(14)^{1.89}}{435\left(\frac{1,000}{1,000}\right)^{0.546}}\right)$$

$$q = \frac{(400)}{435}\frac{(147)}{1} = 135 \text{ bpd}$$

(2) Gilbert's solution from prepared charts of Gilbert. (Fig. G.1—Appendix)
These two charts actually represent one chart and must be used together.
(a) On part II find intersection of bean size $\left(\frac{14}{64}\right)$ and tubing head pressure (400 psi).
(b) Proceed vertically upwards to $^{10}/_{64}$ in. choke line.
(c) Proceed horizontally to the left entering Part I until intersecting the gas-liquid ratio line (1,000 scf/bbl = 1.0 Mcf/bbl)
(d) Proceed vertically upwards to the rate in bpd = 135 bpd.
This checks the formula which gives 135 bpd.

Gilbert's solution gives 135 bpd compared to 155 pd by the Poettmann and Beck Method. Additional eld data is needed for further verification, but we ould presently recommend the Poettmann and Beck olution.

LASS PROBLEM #4-A: TO DETERMINE THE FLOW ATE THROUGH A BEAN

Given data: °API = 40
p_{wh} = 600 psia
Bean size = $^{12}/_{64}$ in.
G/L = 1,500 scf/bbl
Pressure downstream = 200 psia

ind the flow rate in bpd.

LASS PROBLEM #4-B: TO FIND THE FLOW RATE HROUGH A BEAN

Given data: °API = 30
p_{wh} = 500 psia
Bean size = $^{10}/_{64}$ in.
G/L = 800 scf/bbl
Downstream pressure = 200 psia

Find the flow rate in bpd.

EXAMPLE PROBLEM #5: HOW TO SELECT THE CORRECT BEAN SIZE

Given data: °API = 40
p_{wh} = 500 psia
G/L = 900 scf/bbl
q = 600 bpd

Find the correct bean size.

Solution: (Refer to Fig. G.4, Appendix)
(1) Find G/L = 900 scf/bbl on abicissa
(2) Proceed vertically upward to p_{wh} = 500
(3) Find rate at top of nomograph = 600 bpd
(4) From q = 600 bpd proceed vertically downwards until intersecting the same horizontal line encountered from step (2)
(5) Read the choke size = $^{24}/_{64}$ in. on the diagonal

As a matter of interest the Lake Maracaibo chart (Fig. G.5) gives the same answer.

CLASS PROBLEM #5-A & 5-B: TO FIND BEAN SIZE

Given data: (A) °API = 30
p_{wh} = 350 psia
G/L = 1,000 scf/bbl
q = 200 bpd
(B) °API = 30
q = 2,500 bpd
p_{wh} = 300 psi
G/L = 300 scf/B

Find the bean size by:

(a) Poettmann & Beck method (Ros)
(b) Gilbert method

CLASS PROBLEM #5-C & 5-D: TO FIND BEAN SIZE

Given data: (A) °API = 40
p_{wh} = 1,000 psia
G/L = 500 scf/bbl
q = 1,000 bpd
(B) °API = 30
p_{wh} = 200
q = 2,000 bpd
G/L = 400 scf/B

Find the bean size by:

(a) Poettmann & Beck method (Ros)
(b) Gilbert method
(c) Lake Maracaibo chart

EXAMPLE PROBLEM #6: HOW TO DETERMINE BEAN SIZE TO GIVE CORRECT FLOW RATE INTO SURFACE SYSTEM

In many cases the separator pressure and length and size of flow line is known. The correct bean size in order to produce into this system must be selected.

Given data: Separator pressure = 80 psi
Length flow line = 4,000 ft
Dia. flow line = 2 in.
G/L = 600 scf/bbl
q = 800 bpd (all oil), 40° API

Find the bean size and necessary well head pressure in order to produce 800 bpd into the surface facilities.

Solution:

(1) From Fig. D.102 find the pressure downstream from the well-head choke (p_{wh_2}) necessary to move the fluids to the separator.
(Solution given previously)

$$p_{wh_2} = 310 \text{ psi}$$

(2) Assume critical flow through the choke then the pressure upstream from the choke (p_{wh_1}) = (2)(310) = 620 psia.
(3) From Fig. G.4 select the correct choke size to be $^{25}/_{64}$ in.

This solution assumes that the well can tolerate a flowing well-head pressure of 620 psia which in turn will drop to 620/2 = 310 psi immediately downstream from the choke. However, if after checking the well's flow ability (inflow performance) it is found that a well head pressure of less than 620 psi is necessary to permit a flow rate of 800 bpd, then the choke size must be increased until the desired rate is achieved. Unfortunately, multiphase flow through chokes for subsonic flow has not been properly accounted for. This choke size is generally determined by trial and error in the field until the desired rate is achieved.

CLASS PROBLEM #6-A: TO DETERMINE BEAN SIZE TO GIVE CORRECT FLOW RATE INTO SURFACE SYSTEM

Given data: Press. separator = 100 psi
Length flow line = 5,000 ft
Size flow line = 3 in.
G/L = 1,000 scf/bbl
q = 2,000 bpd, (40° API oil)

Find the necessary bean size assuming critical flow across bean.

CLASS PROBLEM #6-B: TO DETERMINE BEAN SIZE TO GIVE CORRECT FLOW RATE INTO SURFACE SYSTEM

Given data: Press. separator = 50 psi
Length flow line = 2,000 ft
Size flow line = 2 in.
G/L = 1,000 scf/bbl
q = 600 bpd

Find the bean size necessary assuming critical flow across bean.

3.53 Flow of fluids through valves and fittings

3.531 Introduction

The Crane Company[7] has conducted numerous tests to find the resistance of valves and fittings to flow. Their results are for single phase flow and additional work is needed in the area of multiphase flow. Crane[7] classified fittings as branching, reducing, expanding, or deflecting. Some examples of branching fittings are tees, crosses, side outlet elbows. Reducing or expanding fittings include swages, reducers, choke bodies, and bushings. Those fittings which change the direction of flow are called deflecting fittings and can be bends, elbows, return bends, etc.

Some piping systems may be combinations of the various types of fittings. Unions and couplings offer no appreciable resistance to flow.

3.532 Equivalent length concept

The usual manner for handling this type of problem is to convert to equivalent length in pipe diameters, or equivalent length in pipe feet from which we can find the corresponding pressure loss or pipe size. Various charts can be found in Appendix G.6 through G.10 for making these determinations for single phase flow. It is recommended that the equivalent length as determined for single phase flow be multiplied by a factor of from 3 to 5 for multiphase flow. For gas oil ratios greater than 1,000 use 5, and for gas oil ratios less than 500, a value of 3 may be used.

For example, Fig. 3.21 shows two flow lines, one with a restricted valve and one without. The pressure drop ΔP_1 is greater than ΔP_2.

Crane[7] presented a very simple explanation of the equivalent length concept.

$$h = \frac{v^2}{2g}$$

h = decrease in static head (ft) due to velocity, v(ft/sec) and is defined as the "velocity head."

If we now incorporate a valve or fitting in the line we have:

$$\left[h_L = K \frac{v^2}{2g}\right]$$ where K is the resistance coefficient which is defined as the number of velocity heads lost due to the valve or fitting.

We can express the same head loss in straight pipe by the Darcy equation:

$$h_L = \left(f \frac{L}{D}\right) \frac{v^2}{2g} \qquad (3.20)$$

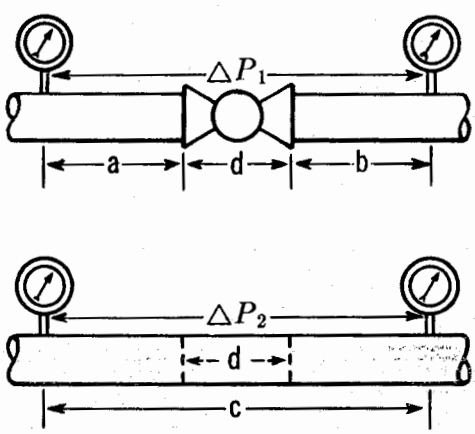


Fig. 3.21 Types of valves and fittings used in pipe systems.

Therefore:

$$K = f\frac{L}{D} \tag{3.21}$$

The ratio L/D is the equivalent length in pipe diameters of straight pipe which will cause the same pressure drop as the fitting under the same flow conditions.

Crane[7] prepared Fig. 3.22 and Table 3.5 from extensive test data. As noted from the figure, the value of K varies with size of fitting similar to friction factor for pipe, and that equivalent length L/D tends toward a constant for the various sizes of a given line of fittings. In the flow range of complete turbulence the K value for a given size and L/D value are constant. In the transition zone it is assumed that L/D is constant and that K varies in the same manner as the friction factor.

3.533 Flow coefficient

Some companies have chosen to express valve capacity and flow characteristics in terms of flow coefficient C_v. The C_v coefficient is defined as the flow of water at 60°F, in gallons per minute, for a pressure drop of 1 psi. Crane gave the following equation:

$$C_v = \frac{29.9d^2}{\sqrt{f\,L/D}} = \frac{29.9d^2}{\sqrt{K}} \tag{3.22}$$

d = dia, in.
L = length, ft
D = dia, ft

Also the flow rate is:

$$q = C_v\sqrt{\Delta P\,\frac{62.4}{\rho}} = 7.9\sqrt{C_v\,\frac{\Delta P}{\rho}} \tag{3.23}$$

ΔP = psi
ρ = lb_m/ft^3
q = gal/min

Crane[7] noted that the difference between flanged, welded, or screwed ends was insignificant. Also the pressure drop due to unions, couplings and flanged joints was insignificant.

3.534 Secondary flow in bends

Crane[7] noted that when fluid passes around a bend in either viscous or turbulent flow a condition known as "secondary flow" is set up in the bend. Fig. 3.23 illustrates this rotating motion.

Crane gave the following equations and explanations for handling this problem:

Resistance of bends to flow: The resistance or head loss in a bend is conventionally assumed to consist of . . . . (1) the loss due to curvature . . . . (2) the excess loss in the downstream tangent . . . . and (3) the loss due to length, thus:

$$h_t = h_p + h_c + h_L \tag{3.24}$$

where:

h_t = total loss, in ft of fluid
h_p = excess loss in downstream tangent, in ft of fluid
h_c = loss due to curvature, in ft of fluid
h_L = loss in bend due to length, in ft of fluid

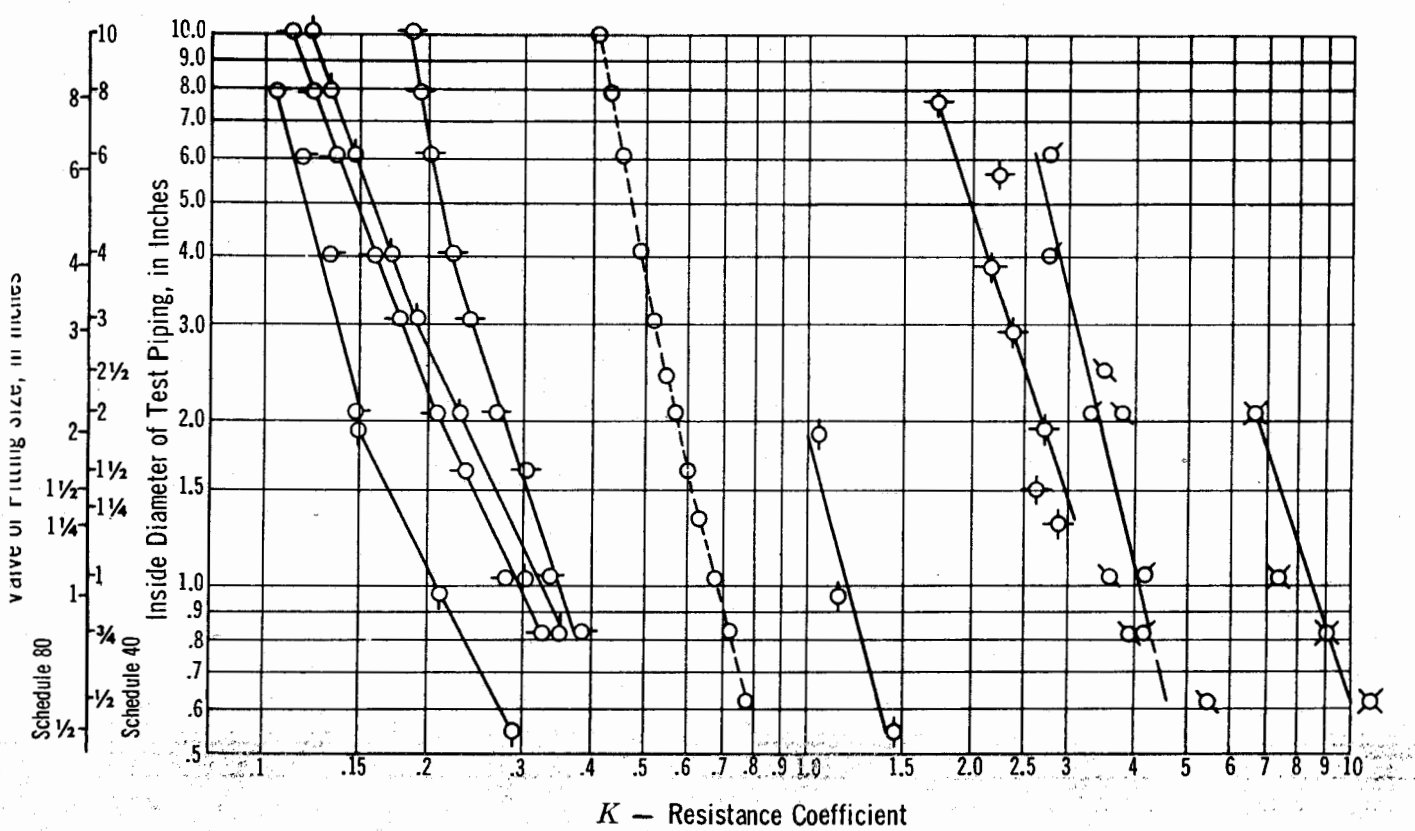


Fig. 3.22 Variation of resistance coefficient K (after Crane).

Table 3.5 Variation of Resistance Coefficient K (= f L/D) with Size

Symbol	Product Tested	Authority
o	— Schedule 40 Pipe, 30 Diameters Long ($K = 30f$)	Moody A.S.M.E. Trans., Nov.-1944[1]
o-	— 125-Pound Iron Body Wedge Gate Valves	Univ. of Wisc. Exp. Sta. Bull., Vol. 9, No. 1, 1922[16]
♀	— 600-Pound Steel Wedge Gate Valves	Crane Tests
-o	— 90 Degree Pipe Bends, $R/D = 2$	Pigott A.S.M.E. Trans., 1950[6]
o	— 90 Degree Pipe Bends, $R/D = 3$	Pigott A.S.M.E. Trans., 1950[6]
-o-	— 90 Degree Pipe Bends, $R/D = 1$	Pigott A.S.M.E. Trans., 1950[6]
♀	— 600-Pound Steel Wedge Gate Valves, Seat Reduced	Crane Tests
-◇-	— 300-Pound Steel Venturi Ball-Cage Gate Valves	Crane-Armour Tests
♂	— 125-Pound Iron Body Y-Pattern Globe Valves	Crane-Armour Tests
♌	— 125-Pound Brass Angle Valves, Composition Disc	Crane Tests
♋	— 125-Pound Brass Globe Valves, Composition Disc	Crane Tests

if:

$$h_b = h_p + h_c \tag{3.25}$$

then:

$$h_t = h_b + h_L \tag{3.26}$$

However, the quantity h_b can be expressed as a function of velocity head in the formula:

$$h_b = K_b \frac{v^2}{2g} \tag{3.27}$$

where:

K_b = the bend coefficient
v = velocity through pipe, ft/sec
g = 32.2 ft/sec^2

Attempts have been made to correlate K_b with relative radius, r/d, (ratio of the radius of the bend axis to the internal diameter of the pipe). Note Fig. 3.24 and the included Table 3.6.

Resistance of 90° bends can be found in Appendix G.7. Crane[7] also showed that the loss due to continuous bends greater than 90°, such as in pipe coils, is less than the summation of the losses in the 90° bends contained in the coil considered separately.

Crane noted that reasonably accurate results for pipe coils and expansion loops consisting of continuous bends can be obtained by the use of the chart (Appendix G.7) . . . if the number of 90° bends contained in the coil minus one, multiplied by the resistance due to length plus one-half of the bend resistance, is added to the total resistance of a 90° bend.

For example, a pipe coil consisting of four complete turns . . . sixteen 90° bends . . . and having a relative radius of five pipe diameters, would have a total equivalent length, in pipe diameters, of:

$$15\ (8 + 4) + 16 = 196$$

It will be noted that this assumes $h_p = h_c$ in Equation 3.24. This relationship has not been established by tests but is believed to represent the most accurate estimate that can be made until further experimental data are available.

Resistance of miter bends: The equivalent length of miter bends, based on the work of H. Kirchbach[20] is also shown on Fig. G.7 of Appendix G.

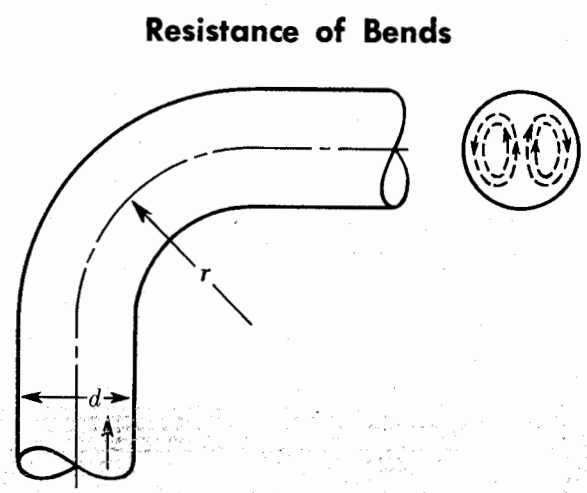


Fig. 3.23 Secondary flow in bends (after Crane).

3.535 Other resistances to flow

There can be losses due to enlargements and contractions such as swages or reducers. These can also be expressed as:

$$h_L = \frac{Kv^2}{2g} \tag{3.28}$$

As stated by Crane, "there is no length involved in losses due to these conditions; thus, relative roughness is not a factor in these resistances, and geometric similarity does exist. The resistance due to sudden enlargement and sudden contraction, as well as entrance and exit losses expressed in terms of velocity head or K factor, are therefore independent of pipe size. Resistance coefficients, K, for such conditions are given in Appendix G.6."

"Equivalent lengths corresponding to these resistance coefficients for any size can be readily determined from Appendix G.9. For example, the equivalent lengths of sharp edged entrances (K = 0.5) to 2 and 6-in. pipes can be read from the nomograph of Appendix G.9 as 26 diameters of 2-in. pipe and 33 diameters of 6-in. pipe, respectively."

3.54 Summary

Unfortunately the effect of bends, fittings, and other standard piping configuration changes have not been

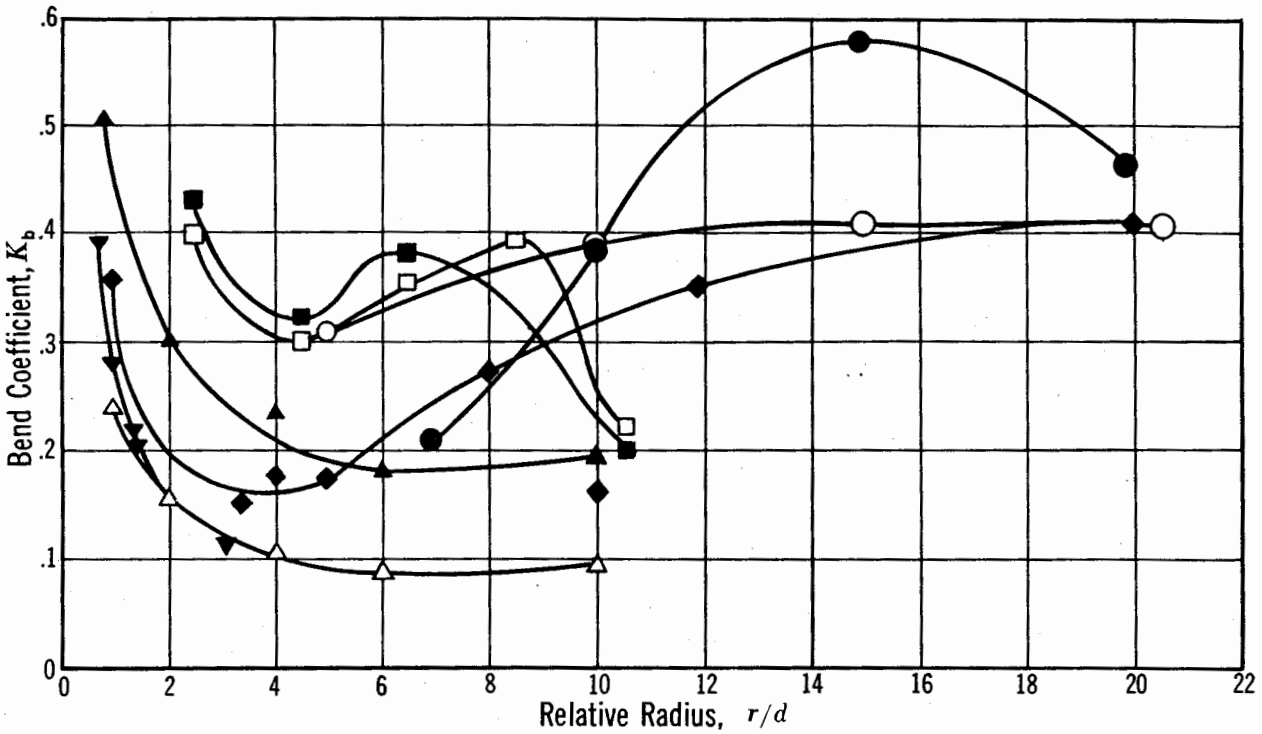


Fig. 3.24 Resistance of bends—continued (after Crane).

TABLE 3.6 Bend Coefficients Found by Various Investigators (Beij)

From "Pressure Losses for Fluid Flow in 90° Pipe Bends" by K. H. Beij. Courtesy of Journal of Research of National Bureau of Standards.

Investigator	Diameter	Symbol (see above figure)
Balch	3-inch	●
Davis	2-inch	○
Brightmore	3-inch	■
Brightmore	4-inch	□
Hofmann	1.7-inch (rough pipe)	▲
Hofmann	1.7-inch (smooth pipe)	△
Vogel	6, 8, and 10-inch	▼
Beij	4-inch	◆

stablished for multiphase flow. I would like to suggest the following solution to this problem:

(1) Establish single phase pressure loss for a section of the piping used.
(2) Establish multiphase pressure loss for the same section of pipe as in step (1).
(3) Establish equivalent lengths of fittings, bends, etc. for single phase flow.
(4) Multiply (3) by ratio of step 2/step 1 and use for equivalent length to find multiphase pressure loss.

6 WELLS FLOWING WITH NO SURFACE CHOKES (UNRESTRICTED PRODUCTION)

3.61 Introduction

There are numerous instances where it is desirable to produce a flowing well at its maximum capacity, that is, where all surface restrictions are removed. Also the surface well head pressure may be kept constant for one choke size, and the following discussions would apply for both cases.

The future production potential can be predicted by the construction of a series of inflow performance curves for different time intervals and/or cumulative productions of a well. The decline of the static bottom hole pressures must be known and the shape and/or change in shape of the IPR curves must be known. The

future trends in static reservoir pressure, PI, G/L and water-cut should be available (Fig. 3.13A). The production engineer should work very closely with the reservoir engineer in this regard.

If proper plots can be made such as Fig. 3.13B, then future predictions can be made. If the bottom hole static pressure declines from $\bar{p}_{R_1}$ to $\bar{p}_{R_2}$ and for a constant G/L the equilibrium point will move from A to B and the production rate will decline from q_1 to q_2. However, it is also likely that the G/L will change. If the G/L declines the equilibrium point will move to C, and if it increases the equilibrium point will move to point D.

If good pressure-rate diagrams can be constructed then we can predict:

(a) Future production rates
(b) When the well will die
(c) Gains in production if secondary recovery or pressure maintenance is applied
(d) Gains in production if artificial lift methods are applied
(e) Prediction of the effects of stimulation, workover, change of tubing sizes, etc.
(f) Effect of water-cut

In order to optimize the production rates from a group of wells in one field we may encounter a different problem as compared to one well since we have the influence of static bottom hole pressures, G/L's, water-cut, etc. There may be alternatives in how to produce the same total amount, especially in artificial lift methods. Also any secondary recovery methods or pressure maintenance programs will influence the individual wells in a different manner. Several alternative production forecasts will probably need to be made and of course an economic evaluation must be made.

3.62 Effect of variables

The effect of variables in a flowing well have already been discussed in Chapter 2 on Multiphase Flow. Such things as the effect of tubing size, rate, gas liquid ratio, kinetic energy, viscosity, surface tension, density, and water-oil ratio have already been shown. Brown discussed these variables in some detail and a portion of his work is discussed here. The effect of tubing sizes and surface flow line sizes have been discussed in Chapter 2.

Fig. 3.25 shows the various well-head pressures that can be encountered in a flowing well. For example, curve #1 illustrates how as the rate is reduced the well-head pressure increases until excess fall-back is encountered at which time the well-head pressure decreases as the rate decreases. This is accomplished by closing the well in very slowly and perhaps allowing it to flow until stabilized on each incremental decrease in choke size. If the well is closed in very rapidly it will show an increased p_{wh} as shown on Fig. 3.25 and p_{wh} may continue to rise after shut-in depending upon the redistribution of fluids in the tubing string.

Curve #2 shows the well-head pressures necessary to move various rates through the horizontal flow line and is determined independently of the well's capability. It is dependent on the separator pressure, flow

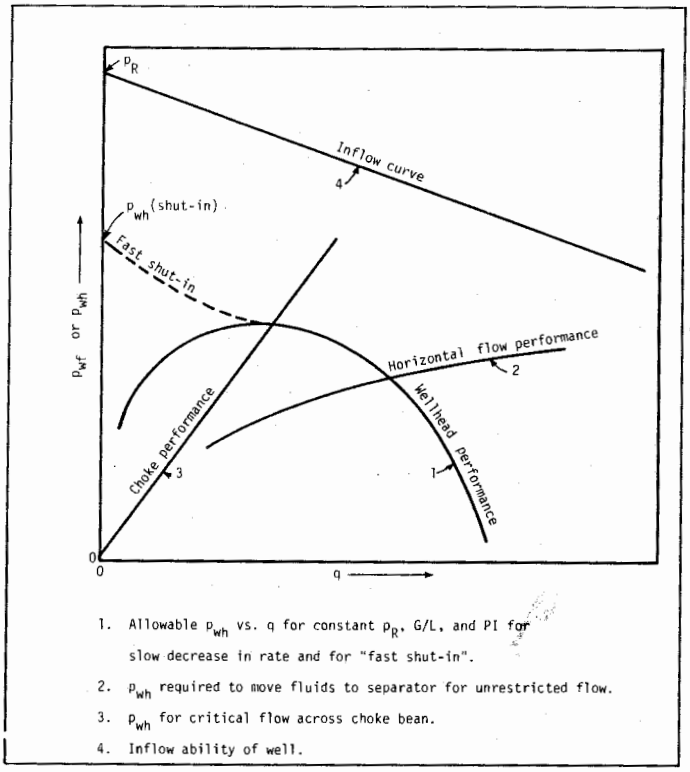


Fig. 3.25 Illustrations of various well-head pressures.

rates of liquid and gas, and length and diameter of flow line. This is for non-restricted flow.

Curve #3 is for one choke size and critical flow is assumed.

3.63 Method to predict rate possible from a flowing well (for no restrictions)

3.631 Introduction

If we are looking for the maximum rate possible from a well, then there must be no chokes or restrictions at the surface. We also have two possibilities that exist: (1) constant well-head pressure and (2) variable well-head pressure depending upon the rate. The problem differs considerably depending upon whether or not the well-head pressure remains a constant value with rate.

For wells producing, for example, on off-shore platforms with very short flow lines, the well-head pressure is mostly dependent on the separator pressure and may remain fairly constant with rate. For wells producing into long surface flow lines, the well-head pressure varies considerably as the rate changes. First, we will look at an example where the well-head pressure remains constant.

EXAMPLE PROBLEM #7-A: HOW TO DETERMINE THE FLOW RATE POSSIBLE FROM A WELL WITH CONSTANT WELLHEAD PRESSURE

There are instances when it is desirable to know the exact flow rate from a flowing well for set constant conditions. For this problem the IPR curve for the well must be known and its intersection with the vertical multiphase flow performance curve gives the exact rate.

Given: $d = 8{,}000$ ft
$\bar{p}_R = 2{,}300$ psi
PI = 2 (assume linear)
$p_{wh} = 120$ psi
G/O = 600 scf/bbl
Tubing = 2½ in.

Determine the flow rate (all oil).

lution:

(1) Construct a plot as follows of p_{wf} vs. q (Fig. 3.26)
(2) Draw in the inflow performance curve. Since this is linear (straight line) assume any two rates and find the corresponding flowing pressures.

Assume $q = 0$ then $p_{wf} = \bar{p}_R = 2{,}300$

Assume $q = 1{,}000$ bpd then $p_{wf} = 2{,}300 - \frac{1{,}000}{2}$
$= 1{,}800$ psi

Draw a straight line between (0, 2,300) and (1,000, 1,800). Label this line as the IPR curve.
(3) Assume various flow rates and determine the corresponding flowing bottom hole pressures from the appropriate vertical multiphase flow correlation or working curves.

Assumed rate	p_{wf}
1,000	1,360
1,200	1,440
1,500	1,570

This information is plotted in Fig. 3.26. The answer is 1,480 bpd and the corresponding flowing bottom hole pressure is 1,560 psi.
(4) Check this rate and pressure against the PI.

$$p_{wf} = \bar{p}_R - \frac{q}{PI} = 2{,}300 - \frac{1{,}480}{2} = 1{,}560 \text{ psi}$$

(Checks OK)

3.632 Selection of tubing sizes for constant wellhead pressure

In the manner of the preceding example problem we can determine the rates possible for several tubing sizes (in this case, for constant well-head pressure). Reference should be made to Fig. 3.27 which shows the flow rates possible for 2 in., 3 in., and 4 in. nominal tubing sizes.

This example is for wells completed in the Cook Inlet of Alaska, which utilize 3½ in.–4½ in. O.D. tubing. Some of these are also completed as duals and utilize large casing sizes depending upon the dual combination. These wells produce from 4,000 – 8,000 bpd.

Fig. 3.28 shows an original well bore hydraulics curve that was prepared for the Trading Bay Unit in the Cook Inlet of Alaska. Inflow curves are given for each 200 psi drop in static pressure and for PIs of 4 and 2. Tubing sizes of 2½ in., 3 in. and 4 in. nominal sizes are given. Again it is quite easy to determine the flow

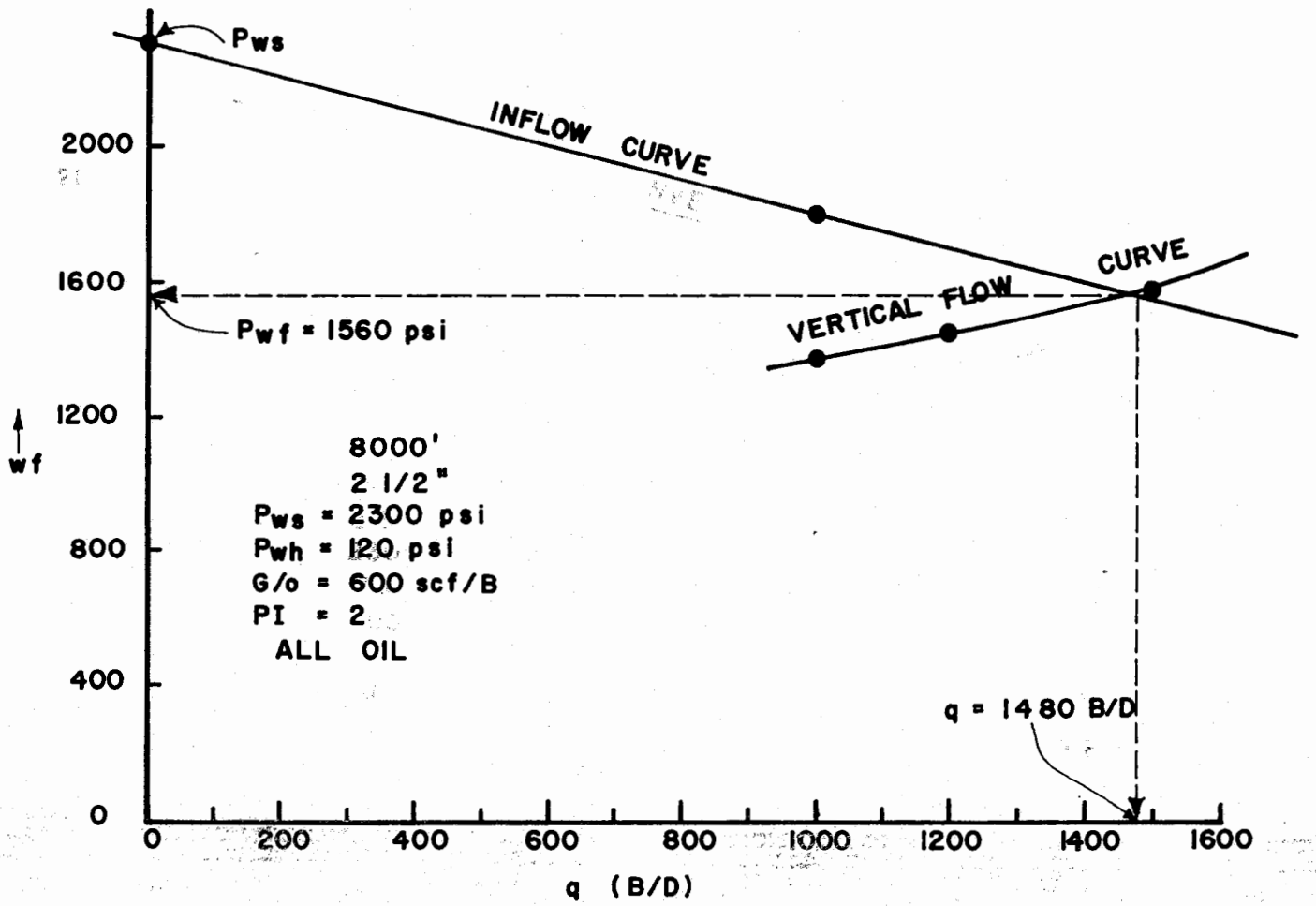


Fig. 3.26 Method for determining flow rate.

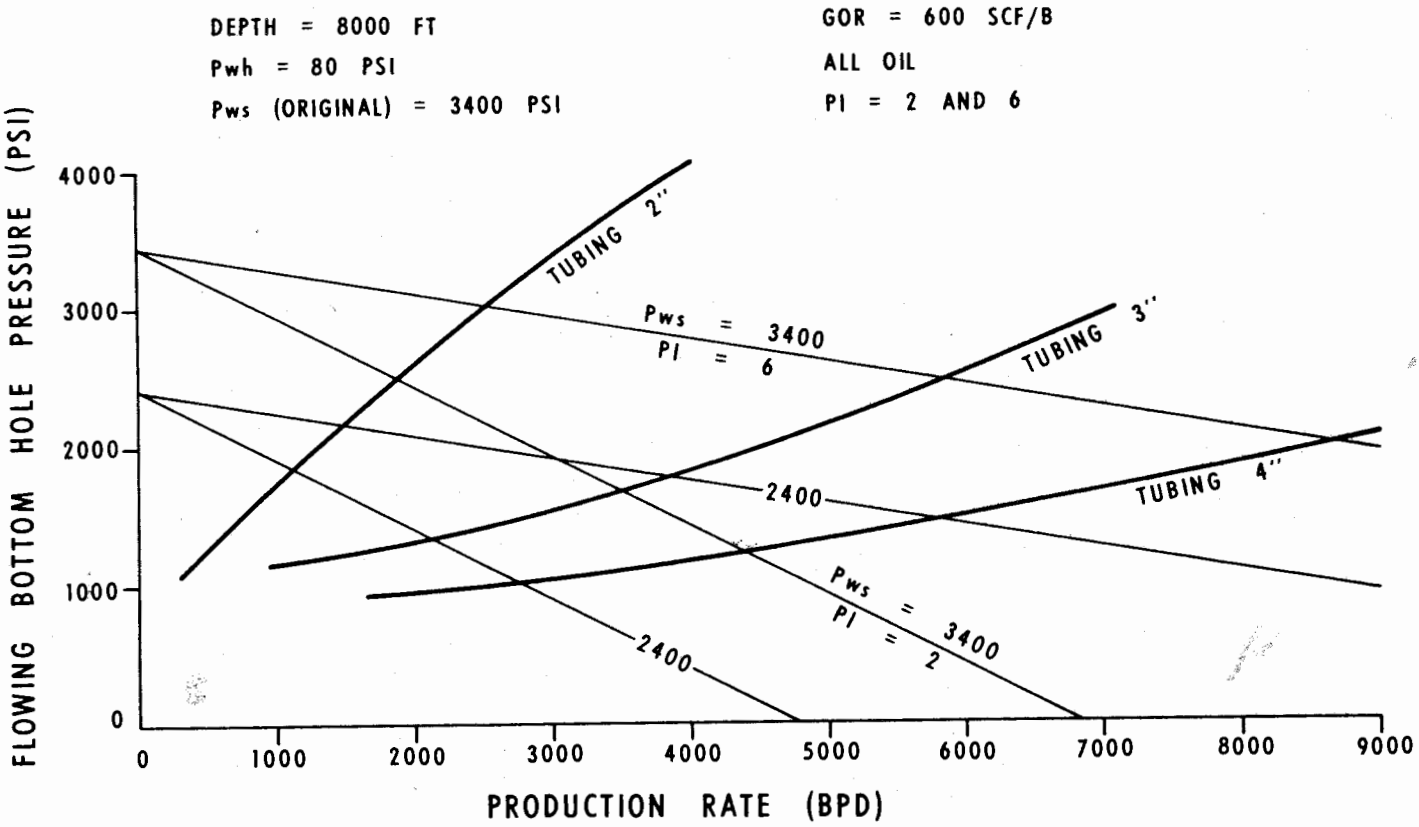


Fig. 3.27 Effects of tubing sizes.

rates for each tubing size and at any static pressure for a PI of 4 or 2.

The selection of a particular tubing size can be made by preparing plots similar to Figs. 3.27 and 3.28. The completion program on these types of wells should be made prior to drilling, and analyses of these types help in making these decisions. For example, we have those wells in the Middle East and Africa that produce very high rates that may vary from 10,000–30,000 bpd and even as high as 60,000–80,000 bpd. The completion casing strings must be decided upon prior to drilling operations. Present day multiphase flow correlations are not yet proven for the larger producing strings in particular in excess of 4 in. tubing or annular flow where 7 in. casing or larger is being used. These Figures show the flow rate possible for changing bottom hole pressures, differing PI's, and variable tubing sizes.

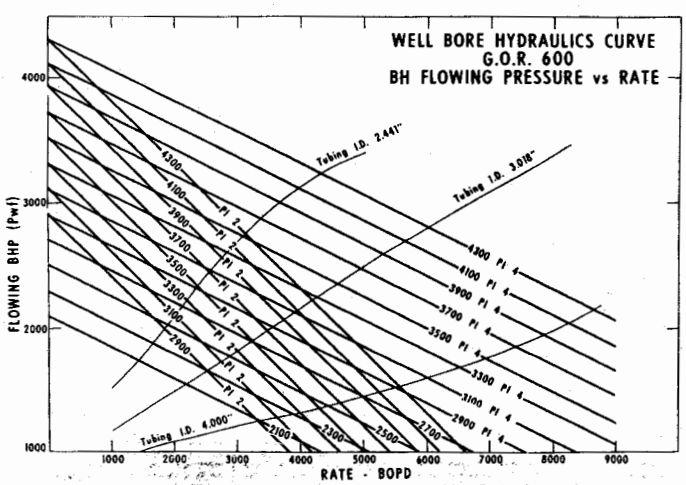


Fig. 3.28 Well bore hydraulics curve g.o.r. 600 bh flowing pressure vs rate.

For example, we note that for a PI of 6 and $\bar{p}_R$ of 3,400 in Fig. 3.27, the flow rates possible for 4 in., 3 in., and 2 in. tubing respectively are 8,700 bpd, 5,900 bpd, and 2,600 bpd. This immediately shows how the tubing size affects the rate. This same type of analysis resulted in 4 in. tubing being selected for wells in the Cook Inlet area of Alaska.

EXAMPLE PROBLEM #7-B: HOW TO ANALYZE TUBING SIZE SELECTION

Given data: Depth = 8,000 ft
p_{wh} = constant at 80 psi
Wells on offshore platform with short large surface flow systems, producing all oil.
$\bar{p}_R$ = 3,000 psi, but will drop 500 psi every year until reaching 2,000 psi at which time a water-flood operation will start increasing $\bar{p}_R$.
G/L = 600 scf/bbl

The PIs for similar wells may vary between 2 and 10. Prepare a graph of flowing pressure vs. flow rate which illustrates effect of PI, $\bar{p}_R$, and tubing size.

Solution procedure:

(1) Plot p_{wf} vs. q (Fig. 3.29)

(2) Construct inflow performance curves for $\bar{p}_R$ = 3,000, 2,500, and 2,000 psi, and for PIs of 10, 5, and 2. (Assume linear for this problem).

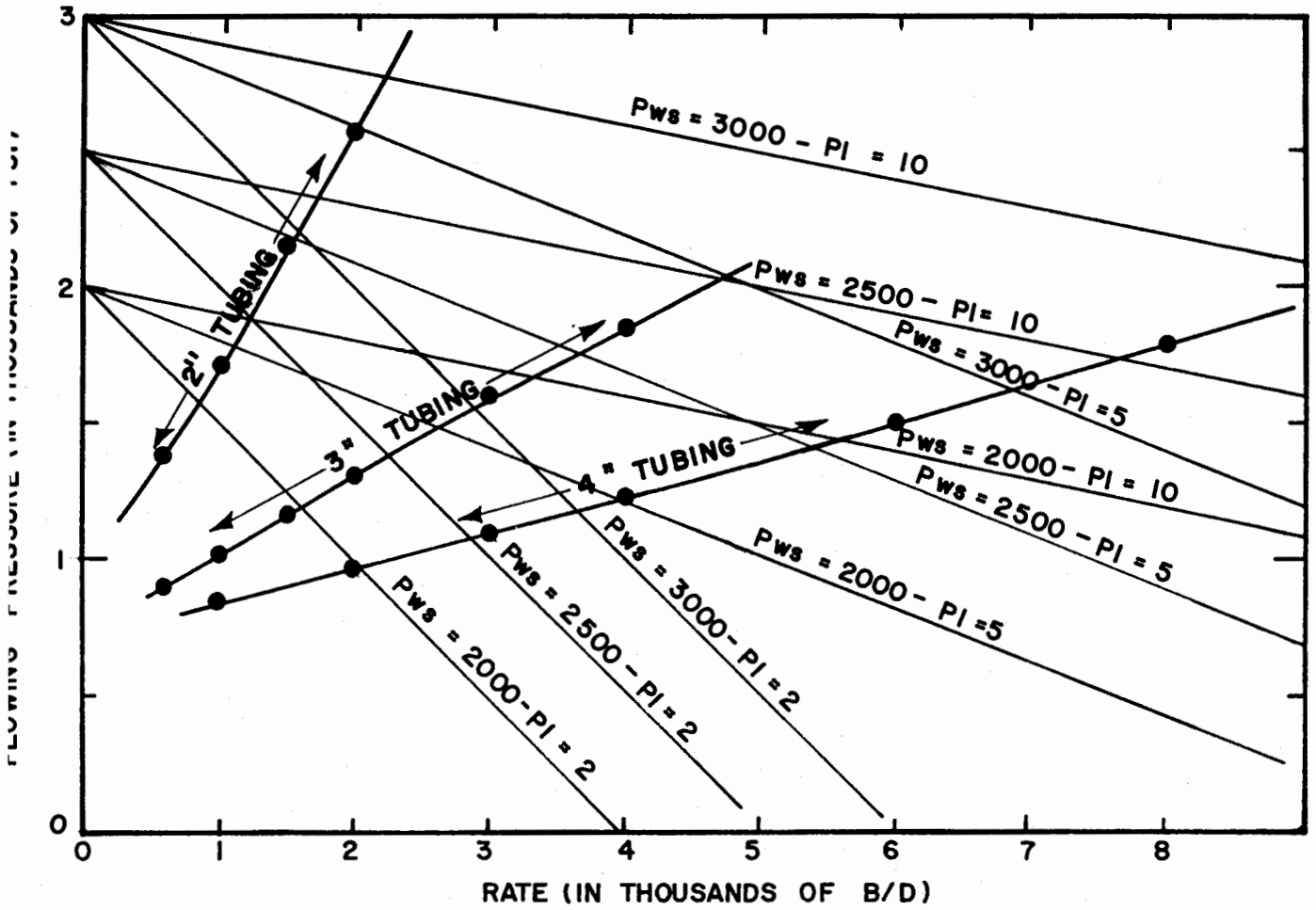


Fig. 3.29 Tubing selection analysis.

(3) From the well-head pressure and for different tubing sizes, assume several rates and determine the corresponding bottom hole flowing pressures.

Rate, bpd	Flowing pressure 2 in.	3 in.	4 in.
10,000			2,020
8,000			1,760
6,000			1,500
4,000		1,850	1,205
3,000		1,570	1,080
2,000	2,560	1,280	940
1,500	2,120	1,160	880
1,000	1,700	1,010	840
600	1,360	885	

(4) Plot rate vs. flowing pressure for each tubing size as noted on Fig. 3.29.

The following Table 3.7 shows typical rates from Fig. 3.26.

Depending upon the most efficient and economical rate (provided it is permissible by regulatory bodies) then the tubing size can be easily selected from Fig. 3.29.

For example, if our well has a PI = 5 and we want a rate of at least 3,500 bpd when $\bar{p}_R$ = 2,500 psi, then we would select 3 in. tubing.

TABLE 3.7

	For $\bar{p}_R$ = 2,500 psi Tubing size		
PI	2 in.	3 in.	4 in.
10	1,700	4,700	7,700
5	1,550	3,700	5,400
2	1,225	2,300	2,850

Fig. 3.30 was prepared to show the effect of slippage, and to give some indication of when a tubing size becomes too large. Various correlations for multiphase flow will differ some or perhaps significantly in that flow regime where heading begins to occur. The example of Fig. 3.30 was prepared from the Hagedorn and Brown correlation, and for the well in question shows 3½ in. tubing becoming more efficient than 4½ in. at 480 bpd, 2⅞ in. more efficient than 3½ in. at 180 bpd, and 2⅜ in. more efficient than 2⅞ in. at 100 bpd. These values will differ depending on the individual well.

Fig. 3.31 shows the same well as presented by Juch using another multiphase flow correlation. In this illustration the tubing sizes never cross, that is 3½ in. appears to be always more efficient than 2⅞ in. or 2⅜ in. I do not think this is the case, hence some error

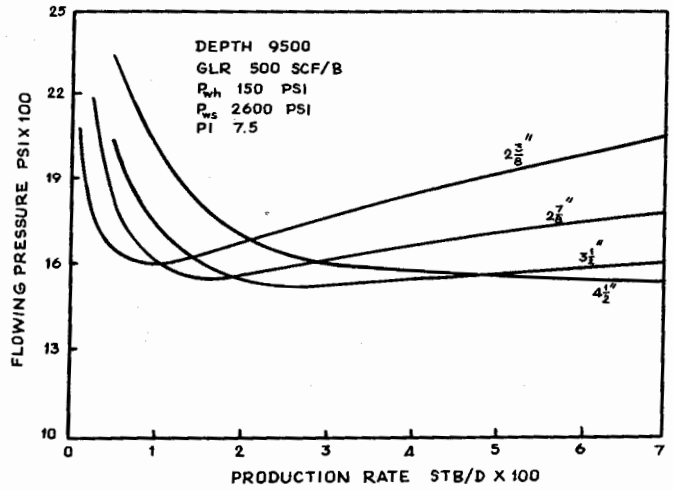


Fig. 3.30 Comparison of flow configuration size.

in the multiphase flow correlation exists in this range of flow.

Fig. 3.31 is presented to show an actual well and the flow rates possible by different flow configurations including annular flow. As can be seen, $4\frac{1}{2}$ in. tubing and a 7 in. × $3\frac{1}{2}$ in. annulus will produce approximately the same.

CLASS PROBLEM #7-A: TO DETERMINE THE FLOW RATE POSSIBLE FROM A WELL

(1) Data same as for example problem #7-A, except use 2 in. tubing.
(2) Work example problem #7-A for all water for both 2 in., $2\frac{1}{2}$ in., and 3 in. tubing.

CLASS PROBLEM #7-B: TO DETERMINE THE FLOW RATE POSSIBLE FROM A WELL

Given: $\bar{p}_R = 3{,}000$
PI = 1.3 (assume linear)
$p_{wh} = 120$
G/L = 400
d = 8000 ft
2 in. = tubing

Determine the flow rate (all oil).

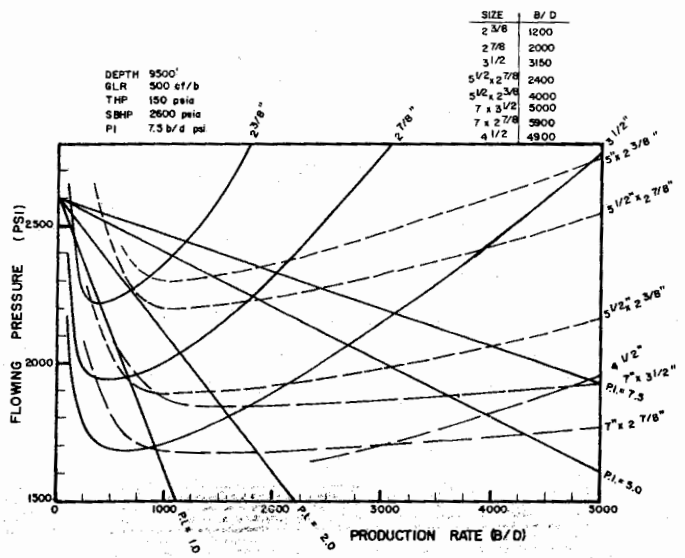


Fig. 3.31 Comparison of flow configuration size (after Juch).

CLASS PROBLEM #7-C: TO DETERMINE THE FLOW RATE POSSIBLE FROM A WELL

Given: d = 7000 ft
G/O = 700 scf/bbl
$p_{wh} = 100$
$\bar{p}_R = 2{,}700$
PI = 1.6 (assume linear)

Find flow rate possible for 2 in. and $2\frac{1}{2}$ in. tubing for all oil.

CLASS PROBLEM #7-D: TO DETERMINE THE FLOW RATE POSSIBLE FROM A WELL

Given: d = 8500 ft
PI = 6
$\bar{p}_R = 3{,}000$ psi
G/L = 1,000 scf/bbl
$p_{wh} = 160$
3 in. tubing

Find flow rate possible for (1) all oil, (2) all water, (3) 50% oil.

CLASS PROBLEM #7-E

Given: d = 8000 ft
GOR = 400 scf/bbl
$p_{wh} = 120$
PI = 6
$\bar{p}_R = 3{,}000$ psi

Find q_o (all oil) for: (a) 4 in. tubing
(b) 3 in. tubing
(c) 2 in. tubing

CLASS PROBLEM #7-F

The following test was observed on a well known to be producing below the bubble point on a solution gas drive well.

Given: $\bar{p}_R = 2{,}400$ psi
d = 7,000 ft
GOR = 400 scf/bbl
$2\frac{1}{2}$ in. tubing
$p_{wh} = 80$ psi

One test was taken on this well while flowing against a choke showing a rate of 500 bpd for a flowing pressure of 2,100 psi. Assuming the well behaves according to Vogel's Reference Curve, find the flow rate possible for $p_{wh} = 80$ psi.

CLASS PROBLEM #7-G

Given data: Depth = 8,000 ft
$\bar{p}_R = 2{,}800$ psi
PI = 10 (originally)
Tubing size = $2\frac{1}{2}$ in. and 4 in.
$p_{wh} = 80$ psi
G/O = 500 scf/bbl

In two years time $\bar{p}_R$ decreases to 2,000 psi and PI = 7. In four years time $\bar{p}_R$ decreases to 1,000 psi and PI = 4. In five years time $\bar{p}_R$ decreases to 700 psi and PI = 2. (Assume PI Linear in all cases)

Find: (For both 2½ in. and 4 in. Tubing)
(a) The rate at which the well will produce for each year.
(b) Find that time in years when the well will no longer flow for each tubing size.

CLASS PROBLEM #7-H: TO ANALYZE TUBING SIZE SELECTION GRAPH

Given data for typical well in field:
Depth = 7,500 ft
Wellhead pressure = 100 psi (constant)
G/L = 800 scf/bbl
$\bar{p}_R$ = 2,800 psi now and drops to 2,300 psi in 2 years
PI's vary from 8 to 4
Tubing sizes available = 2 in., 2½ in., and 4 in.

Prepare a figure to illustrate rates possible for the following conditions:

$\bar{p}_R$ = 2,800	PI = 8 and 4
$\bar{p}_R$ = 2,300	PI = 8 and 4

CLASS PROBLEM #7-I: TO ANALYZE TUBING SIZE SELECTION GRAPH

Given data: Depth = 7,000 ft
Wellhead pressure = 120 psi (constant)
G/L = 500 scf/bbl
$\bar{p}_R$ = 2,500 psi now and 2,100 psi in 1 year
PI's vary from 4 to 1

Prepare a figure to illustrate rates possible from tubing sizes of 2 in., 3 in., and 4 in.
Show the effect of producing all water (same G/L = 500) as compared to producing all oil for the 3 in. tubing size only.

3.633 Determination of flow rates and selection of tubing sizes for wells with variable wellhead pressures

In a real case of field wells having surface flowlines of any significant length (500 ft or more) the loss in the surface flowline must be included in the solution. In order to accurately predict the flow rate possible from a well, it is necessary to utilize both the vertical and horizontal multiphase flow correlations. This solution assumes no wellhead restrictions.

In general, we know the separator pressure, the length and diameter of flow line, the length and diameter of tubing, rate, G/L, necessary fluid properties, the static bottom hole pressure and the PI. The preferred solution is to prepare a pressure-flowrate diagram in which we construct the inflow performance curve vs. flowing bottom hole pressures required to move the fluids from the bottom of the tubing string to the wellhead and on to the separator.

This solution can be best illustrated by working an example problem. Reference should be made to Fig. 3.32.

EXAMPLE PROBLEM #8-A

The following data is given:
Depth of well = 7,000 ft
Flowline length = 3,000 ft

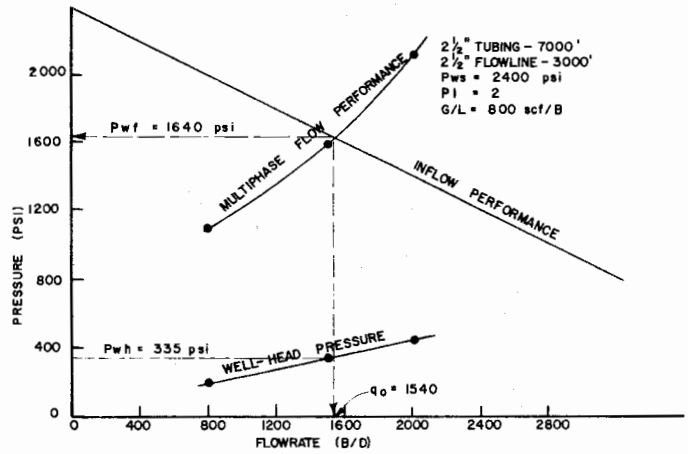


Fig. 3.32 Predicting unrestricted flow rate (solution 1).

$\bar{p}_R$ = 2,400 psi
PI = 2 (assume linear)
G/L = 800 scf/bbl
Separator pressure = 100 psi
Tubing diameter = 2½ in.
Flowline size = 2½ in.
Production = 100% oil

Temperatures and fluid properties are such that we will use gradient curves of Appendix C.8 and D.10. Find flow rate possible if producing all oil.

Solution procedure #1:
(1) Plot pressure rate diagram as noted in Fig. 3.32.
(2) Construct inflow performance curve by assuming two rates, finding the corresponding flowing pressures and draw in a straight line. First, use zero rate at which rate the flowing pressure is equal to the static pressure. Then assume any other rate, for example 2,000 bpd and find the corresponding flowing pressure from the PI (assume linear)

For 2,000 bpd:

$$p_{wf} = \bar{p}_R - \frac{q}{PI} = 2{,}400 - \frac{2{,}000}{2} = 1{,}400 \text{ psi}$$

We now have the following 2 points.

$\frac{q}{0}$	$\frac{p_{wf}}{2{,}400}$
2,000	1,400

Plot this inflow performance curve as a straight line on Fig. 3.17 as shown. For a solution drive reservoir the inflow drive may be constructed by method of Vogel.
(3) Assume various production rates such as 800, 1,500, and 2,000 bpd.
(4) For each production rate and beginning with the separator pressure determine the necessary well head pressures to move these flow rates through the surface flow line to the separator.
These are tabulated as follows:

Assumed rate	Required p_{wh}	Fig. in Appendix
800	195	D.107
1,500	330	D.109
2,000	450	D.1010

(5) Using the assumed rates, and the corresponding well head pressures, find the flowing bottom holes pressures required from vertical flow correlations.

Rate	p_{wh}	p_{wf}	Fig. in Appendix
800	195	1080	C.104
1,500	330	1600	C.106
2,000	450	1980	C.107

(6) Plot the multiphase flow performance curve on Fig. 3.32 as noted.
(7) The flow rate possible (also the maximum rate possible from this well since we have no restrictions) is found from the intersection of the inflow performance curve and the multiphase flow performance curve, and is 1,540 bpd with a flowing pressure of 1,640 psi.
(8) The well head pressures can also be plotted on Fig. 3.32 and the well head pressure at which this rate occurs is noted to be 335 psi.

Solution procedure #2:

This problem can be worked in another manner as follows. A plot of flow rate vs well head pressure is made as noted in Fig. 3.33.

(1) Prepare plot of well-head pressure vs. flow rate as noted in Fig. 3.33.

(2) Assume various flow rates such as 800, 1,500, and 2,000 bpd.
(3) For each assumed rate and beginning with the separator pressure determine the necessary well head pressures to move these flow rates through the surface flow line to the separator. These are the same as found in Step (4) of Solution Procedure 1 and are tabulated as follows:

Assumed rate	Required p_{wh}
800	195
1,500	330
2,000	450

Plot q vs. p_{wh} as noted in Fig. 3.33.
(4) For each flow rate, determine the required flowing pressure from the inflow ability of the well.

For example, for 800 bpd the p_{wf} necessary for the well to give up this rate is $2{,}400 - \frac{800}{2} = 2{,}000$ psi (Assume PI = linear)

Tabulate each p_{wf} required from the well's inflow ability. (PI = 2)

Assumed rate	Required p_{wf}
800	2,000
1,500	1,650
2,000	1,400

(5) From vertical flow correlations determine the permissible well-head pressure for each of the assumed rates. For example, for q = 1,500 bpd find 1,650 psi on the 800 scf/bbl G/L line of Fig. C.92 in the Appendix. Note this depth to be at 9,650 ft. Subtract 7,000 ft from 9,650 ft = 2,650 ft. Find p_{wh} at 2,650 ft on the 800 scf/bbl line to be 360 psi. The tabulation for all rates is as follows:

Assumed q	Required p_{wf}	p_{wh} allowed
800	2,000	–
1,500	1,650	360
2,000	1,400	70
1,200	1,800	500

Plot q vs. p_{wh} as noted in Fig. 3.33.
(6) The intersection of these two curves gives the flow rate of 1,540 bpd for p_{wh} = 335 psi.

This checks with Solution #1.

This problem lends itself well to machine computation and should be programmed for greater accuracy.

The solution procedure #2 as noted in Fig. 3.33 lends itself well to looking at the various combinations of tubing and flow line sizes available as noted in Fig. 3.34 and from data as given in the following example problem #8-B.

EXAMPLE PROBLEM #8-B: HOW TO SELECT THE CORRECT COMBINATION OF TUBING AND FLOWLINE SIZES

Given data: (Refer to Fig. 3.34)
Depth = 7,000 ft

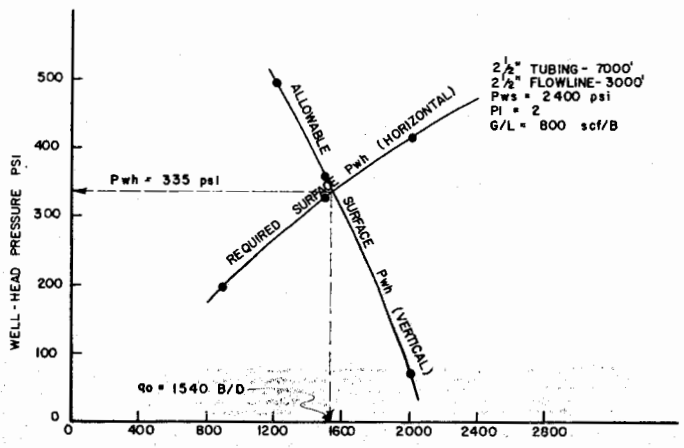


Fig. 3.33 Predicting unrestricted flow rate (solution 2).

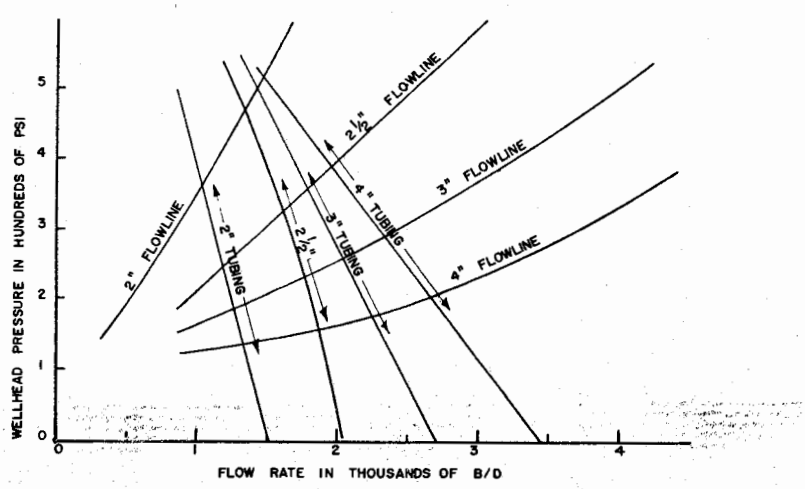


Fig. 3.34 Tubing and flowline analysis chart.

Flowline length = 4,000 ft
$\bar{p}_R$ = 2,800 psi
PI = 2
Pressure of separator = 100 psi
q_L = 2,000 bpd (90% water)
G/L = 500 scf/bbl

Find the most economical size of tubing and flowline to produce this well if the following sizes are available.

Tubing sizes available, in.	Flowline sizes available, in.
2	2
2½	2½
3	3
4	4

This example represents a typical production problem in which the selection of a tubing size as well as a flowline size must be made from the available supply of tubing and flowlines.

The problem can be worked in two different ways but the following is recommended in that the rate possible for any combination of flowline and tubing size can be easily noted.

(1) Assume various rates possible for the separator pressure of 100 psi and 4000 ft of flowline. Tabulate p_{wh} vs. rate for each flowline size.

	p_{wh} flowline size			
Assumed rate	2 in.	2½ in.	3 in.	4 in.
600	220	160	150	
1,000	355	210	160	125
1,500	535	300	205	
2,000	720	400	250	160
3,000		610	360	
4,000		770	510	235
5,000			610	
6,000				355

(2) For the same assumed rates as noted in Step 1, find the permissible well-head pressures for each tubing size. The flowing pressure for each rate is calculated from $\bar{p}_R$ and PI.

Assumed	p_{wh} tubing size				
rate	2 in.	2½ in.	3 in.	4 in.	p_{wf}
600	660	660			2,500
1,000	380	540	540		2,300
1,500	20	380	460	480	2,150
2,000		40	265	360	1,800
3,000				120	1,300
4,000					800
5,000					
6,000					

(3) Plot this information as noted on Fig. 3.34. The various combinations to produce 2,000 bpd can then be taken from the plot. As noted, all intersections of flowline and tubing plots to the right of a vertical line through 2000 bpd will produce the well satisfactorily.

These are noted to be:

Tubing, in.	Flowline, in.
3	3
3	4
4	3
4	4

The most economical selection is 3 in. tubing and a 3 in. flowline.

Another solution to this problem is to plot a pressure-flowrate diagram with an inflow performance curve and all combinations of overall multiphase flow curves. That is, starting from the separator pressure and one assumed rate, assume one flowline size (say 3 in.) and obtain p_{wh}, then with that p_{wh} find the corresponding p_{wf} for another tubing size (say 3 in.). Repeat this procedure for several logical combinations and plot on the pressure flowrate diagram. Those combinations giving greater than 2,000 bpd represent possibilities from which the most economical or available stock sizes can be selected. Further illustrations show the effect of maintaining either the tubing or flow line size constant and varying the other. These effects are shown in Figs. 3.35, 3.36, 3.37, 3.38 and 3.39.

Figs. 3.35 and 3.36 show the effect of flowline size on a well with a set tubing size. In this example an

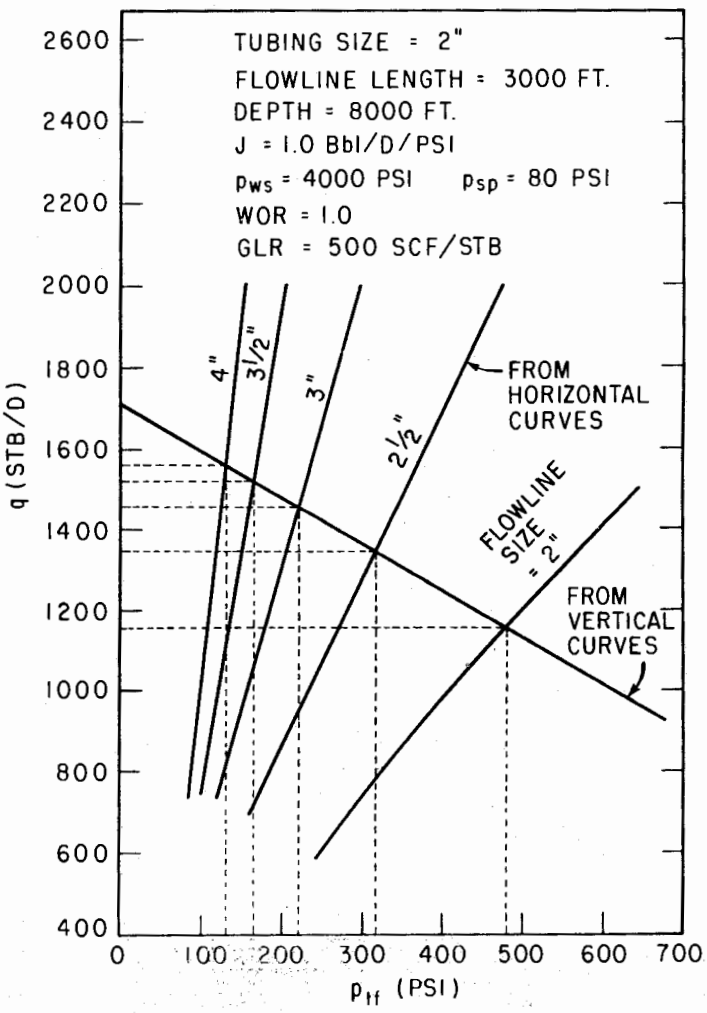


Fig. 3.35 Effect of flowline size (flowing well).

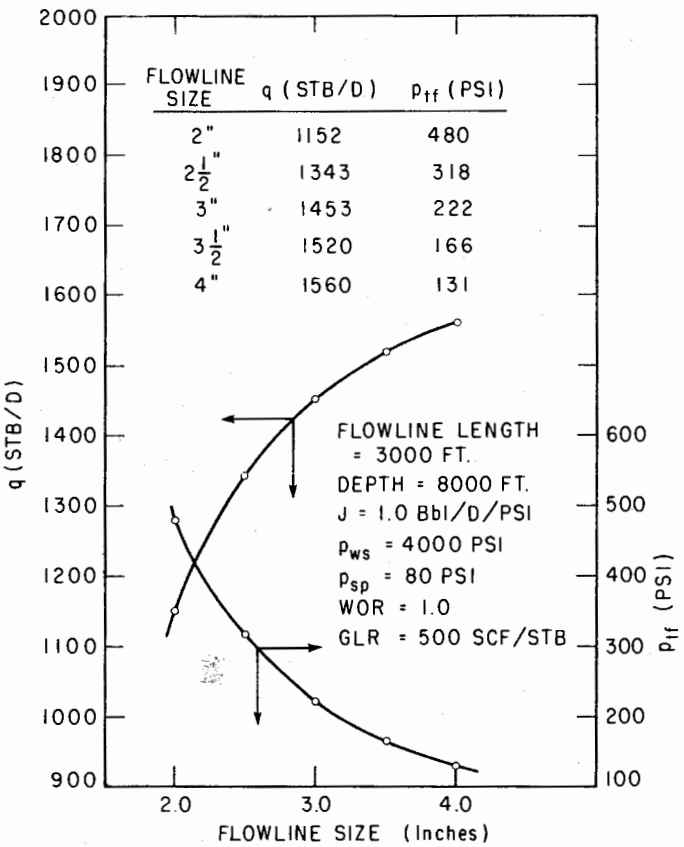


Fig. 3.36 Effect of flowline size (flowing well).

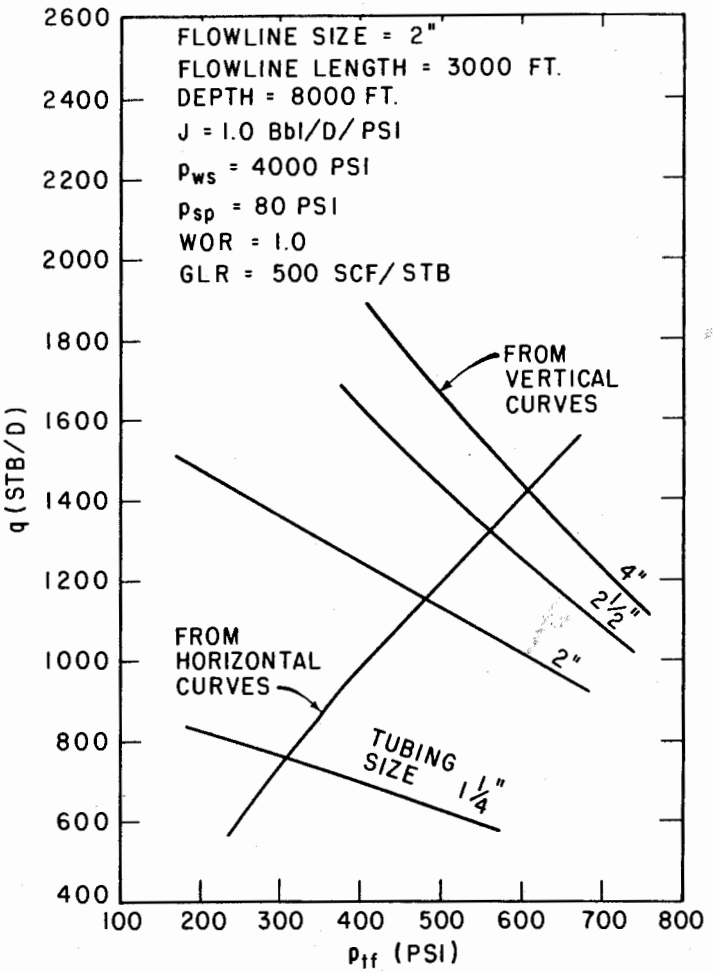


Fig. 3.38 Effect of tubing size (flowing well).

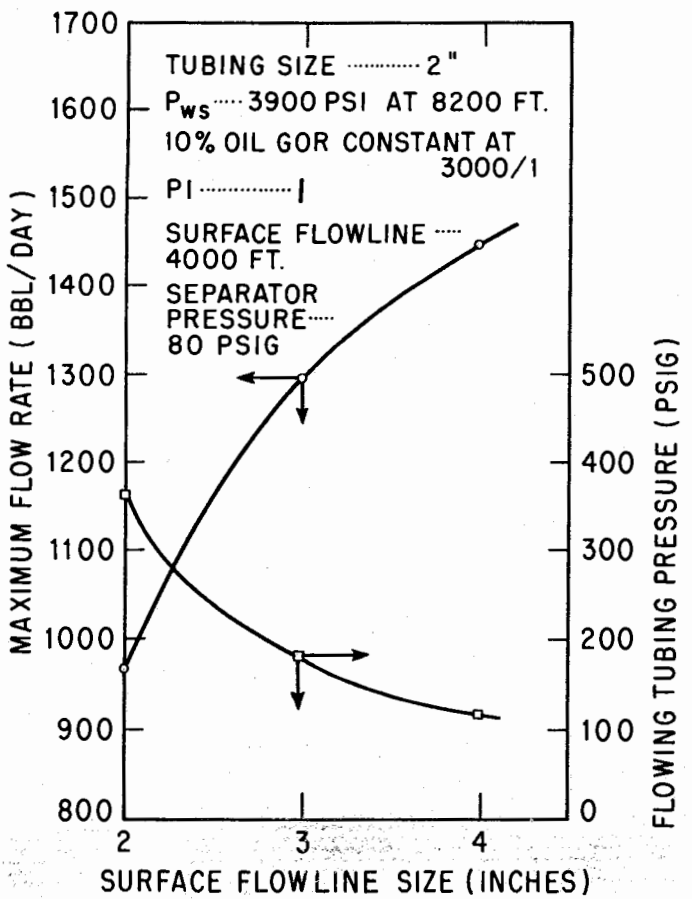


Fig. 3.37 Maximum production rate vs. flowline size (flowing well).

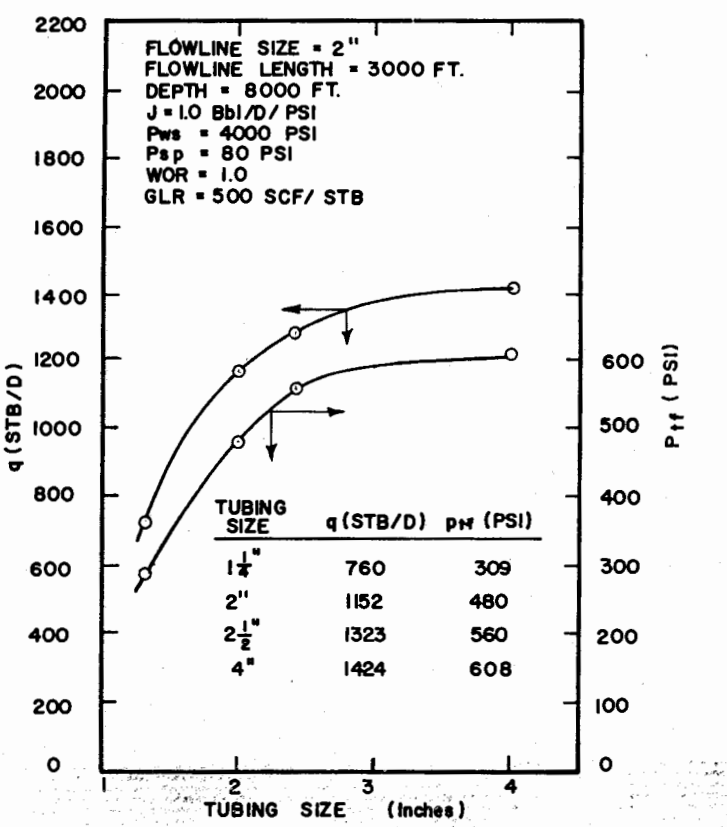


Fig. 3.39 Effect of tubing size (flowing well) (after Brill, et al., courtesy SPE).

000 ft well is equipped with 2 in. tubing. It is desired
o obtain the maximum flow rate possible. Fig. 3.35
ows that 1,152 stk/bpd can be produced through a
in. flowline compared to 1,560 stk/bpd through a
in. flowline. Fig. 3.36 shows a tendency for the pro-
uction rate to level off as the flowline size increases.
'e would eventually reach a point where any further
ncrease in size would result in a negligible increase
n production. We would also be approaching the maxi-
um capacity of the tubing size given for this well.

Fig. 3.37 shows the effect of changing out the sur-
ace flowline while maintaining a constant tubing size.

Figs. 3.37 and 3.38 show the effect of various ver-
cal tubing sizes. The surface flowline size and length
set at a constant value. An example problem is
own in Figs. 3.38 and 3.39. The production rate in-
eases with increased vertical tubing size. However, a
oint is reached where the production capacity of the
owline prevents any additional increase in flow rate
egardless of the tubing size.

The selection of the tubing sizes and flowline sizes
s extremely important in oil field production problems.
'e have wells that produce in ranges of tubing sizes
om 1 in. — $1\frac{1}{4}$ in. to 5 in. tubing sizes or even pro-
ucing from $9\frac{5}{8}$ in. casing or larger with no tubing.
)ther combinations exist such as annular flow be-
ween 2 in. tubing and 7 in. casing or flowing simul-
aneously from both the tubing and the annulus.
'roduction rates vary from 15–20 bpd to 30,000 —
0,000 bpd.

For example, in areas of the southwest United
tates, there are wells completed with small casing
nd tubing sizes (ultra-slim-hole completions). In
xtreme cases these wells may utilize $2\frac{3}{8}$ in. O.D.
asing and 1 in. or $1\frac{1}{4}$ in. tubing. However, normally
he wells are flowed out the $2\frac{3}{8}$ in. O.D. casing until
rtificial lift is needed. These are low volume wells
ormally in solution gas drive reservoirs and flow
ates in excess of 100–200 bpd are not expected.

CLASS PROBLEM #8-A: TO FIND FLOW RATE POSSIBLE

Given data:
- Separator pressure = 80 psig
- Length flow line = 5,000 ft
- Well depth = 7,000 ft
- G/L = 300 scf/bbl
- PI = 2.0 (work also for PI = 1.0 and 10.0)
- $\bar{p}_R$ = 2,400 psi
- $2\frac{1}{2}$ in. flowline
- $2\frac{1}{2}$ in. Tubing

Find the flow rate possible for:

(a) All oil
(b) 50% oil — 50% water
(c) All water
(d) What is the most economical way to double the production for PI = 1.0?; for PI = 10.

Plot all on one graph. (Use Procedures 1 and 2.)

CLASS PROBLEM #8-B: TO FIND FLOW RATE POSSIBLE

Given data:
- Depth = 7,000 ft
- $\bar{p}_R$ = 2,800 psi
- PI = 2 (assume linear)
- G/L = 1,000 scf/bbl
- Flowline length = 4,000 ft
- Separator pressure = 80 psi
- Production — all oil

Find the flow rates possible for the following combinations of tubing and flow line sizes.

Tubing size, in.	Flowline size, in.
2	2
3	3
4	4
2	3
2	4
3	4

CLASS PROBLEM #8-C: TO FIND FLOW RATE POSSIBLE

Given data:
- $2\frac{1}{2}$ in. tubing
- $2\frac{1}{2}$ in. flowline
- Depth = 8,000 ft
- $\bar{p}_R$ = 3,000 psi
- PI = 5
- G/L = 600 scf/bbl
- Flowline length = 5,000 ft
- Separator pressure = 100 psi

Find the flow rate possible and the corresponding wellhead pressure for:

(a) All oil
(b) All water

CLASS PROBLEM #8-D: TO SELECT THE CORRECT COMBINATION OF TUBING AND FLOWLINE SIZES

Given data:
- Depth = 6,000 ft
- Flowline length = 5,000 ft
- $\bar{p}_R$ = 2,300 psi
- PI = 6
- Separator pressure = 80 psi
- G/L = 600 scf/bbl
- q_L = 2,500 bpd (100% oil)

Prepare a tubing a flowline analysis chart and tabulate those possible combinations of tubing and flowline sizes that will handle 2,500 bpd.

STOCK SIZES AVAILABLE

Tubing, in.	Flowline, in.
2	2
$2\frac{1}{2}$	$2\frac{1}{2}$
3	3
4	4

CLASS PROBLEM #8-E: TO SELECT THE CORRECT COMBINATION OF TUBING AND FLOWLINE SIZES

Given data:
- Depth = 8,000 ft
- Flowline length = 3,000 ft
- $\bar{p}_R$ = 3,000 psi
- PI = 8

Separator pressure = 80 psi
GOR = 500 scf/bbl
q_o = 3,000 bpd

Prepare a tubing–flowline analysis chart and tabulate those possible combinations of tubing and flowline sizes that will produce 3,000 bpd.

SIZES AVAILABLE

Tubing, in.	Flowline, in.
2½	2½
3	3
4	4

3.64 Effect of other variables including example problems

3.641 The effect of changing static pressure

In most cases we can forecast $\bar{p}_R$ by either material balance calculations or production history. The decrease in bottom hole pressure of course has a decided effect on when the well will quit flowing or cease to produce at a set rate.

EXAMPLE PROBLEM #9: HOW TO DETERMINE WHEN A WELL WILL QUIT FLOWING DUE TO A DECREASING BOTTOM HOLE PRESSURE

Given data: Depth = 7,500 ft
2½ in. tubing
p_{wh} = 120 psig (lowest permissible p_{wh})
Wells allowable rate = 200 bpd
No water production
GOR = 400 scf/bbl (assume constant)
p_{Ri} = 2,800 psi

The static bottom hole pressure is decreasing at the rate of 100 psi per 10,000 bbl of oil produced.

The PI on the well is also decreasing as the bottom hole pressure decreases and from several tests it is found the PI can be calculated from the following equation:

$PI = 0.002\ (\bar{p}_R)$ for an original PI of $(0.002)(2{,}800) = 5.6$

Find the static bottom hole pressure at which this well will quit flowing its allowable rate along with cumulative production to that time for p_{wh} = 120 psi.

Solution procedure:

(1) Find the flowing pressure necessary to produce the allowable of 200 bpd of oil in 2½ in. tubing. From Fig. C.99 (Appendix) this is found to be 1,280 psi.

(2) Find the static pressure for the conditions of Step 1, that is, when p_{wf} = 1,280 psi.

$$PI = \frac{q}{\bar{p}_R - p_{wf}} = 0.002\ \bar{p}_R$$

$$\frac{0.002\ \bar{p}_R}{1} = \frac{200}{\bar{p}_R - 1{,}280}$$

or: $$0.002\ \bar{p}_R^2 - 2.56\ \bar{p}_R - 200 = 0$$

Recall the quadratic equation:

$$\bar{p}_R = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

where: $0.002\ \bar{p}_R^2 - 2.56\ \bar{p}_R - 200 = 0$
$a = 0.002$, $b = -2.56$ and $c = -200$

Solving we find $\bar{p}_R$ = 1,354 psi. We can also solve by trial and error in the equation:

$$\frac{0.002\ \bar{p}_R}{1} = \frac{200}{\bar{p}_R - 1{,}280}$$

The well no longer flows 200 bpd at static pressures less than 1,354 psi.

(3) Find cumulative production to this time:

$$\left(\frac{2{,}800 - 1{,}354}{100}\right)(10{,}000) = 144{,}600 \text{ bbl}$$

(4) If this well is produced at 100% its allowable capacity on a 30 day per month basis then it will flow at 200 bpd for:

$$\frac{144{,}600}{(30)(200)(12)} = 2.02 \text{ years}$$

Although it would flow longer at a lesser rate, it is recommended that artificial lift be placed on at this time.

CLASS PROBLEM #9-A: TO DETERMINE WHEN A WELL WILL QUIT FLOWING DUE TO A DECREASING STATIC BOTTOM HOLE PRESSURE

Given data: Depth = 8,000 ft
2 in. tubing
p_{wh} = 100 psi
Allowable rate = 100 bpd (oil)
Produces all oil
GOR = 500 scf/bbl (assume constant)
Original $\bar{p}_R$ = 2,600 psi

The static bottom hole pressure decreases at a rate of 150 psi per 10,000 bbl of production. The PI is found to follow the equation $PI = 0.0015\ \bar{p}_R$. Find the static bottom hole pressure at which the well will no longer flow its allowable of 100 bpd. What is the cumulative production at that time?

CLASS PROBLEM #9-B: TO DETERMINE WHEN A WELL WILL QUIT FLOWING DUE TO A DECREASING STATIC BOTTOM HOLE PRESSURE

Given data: Depth = 6,000 ft
2 in. tubing
p_{wh} = 80
$\bar{p}_R$ (original) = 2,200 psi
Allowable rate = 100 bpd (all oil)
Produces 100% oil
GOR = 300 scf/bbl
$\bar{p}_R$ decreases 100 psi per 6,000 bbl of production
The PI is found to be = $0.001\ (\bar{p}_R)$
The GOR is found to increase 100 scf/bbl for each 200 psi drop in $\bar{p}_R$.

Find $\bar{p}_R$ at which this well will no longer flow 100 bpd. Find cumulative production and GOR at this time.

42 The effect of water-cut on a flowing well

421 Introduction

The encroachment of water into a flowing oil well
ı cause any number of problems. Some of these in-
de emulsions, surface separation problems, addi-
nal slippage, well loading conditions, and finally
› dying of the well.

Our multiphase flow correlations are not sufficiently
:urate to properly predict pressure loss calculations
mixtures of oil, water, and gas. One of the main
ısons for this is our failure in being able to predict
:h things as the viscosity and surface tension of a
xture of oil and water. They do not truly mix and
›n in the form of an emulsion we do not have a homo-
ıeous mixture. Also, the viscosity term is further
nplicated by defining the viscosity of a mixture of
ȝ, oil, and water.

f, by any chance, a stable emulsion forms, then all
ıltiphase flow correlations can be thrown out, be-
ıse the pressure loss will be in excess of what any
xture properties will predict. Emulsions are a
›arate and difficult problem and no good solution is
ailable.

t is suggested that mixture properties be handled
a weighted average basis. For example, for a well
›ducing 70% oil and 30% water where the viscosity
the oil is 10 cp, and the water 1 cp at a defined point
the tubing string, then the viscosity of the liquid
xture is:

$$\mu_L = \mu_o\ (\%\ \text{oil}) + \mu_w\ (\%\ \text{water})$$
$$\mu_L = 10\ (0.70) + (1)\ (0.30) = 7 + 0.3 = 7.3\ \text{cp}$$

The preceding viscosity of the mixture of 7.3 cp has
real meaning and should not be thought of as
›resenting the true viscosity of the mixture.

Sherman[21] has proposed a method for calculating
xture viscosities and has written a book entitled
mulsion Science." In 1942, Woelflin[22] published a
per in API Drilling and Production Practice showing
w the viscosity increased with water-cut up to
›ut 70% brine at which time the emulsion reversed
th an abrupt reduction in viscosity as the water
:ame a continuous phase. Reference should be
ıde to Chapter II.

422 Physical significance of water-cut

Reference should be made to Fig. 3.40 which shows
),000 ft well flowing 600 bpd each of oil and water—
% oil—50% water in 2 in. tubing into a well head
:ssure of 120 psi. The flowing bottom pressures re-
ired for each case is noted as follows:

100% oil: 1,765 psi
50% oil—50% water: 1,925 psi
100% water: 2,125 psi

e difference between 2,125 psi for all water and 1,765
i for all oil is 360 psi. This can be enough difference
cause a well to die or have a considerable decrease
production. The gas-liquid ratio has been assumed
ıstant to illustrate effect of water-cut only. Actually
e G/L will decrease rapidly as the water-cut in-
:ases causing the flowing pressure required to in-

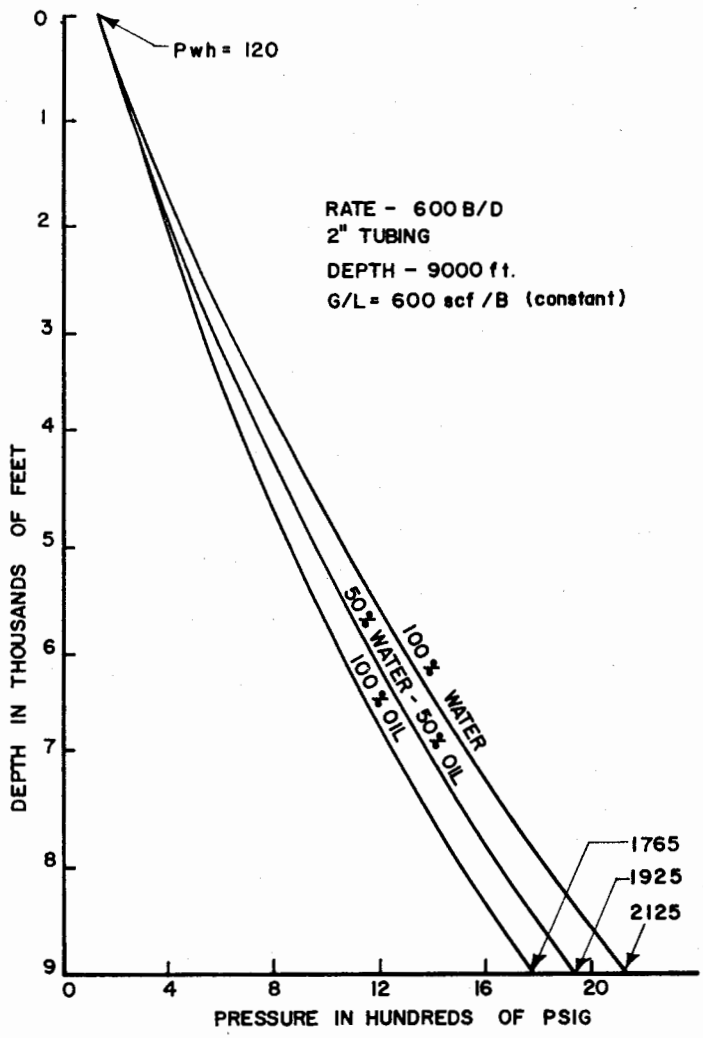


Fig. 3.40 Effect of water-cut on required flowing pressure.

crease much more rapidly than noted in Fig. 3.40.

The same problem is shown in Fig. 3.41 whereby the changing G/L is accounted for. We note that the flowing pressure required for 100% oil is the same as in Fig. 3.40 at 1,765 psi. However, for 50% water the G/L is reduced to 300 scf/bbl and the flowing pressure required is now 2,585 psi. For 100% water and for a G/L = 0, the flowing pressure is extremely high. The reduction in total gas-liquid ratio is the principal reason why the increased water-cut kills the well.

No attempt has been made to properly account for the forming of emulsions and its effect on viscosity for various water-cuts. The effect of density only has been accounted for. Woelflin[22] showed that the viscosity increased rapidly after a 50% water-cut, and this will cause an additional increase in pressure loss. If emulsions are properly accounted for, it is entirely possible that a well with 70% cut will make less than a well with 100% water due to increased emulsion viscosity.

The maximum rate possible from a well making all oil as compared to all water can be noted in Fig. 3.42 and is 5,500 bpd for all oil compared to 4,000 bpd for all water. This is a significant difference for an 8,000 ft well having a static pressure of 2,800 psi and a PI of 6.0. This is for a gas lift well and the G/L was maintained constant.

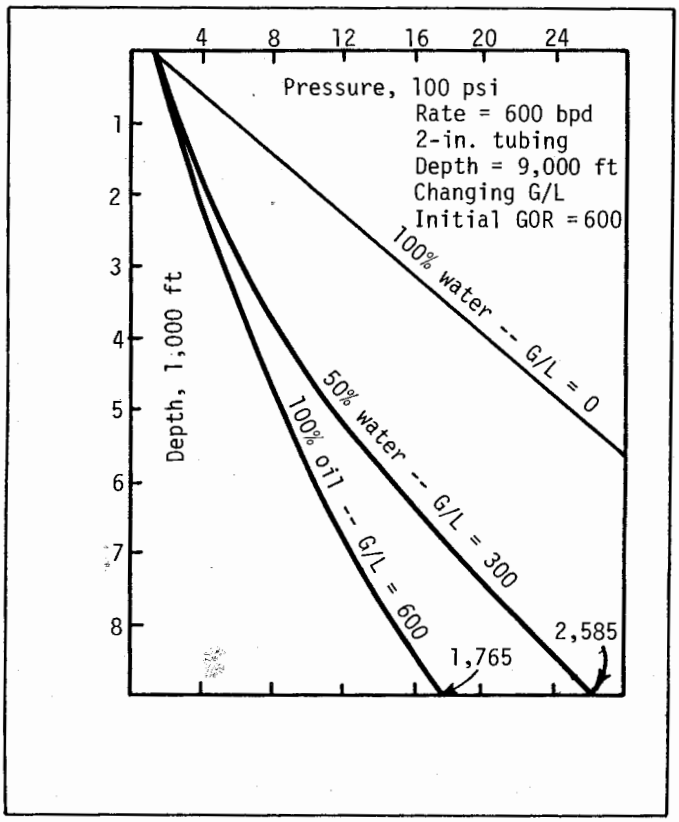


Fig. 3.41 Effect of water-cut on required flowing pressure.

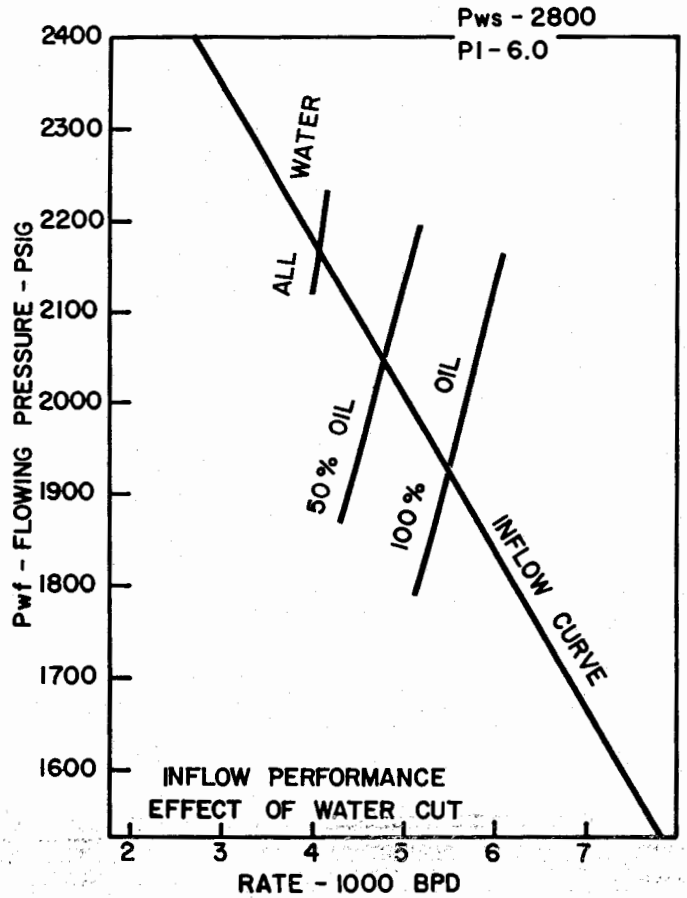


Fig. 3.42 Effect of water-cut.

CLASS PROBLEM #10-A

Given data: Depth = 8,000 ft
Rate = 1,000 bpd
$2\frac{1}{2}$ in. tubing
G/O = 500 scf/bbl
p_{wh} = 120

(1) Find the flowing bottom hole pressures required in this well for a rate of 1,000 bpd of liquid for:
(a) 100% oil
(b) 50% oil
(c) 100% water
Work for a constant gas-liquid ratio and for a changing gas-liquid ratio.

(2) For $\bar{p}_R$ = 3,100 psia and a PI = 6, find the flow rate possible for:
(a) 100% oil
(b) 50% oil
(c) 100% water
Work for constant G/L and for changing G/L.

3.6423 How to determine at what water-cut a well will quit flowing a particular rate

EXAMPLE PROBLEM #11

Given data: Depth = 7000 ft
p_{wh} = 120 psi
$2\frac{1}{2}$ in. tubing
$\bar{p}_R$ = 2,400 psi (constant)
PI = 1.5 (assume constant)
GOR = 800 scf/bbl
Oil allowable = 200 bpd

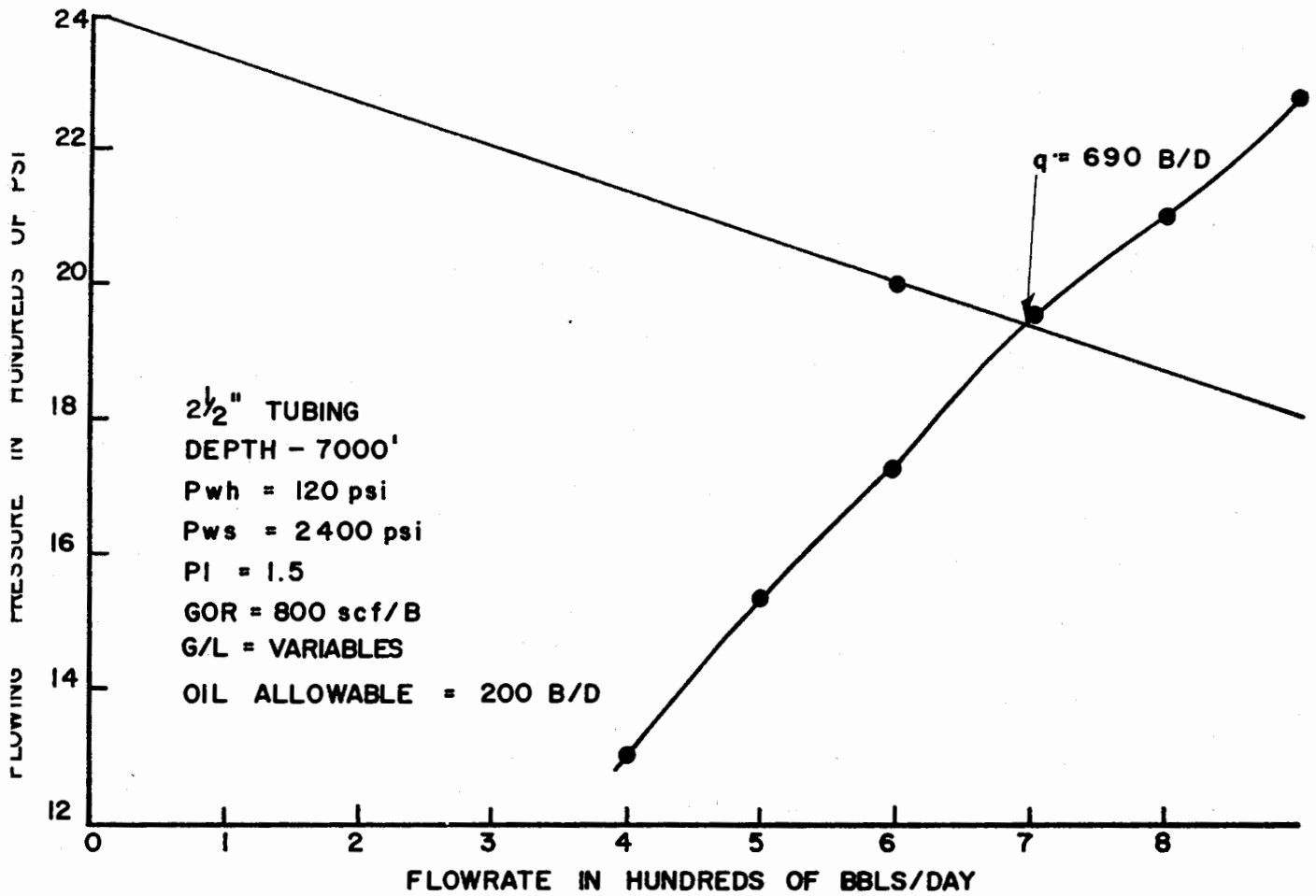
For this example we are assuming a constant PI and a constant static bottom hole pressure. The water-cut is increasing and the flowing G/L is changing accordingly. Solution procedure:

A computer solution to this problem is much preferred because working curves are not available for all water-cuts, however interpolations can be made to illustrate this change. Refer to Appendix C.8.

(1) Construct a Pressure Flowrate Diagram (Fig. 3.43).
(2) Draw in the inflow performance curve for a constant PI of 1.5.
(3) Construct the vertical flow performance curve for different rates accounting for the change in gas-liquid ratio.

Oil rate	Water rate	Total rate	G/L	p_{wf}	% water cut
200	0	200	800	760	0
200	100	300	533	1,026	33
200	200	400	400	1,300	50
200	300	500	320	1,530	60
200	400	600	267	1,725	67
200	500	700	228	1,950	71.5
200	600	800	200	2,100	75
200	700	900	178	2,280	78
200	800	1,000	160	2,300	80

p_{wf} is interpolated where necessary between 100% oil, 50% oil–50% water, and 100% water from working



ig. 3.43 Method to predict water-cut at which a well will no longer produce its allowable.

:urves. No attempt has been made to account for in-:reased viscosity due to the forming of either stable ›r loose emulsions due to various percentages of water.

Plot the flowing bottom hole pressures vs. total rate:

(4) The rate above which the well will no longer make its oil allowable is 690 bpd total liquid.

(5) Determine the water percentage at this rate.

$$\% \text{ water-cut} = \frac{690 - 200}{690} = \frac{490}{690} = 71\%$$

For water percentages greater than 71% the well vould no longer produce its allowable of 200 bpd of ›il. However, it would continue to flow for a longer ›eriod of time but at a lower total liquid rate and a ower oil rate. As the water percentage continued to ncrease the G/L would decrease and the oil rate would lecrease rapidly.

CLASS PROBLEM #11-A: TO DETERMINE AT WHAT WATER-CUT A WELL WILL QUIT FLOWING ITS OIL ALLOWABLE

Given data: Depth = 8,000 ft
p_{wh} = 80 psi
2 in. tubing
$\bar{p}_R$ = 2,600 psi (remains constant)
PI = 2 (assume linear)
GOR = 600 scf/bbl
Oil allowable = 100 bpd

Water starts moving into the well and continues to increase until the well will no longer make its allowable. Find the water-cut at which the well will no longer flow 100 bpd oil. Assume that $\bar{p}_R$ and PI remain constant. The G/L varies but GOR remains constant.

CLASS PROBLEM #11-B: TO DETERMINE AT WHAT WATER-CUT A WELL WILL QUIT FLOWING ITS OIL ALLOWABLE

Given data: Depth = 7,500 ft
p_{wh} = 100 psi
Tubing size = 2½ in.
$\bar{p}_R$ = 2,500 psi (constant)
PI = 4 (constant)
GOR = 1,000 scf/bbl
Oil allowable = 150 bpd

Find that water-cut at which the well no longer flows its allowable of 150 bpd oil.

3.643 Combination of variables that affect a flowing well

3.6431 Introduction

In order to accurately predict the flowing life of a well several changes may need to be accounted for at any one time. For example, for a particular well the following changes are possible:

(a) Decreasing static bottom hole pressure
(b) Decreasing productivity index
(c) Increasing water-cut
(d) Increasing GOR

All of these may or may not be occurring for one well, but it is entirely possible. For a solution gas drive it is very likely that the static pressure is decreasing rapidly, the PI decreasing, and the GOR increasing. For a water-drive well it is likely that the water percentage is increasing while the G/L is decreasing. However, there are occasions whereby a solution gas drive field may be water-flooded and all the preceding changes are taking place.

This type of problem should be worked on the computer to properly illustrate all variables. The general procedure will be given in an example and the solution can be easily programmed.

EXAMPLE PROBLEM #12: HOW TO ACCOUNT FOR SEVERAL VARIABLES IN A FLOWING WELL

Given data: Depth = 7,000 ft
2 in. tubing
Oil allowable = 200 bpd
GOR = 600 scf/bbl
p_{wh} = 80 psi (constant)
$\bar{p}_R$ original = 2,600 psi

The static bottom hole pressure is decreasing at the rate of 200 psi per 10,000 bbl of oil production. The PI follows in general the decrease in bottom hole pressure and is equal to 0.002 $\bar{p}_R$. In general the GOR shows an increase of 100 scf/bbl per 10,000 bbl of oil recovered. Water production starts after 30,000 bbl of recovery due to initiating water flood operations. At this time the static pressure decreases at 50 psi per 10,000 bbl with the PI following the same at 0.002 $\bar{p}_R$. The water production is noted to increase on the average at the rate of 200 B/D after each 10,000 bbl of recovery, starting with 300 B/D total fluid after 40,000 bbl of recovery. Note Table 3.8.

Prepare a pressure flow rate diagram showing the effect of all changes, and showing when the well will no longer make its allowable.

Solution procedure:

(1) Plot p_{wf} vs. q as noted in Fig. 3.44.
(2) Construct IPR curves for each 10,000 bbl of oil production taking into account the changing PI = 0.002 $\bar{p}_R$ and the changing static bottom hole pressure. This series of lines (assumed linear) is noted on Fig. 3.44.
(3) Assume various total liquid rates and calculate the corresponding flowing bottom hole pressure needed accounting for the changing water-cut and changing G/Ls. Prepare a table as follows:

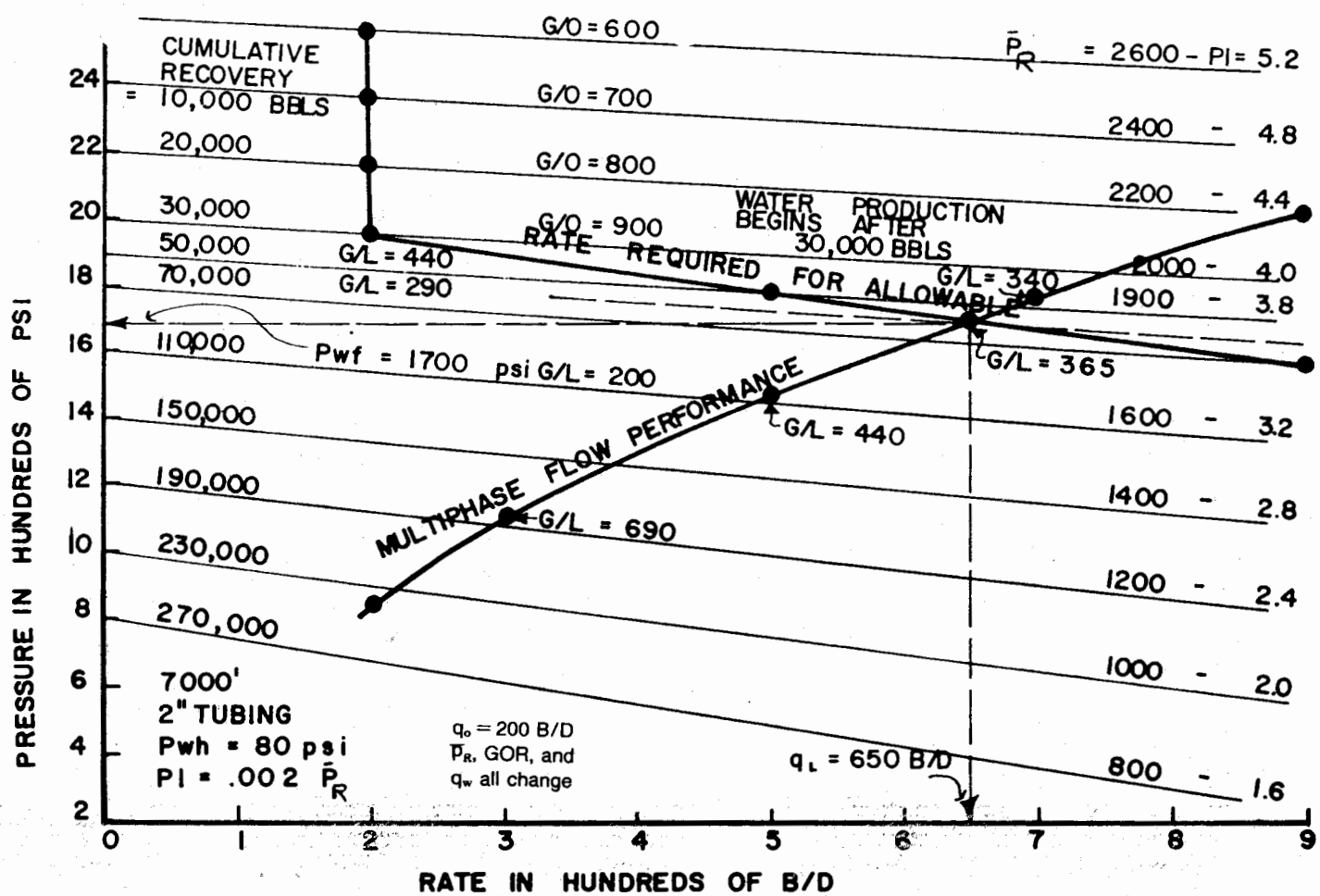


Fig. 3.44 Pressure flowrate diagram showing effect of changing $\bar{p}_r$, PI, and GOR.

TABLE 3.8

Cum recovery	GOR	Total rate	G/L	p_{wf}
0	600	0	600	0
10,000	700	200	200	850
20,000	800	200	200	820
30,000	900	200	200	760
40,000	1,000	300	670	1,105
50,000	1,100	500	440	1,460
60,000	1,200	700	340	1,770
70,000	1,300	900	288	2,000
80,000	1,400	1,100	255	
90,000	1,500	1,300		
100,000	1,600	1,500		
110,000	1,700	1,700		

(4) Plot the multiphase flow performance curve (p_{wf} vs. q) on Fig. 3.44.
(5) Note the G/Ls at each point
(6) Determine the required rate that the well must produce at the end of each cumulative recovery in order for the well to produce its allowable and plot this information on Fig. 3.44.
(7) The intersection of the two curves shows the cumulative oil recovery above which the well can no longer produce its allowable to be 57,000 bbl at which time the total liquid rate is 650 bpd (200 bpd oil—450 bpd water), the static bottom hole pressure is 1,860 psi the flowing bottom hole pressure is 1,700 psi and the G/L is 365 scf/bbl.

This type of problem should be placed on computer. The $\bar{p}_R$, GOR, G/L, PI, q_o, q_w, q_L and p_{wf} can be determined in increments of cumulative recovery. The p_{wf} required at each increment of cumulative recovery can then be compared to the p_{wf} necessary from the multiphase vertical flow performance. When these two values of p_{wf} coincide at some rate, it is then the rate at which the well will no longer continue to make its allowable. In reference to Fig. 3.44 any values of rate vs. p_{wf} that fall to the right and below the multiphase flow performance curve represent impossible rates. As long as the plot of rate required for allowable falls to the left and above the multiphase flow performance curve, then the well will flow its allowable rate.

Other water-cuts for typical wells can be placed on the same plot; however, the gas-liquid ratios available and gas liquid ratios required must also coincide. These two coincide at the point of intersection on Fig. 3.44 at approximately 365 scf/bbl. In other words, a certain GOR is coming from the well at each increment of cumulative recovery, and this must be the same G/L required to produce the well at the point of intersection of the rate required and multiphase flow performance curve.

Fig. 3.44 allows the tabulation of cumulative recoveries vs. $\bar{p}_R$, and most important signals the time for some type of artificial lift method. That is, at approximately 57,000 bbl of oil, the well will no longer flow its allowable.

With good reservoir information, the equation describing changing GORs, $\bar{p}_R$, q_o, q_w, PI, etc. can be written into a computer program which combines reservoir flow equations with multiphase flow equations.

CLASS PROBLEM #12-A: TO DETERMINE FLOW RATE AT WHICH A WELL WILL NO LONGER PRODUCE ITS ALLOWABLE WITH CHANGING $\bar{p}_R$, GOR, G/L, PI AND WATER-CUT

Given data: Depth = 8,000 ft
$2\frac{1}{2}$ in. tubing
Oil allowable = 200 bpd
GOR original = 500 scf/bbl
$\bar{p}_R$ original = 2,800 psi
p_{wh} = 100 psi (assume constant)
$\bar{p}_R$ is decreasing at rate of 100 psi per 10,000 bbl at recovery
PI = 0.0015 $\bar{p}_R$
GOR is increasing 100 scf per 10,000 bbl of recovery

The well makes no water for first 40,000 bbl of recovery. The water production then follows the form of (0.50)(previous rate). For example, at 50,000 bbl cumulative recovery the water production = (0.5)(200) = 100 bpd giving a total rate of 300 bpd. At 60,000 bbl total rate = 300 + 0.5(300) = 450 bpd.

Plot a pressure flow rate diagram finding that rate at which the well will no longer flow its allowable.

Find the G/L, p_{wf}, q_o, and q_w at this cumulative recovery.

CLASS PROBLEM #12-B: TO DETERMINE FLOW RATE AT WHICH A WELL WILL NO LONGER PRODUCE ITS ALLOWABLE WITH CHANGING $\bar{p}_R$, GOR, G/L, PI, WATER-CUT AND p_{wh}.

In this problem we have added the additional variable of changing well-head pressure depending on the rate.

Given data: Depth = 6,500 ft
2 in. tubing
Oil allowable = 100 bpd
GOR original = 400 scf/bbl
$\bar{p}_R$ original = 2,200 psi
Separator pressure = 80 psi
Flowline = 3,000 ft of $2\frac{1}{2}$ in. line
$\bar{p}_R$ is decreasing at rate of 200 psi per 10,000 bbl of recovery
PI = 0.0025 $\bar{p}_R$
GOR is increasing 200 scf per 10,000 bbl of recovery.
The well starts making water after 20,000 bbl of recovery and the water increases at (0.75)(previous rate).

Plot a pressure flow rate diagram finding that rate at which the well will no longer produce its allowable. Find the GOR, G/L, p_{wf}, p_{wh}, q_o, and q_w at this cumulative recovery.

The problems presented here have been simplified a great deal. Recent publications show that a multiphase wellbore flow program can be combined with a multiphase reservoir flow program to more accurately predict production behavior. A paper by Aziz et al[23] described such a piece of work. However, if good information is available on the inflow performance ability of the well, along with pressure decline information, gas oil ratio change, and water-cut change then the previous solutions are valid.

3.7 FLOWING WELLS WITH SURFACE CHOKES

3.71 Introduction

Surface choke performance has already been discussed in section 3.5 of this Chapter. Recall that up to $^{14}/_{64}$ in. chokes and for critical flow the correlation of Omana's is very good although his work was performed through a dual flow choke. This probably means that a slightly different coefficient would make his correlation better for positive type chokes. The purpose of this section is to show how to analyze the over-all system of the flowing well by including a surface choke and to make a surface choke selection to allow a certain producing rate.

3.72 Determining choked flow rates

EXAMPLE PROBLEM #13-A: HOW TO DETERMINE FLOW RATE POSSIBLE AND CHOKE SIZE THAT GIVES CRITICAL FLOW ACROSS CHOKE

Given data: Depth = 6,500 ft
2 in. tubing
$\bar{p}_R$ = 2,500 psi
PI = 1.5 (assume linear)
GOR = 500 scf/bbl
Length flowline = 2,000 ft
Size flowline = 2 in.
Separator pressure = 100 psi
Production = 100% oil (API = 30°)

Find production rate for critical flow across choke, then find necessary choke size.

Solution procedure:

(1) Construct inflow performance curve on pressure-flowrate diagram as described previously (Fig. 3.45)

(2) Assume flow rates and find p_{wh_D} (downstream well-head pressure from choke). Prepare Table 3.9.

TABLE 3.9

Assumed rate	p_{wh_D}
200	110
400	135
600	170
800	205
1,000	260
1,500	360

(3) Assume critical flow such that the pressure upstream from choke (p_{wh_U}) is approximately twice that of the downstream pressure

(4) Find the flowing bottom hole pressures. The table of Step 2 is continued.

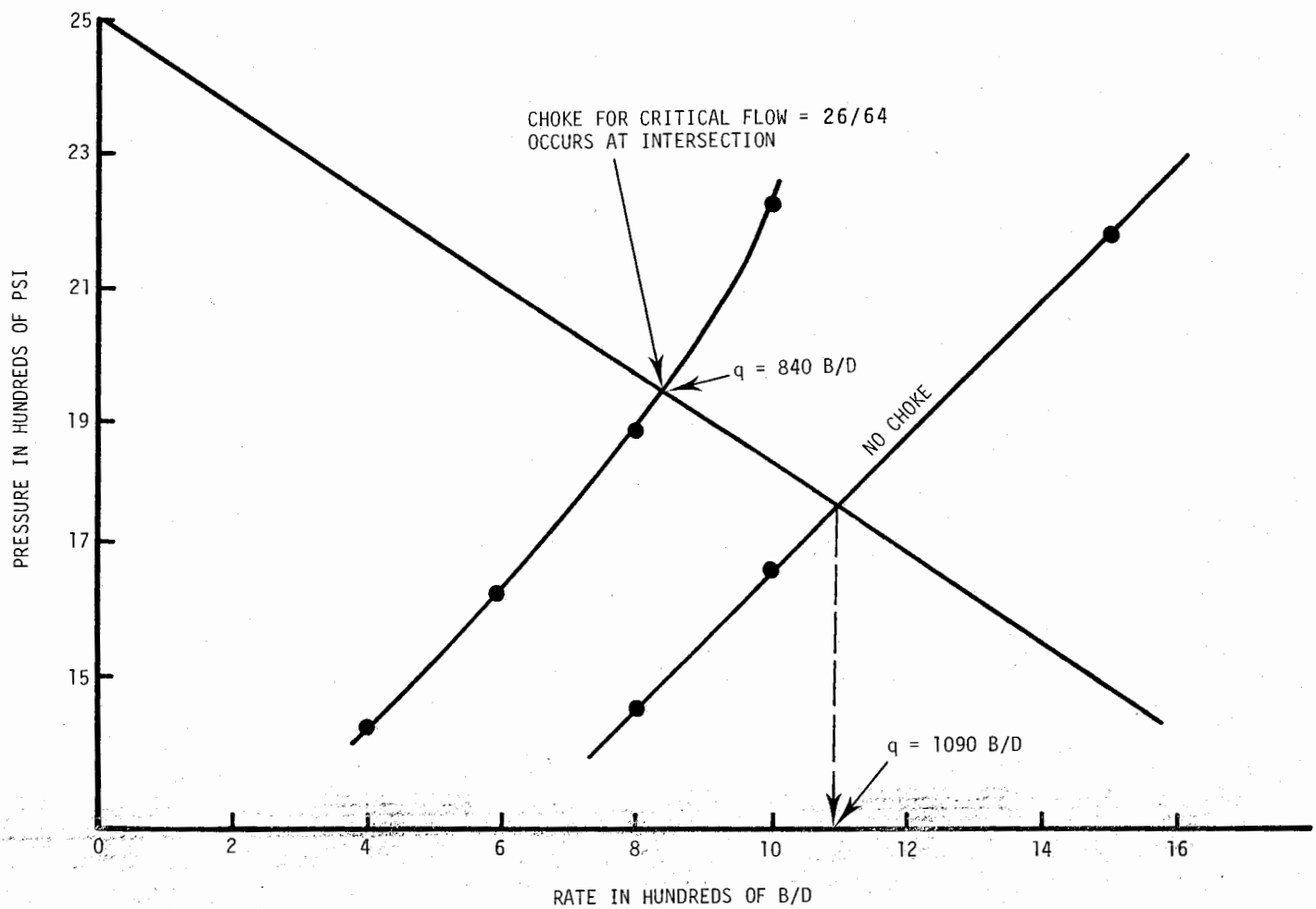


Fig. 3.45 Pressure flowrate diagram to find choke size at critical flow.

TABLE 3.10

Assumed rate	p_{wh_D}	$p_{wh_U} = 2p_{wh_D}$	p_{wf}
200	110	220	1,260
400	135	270	1,440
600	170	340	1,650
800	205	410	1,880
1,000	260	520	2,220
1,500	360	720	

(5) The rate vs. p_{wf} required for a choke is then plotted on Fig. 3.45.

It is noted that for a choke size (yet to be determined) ne flow rate is 840 bpd with a flowing bottom hole ressure of 1,940 psi. This compares to a non-restricted ow rate of 1,090 bpd (Fig. 3.45).

(6) Determine the flowing well-head pressure for the rate of 840 bpd. We can plot p_{wh_U} vs. rate on the same graph, but the scale does not permit, so refer to Fig. 3.46. The necessary flowing pressure is found to be 425 psi.

(7) The choke size necessary can now be selected from the following known information:

$q = 840$ bpd
GOR = 500 scf/bbl
$p_{wh_U} = 425$ psi

Referring to Fig. G.2 (Appendix) of Poettmann and Beck we note that a $^{26}/_{64}$ in. choke is required. We can also check this by Gilbert's formula where:

$$S^{1.89} = \frac{435(GOR)^{0.546}q}{P}$$

$$S^{1.89} = \frac{435\ (0.500)^{0.546}\ (840)}{425}$$

$$S^{1.89} = \frac{(435)\ (0.685)\ (840)}{425} = 589$$

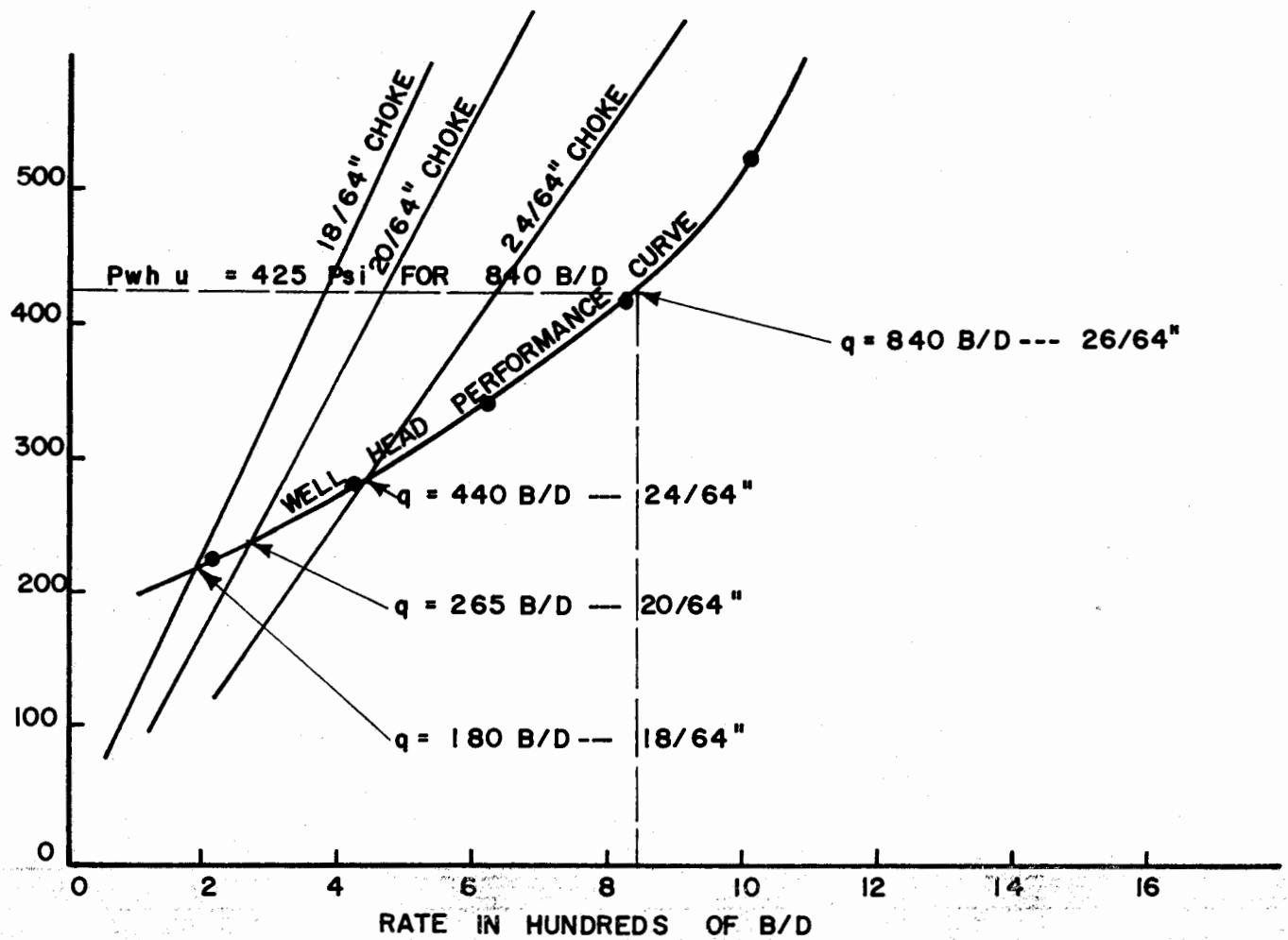
$$S = 29.2/64\text{ths}$$

EXAMPLE PROBLEM #13-B: FOR SAME PROBLEM, FIND THE PRODUCTION RATE POSSIBLE FOR A $^{24}/_{64}$ in., $^{20}/_{64}$ in., and $^{18}/_{64}$ in. CHOKE

(1) Refer to the plot of p_{wh_U} (wellhead pressure) to give critical flow vs. rate (Fig. 3.46)

(2) From Poettmann and Beck chart assume rates and find p_{wh_U} necessary.

Assumed rate	p_{wh_U} $^{18}/_{64}$	p_{wh_U} $^{20}/_{64}$	p_{wh_U} $^{24}/_{64}$
200	240	180	
400	450	360	260
600	650	550	400



g. 3.46 Rates possible for various choke sizes.

(3) Plot the wellhead pressure vs rate for each choke size as noted on Fig. 3.46. The production rate from each choke size is that point where the choke line intersects the wellhead performance curve.

3.73 Choke performance analysis

Fig. 3.47 is presented to further clarify the choke flow problem. The following well data is given.

EXAMPLE PROBLEM #13-C

Well depth = 5,000 ft
Tubing = 2½ in.
Flowline length = 2,000 ft
Flowline size = 2½ in.
GOR = 300 scf/bbl
Separator pressure = 80 psi
Producing 100% oil
$\bar{p}_R$ = 2,500 psi
PI = 1.0 (assume constant)

The purpose of this illustrative problem is to show the unrestricted flow rate (no choke), the flow rate and choke size for critical flow, and the necessary choke sizes and corresponding well-head pressures for flow rates where the ratio of downstream pressure to well-head pressure is less than 0.5.

Solution procedure:

(1) Construct the well-head pressure line for unrestricted flow. Assume flow rates and from the separator pressure of 80 psi and 2000 ft of 2½ in. flowline find the well-head pressures necessary for unrestricted flow.
(2) In the manner described previously construct the well-head performance curve by assuming rates, finding the flowing bottom hole pressure from $\bar{p}_R$ and PI and determining the permissible well-head pressure for a particular rate.
(3) The intersection of these two curves gives the unrestricted flow rate of 1,300 bpd.
(4) Check that rate where the pressure from the well-head performance curve is double that of the required well-head pressure from flow through the horizontal line. This is 1,040 bpd at p_{wh} = 280 = 2 p_{horiz}. (p_{horiz}/p_{wh} = 0.5).
(5) From Poettmann's charts the choke size is found to be ½ in.
(6) Other choke sizes and their corresponding rates are noted on Fig. 3.47.

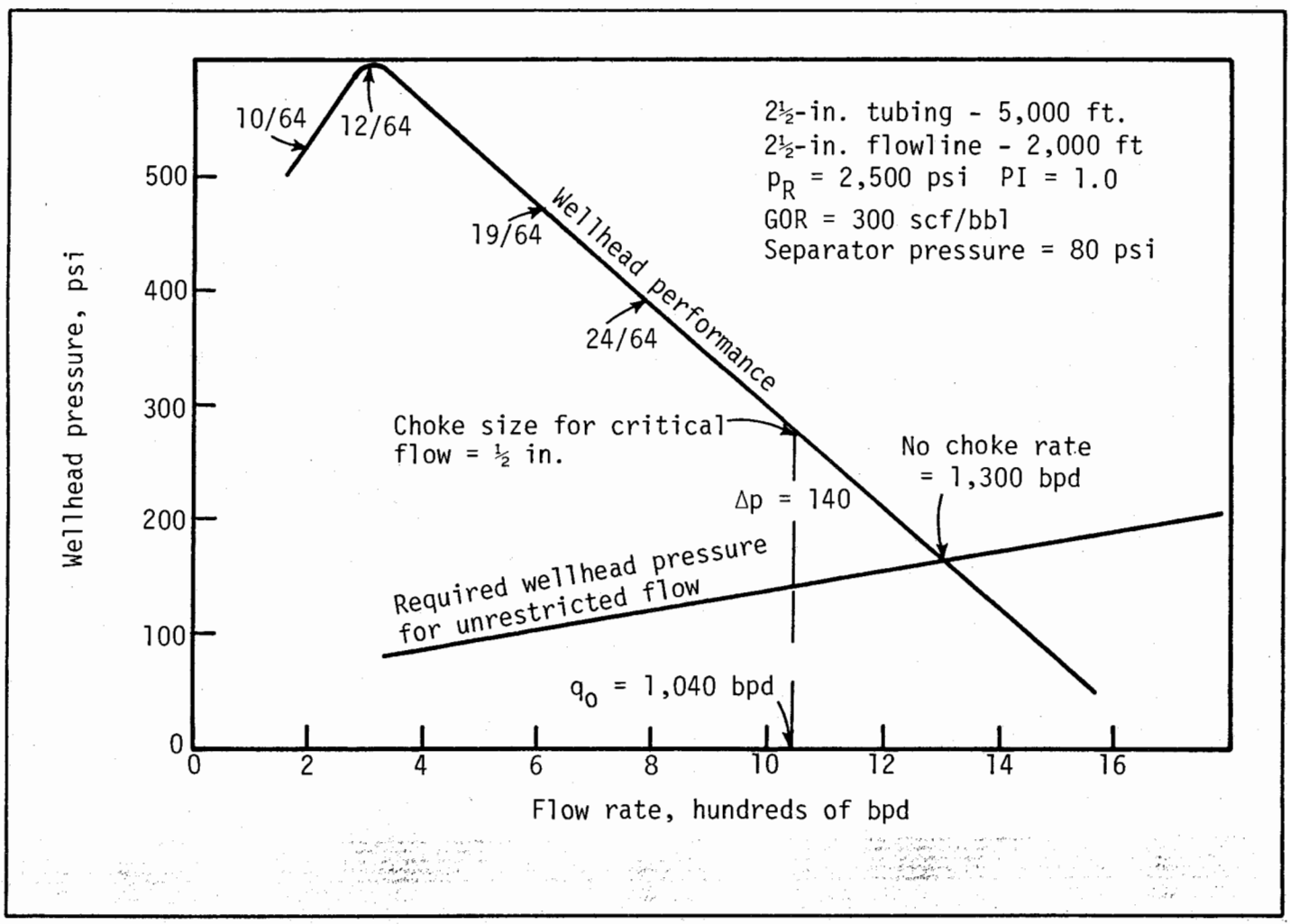


Fig. 3.47 Choke performance analysis.

3.8 COMPARISON OF WELLS FLOWING WITH AND WITHOUT A CHOKE

3.81 Introduction

Fonseca[1] prepared a study to show the effect of several variables in a flowing well. In general, he showed how variables such as changing bottom hole-pressure, diameter, etc. affected the flow rate for wells flowing with no surface restrictions and for wells flowing with chokes. He also showed the effect of assuming a linear IPR as compared to the IPR curve shape predicted by Vogel. He looked at the overall system including the surface flow line. For the analysis in this section, the vertical multiphase flow correlation of Hagedorn and Brown[15] and the horizontal multiphase flow correlation of Dukler[16] is used.

Fonseca[1] used the following well data to solve the problem examples to follow:

Given data:

$\gamma_g = 0.650$	API = 35.0
$\gamma_w = 1.074$	Water cut = 50.0%
Surface temp = 100.0°F	Surface tension = 70.0 dynes/cm
Flowing temp = 140.0°F	Depth = 8,000 ft
Bottom hole temp = 180.0°F	Flow line length = 3,000 ft
Separator pressure = 80.0 psig	Tubing size = 1.995 in.
GLR = 500.0 scf/stk bbl	Productivity index = 1.0 bpd/psi and 10.0 bpd/psi

The ideal rate of flow is to be interpreted as the equilibrium rate in case of no restriction at the well head (no choke bean installed), and as the maximum liquid production if a bean is installed at the well head.

3.82 Effect of flowline size for changing static bottom hole pressures

Fonseca[1] worked the previous problem for static pressures of 4,000, 3,000, and 2,000 psia. He included four different curves which showed the following:

(1) Unrestricted flow (no bean) and a linear IPR curve.
(2) Unrestricted flow (no bean) and an IPR curve predicted by Vogel for solution gas drive reservoirs.
(3) A surface choke bean of the exact size to give critical flow across the bean (Gilbert's formula) and linear IPR.
(4) A surface choke bean of the exact size to give critical flow across the bean (Gilbert's formula) and an IPR curve predicted by Vogel.

The results are shown in Figs. 3.48, 3.49, and 3.50 which show the flow rate vs. flow-line sizes for static pressures of 4,000, 3,000 and 2,000 psi. We note several things. As the flow line size exceeds 3 in. there is less benefit from further increasing the size.

A change in diameter from 3.124 in. to 3.624 in., for instance, will cause the following variations in liquid production:

For $\bar{p}_R = 4{,}000$ psia, $\Delta q = 40$ bpd (Fig. 3.48, curve 1)
For $\bar{p}_R = 3{,}000$ psia, $\Delta q = 23$ bpd (Fig. 3.49, curve 1)
For $\bar{p}_R = 2{,}000$ psia, $\Delta q = 6$ bpd (Fig. 3.50, curve 1)

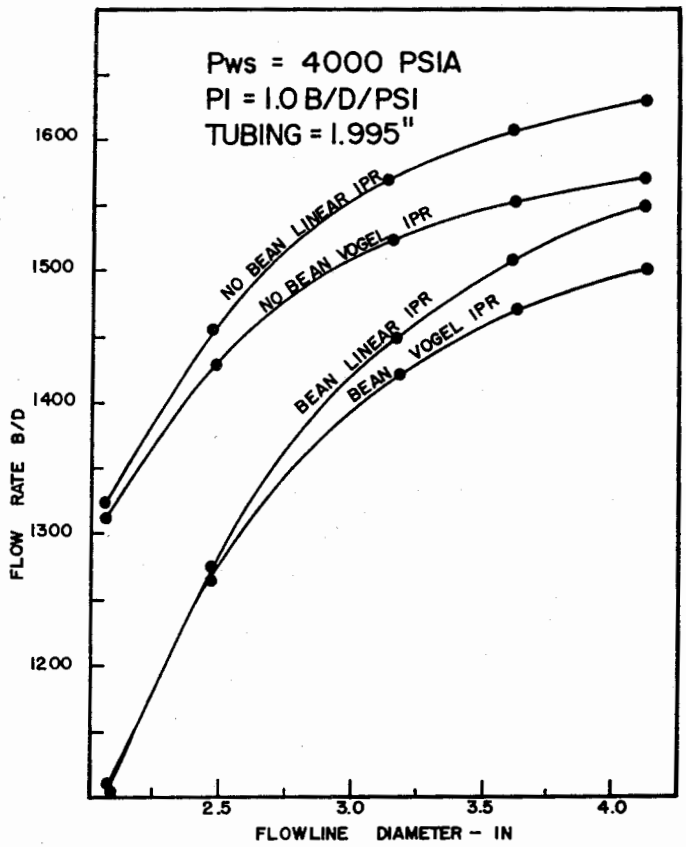


Fig. 3.48 Effect of flow line size (after Fonseca).

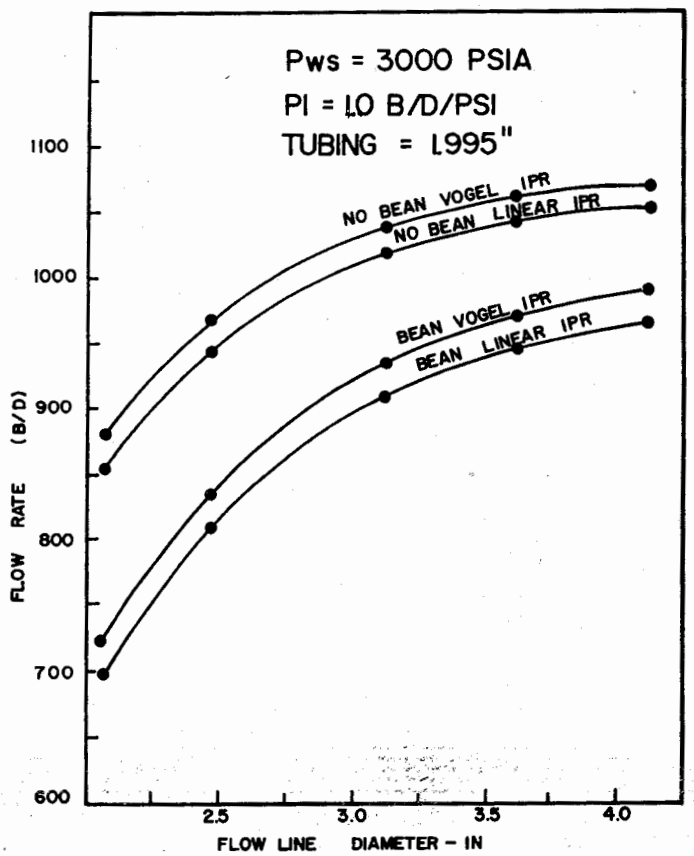


Fig. 3.49 Effect of flow line size (after Fonseca).

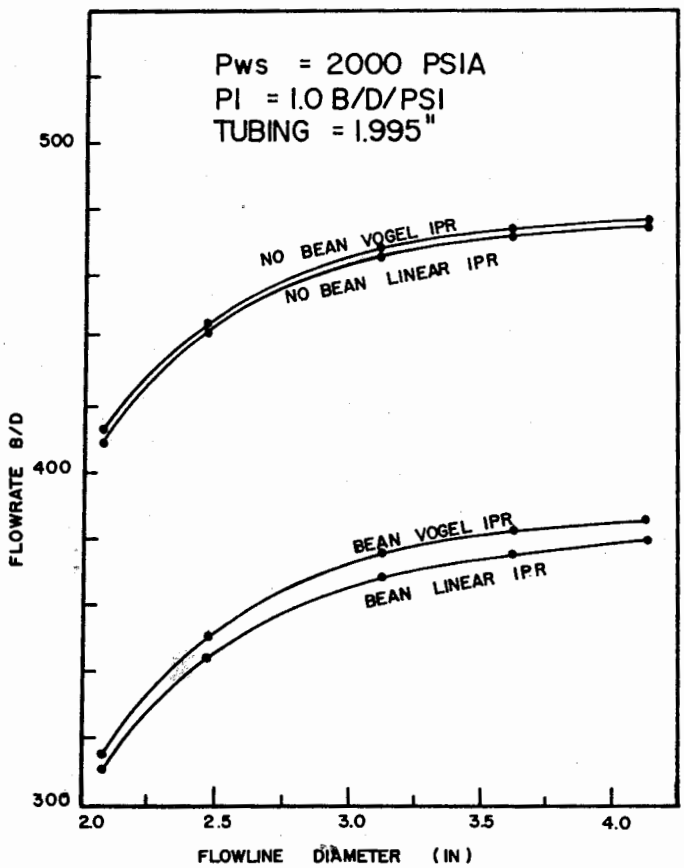


Fig. 3.50 Effect of flow line size (after Fonseca).

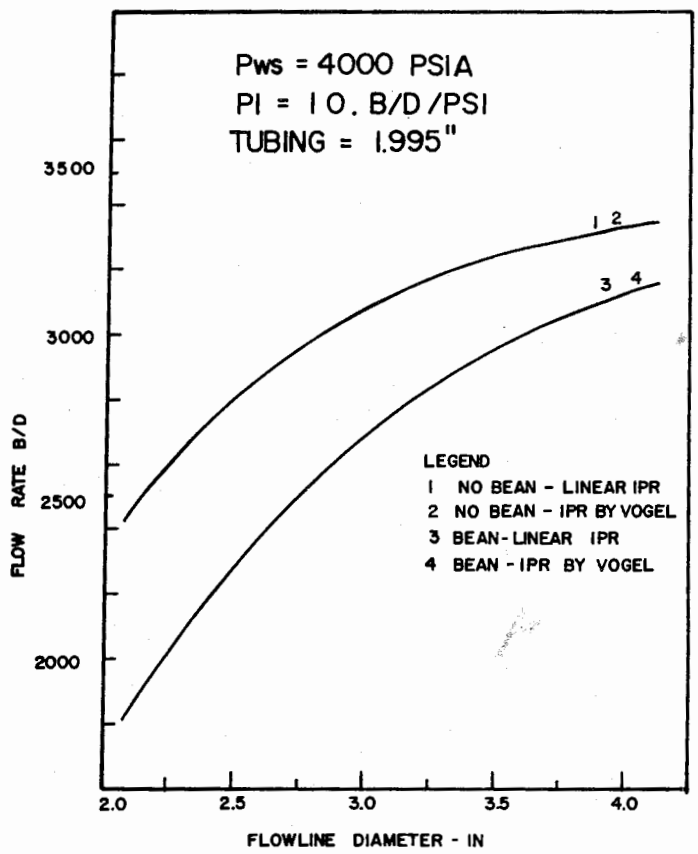


Fig. 3.51 Effect of flow line size (after Fonseca).

While an increase of 40 bpd is worthy of consideration, it is quite probable that economic reasons would not justify the flow line size change for the case in which $\bar{p}_R = 2{,}000$ psia, where the production increase is only 6 bpd.

A change from 3.624 in. to 4.124 in. certainly would not be justified since the production increase is too small. It would be only 3 bpd for $\bar{p}_R = 2{,}000$ psia (Fig. 3.50) for no bean-linear IPR.

The same problem was worked for PI = 10.0 bpd/psi and static pressures of 4,000, 3,000, and 2,000 psia. Results are presented in Figs. 3.51, 3.52 and 3.53 as plots of diameter vs. flowrate.

All figures presented so far show a tendency for the rates of production to level off with any increase in flow line size. Fig. 3.54 is presented to emphasize the discussion on the selection of the flow line size. For a PI = 10 b/d/psi, diameters greater than 4.0 in. would be selected for $\bar{p}_R = 4{,}000$ psia, since a reasonable rate of production increase is still being observed for diameters above 4.0 in.

If a choke bean is to be installed in the well head, less production loss will be experienced with greater flow line sizes than with small ones, since the formula used to calculate bean sizes is based on the assumption that critical flow is established within the choke bean. Fig. 3.51 shows that the production loss due to the choke bean is 607 bpd for a flow line size equal to 2.067 in., and only 190 bpd for 4.124 in.

The difference between the calculated rates of production for linear IPRs and for IPRs calculated by Vogel's formula, increases for higher rates of production (Fig. 3.48) with the one calculated by the linear

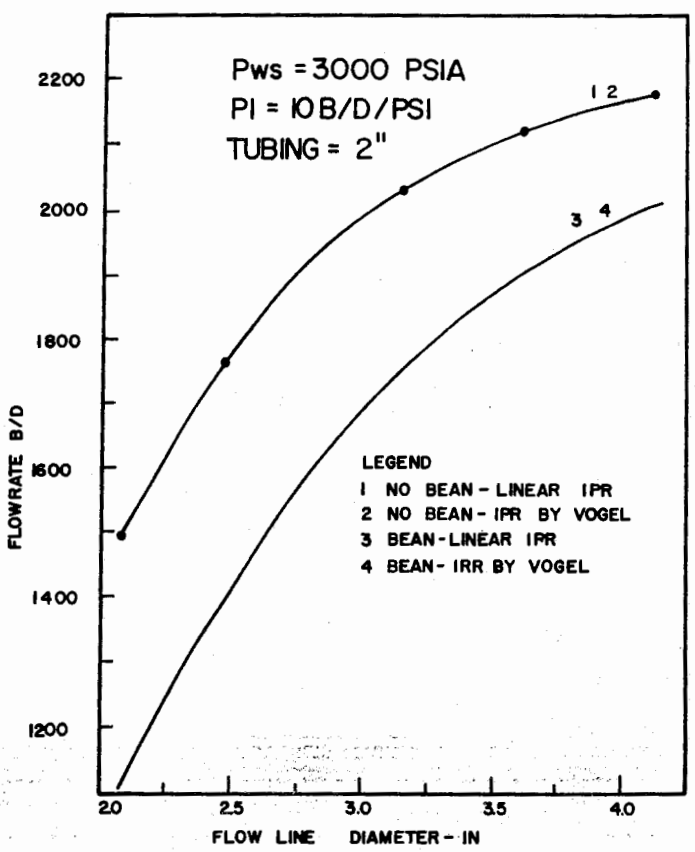


Fig. 3.52 Effect of flow line size (after Fonseca).

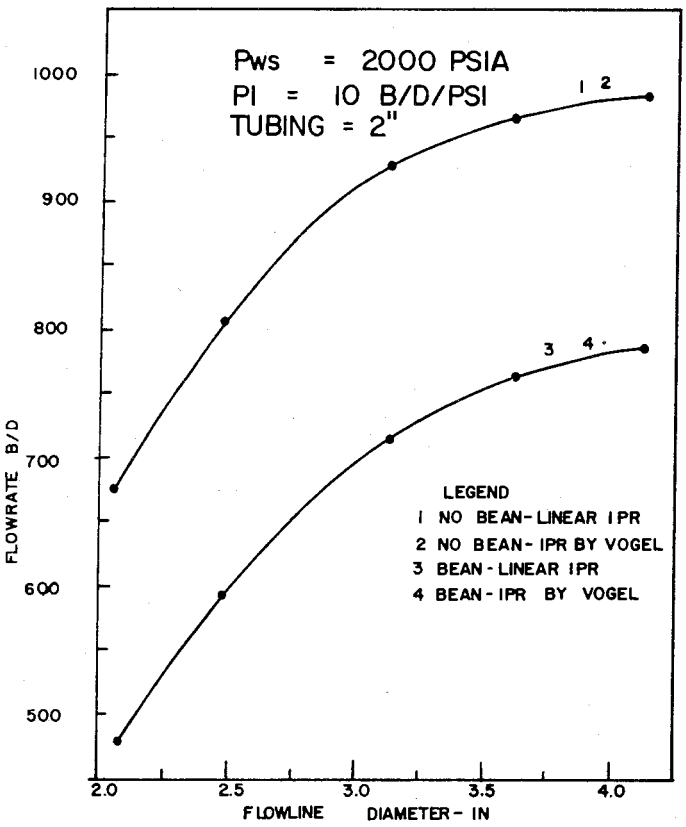


Fig. 3.53 *Effect of flow line size (after Fonseca).*

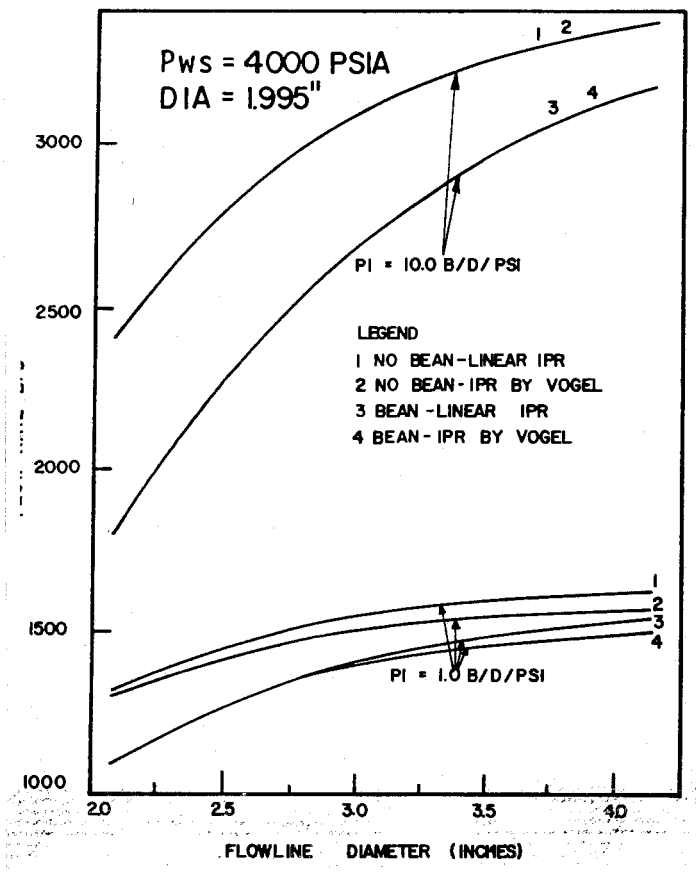


Fig. 3.54 *Effect of flow line size (after Fonseca).*

IPR being greater. However, for lower rates of production, rates calculated by Vogel's formula are slightly greater than those calculated by assuming linear IPR (Fig. 3.49 and 3.50). This is to be expected because IPR curves as calculated by Vogel's formula start from the same point (extrapolated static pressure at infinite boundary) as linear IPR. The graph can be divided into two regions: one in which the linear IPR yields smaller rates of production and the other in which it yields greater rates of production.

For the productivity index of 10.0 bpd/psi, the difference between rates of liquid flow as calculated assuming linear IPR or Vogel's formula are very small (Figs. 3.51, 3.52 and 3.53). This is due to the fact that in this example the tubing and flow line sizes are too small to handle the higher rates of flow the well is capable of producing, and in all situations the results are concentrated in the region where the linear IPR yields smaller rates of flow.

3.83 Effect of tubing size

The general tendency of increased tubingless completions, the higher number of offshore wells and the constant increasing of well depths makes the study of the effect of tubing sizes in well performance more and more important. Although, we have discussed tubing sizes previously, Fonseca[1] showed the effect of pressures, flowline sizes, and type of IPR.

Two example problems were solved in order to show the effect of tubing size on the performance of an oil well. The same problem as previously worked is again used except the flow line size was set at 3.124 in.

$\gamma_g = 0.650$
$\gamma_w = 1.074$
API = 35.0
Water cut = 50.0%
Surface temp = 100.0°F
Flowing wellhead temp = 140.0°F
Bottom hole temp = 180.0°F
Separator pressure = 80.0 psig
GLR = 500.0 scf/stk bbl
Surface tension = 70.0 dynes/cm
Depth = 8,000 ft
Flowline Length = 3,000 ft
Flowline size = 3.124 in.
Productivity index = 1.0 bpd/psi and 10.0 bpd/psi

In the examples studied, tubing sizes greater than flow line sizes were not used, since it is not a practical situation.

As in the case of flow line sizes, Figs. 3.55 and 3.56, which are plots of tubing size vs. flowrate show the tendency of the curves to level off as the tubing size increases; consequently, similar discussions and conclusions can be drawn for the selection of adequate tubing sizes.

Fig. 3.57 shows the effect of the static reservoir pressure for the same PI of 10.0 bpd/psi. Approximately 1,000 bpd is the production increase observed for each ½ in. increase in tubing size in the case of $\bar{p}_R = 4{,}000$ psia. Additional production can be obtained if greater tubing sizes are used provided adequate flow line sizes are also used. It appears that a 2.992 in. ID tubing is an adequate size to handle the production in the case of $\bar{p}_R = 3{,}000$ psia. For $\bar{p}_R = 2{,}000$ psia it seems arti-

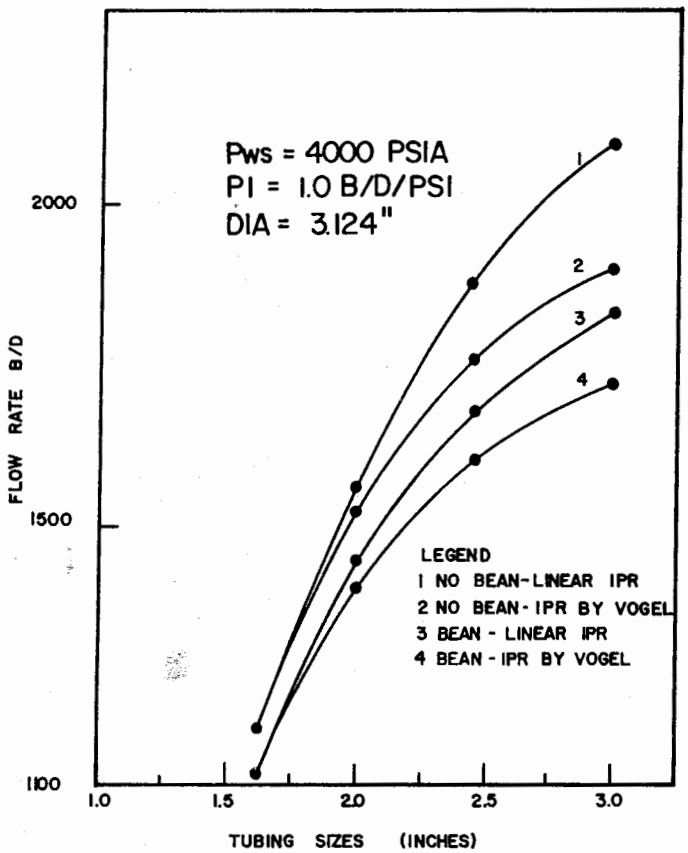


Fig. 3.55 Effect of tubing size (after Fonseca).

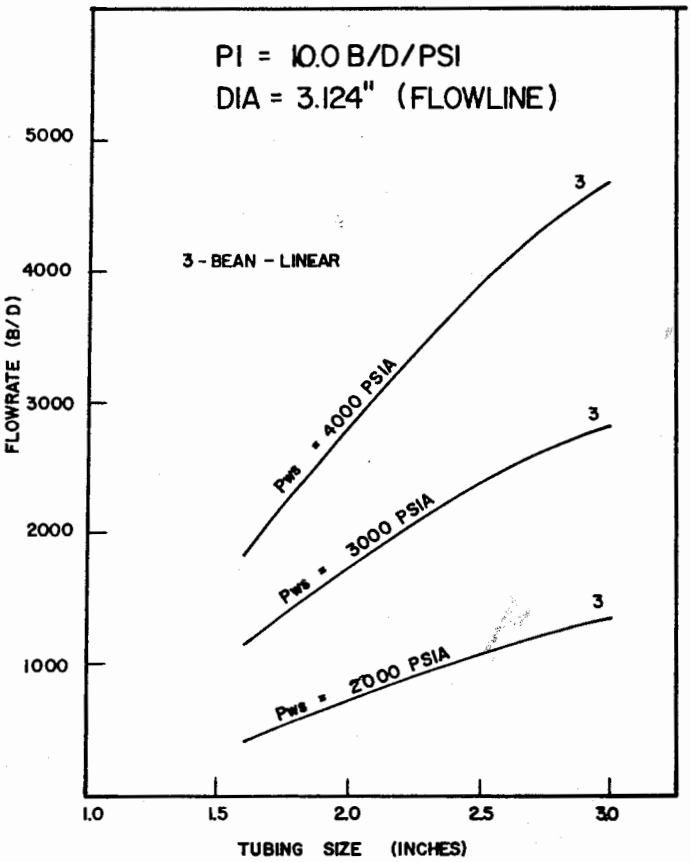


Fig. 3.57 Effect of tubing size (after Fonseca).

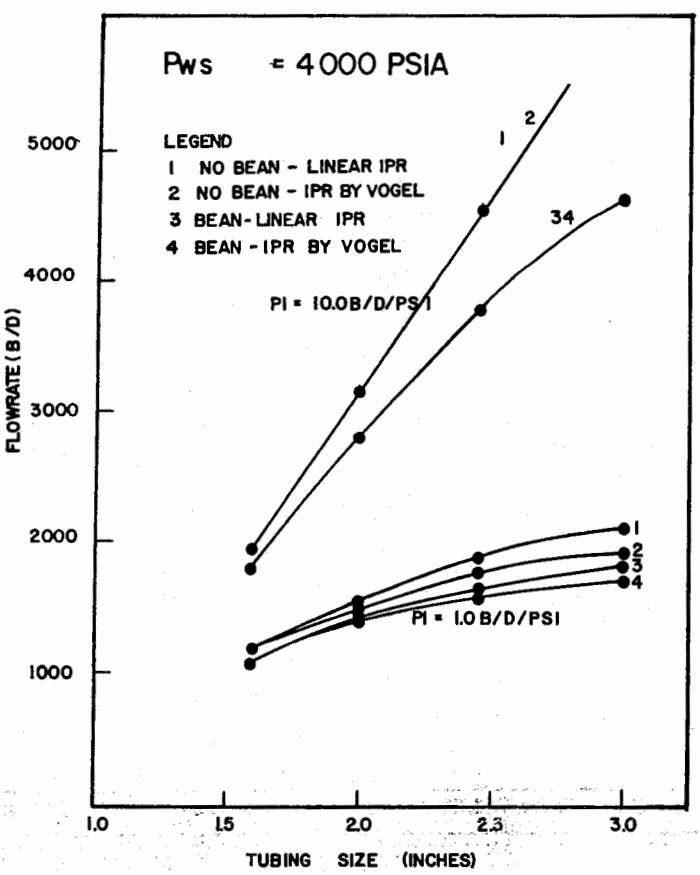


Fig. 3.56 Effect of tubing size (after Fonseca).

ficial lift is strongly recommended for any substantial increase in production.

Fig. 3.58 shows the amount of pressure drop to be experienced for a constant rate of production when the tubing diameter varies. For a rate of production of 256 bpd, a ΔP change of 483 psi is observed for a tubing size change from 1.610 in. to 2.992 in., while 1,408 psi is the value observed for a rate of 1,282 bpd.

Fig. 3.59 is also a plot showing the influence of the tubing size on the rate of increasing pressure loss for increasing rates of production. A variation of 200 bpd causes an increase of 80 psi in a 2.992 in. ID tubing and of 260 psi in a 1.610 in. ID tubing.

Fig. 3.55 is a good example of how small the influence of the choke bean is when small tubing diameters are combined with greater flow line sizes. A difference of 66 bpd is observed for a combination 1.610 in. × 3.124 in., while 262 bpd is noted for a combination of 2.992 in. × 3.124 in.

3.84 Effect of gas-liquid ratio (GLR)

Due to the difference in volumes occupied by gas within the well and within the flow line, the effect of gas liquid ratio on the well performance can be very significant. This is particularly true when a choke bean is installed at the well head and a long and/or undersized flow line is used.

The same problems were solved to show the effect of gas liquid ratio on the performance of an oil well. A 3.124 in. flow line was assumed and the problem again worked for PI = 1.0 and 10.0.

All curves in Fig. 3.60 have an optimum corre-

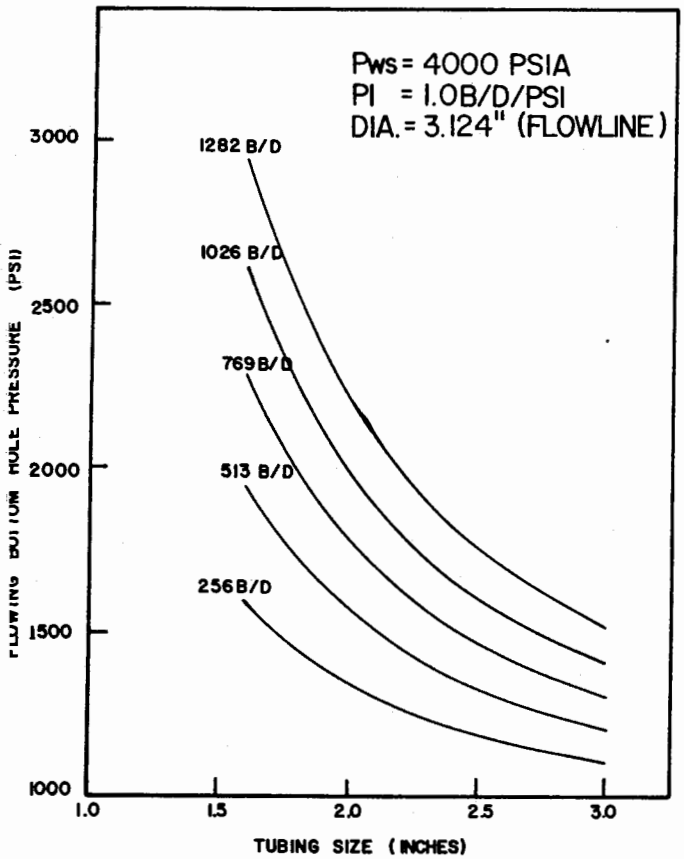


Fig. 3.58 Effect of tubing size for various rates of production after Fonseca).

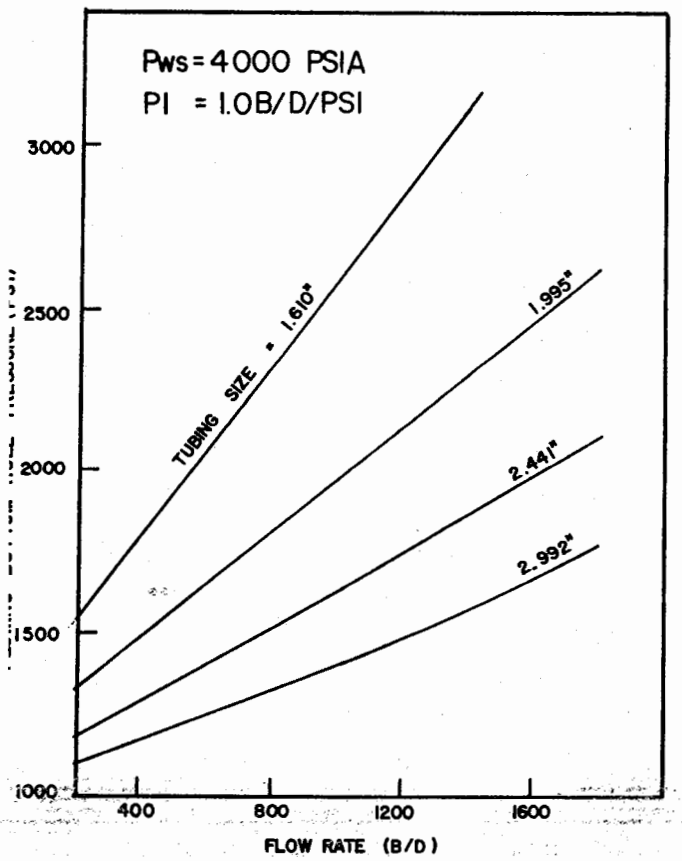


Fig. 3.59 G vs Pwf for various tubing sizes (after Fonseca).

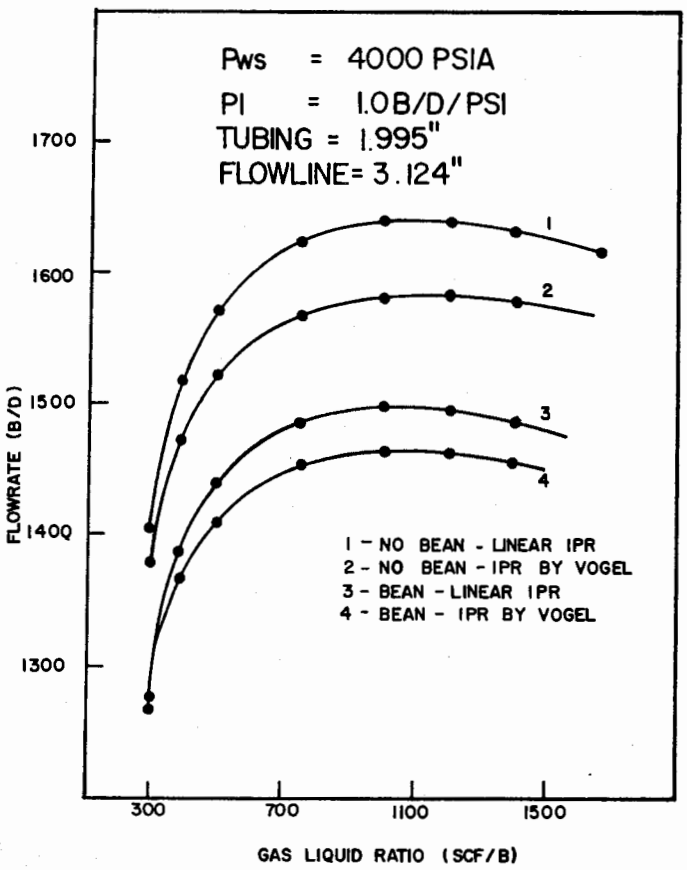


Fig. 3.60 Effect of G/L on flowrate (after Fonseca).

sponding to a GLR between 1,000 and 1,100 scf/stk bbl, although these values are less than those which would give a minimum gradient in the vertical flow string (1,200 scf/stk bbl for this particular case). The conclusion is that as the GLR increases, the liquid production increases due to the decreasing gradients in vertical flow. At the same time, however, the tubing pressures increase due to more and more volumes that have to be handled in the flow line. A point is reached in which the lightening effect in the vertical flow is overcome by the pressure loss in the flow line and the production starts decreasing.

Fig. 3.61 for a PI of 10 bpd/psi shows the combined influence of the GLR and the static reservoir pressure. We note that for $\bar{p}_R = 2{,}000$ psia the maximum production is not reached even for a GLR = 1,400 scf/stk bbl, while it is reached for a GLR = 500 scf/stk bbl in the case of $\bar{p}_R = 4{,}000$ psia. The maximum production shifts to the right as the static pressure decreases, corresponding to higher gas liquid ratios.

Fig. 3.62 is presented to show the influence of the productivity Index combined with variations in gas liquid ratio.

3.85 Effect of well depth

The same problems were solved to show the effect of depth on well performance for flowline diameter = 3.124 in. and PIs = 1.0 and 10.0.

It is obvious that production decreases with any increase in well depth, with other conditions remaining constant. Figs. 3.63, 3.64, and 3.65 are presented to show the effect of well depth combined with the static

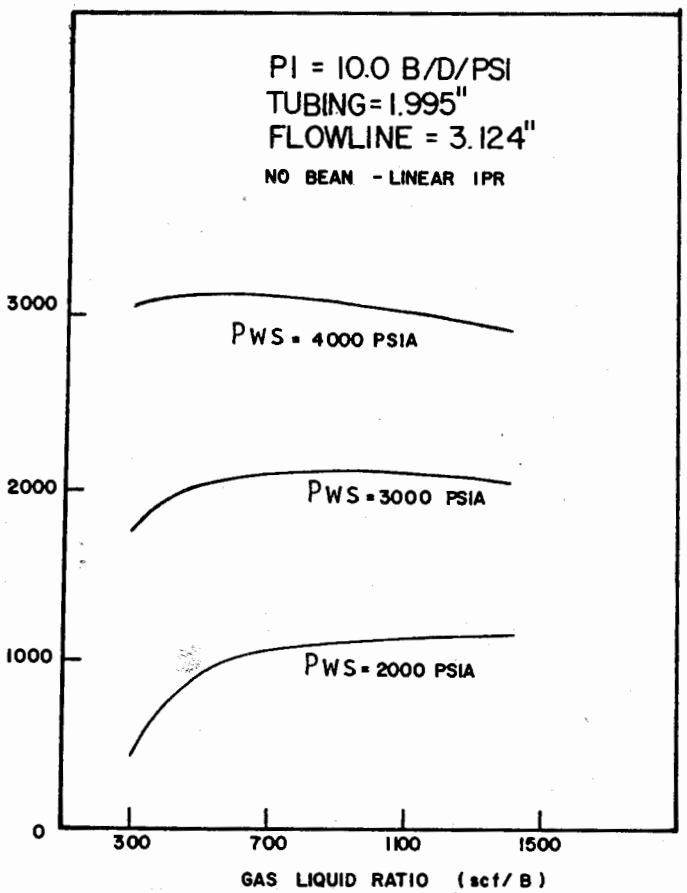


Fig. 3.61 Effect of G/L and Pws on flowrate (after Fonseca).

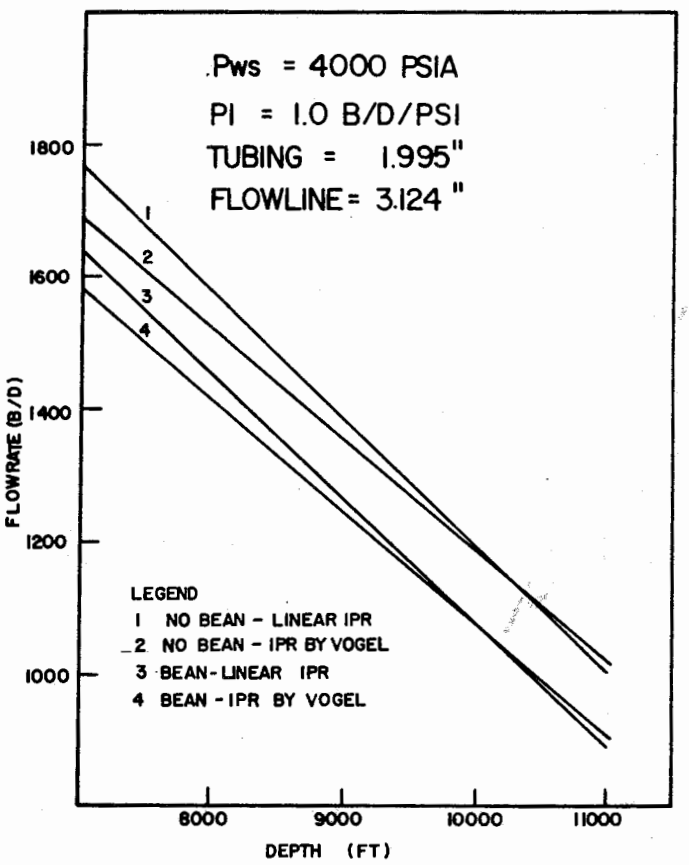


Fig. 3.63 Effect of depth on flowrate (after Fonseca).

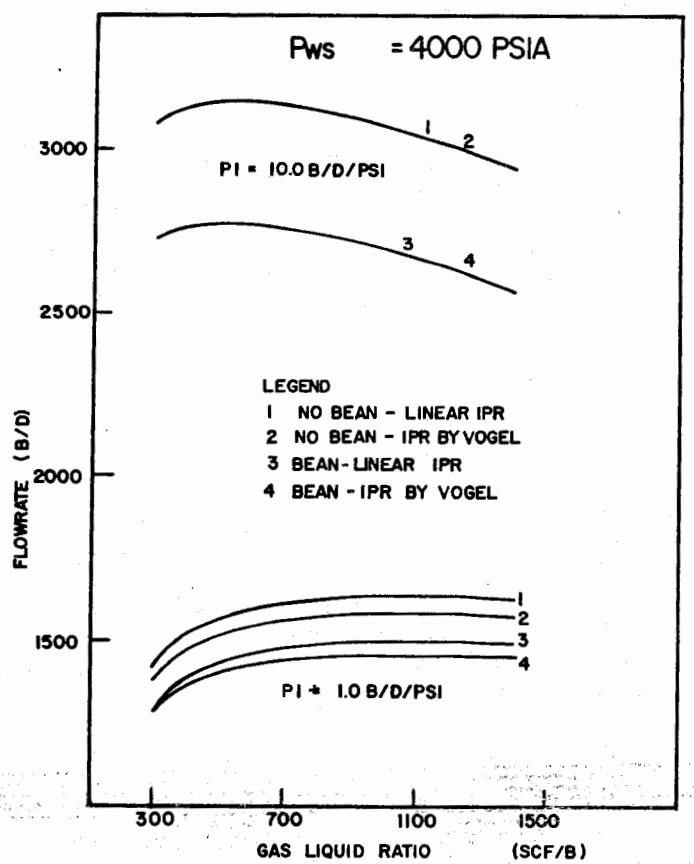


Fig. 3.62 Effect of G/L for PI = 10.0 and PI = 1.0 (after Fonseca).

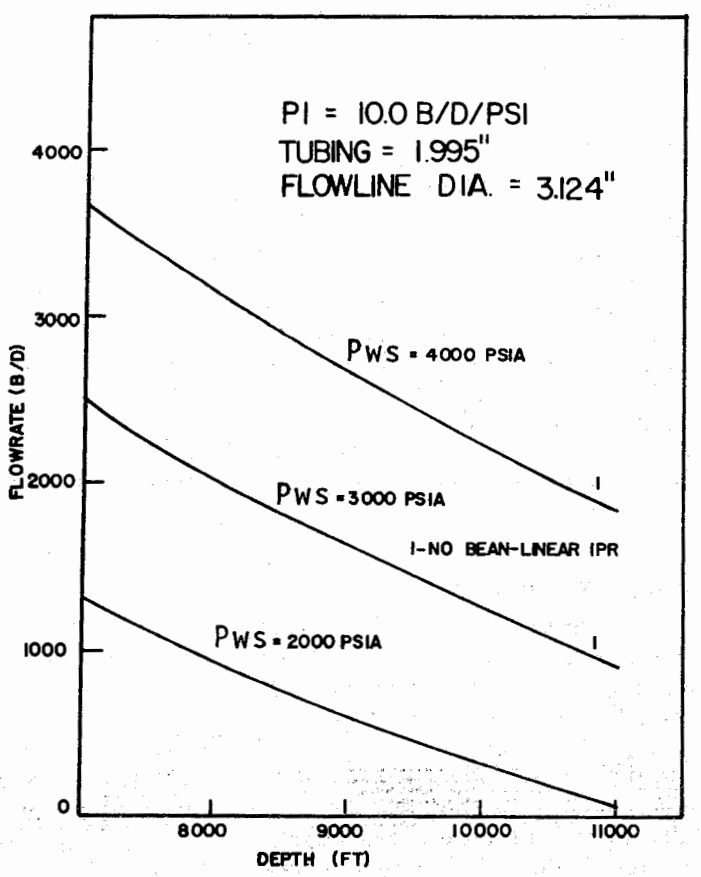


Fig. 3.64 Effect of depth on flowrate (after Fonseca).

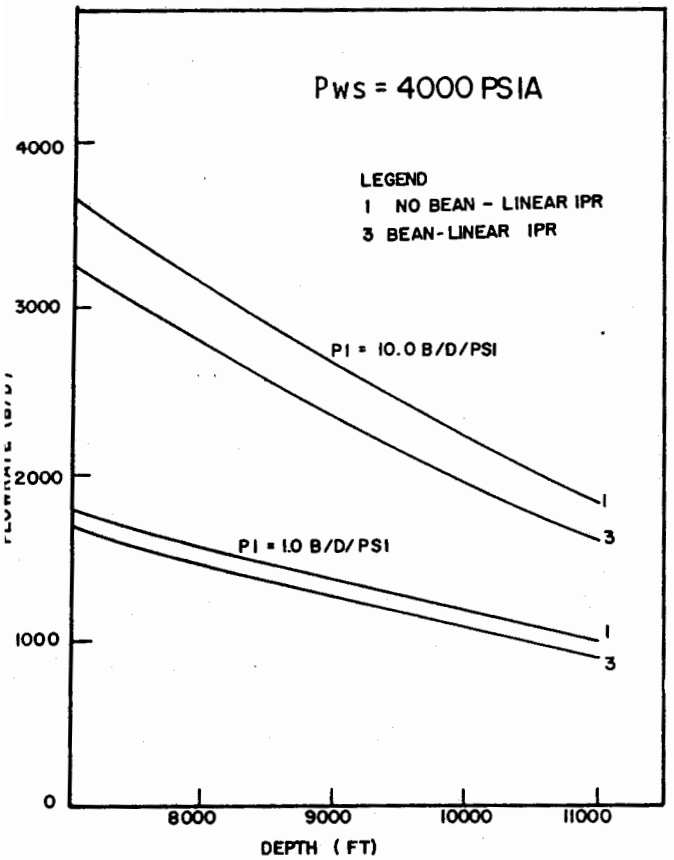


Fig. 3.65 *Effect of depth on flowrate (after Fonseca).*

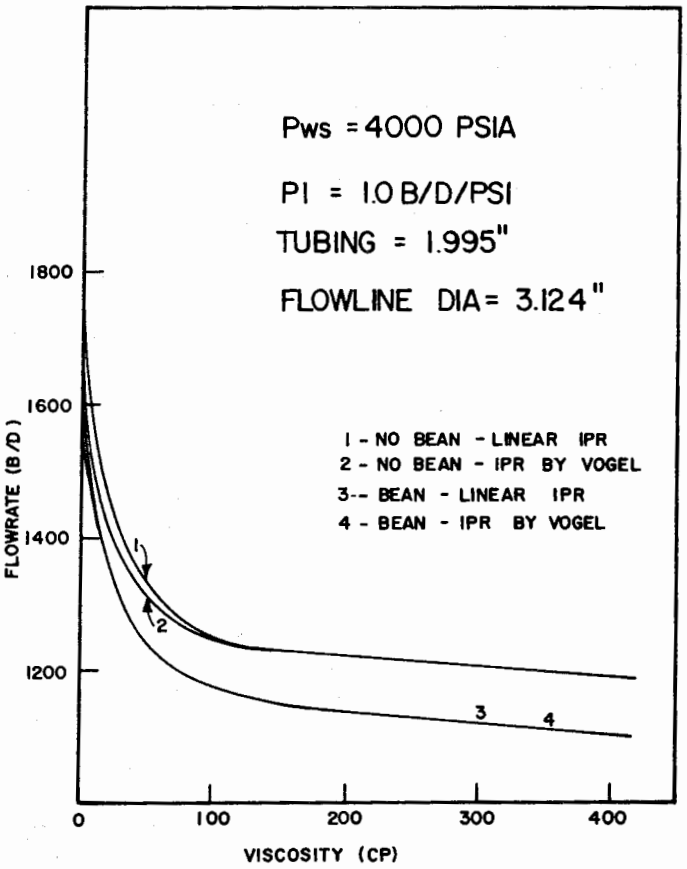


Fig. 3.66 *Effect of viscosity (after Fonseca).*

reservoir pressure and PI. Fig. 3.63 shows the reverse n production rates when calculated assuming linear IPR and IPR calculated by Vogel's formula. Higher values for the Vogel's formula are found at lower rates of production.

3.86 Effect of viscosity

The same problem was solved to show the effect of viscosity on the well performance with PI = 1.0 and flowline diameter = 3.124 in. and tubing size = 2 in.

Fig. 3.66 shows how liquid production decreases with increasing viscosities. It is apparent that with increasing viscosities and decreasing rates of production, laminar flow is established. This appears to occur for values above 100 cp and then further increases in viscosity have less effect on the rate of production. It is interesting to note that in this case there is no appreciable difference between the results obtained by assuming linear IPR or using Vogel's formula.

3.87 Effect of water cut

In order to show the effect of the percentage of water present in the liquid produced by the well, the same example problem was solved using tubing size = 2 in. and flow line diameter = 3.124 in., and for PI = 1.0.

Fig. 3.67 shows how water cut affects the rate of liquid production. As was expected, the production decreases with the increase in percentage of water due to the influence on the density of the liquid, which increases as water cut increases. Greater densities are responsible for greater hydrostatic pressures and

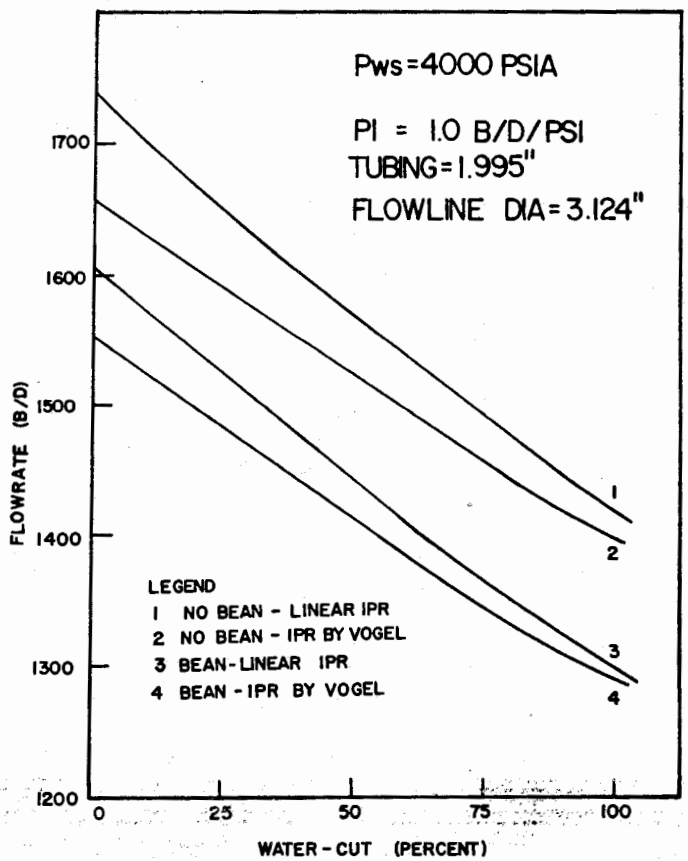


Fig. 3.67 *Effect of water-cut (after Fonseca).*

as a consequence, greater flowing bottom hole pressures are necessary to produce the well. The convergence observed between curves 1 and 2, and 3 and 4 as water percentages increases (rates of production decreases) is due to the fact that Vogel's formula (reference curve) is for the two-phase flow of oil and gas only and should not be considered valid when three phases are flowing. No attempt was made to consider increase in viscosity due to emulsions.

3.9 ECONOMIC ANALYSIS OF THE FLOWING WELL

3.91 Introduction

In all our selection of tubing and flow line sizes we are confronted with having to make an economic analysis. For example, the rate increase in using 3 in. tubing instead of $2\frac{1}{2}$ in. tubing may not be sufficient to offset the additional costs for completing the well. We can use the standard measures of value to make these decisions.

3.92 Measures of value

3.921 Introduction

Measures of value are used by the decision-maker to order, accept, or reject various proposals. In our case, we are looking only at the no-risk measures of value, that is, pay-out, rate of return, net present worth, and others. A good measure of value should be suitable for comparing and ranking the profitability of investment opportunities.

Newendorp[27] listed the following measures of value:

(1) Pay-out
(2) Profit-to-investment ratio
(3) Time-value of money
(4) Rate of return
(5) Net present value
(6) Discounted profit-to-investment ratio
(7) Appreciation of equity
(8) Percentage gain and investment
(9) Analysis of rate acceleration projects

Although it may be difficult to rank the desirability of the various profit indicators, most economists use pay-out, net present value, and rate of return more commonly than the others and probably in that order. Where applicable, we use all indicators possible in making a decision.

3.922 Example problem

In order to best illustrate these measures of value, the following example problem will be worked. Although this problem is very short and covers only a few years, the same procedure would be used for other problems.

A dual well is being drilled and completed to 15,000 ft and a decision is to be made as to whether to design the drilling program to be able to run two strings of 2 in. tubing in parallel or two strings of 3 in. tubing. This decision influences the entire drilling program since 7 in. casing will be used for 2 in. tubings and $10\frac{3}{4}$ in. casing for 3 in. tubings. This, of course, influences the other casing string sizes such as surface and any intermediate or protection strings which controls the hole sizes that must be drilled.

For this problem the following assumptions are made:

(1) Additional cost to drill and complete with two strings of 3 in. tubing = \$1,800,000.
(2) The total change in production rate for six years is according to the following Table 3.11:

TABLE 3.11

Year	3″ × 3″ rate	2″ × 2″ rate	Average rate increase bpd
1	4,000	3,600	400
2	3,000	2,700	300
3	2,500	2,300	200
4	2,000	1,900	100
5	1,800	1,750	50
6	1,500	1,500	0

Use average price for oil = \$10.00/bbl. Assuming no workovers during this period and equal operating costs of \$2.00/bbl, we will compare these two completions from an economic analysis standpoint.

3.9221 Pay-out

Pay-out time is defined as the length of time required to receive accumulated net revenues equal to the investment, or in reality it is the length of time to get the investment back. It is a good idea to prepare a graph similar to Fig. 3.68. The pay-out time is noted to be 21 months.

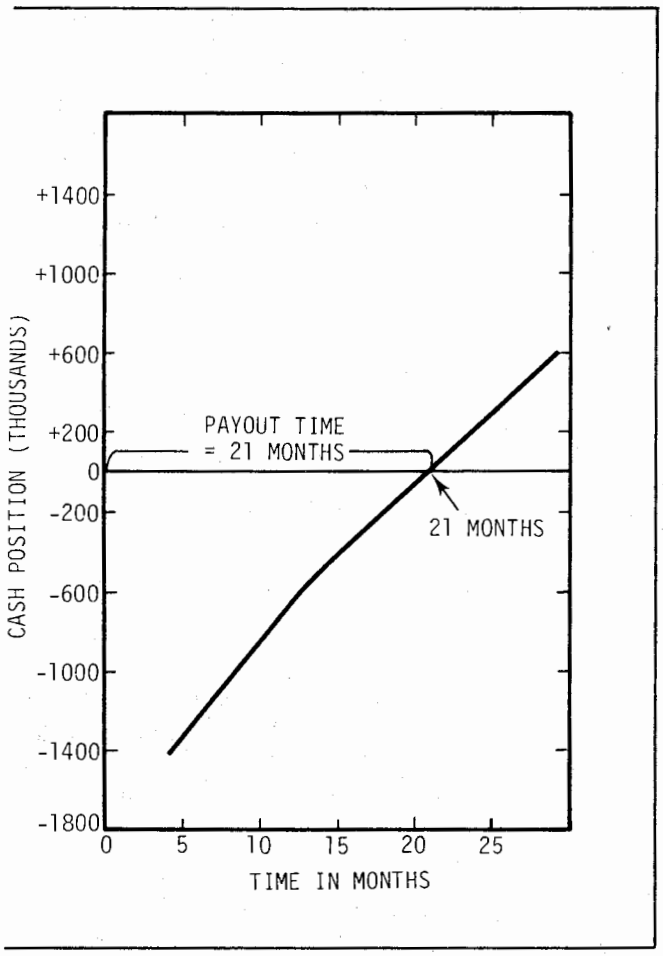
3.9222 Net present value

For obtaining net present value a single, previously specified discount rate is used for all economic analyses. It is called the average opportunity rate and represents the average earnings rate at which future revenues can be invested. It does not represent the interest rate on invested money in savings unless that is a realistic return for a particular company on investments.

Let us assume an average investment opportunity rate of 12% for this company. We will prepare the following Table 3.12. Discount tables can be found in Appendix H. NPV = \$731,529.00

3.9223 Rate of return

Rate of return is the interest rate which makes the present day value of net receipts equal to the present value of the investments. The term "rate of return" is quite commonly used to mean other things used in a similar manner to payout. This represents a trial and error solution. First we select an interest rate and discount all of the cash revenues back to time zero. If the sum of the present values of future cash flow is exactly equal to the initial investment, the rate selected is the rate of return (ROR).



g. 3.68 Cash position curve (after Regnault).

TABLE 3.12

ear	Net cash flow	Discount factor (12%)	12% discounted cash flow
0	− 1,800,000	1.000	− 1,800,000
1	+ 1,152,000	0.94491	+ 1,088,536
2	+ 864,000	0.84367	+ 728,931
3	+ 576,000	0.75328	+ 433,889
4	+ 288,000	0.67257	+ 193,700
5	+ 144,000	0.60051	+ 86,473
6	+ 0	0.53617	+ 0
			+ $ 731,529

Net present value = $731,529.00

A Table 3.13 is prepared as follows: (See Appendix H)

TABLE 3.13

ear	Net cash flow	Discount factor (45%)	Discounted cash flow (45%)
0	− 1,800,000	1.000	− 1,800,000
1	1,152,000	0.83045	+ 956,678
2	864,000	0.57273	+ 494,839
3	576,000	0.39498	+ 227,508
4	288,000	0.27240	+ 78,451
5	144,000	0.18786	+ 27,052
6	0		+ 0
			− 15,472

The above table shows 45% to be slightly too large a value. By comparison 50% gives a value of −87,195 and 40% gives a value of +63,948.

The final value is very close to 44% and represents an excellent ROR on this investment. The problem is easily handled on computer, but may become tedious by long hand if cash flow periods extend over a long period of time. It also gives a profit indicator independent of the absolute size of the cash flow. Also cash flows received early in the project (first two years in our example) are weighted more heavily than later cash flows. This becomes very pronounced for high ROR values (in our example for 44%). The ROR is very sensitive to errors in the original estimate of additional investment (1,800,000 for our example).

Rate of return assumes that all cash flows are reinvested at the computed rate of return when received. For our example, the revenues must be reinvested at 44% when they are received. This is the reason that ROR may not be a realistic measure of true profitability for our example. It is a more realistic measure of value than payout time because it includes the time-value of money concept. It is also useful in comparing one investment to another.

3.9224 Profit-to-investment ratio

The profit-to-investment ratio differs from pay-out in that it does reflect total profitability. It is defined as the ratio of the total undiscounted net profit to investment. It may be called the return-on-investment, or ROI.

For our example we have an investment of $1,800,000, and a total net cash flow of $3,024,000. Undiscounted net profit = $3,024,000 − $1,800,000 = $1,224,000.

$$\text{Profit to investment ratio} = \frac{1{,}224{,}000}{1{,}800{,}000} = 0.68$$

In some cases we use the ratio of net income to investment, which is for our example:

$$\text{Net income-to-investment ratio} = \frac{3{,}024{,}000}{1{,}800{,}000} = 1.68$$

This ratio is sometimes called "leverage." The ratio is simple to calculate. It is a measure of total profit from the investment and does not require a detailed cash flow.

3.9225 Discounted profit-to-investment ratio (DPR)

This is a particularly good measure of value for selecting projects from a list which contains more opportunities than the available funds can cover. The DPR is the amount of discounted net present value profit generated in excess of the average opportunity rate per dollar invested.

The NPV previously calculated for our example was $731,529.00.

Investment = 1,800,000

$$\text{Ratio of discounted profit-to-investment} = \frac{731{,}529}{1{,}800{,}000} = 0.4064$$

3.9226 Other measures of value

There are other measures of value used, but most are adaptations or modifications of the basic concepts of rate of return and net present value. Some of these are (1) appreciation of equity, (2) percentage gain on investment and "risk weighted" rate of return. I suggest that reference be made to Newendorp for further detailed discussions on these topics.[27]

EXAMPLE PROBLEM #15

The following well is to be dually completed and we have available different flowline and tubing sizes. The purpose of this problem is to illustrate the manner in looking at all the tubing and flowline sizes for each zone and then to show how quickly these high rate zones will payout from an economic standpoint.

Given data:

A well is to be dually completed:

1. Zone one at 7,500 ft
2. Zone two at 8,000 ft
3. Casing available size $9^5/_8$ in., 47 lb/ft
4. Flow lines and tubing available: 2 in., 2.5 in., 3 in., 4 in. (ID.)

Zone I:

$\bar{p}_R$ original = 3,000 psi
G/O = 400 scf/bbl
Horizontal flow line length = 3,000 ft
Pressure of the separator = 80 psi
P.I. original = 4 bpd/psi
$\bar{p}_R$ decreases 100 psi each 100,000 bbl of recovery
P.I. = 0.0013334 $\bar{p}_R$ (assumed linear)

Zone II:

$\bar{p}_R$ original = 3,400 psi
G/O = 600 scf/bbl
Horizontal flow line length = 3,000 ft
Pressure of the separator = 80 psi
P.I. original = 10 bpd/psi
$\bar{p}_R$ decreases 100 psi each 250,000 bbl of recovery
P.I. = 0.00294 $\bar{p}_R$ (assumed linear)
Zone II starts producing 25% water after 2,000,000 bbls recovery and 50% water after 3,000,000 bbls of oil recovery.

Required

Select the tubing and flowline sizes for both zones with no regard for artificial lift method. Plan on producing wells at maximum flow rates possible. Select optimum tubing and flowline sizes. Determine the time in years, the cumulative recovery and static pressure when the production of 200 bpd can no longer be obtained from each zone.

Assuming that total cost of drilling and completion is $5,000,000 and interest on money is 10%, determine at what year the investment will be paid for and what will be the present worth of total production when both zones quit flowing.

Additional assumptions are:

Year	Lifting cost, $/bbl	
1	0.05	Workover every three years
2	0.06	costing: $150,000 each time.
3	0.07	
4	0.07	Gross value of bbl of oil = $11.00
5	0.08	
6 thru 10	0.10	
10 thru 15	0.15	
15 thru 20	0.25	
20	0.50	

Average investment opportunity rate is 20%.

Also determine:

(a) Rate of return
(b) Present worth
(c) Payout

In the same manner as described previously Figs. 3.69 and 3.70 are prepared in order to look at all pipe sizes for each zone. Table 3.14 summarizes this information.

From these two tables it is quite obvious that the maximum flow rate of any tubing is associated with a 4 in. flowline, and vice versa. However, we recall that we are limited by having a casing size of $9^5/_8$ in. In order to select the tubing size possibilities we need the following information on coupling sizes. (See Table 3.15).

In order to find the maximum possible flow rates from the tubing of both zones, we prepare the following Table 3.16 where we combine the flow rates of different tubing sizes of both zones with flowlines of 4 in. diameters only.

From Table 3.16 we determine that the maximum flow rate is 10,850 bpd, which is the optimum tubing sizes that fit the well's casing. These two selections are:

2 in. tubing from Zone I associated with a 4 in. flowline.

4 in. tubing from Zone II associated with a 4 in flowline.

These combinations of tubing sizes will give a clearance of: 8.681 in. − (2.875 in. + 5.200 in.) = 0.606 in., which is sufficient for a flowing well, but may have to be changed for artificial lift purposes.

In order to determine the static pressure, cumulative recovery and time when the well no longer flows 200 bpd we prepare Fig. 3.71, Table 3.17 and Fig. 3.72 for Zone 1. For Zone II we prepare Fig. 3.73, Table 3.18 and Fig. 3.74.

The economic analysis is obtained by preparing Tables 3.19 and 3.20, Fig. 3.75, Table 3.21, and Fig. 3.76.

Finally a summary of results is given in Table 3.22.

Of course, the economics of this venture are extremely attractive because of the high flow rate zones.

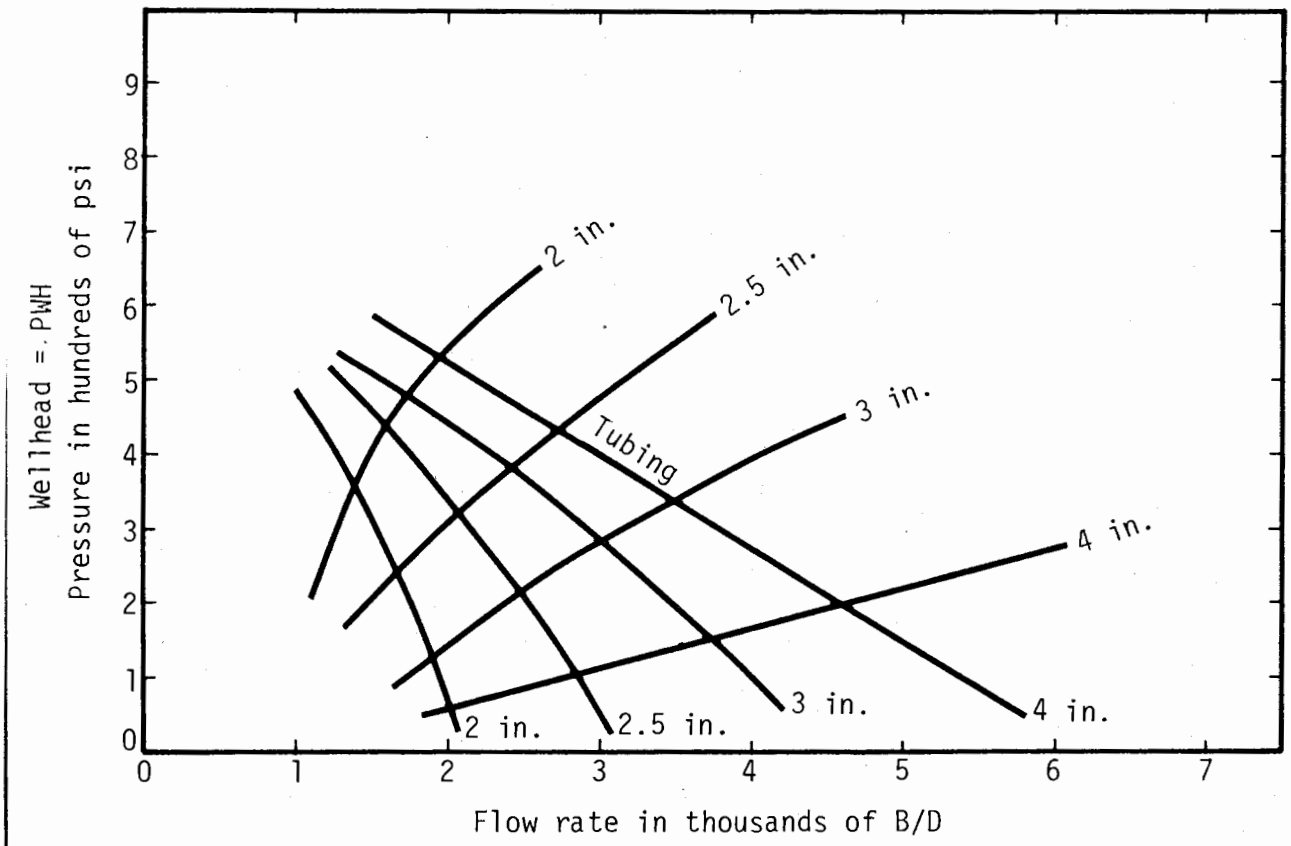


Fig. 3.69 Flowing analysis chart of different tubing and flowline of zone I (after Regnault).

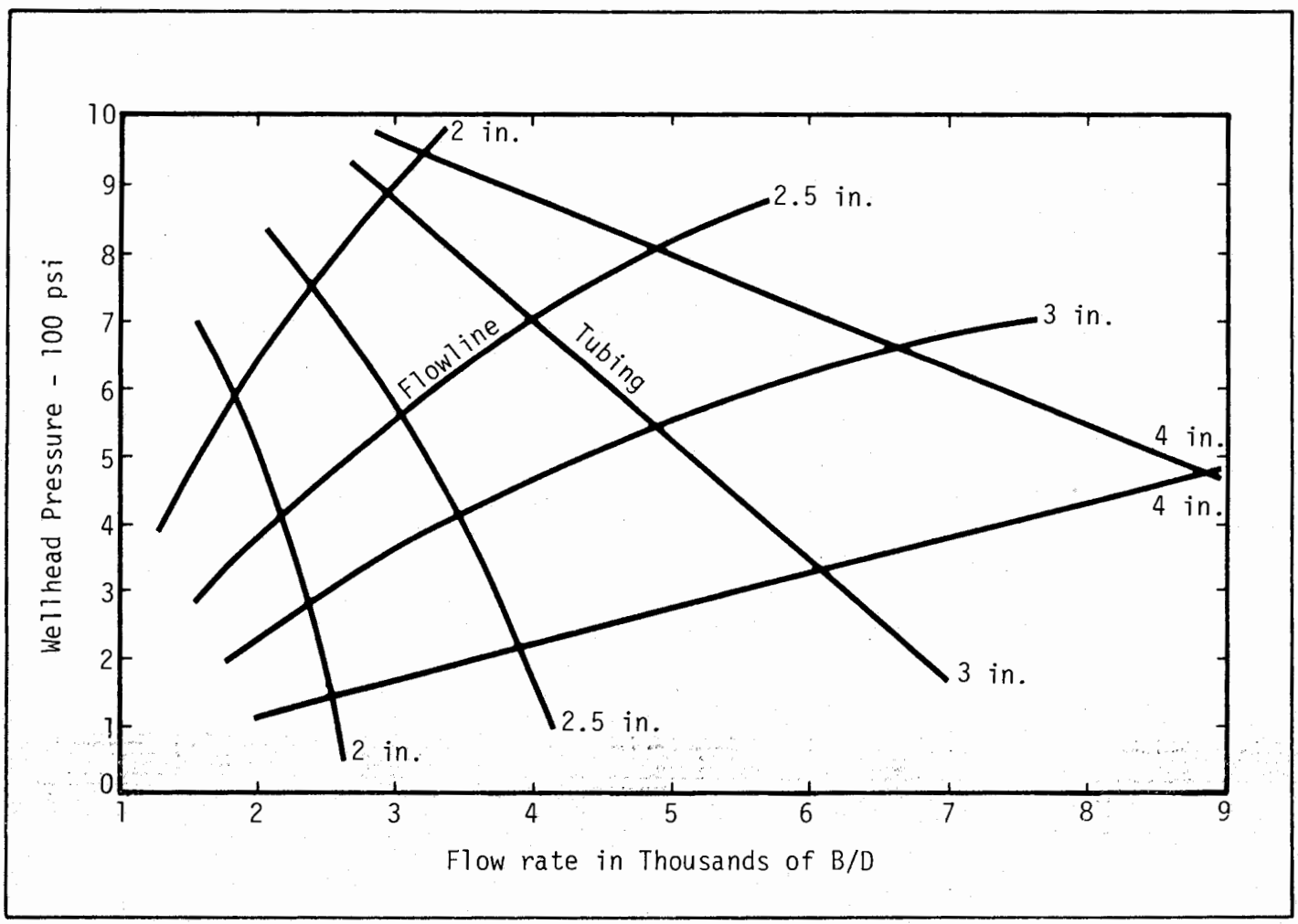


Fig. 3.70 Flowing analysis chart of different tubing and flowline of zone II (after Regnault).

TABLE 3.14
MAXIMUM FLOW RATE FOR EACH COMBINATION OF SIZES OF FLOWLINE AND TUBING

Zone I:

Tubing \ Flowline	2 in.	2.5 in.	3 in.	4 in.
2 in.	1,375	1,650	1,900	2,000
2.5 in.	1,600	2,075	2,500	2,850
3 in.	1,700	2,400	3,000	3,725
4 in.	1,925	2,700	3,475	4,625

Zone II:

Tubing \ Flowline	2 in.	2.5 in.	3 in.	4 in.
2 in.	1,825	2,150	2,350	2,550
2.5 in.	2,400	3,025	3,450	3,925
3 in.	2,900	3,975	4,850	6,075
4 in.	3,175	4,850	6,600	8,850

TABLE 3.15

ID of tubing, in.	OD coupling, in.
2	2.875
2.5	3.500
3	4.250
4	5.200

TABLE 3.16
MAXIMUM FLOW RATE OF THE TUBINGS OF BOTH ZONES FOR CONSTANT FLOWLINE SIZE OF 4 IN.

Tubing Zone I \ Tubing Zone II	2 in.	2.5 in.	3 in.	4 in.
2 in.	4,550	5,925	8,075	10,850
2.5 in.	5,400	6,775	8,925	11,700
3 in.	6,275	7,650	9,800	12,575
4 in.	7,175	8,550	10,700	13,475

TABLE 3.17
DETERMINATION OF FLOW RATE (B/D) AND TIME (MONTH) FOR ZONE I FROM FIG. 3.72

$\overline{PR}$ (psi)	Np (Bbls)	PI (B/D/psi)	q_o (B/D)	Time (Month)
3000	–	4.00	–	–
2900	100,000	3.87	1800	1.83
2800	200,000	3.74	1688	3.78
2700	300,000	3.60	1575	5.87
2600	400,000	3.47	1462	8.12
2500	500,000	3.34	1362	10.53
2400	600,000	3.20	1250	13.16
2300	700,000	3.07	1150	16.02
2200	800,000	2.94	1038	19.19
2100	900,000	2.80	938	22.69
2000	1,000,000	2.67	838	26.61
1900	1,100,000	7.54	738	31.06
1800	1,200,000	2.40	650	36.12
1700	1,300,000	2.27	550	42.10
1600	1,400,000	2.13	438	49.61
1500	1,500,000	2.00	338	59.34
1400	1,600,000	1.87	212	74.85

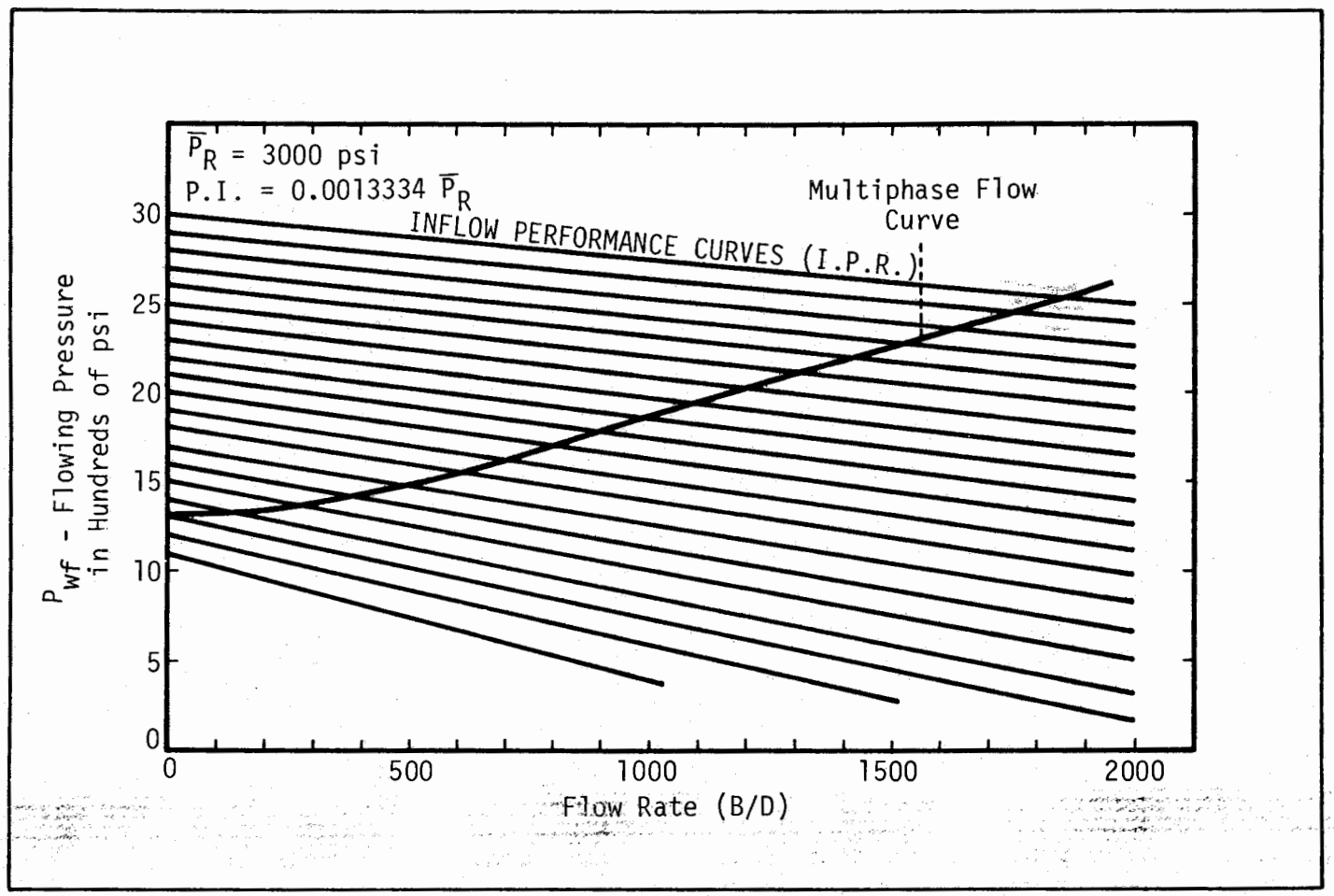


Fig. 3.71 Plot of flow rate vs. PWF for zone I (after Regnault).

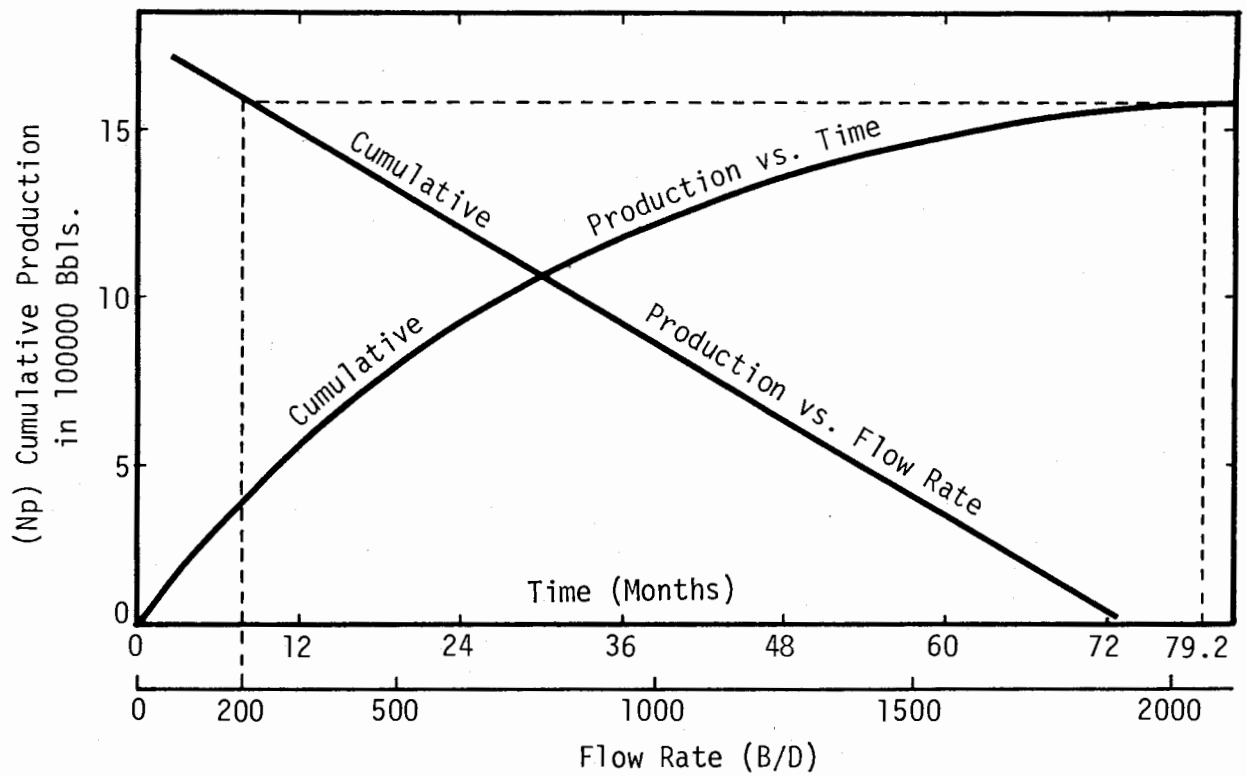


Fig. 3.72 Plot of cumulative production versus time and flow rate zone I (after Regnault).

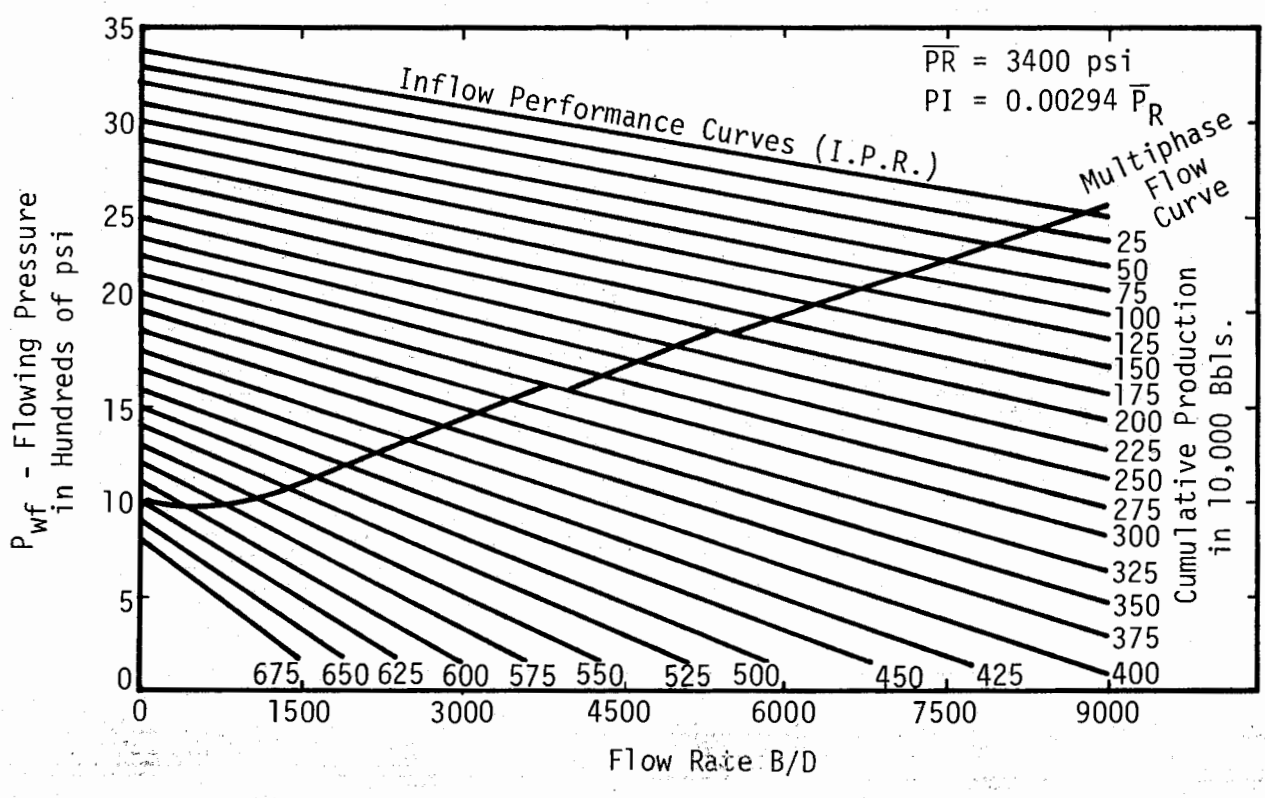


Fig. 3.73 Plot of flow rate vs. P_{wf} for zone II (after Regnault).

TABLE 3.18
DETERMINATION OF FLOW RATE (B/D) AND TIME (MONTH) FOR ZONE II FROM FIG. 3.74

$\overline{PR}$ (psi)	Np (Bbls)	PI (B/D/psi)	q_o (B/D)	Time (Month)
3400	–	10.00	–	–
3300	250,000	9.70	8580	0.96
3200	500,000	9.41	8160	1.97
3100	750,000	9.11	7710	3.04
3000	1,000,000	8.82	7260	4.17
2900	1,250,000	8.53	6900	5.36
2800	1,500,000	8.23	6480	6.63
2700	1,750,000	7.94	6060	7.99
2600	2,000,000	7.64	5730	9.42
2500	2,187,500	7.35	5160	10.61
2400	2,375,000	7.06	4800	11.89
2300	2,562,500	6.76	4500	13.26
2200	2,750,000	6.47	4140	14.75
2100	2,875,000	6.17	3600	15.89
2000	3,000,000	5.88	3270	17.15
1900	3,125,000	5.59	2940	18.55
1800	3,250,000	5.29	2640	20.11
1700	3,375,000	5.00	2340	21.87
1600	3,500,000	4.70	2040	23.88
1500	3,625,000	4.41	1740	26.24
1400	3,750,000	4.12	1500	28.98
1300	3,875,000	3.82	1230	32.32
1200	4,000,000	3.53	960	36.60
1100	4,125,000	3.23	720	42.31

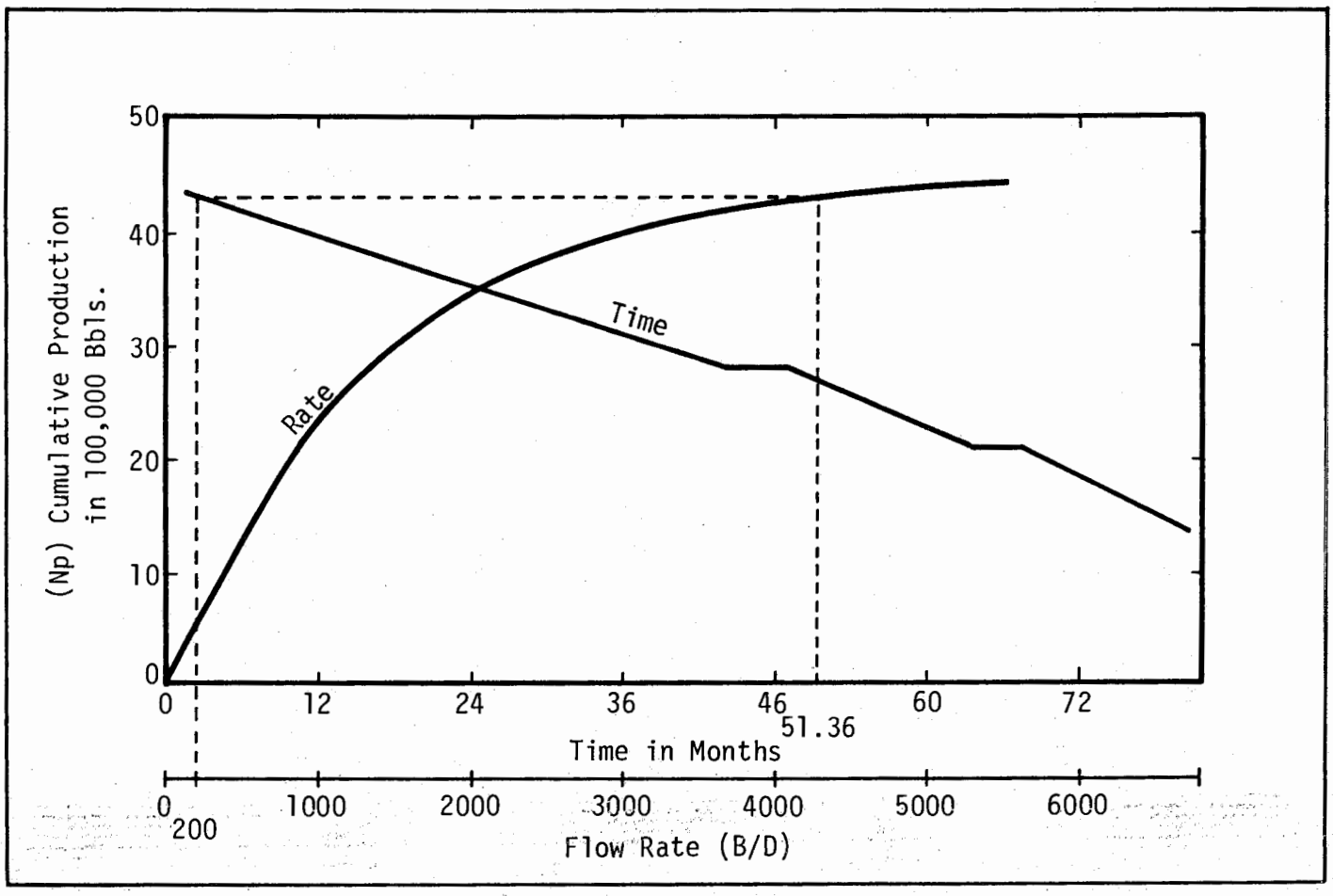


Fig. 3.74 Plot of cumulative production versus time and flow rate zone II (after Regnault).

TABLE 3.19
PRODUCTION AND PROJECTED NET CASH FLOW FOR ZONE I AND ZONE II

Year	Cumulative Oil Production (bbl) Zone I	Cumulative Oil Production (bbl) Zone II	Total Production (bbl) Each Year	Annual Revenue $11.00/bbl	Lifting Cost $	Future Expenditures $	Net Cash Flow $
0	—	—	—	—	5,000,000	—	−5,000,000
1	560,000	2,368,421	2,928,421	32,212,631	146,421		32,066,210
2	940,000	3,526,315	1,537,894	16,916,834	92,274		16,824,560
3	1,200,000	4,000,000	733,685	8,070,535	51,358	150,000	7,869,177
4	1,380,000	4,263,158	443,158	4,874,738	31,021		4,843,717
5	1,520,000	4,315,789	192,631	2,118,941	15,410		2,103,531
6	1,600,000	—	80,000	880,000	8,000	150,000	722,000
7	1,630,000	—	30,000	330,000	3,000		327,000

TABLE 3.20
PAYOUT TIME CALCULATION FOR ZONE I AND ZONE II

Year	Net Cash Flow	Discount Factor 10%	Discount Factor 20%	Discounted Cash Flow 10%	Discounted Cash Flow 20%	Cumulative Net Cash Flow 10%	Cumulative Net Cash Flow 20%
0	−5,000,000	1.00000	1.00000	−5,000,000	−5,000,000	−5,000,000	−5,000,000
1	32,066,210	0.95346	0.91287	30,573,848	29,272,281	25,573,848	24,272,281
2	16,824,560	0.86678	0.76073	14,583,192	12,798,947	40,157,040	37,071,228
3	7,869,177	0.78799	0.63394	6,200,832	4,988,586	46,357,872	42,059,814
4	4,843,717	0.71635	0.52828	3,469,797	2,558,839	49,827,669	44,618,653
5	2,103,531	0.65123	0.44023	1,369,882	926,037	51,197,551	45,544,690
6	722,000	0.59203	0.36686	427,446	264,873	51,624,997	45,809,563
7	327,000	0.53820	0.30572	175,991	99,970	51,800,988	45,909,533
		Net Present Worth		51,800,988	45,909,533		

TABLE 3.21
RATE OF RETURN CALCULATION FOR ZONE I AND ZONE II

Year	Net Cash Flow	Discount Factor 50%	Discount Factor 100%	Discount Factor 200%	Discounted Cash Flow 50%	Discounted Cash Flow 100%	Discounted Cash Flow 200%
0	−5,000,000	1.00000	1.00000	1.00000	−5,000,000	−5,000,000	−5,000,000
1	32,066,210	0.81650	0.70711	0.57735	26,182,060	22,674,338	18,513,426
2	16,824,560	0.54433	0.35355	0.19245	9,158,112	5,948,323	3,237,886
3	7,869,177	0.36289	0.17673	0.06415	2,855,646	1,390,720	504,808
4	4,843,717	0.24192	0.08839	0.02138	1,171,792	428,136	103,558
5	2,103,531	0.16128	0.04419	0.00713	339,257	92,955	14,998
6	722,000	0.10752	0.02210	0.00238	77,629	15,956	11,718
7	327,000	0.07168	0.01105	0.00079	23,439	3,613	258
			Net Present Worth		34,807,935	25,554,041	17,376,652

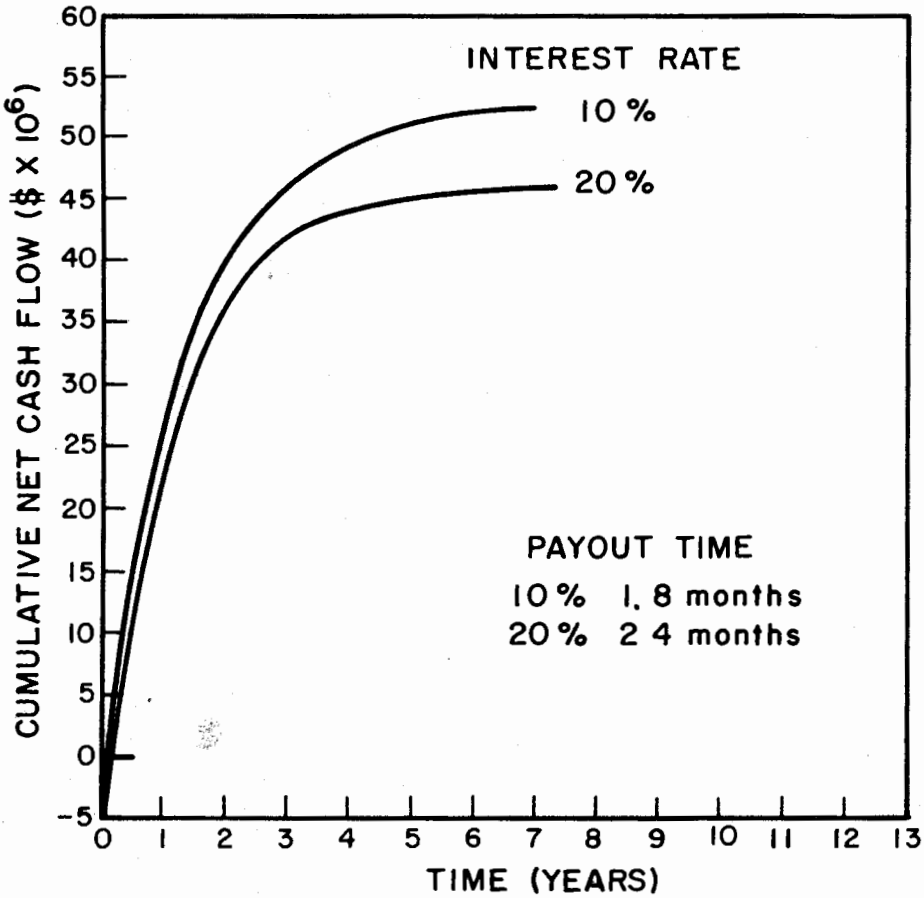
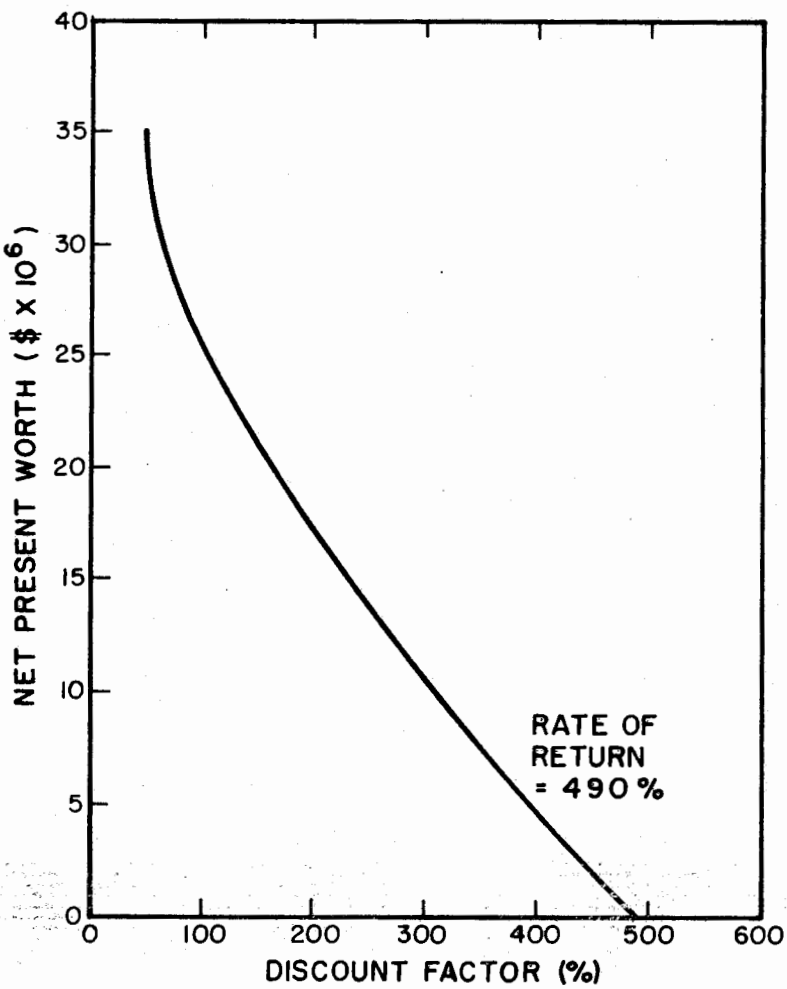


TABLE 3.22
SUMMARY OF RESULTS

	Zone I	*Zone II*
Flowline size	4 in.	4 in.
Tubing size	2 in. (ID)	4 in. (ID)
Production less than 200 bpd		
Time, years	6.60	4.28
Cumulative recovery, bbl	1.63×10^6	4.32×10^6
Static pressure, psi	1,333	966
Economic analysis	*Both zones combined*	
At 10% discount rate	Payout	0.15 year
	Net present worth	$51,800,988
At 20% discount rate	Payout	0.20 year
	Net present worth	$45,909,533
Rate of return	490%	



CLASS PROBLEM #15-A

Work example problem #15 with the following changes. Work Zone I only for 4 in. tubing and 4 in. flowline and change $\bar{p}_R$ initial to 3,600 psia and show $\bar{p}_R$ decreasing 100 psi for each 300,000 bbls of recovery.

CLASS PROBLEM #15-B

Work #15-A for 2 in. tubing and $2\frac{1}{2}$ in. flowline.

3.10 Summary and conclusions

The problem of the flowing well is not simple. The numerous variables and rapid changes that can occur complicate the problem. However, in general, if we have good information on the well and can reasonably predict future changes in static pressure, inflow performance, gas liquid ratio and water-cut, we can properly size the tubing and flowline, predict needed choke sizes, maximum flow rates, and flowing life. Also we can make good economic judgments as to sizes of tubing strings and flowline sizes.

Finally, the total system must be considered. It may very well be that the system (flowline and tubing) is the limiting factor on the production rate, not the reservoir. An expensive stimulation treatment may not be necessary, since the system may not be able to handle any additional production. However, we may also find the opposite—that is, larger flowline and tubular size, or even annular flow, may be beneficial. Only by looking critically at the entire system can this be properly evaluated.

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Appendices

Appendix A

Appendix A.1: Solving for FE from two flow tests

Couto[1] presented a procedure to solve for flow efficiency (FE) from two flow tests on the well. This procedure makes use of Vogel's equation and does require that we know $\bar{p}_R$.

From Standing's work in Section 1.343 we recall that $FE = \frac{\bar{p}_R - p'_{wf}}{\bar{p}_R - p_{wf}}$ where p_{wf} is the ideal flowing pressure and p_{wf} is the actual flowing pressure. Remember also that FE is found from:

$$FE = \frac{\bar{p}_R - p_{wf} - \Delta p_{skin}}{\bar{p}_R - p_{wf}}$$

Since Standing assumed a constant skin value (independent of rate and time) we should obtain the same FE value from each flow test. Therefore, in general, this solution is trial and error in that a value of FE is assumed and a value of $q_{o(max)}$ is solved for from each flow test. Other values are assumed until we obtain the same value of $q_{o(max)}$ from each test.

The following equations are valid:

$$p_{wf} = \bar{p}_R - FE(\bar{p}_R - p_{wf}) \qquad (A.1)$$

$$\frac{p_{wt}}{\bar{p}_R} = 1 - FE + FE\left(\frac{p_{wf}}{\bar{p}_R}\right) \qquad (A.2)$$

This gives us the relationship between the flow rate $q_{o,p_{wf}}^{FE=j}$ for a well with a flow efficiency j, producing with a bottom hole flowing pressure p_{wf} and the maximum flow rate $q_{o(max)}^{FE=1}$ of the image well (FE = 1, well producing with $p_{wf} = 0$), for each $R' = 1 - FE + (R)(FE) = 0$.

$$\left(\text{where } R = \frac{p_{wf}}{\bar{p}_R}\right) \text{ and } \left(R' = \frac{p'_{wf}}{\bar{p}_R}\right)$$

Vogel's curve is that obtained from the generalized equation for FE = 1. If we use the generalized equation (FE = 1) we obtain Standing's set of curves.

We have a trial and error solution in that only one value of FE will satisfy both tests. We then assume values of FE and find that value that is the same from both tests. We can also prepare a graphical plot as noted in Fig. A.1 to simplify the solution. An example problem will best illustrate this procedure:

Given: $\bar{p}_R = 2{,}000$ psia

Test	q_o	p_{wf}
1	165	1,500
2	298	1,000

The problem is to find FE for the well.

Recalling Vogel's equation:

$$q_o/q_{o(max)} = 1 - 0.2\left(\frac{p_{wf}}{\bar{p}_R}\right) - 0.8\left(\frac{p_{wf}}{\bar{p}_R}\right)^2$$

In the form used by Standing we can write the equation:

$$\frac{q_o^{FE=j}}{q_{o(max)}^{FE=1.0}} = 1 - 0.20\left(\frac{p'_{wf}}{\bar{p}_R}\right) - 0.80\left(\frac{p'_{wf}}{\bar{p}_R}\right)^2 \qquad (A.3)$$

where j = value of FE and p'_{wf} is the ideal flowing pressure. By substituting equation A.2: $\frac{p_{wf}}{\bar{p}_R} = 1 - FE + FE\left(\frac{p_{wf}}{\bar{p}_R}\right)$ we have:

$$\frac{q_o^{FE=j}}{q_{o(max)}^{FE=1.00}} = 1 - 0.20\left[1 - FE + FE\left(\frac{p_{wf}}{\bar{p}_R}\right)\right] - 0.80\left[1 - FE + FE\left(\frac{p_{wf}}{\bar{p}_R}\right)\right]^2 \qquad (A.4)$$

Solution:

$$\frac{p_{wf_1}}{\bar{p}_R} = \frac{1{,}500}{2{,}000} = 0.75$$

$$\frac{p_{wf_2}}{\bar{p}_R} = \frac{1{,}000}{2{,}000} = 0.50$$

$$\frac{p'_{wf_1}}{\bar{p}_R} = 1 - FE + \frac{p_{wf_1}}{\bar{p}_R}FE = 1 - FE + 0.75FE = 1 - .25FE$$

$$\frac{p'_{wf_2}}{\bar{p}_R} = 1 - FE + FE\ \frac{p_{wf_2}}{\bar{p}_R} = 1 - 0.5FE$$

Assume a value of FE = 0.6

Then:

$$\frac{p'_{wf_1}}{\bar{p}_R} = 1 - 0.25FE = 0.85$$

$$\frac{p'_{wf_2}}{\bar{p}_R} = 1 - 0.5FE = 0.70$$

We now use Equation A.3:

$$\frac{q_o^{FE=j}}{q_{o(max)}^{FE=1.00}} = 1 - 0.2\left(\frac{p'_{wf}}{\bar{p}_R}\right) - 0.80\left(\frac{p'_{wf}}{\bar{p}_R}\right)^2$$

For FE = 0.6 and Test #1 where $\frac{p'_{wf}}{\bar{p}_R} = 0.85$

$$\frac{q_o^{FE=0.6}}{q_{o(max)}^{FE=1.00}} = 1 - 0.2\ (0.85) - 0.80\ (0.85)^2 = 0.252$$

For FE = 0.6 and Test #2:

$$\frac{q_o^{FE=0.6}}{q_{o(max)}^{FE=1.00}} = 1 - 0.2(0.7) - 0.80(0.7)^2 = 0.468$$

In the same manner values are calculated for assumed values of FE = 0.7, 0.8, 0.9, 1.0, 1.1, 1.2. These are noted in the following table:

FE values	$q_o/q_{o(max)}$ Test 1	$q_o/q_{o(max)}$ Test 2	Ratio q_o/q_{max} Test 1/Test 2
0.6	0.252	0.468	0.539
0.7	0.290	0.532	0.545
0.8	0.328	0.592	0.554
0.9	0.364	0.648	0.562
1.0	0.400	0.700	0.571
1.1	0.434	0.748	0.580
1.2	0.468	0.792	0.591

For our problem the ratio of $\frac{q_{o_1}^{FE=j}}{q_{o_2}^{FE=j}} = \frac{165}{298} = 0.554$ and this occurs at a value of j = 0.8 as noted in the tables.

A graphical solution is noted on Fig. A.1 where the intersection of the curves for Test 1 and Test 2 give the value of FE = 0.8 and for a $q_{o(max)}$ rate of 503 bpd. This graph is prepared by assuming values of FE and calculating the corresponding rates. For example for FE = 0.6, we have

Test 1: $q_{o(max)}^{FE=1.00} = \frac{q_o^{FE=0.6}}{0.252} = 654.7$ bpd

Test 2: $q_{o(max)}^{FE=1.00} = \frac{q_o^{FE=0.6}}{0.469} = \frac{298}{0.469} = 636.8$ bpd

We also find $q_{o(max)}$ values for FE = 0.7, 0.8, 0.9, 1.0, 1.1, 1.2, and the results are plotted in Figure A.1.

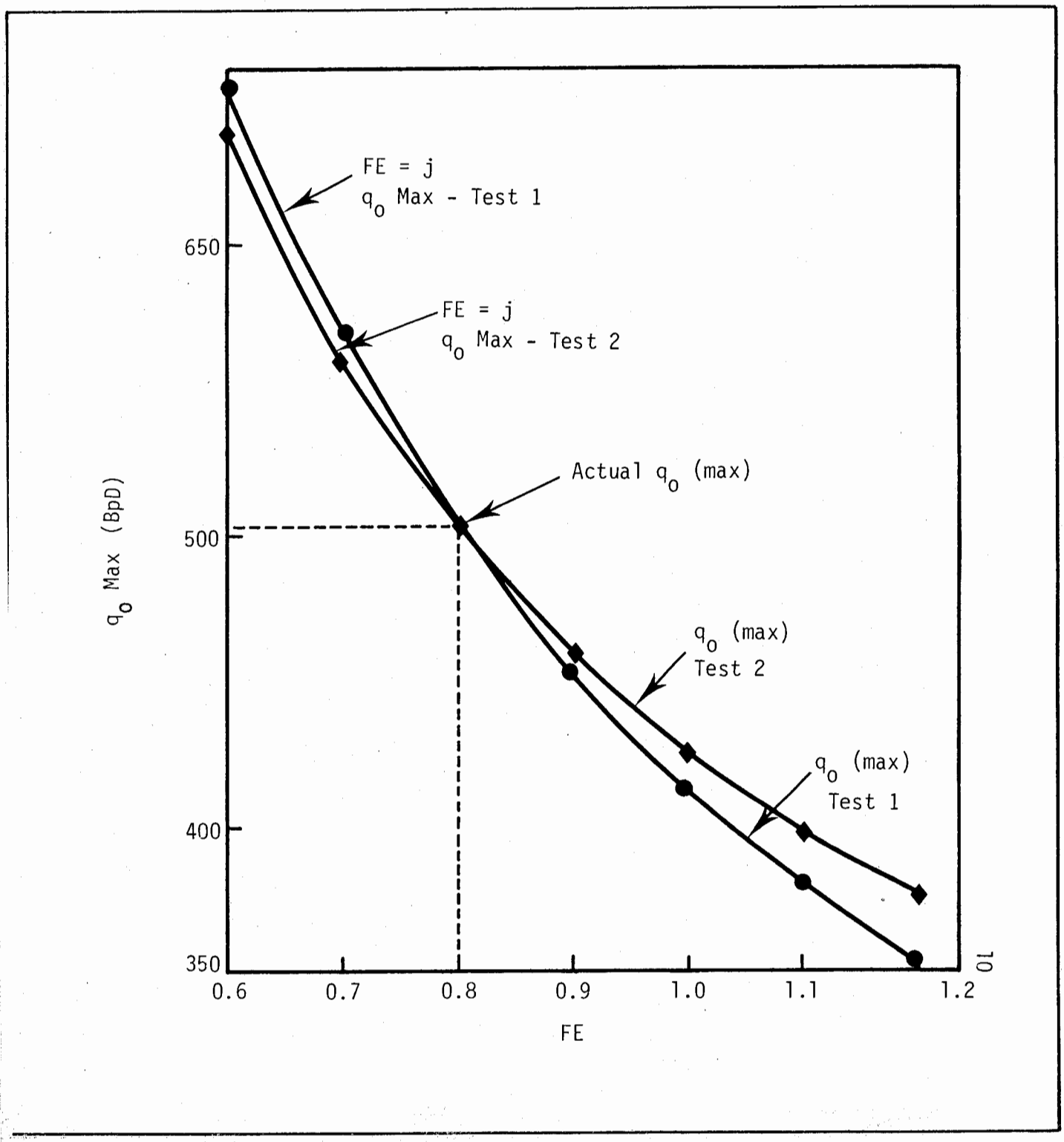

Fig. A.1 Graphical solution for FE from two flow tests.

Summary of equations for solving for FE from two flowing tests on a well

$$FE = \frac{\bar{p}_R - p'_{wf}}{\bar{p}_R - p_{wf}}$$

$$p'_{wf} = \bar{p}_R - FE(\bar{p}_R - p_{wf})$$

$$\frac{p'_{wf}}{\bar{p}_R} = 1 - FE + FE\left(\frac{p_{wf}}{\bar{p}_R}\right) \qquad \text{Let} \quad \frac{p_{wf}}{\bar{p}_R} = A$$

Substituting into Vogel's eqn.:

$$\frac{q_o}{q_{o(max)}} = 1 - 0.2\left[1 - FE + FE\left(\frac{p_{wf}}{\bar{p}_R}\right)\right] - 0.8\left[1 - FE + FE\left(\frac{p_{wf}}{\bar{p}_R}\right)\right]^2$$

$$\frac{q_o}{q_{o(max)}} = 0.8\ FE^2\ A^2 - 1.8\ A\ FE \times 1.6\ FE^2\ A + 1.8\ FE - 0.8\ FE^2$$

$$FE^2(-0.8 + 1.6\ A - 0.8\ A^2) + FE(1.8 - 1.8\ A) = \frac{q_o}{q_{o(max)}}$$

For 2 flow tests we can write two equations:

$$\text{Let} \quad \frac{q_1}{q_2} = C \qquad A_1 = \frac{p_{wf_1}}{\bar{p}_R};\ A_2 = \frac{p_{wf_2}}{\bar{p}_R}$$

Then:

$$FE = (2.25)\,\frac{(1 - A_2)(C) - (1 - A_1)}{(-1 + 2\ A_1 - A_1^2) - C(-1 + 2\ A_2 - A_2^2)}$$

REFERENCES

Couto, Luiz Evanio Dias, "Private Communication," Petrobras, Rio de Janeiro, 1975.

Appendix A.2: Equations presented by Fetkovich

Starting with Equation 6 in the publication by Fetkovich, which is Equation 1.60 of Chapter 1, we have:

$$q_o = \frac{7.08\ kh}{\left[\ln\left(\frac{r_e}{r_w}\right) + s'\right]} \cdot \left[\int_{p_{wf}}^{p_b} \frac{k_{ro}(S,p)}{\mu_o\ B_o}\,dp + \frac{(p_e - p_b)}{(\mu_o B_o)}\right] \quad (6)$$

can be used to describe all three possible flow conditions that could exist for a producing well at some time during the life of an initially undersaturated oil reservoir by eliminating any terms that do not apply over appropriate pressure ranges.

I. Single-phase flow: $p_{wf} > p_b$, $p_e > p_b$ or $\bar{p}_R > p_b$

A. Steady-state flow, constant pressure at outer boundary

$$q_o = \frac{7.08\ kh}{\left[\ln\left(\frac{r_e}{r_w}\right) + s'\right]} \left[\frac{(p_e - p_{wf})}{(\mu_o\ B_o)}\right] \quad (A\text{-}1)$$

B. Pseudo-steady state flow, closed (no flow) at outer boundary

a) Boundary pressure p_e is known at r_e (initial isochronal test)

$$q_o = \frac{7.08\ kh\ (p_e - p_{wf})}{\left[\ln\left(\frac{r_e}{r_w}\right) - \frac{1}{2} + s'\right]\ (\overline{\mu_o B_o})} \quad (A\text{-}2)$$

b) Average pressure $\bar{p}_R$ is known ($\bar{p}_R$ = shut-in pressure)

$$q_o = \frac{7.08\ kh\ (\bar{p}_R - p_{wf})}{\left[\ln\left(\frac{r_e}{r_w}\right) - \frac{3}{4} + s'\right]\ (\overline{\mu_o B_o})} \quad (A\text{-}3)$$

c) Transient flow

$$q_o = \frac{7.08\ kh\ (p_i - p_{wf})}{\left[\ln\sqrt{\frac{14.23\ k_i t}{\phi(\mu c_t)_i\ r_w^2}} + s'\right]\ (\overline{\mu_o B_o})} \quad (A\text{-}4)$$

II. Two-phase flow: $p_{wf} < p_b$, $p_e \leqq p_b$ or $\bar{p}_R \leqq p_b$, and $S_g > S_{gc}$

A. Steady-state flow [constant pressure at outer boundary]

$$q_o = \frac{7.08\ kh}{\left[\ln\left(\frac{r_e}{r_w}\right) + s'\right]} \int_{p_{wf}}^{p_e} \frac{k_{ro}\ (S,p)}{\mu_o B_o}\,dp \quad (A\text{-}5)$$

B. Pseudo-steady state flow [closed (no flow) at outer boundary]

a) Boundary pressure p_e is known at r_e (initial isochronal test)

$$q_o = \frac{7.08\ kh}{\left[\ln\left(\frac{r_e}{r_w}\right) - \frac{1}{2} + s'\right]} \int_{p_{wf}}^{p_e} \frac{k_{ro}(S,p)}{\mu_o B_o}\,dp \quad (A\text{-}6)$$

b) Average pressure $\bar{p}_R$ is known ($\bar{p}_R$ = shut-in pressure)

$$q_o = \frac{7.08\ kh}{\left[\ln\left(\frac{r_e}{r_w}\right) - \frac{3}{4} + s'\right]} \int_{p_{wf}}^{p_R} \frac{k_{ro}(S,p)}{\mu_o B_o}\,dp \quad (A\text{-}7)$$

C. Transient flow

$$q_o = \frac{7.08\ kh}{\left[\ln\sqrt{\frac{14.23\ k_i t}{\phi(\mu c_t)_i\ r_w^2}} + s'\right]} \cdot \int_{p_{wf}}^{p_i} \frac{k_{ro}(S,p)}{\mu_o B_o}\,dp \quad (A\text{-}8)$$

III. Two-phase and single-phase flow: $p_{wf} < p_b$, $p_e > p_b$ or $\bar{p}_R > p_b$

A. Steady-state flow (constant pressure at outer boundary)

$$q_o = \frac{7.08\ kh}{\left[\ln\left(\frac{r_e}{r_w}\right) + s'\right]} \left[\int_{p_{wf}}^{p_b} \frac{k_{ro}(S,p)}{\mu_o B_o}\,dp + \frac{(p_e - p_b)}{(\overline{\mu_o B_o})p_e,p_b}\right] \quad (A\text{-}9)$$

B. Pseudo-steady state flow (closed (no flow) at outer boundary)

a) Boundary pressure p_e is known at r_e (initial isochronal test)

$$q_o = \frac{7.08\ kh}{\left[\ln\left(\frac{r_e}{r_w}\right) - \frac{1}{2} + s'\right]} \left[\int_{p_{wf}}^{p_b} \frac{k_{ro}(S,p)}{\mu_o B_o}\,dp + \frac{(p_e - p_b)}{(\overline{\mu_o B_o})p_e,p_b}\right] \quad (A\text{-}10)$$

b) Average pressure $\bar{p}_R$ is known ($\bar{p}_R$ = shut-in pressure during depletion)

$$q_o = \frac{7.08\ kh}{\left[\ln\left(\frac{r_e}{r_w}\right) - \frac{3}{4} + s'\right]} \cdot \left[\int_{p_{wf}}^{p_b} \frac{k_{ro}(S,p)}{\mu_o B_o}\,dp + \frac{(\bar{p}_R - p_b)}{(\mu_o B_o)_{\bar{p}_R,p_b}}\right] \quad (A\text{-}11)$$

C. Transient flow

$$q_o = \frac{7.08\ kh}{\left[\ln\sqrt{\frac{14.23\ k_i t}{\phi(\mu c_t)_i\ r_w^2}} + s'\right]} \cdot \left[\int_{p_{wf}}^{p_b} \frac{k_{ro}(S,p)}{\mu_o B_o}\,dp + \frac{(p_i - p_b)}{(\bar{\mu}_o \bar{B}_o)p_i,p_b}\right] \quad (A\text{-}12)$$

All of the preceeding flow equations could be more simply expressed in terms of a pseudo-pressure[9] $m_o\,(p)$ where

$$\int_{p_{wf}}^{p_e} \frac{k_{ro}(S,p)}{\mu_o B_o}\,dp = \int_0^{p_e} \frac{k_{ro}(s,p)}{\mu_o B_o}\,dp - \int_0^{p_{wf}} \frac{k_{ro}(S,p)}{\mu_o\ B_o}\,dp \quad (A\text{-}13)$$

or

$$\int_{p_{wf}}^{p_e} \frac{k_{ro}(S,p)}{\mu_o B_o}\,dp = m_o\,(p_e) - m_o\,(p_{wf}) \quad (A\text{-}14)$$

For the limiting case of at least using known PVT properties ($\mu_o B_o$), assuming $k_{ro}(S,p) = 1$, we have:

$$m_o\,(p_e) - m_o(p_{wf}) = \frac{p_b - p_{wf}}{(\bar{\mu}_o \bar{B}_o)\ p_b,\ p_{wf}} + \frac{p_e - p_b}{(\bar{\mu}_o \bar{B}_o)\ p_e,\ p_b} = \frac{p_e - p_{wf}}{(\bar{\mu}_o B_o)_{avg}} \quad (A\text{-}15)$$

Note that $(\mu_o B_o)$ normally evaluated at the average pressure $(p_e + p_{wf})/2$ would not result in a properly weighted average. But for the decline in $k_{ro}(S,p)$, a plot of q_o vs $(p_e - p_{wf})/(\mu_o B_o)_{avg}$ would plot a straight line with a slope of $7.08\ kh/\left[\ln\left(\frac{r_e}{r_w}\right) + s'\right]$ and intercept 0.

Let us now consider the case where $k_{ro}(S,p)$ decreases with increased drawdown, k_{ro} should approach 0, resulting in $k_{ro}/(\mu_o B_o)$ approaching 0. Assuming $k_{ro}/(\mu_o B_o)$ could be approximated by straight line functions as depicted in Fig. 2, we could write for the two-phase region.

$$\int_{p_{wf}}^{p_b} f(p)dp = \int_{p_{wf}}^{p_b} [a_2\ p + b_2]\ dp \quad (A\text{-}16)$$

which when integrated between limits yields:

$$\int_{p_{wf}}^{p_b} f(p)\,dp = \frac{a_2}{2}(p_b^2 - p_{wf}^2) + b_2(p_b - p_{wf}) \quad \text{(A-17)}$$

To approximate Vogel's IPR equation we set $b_2 = 0$, then:

$$\int_{p_{wf}}^{p_b} f(p)\,dp = \frac{a_2}{2}(p_b^2 - p_{wf}^2) \quad \text{(A-18)}$$

Replacing p_b with $\bar{p}_R$ for the two-phase flow equation ($\bar{p}_R \leqq p_b$), we have:

$$q_o = \frac{7.08\,kh}{\left[\ln\left(\frac{r_e}{r_w}\right) + s'\right]}\left[\frac{a_2}{2}(\bar{p}_R^2 - p_{wf}^2)\right] \quad \text{(A-19)}$$

The slope a_2, for $b_2 = 0$, is simply $(k_{ro}/\mu_o B_o)/\bar{p}_R$. We then can write:

$$q_o = \frac{7.08\,kh}{\left[\ln\left(\frac{r_e}{r_w}\right) + s'\right]} \cdot \left[\left(\frac{k_{ro}}{\mu_o B_o}\right)_{\bar{p}_R} \frac{(\bar{p}_R^2 - p_{wf}^2)}{2\bar{p}_R}\right] \quad \text{(A-20)}$$

Defining:

$$J_o' \equiv \frac{7.08\,kh}{\left[\ln\left(\frac{r_e}{r_w}\right) + s'\right]} \cdot \left[\left(\frac{k_{ro}}{\mu_o B_o}\right)_{\bar{p}_R}\left(\frac{1}{2\bar{p}_R}\right)\right] \quad \text{(A-21)}$$

Then:

$$q_o = J_o'(\bar{p}_R^2 - p_{wf}^2) \quad \text{(A-22)}$$

Similarly treating the single-phase flow region as depicted in Fig. 2 ($p_{wf} \geq p_b$):

$$q_o = \frac{7.08\,kh}{\left[\ln\left(\frac{r_e}{r_w}\right) + s'\right]}\left[b_1(p_e - p_{wf}) - \frac{a_1}{2}(p_e^2 - p_{wf}^2)\right] \quad \text{(A-23)}$$

In terms of PI at a vanishing ΔP:

$$J_{o\,\Delta p \to o} = \frac{7.08\,kh}{\left[\ln\left(\frac{r_e}{r_w}\right) + s'\right]}[b_1 - a_1 p_e] \quad \text{(A-24)}$$

where a_1 or b_1, if $a_1 = 0$, is simply $(k_{ro}/\mu_o B_o)$ evaluated at p_e.

For the combined single-phase and two-phase flow case we can write:

$$q_o = \frac{7.08\,kh}{\left[\ln\left(\frac{r_e}{r_w}\right) + s'\right](\overline{u_o B_o})_{p_e,p_b}}\left[(\overline{\mu_o B_o})_{p_e,p_b}\frac{a_2}{2}(p_b^2 - p_{wf}^2) + (p_e - p_b)\right] \quad \text{(A-25)}$$

where $(\overline{\mu_o B_o})\ p_e, p_b$ is evaluated at the average pressure $(p_e + p_b)/2$.

In terms of PI definition:

$$q_o = J_o(\overline{\mu_o B_o})\ p_e, p_b\left(\frac{a_2}{2}\right)(p_b^2 - p_{wf}^2) + J_o(p_e - p_b) \quad \text{(A-26)}$$

or:

$$q_o = J_o'(p_b^2 - p_{wf}^2) + J_o(p_e - p_b) \quad \text{(A-27)}$$

Nomenclature

- a = slope of pressure function f(p), $(psi - cp)^{-1}$
- b = intercept of pressure function f(p), cp^{-1}
- B = formation volume factor, reservoir vol/surface vol.
- c_t = total compressibility, psi^{-1}
- C = back-pressure curve coefficient
- D = non-Darcy flow constant, $(stk\ bopd)^{-1}$
- h = thickness, ft.
- J = productivity index, stk bbl/day/psi
- J' = productivity index (back-pressure curve coefficient) stk bbl/day/$(psi)^{2n}$
- k = effective permeability, Darcy
- k_a = permeability of altered or damaged zone, Darcy
- k = relative permeability to oil, fraction
- $m(p)$ = pseudo-pressure, (See Eq. 14), psi/cp
- n = exponent of back-pressure curve
- $\bar{p}$ = average pressure, psia
- p_b = bubble point pressure, psia
- p_e = external boundary pressure, psia
- $\bar{p}_R$ = reservoir average pressure (shut-in pressure), psia
- p_i = initial formation pressure, psia
- p_{wf} = bottom-hole flowing pressure, psia
- PI = productivity index (J), stk bbl/day/psi
- q = surface rate of flow, stk bopd
- r_a = radius of altered or damaged zone, ft
- r_e = external boundary radius, ft
- r_w = wellbore radius, ft
- R_L = Gas-oil ratio liberated per barrel of residual oil, scf/stk bbl
- s = skin effect, dimensionless
- s_b = skin effect caused by partial penetration of formation, dimensionless
- s' = total effective skin effect (see Eq. 22), dimensionless
- s'' = total effective skin effect (see Eq. 23), dimensionless
- $s(q,t)$ = rate and time dependent skin effect (see Eqs. 18 and 19) dimensionless
- S = saturation, fraction of pore volume
- S_{clh} = hydrocarbon liquid saturation to achieve mobility, fraction of pore volume
- t = time, days
- T = reservoir temperature, °R
- X = reservoir cu ft of gas evolved in the reservoir/stk bbl produced/psi, (dc/dp) in Eq. 15
- Y = reservoir cu ft of condensate accumulation in the reservoir/Mscf full wellstream gas produced/psi, (dc/dp) in Eq. 15
- z = gas deviation factor, dimensionless
- μ = viscosity, cp.
- ϕ = porosity, fraction of bulk volume

Subscripts

- i = initial
- o = oil
- g = gas

Note: All references, figures and equation numbers in nomenclature refer to the original paper by Fetkovich.

FIG. A.3(A)
PHYSICAL CONSTANTS FOR HYDROCARBONS

Abbreviation of formula	Compound	Mol. wt.	Liq. lb/gal	Gas ft³/lb	γ_g. gas gravity	Gas, ft³/gal	P_c psia	T_c, ^oR
C_1	Methane	16.04	2.50	23.61	0.554	59.00	673.1	343.3
C_2	Ethane	30.07	3.11	12.62	1.038	39.25	708.3	549.8
C_3	Propane	44.09	4.22	8.61	1.522	36.35	617.4	666.0
iC_4	i-Butane	58.12	4.69	6.52	2.006	30.59	529.1	734.7
nC_4	n-Butane	58.12	4.86	6.53	2.006	31.75	550.7	765.3
iC_5	i-Pentane	72.15	5.20	5.27	2.491	27.40	483.0	829.8
nC_5	n-Pentane	72.15	5.25	5.27	2.491	27.68	489.5	845.6
nC_6	n-Hexane	86.17	5.53	4.41	2.975	24.38	439.7	914.1
nC_7	n-Heptane	100.20	5.73	3.79	3.459	21.73	396.9	972.3
	Heptanes+	M	8.334S	379/M	M/28.97	3157 S/M		
	Where M—molecular wt. (ave.) S—specific gravity (water = 1.0) All gas volumes at standard conditions (14.70 psia and 60°F)							
–	Air	28.97	–	–	1.000	–	547.0	238.4
N_2	Nitrogen	28.02	–	–	0.967	–	730	278
O_2	Oxygen	32.00	–	–	1.105	–	730.0	277.9
CO_2	Carbon Dioxide	44.01	–	–	1.519	–	1073.0	584
H_2S	Hydrogen Sulfide	34.08	6.58	11.12	1.176	73.28	1306.0	672.4
H_2O	Water	18.02	8.33	21.10	0.622	175.44	3206	1165
H	Helium	4.00	–	–	–	–	33.2	9.4
CO	Carbon Monoxide	28.01					514	242
H_2	Hydrogen	2.02					188	60
	Olefins							
C_2H_4	Ethylene	28.05	–	–	–	–	748	50
C_3H_6	Propylene	42.1	–	–	–	–	661	198
	Aromatics							
C_6H_6	Benzene	78.1	–	–	–	–	704	551
C_7H_8	Touluene	92.2					611	609

FIG. A.3(B)
PHYSICAL PROPERTIES OF GASES[13]

c_p = specific heat at constant pressure
c_v = specific heat at constant volume

Name of gas	Chemical formula or symbol	Approx. molecular weight M	Weight density, pounds per cubic foot* ρ	Specific gravity relative to air S_g	Individual gas constant R	Specific heat per pound at room temperature		Heat capacity per cubic foot at atmospheric pressure and 68 F		k equal to c_p/c_v
						c_p	c_v	c_p	c_v	
Acetylene	C_2H_2	26.0	.06754	.897	59.4	.350	.2737	.0236	.0185	1.28
Air	–	29.0	.07528	1.000	53.3	.241	.1725	.0181	.0130	1.40
Ammonia	NH_3	17.0	.04420	.587	90.8	.523	.4064	.0231	.0179	1.29
Argon	A	40.0	.1037	1.377	38.7	.124	.0743	.0129	.0077	1.67
Carbon Dioxide	CO_2	44.0	.1142	1.516	35.1	.205	.1599	.0234	.0183	1.28
Carbon Monoxide	CO	28.0	.07269	.965	55.2	.243	.1721	.0177	.0125	1.41
Ethylene	C_2H_4	28.0	.0728	.967	55.1	.40	.3292	.0291	.0240	1.22
Helium	He	4.0	.01039	.138	386.	1.25	.754	.0130	.0078	1.66
Hydrochloric Acid	HCl	36.5	.09460	1.256	42.4	.191	.1365	.0181	.0129	1.40
Hydrogen	H_2	2.0	.005234	.0695	767.	3.42	2.435	.0179	.0127	1.40
Methane	CH_4	16.0	.04163	.553	96.4	.593	.4692	.0247	.0195	1.26
Methyl Chloride	CH_3Cl	50.5	.1309	1.738	30.6	.24	.2006	.0314	.0263	1.20
Nitrogen	N_2	28.0	.07274	.966	55.2	.247	.1761	.0179	.0128	1.40
Nitric Oxide	NO	30.0	.07788	1.034	51.5	.231	.1648	.0180	.0128	1.40
Nitrous Oxide	N_2O	44.0	.1143	1.518	35.1	.221	.1759	.0253	.0201	1.26
Oxygen	O_2	32.0	.08305	1.103	48.3	.217	.1549	.0180	.0129	1.40
Sulphur Dioxide	SO_2	64.0	.1663	2.208	24.1	.154	.1230	.0256	.0204	1.25

* Weight density values are at atmospheric pressure and 68 F. For values at 60 F, multiply by 1.0154.

FIG. A.3(C)
VOLUMETRIC COMPOSITION AND SPECIFIC GRAVITY OF GASEOUS FUELS[13]

Type of gas	Chemical composition percent by volume: Hydrogen	Carbon monoxide	Paraffin hydrocarbons: Methane	Paraffin hydrocarbons: Ethane	Illuminants: Ethylene	Illuminants: Benzene	Oxygen	Nitrogen	Carbon dioxide	Specific gravity relative to air S_g
Natural Gas, Pittsburgh	...	...	83.4	15.8	...	...	...	0.8	...	0.61
Producer Gas from Bituminous Coal	14.0	27.0	3.0	...	...	...	0.6	50.9	4.5	0.86
Blast Furnace Gas	1.0	27.5	...	...	...	...	...	60.0	11.5	1.02
Blue Water Gas from Coke	47.3	37.0	1.3	...	...	...	0.7	8.3	5.4	0.57
Carbureted Water Gas	40.5	34.0	10.2	...	6.1	2.8	0.5	2.9	3.0	0.63
Coal Gas (Cont. Vertical Retorts)	54.5	10.9	24.2	...	1.5	1.3	0.2	4.4	3.0	0.42
Coke-Oven Gas	46.5	6.3	32.1	...	3.5	0.5	0.8	8.1	2.2	0.44
Refinery Oil Gas (Vapor Phase)	13.1	1.2	23.3	21.7	39.6	...	1.0	...	0.1	0.89
Oil Gas, Pacific Coast	48.6	12.7	26.3	...	2.7	1.1	0.3	3.6	4.7	0.47

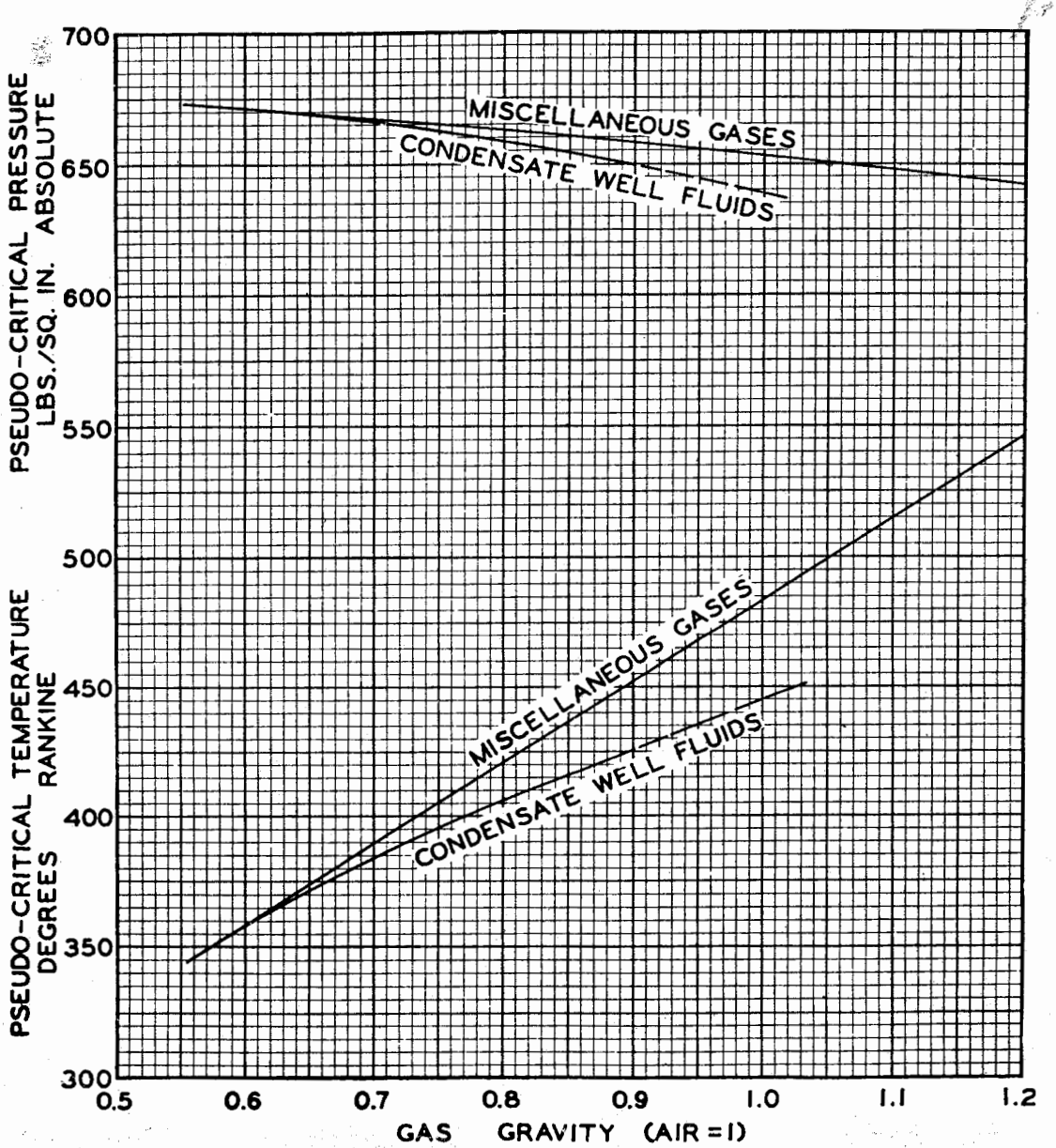

Fig. A.4a Pseudo-critical properties of natural gases as functions of gas gravity (courtesy GG Brown, et al.).

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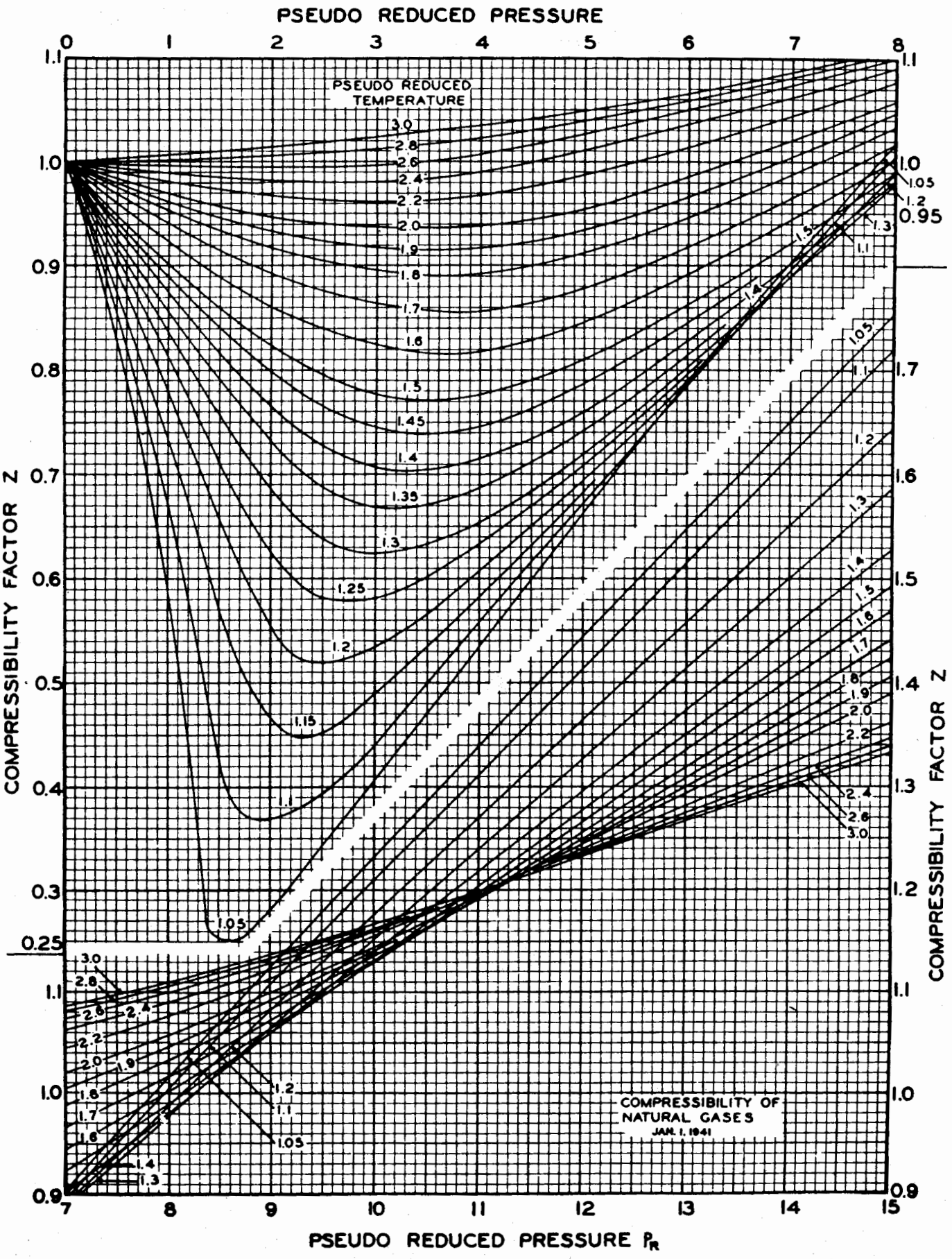


Fig. A.4b Compressibility of natural gases as a function of reduced pressure and temp (after Gatlin, courtesy API).

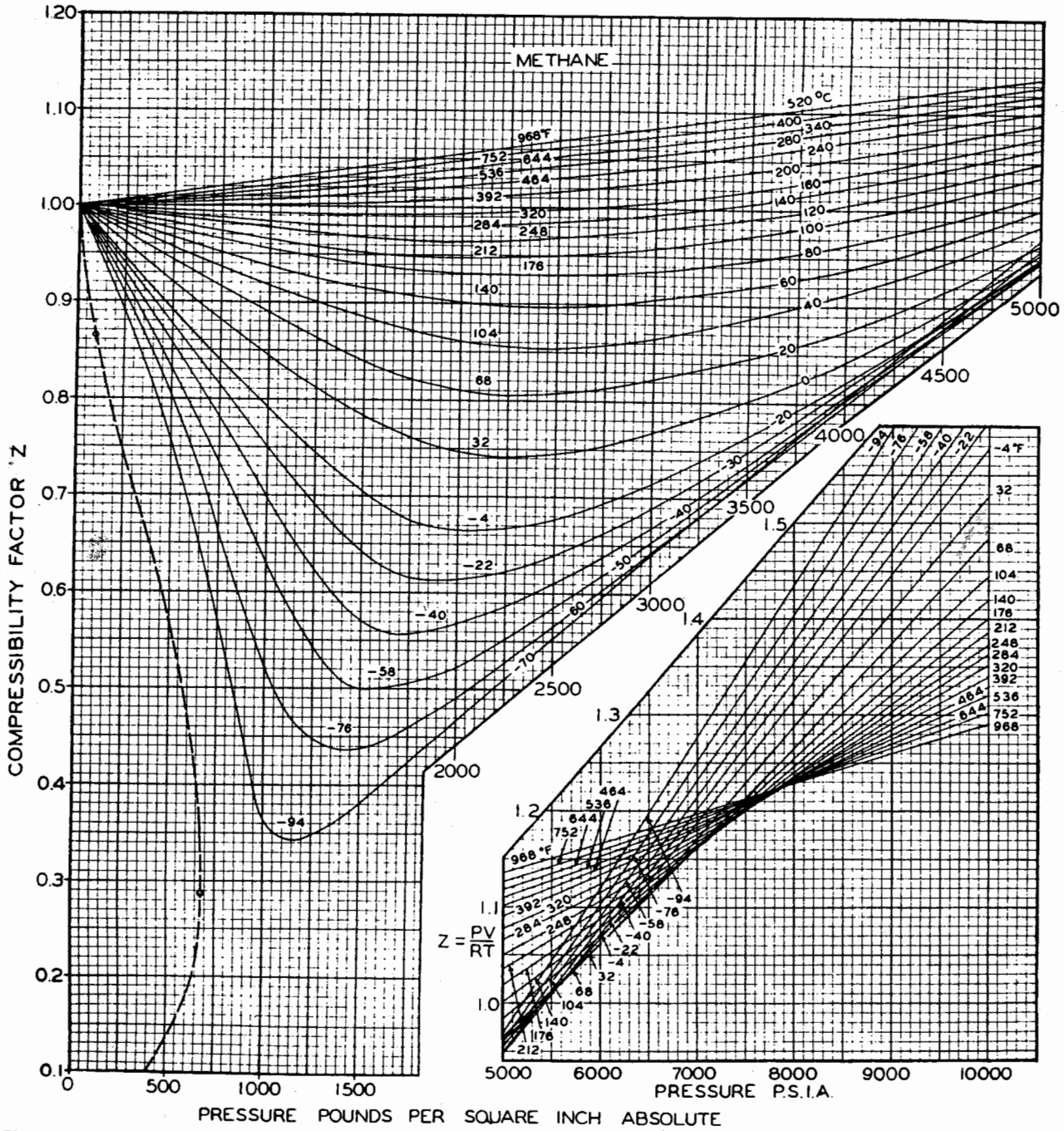


Fig. A.5 Compressibility factors of methane (courtesy GG Brown, et al.)

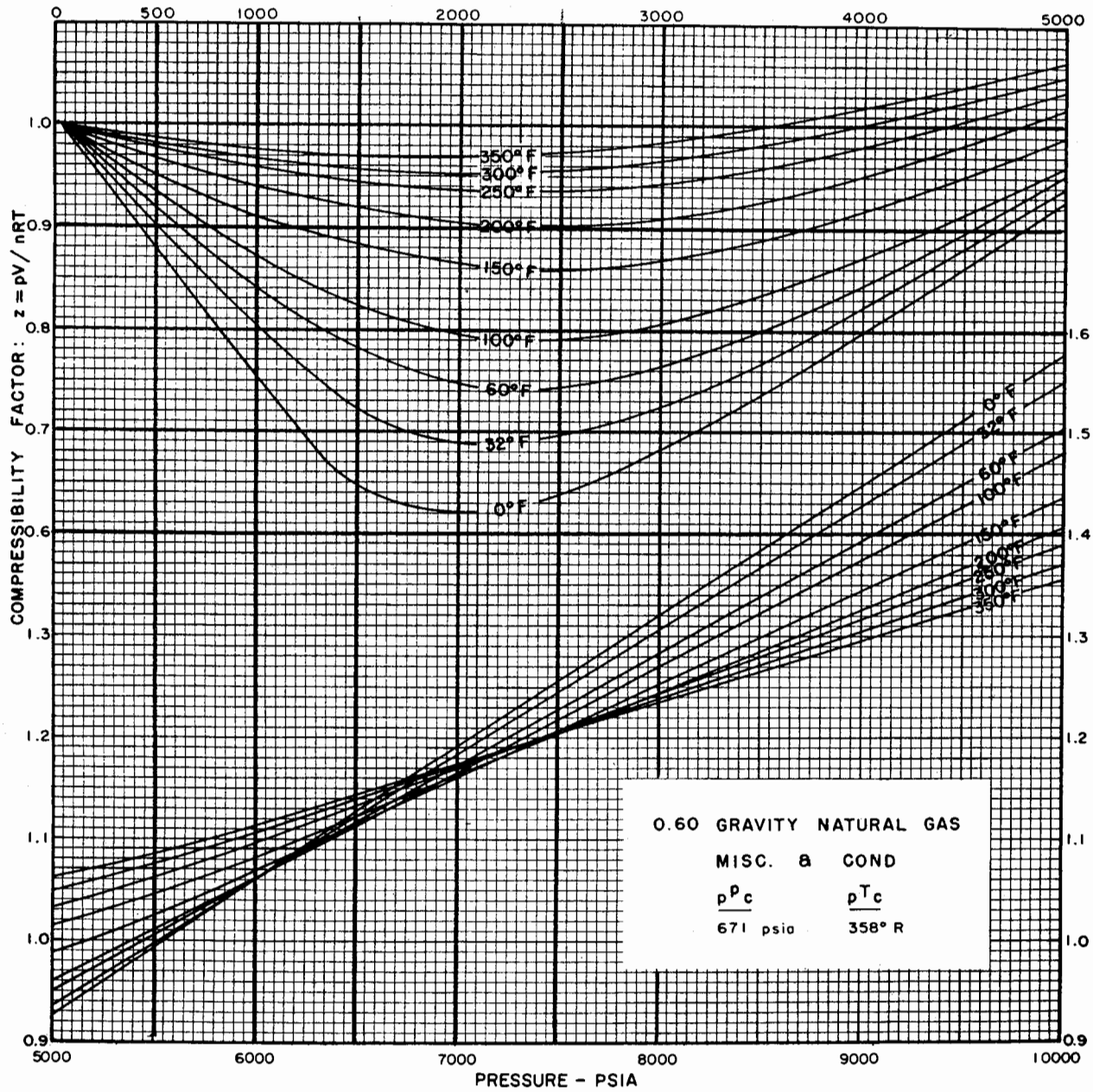


Fig. A.6 Compressibility factors for a 0.60 natural gas (after Gatlin, courtesy API).

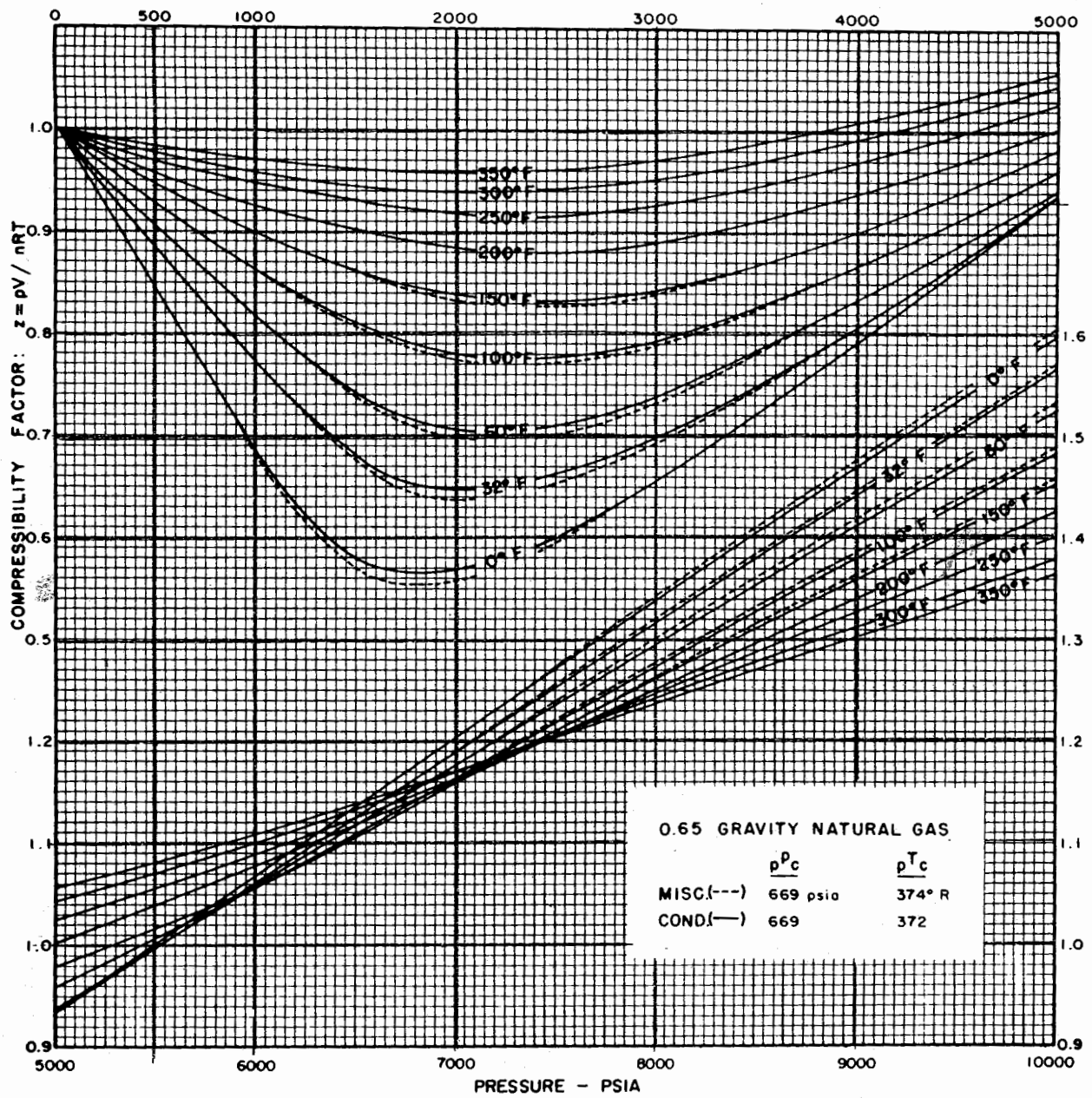


Fig. A.7 Compressibility factors for a 0.65 natural gas (after Gatlin, courtesy API).

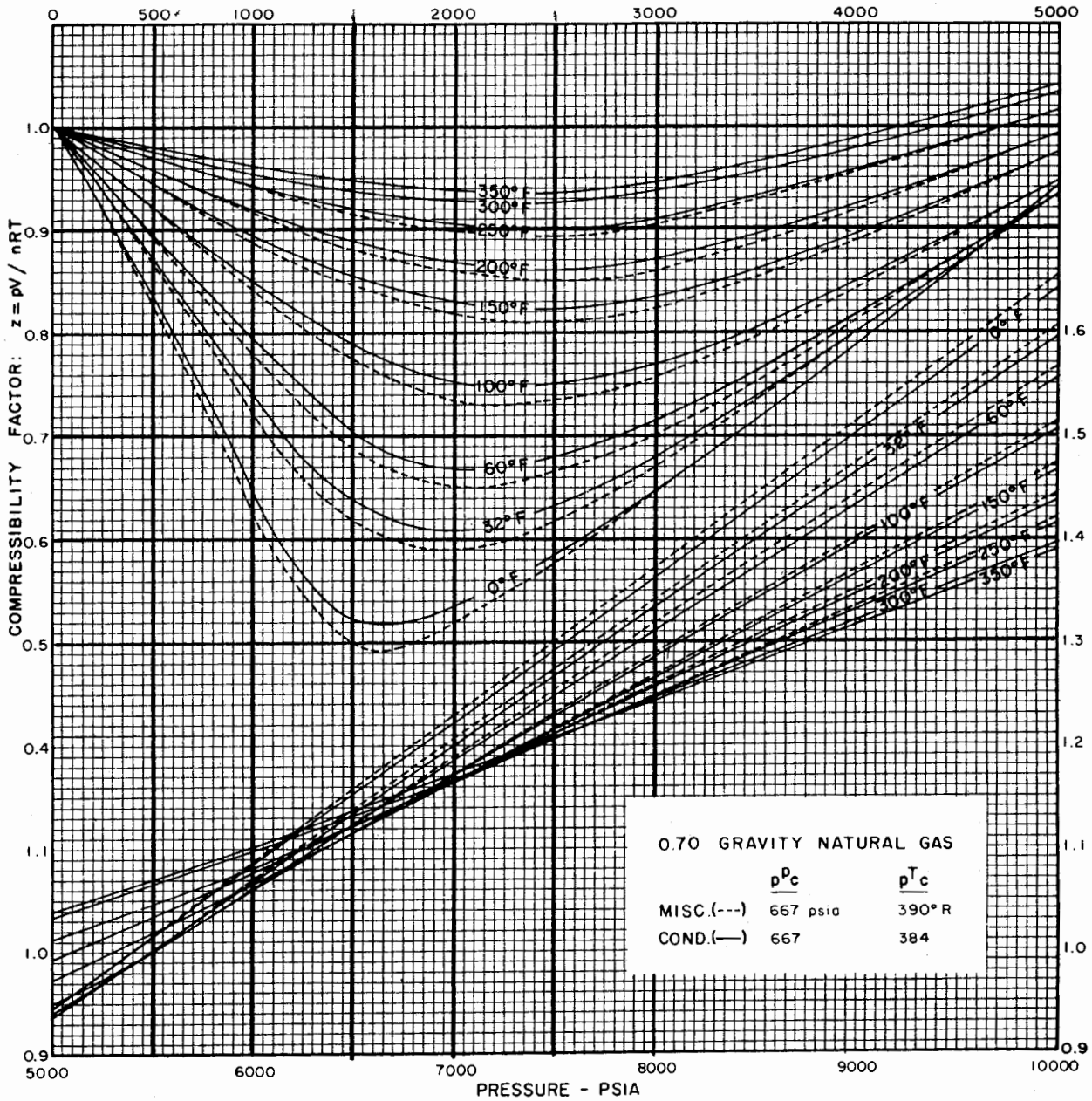


Fig. A.8 Compressibility factors for a 0.70 natural gas (after Gatlin, courtesy API).

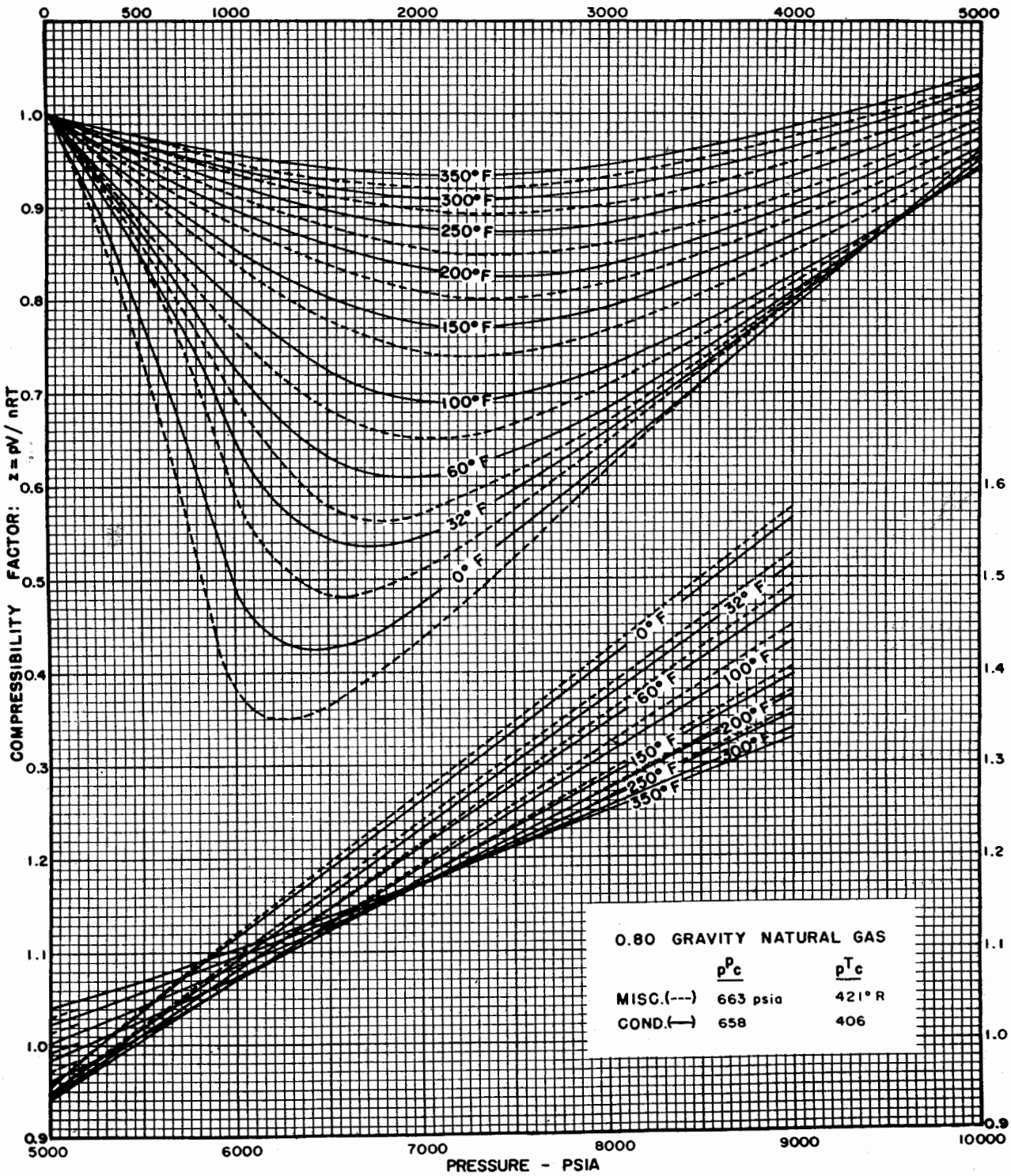


Fig. A.9 Compressibility factors for a 0.80 natural gas (after Gatlin, Courtesy API).

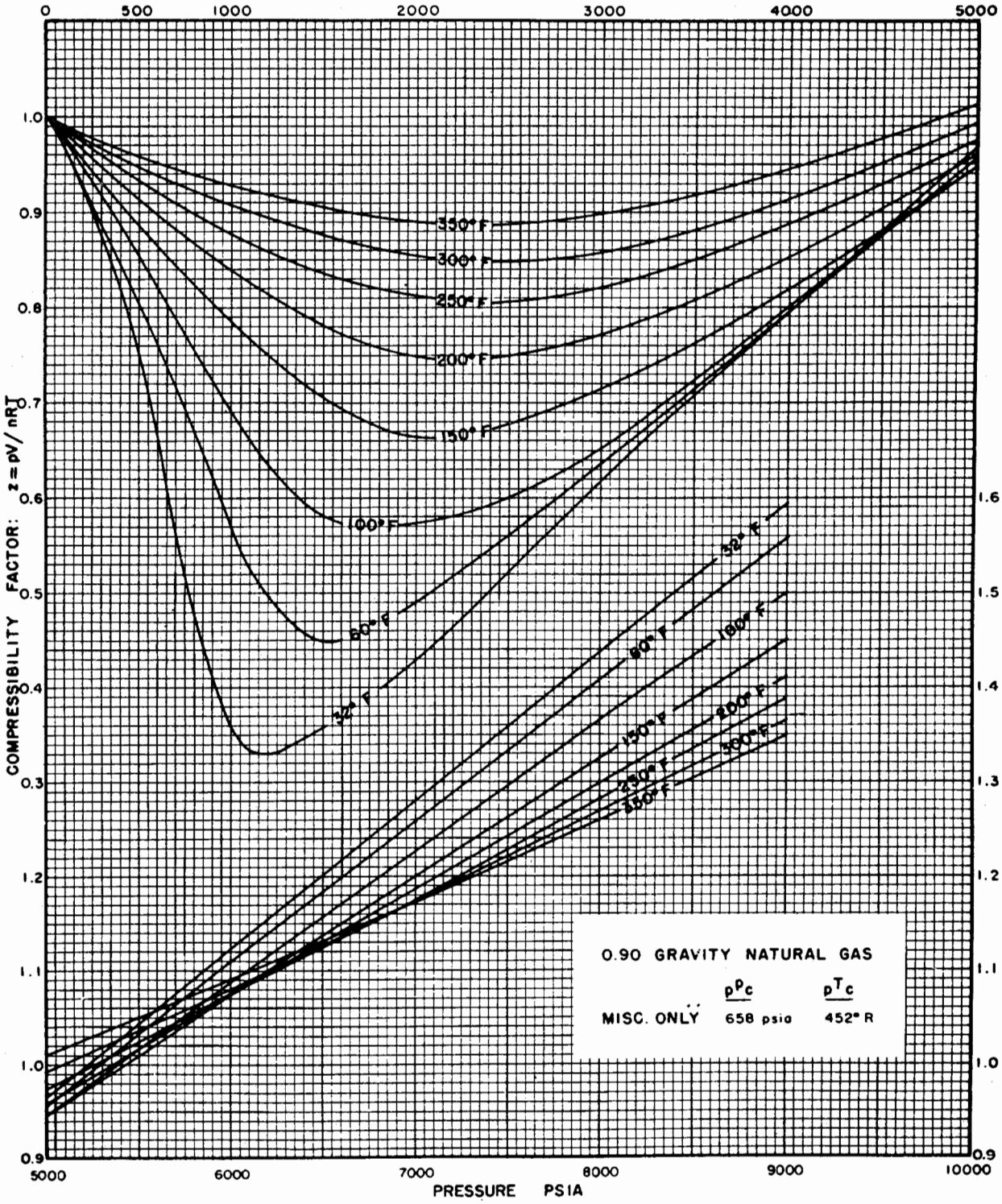


Fig. A.10 Compressibility factors for a 0.90 natural gas, miscellaneous only (after Gatlin, courtesy API).

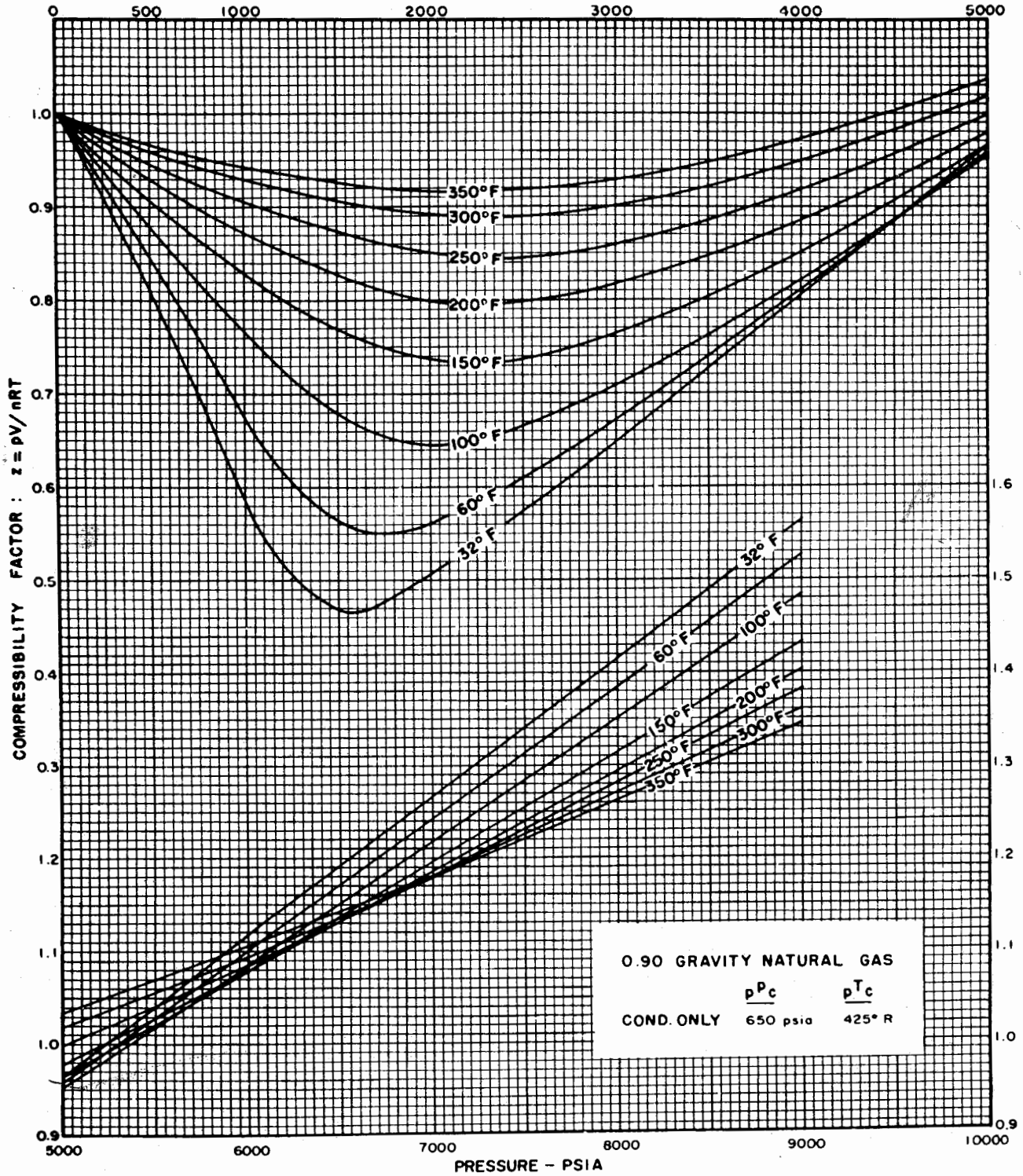


Fig. A.11 Compressibility factors for a 0.90 natural gas, condensate only (after Gatlin, courtesy API).

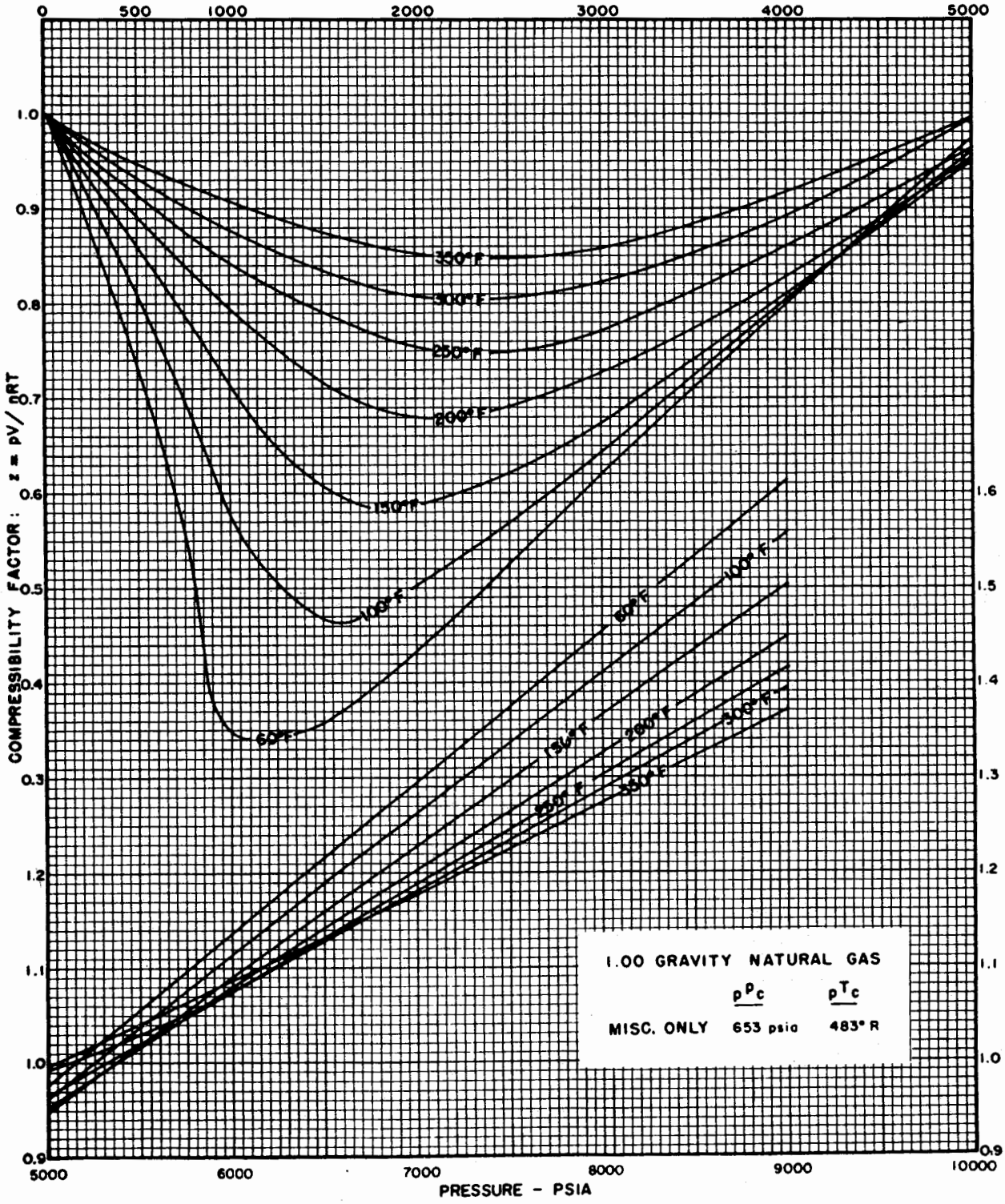


Fig. A.12 Compressibility factors for a 1.00 natural gas, miscellaneous only (after Gatlin, courtesy API).

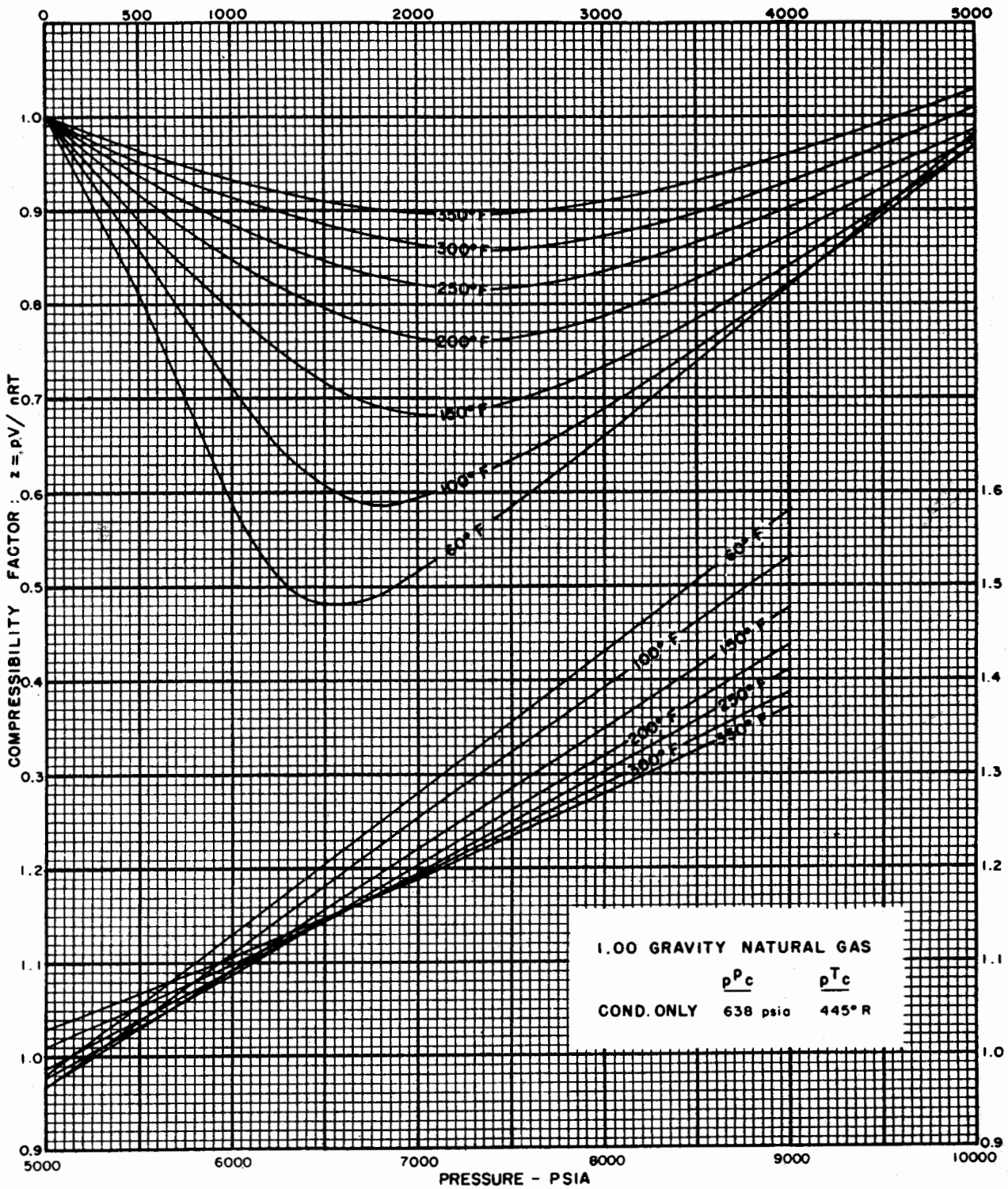


Fig. A.13 Compressibility factors for a 1.00 natural gas, condensate only (after Gatlin, courtesy API).

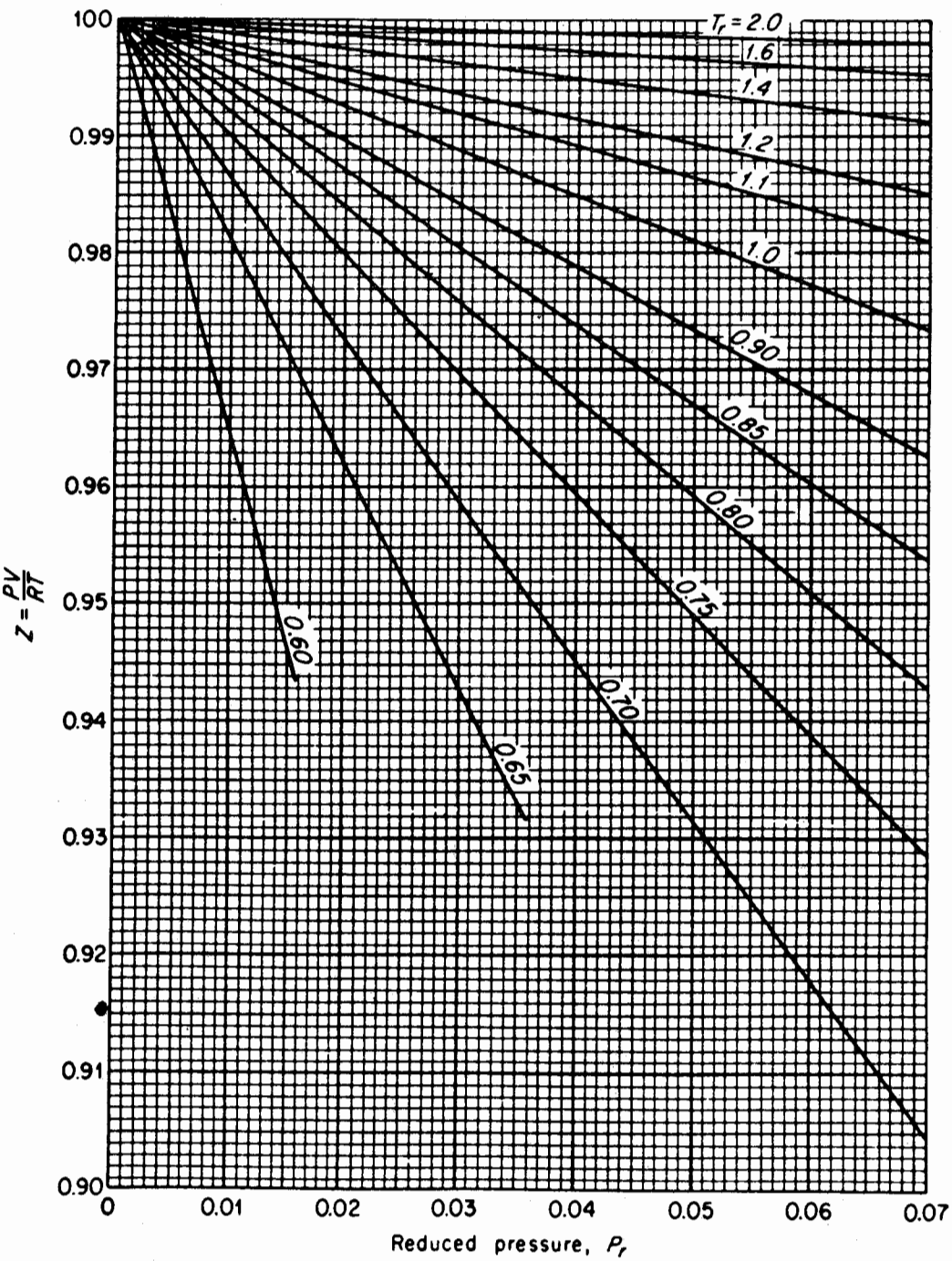


Fig. A.14 Compressibility factors for gases near atmospheric pressure (from Brown, et al., courtesy National Gasoline Association of America).

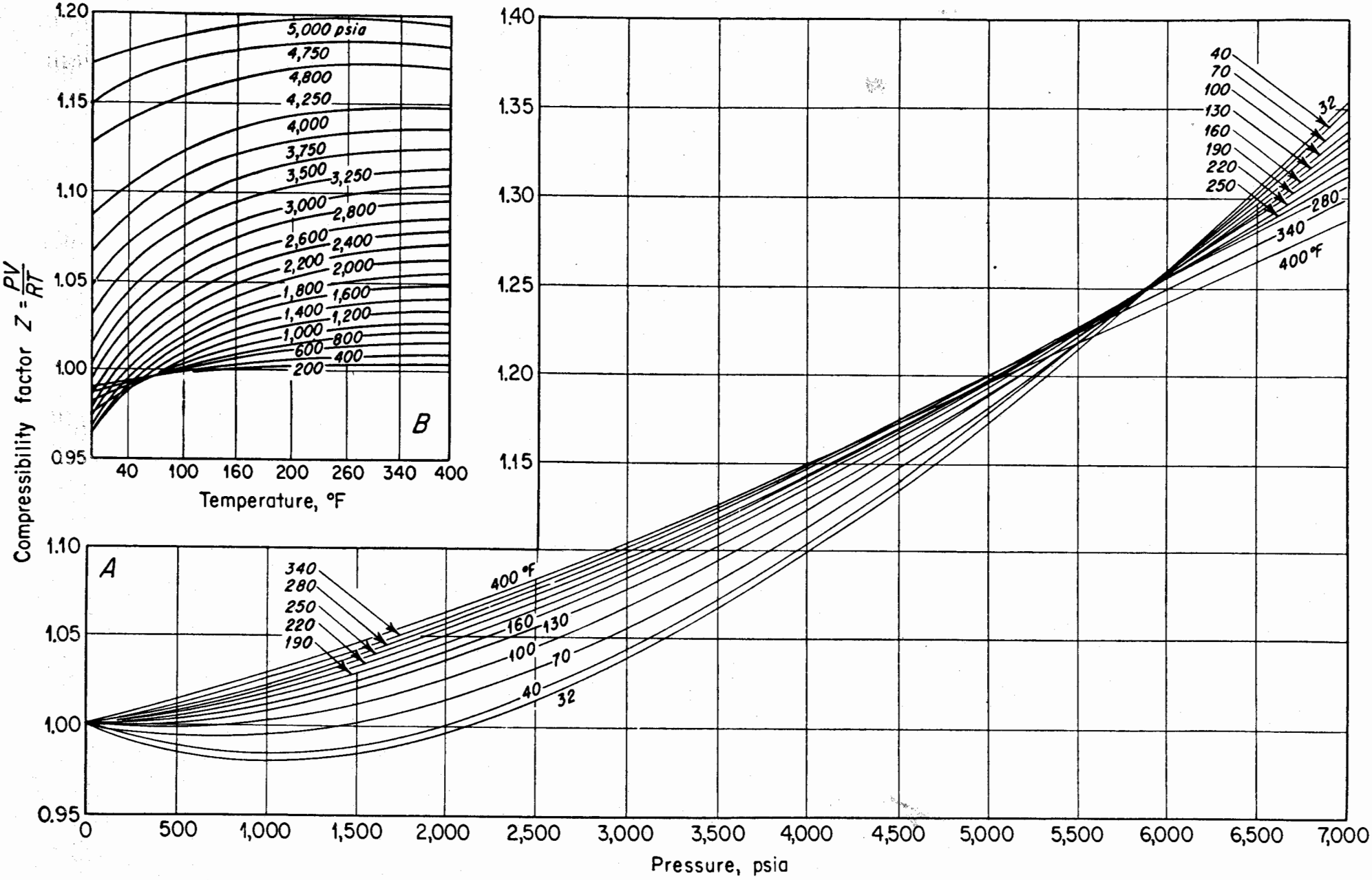


Fig. A.15 Compressibility factors z for nitrogen (from Eilerts, et al., courtesy World Oil).

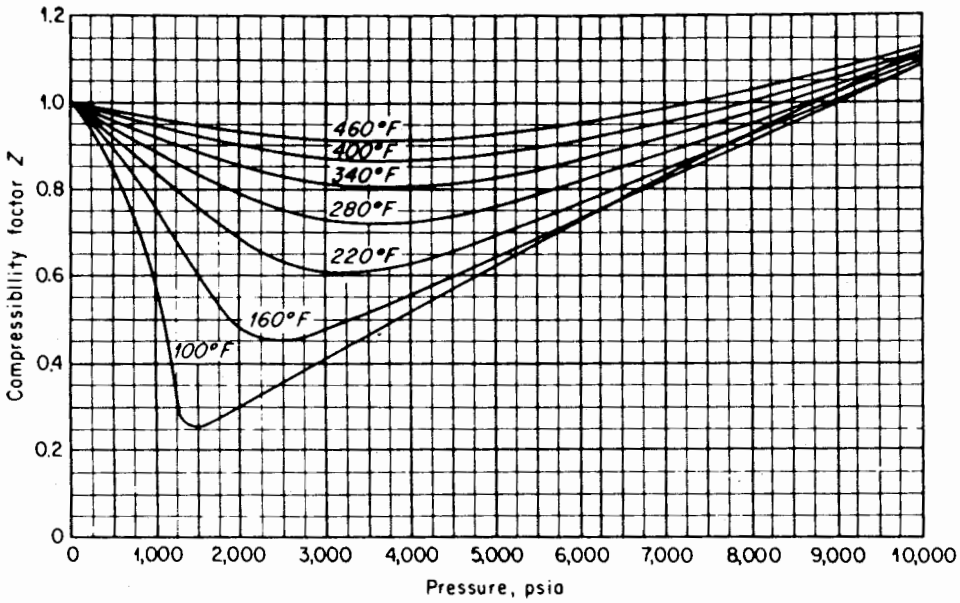


Fig. A.16 Compressibility factors z for carbon dioxide (after Olds, et al., courtesy API).

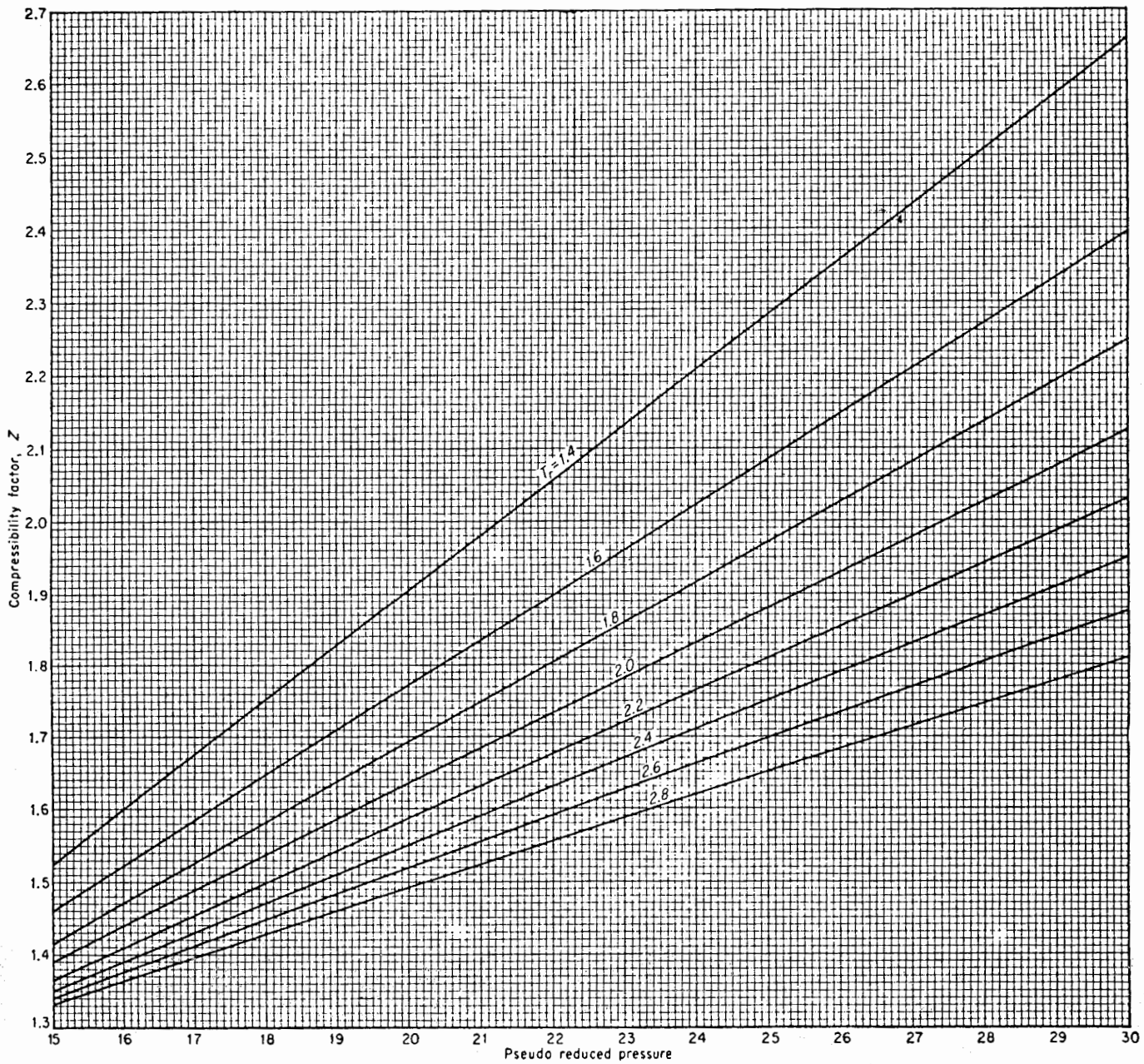


Fig. A.17 Compressibility factors for natural gases at pressures of 10,000 to 20,000 psia.

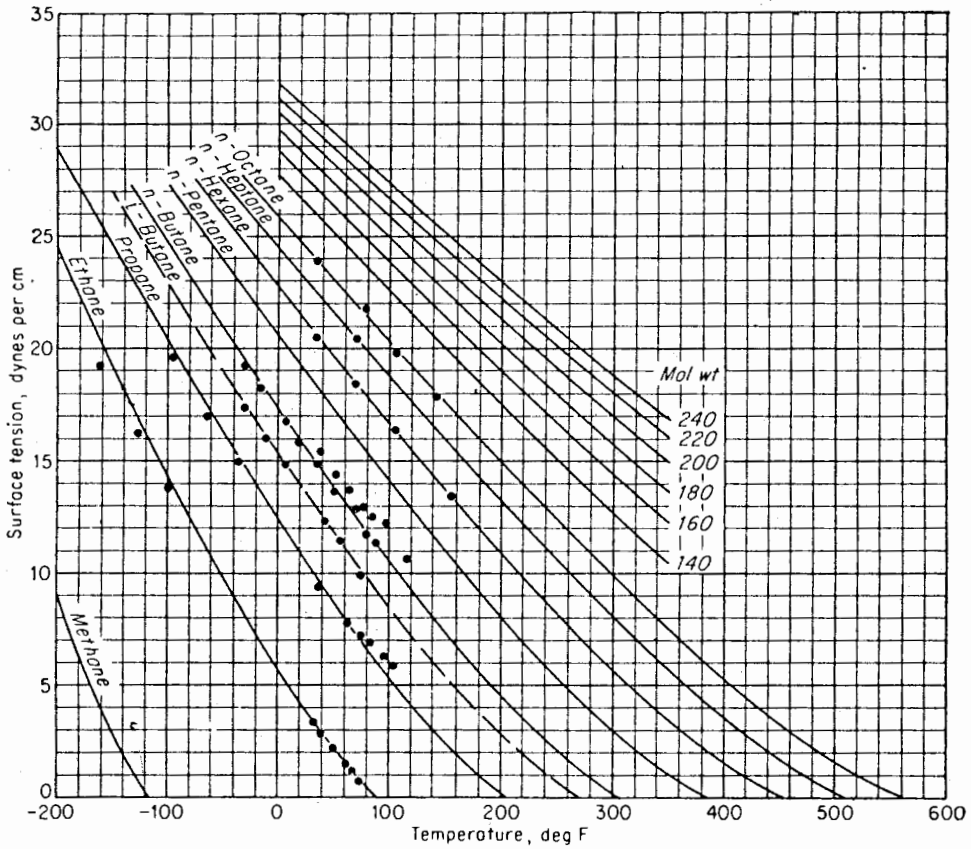


Fig. A.18 Surface tension of hydrocarbons (Katz et al., 4–49, 4–48. courtesy AIME.).

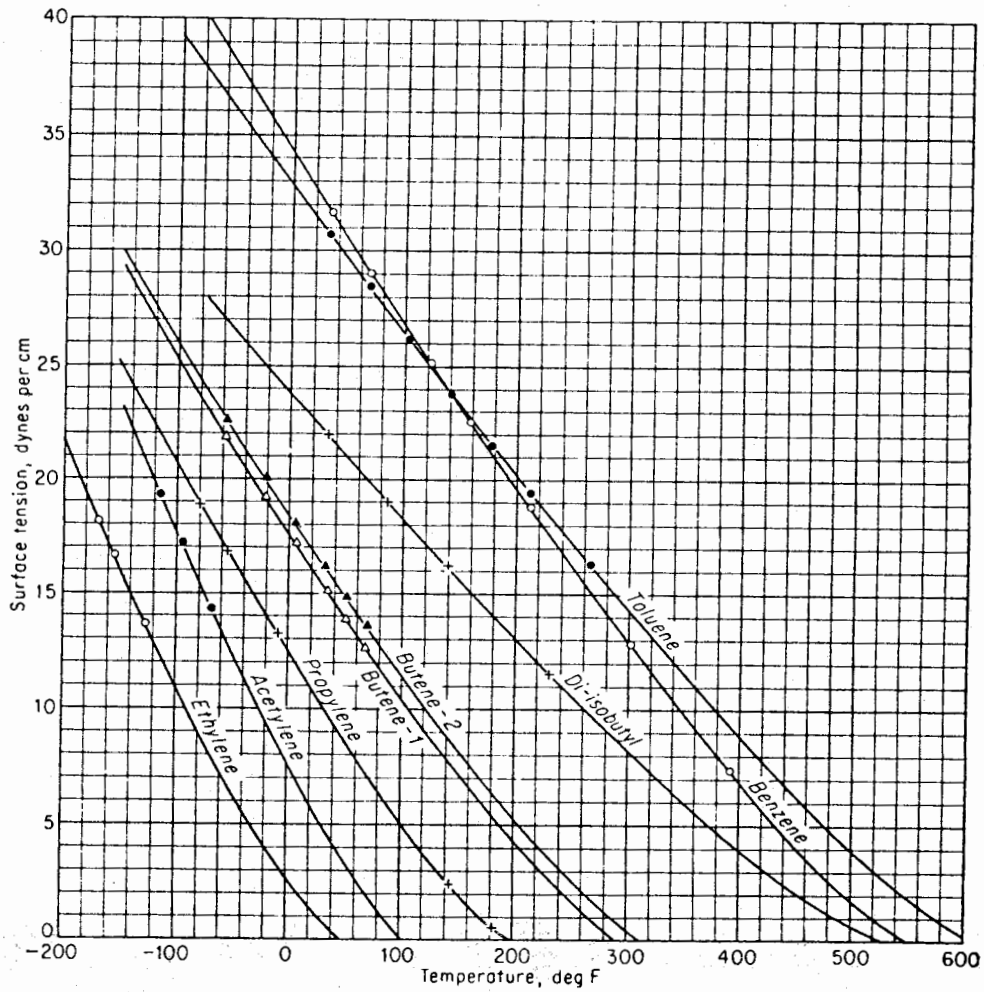


Fig. A.19 Surface tension of hydrocarbons.

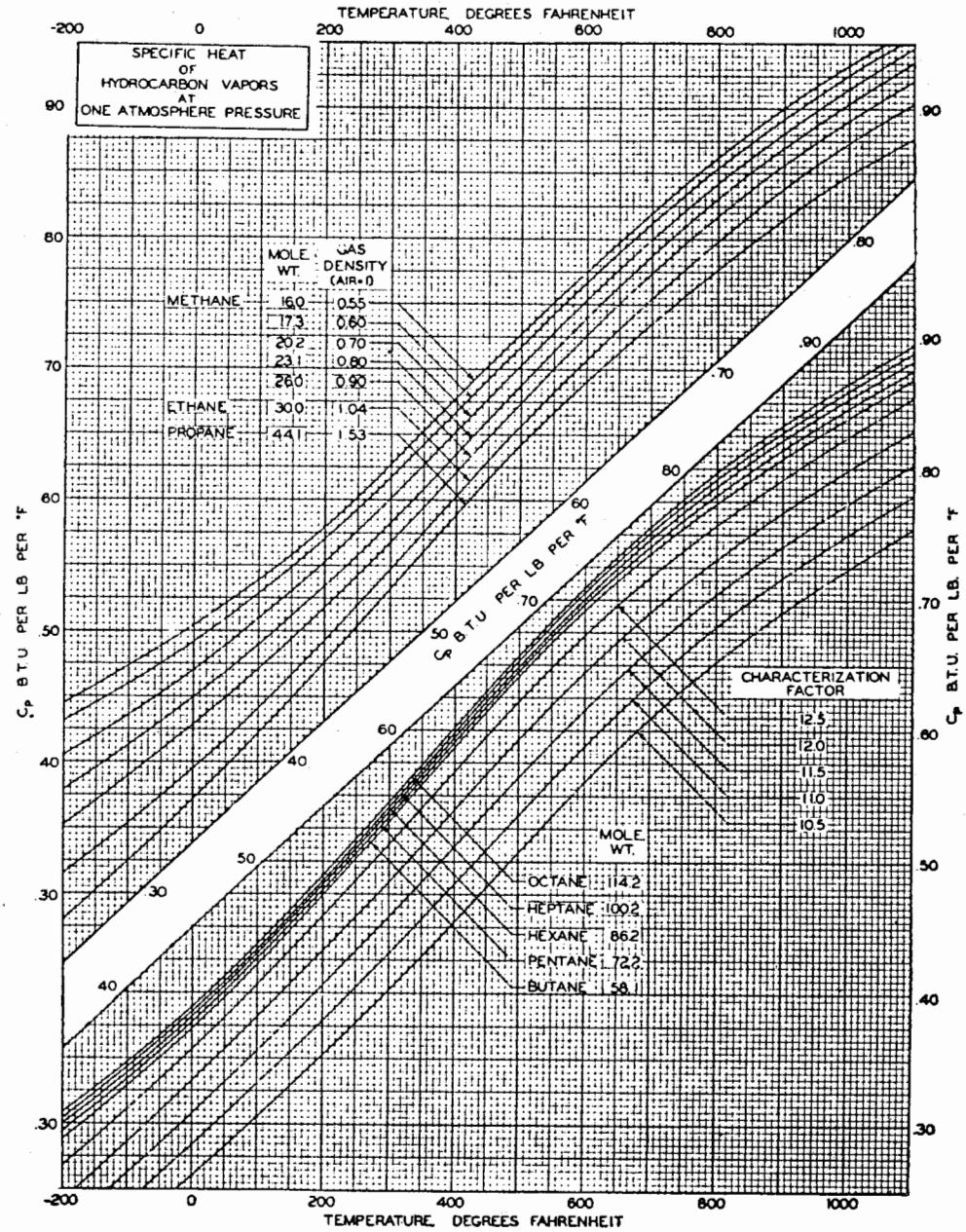


Fig. A.20 Specific heat of hydrocarbon gases. (Brown 4–16 courtesy Aime).

Appendix B

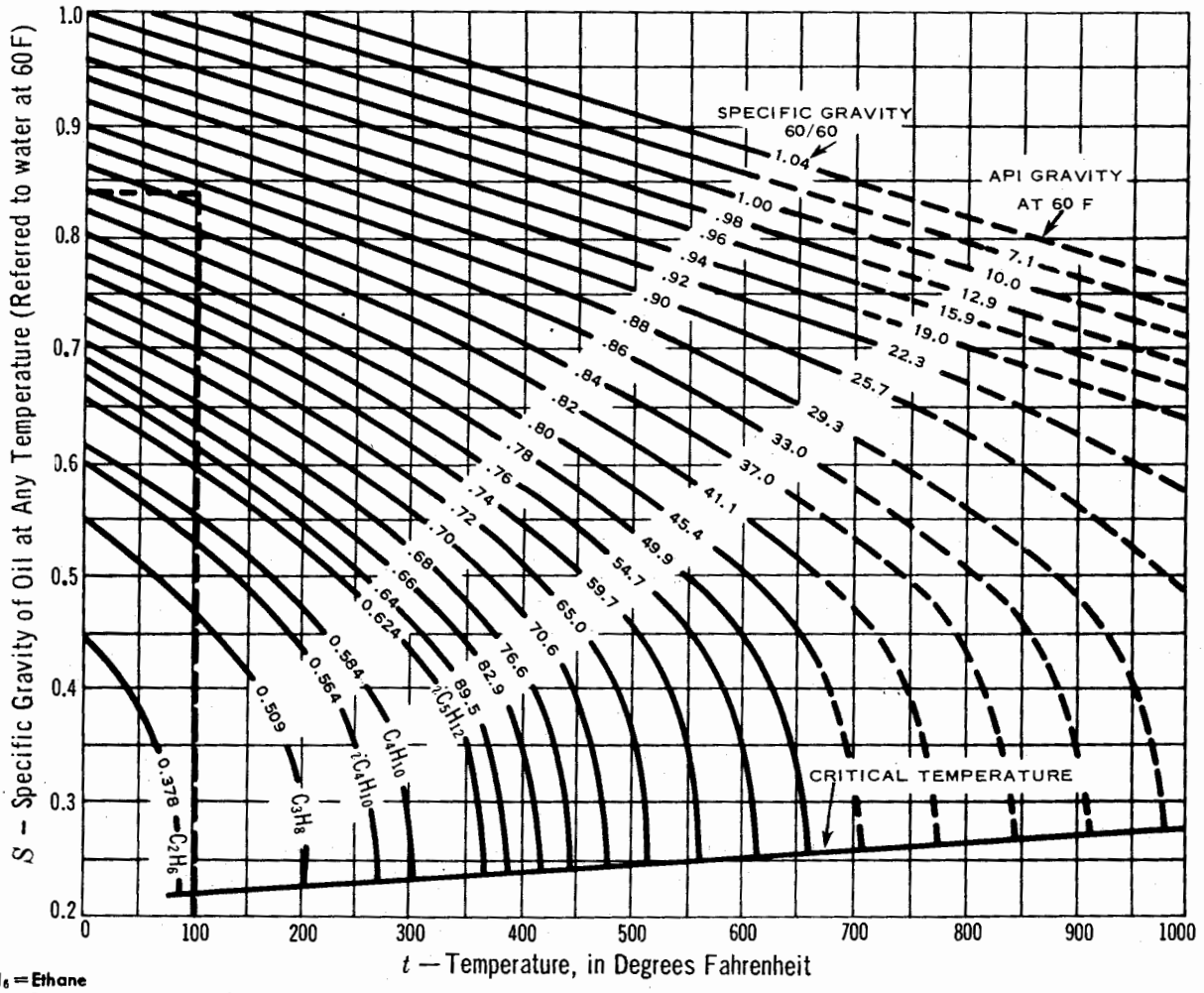
FIG. B.1
PHYSICAL PROPERTIES OF WATER

Temperature of water t Degrees Fahrenheit	Saturation pressure P′ Pounds per square inch absolute	Specific volume $\overline{V}$ Cubic feet per pound	Weight density ρ Pounds per cubic foot	Weight Pounds per gallon
32	.08854	.01602	62.42	8.345
40	.12170	.01602	62.42	8.345
50	.17811	.01603	62.38	8.340
60	.2563	.01604	62.34	8.334
70	.3631	.01606	62.27	8.325
80	.5069	.01608	62.19	8.314
90	.6982	.01610	62.11	8.303
100	.9492	.01613	62.00	8.289
110	1.2748	.01617	61.84	8.267
120	1.6924	.01620	61.73	8.253
130	2.2225	.01625	61.54	8.227
140	2.8886	.01629	61.39	8.207
150	3.718	.01634	61.20	8.182
160	4.741	.01639	61.01	8.156
170	5.992	.01645	60.79	8.127
180	7.510	.01651	60.57	8.098
190	9.339	.01657	60.35	8.068
200	11.526	.01663	60.13	8.039
210	14.123	.01670	59.88	8.005
212	14.696	.01672	59.81	7.996
220	17.186	.01677	59.63	7.972
240	24.969	.01692	59.10	7.901
260	35.429	.01709	58.51	7.822
280	49.203	.01726	57.94	7.746
300	67.013	.01745	57.31	7.662
350	134.63	.01799	55.59	7.432
400	247.31	.01864	53.65	7.172
450	422.6	.0194	51.55	6.892
500	680.8	.0204	49.02	6.553
550	1045.2	.0218	45.87	6.132
600	1542.9	.0236	42.37	5.664
700	3093.7	.0369	27.10	3.623

Specific gravity of water at 60 F = 1.00

Weight per gallon is based on 7.48 gallons per cubic foot.

All data on volume and pressure abstracted from Keenan and Keyes' Steam Tables (1936).



C_2H_6 = Ethane
C_3H_8 = Propane iC_4H_{10} = Isobutane
C_4H_{10} = Butane iC_5H_{12} = Isopentane

Example: The specific gravity of an oil at 60 F is 0.85 The specific gravity at 100 F = 0.83.

To find the weight density of a petroleum oil at its flowing temperature when the specific gravity at 60 F/60 F is known, multiply the specific gravity of the oil at flowing temperature (see chart above) by 62.4, the density of water at 60 F.

Fig. B.2a Specific gravity-temperature relationship for petroleum oils.

Liquid	Temp. t Deg. Fahr.	Weight Density ρ Lbs. per Cu. Ft.	Specific Gravity S
Acetone	60	49.4	0.792
Ammonia, Saturated	10	40.9	...
Benzene	32	56.1	...
Brine, 10% Ca Cl	32	68.05	...
Brine, 10% Na Cl	32	67.24	...
Bunkers C Fuel Max.	60	63.25	1.014
Carbon Disulphide	32	80.6	...
Distillate	60	52.99	0.850
Fuel 3 Max.	60	56.02	0.898
Fuel 5 Min.	60	60.23	0.966
Fuel 5 Max.	60	61.92	0.993
Fuel 6 Min.	60	61.92	0.993
Gasoline	60	46.81	0.751
Gasoline, Natural	60	42.42	0.680
Kerosene	60	50.85	0.815
M. C. Residuum	60	58.32	0.935
Mercury	20	849.74	...
Mercury	40	848.03	...
Mercury	60	846.32	13.570
Mercury	80	844.62	...
Mercury	100	842.93	...
Milk	...	†	...
Olive Oil	59	57.3	0.919
Pentane	59	38.9	0.624
SAE 10 Lube‡	60	54.64	0.876
SAE 30 Lube‡	60	56.02	0.898
SAE 70 Lube‡	60	57.12	0.916
Salt Creek Crude	60	52.56	0.843
32.6° API Crude	60	53.77	0.862
35.6° API Crude	60	52.81	0.847
40° API Crude	60	51.45	0.825
48° API Crude	60	49.16	0.788

*Liquid at 60 F referred to water at 60 F.

†Milk has a weight density of 64.2 to 64.6.

‡100 Viscosity Index.

Values in the table at the left were taken from *Smithsonian Physical Tables*, Mark's *Engineers' Handbook*, and [12]Nelson's *Petroleum Refinery Engineering*.

Fig. B.2b Weight density and specific gravity of various liquids.

FIG. B.3(A)
EQUIVALENTS OF ABSOLUTE VISCOSITY

Absolute or dynamic viscosity		Centipoise (μ)	Poise $\frac{\text{Gram}}{\text{Cm Sec}}$ $\frac{\text{Dyne Sec}}{\text{Cm}^2}$ (100 μ)	$\frac{\text{Slugs}}{\text{Ft Sec}}$ $\frac{\text{*Pound}_f\text{ Sec}}{\text{Ft}^2}$ (μ'_e)	$\frac{\text{†Pound}_m}{\text{Ft Sec}}$ $\frac{\text{Poundal Sec}}{\text{Ft}^2}$ (μ_e)
Centipoise	(μ)	1	0.01	2.09 (10^{-5})	6.72 (10^{-4})
Poise $\frac{\text{Gram}}{\text{Cm Sec}}$ $\frac{\text{Dyne Sec}}{\text{Cm}^2}$	(100 μ)	100	1	2.09 (10^{-3})	0.0672
$\frac{\text{Slugs}}{\text{Ft Sec}}$ $\frac{\text{*Pound}_f\text{ Sec}}{\text{Ft}^2}$	(μ'_e)	47 900	479	1	g or 32.2
$\frac{\text{†Pound}_m}{\text{Ft Sec}}$ $\frac{\text{Poundal Sec}}{\text{Ft}^2}$	(μ_e)	1487	14.87	$\frac{1}{g}$ or .0311	1

*Pound$_f$ = Pound of Force †Pound$_m$ = Pound of Mass

To convert absolute or dynamic viscosity from one set of units to another, locate the given set of units in the left hand column and multiply the numerical value by the factor shown horizontally to the right under the set of units desired.

As an example, suppose a given absolute viscosity of 2 poise is to be converted to slugs/foot second. By referring to the table, we find the conversion factor to be 2.09 (10^{-3}). Then, 2 (poise) times 2.09 (10^{-3}) = 4.18 (10^{-3}) = 0.00418 slugs/foot second.

FIG. B.3(B)
EQUIVALENTS OF KINEMATIC VISCOSITY

Kinematic viscosity		Centistokes (ν)	Stokes $\frac{\text{Cm}^2}{\text{Sec}}$ (100 ν)	$\frac{\text{Ft}^2}{\text{Sec}}$ (ν')
Centistokes	(ν)	1	0.01	1.076 (10^{-5})
Stokes $\frac{\text{Cm}^2}{\text{Sec}}$	(100 ν)	100	1	1.076 (10^{-3})
$\frac{\text{Ft}^2}{\text{Sec}}$	(ν')	92 900	929	1

To convert kinematic viscosity from one set of units to another, locate the given set of units in the left hand column and multiply the numerical value by the factor shown horizontally to the right, under the set of units desired.

As an example, suppose a given kinematic viscosity of 0.5 square foot/second is to be converted to centistokes. By referring to the table, we find the conversion factor to be 92,900. Then, 0.5 (sq ft/sec) times 92,900 = 46,450 centistokes.

FIG. B.4(A)
EQUIVALENTS OF KINEMATIC AND SAYBOLT UNIVERSAL VISCOSITY

Kinematic viscosity Centistokes ν	Equivalent Saybolt Universal viscosity, sec: At 100 F basic values	Equivalent Saybolt Universal viscosity, sec: At 210 F	Kinematic viscosity Centistokes ν	Equivalent Saybolt Universal viscosity, sec: At 100 F basic values	Equivalent Saybolt Universal viscosity, sec: At 210 F
2.0	32.6	32.9	29	136.9	137.9
2.5	34.4	34.7	30	141.3	142.3
3.0	36.0	36.3	31	145.7	146.8
3.5	37.6	37.9	32	150.2	151.2
4.0	39.1	39.4	33	154.7	155.8
4.5	40.8	41.0	34	159.2	160.3
5	42.4	42.7	35	163.7	164.9
6	45.6	45.9	36	168.2	169.4
7	48.8	49.1	37	172.7	173.9
8	52.1	52.5	38	177.3	178.5
9	55.5	55.9	39	181.8	183.0
10	58.9	59.3	40	186.3	187.6
11	62.4	62.9	41	190.8	192.1
12	66.0	66.5	42	195.3	196.7
13	69.8	70.3	43	199.8	201.2
14	73.6	74.1	44	204.4	205.9
15	77.4	77.9	45	209.1	210.5
16	81.3	81.9	46	213.7	215.2
17	85.3	85.9	47	218.3	219.8
18	89.4	90.1	48	222.9	224.5
19	93.6	94.2	49	227.5	229.1
20	97.8	98.5	50	232.1	233.8
21	102.0	102.8	55	255.2	257.0
22	106.4	107.1	60	278.3	280.2
23	110.7	111.4	65	301.4	303.5
24	115.0	115.8	70	324.4	326.7
25	119.3	120.1	Over 70	Saybolt Seconds equal Centistokes ×4.635	×4.667
26	123.7	124.5			
27	128.1	129.0			
28	132.5	133.4			

ote: To obtain the Saybolt Universal viscosity, equivalent to a nematic viscosity determined at t, multiply the equivalent Saybolt niversal viscosity at 100 F by $1 + (t - 100)\ 0.000\ 064$.

or example, 10 ν at 210 F are equivalent to 58.9 multiplied by 0070 or 59.3 sec Saybolt Universal at 210 F.

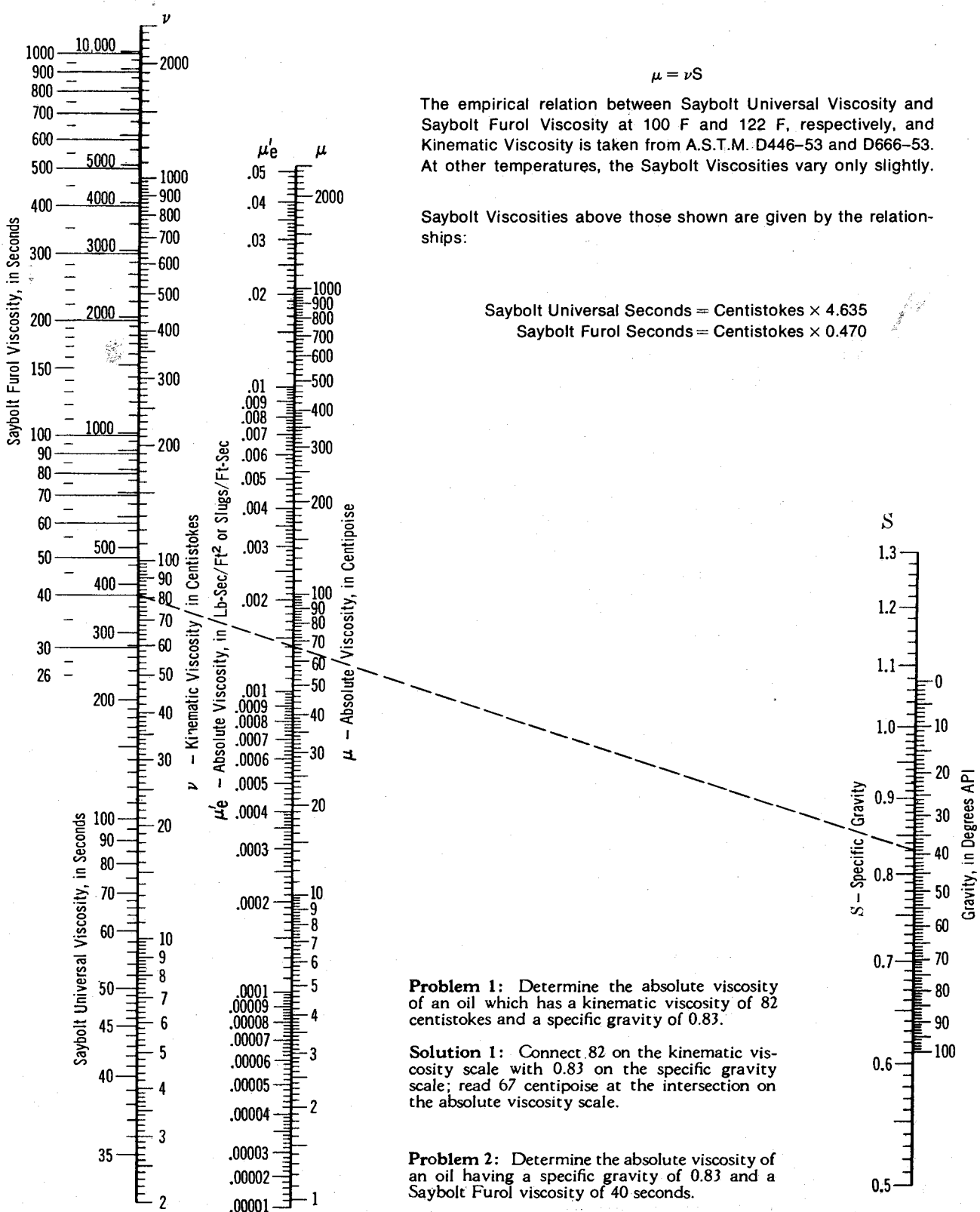
FIG. B.4(B)
EQUIVALENTS OF KINEMATIC AND SAYBOLT FUROL VISCOSITY AT 122 F

Kinematic viscosity Centistokes ν	Equivalent Saybolt furol viscosity, sec	Kinematic viscosity Centistokes ν	Equivalent Saybolt furol viscosity, sec
48	25.3	150	71.7
50	26.1	155	74.0
52	27.0	160	76.3
54	27.9	165	78.7
56	28.8	170	81.0
58	29.7	175	83.3
60	30.6	180	85.6
62	31.5	185	88.0
64	32.4	190	90.3
66	33.3	195	92.6
68	34.2	200	95.0
70	35.1	210	99.7
72	36.0	220	104.3
74	36.9	230	109.0
76	37.8	240	113.7
78	38.7	250	118.4
80	39.6	260	123.0
82	40.5	270	127.7
84	41.4	280	132.4
86	42.3	290	137.1
88	43.2	300	141.8
90	44.1	310	146.5
92	45.0	320	151.2
94	45.9	330	155.9
96	46.8	340	160.6
98	47.7	350	165.3
100	48.6	360	170.0
105	50.9	370	174.7
110	53.2	380	179.4
115	55.5	390	184.1
120	57.8	400	188.8
125	60.1	Over 400	Saybolt Furol Seconds = Centistokes ×0.4717
130	62.4		
135	64.7		
140	67.0		
145	69.4		

These tables are reprinted with the permission of the American Society for Testing Materials (ASTM). The table at the left was abstracted from D446–53, as shown on page 226 of ASTM Standards, Part 5, 1955 edition. The table at the right was abstracted from D 666–53, as shown on page 302 of ASTM Standards, Part 5, 1955 edition.

FIG. B.5
EQUIVALENTS OF KINEMATIC, SAYBOLT UNIVERSAL, SAYBOLT FUROL, AND ABSOLUTE VISCOSITY

Equivalents of Kinematic, Saybolt Universal, Saybolt Furol, and Absolute Viscosity



$$\mu = \nu S$$

The empirical relation between Saybolt Universal Viscosity and Saybolt Furol Viscosity at 100 F and 122 F, respectively, and Kinematic Viscosity is taken from A.S.T.M. D446–53 and D666–53. At other temperatures, the Saybolt Viscosities vary only slightly.

Saybolt Viscosities above those shown are given by the relationships:

Saybolt Universal Seconds = Centistokes × 4.635
Saybolt Furol Seconds = Centistokes × 0.470

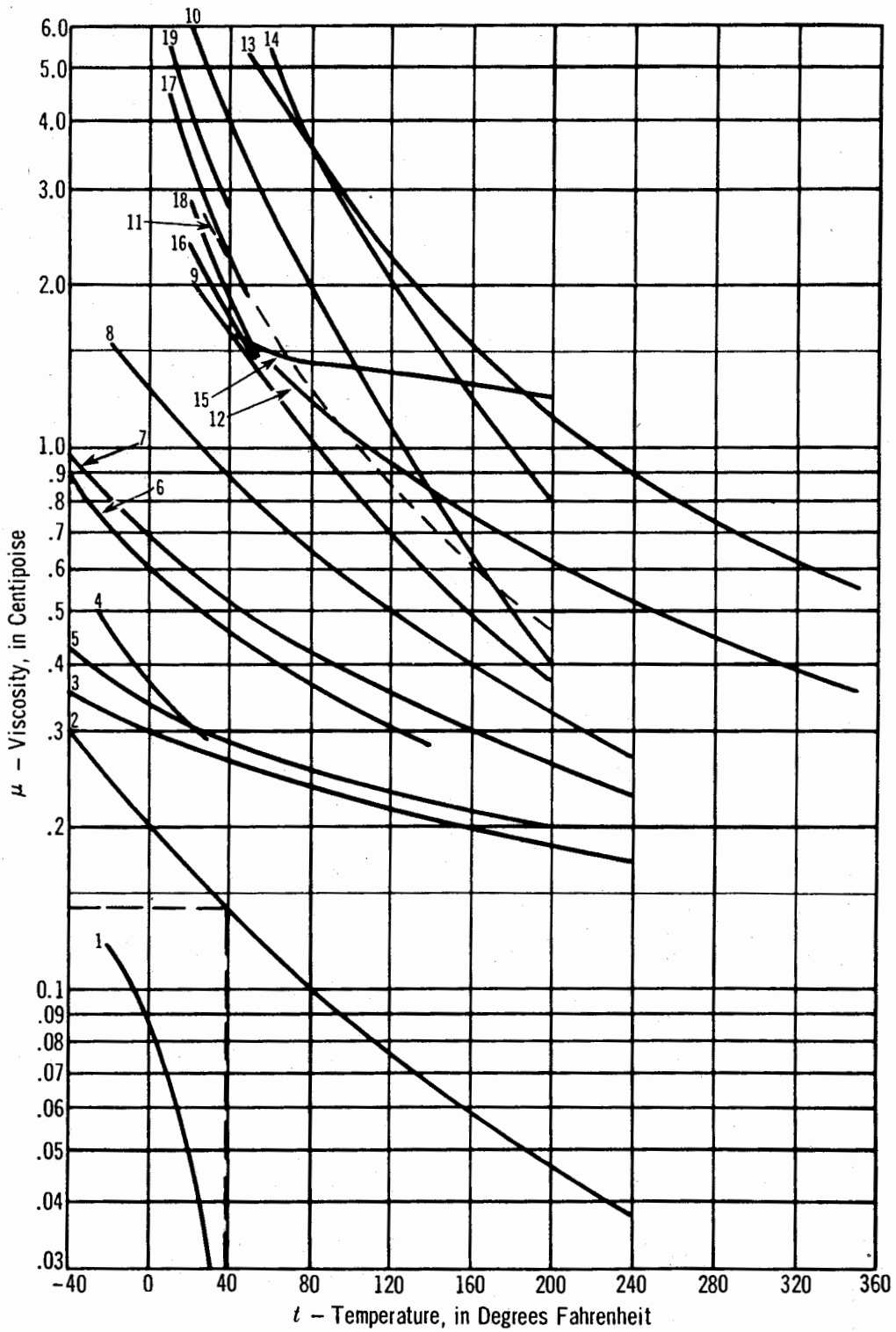
Problem 1: Determine the absolute viscosity of an oil which has a kinematic viscosity of 82 centistokes and a specific gravity of 0.83.

Solution 1: Connect 82 on the kinematic viscosity scale with 0.83 on the specific gravity scale; read 67 centipoise at the intersection on the absolute viscosity scale.

Problem 2: Determine the absolute viscosity of an oil having a specific gravity of 0.83 and a Saybolt Furol viscosity of 40 seconds.

Solution 2: Connect 0.83 on the specific gravity scale with 40 seconds on the Saybolt Furol scale; read 67 centipoise at the intersection on the absolute viscosity scale.

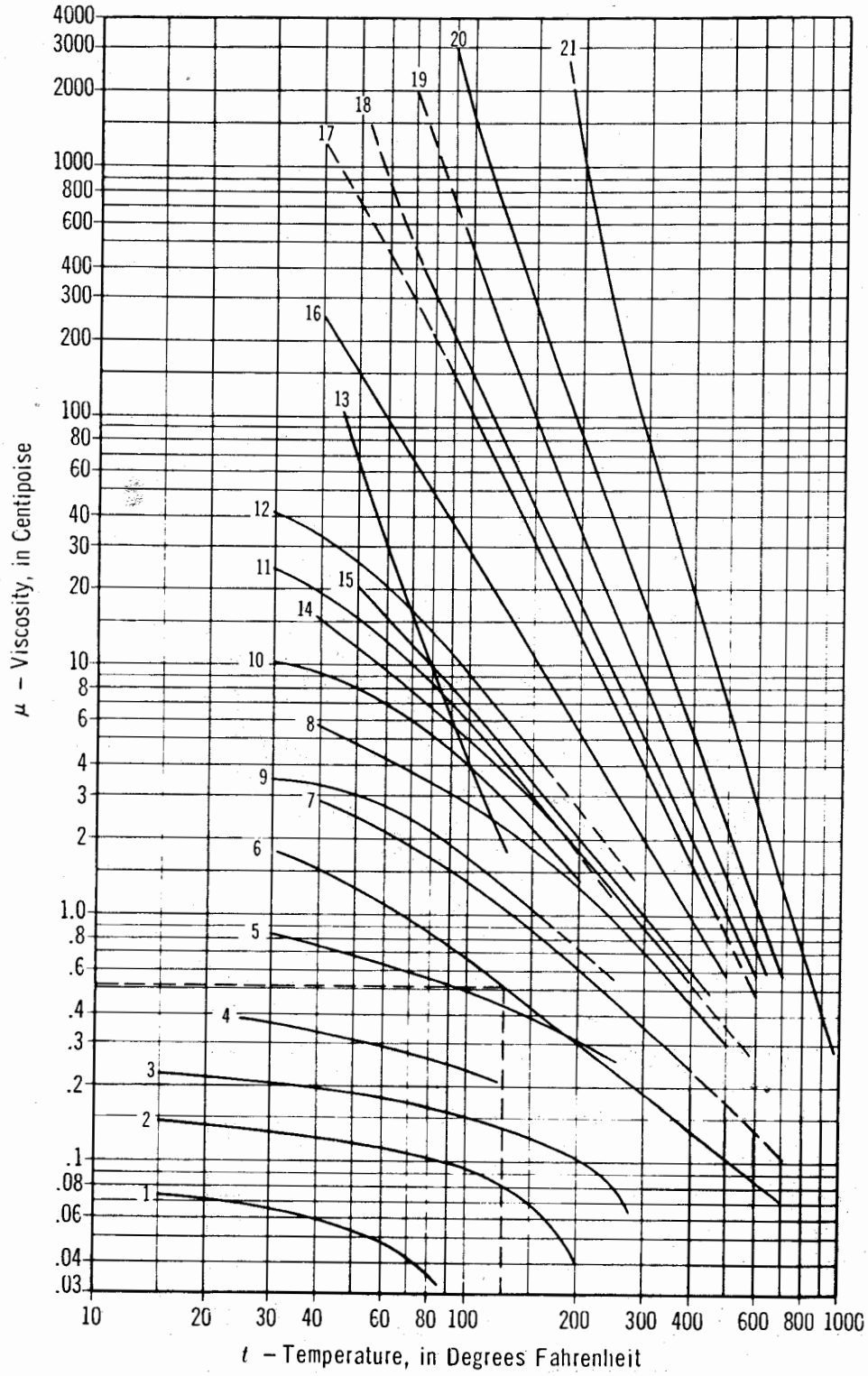
FIG. B.6
VISCOSITY OF VARIOUS LIQUIDS[5,8,11]



1. Carbon Dioxide..CO_2
2. Ammonia.....NH_3
3. Methyl Chloride..CH_3Cl
4. Sulphur Dioxide..SO_2
5. Freon 12.....F-12
6. Freon 114.....F-114
7. Freon 11.....F-11
8. Freon 113.....F-113
9. Ethyl Alcohol
10. Isopropyl Alcohol
11. 20% Sulphuric Acid.....20% H_2SO_4
12. Dowtherm E
13. Dowtherm A
14. 20% Sodium Hydroxide..20% NoOH
15. Mercury
16. 10% Sodium Chloride Brine...10% NaCl
17. 20% Sodium Chloride Brine...20% NaCl
18. 10% Calcium Chloride Brine..10% $CaCl_2$
19. 20% Calcium Chloride Brine..20% $CoCl_2$

Example: The viscosity of ammonia at 40 F is 0.14 centipoise.

FIG. B.7
VISCOSITY OF WATER AND LIQUID PETROLEUM PRODUCTS[8,12,23]

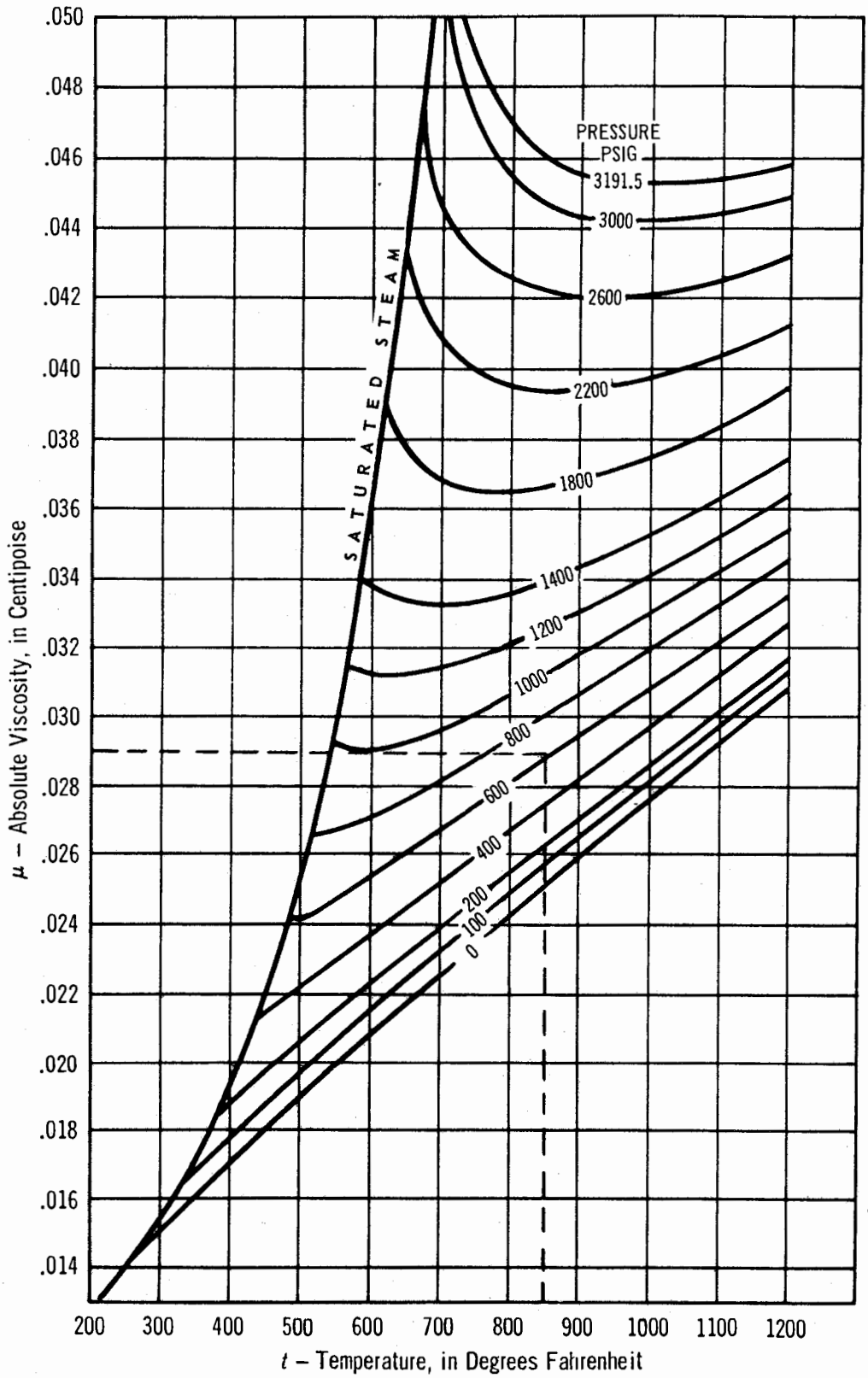


1. Ethane (C_2H_6)
2. Propane (C_3H_8)
3. Butane (C_4H_{10})
4. Natural Gasoline
5. Gasoline
6. Water
7. Kerosene
8. Distillate
9. 48 Deg. API Crude
10. 40 Deg. API Crude
11. 35.6 Deg. API Crude
12. 32.6 Deg. API Crude
13. Salt Creek Crude
14. Fuel 3 (Max.)
15. Fuel 5 (Min.)
16. SAE 10 Lube (100 V.I.)
17. SAE 30 Lube (100 V.I.)
18. Fuel 5 (Max.) or Fuel 6 (Min.)
19. SAE 70 Lube (100 V.I.)
20. Bunker C Fuel (Max.) and M.C. Residuum
21. Asphalt

Data extracted in part by permission from the Oil and Gas Journal.

Example: The viscosity of water at 125 F is 0.52 centipoise (Curve No. 6).

FIG. B.8
VISCOSITY OF STEAM[14]



Example: Viscosity of 600 psig, 850 F steam is 0.029 centipoise.

Adapted from:
Philip J. Potter — *Steam Power Plants*,
Copyright 1949, The Ronald Press Company.

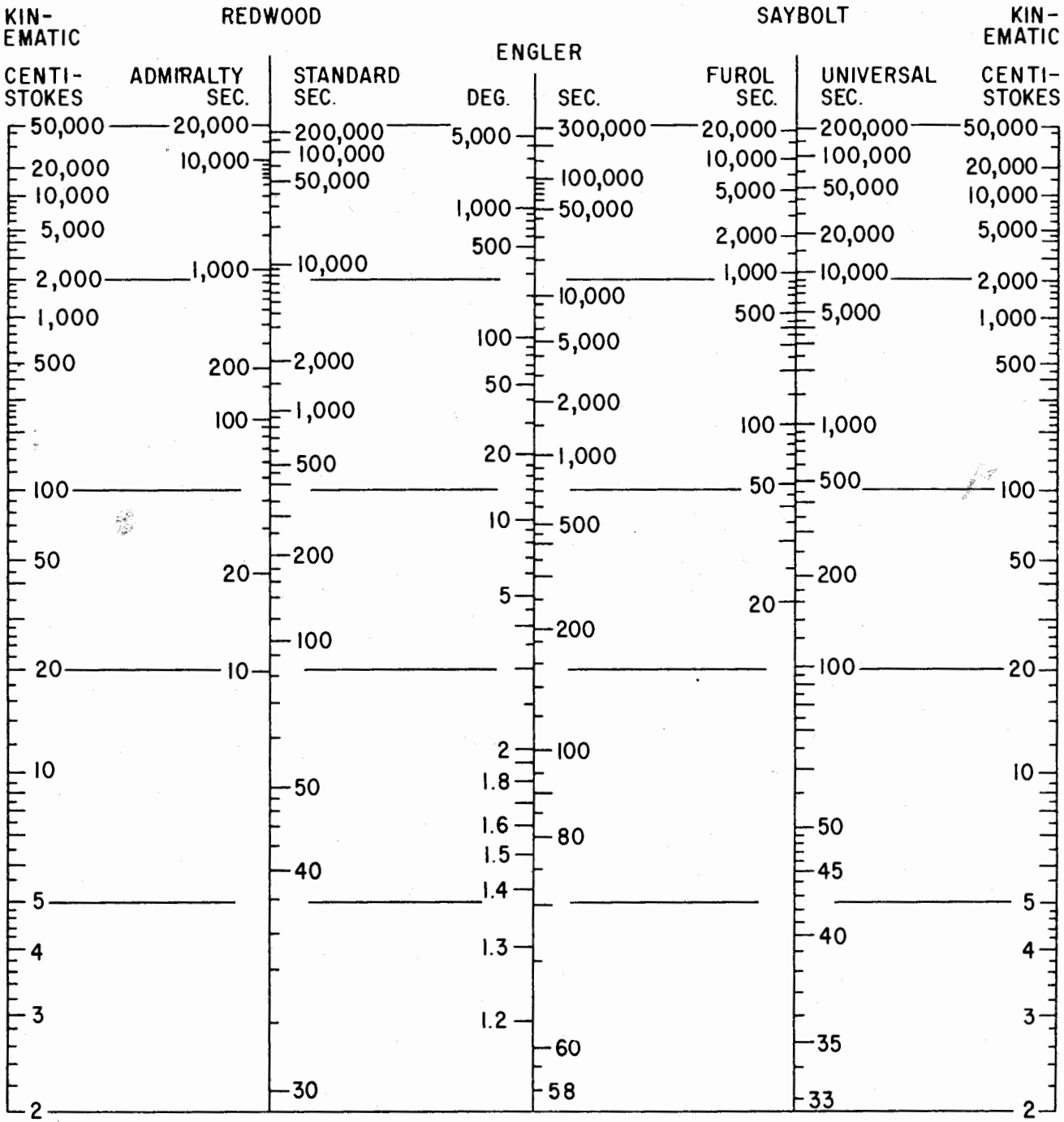


Fig. B.9 Convert viscosity values to various units.

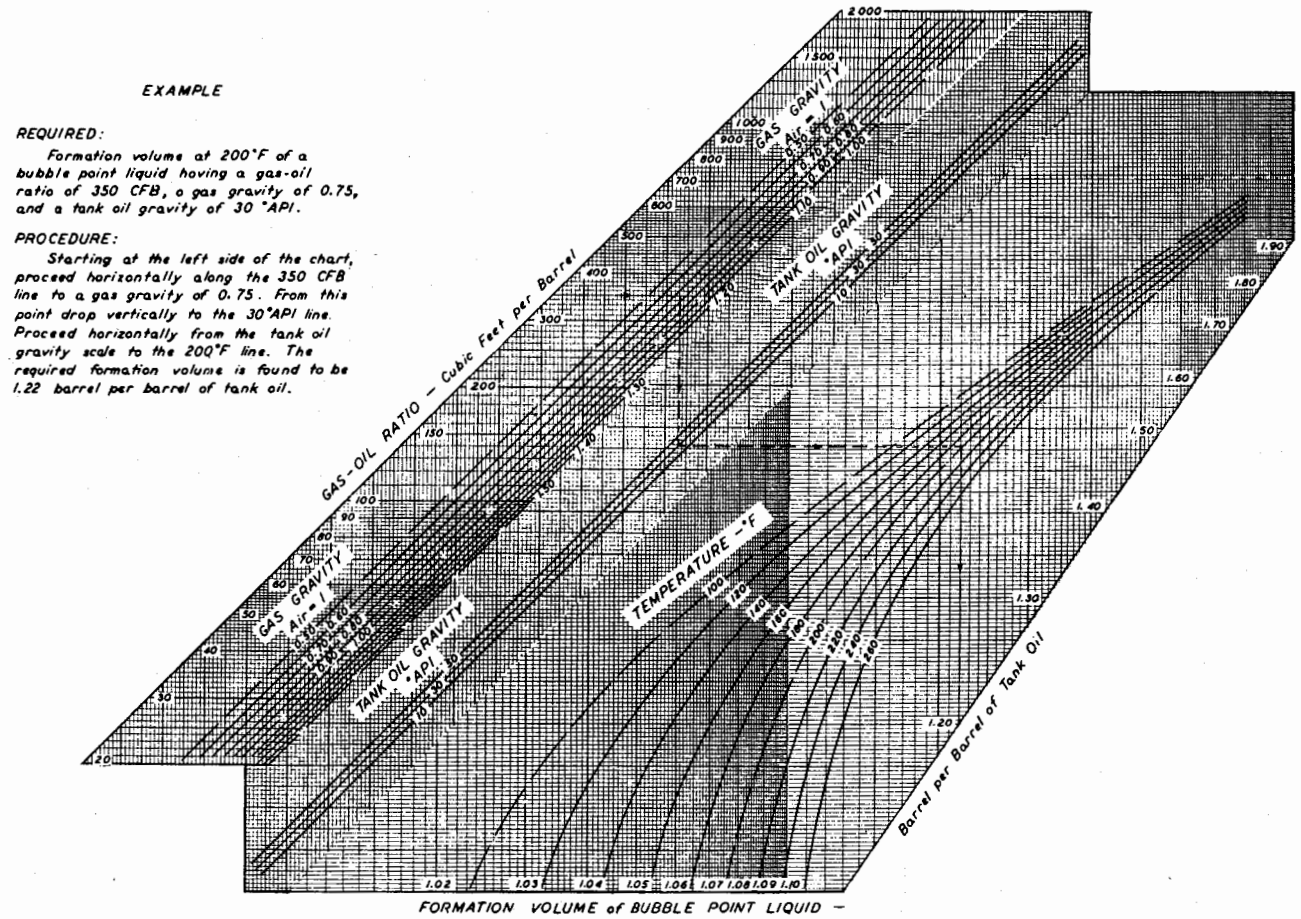
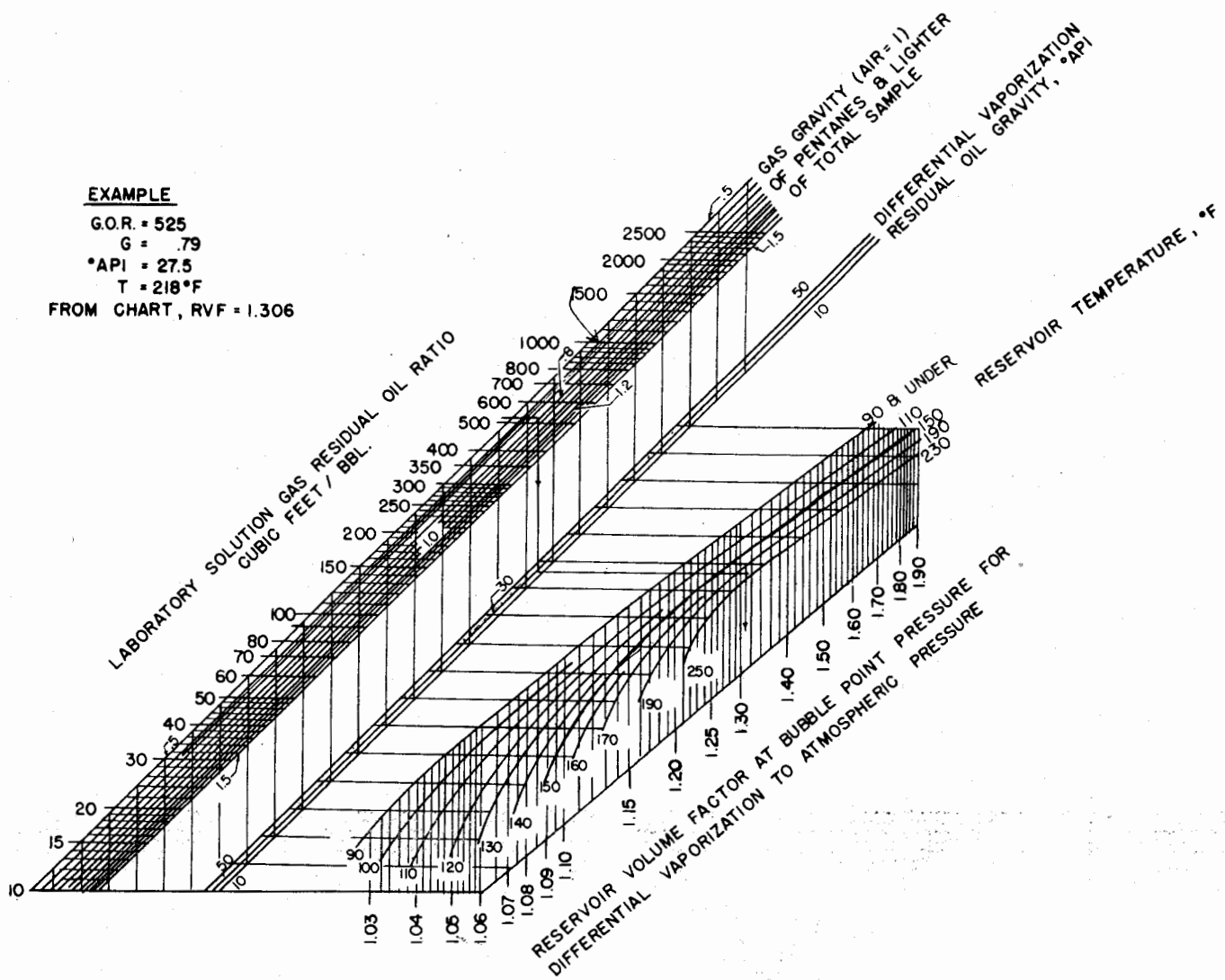


Fig. B.10 Properties of natural hydrocarbon mixtures of gas and liquid; formation volume of bubble point liquids (compliments of Chevron Research Co.).



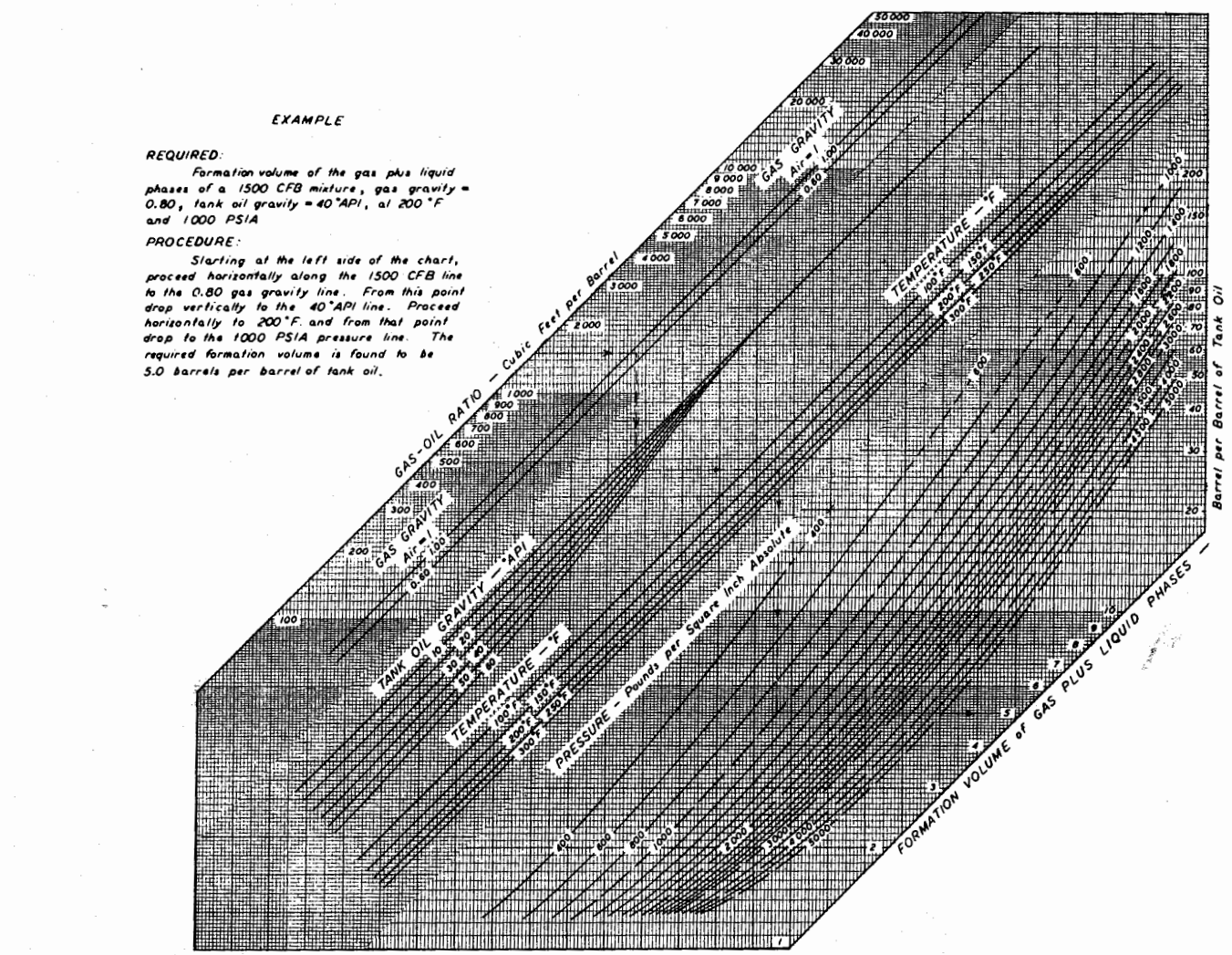
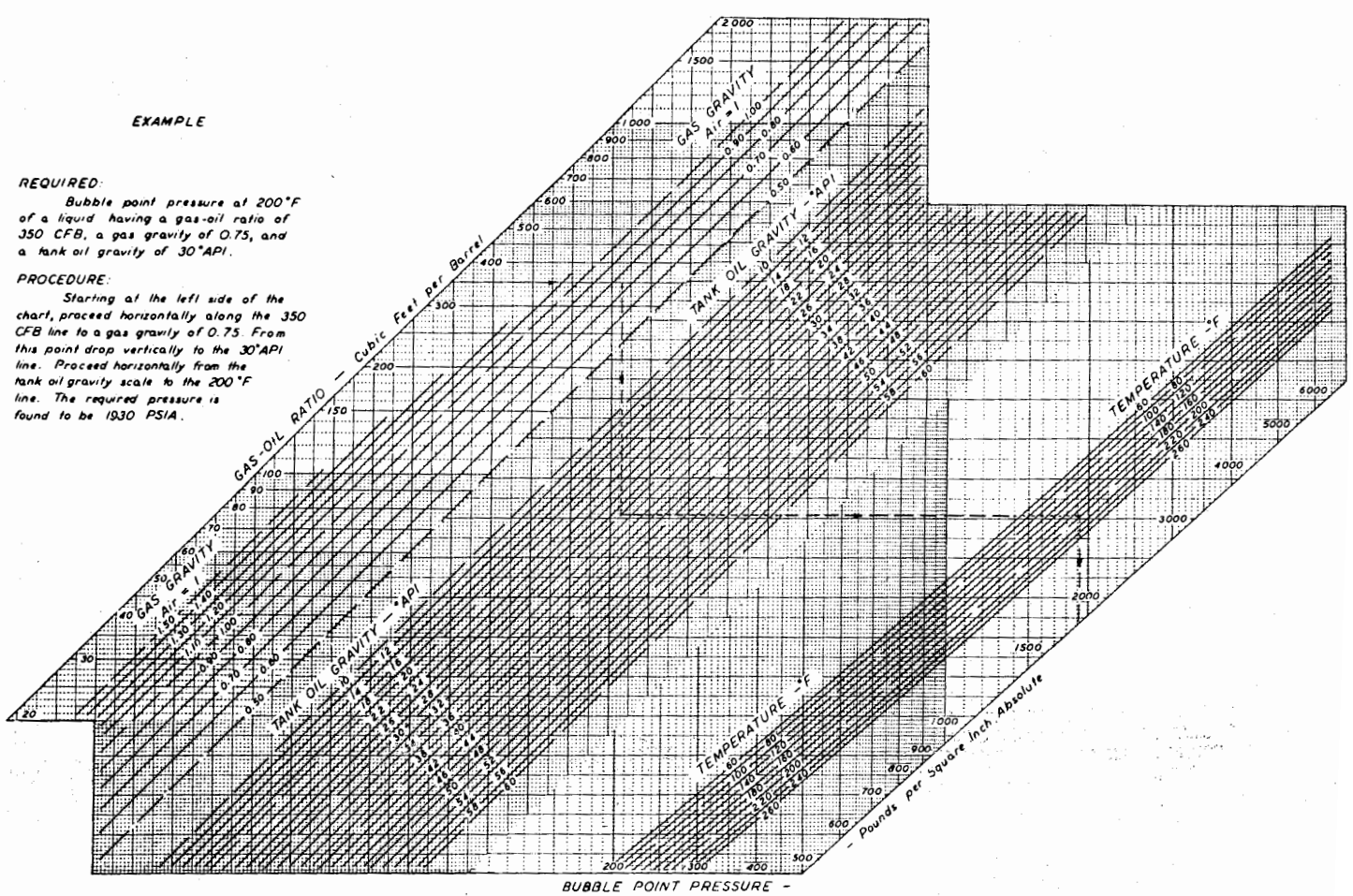


Fig. B.12 Properties of natural hydrocarbon mixtures of gas and liquid; formation volume of gas plus liquid phase (compliments of Chevron Research Co.).



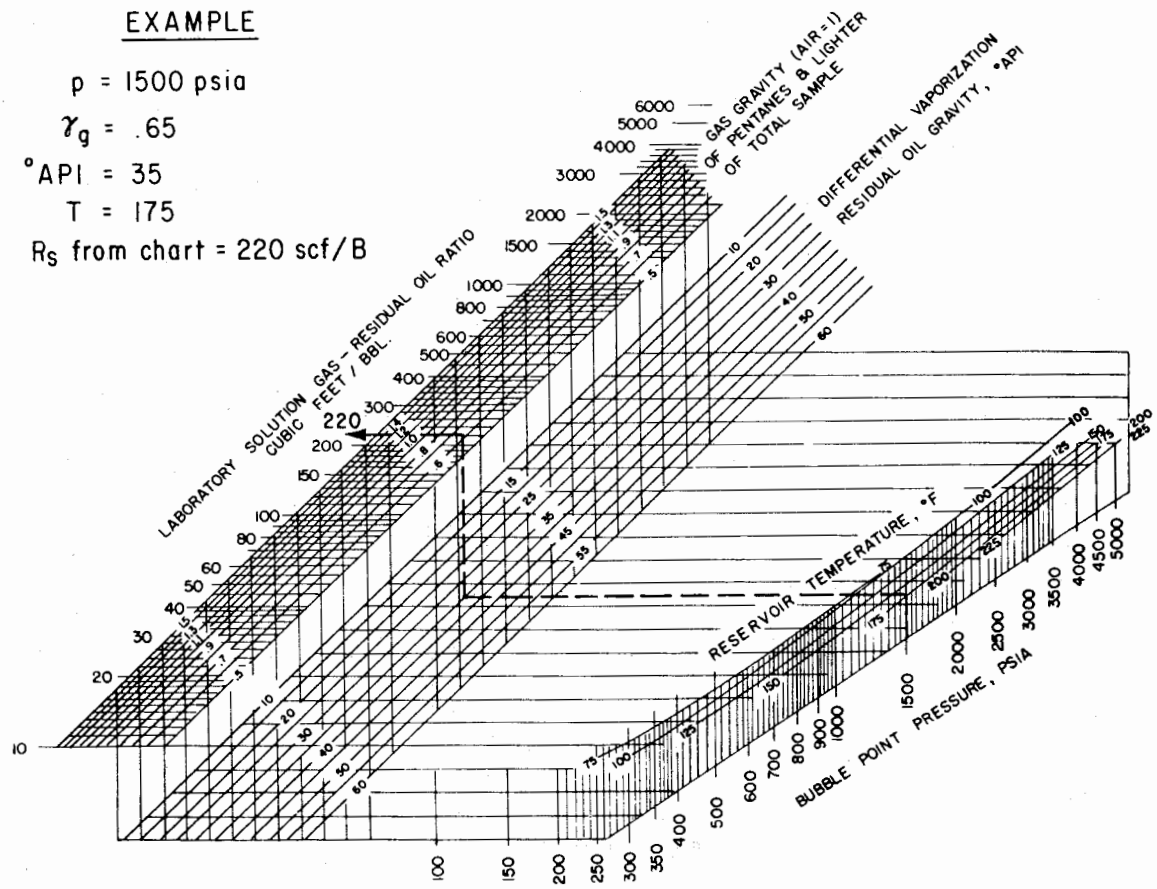


Fig. B.14 Reservoir liquid bubble point pressure correlation (from Aime Jour. Pet. Tech. T. P. 2931, 1950).

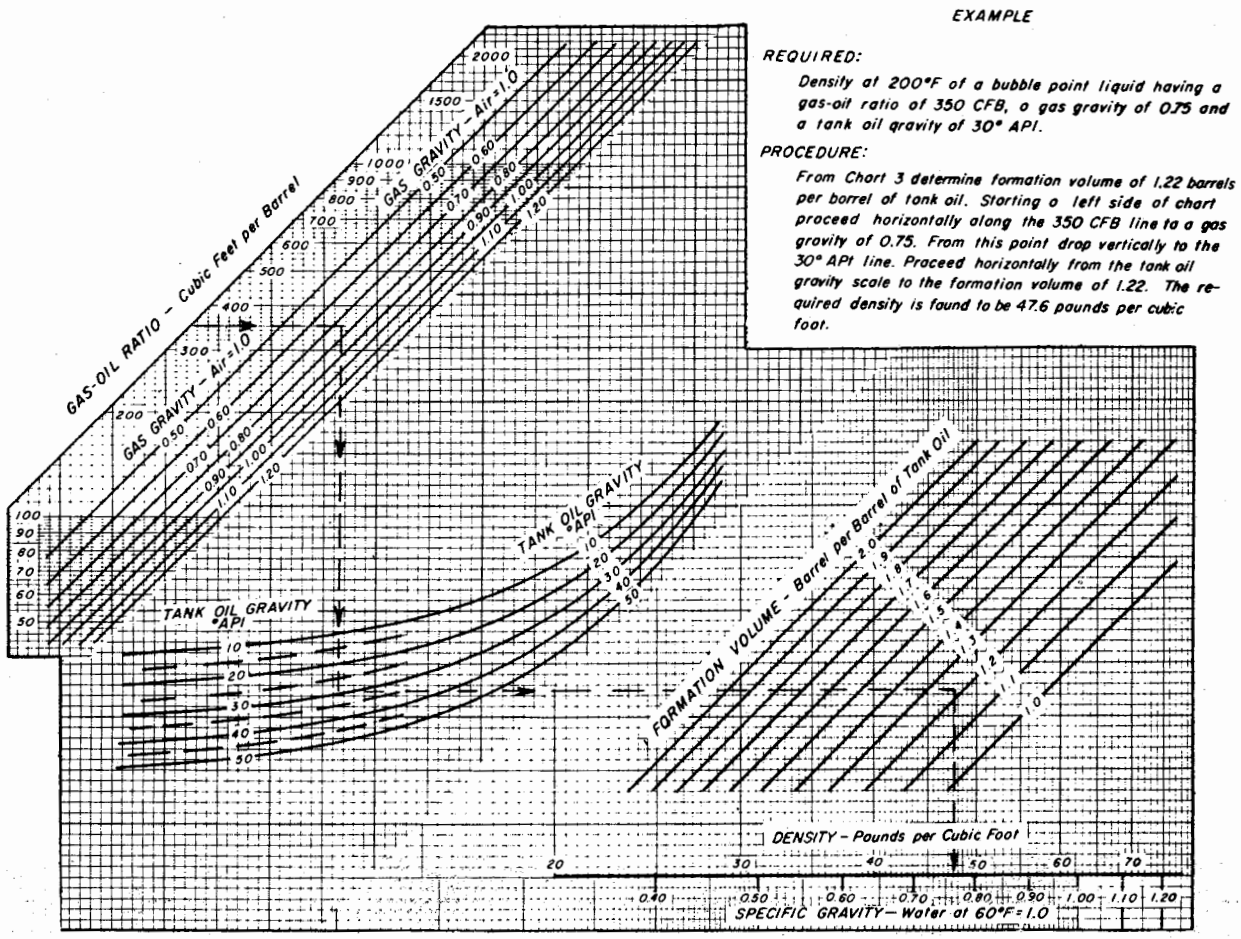


Fig. B.15 Properties of natural hydrocarbon mixtures of gas and liquid; density and sp gr of mixtures (compliments of Chevron Research Co.).

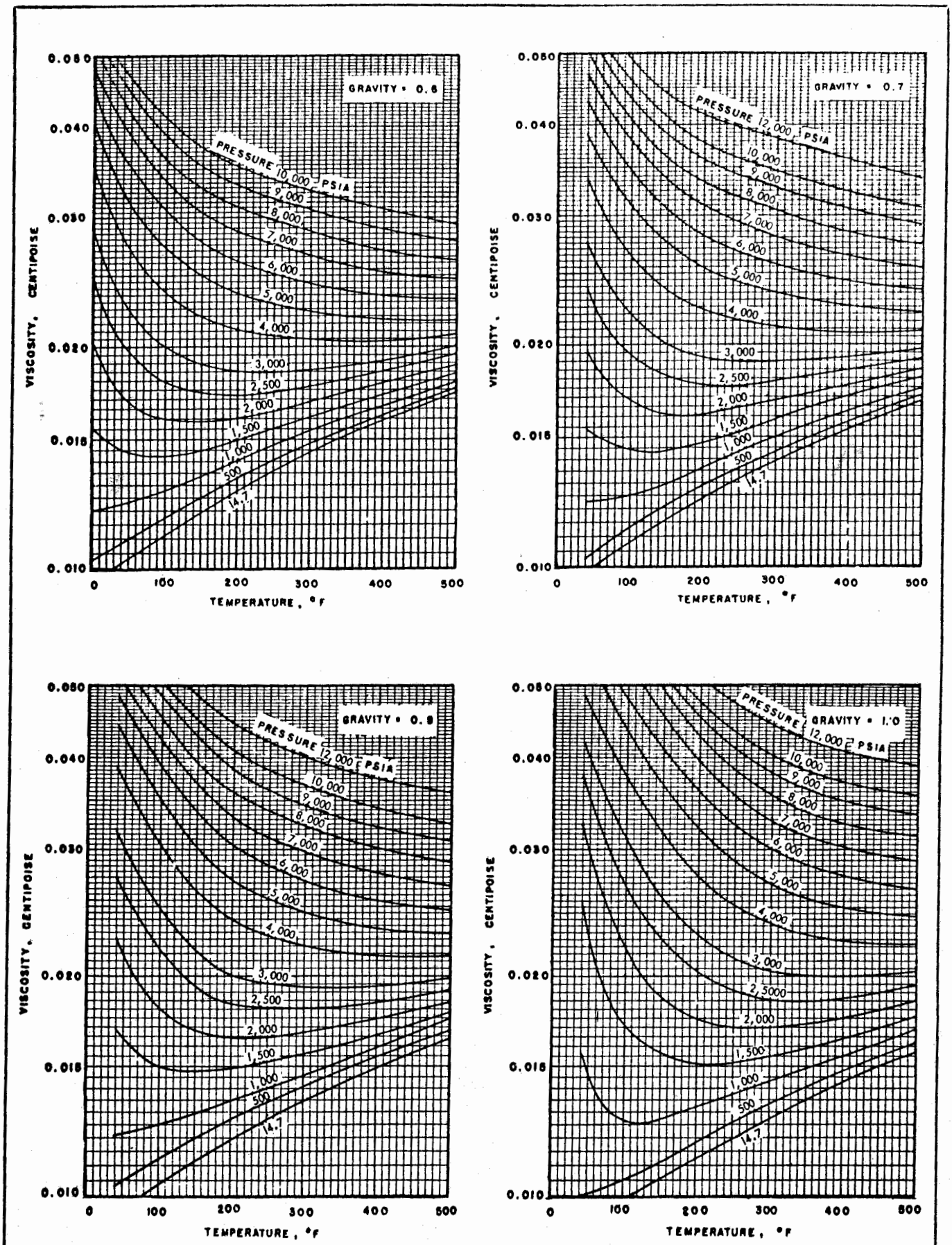
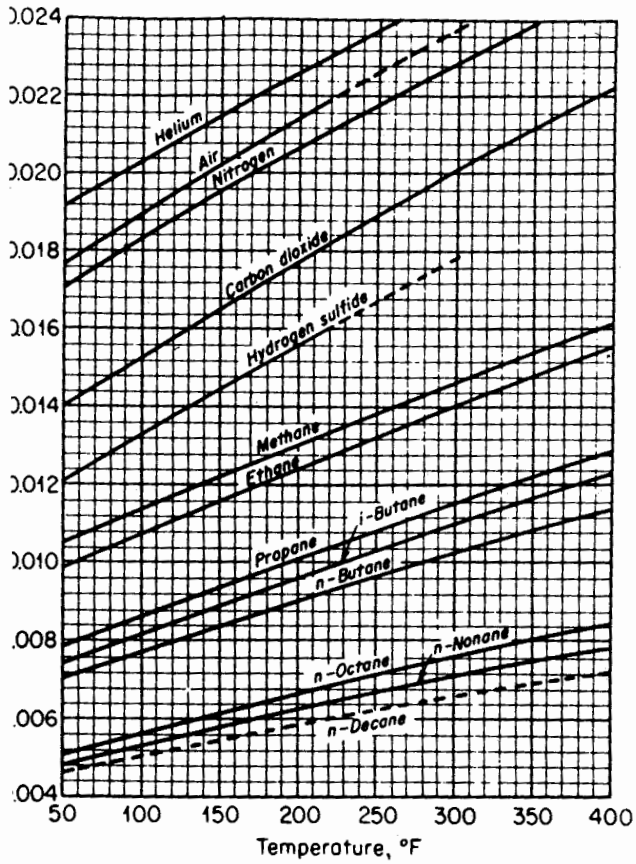


Fig. B.16 Viscosity of natural gas.

(Charts were presented at summer session on Flow of Natural Gas from Reservoirs, conducted by D. L. Katz and David Cornell, University of Michigan, College of Engineering, 1955)



ig. B.17 Viscosity of natural gases at atmospheric pressure (after Carr, et al., courtesy Aime).

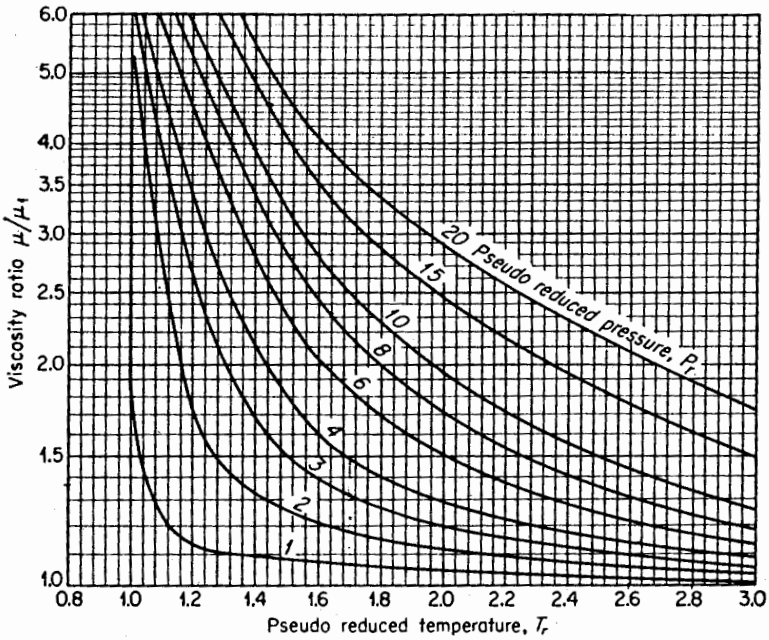
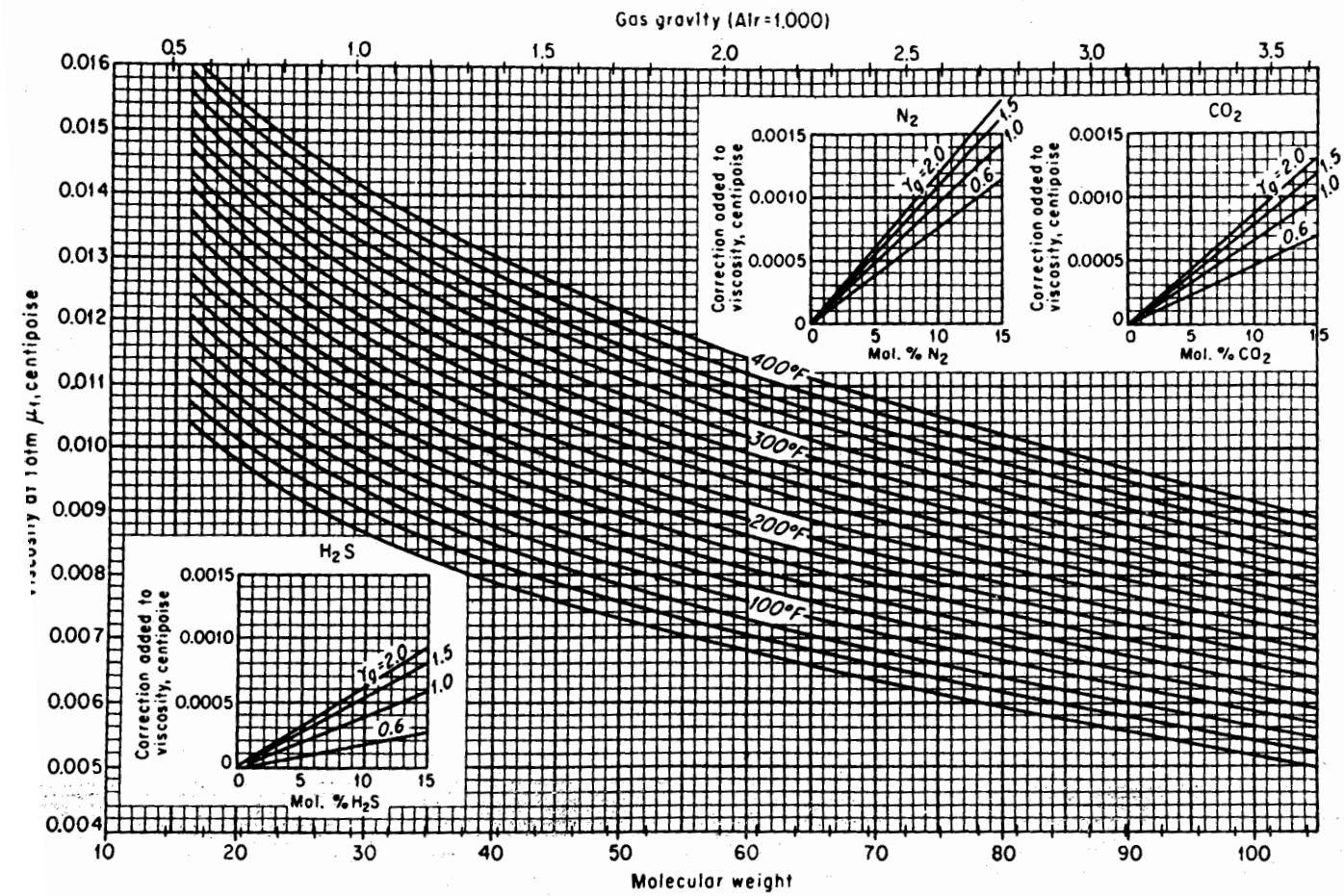
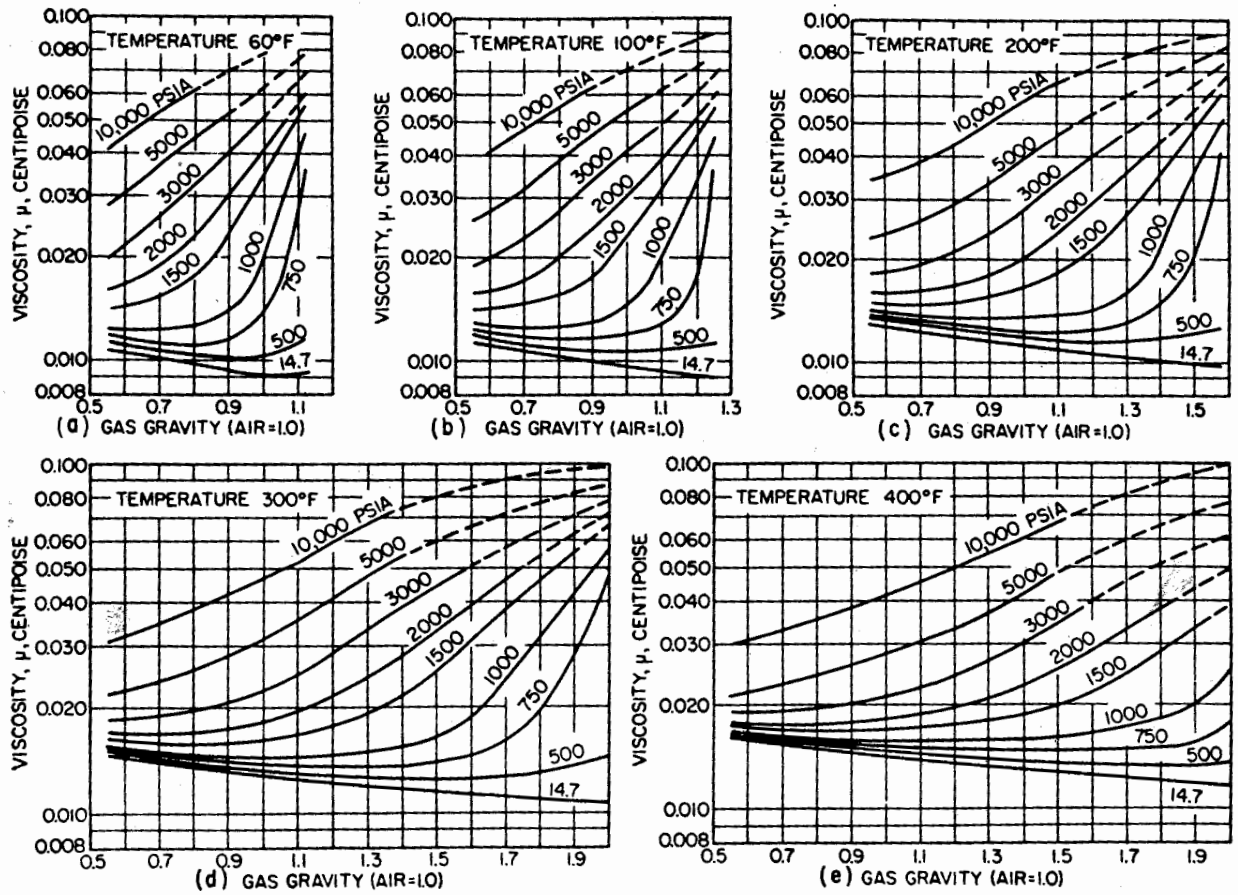


Fig. B.18A Viscosity ratio vs. pseudoreduced temperature (from Carr et al.)



. B.18 Viscosity of paraffin hydrocarbon gases at 1 atm (after Carr, et al., courtesy Aime).



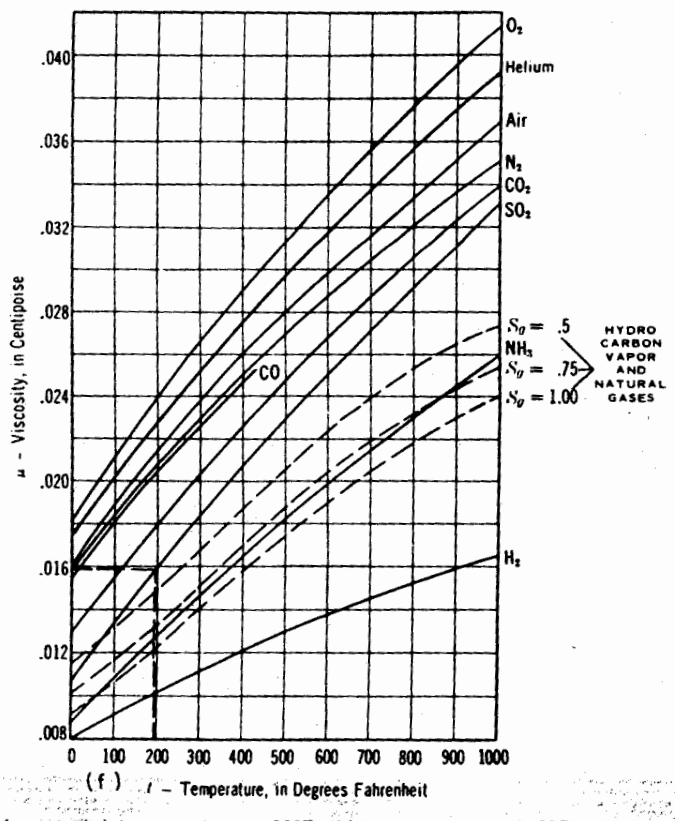


Fig. B.19 Viscosity of natural gases (a) temperature = 60°F; (b) temperature = 100°F; (c) temperature = 200°F; (d) temperature = 300°F; (e) temperature = 400°F; (f) viscosity of various gases (a)–(e) by permission from Handbook of Natural Gas Engineering, *by Donald L. Katz, et al., McGraw-Hill Book Co, Inc.; (f) courtesy Crane Co.*

The curves for hydrocarbon vapors and natural gases in the chart at the upper right are taken from Maxwell[15]; the curves for all other gases (except helium[27]) in the chart are based upon Sutherland's formula, as follows:

$$\mu = \mu_0 \left(\frac{0.555\ T_0 + C}{0.555\ T + C}\right)\left(\frac{T}{T_0}\right)^{3/2}$$

where:

μ = viscosity, in centipoise at temperature T.

μ_0 = viscosity, in centipoise at temperature T_0.

T = absolute temperature, in degrees Rankine (460 + deg. F) for which viscosity is desired.

T_0 = absolute temperature, in degrees Rankine, for which viscosity is known.

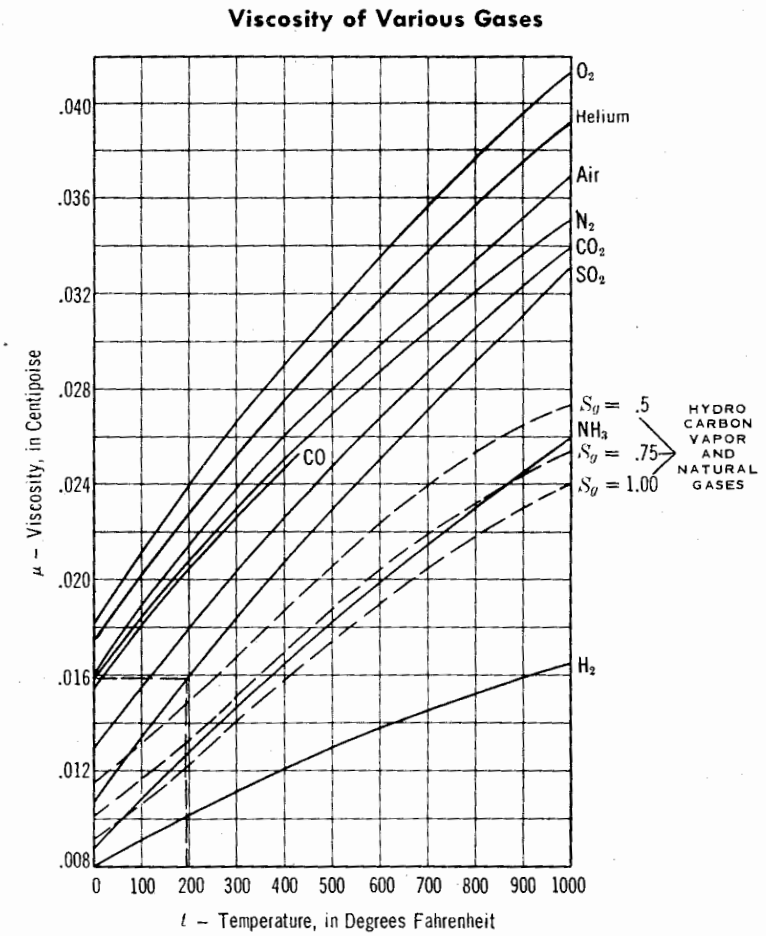
C = Sutherland's constant.

Note: The variation of viscosity with pressure is small for most gases. For gases given on this page, the correction of viscosity for pressure is less than 10 per cent for pressures up to 500 pounds per square inch.

Fluid	Approximate Values of "C"
O_2	127
Air	120
N_2	111
CO_2	240
CO	118
SO_2	416
NH_3	370
H_2	72

Upper chart example: The viscosity of sulphur dioxide gas (SO_2) at 200 F is 0.016 centipoise.

Lower chart example: The viscosity of carbon dioxide gas (CO_2) at about 80 F is 0.015 centipoise.



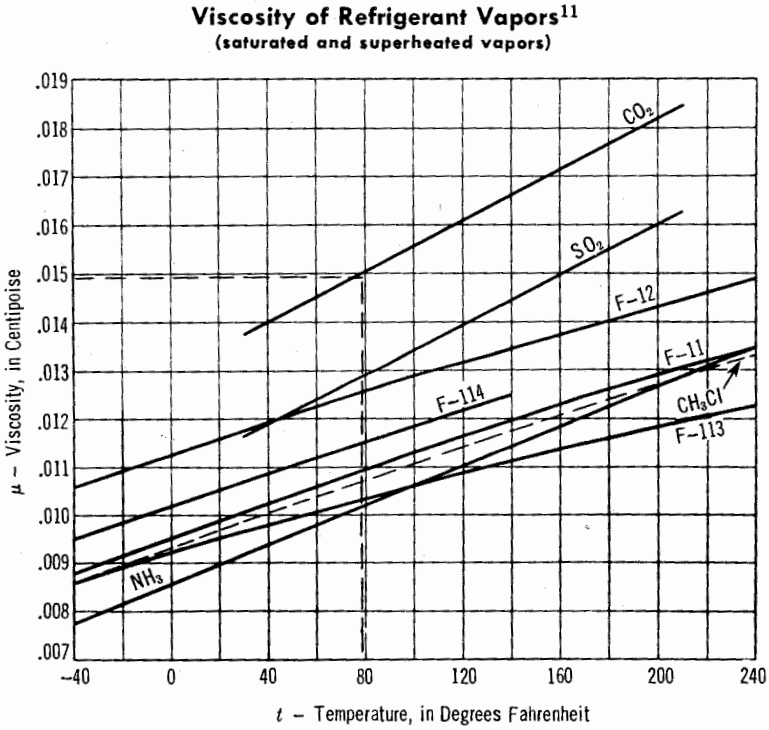
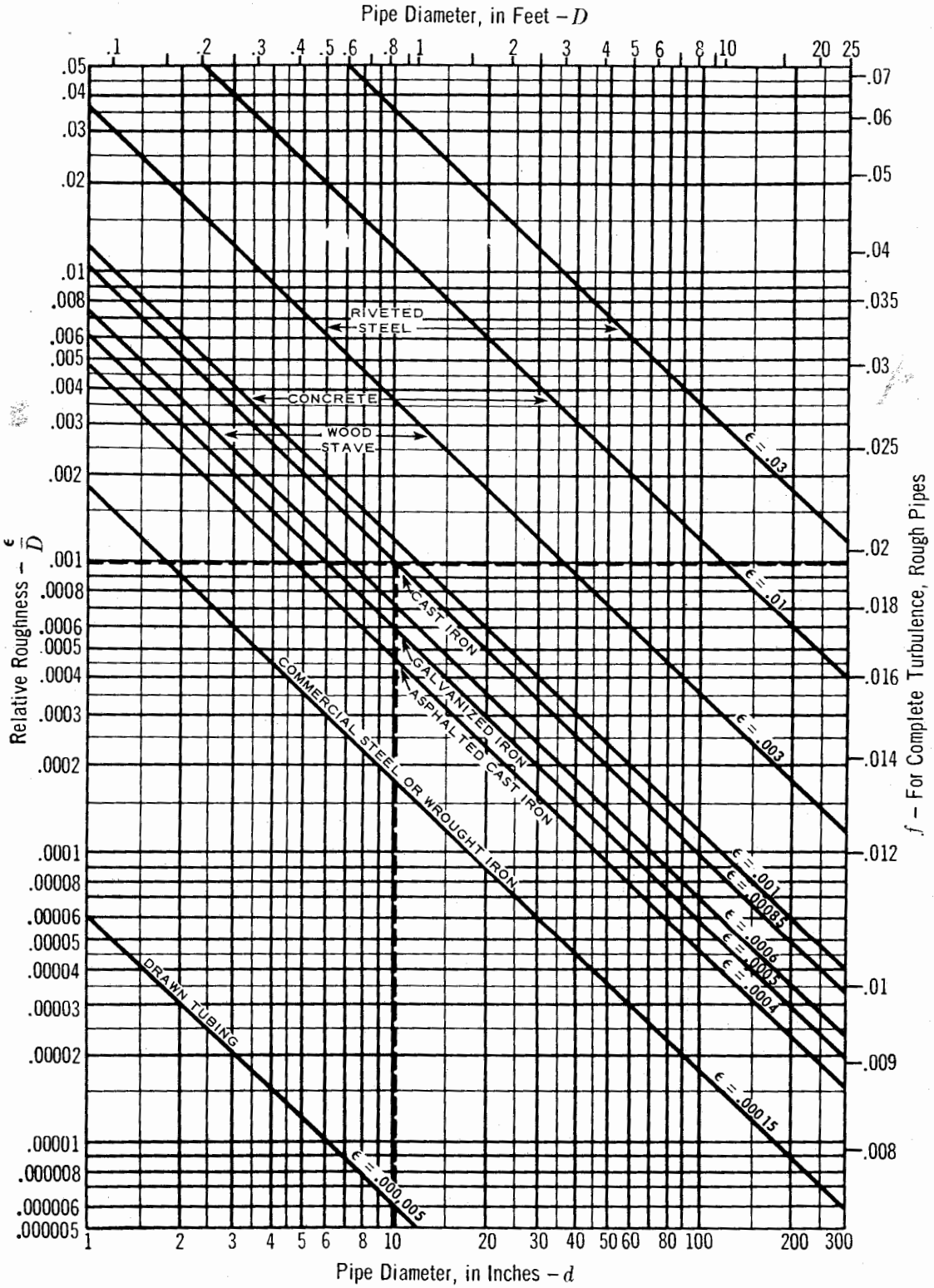


Fig. B.20 Viscosity of gases and vapors.

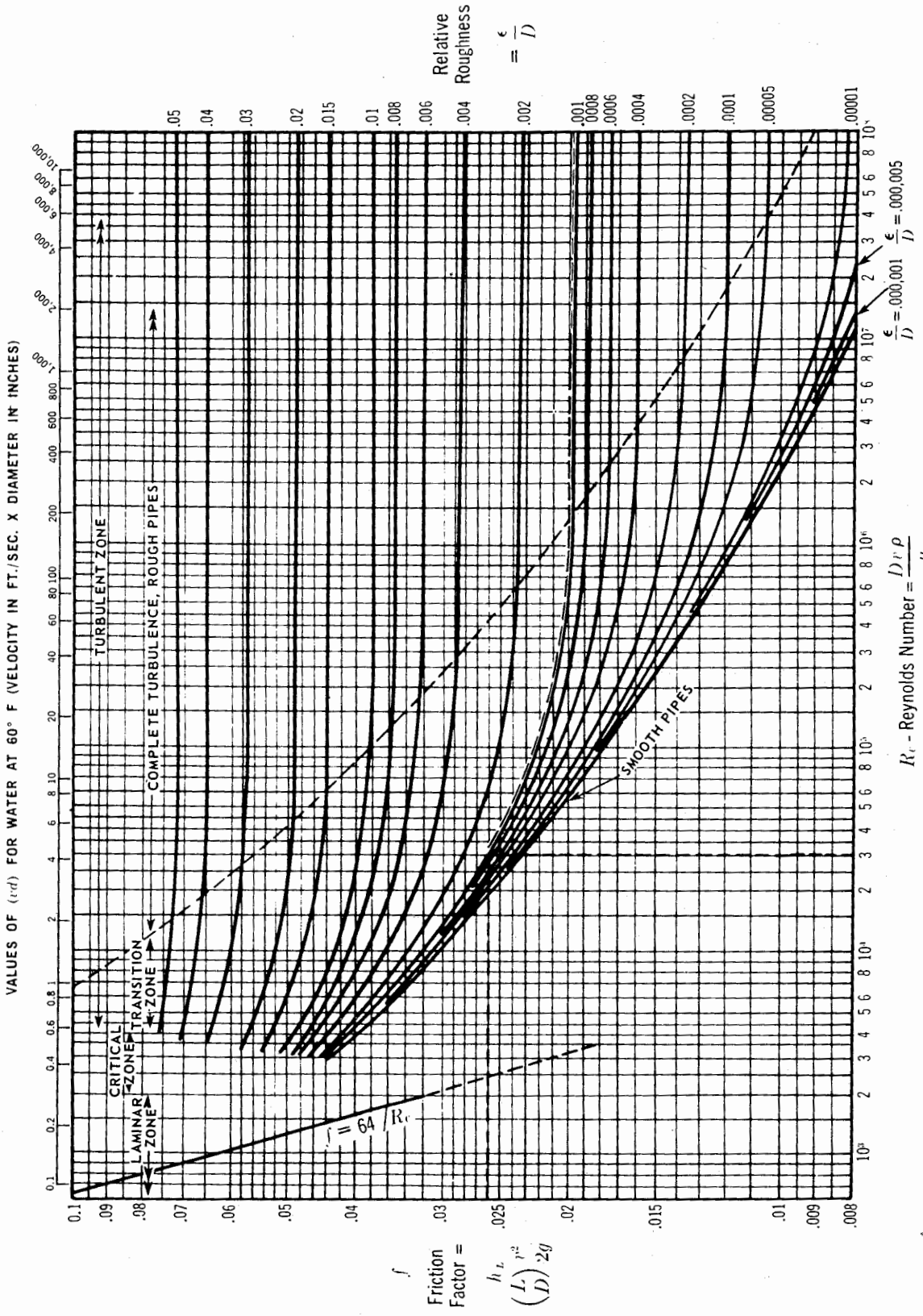
Relative Roughness of Pipe Materials and Friction Factors For Complete Turbulence[18]



Data extracted from *Friction Factors for Pipe Flow* by L.F. Moody, with permission of the publisher, The American Society of Mechanical Engineers, 29 West 39th Street, New York.

Problem: Determine absolute and relative roughness, and friction factor, for fully turbulent flow in 10-inch cast iron pipe (I.D. = 10.16″).

Solution: Absolute roughness (ϵ) = 0.00085. . . . Relative roughness (ϵ/D) = 0.001 . . . . Friction factor at fully turbulent flow (f) = 0.0196.



For other forms of the R_e equation, see page 3-2.

Data extracted from *Friction Factors for Pipe Flow* by L. F. Moody, with permission of the publisher, The American Society of Mechanical Engineers, 29 West 39th Street, New York 18, N. Y.

Problem: Determine the friction factor for 10-inch cast iron pipe (10.16" I.D.) at a Reynolds number flow of 30,000.

Solution: The relative roughness (see page A-23) is 0.001. Then, the friction factor (f) equals 0.026.

Fig. B.22 Friction factors for any type of commercial pipe.

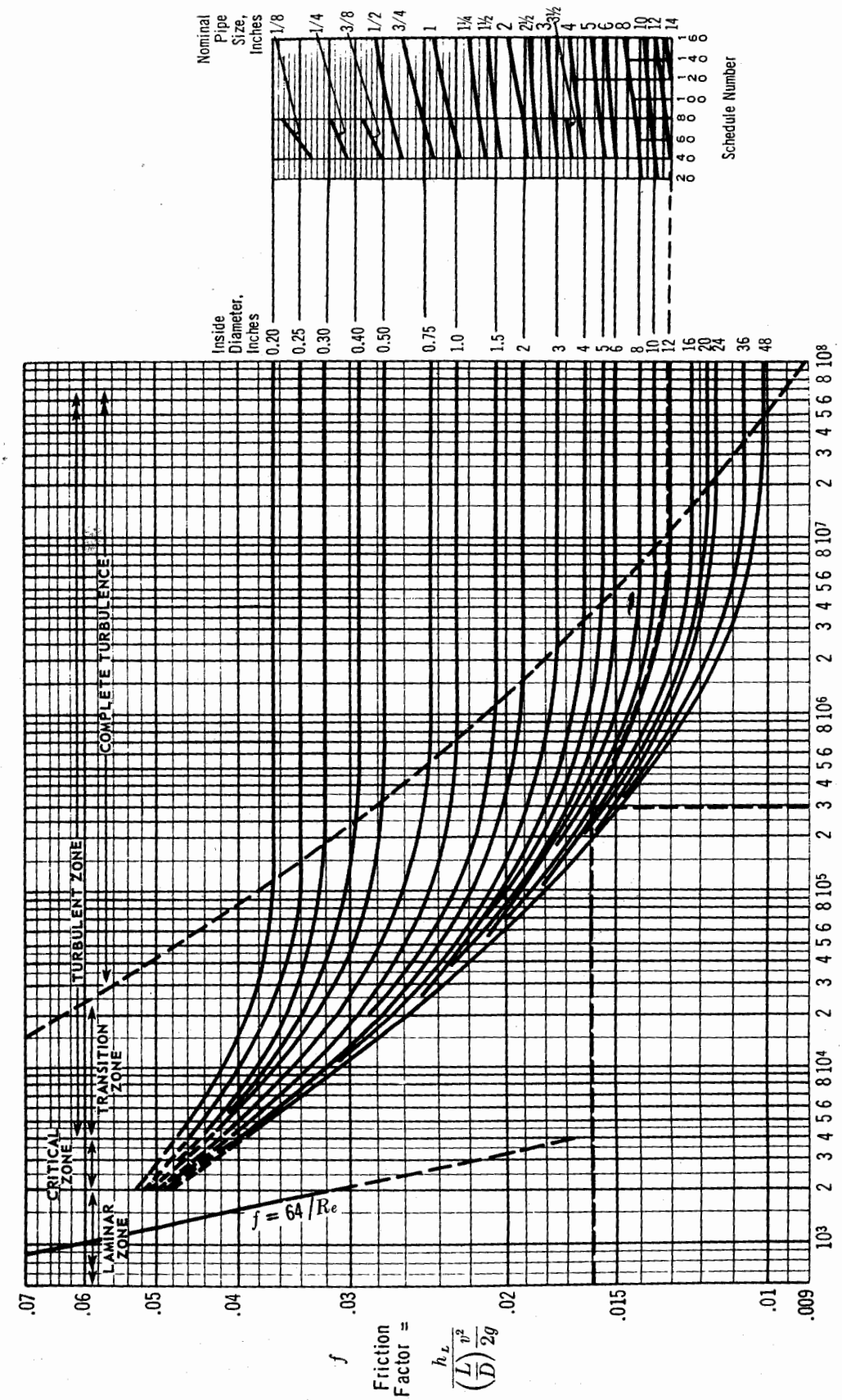


Fig. B.23 Friction factors for clean commercial steel and wrought iron pipe.

Note: To find viscosity in centipoise multiply kinematic viscosity by the oil specific gravity. Example: Kinematic viscosity = 10 centistokes 42° API oil γ_{oil} = .816 viscosity in centipoise = (10)(.816) = 8.16 cp.

Problem: Determine the friction factor for 12-inch Schedule 40 pipe at a flow having a Reynolds number of 300,000.

Solution: The friction factor (f) equals 0.016.

For other forms of the R_e equation, see page 3-2.

Appendix C

C.1 Poettmann and Carpenter Method
- C.11 Derivation of the Poettmann and Carpenter Equation for Multiphase Vertical Flow
- C.12 Detailed Procedure for the Calculation of a Vertical Pressure Traverse by the Poettmann and Carpenter Method
- C.13 Example Problem Worked by the Poettmann and Carpenter Method

C.2 The Fancher and Brown Method
- C.21 Stepwise Procedure for the Calculation of a Pressure Traverse by the Fancher and Brown Method
- C.22 Example Problem Worked by the Fancher and Brown Method for $2^{3}/_{8}$ in. O.D. Tubing (same data as given for Ex. C.13)

C.3 First Method of Hagedorn and Brown (To Account for Viscous Effects) (For $1^{1}/_{4}$ in Tubing Only)
- C.31 Stepwise Procedure for the Calculation of a Pressure Traverse by the First Method of Hagedorn and Brown (Use Only for $1^{1}/_{4}$ in. Tubing)
- C.32 Example Problem Worked by First Method of Hagedorn and Brown (Recommended for $1^{1}/_{4}$ in. Tubing Only)

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C.6 Method of Orkiszewski
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- C.71 Procedure for Calculating a Pressure Traverse by the Beggs and Brill Method
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C.8 Vertical Multiphase Flow Curves

Appendix C.1: Poettmann and Carpenter Method

C.11 DERIVATION OF THE POETTMANN AND CARPENTER EQUATION FOR MULTIPHASE VERTICAL FLOW

Starting with the general energy equation:

$$\int_{p_1}^{p_2} V\,dp + \Delta h \frac{g}{g_c} + \Delta\left(\frac{v^2}{2g_c}\right) + w + lw = 0$$

Assume work done by the fluid to be zero and kinetic energy to be negligible and that g/g_c is numerically equal to 1.000, then:

$$\int_{p_1}^{p_2} V\,dp + \Delta h + lw = 0$$

Assume that total losses due to irreversibilities can be correlated by the Fanning equation:

$$lw = \frac{4f(\Delta h)\,\bar{v}^2}{2\,g_c d}$$

where f = correlating energy loss factor.
Then:

$$\int_{p_1}^{p_2} V\,dp + \Delta h + \frac{4f\,\bar{v}^2\,\Delta h}{2\,g_c d} = 0$$

Solving for Δh:

$$\Delta h = \frac{-\int_{p_1}^{p_2} V\,dp}{1 + \frac{4\,\bar{v}^2 f}{2\,g_c d}}$$

By reversing the limits on the $\int_{p_1}^{p_2} V\,dp$ to $\int_{p_2}^{p_1} V\,dp$, eliminate the negative sign.

$$\Delta h = \frac{\int_{p_2}^{p_1} V\,dp}{1 + \frac{4\,\bar{v}^2 f}{2\,g_c d}}$$

By allowing $V = \frac{V_m}{m}$ and $\bar{v} = \frac{q\,\bar{V}_m}{A}$ rewrite the above equation as follows:

$$\Delta h = \frac{\frac{1}{m}\int_{p_2}^{p_1} V_m\,dp}{1 + \frac{4\,f\,q^2\,\bar{V}_m^2}{2\,(A)^2\,g_c d}}$$

By letting $A = \frac{\pi d^2}{4}$ and converting to field units:

$$\Delta h = \frac{\frac{1}{m}\int_{p_2}^{p_1} V_m\,dp}{1 + \frac{f\,q^2\,\bar{V}_m^2}{7.413 \times 10^{10}\,d^5}}$$

Let:
$$\bar{K} = \frac{f q^2\,\bar{V}_m^2}{7.413 \times 10^{10}\,d^5}$$

Since $\bar{V}_m = \frac{m}{\bar{\rho}}$

Then $\bar{K} = \frac{f\,w^2}{7.413 \times 10^{10}\,\bar{\rho}^2\,d^5}$

where $w = qm$ = mass flow rate.
or:

$$\Delta h = \frac{\frac{1}{m}\int_{p_2}^{p_1} V_m\,dp}{1 + \bar{K}}$$

Dividing the above equation into Δp:

$$\frac{\Delta p}{\Delta h} = \frac{\Delta p}{\frac{\frac{1}{m}\int_{p_2}^{p_1} V_m\,dp}{1 + \bar{K}}}$$

$$\frac{\Delta p}{\Delta h} = \frac{\Delta p\,(1 + \bar{K})}{\frac{1}{m}\int_{p_2}^{p_1} V_m\,dp}$$

The above equation can be written as:

$$\frac{\Delta p}{\Delta h} = \frac{p_1 - p_2}{\frac{1}{m}\int V_m\,dp} + \frac{(p_1 - p_2)\,\bar{K}}{\frac{1}{m}\int_{p_2}^{p_1} V_m\,dp}$$

It is noted that:

$$\bar{V}_m = \frac{\int_{p_2}^{p_1} V_m\,dp}{p_1 - p_2}$$

which is guaranteed by the theorem of the mean for integrals.
Thus:

$$\frac{\Delta p}{\Delta h} = \frac{m}{\bar{V}_m} + \frac{m\,\bar{K}}{\bar{V}_m}$$

It is know that $\bar{\rho} = \frac{m}{\bar{V}_m}$

so:

$$\frac{\Delta p}{\Delta h} = \bar{\rho} + \bar{K}\,\bar{\rho}$$

$$\frac{dp}{dh} = \rho + K\,\rho$$

or the entire equation may be written as:

$$\frac{dp}{dh} = \rho + \frac{f\,w^2}{7.413 \times 10^{10}\,\rho\,d^5}$$

Poettmann and Carpenter developed their energy loss factor correlation by knowing $\frac{dp}{dh}$ in psi/ft and solving the above equation for the energy loss factor as follows:

$$f = 7.413 \times 10^{10}\,\rho\,d^5\left[144\frac{dp}{dh} - \rho\right]\Big/ w^2$$

C.12 DETAILED PROCEDURE FOR THE CALCULATION OF A VERTICAL PRESSURE TRAVERSE BY THE POETTMANN AND CARPENTER METHOD

1. The following information must be obtained:
   a. The gas-liquid ratio
   b. The specific gravity of the gas
   c. The formation volume factors at different pressures
   d. The gas in solution at different pressures
   e. The API gravity of the oil
   f. The specific gravity of the water
   g. The flow rates of oil and water
   h. The flowing surface pressure
   i. Temperature (surface and bottom) and temperature gradient
   j. Depth
   k. Tubing size

   All of this information may not be available. For example, it may be necessary to use generalized correlations

such as those of Standing for gas in solution and oil volume factor.

. On a sheet of rectangular graph paper, plot depth on the ordinate vs pressure, starting with the surface flowing tubing pressure at zero depth or with the flowing bottom-hole pressure at total depth. (Optional) A print-out may be obtained on computer.

. Using a basis of one stock tank bbl of oil, determine the mass of oil, gas, and water per stock tank bbl.

$$m = \text{wt of oil} + \text{wt of gas} + \text{wt of water}$$
$$m = (350)(\gamma_o) + (0.0764)(\gamma_g)(R) + (350)(\gamma_w)(WOR)$$

. Determine the total lbs of produced fluid per day by multiplying (3) by the production rate in stock tank bbl of oil per day.

. Starting with the flowing tubing pressure, assume various pressure points down the tubing string. The pressure increments should be near enough to obtain a good curve. If a computer is being used, these increments may be set very close (say 25 psi) without serious time loss. However, if hand calculations are to be used, the small increments will be too time consuming.

. Calculate the volume of the mixture of oil, gas, and water at the particular pressure point in question per bbl of stock tank oil (units, cu ft). Start with the surface pressure.

Total volume = vol of oil + vol of water + vol of gas

$$V_m = 5.61\ B_o + 5.61\ (\text{water-oil ratio}) + (\text{vol free gas})\left(\frac{14.7}{p}\right)\left(\frac{T}{520}\right)\left(\frac{Z}{1}\right)$$

The volume of free gas is determined by subtracting gas in solution at the particular pressure and temperature in question from the GOR.

Determine the density at this pressure point where $\rho = \frac{m}{V_m}$.

From Fig. 2.37 select the flowing pressure gradient for the particular mass flow rate and density.

. Once the flowing pressure gradient has been established for the tubing pressure, select the next pressure increment down the tubing string and repeat the procedure. Determine a pressure gradient for the pressure selected.

. From the two flowing gradients, determine an average flowing gradient between the two.

$$\frac{\left(\frac{\Delta p}{\Delta h}\right)_1 + \left(\frac{\Delta p}{\Delta h}\right)_2}{2} = \left(\frac{\Delta p}{\Delta h}\right)_{avg}$$

From the difference in pressure between these two points, determine distance between these two pressure points.

$$\frac{P_1 - P_2}{\left(\frac{\Delta p}{\Delta h}\right)_{avg}} = \frac{\Delta P}{\left(\frac{\Delta p}{\Delta h}\right)_{avg}} = \Delta h \text{ in feet}$$

This distance is then plotted down from top.

The same preceding series of steps is repeated from the next increased pressure point down the hole. It should be noted that the mass rate of flow remains the same for all pressure points. However, the flowing density changes because of the gas being compressed and gas going into solution.

Using the average flowing gradient between the point of pressure just determined and the one immediately preceding it, another distance is calculated. This point is then plotted on the graph.

. Continue this operation until the depth of the well is reached.

15. The flowing bottom-hole pressure will be the pressure read at the total depth of the well.

16. The reverse of this procedure may be utilized by starting from bottom with a known flowing bottom-hole pressure, and extending the curve upward to predict whether or not the well will flow and, if so, with what flowing tubing pressure.

If a computer is available, the procedure is changed somewhat. The pressure traverse is calculated from the equation:

$$\frac{dp}{dh} = \frac{1}{144}\left[\rho + \frac{f\,(w)^2}{7.413 \times 10^{10}\ \rho\ d^5}\right] \qquad \text{C.12}$$

It is still necessary to assume pressure points and calculate a density of the mixture at each pressure point. However, instead of going to Fig. 2.37 to select a pressure gradient in psi/ft, the gradient is calculated directly from the above equation. In noting the equation, it is seen that an f factor must be determined. It is quite easy to fit the curve of Fig. 2.36 with a least squares fit and then have the computer select f for each set of conditions. Once f has been selected and ρ determined, the gradient is calculated directly. This same procedure is repeated for the next pressure point. As in the outlined procedure, an average gradient is then determined for the two pressure points and the distance between them.

The above equation may be used in finite difference form whereby:

$$\frac{\Delta p}{\Delta h} = \frac{1}{144}\left(\bar{\rho} + \frac{f\,(w)^2}{7.413 \times 10^{10}\ \bar{\rho}\ d^5}\right) \qquad \text{C.13}$$

An average pressure and temperature between the two pressure points may first be calculated. The average density is calculated and then $\frac{\Delta p}{\Delta h}$ (average) is determined. For very close pressure increments, the results will be the same.

A pressure traverse may also be calculated by a direct method which involves either graphical or numerical integration.

The length of pipe can be defined as the integral of dh. The variable of integration may be changed to dp as follows:

$$\Delta h = \int_{h_1}^{h_2} dh = \int_{p_1}^{p_2} \left(\frac{dh}{dp}\right) dp = \int_{p_1}^{p_2} \frac{dp}{(dp/dh)}$$

From this integral it can be seen that the area under a curve generated by plotting p vs. $\frac{1}{(dp/dh)}$ is equal to Δh.

In using this method, dp/dh is calculated at the wellhead pressure and at several other pressures using Eq. C.12. The reciprocal of dp/dh is then plotted vs. pressure and integrated either graphically or numerically until the total area under the curve is equal to the length of the tubing. No trial and error is involved in this procedure.

C.13 EXAMPLE PROBLEM WORKED BY THE POETTMANN AND CARPENTER METHOD

Given:

$d = 1.995$ in. or 1.380 in.
$p_1 = 500$ psig
$p_2 = 1{,}000$ psig
$p_a = 14.7$ psia
$T_1 = 120°F$
$T_2 = 150°F$
$\gamma_g = 0.65$
$\gamma_w = 1.07$
$\rho_o = 22°API$
$q_o = 400$ stk bpd
$q_w = 600$ stk bpd
$\mu_g = 0.018$ cp (assume constant)
$\sigma_o = 30$ dynes/cm (assume constant)

$\sigma_w = 70$ dynes/cm (assume constant)
Gas-liquid ratio = 500 scf/stk bbl

A certain number of factors must be determined prior to starting these calculations.

$$\gamma_o = \frac{141.5}{131.5 + 22} = 0.922$$

$$\text{GOR} = \frac{600 + 400}{400}(500) = 1{,}250 \text{ cu ft/bbl}$$

2. No plot is made in this example.
3. Determine the mass per stk bbl of oil.

$$\begin{aligned} m &= \text{wt of oil} + \text{wt of gas} + \text{wt of water} \\ m &= (350)(0.922) + (0.0764)(1{,}250)(0.65) \\ &\quad + (350)(1.07)(1.5) \\ &= 323 + 62 + 562 \\ &= 947 \text{ lb}_m\text{/stk bbl of oil} \end{aligned}$$

4. The total mass flow rate per day (w) is (400)(947) = 378,800 lb_m/day.
5. Start calculations with the 500 psig pressure point.
6. Calculate the volume of oil, gas and water at 500 psig.

$B_o = 1.043 \qquad R_s = 59$ scf/bbl (Figs. 218 and 2.19)

$$\begin{aligned} \text{vol total} &= \text{vol oil} + \text{vol water} + \text{vol gas} \\ &= (5.61)(1.043) + (5.61)(1.5) \\ &\quad + (1250 - 59)\left(\frac{14.7}{514.7}\right)\left(\frac{580}{520}\right)(0.935) \\ &= 5.88 + 8.43 + 35.69 \\ &= 50 \text{ cu ft} \end{aligned}$$

7. Density at 500 psig

$$\rho_m = \frac{947}{50} = 18.96 \text{ lb}_m \text{ cu ft}$$

8. Flowing pressure gradient = 0.176 psi/ft at 500 psig (Refer to Fig. 2.37).
9. Determine pressure gradient at 1,000 psig.

$$\begin{aligned} B_o &= 1.083 \\ R_s &= 120 \text{ scf/bbl} \\ z &= 0.90 \end{aligned}$$

$$\begin{aligned} V &= (1.083)(5.61) + (1.5)(5.61) \\ &\quad + (1130)\left(\frac{14.7}{1014.7}\right)\left(\frac{610}{520}\right)\left(\frac{0.90}{1}\right) = 31.8 \text{ cu ft} \end{aligned}$$

$$\rho \text{ @ } 1{,}000 = \frac{947}{31.8} = 29.8 \text{ lb}_m\text{/cu ft}$$

Flowing pressure gradient = 0.236 psi/ft at 1,000 psig (Refer to Fig. 2.37).

10. Average pressure gradient $= \dfrac{0.176 + 0.236}{2} = 0.206$ psi/ft
11. The distance between 500 and 1000 psig $= \dfrac{1{,}000 - 500}{0.206} =$ 2,425 ft

This same problem can be worked by obtaining a friction factor and solving for dp/dh by Eq. C.12.
As noted in Fig. 2.36:

$$d\,v\,\rho = \frac{1.4737 \times 10^{-5}\, q\, m}{d} = \frac{(1.4737 \times 10^{-5})(378{,}800)}{0.1662} = 33.6$$

From Fig. 2.36, we find f = 0.0080.

$$\frac{dp}{dh} = \frac{1}{144}\left[\rho + \left(\frac{f\,w^2}{7.413 \times 10^{10}\,\rho d^5}\right)\right]$$

$$\left(\frac{dp}{dh}\right)_{500} = \frac{1}{144}\left[18.96 + \frac{(0.008)(378{,}800)^2}{7.413 \times 10^{10}\,(18.96)(0.1662)^5}\right] = 0.177 \text{ psi/ft}$$

$$\left(\frac{dp}{dh}\right)_{1{,}000} = \frac{1}{144}\left[29.8 + \frac{(0.008)(378{,}800)^2}{7.413 \times 10^{10}\,(29.8)(0.1662)^5}\right] = 0.235 \text{ psi/ft}$$

The average gradient is $\dfrac{0.177 + 0.235}{2} = 0.206$ psi/ft

This calculation is simplified by first obtaining the average density:

$$\frac{18.96 + 29.8}{2} = 24.38 \text{ lb}_m\text{/cu ft}$$

Then: $$\frac{dp}{dh} = \frac{1}{144}\left[24.38 + \frac{(0.008)(378{,}800)^2}{7.413 \times 10^{10}\,(24.38)(0.1662)^5}\right] = 0.205 \text{ psi/ft}$$

Also for the average density of 24.38 lb_m/cu ft, the average gradient may be looked up on Fig. 2.37 and found to be 0.204 psi/ft.
Again, the average gradient is 0.206 psi/ft, giving a distance of 2,425 ft between 500 and 1,000 psig.

Based on density alone, the pressure gradients are 0.132 psi/ft at 500 psi and 0.207 psi/ft at 1,000 psi. This is an average of 0.1695 psi/ft, giving a distance of $\dfrac{500}{0.1695} = 2{,}950$ ft.

For 1¼ in. tubing (1.380 ID):

$$d\,v\,\rho = \frac{(1.4737 \times 10^{-5})(378{,}800)}{0.1150} = 48.60$$

From Fig. 2.36 f = 0.0046.

$$\frac{dp}{dh} \text{ at 500 psig} = 0.293 \text{ psi/ft}$$

$$\frac{dp}{dh} \text{ at 1,000 psig} = 0.310 \text{ psi/ft}$$

$$\text{Avg } \frac{dp}{dh} = 0.302 \text{ psi ft}$$

$$\text{Distance} = \frac{500}{0.302} = 1{,}650 \text{ ft}$$

This problem may also be worked using the direct method. The gradients at 500 psi and 1,000 psi are calculated as before to be 0.177 psi/ft and 0.235 psi/ft respectively for 2".

Calculate $\left(\dfrac{1}{\dfrac{dp}{dh}}\right)_{500} = \dfrac{1}{0.177} = 5.65$

Calculate $\left(\dfrac{1}{\dfrac{dp}{dh}}\right)_{1{,}000} = \dfrac{1}{0.235} = 4.26$

On a graph of P vs $\dfrac{1}{\dfrac{dp}{dh}}$ plot 5.65 vs 500 and 4.26 vs 1,000.

Graphical integration between P = 500 and P = 1,000 gives $\Delta h = 2{,}480$ ft.

Appendix C.2: The Fancher and Brown method

STEPWISE PROCEDURE FOR THE CALCULATION OF A PRESSURE TRAVERSE BY THE FANCHER AND BROWN METHOD

Determine the mass, m, in lb_m per stk bbl of oil.
Determine w, the total mass flow rate per day.
Select the first pressure point (p_1) to start the calculation. This may be from the surface downwards or from total depth upwards.
Calculate the volume, V, of oil, gas, and water associated with one stk bbl of oil at the pressure selected in 3. (Assume no slippage)

Determine the density at p_1 where $\rho_m = \frac{m}{V}$ = step 1/step 4.

Calculate the abscissa of Fig. 2.41 which is $dv\rho = \frac{1.4737 \times 10^{-5}\ w}{d}$

Select an "f" factor from Fig. 2.41 accounting for the correct gas-liquid ratio. (Interpolation should be performed)

Calculate $\frac{dp}{dh}$ from

$$\frac{dp}{dh} = \frac{1}{144}\left[\rho + f\left(\frac{w^2}{7.413 \times 10^{10}\ \rho\ d^5}\right)\right]$$

Choose the next pressure point (p_2). For calculating a pressure traverse from 0 psig at the surface to higher pressures, take 10 psi increments until reaching 100 psi, then 25 psi increments to 500 psig; 100 psig increments between 500 and 1,000 psig and 200 psig from there on.
Calculate the volume of oil, gas, and water associated with one stk bbl of oil at this pressure (p_2).
Determine the density at p_2.

Using the same "f" value as step 7, calculate $\frac{dp}{dh}$ at p_2 from the equation in Step 8.

Average $\frac{dp}{dh}$ at p_1 and p_2.

From the difference in pressure, that is, Δp, determine the distance between these two points, by dividing Δp by the average $\frac{dp}{dh}$ of Step 13.

EXAMPLE PROBLEM WORKED BY THE FANCHER AND BROWN METHOD FOR 2⅞ IN. OD TUBING (SAME DATA AS GIVEN FOR EX. C.13)

Determine the mass per stk bbl of oil

$$\begin{aligned} m &= \text{wt oil} + \text{wt water} + \text{wt gas} \\ &= (350)(0.922) + (350)(1.07)(1.5) \\ &\quad + (0.0764)(1250)(0.65) \\ &= 323 + 562 + 62 = 947\ lb_m/\text{stk bbl of oil} \end{aligned}$$

Total mass flow rate per day

$$w = (400)(947) = 378{,}800\ lb_m/\text{day}.$$

Select the first pressure point to be 500 psig.
Calculate the volume of oil, gas, and water at 500 psia.

$B_o = 1.043$, $R_s = 59$ scf/bbl

$$\begin{aligned} \text{vol total} &= \text{vol oil} + \text{vol water} + \text{vol gas} \\ V &= (5.61)(1.043) + (5.61)(1.5) \\ &\quad + (1250 - 59)\left(\frac{14.7}{514.7}\right)\left(\frac{580}{520}\right)\left(\frac{0.935}{1}\right) \\ V &= 5.88 + 8.43 + 35.69 = 50\ \text{cu ft} \end{aligned}$$

5. Density at p_1, $\rho_m = \frac{947}{50} = 18.96\ lb_m/\text{cu ft}$

6. Calculate the abscissa of Fig. 2.41.

$$\begin{aligned} dv\rho_m &= \frac{1.4737 \times 10^{-5}\ w}{d} \\ &= \frac{(1.4737)(10^{-5})(378{,}800)}{0.1661} \\ &= 33.60 \end{aligned}$$

7. From Fig. 2.41, $f = 0.0095$

8. Calculate $\frac{dp}{dh}$

$$\begin{aligned} \frac{dp}{dh} &= \frac{1}{144}\left[\rho_m + f\left(\frac{w^2}{7.413 \times 10^{10}\ \rho_m\ d^5}\right)\right] \\ &= \frac{1}{144}\left[18.96 + \frac{(0.0095)(378{,}800)^2}{(7.413 \times 10^{10})(18.96)(0.1661)^5}\right] \\ &= \frac{1}{144}\left[18.96 + 7.65\right] = \frac{1}{144}\left[26.61\right] \\ &= 0.185\ \text{psi/ft} \end{aligned}$$

9. Choose p_2 for this example to be 1,000 psig.
10. Calculate vol of oil, gas, and water at 1,000 psia

$B_o = 1.083$
$R_s = 120$ scf/bbl
$z = 0.90$

$$\begin{aligned} V &= (1.083)(5.61) + (1.5)(5.61) \\ &\quad + (1130)\left(\frac{14.7}{1014.7}\right)\left(\frac{610}{520}\right)\left(\frac{0.90}{1}\right) \\ &= 31.8\ \text{cu ft} \end{aligned}$$

11. $\rho_m(p_2) = \frac{947}{31.8} = 29.8\ lb_m/\text{cu ft}$

12. Calculate $\frac{dp}{dh}$ at p_2

$$\begin{aligned} \frac{dp}{dh} &= \frac{1}{144}\left[29.8 + \frac{(0.0095)(378{,}800)^2}{7.413 \times 10^{10}(29.8)(0.1661)^5}\right) \\ &= \frac{1}{144}\left[29.8 + 4.90\right] = 0.2406\ \text{psi/ft} \end{aligned}$$

13. Average $\frac{dp}{dh}$ = between p_1 and p_2

$$\frac{\Delta p}{\Delta h_{(avg)}} = \frac{0.185 + 0.2406}{2} = 0.2128$$

14. Distance between 1,000 and 500 psi pressure points

$$= \frac{1{,}000 - 500}{0.2128} = 2{,}350\ \text{ft}$$

Appendix C.3: First method of Hagedorn and Brown (to account for viscous effects) (for 1¼ in. tubing only)

C.31 STEPWISE PROCEDURE FOR THE CALCULATION OF A PRESSURE TRAVERSE BY THE FIRST METHOD OF HAGEDORN AND BROWN (USE ONLY FOR 1¼ IN. TUBING)

1. Determine the mass, m, in lb_m per stk bbl of oil.
2. Determine the total mass flow rate, w, in lb_m per day.
3. Select the first pressure point (p_1).
4. Calculate the volume of oil, gas, and water (V) associated with one stk bbl of oil at p_1 (assume no slippage).
5. Determine the density at p_1 where $\rho_m = \frac{m}{V}$ = Step 1/ Step 4.
6. Calculate the abscissa of Fig. 2.54 or 2.55 depending upon whether the viscosity is less or greater than 12 cp.
7. Select an "f" factor from Fig. 2.54 or 2.55.
8. Calculate $\frac{dp}{dh}$ from the equation:

$$\frac{dp}{dh} = \frac{1}{144}\left[\rho_m + f\left(\frac{w^2}{7.413 \times 10^{10}\ \rho_m\ d^5}\right)\right]$$

9. Choose the next pressure point (p_2).
10. Calculate the volume of oil, gas, and water associated with one stk bbl of oil at this pressure (p_2).
11. Determine the density at p_2.
12. Using the same "f" value as Step 7, calculate dp/dh at p_2 from the equation in Step 8.
13. Average $\frac{dp}{dh}$ at p_1 and p_2.
14. From the difference in pressure, that is, Δp, determine the distance between these two points by dividing Δp by the average $\frac{dp}{dh}$ of Step 13.

This procedure may be simplified somewhat by changing Step 3 to the determination of the average pressure between p_1 and p_2. The volume of the mixture of oil, gas, and water is then calculated at p (avg) and $\frac{dp}{dh}$ calculated from the equation

$$\frac{\Delta p}{\Delta h} = \frac{1}{144}\left[\rho_m + f\left(\frac{w^2}{7.413 \times 10^{10}\ \rho_m\ d^5}\right)\right]$$

C.32 EXAMPLE PROBLEM WORKED BY FIRST METHOD OF HAGEDORN AND BROWN (RECOMMENDED FOR 1¼ IN. TUBING ONLY)

Given the same data as in Ex. C.13 (except for 1¼ in. tubing).

1. Determine the mass per stk bbl of oil.

$$m = \text{wt oil} + \text{wt water} + \text{wt gas}$$
$$= (350)(0.922) + (350)(1.07)(1.5) + (0.0764)(1{,}250)(0.65)$$
$$m = 323 + 562 + 62 = 947\ lb_m/\text{stk bbl of oil}$$

2. Total mass flow rate per day, $w = (400)(947) = 378{,}800\ lb_m/\text{day}$.
3. Select the first pressure point to be 500 psig.
4. Calculate the volume of oil, gas, and water at 500 psia.

$$B_o = 1.043;\ R_s = 59\ \text{scf/bbl}$$
$$\text{total vol} = \text{vol oil} + \text{vol water} + \text{vol gas}$$
$$V_m = (5.61)(1.043) + (5.61)(1.5) + (1250 - 59)\left(\frac{14.7}{514.7}\right)\left(\frac{580}{520}\right)\left(\frac{0.935}{1}\right)$$
$$V_m = 5.88 + 8.43 + 35.69 = 50\ \text{cu ft/stk bbl of oil}$$

5. Density at p_1

$$\rho_m = \frac{947}{50} = 18.96\ lb_m/\text{cu ft}$$

6. Calculate the abscissa of Fig. 2.54 where:

$$N_{Re} = \left[\frac{1.10\ w}{d} + 280\ G/L\right] \times 10^{-5}$$
$$= \left[\frac{(1.10)(378{,}800)}{0.115} + 280\ (500)\right] \times 10^{-5}$$
$$= 37.65$$

7. Select an "f" value of 0.00710 from Fig. 2.54. (Viscosity less than 12 cp.)
8. Calculate $\frac{dp}{dh}$ from the following equation:

$$\frac{dp}{dh} = \frac{1}{144}\left[\rho_m + f\left(\frac{w^2}{7.413 \times 10^{10}\ \rho_m\ d^5}\right)\right]$$
$$\frac{dp}{dh} = \frac{1}{144}\left[18.96 + \frac{(0.0071)(378{,}800)^2}{(7.413 \times 10^{10}\ (18.96)(0.115)^5}\right]$$
$$= 0.381\ \text{psi/ft}$$

9. Choose $p_2 = 1{,}000$ psig.
10. Calculate V_m at 1,000 psig

$$B_o = 1.083$$
$$R_s = 120\ \text{scf/bbl}$$
$$z = 0.90$$
$$V_m = (1.083)(5.61) + (1.5)(5.61) + (1130)\left(\frac{14.7}{1014.7}\right)\left(\frac{610}{520}\right)\left(\frac{0.90}{1}\right)$$
$$= 31.8\ \text{cu ft}$$

11. $\rho_m = \frac{947}{31.8} = 29.8\ lb_m/\text{cu ft}$

12. Calculate $\frac{dp}{dh}$ at 1,000 psig.

$$\frac{dp}{dh} = \frac{1}{144}\left[29.8 + \frac{(.0071)(378{,}800)^2}{7.413 \times 10^{10}(29.8)(0.115)^5}\right]$$
$$= 0.366\ \text{psi/ft}$$

13. Average $\frac{dp}{dh} = \frac{0.381 + 0.366}{2} = 0.374$ psi/ft.

14. $\Delta H = \frac{1{,}000 - 500}{0.374} = 1{,}335$ ft

Although this correlation is not recommended for 2 in. tubing, the calculation procedure would be the same and gives a value of 2,050 ft, which is about 300 ft off from both Poettmann and Carpenter and Fancher and Brown correlations.

Appendix C.4: Generalized correlation of Hagedorn and Brown

C.41 DERIVATION OF HAGEDORN AND BROWN'S EQUATION FOR MULTIPHASE VERTICAL FLOW

Starting with the basic flow equation in symbolic differential form based on one pound mass of flowing fluid:

$$144\frac{g_c}{g}V\,dp + dh + \frac{vdv}{g} + dW_f + dW_e = 0 \qquad C.41$$

This equation assumes steady-state flow and its units are in feet. Hagedorn and Brown chose to treat the mixture of gas and liquid as a homogeneous mixture of combined properties.

Assuming that no work is done by the flowing fluid, the equation becomes:

$$144\frac{g_c}{g}V\,dp + dh + \frac{vdv}{g} + dW_f = 0 \qquad C.42$$

Assuming that the frictional loss can be defined by the Darcy-Weisbach equation:

$$dW_f = \frac{f\,dh\,\bar{v}_m^2}{d2g} \qquad C.43$$

The following equation for the mixture results:

$$144\frac{g_c}{g}V\,dp + dh + \frac{v_m d\,v_m}{g} + \frac{f\,\bar{v}_m^2\,dh}{2\,g\,d} = 0 \qquad C.44$$

where $\bar{v}_m$ is an average velocity of the mixture.

Writing this equation in integral form to solve:

$$144\frac{g_c}{g}\int_{p_1}^{p_2}V\,dp + \int_{h_1}^{h_2}dh + \int_{v_{m_1}}^{v_{m_2}}\frac{v_m d\,v_m}{g} + \int_{h_1}^{h_2}\frac{f\,\bar{v}_m^2\,dh}{2\,gd} = 0 \qquad C.45$$

Evaluating three of the terms:

$$144\frac{g_c}{g}\int_{p_1}^{p_2}V\,dp + (h_2 - h_1) + \frac{(v_{m_2}^2 - v_m^2)}{2\,g} + \frac{f\,\bar{v}_m^2\,(h_2 - h_1)}{2\,gd} = 0 \qquad C.46$$

Solving Eq. C.46 for $h_2 - h_1$, we have:

$$h_2 - h_1 = \frac{144\frac{g_c}{g}\int_{p_2}^{p_1}V\,dp - \frac{v_{m_2}^2 - v_{m_1}^2}{2\,g}}{1 + \frac{f\,\bar{v}_m^2}{2\,g\,d}} \qquad C.47$$

Similar to the method of Poettmann and Carpenter, $\bar{V}_m$ can be approximated between p_1 and p_2 by the equation:

$$\bar{V}_m = \frac{\int_{p_2}^{p_1}V\,dp}{p_1 - p_2} \qquad C.48$$

Substituting $V_m\,(p_1 - p_2)$ for $\int_{p_2}^{p_1}V\,dp$ in Eq. C.47

$$h_2 - h_1 = \frac{144(p_1 - p_2)\,\bar{V}_m\frac{g_c}{g} - \frac{(v_{m_2}^2 - v_{m_1}^2)}{2\,g}}{1 + \frac{f\,v_m^2}{2\,gd}} \qquad C.49$$

$$\text{or } \Delta h = \frac{144\,\Delta p\,\bar{V}_m\frac{g_c}{g} - \Delta\left(\frac{v_m^2}{2\,g}\right)}{1 + \frac{f\,\bar{v}_m^2}{2\,gd}} \qquad C.410$$

The average velocity $\bar{v}_m$ is as follows:

$$\bar{v}_m = \bar{v}_{sL} + \bar{v}_{sg} \qquad C.411$$

Also the velocity of the mixture is:

$$v_m = v_{sL} + v_{sg} \qquad C.412$$

Making these substitutions in Eq. C.49,

$$\Delta h = \frac{144\,\Delta p\,\bar{V}_m\frac{g_c}{g} - \Delta\left[\frac{(v_{sL} + v_{sg})^2}{2g}\right]}{1 + \frac{f(\bar{v}_{sL} + \bar{v}_{sg})^2}{2\,g_c d}}$$

The average integrated density of the mixture between points 1 and 2 is:

$$\bar{\rho}_m = \frac{1}{\bar{V}_m} \qquad C.414$$

Substituting into Eq. C.413

$$\Delta h = \frac{\frac{144\,\Delta p\,g_c}{\bar{\rho}_m g} - \Delta\left[\frac{(v_{sL} + v_{sg})^2}{2\,g}\right]}{1 + \frac{f(\bar{v}_{sL} + \bar{v}_{sg})^2}{2\,g_c d}} \qquad C.415$$

Solving for the average pressure gradient between pressure points (1) and (2):

$$144\frac{\Delta p}{\Delta h} = \frac{g}{g_c}\bar{\rho}_m + \frac{f\,\bar{\rho}_m\,(\bar{v}_{sL} + \bar{v}_{sg})^2}{2\,g_c d} + \frac{\bar{\rho}_m\,\Delta\,[(v_{sL} + v_{sg})^2]}{\Delta h} \qquad C.416$$

This leaves the units of each term in lb_f/sq ft − ft with the gradient $\frac{\Delta p}{\Delta h}$ having units of lb_f/sq in − ft.

If g is assumed to be numerically equal to g_c and the conversion to units commonly used in the field is made:

$$144\frac{\Delta p}{\Delta h} = \frac{f\,w^2}{2.9652\times10^{11}\,d^5\,\bar{\rho}_m} + \bar{\rho}_m + \bar{\rho}_m\frac{\Delta\left[\frac{(5.61\,q + q_g)^2}{2.9652\times10^{11}\,d^4}\right]}{\Delta h} \qquad C.417$$

Based on one bbl of liquid, the mass is:

$$m = \frac{1}{1 + WOR}(\gamma_o)(5.61\times62.4) + (0.0764)(\gamma_g)(GLR) + \left(\frac{WOR}{1 + WOR}\right)(\gamma_w)(5.61\times62.4) \qquad C.418$$

and w = qm = mass flow rate.

In this procedure we need to determine the average integrated density, $\bar{\rho}_m$. It cannot be calculated directly because of the slippage or fallback of the liquid phase.

If V_e is the volume of a section of pipe, then $H_L V_e$ is that portion occupied by liquid and $(1 - H_L)\,V_e$ is that portion occupied by gas.

$$\text{And: } \bar{\rho}_m = \frac{\gamma_L\rho_w H_L V_e + \gamma_g\rho_{air}\left(\frac{\bar{p}}{14.7}\frac{520}{\bar{T}}\frac{1}{\bar{Z}}\right)V_e\,(1 - H_L)}{V_e} \qquad C.419$$

$$\text{or } \bar{\rho}_m = \bar{\rho}_L H_L + \bar{\rho}_g(1 - H_L) \qquad C.420$$

By substituting Eq. C.420 into Eq. C.416:

$$144\frac{\Delta p}{\Delta h} = \frac{g}{g_c}\left[\bar{\rho}_L H_L + \bar{\rho}_g(1 - H_L)\right]\left\{1 + \frac{f(\bar{v}_{sL} + \bar{v}_{sg})^2}{2\,g_c d} + \Delta\frac{\left[\frac{(v_{sL} + v_{sg})^2}{2g}\right]}{\Delta h}\right\} \qquad C.421$$

This equation will reduce to single phase liquid flow by taking the limit of both sides as $H_L \to 1$, $\bar{v}_{sg} \to 0$, $v_{sg} \to 0$, i.e., the gas rate becomes zero. It reduces to:

$$144\frac{\Delta p}{\Delta h}=\frac{g}{g_c}\bar{\rho}_L+\frac{f\bar{\rho}_L\bar{v}_L^2}{2\,g_c d}+\bar{\rho}_L\frac{\Delta\left(\frac{v_g^2}{2\,g_c}\right)}{\Delta h} \qquad C.422$$

Eq. C.421 will reduce to an equation describing single phase gas flow if the limit of both sides is taken as $H_L \rightarrow 0$, $v_{sL} \rightarrow 0$, $v_{sL} \rightarrow 0$, i.e., the liquid rate becomes zero.

It reduces to:

$$144\frac{\Delta p}{\Delta h}=\frac{g}{g_c}(\bar{\rho}_g)+\frac{f\bar{\rho}_g v_g^2}{2\,g_c d}+\bar{\rho}_g\frac{\Delta\left(\frac{v_g^2}{2\,g_c}\right)}{\Delta h} \qquad C.423$$

C.42 DETAILED PROCEDURE FOR CALCULATING A VERTICAL PRESSURE TRAVERSE BY THE METHOD OF HAGEDORN AND BROWN

Starting with the equation:

$$144\frac{\Delta p}{\Delta h}=\bar{\rho}_m+\frac{f\,w^2}{2.9652\times10^{11}\,d^5\,\bar{\rho}_m}+\bar{\rho}_m\frac{\Delta\left(\frac{v_m^2}{2\,g_c}\right)}{\Delta h}$$

Solving for the depth increment, Δh

$$\Delta h=\frac{144\,\Delta p-\bar{\rho}_m\,\Delta\left(\frac{v_m^2}{2g_c}\right)}{\bar{\rho}_m+\frac{f\,w^2}{2.9652\times10^{11}d^5\bar{\rho}_m}}$$

Start with a known pressure p_1, assume a value for p_2 and calculate the depth increment.

1. Calculate the average pressure between the two pressure points, psia

$$\bar{p}=\frac{p_1+p_2}{2}+14.7$$

Depending upon the requirements of the problem, i.e., whether or not a flowing bottom-hole pressure is to be determined from surface information, or whether the calculations are to start from total depth and come up the pipe, the starting pressure must be known. Pressure increments or decrements must then be assumed from which the distance between pressure points (1) and (2) will be calculated.

As a word of precaution, if starting from the surface with a pressure lower than 100 psi, increments of 25 psi should be taken until reaching 400 psi. This type of calculation is practically forbidden by long hand but lends itself readily to machine computation. If starting from bottom with pressures in excess of 1,000 psi, the pressure decrements may be as great as 200 psi.

2. Calculate the specific gravity of the oil γ_o

$$\gamma_o=\frac{141.5}{131.5+°API}$$

3. Find total mass associated with one bbl of stock tank liquid

$$m=\gamma_o(350)\left(\frac{1}{1+WOR}\right)+\gamma_w(350)\left(\frac{WOR}{1+WOR}\right)+(0.0764)(GLR)\,\gamma_g$$

4. Calculate the mass flow rate $w = qm$
5. From Fig. 2.19 obtain R_s at $\bar{p}$ and $\bar{T}$.
6. Calculate the density of liquid phase:

$$\rho_L=\left[\frac{\gamma_o(62.4)+R_s\,\gamma_g(0.0764)/5.614}{B_o}\right]\left(\frac{1}{1+WOR}\right)+\left[\gamma_w(62.4)\left(\frac{WOR}{1+WOR}\right)\right]$$

7. Assuming $\bar{T}$ = constant, find a value of $\bar{Z}$ for a constant $\bar{T}$, $\bar{p}$ and γ_g. (Fig. 2.17) If $\bar{T}$ is to be a variable, then a single trial and error solution develops. Although the temperature gradient may be known, the depth at which the pressure increment occurs is not known and, therefore, the temperature at the next pressure point is not known.
8. Calculate the average density of the gas phase

$$\bar{\rho}_g=\gamma_g(0.0764)\left(\frac{\bar{p}}{14.7}\right)\left(\frac{520}{\bar{T}}\right)\left(\frac{1}{\bar{Z}}\right)$$

9. Calculate the average viscosity of the oil from appropriate correlations. As noted, a knowledge of the fluid properties of the oil, $\bar{p}$, and/or $\bar{T}$ is required.
10. Determine the average water viscosity from Fig. 2.9.
11. Calculate the liquid mixture viscosity:

$$\mu_L=\mu_o\left(\frac{1}{1+WOR}\right)+\mu_w\left(\frac{WOR}{1+WOR}\right)$$

This can only be an approximation since the viscosity of two immiscible liquids is quite complex.

12. Assuming constant surface tensions at each pressure point, calculate the liquid mixture surface tension.

$$\sigma_L=\sigma_o\left(\frac{1}{1+WOR}\right)+\sigma_w\left(\frac{WOR}{1+WOR}\right)$$

Again, this represents only an approximation of the surface tension of the liquid phase.

13. Calculate the liquid viscosity number:

$$N_L=0.15726\,\mu_L\left(\frac{1}{\rho_L\,\sigma_L^3}\right)^{1/4}$$

14. From Fig. 2.56, determine CN_L.
15. Calculate area of tubing, A_P.

$$A_P=\frac{\pi\,d^2}{4}$$

16. From Fig. 2.18, obtain β_o at $\bar{p}$, $\bar{T}$
17. Assuming $B_w = 1.0$, calculate the superficial liquid velocity v_{sL}, ft/sec:

$$v_{sL}=\frac{5.61\,q_L}{86{,}400\,A_P}\left\{B_o\left(\frac{1}{1+WOR}\right)+B_w\left(\frac{WOR}{1+WOR}\right)\right\}$$

18. Calculate the liquid velocity number, N_{LV}:

$$N_{LV}=1.938\,v_{sL}\left(\frac{\rho_L}{\sigma_L}\right)^{1/4}$$

19. Calculate the superficial gas velocity, v_{sg}:

$$v_{sg}=\frac{q_L\left\{GLR-R_s\left(\frac{1}{1+WOR}\right)\right\}}{86{,}400\,A_P}\left(\frac{14.7}{\bar{p}}\right)\left(\frac{\bar{T}}{520}\right)\left(\frac{\bar{Z}}{1}\right)$$

20. Determine the gas velocity number, N_{Gv}:

$$N_{Gv}=1.938\,v_{sg}\left(\frac{\rho_L}{\sigma_L}\right)^{1/4}$$

21. Check the flow regime to determine whether to continue with the Hagedorn and Brown correlation or proceed to the Griffith correlation for bubble flow.

A number "A" must be calculated by the following formula:

$$A=1.071-[0.2218\,(v_{sL}+v_{sg})^2]/d$$

If $A \geq 0.13$, then use the calculated value and if less than 0.13, use $A = 0.13$.

Another number B is calculated by the following formula:

$$B=\frac{v_{sg}}{v_{sL}+v_{sg}}$$

If (B-A) is positive or has a value of zero, continue with the Hagedorn and Brown correlation. If (B-A) is negative, proceed with the Griffith correlation. This detailed procedure is explained in Appendix C.6 under the Orkiszewski method.

Note: This procedure is followed through each pressure increment. It is possible that the calculation procedure will change from one regime to another in any one pressure traverse.

22. Find the pipe diameter number, N_d:

$$N_d = 120.872\ d \sqrt{\frac{\rho_L}{\sigma_L}}$$

23. Calculate the holdup correlating function ϕ:

$$\phi = \left(\frac{N_{Lv}}{N_{gv}^{0.575}}\right)\left(\frac{\bar{p}}{14.7}\right)^{0.10}\left(\frac{CN_L}{N_d}\right)$$

24. Obtain $\frac{H_L}{\psi}$ from Fig. 2.34.

25. Determine the secondary correction factor correlating parameter, ϕ:

$$\phi = \left(\frac{N_{gv}\ N_L^{0.380}}{N_d^{2.14}}\right)$$

26. Obtain ψ from Fig. 2.57.

27. Calculate a value for H_L.

$$H_L = \left(\frac{H_L}{\psi}\right)(\psi)$$

For low viscosities there will be no correction and $\psi = 1.00$.

28. In order to obtain a friction factor, determine a value for the two-phase Reynolds number, $(N_{Re})_{TP}$:

$$(N_{Re})_{TP} = \frac{2.2 \times 10^{-2} w}{(d)(\mu_L^{H_L})(\mu_g^{(1-H_L)})}$$

29. Determine a value for ϵ/d.
If the value for ϵ is not known, a good value to use is 0.00015 ft which is an average value given for commercial steel.

30. Obtain the friction factor from Fig. 2.35.

31. Calculate the average two-phase density of the mixtures $\bar{\rho}_m$ by two methods.
(a) Using the value of H_L from Step 27, calculate $\bar{\rho}_m$ as follows:

$$\bar{\rho}_m = \bar{\rho}_L H_L + \bar{\rho}_g (1 - H_L)$$

(b) Calculate a value of $\bar{\rho}_m$ assuming no slippage. This is identical to the calculation procedure found in Steps 1 through 7 of Appendix C.12 except that the average conditions of pressure and temperature between pressure points is used.

Compare the two values of $\bar{\rho}_m$ from (a) and (b) and use whichever is greater.

32. Repeat steps 5, 7, 16, 17, and 19 for both p_1 and p_2.

33. Calculate the two-phase mixture velocity at both p_1 and p_2.

$$v_{m_1} = v_{sL_1} + v_{sg_1}$$
$$v_{m_2} = v_{sL_2} + v_{sg_2}$$

34. Determine a value for $\Delta (v_m^2)$

$$\Delta (v_m^2) = [v_{m_1}^2 - v_{m_2}^2]$$

35. Calculate Δh corresponding to $\Delta p = p_1 - p_2$.

$$\Delta h = \frac{144\ \Delta p - \rho_m \Delta\left(\frac{v_m^2}{2g_c}\right)}{\bar{\rho}_m + \frac{f w^2}{2.9652 \times 10^{11} d^5 \bar{\rho}_m}}$$

36. Starting with p_2 and the known depth at p_2, assume another pressure point and repeat the procedures until reaching total depth, or until reaching the surface depending upon whether you are starting from the bottom or top of the tube.

C.43 EXAMPLE PROBLEM BY GENERALIZED METHOD OF HAGEDORN AND BROWN

The following problem will be worked by the method of Hagedorn and Brown for both 2 in. and 1¼ in. tubing having internal diameters of 1.995 and 1.380 in., respectively. In order to illustrate the procedure more clearly we will determine the distance between a 500 psig and 1,000 psig pressure point. In actual practice we would select pressure increments much closer than 500 psi.

Given: $d = 1.995$ in.
$p_1 = 500$ psig
$p_2 = 1{,}000$ psig
$p_a = 14.7$ psia
$T_1 = 120°F$
$T_2 = 150°F$
$\gamma_g = 0.65$
$\gamma_w = 1.07$
$\rho_o = 22°API$ (μ_o obtained from °API versus μ correlation)
$q_o = 400$ stk bpd
$q_w = 600$ stk bpd (WOR = 1.5)
$\mu_g = 0.018$ cp (constant)
$\sigma_o = 30$ dynes/cm (assume constant)
$\sigma_w = 70$ dynes/cm (assume constant)
GLR = 500 scf/stk bbl

Required: Distance between the 500 and 1,000 psig pressure points.

Solution:

1. Find specific gravity of the oil:

$$\gamma_o = \frac{141.5}{131.5 + °API} = \frac{141.5}{131.5 + 22} = 0.922$$

2. Find mass associated with one bbl of stock tank liquid:

$$m = \gamma_o(350)\left(\frac{1}{1 + WOR}\right) + \gamma_w(350)\left(\frac{WOR}{1 + WOR}\right) + (0.0764)(GLR)(\gamma_g)$$

$$m = (0.922)(350)\left(\frac{1}{1 + 1.5}\right) + (1.07)(350)\left(\frac{1.5}{1 + 1.5}\right) + (0.0764)(500)(0.65)$$

$$m = 129 + 224.5 + 24.8$$
$$m = 378.3\ lb_m/stk\ bbl\ liquid$$

3. Calculate mass flow rate:

$$w = qm$$
$$w = (378.3)(1000) = 378{,}300\ lb_m/day$$

4. Calculate the density of the liquid phase:

$$\rho_L = \left[\frac{\gamma_o(62.4) + \frac{R_s\ \gamma_g(0.0764)}{5.614}}{B_o}\right]\left(\frac{1}{1 + WOR}\right) + \left[(\gamma_w)(62.4)\left(\frac{WOR}{1 + WOR}\right)\right]$$

where $\bar{R}_s = 92$ scf/bbl

$\bar{B}_o = 1.063$

$$\rho_L = \left[\frac{(0.922)(62.4) + \frac{92(0.65)(0.0764)}{5.614}}{1.063}\right]\left(\frac{1}{2.5}\right) + \left[(1.07)(62.4)\left(\frac{1.5}{2.5}\right)\right]$$

$$= 21.95 + 40 = 61.95 \text{ lb/cu ft}$$

5. Calculate the average pressure:

$$\bar{p} = \frac{p_1 + p_2}{2} + 14.7 = \frac{500 + 1{,}000}{2} + 14.7 = 764.7 \text{ psia}$$

6. Find Z factor from Fig. 2.17:

Average temperature $= \bar{T} = \frac{120 + 150}{2} = 135°$ F, $\bar{p}$ $= 764.7$ psia

$Z = 0.91$

7. Calculate average density of the gas phase:

$$\rho_g = \gamma_g\,(0.0764)\left(\frac{\bar{p}}{14.7}\right)\left(\frac{520}{\bar{T}}\right)\left(\frac{1}{\bar{Z}}\right)$$

$$\rho_g = (0.65)(0.0764)\left(\frac{764.7}{14.7}\right)\left(\frac{520}{595}\right)\left(\frac{1}{0.91}\right) = 2.47 \text{ lb}_m\text{/cu ft}$$

8. From Fig. 2.19 obtain R_s at $\bar{p}$ and $\bar{T}$ (also from PVT data). Also obtain R_s at points (1) and (2):

R_s at $\bar{p}$ and $\bar{T} = 92$ scf/bbl
R_s at p_1, $T_1 = 58$ scf/bbl
R_s at p_2, $T_2 = 118$ scf/bbl

9. Calculate the average viscosity of the oil (See Figs. 2.2 through 2.6):

From Fig. 2.2 $\mu_{o1} = 38$ cp
$\mu_{o2} = 18$ cp

Accounting for gas in solution from Fig. 2.6

$\mu_{o1} = 23$ cp
$\mu_{o2} = 11$ cp

Average viscosity $= \frac{23 + 11}{2} = 17$ cp

10. Determine the average water viscosity from Fig. 2.9.

$\mu_{w_1} = 0.72$
$\mu_{w_2} = 0.54$

Average viscosity water $= \frac{0.72 + 0.54}{2} = 0.63$ cp

11. Determine the liquid mixture viscosity

$$\mu_L = \mu_o\left(\frac{1}{1 + \text{WOR}}\right) + \mu_w\left(\frac{\text{WOR}}{1 + \text{WOR}}\right) = 17\left(\frac{1}{1 + 1.5}\right) + 0.63\left(\frac{1.5}{1 + 1.5}\right)$$

$$= 6.8 + 0.378$$
$$= 7.178 \text{ cp}$$

12. Assuming constant surface tensions at each pressure point, calculate the liquid mixture surface tension:

$$\sigma_L = \sigma_o\left(\frac{1}{1 + \text{WOR}}\right) + \sigma_w\left(\frac{\text{WOR}}{1 + \text{WOR}}\right)$$

$$= 30\left(\frac{1}{1 + 1.5}\right) + 70\left(\frac{1.5}{1 + 1.5}\right)$$

$$= 12 + 42 = 54 \text{ dynes/cm}$$

13. Calculate the liquid viscosity number:

$$N_L = 0.15726\,\mu_L\left(\frac{1}{\rho_L\,\sigma_L^3}\right)^{1/4}$$

$$N_L = (0.15726)(7.178)\left[\frac{1}{(61.95)(54)^3}\right]^{1/4}$$

$$= 1.128\left[\frac{1}{9{,}950{,}000}\right]^{1/4}$$

$$N_L = \frac{1.128}{56.2} = 0.0206$$

14. From Fig. 2.56 determine:

$$CN_L = 0.003$$

15. Calculate area of tubing:

$$A_P = \frac{\pi\,d^2}{4} = \frac{\pi\,(1.995)^2}{(144)\,4} = 0.0217 \text{ sq ft}$$

16. From Fig. 2.8 obtain $\bar{B}_o$:

$$\bar{B}_o = 1.063$$

17. Calculate the superficial liquid velocity v_{sL}, ft/sec:

$$v_{sL} = \frac{(5.61)(1{,}000)}{(86{,}400)(0.0217)}\left\{1.063\left(\frac{1}{1 + 1.5}\right) + 1.0\left(\frac{1.5}{1 + 1.5}\right)\right\}$$

$v_{sL} = 2.99\ \{0.426 + 0.6\} = (2.99)\ (1.026)$
$v_{sL} = 3.07$ ft/sec

18. Calculate the liquid velocity number, N_{LV}:

$$N_{LV} = (1.938)(v_{sL})\left(\frac{\rho_L}{\sigma_L}\right)^{1/4}$$

$$= (1.930)(3.07)\left(\frac{61.95}{54}\right)^{1/4} = 6.2$$

19. Calculate the superficial gas velocity, v_{sg}:

$$v_{sg} = \frac{q_L\left\{\text{GLR} - R_s\left(\frac{1}{1 + \text{WOR}}\right)\right\}}{86{,}400\,A_P}\left(\frac{14.7}{\bar{p}}\right)\left(\frac{\bar{T}}{520}\right)\left(\frac{\bar{Z}}{1}\right)$$

$$v_{sg} = \frac{1{,}000\left\{500 - 92\left(\frac{1}{1 + 1.5}\right)\right\}}{(86{,}400)(0.0217)}\left(\frac{14.7}{764.7}\right)\left(\frac{595}{520}\right)\left(\frac{0.91}{1}\right)$$

$$= 4.93 \text{ ft/sec}$$

20. Determine the gas velocity number, N_{Gv}:

$$N_{Gv} = 1.938\,v_{sg}\left(\frac{\rho_L}{\sigma_L}\right)^{1/4}$$

$$= (1.938)(4.93)\left(\frac{63.2}{54}\right)^{1/4} = 9.95$$

21. Check the flow regime:

$A = 1.071 - [0.2218\,(v_{sL} + v_{sg})^2]/d$
$v_{sL} = 3.07$ ft/sec from step 17
$v_{sg} = 4.93$ ft/sec from step 19
$d = 1.995$ in. $= 0.166$ ft
$A = 1.071 - [0.2218\,(8)^2]/0.166 = -88$

Therefore we use A = 0.13 since 0.13 = min limit. Find B:

$$B = \frac{v_{sg}}{v_{sL} + v_{sg}} = \frac{4.93}{8} = 0.616$$

B-A = 0.616 − 0.13 = +0.486 and is positive; therefore, continue with the Hagedorn and Brown procedure.

22. Find the pipe diameter number, N_d:

$$N_d = 120.872\,d\sqrt{\frac{\rho_L}{\sigma_L}}$$

$$= (120.872)\left(\frac{1.995}{12}\right)\sqrt{\frac{61.95}{54}} = 21.7$$

23. Calculate the holdup correlating function:

$$\phi = \frac{N_{Lv}}{N_{gv}^{0.575}} \left(\frac{\bar{p}}{14.7}\right)^{0.10} \left(\frac{CN_L}{N_d}\right)$$

$$\phi = \frac{6.2}{(9.95)^{0.575}} \left(\frac{764.7}{14.7}\right)^{0.10} \left(\frac{0.003}{21.7}\right) = 0.00034$$

24. Obtain $\frac{H_L}{\psi}$ from Fig. 2.34.

$$\frac{H_L}{\psi} = 0.56$$

25. Determine the secondary correction factor correlating parameter.

$$= \frac{N_{gv}\, N_L^{0.380}}{N_d^{2.14}}$$

$$= \frac{9.95\ (0.0206)^{0.380}}{(21.7)^{2.14}} = 0.00315$$

26. Obtain ψ from Fig. 2.57.

$$\psi = 1.00$$

27. Calculate a value for H_L.

$$H_L = \frac{H_L}{\psi} = (0.56)(1.00) = 0.56$$

28. Determine a value for the two-phase Reynolds number $(N_{Re})_{TP}$:

$$(N_{Re})_{TP} = \frac{2.2 \times 10^{-2}\ w}{d\ (\mu_L)^{H_L}\ (\mu_g)^{(1-H_L)}}$$

$$= \frac{(2.2 \times 10^{-2})(378{,}300)}{(0.1661)(7.178)^{0.56}(0.018)^{0.44}} = 9.67 \times 10^4$$

29. $\epsilon/d = \frac{(0.00015)(12)}{1.995} = 0.000904.$

30. Obtain the friction factor from Fig. 2.35:

$$f = 0.022$$

31. Calculate $\bar{\rho}_m$ by two methods.
(a) Using $H_L = 0.56$ from Step 27:

$$\rho_m = \bar{\rho}_L H_L + \bar{\rho}_g (1 - H_L)$$
$$= (61.95)(0.56) + (2.47)(0.44) = 35.7\ lb_m/cu\ ft$$

(b) Calculate $\bar{\rho}_m$ assuming no slippage. Since this is the same problem as worked in Ex. C.12, the same steps 1 through 7 of Appendix C.12 show $\bar{\rho}_m$ for no slippage to be 18.96 lb_m/cu ft.
Therefore, use procedure (a) where $\bar{\rho}_m = 35.7\ lb_m$/cu ft.

32. $T_1 = 120°F$ $\quad T_2 = 150°F$
$Z_1 = 0.932$ $\quad Z_2 = 0.895$
$B_{o_1} = 1.044$ $\quad R_{s_2} = 122$
$R_{s_1} = 60$ $\quad v_{sL_2} = 3.1$
$v_{sL_1} = 3.05$ $\quad v_{sg_2} = 3.66$
$v_{sg_1} = 7.54$

33. $v_{m_1} = v_{sL_1} + v_{sg_1} = 10.59$
$v_{m_2} = v_{sL_2} + v_{sg_2} = 6.76$

34. $\Delta (v_m)^2 = 66.3$

35. $$\Delta h = \frac{144\ \Delta p - \rho_m\ \Delta \left(\frac{v_m^2}{2\ g_c}\right)}{\bar{\rho}_m + \frac{f\, w^2}{2.9652 \times 10^{11}\ d^5\ \bar{\rho}_m}}$$

$$\Delta h = \frac{(144)(500) - (35.7)\left(\frac{66.3}{64.4}\right)}{35.7 + \frac{(0.022)(378{,}300)^2}{(2.9652 \times 10^{11})(0.166)^5(35.7)}}$$

$$= 1{,}870\ ft$$

36. This procedure would be repeated for additional pressure points. The same procedure would be used for 1-¼ in. tubing and a distance of 1,250 ft is found.

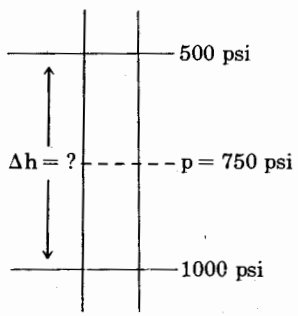
C.44 EXAMPLE PROBLEM BY HAGEDORN AND BROWN CORRELATION SHOWING THE DIFFERENCE OF INCLUDING AND EXCLUDING THE KINETIC ENERGY TERM

Given:
$d_p = 1.995$ in. (ID)
$T = 150°F$
$\mu_o = 1$ cp (constant)
$\mu_g = 0.02$ cp (constant)
$\sigma_o = 30$ dynes/cm (constant)
$\gamma_o = 42°API = 0.815$
$\gamma_g = 0.65$
$q_o = 600$ stk bopd
WOR = 0
GOR = 1,000 scf/stkb

$$144\ \frac{\Delta p}{\Delta h} = \bar{\rho}_m + \frac{f\, w^2}{2.9652 \times 10^{11}\ d^5\ \rho_m} + \bar{\rho}_m \frac{\Delta \left(\frac{vm^2}{2g_c}\right)}{\Delta h}$$

or:

$$\Delta h = \frac{144\ \Delta p - \bar{\rho}_m\ \Delta \left(\frac{vm^2}{2g_c}\right)}{\bar{\rho}_m + \frac{f\, w^2}{2.9652 \times 10^{11}\ d^5\ \rho_m}}$$



C.441 ASSUME KINETIC ENERGY IS NEGLIGIBLE

$m = (\gamma_o)(62.4)(5.614) + (\gamma_g)(0.0764)(GOR)$
$= (0.815)(62.4)(5.614) + (0.65)(0.0764)(1{,}000)$
$= 285.5 + 49.6$
$= 335.1$ lb/stk bbl of oil
$w = q_o m = (600)(335.1) = 2.01 \times 10^5\ lb_m/day$
$w^2 = (2.01 \times 10^5)^2 = 4.04 \times 10^{10}\ lb_m^2/day^2$

$$d_p = 1.995\ in. \times \frac{1\ ft}{12\ in.} = 0.1661\ ft$$

$$d_p^5 = (0.1661)^5 = 1.27 \times 10^{-4}$$

$$\Delta h = \frac{(144)(500)}{\bar{\rho}_m + \frac{f}{\bar{\rho}_m} \frac{(4.04 \times 10^{10})}{(2.9652 \times 10^{11})(1.27 \times 10^{-4})}}$$

$$\Delta h = \frac{7.2 \times 10^4}{\bar{\rho}_m + 1.07 \times 10^3 \frac{f}{\bar{\rho}_m}}$$

$$\bar{\rho}_m = \bar{\rho}_L \bar{H}_L + \bar{\rho}_g(1 - \bar{H}_L) \quad \bar{\rho}_L = \bar{\rho}_o \text{ since WOR} = 0$$

$$\bar{\rho}_o = \frac{\gamma_o \times 62.4 + [\bar{R}_s \gamma_g \times 0.0764]/5.614}{B_o}$$

$$\bar{\rho}_o = \frac{(0.815)(62.4) + (175)(0.65)(0.0764)/5.6.4}{1.11}$$

$$= \frac{50.90 + (8.69)/5.614}{1.11} = \frac{50.90 + 1.55}{1.11}$$

$$= 47.2 \text{ lb}_m/\text{ft}^3$$

$$\bar{\rho}_g = \frac{(\gamma_g)(0.0764)}{\bar{B}_g}$$

$$\bar{B}_g = \frac{(14.7)\, z\, T}{\bar{p}\,(520)}$$

at: $\left.\begin{array}{l}\bar{p} = 750 \text{ psi} \\ \bar{T} = 150°\text{F} \\ \gamma_g = 0.65\end{array}\right\} \bar{z} = 0.92$

$$\bar{B}_g = \frac{(14.7)(0.92)(610)}{(764.7)(520)} = 0.02077 \frac{\text{cu ft}}{\text{scf}}$$

$$\bar{\rho}_g = \frac{(0.65)(0.0764)}{(0.02077)} = 2.39 \frac{\text{lb}_m}{\text{cu ft}}$$

Therefore:

$$\rho_m = 47.2\, \bar{H}_L + 2.39\, (1 - \bar{H}_L)$$

Determine $\bar{H}_L$:

$$\bar{N}_L = 0.15726\, \mu \cdot \sqrt[4]{\frac{1}{\bar{\rho}_L\, \sigma^3}} = (0.15726)(1) \left(\frac{1}{(47.2)(30)^3}\right)^{1/4}$$

$$= 0.00469$$

From CN_L vs N_L curve: $\bar{CN}_L = 0.0022$ (Fig. 3.27)

$X = H_L$ Correlating Function $= 0.00326 \times$

$$\frac{q_L^{0.425}\left[\left(\frac{1}{1+\text{WOR}}\right)\bar{B}_o + \left(\frac{\text{WOR}}{1+\text{WOR}}\right)\bar{B}_w\right]\bar{p}^{0.675}\rho^{0.394}\, CN_L}{d^{1.850}\,(\bar{T}\,\bar{z})^{0.575}\,\bar{\rho}_L^{0.394}\left[\text{GLR} - \bar{R}_s\left(\frac{1}{1+\text{WOR}}\right)\right]^{0.575}}$$

WOR = 0.0; $\therefore \bar{\rho}_L = \bar{\rho}_o$ and GLR = GOR.

Therefore:

$$X = \frac{(0.00326)(600^{0.425}(1.11)(764.7)^{0.675}(30)^{0.394}(0.0022)}{(0.1661)^{1.850}\,[(610)(0.92)]^{0.575}(47.2)^{0.394}(1000 - 175)^{0.575}}$$

$$= \frac{(0.00326)(1.11)(15.15)(88.5)(3.82)(0.0022)}{(0.0361)(38)(4.57)(47.5)}$$

$$= 1.365 \times 10^{-4}$$

From $\frac{H_L}{\psi}$ curve: $\frac{\bar{H}_L}{\bar{\psi}} = 0.37$ (Fig. 2.34).

Determine $\bar{\psi}$:

$$\bar{N}_L = 0.00469$$

$$\bar{N}_{GV} = 1.938\, v_{sg} \sqrt{\frac{\bar{\rho}_L}{\sigma}}$$

$$v_{sg} = \frac{q_o(\text{GOR} - \bar{R}_s)}{86{,}400\, \bar{B}_g\, A_T}$$

$$v_{sg} = \frac{(600)(1{,}000 - 175)}{(86{,}400)\,\frac{\pi}{4}\,(0.1661)^2(0.02077)}$$

$$= 5.48 \text{ ft/sec}$$

Therefore:

$$\bar{N}_{GV} = (1.938)(5.48)\left(\frac{47.2}{30}\right)^{1/4} = 11.90$$

$$\bar{N}_D = 120.872\, d_p \sqrt{\frac{\rho_L}{\sigma}} = (120.872)(0.1661)\left(\frac{47.2}{30}\right)^{1/2}$$

$$= 25.2$$

$$\text{Abscissa function} = \frac{\bar{N}_{GV}\, \bar{N}_L^{0.380}}{\bar{N}_o^{2.14}} = \frac{(11.90)(0.00469)^{0.380}}{(25.2)^{2.14}}$$

$$= 1.55 \times 10^{-3} < 0.01$$

From ψ curve: $\bar{\psi} = 1.0$ (Fig. 2.57).

$$\bar{H}_L = \frac{\bar{H}_L}{\bar{\psi}} \times \bar{\psi} = (0.37)(1.0) = 0.37$$

$$\bar{\rho}_m = (47.2)(0.37) + (2.39)(1 - 0.37)$$

$$= 17.45 + 1.50$$

$$= 18.95 \text{ lb}_m/\text{cu ft}$$

Determine friction factor:

$$N_{Re_{TP}} = 2.2 \times 10^{-2} \frac{q_o\, M}{d_p\, \mu_L^{H_L}\, \mu_g^{(1-H_L)}}$$

$$= (2.2 \times 10^{-2}) \frac{(2.01 \times 10^5)}{(0.1661)(1.0)^{0.36}(0.02)^{(1-0.36)}}$$

$$= 3.13 \times 10^5$$

$$\frac{\epsilon}{d_p} = \frac{0.00015}{0.1661} = 0.0009$$

From Moody Diagram (Fig. 2.35): f = 0.02.

$$\Delta h = \frac{7.2 \times 10^4}{18.95 + \frac{(0.02)(1.07 \times 10^3)}{(18.9)}}$$

$$= \frac{7.2 \times 10^4}{18.95 + 1.03} = \frac{7.2 \times 10^4}{20.08}$$

$$\Delta h = 3{,}580 \text{ ft}$$

$$\text{Avg. gradient} = \left(\frac{\Delta p}{\Delta h}\right) = \frac{500}{3{,}580} = 0.1397 \frac{\text{psi}}{\text{ft}}$$

C.442 INCLUDING KINETIC ENERGY

$$\Delta h = \frac{7.2 \times 10^{-4} - \bar{\rho}_m\, \Delta\left(\frac{v_m^2}{2\,g}\right)}{18.95 + 1.03}$$

$$= 3.580 - \frac{18.95}{(20.08)(64.4)}\, \Delta\left(\frac{v_m^2}{2\,g}\right)$$

$$= 3580 - 0.01465\, \Delta\,(v_m^2)$$

$$v_m = v_{sL} + v_{sg}$$

$$v_{sL} = \frac{5.614\, q_o\, B_o}{86{,}400\, A_p}$$

$$v_{sg} = \frac{q_o(\text{GOR} - R_s)\, B_g}{86{,}400\, A_p}$$

$p = 500$ psi	$p = 1{,}000$ psi
$B_o = 1.085$	$B_o = 1.145$
$R_s = 118$	$R_s = 250$
$z = 0.941$	$z = 0.892$

$$= \frac{(14.7)(610)(0.941)}{(514.7)(520)}$$

$$= 0.0316 \text{ cu ft/scf}$$

$$= 500 \text{ psi}$$

$$= \frac{(5.614)(600)(1.085)}{(86{,}400)\frac{\pi}{4}(0.1661)^2}$$

$$= 1.95 \text{ ft/sec}$$

$$= \frac{(600)(1{,}000 - 118)(0.0316)}{(86{,}400)\frac{\pi}{4}(0.1661)^2}$$

$$= 8.90 \text{ ft/sec}$$

$$= 1.95 + 8.90$$

$$= 10.85 \text{ ft/sec}$$

$$= 1{,}000 \text{ psi}$$

$$B_g = \frac{(14.7)(610)(0.892)}{(1014.7)(520)}$$

$$= 0.01515 \text{ cu ft/scf}$$

$$v_{sL} = \frac{(5.614)(600)(1.145)}{(86{,}400)\frac{\pi}{4}(0.1661)^2}$$

$$= 2.06 \text{ ft/sec}$$

$$v_{sg} = \frac{(600)(1{,}000 - 250)(0.01515)}{(86{,}400)\frac{\pi}{4}(0.1661)^2}$$

$$= 3.63 \text{ ft/sec}$$

$$v_m = 2.06 + 3.63$$

$$= 5.69 \text{ ft/sec}$$

$$\Delta v_m^2 = (10.85)^2 - (5.69)^2$$

$$= 117.8 - 32.4$$

$$= 85.4$$

Therefore:

$$\Delta h = 3580 - (0.01465)(85.4)$$

$$= 3580 - 1.25$$

$$\Delta h \approx 3579 \text{ ft}$$

Appendix C.5: Duns and Ros method

C.51 STEPWISE PROCEDURE FOR CALCULATION OF A PRESSURE TRAVERSE BY METHOD OF DUNS AND ROS

1. Determine the specific gravity of the oil:

$$\gamma_o = \frac{141.5}{131.5 + °API}$$

2. Find mass associated with one bbl of stock tank liquid:

$$m = \gamma_o(350)\left(\frac{1}{1+WOR}\right) + \gamma_w(350)\left(\frac{WOR}{1+WOR}\right) + (0.0764)(GLR)(\gamma_g)$$

3. Determine the density of the liquid phase:

$$\rho_L = 62.4\left[\gamma_o\left(\frac{1}{1+WOR}\right) + \gamma_w\left(\frac{WOR}{1+WOR}\right)\right]$$

4. Find the average pressure:

$$\bar{p} = \frac{p_1 + p_2}{2} + p_a$$

5. Find the average temperature:

$$\bar{T} = \frac{T_1 + T_2}{2}$$

6. Find $\bar{Z}$
7. Find the average density of the gas phase:

$$\rho_g = \gamma_g(0.0764)\left(\frac{\bar{p}}{14.7}\right)\left(\frac{520}{\bar{T}}\right)\left(\frac{1}{\bar{Z}}\right)$$

8. Calculate the average viscosity of the oil from correlations as noted in Figures 2.2 through 2.6.
9. Determine the average water viscosity (Fig. 2.9).
10. Calculate the liquid mixture viscosity:

$$\mu_L = \mu_o\left(\frac{1}{1+WOR}\right) + \mu_w\left(\frac{WOR}{1+WOR}\right)$$

11. Find the liquid mixture surface tension:

$$\sigma_L = \sigma_o\left(\frac{1}{1+WOR}\right) + \sigma_w\left(\frac{WOR}{1+WOR}\right)$$

12. Find R_s at $\bar{p}$ and $\bar{T}$.
13. Find B_o at $\bar{p}$ and $\bar{T}$.
14. Find tubing area.
15. Find the liquid viscosity number:

$$N_L = 0.15726\ \mu_L\left(\frac{1}{\rho_L \sigma_L^3}\right)^{1/4}$$

16. Find v_{sL} (assume $B_w = 1.0$).
17. Find the liquid velocity number:

$$N_{Lv} = 1.938\ v_{sL}\left(\frac{\rho_L}{\sigma_L}\right)^{1/4}$$

18. Find the superficial gas velocity:

$$v_{sg} = \frac{q_L\left\{GLR - R_s\left(\frac{1}{1+WOR}\right)\right\}}{86{,}400}\left(\frac{14.7}{p}\right)\left(\frac{\bar{T}}{520}\right)\left(\frac{\bar{Z}}{1}\right)$$

19. Find the gas velocity number:

$$N_{gv} = 1.938\ v_{sg}\left(\frac{\rho_L}{\sigma_L}\right)^{1/4}$$

20. Find the pipe diameter number:

$$N_d = 120.872\ d\sqrt{\frac{\rho_L}{\sigma_L}}$$

21. Select the proper flow regime from Fig. 2.63.
22. Determine the proper slip factor depending upon the region found in Step 21.
    (a) For Region I:
    Slip factor determination
    (1) The slip factor is found by the following formula:

$$S = F_1 + F_2\ N_{Lv} + F_3'\left(\frac{N_{gv}}{1+N_{Lv}}\right)^2$$

F_1 and F_2 are found in Fig. 2.65.

$F_3' = F_3 - \frac{F_4}{N_d}$ where F_3 and F_4 are found in Fig. 2.65.

For annular flow N_d is based on the wetted perimeter, thus: $d = (d_c + d_t)$.
Region I extends from zero N_{Lv} and N_{gv} up to $N_{gv} = L_1 + L_2\ N_{Lv}$, where L_1 and L_2 can be found in Fig. 3.34.

(b) For Region II:

$$S = (1 + F_5)\frac{(N_{gv})^{0.982} + F_6'}{(1 + F_7\ N_{Lv})^2}$$

F_5, F_6 and F_7 can be found in Fig. 2.66 where $F'_6 = 0.029\ N_d + F_6$
Region II extends from the upper limit of Region I to the transition zone to mist flow given by $N_{gv} = 50 + 36\ N_{Lv}$.

(c) For Region III (mist flow):

$$S = 0$$

Therefore $H_L = \frac{1}{1 + v_{sg}/v_{sL}}$

This is valid for $N_{gv} > 75 + 84\ N_{Lv}^{0.75}$

23. Determine the slip velocity if in Region I or II:

$$v_s = \frac{S}{1.938\sqrt{\rho_L/\sigma_L}}$$

24. Determine the liquid holdup:

$$H_L = \frac{v_s - v_{sg} - v_{sL} + \sqrt{(v_s - v_{sg} - v_{sL})^2 + 4v_s\ v_{sL}}}{2\ v_s}$$

25. Determine the liquid Reynolds number:

$$N_{Re} = \frac{1488\ \rho_L\ v_{sL}\ d}{\mu_L}$$

26. Determine the friction gradient according to the flow region.
    (a) For region I and II:

$$G_{fr} = 2f_w\frac{N_{Lv}\ (N_{Lv} + N_{gv})}{N_d}$$

where: $f_w = (f_1)\frac{f_2}{f_3}$

f_1 is found in Fig. 2.67.
f_2 is found in Fig. 2.68.
The abscissa must be determined in Fig. 2.68 and is $f_1 R\ N_d^{2/3}$

where: $R = \frac{v_{sg}}{v_{sL}}$

$f_3 = 1 + f_1\sqrt{R/50}$

The friction factor f_w is valid in regions I and II and covers heading also. It is good from zero N_{Lv} and N_{gv} up to the limit given by $N_{gv} = 50 + 36\ N_{Lv}$

(b) For Region III:

In mist flow where $N_{gv} > 75 + 84\ N_{Lv}^{0.75}$

$$G_{fr} = 2f_w\ N_\rho \frac{(N_{gv})^2}{N_d}$$

where: $N_\rho = \rho_g/\rho_L$

In Region III f_w is taken as f_i and may be taken from Fig. 2.67. For $\epsilon > 0.05d$ the value of f_i is calculated by the formula:

$$f_i = \frac{1}{\{4\log_{10}(0.027\ \epsilon/d)\}^2} + 0.067\ (\epsilon/d)^{1.73}$$

For $\epsilon > 0.05d$, the value of $d - \epsilon$ should be substituted for d throughout the friction gradient calculation, and also this substitution should be made:

$$v_{sg} = \frac{v_{sg}\ d^2}{(d-\epsilon)^2}$$

27. Determine the static gradient:

$$G_{st} = H_L + (1 - H_L)\frac{\rho_g}{\rho_L}$$

28. Determine the total pressure gradient.
(a) For Regions I and II:

$$G = G_{st} + G_{fr}$$

(b) For Region III (accounting for acceleration):

$$G = \frac{G_{st} + G_{fr}}{1 - (\rho_L\ v_{sL} + \rho_g\ v_{sg})(v_{sg}/p)}$$

29. Convert to gradient in psi/ft:

$$\frac{dp}{dh_{st}} = \frac{G_{st}\ \rho_L}{144};\ \frac{dp}{dh_{(fr)}} = \frac{G_{fr}\ \rho_L}{144};\ \text{or}\ \frac{dp}{dh_{total}} = \frac{G\ \rho_L}{144}$$

$$\Delta h = \frac{\Delta p}{\frac{dp}{dh_{total}}}$$

C.52 EXAMPLE PROBLEM BY DUNS AND ROS METHOD

The same problem worked by the other methods is worked except 1¼ in. tubing is used. For convenience the data is given again.

Nominal diameter: 1¼ in.
Flow diameter: 1.38 in. (0.115 ft)
Surface temperature: 120°F
Bottom temperature: 150°F
Surface pressure: 500 psig
Bottom pressure: 1,000 psig
q_o: 400 bpd
q_w: 600 bpd
GLR: 500 scf/bbl
ρ_o: 22°API
Specific gravity of gas: 0.65
Specific gravity of water: 1.07
Surface tension of water: 70 dynes/cm
Surface tension of oil: 30 dynes/cm
Viscosity of gas: 0.02 cp

Procedure:

1. Calculate specific gravity of oil, γ_o:

$$\gamma_o = \frac{141.5}{131.5 + \text{API}} = \frac{141.5}{131.5 + 22} = 0.9218$$

2. Calculate total mass associated with 1 stk bbl liquid:

$$m = 350\ \gamma_o(1/1 + \text{WOR}) + 350\ \gamma_w[\text{WOR}/(1 + \text{WOR})] + 0.0764(\text{GLR})\ \gamma_g$$

$m = 378.80\ lb_m$/stk bbl of oil

3. Calculate density of liquid phase:

$$\rho_L = 62.4(\gamma_o 1/1 + \text{WOR})] + \gamma_w[\text{WOR}/(1 + \text{WOR})])$$
$$\rho_L = 62.4[(0.9218(0.4) + 1.07(0.6)] = 61.95\ \text{lb/cu ft}$$

4. Calculate average pressure (psia):

$$\bar{p} = (p_1 + p_2)/2 + 14.7$$
$$\bar{p} = (500 + 1{,}000)/2 + 14.7 = 764.7\ \text{psia}$$

5. Find the average temperature:

$$\bar{T} = (120 + 150)/2 = 135°F$$

6. Find $\bar{Z}$ from Fig. 2.7:

$$\bar{Z} = 0.91$$

7. Calculate the average density of the gas phase:

$$\bar{\rho}_g = \gamma_m(0.0764)(\bar{p}/p)(520/\bar{T})(1/Z)$$

$$\bar{\rho}_g = 0.65(0.0764)\frac{764.7}{14.7}\left(\frac{520}{595}\right)\left(\frac{1}{0.910}\right) = 2.47\ lb_m/\text{cu ft}$$

8. Calculate the average oil viscosity.

From Fig. 2.2: $\mu_{o1} = 38$ cp
$\mu\ o_2 = 18$ cp

Accounting for gas in solution (Fig. 2.6):

$$\mu_{o1} = 23\ \text{cp}$$
$$\mu_{o2} = 11\ \text{cp}$$
$$\mu_o = \frac{23 + 11}{2} = 17\ \text{cp}$$

9. Average water viscosity (Fig. 2.9):

$$\mu_{w1} = 0.72$$
$$\mu_{w2} = 0.54 \qquad \mu_w = \frac{0.72 + 0.54}{2} = 0.63\ \text{cp}$$

10. Liquid mixture viscosity:

$$\bar{\mu}_L = \bar{\mu}_o\left(\frac{1}{1 + \text{WOR}}\right) + \bar{\mu}_w\left(\frac{\text{WOR}}{1 + \text{WOR}}\right)$$

$$\bar{\mu}_L = 17\left(\frac{1}{1 + 1.5}\right) + 0.63\left(\frac{1.5}{1 + 1.5}\right) = 7.178\ \text{cp}$$

11. Find liquid mixture surface tension.
Assuming water and oil surface tensions are constants, calculate the liquid mixture surface tension:

$$\sigma_L = \sigma_o\left(\frac{1}{1 + \text{WOR}}\right) + \left(\frac{1}{1 + \text{WOR}}\right)\sigma_w$$

$$\sigma_L = 30(0.4) + 70(0.6) = 54\ \text{dynes/cm}$$

12. Find R_s at p and T (Fig. 2.19):

$$R_s = 92\ \text{scf/bbl}$$

13. Find B_o at p and $\bar{T}$ (Fig. 2.18):

$$B_o = 1.06\ \text{bbl/stk bbl of oil}$$

14. Calculate tubing flow area, A_p:

$$A_p = \frac{\pi d^2}{4} = \frac{\pi}{4}(0.115)^2 = 0.01036\ \text{sq ft}$$

15. Calculate the liquid viscosity number:

$$N_L = 0.15726\ \mu_L\left(\frac{1}{\rho_L\ \sigma_L^3}\right)^{1/4}$$

$$N_L = 0.15726 \times 7.178\left(\frac{1}{61.95 \times 54^3}\right)^{1/4} = 0.0202$$

16. Assuming $B_w = 1.0$, calculate v_{sL}

$$v_{sL} = \frac{5.61\ q_L}{86{,}400\ A_p}\left[B_o\left(\frac{1}{1 + \text{WOR}}\right) + B_w\left(\frac{\text{WOR}}{1 + \text{WOR}}\right)\right]$$

$$v_{sL} = \frac{5.61\ (1{,}000)}{86{,}400\ (0.01036)}\left[1.064(0.4) + 1.0(0.6)\right] = 6.427 \text{ ft/sec}$$

17. Calculate liquid velocity number, N_{Lv}:

$$N_{Lv} = 1.938\ v_{sL}\left(\frac{\rho_L}{\sigma_L}\right)^{1/4}$$

$$N_{Lv} = 1.938\ (6.427)\left(\frac{62.95}{54}\right)^{1/4} = 12.8$$

18. Calculate superficial gas velocity, v_{sg}:

$$v_{sg} = q_L \frac{[GLR - R_s\ (1/(1 + WOR)]}{86{,}400\ A_p}\frac{P_{sc}}{P}\frac{\overline{T}}{T_{sc}}(\bar{z})$$

$$v_{sg} = 1{,}000\ \frac{500 - 92(0.4)}{86{,}400\ (0.01036)}\frac{14.7}{764.7}\left(\frac{595}{520}\right)\frac{0.91}{1} = 10.3 \text{ ft/sec}$$

19. Calculate gas velocity number, N_{gv}:

$$N_{gv} = 1.938\ v_{sg}\left(\frac{\rho_L}{\sigma_L}\right)^{1/4}$$

$$N_{gv} = 1.938\ (10.3)\left(\frac{61.95}{54}\right)^{1/4} = 20.76$$

20. Calculate pipe diameter number, N_d:

$$N_d = 120.872\ d\left(\frac{\rho_L}{\sigma_L}\right)^{1/2}$$

$$N_d = 120.872\ (0.115)\ \sqrt{(61.95)/54} = 15.01$$

21. Select the flow regime.

Refer to Fig. 2.63:

$N_{gv} = 20.76$
$N_{Lv} = 12.8$
Falls in Region II.

22. Determine slip velocity.
For Region II:

(From Fig. 2.66)
$F_5 = 0.17$
$F_6 = -0.1$
$F_7 = 0.07$

$$S = (1 + F_5)\frac{N_{gv}^{0.982} + F'_6}{(1 + F_7 N_{gv})^2}$$

$$F'_6 = 0.029\ N_d + F_6$$

$$F'_6 = (0.029)\ (13.4) - 0.1 = 0.3$$

$$S = (1 + 0.17)\frac{(20.76)^{0.982} + 0.3}{[(1 + (0.07)\ (12.8)]^2} = 6.5$$

23. Calculate slip velocity, v_s (Region II):

$$v_s = \frac{S}{1.938\ \sqrt[4]{\rho_L/\sigma_L}} = \frac{6.5}{(1.938)\left(\frac{61.95}{54.0}\right)^{1/4}} = 3.24$$

24. Calculate liquid holdup, H_L

$$H_L = \frac{v_s - v_{sg} - v_{sL} + \sqrt{(v_s - v_{sg} - v_{sL})^2 + (4v_s)\ (v_{sL})}}{2\ v_s}$$

$$H_L = \frac{(3.24 - 10.3 - 6.427) +}{}$$

$$\frac{\sqrt{(3.24 - 10.3 - 6.427)^2 + 4(3.24)\ (6.427)}}{(2)\ (3.24)}$$

$$H_L = 0.43$$

This value can be checked:

$$v_s = \frac{v_{sg}}{1 - H_L} - \frac{v_{sL}}{H_L}$$

$$v_s = \frac{103}{0.57} - \frac{6.427}{0.43} = 3.3 \qquad \text{(Checks)}$$

25. Calculate liquid Reynolds number, $(N_{Re})_L$

$$N_{ReL} = \frac{1488\ (\rho_L)\ (v_{sL})\ (d)}{\bar{\mu}_L}$$

$$(N_{Re})_L = \frac{1488 \times 61.95 \times 6.427 \times 0.115}{7.178} = 9.3 \times 10^3$$

26. Determine the friction gradient for Region II:

$$G_{fr} = 2f_w \frac{N_{Lv}\ (N_{Lv} + N_{gv})}{N_d}$$

$$f_w = (f_1)\frac{f_2}{f_3}$$

From Fig. 2.67 $f_1 = 0.0083$
From Fig. 2.68 $f_2 = 1.05$

where $f_1\ R\ N_d^{2/3} = (0.0083)\left(\frac{10.3}{6.427}\right)(15.01)^{2/3} = 8.1 \times 10^{-2}$

$$f_3 = 1 + f_1\ \sqrt{R/50} = 1 + 0.0083\sqrt{1.62/50} = 1.0$$

$$f_w = (0.0083)\left(\frac{1.05}{1.00}\right) = 0.0087$$

Calculate friction gradient, G_{fr}:

$$G_{fr} = 2f_w \frac{(N_{Lv} + N_{gv})\ N_{Lv}}{N_d}$$

$$G_{fr} = 2(0.0087)\frac{(12.8)\ (12.8 + 20.76)}{15.01}$$
$$= 0.561 \quad \text{(dimensionless)}$$

27. Calculate static gradient, G_{st}:

$$G_{st} = H_L + (1 - H_L)\frac{\rho_g}{\rho_L}$$

$$G_{st} = 0.43 + (0.57)\frac{2.51}{61.95} = 0.43 + 0.0226$$

$$G_{st} = 0.4526 \text{ (dimensionless)}$$

28. Determine total pressure gradient:

$$G = G_{st} + G_{fr} = 0.4526 + 0.561 = 1.0136$$

29. Convert to psi/ft:

$$\frac{dp}{dh} = \frac{1.0136\ (61.95)}{144} = 0.444 \text{ psi/ft}$$

30. Determine distance:

$$\Delta h = \frac{1{,}000 - 500}{0.444} = 1{,}125 \text{ ft}$$

As noted, this problem was worked using the average oil-water mixture properties as if one homogeneous liquid were flowing alone. If separate calculations are made for oil and for water as if they were flowing alone, the distance calculated is 1,550 ft.

In working this problem for 2 in. tubing, the same procedure was followed and a Δh of 2,100 ft was found when using the average oil-water mixture properties and a δh of 2,190 ft was found when calculations were made for water and oil as if they were flowing alone.

It is recommended that the calculations be made using the average oil-water mixture properties.

Appendix C.6: Method of Orkiszewski

C.61 PROCEDURE FOR THE CALCULATION OF A PRESSURE TRAVERSE BY THE ORKISZEWSKI METHOD (AS PRESENTED BY ORKISZEWSKI[22])

1. Select the starting point which is normally at the surface or at the bottom of the tubing string.
2. Determine the temperature gradient of the well.
3. Fix the Δp at about 10% of the measured or previously calculated pressure which may be either at the top or the bottom of the increment. Find the average pressure of the increment.
4. Assume a depth increment, Δh, and find the average depth of the increment.
5. From the temperature gradient, determine the average temperature of the increment.
6. Correct the fluid properties for temperature and pressure.
7. Determine the type of flow regime by the following procedure: Griffith and Wallis have defined the boundary between bubble and slug flow,[21] and Duns and Ros have defined the boundaries for the remaining three regimes.[20] The flow regime may be determined by testing whether the variables q_g/q_t, or $\bar{v}_g$, or both fall within the limits prescribed below:

Limits	Flow regime
$q_g/q_t < (L)_B$	Bubble
$q_g/q_t > (L)_B$, $\bar{v}_g < (L)_S$	Slug
$(L)_M > \bar{v}_g > (L)_S$	Transition
$\bar{v}_g > (L)_M$	Mist

The above variables are defined as:

$$\bar{v}_g = q_g \sqrt[4]{\rho_L/g\,\sigma}/A_p \quad (C.61)$$

$$(L)_B = 1.071 - (0.2218\, v_t\, 2/d_h), \text{ with the limit } (L)_B \geq 0.13 \quad (C.62)$$

$$(L)_S = 50 + 36\, \bar{v}_g\, q_L/q_g \quad (C.63)$$

$$(L)_M = 75 + 84\, (\bar{v}_g\, q_L/q_g)^{0.75} \quad (C.64)$$

where:

$\bar{v}_g$ = dimensionless gas velocity
v_t = total fluid velocity (q_t/A_p), ft/sec
ρ_L = liquid density, lb/cu ft
σ = liquid surface tension, lb/sec^2

8. Based on the flow regime obtained from Step 7, determine the average density ($\bar{\rho}$) and the friction loss gradient (τ_f) as follows:

The variables $\bar{\rho}$ and τ_f are defined in (a) below for bubble flow, in (b) for slug flow, in (c) for transition, and in (d) for mist flow. Section (b), while the most complex, is also the most important since slug flow has been encountered in over 95% of the gas lift or flowing wells studied. Section (e) describes how τ_f was developed for the slug flow regime.

(a) *Bubble flow*

The void fraction of gas (H_g) in bubble flow can be expressed as:

$$H_g = \frac{1}{2}\left[1 + \frac{q_t}{v_s A_p} - \sqrt{(1 + q_t/v_s\, A_p)^2 - \frac{4q_g}{v_s A_p}}\right] \quad (C.65)$$

where:

v_s = slip velocity (bubble rise velocity), ft/sec.

Griffith suggested that a good approximation of an average v_s is 0.8 ft/sec. Thus with Eq. C.75, the average flowing density can be computed as:

$$\bar{\rho} = (1 - H_g)\, \rho_L + H_g\, \rho_g \quad (C.66)$$

The friction gradient is:

$$\tau_f = f\, \rho_L\, v_L^2/2g_c\, d_h \quad (C.67)$$

where:

$$v_L = q_L/[A_p\, (1 - H_g)]$$

The friction factor, f, is obtained from Fig. 2.72 by using a Moody relative roughness factor obtained from Fig. 2.73. The Reynolds number is calculated as:

$$N_{Re} = 1488\, \rho_L\, d_h\, v_L/\mu_L$$

where

d_h = hydraulic pipe diameter, ft
μ_L = liquid viscosity, cp

(b) *Slug flow*

The average density term is:

$$\bar{\rho} = \frac{w_t + \rho_L\, v_s\, A_p}{q_t + v_s\, A_p} + \delta\, \rho_L \quad (C.68)$$

where δ is a coefficient correlated from oil field data. Griffith and Wallis correlated bubble rise velocity by the relationship:

$$v_s = C_1 C_2 \sqrt{g\, d_h} \quad (C.69)$$

where C_1 is expressed in Fig. 2.74 as a function of bubble Reynolds number ($N_b = 1488\, v_s\, d_h\, \rho_L/\mu_L$), and C_2 is expressed in Fig. 2.75 as a function of both N_b and liquid Reynolds number:

$$N_{Re} = 1488\, \rho_L\, d_h\, v_l/\mu_L \quad (C.610)$$

where v_t = total velocity of liquid and gas (q_t/A_p), ft/sec.

Fig. 2.75 was extrapolated* so that v_s could be evaluated at the higher Reynolds numbers. When C_2 cannot be read from Fig. 2.75, the extrapolated values of v_s may be calculated from the following set of equations:

when: $N_b \leq 3{,}000$:

$$v_s = (0.546 + 8.74 \times 10^{-6}\, N_{Re}) \sqrt{g d_h} \quad (C.611)$$

when: $N_b \geq 8{,}000$:

$$v_s = (0.35 + 8.74 \times 10^{-6}\, N_{Re}) \sqrt{g d_h} \quad (C.612)$$

when: $3{,}000 < N_b < 8{,}000$:

$$v_{si} = (0.251 + 8.74 \times 10^{-6}\, N_{Re}) \sqrt{g d_h}$$

$$v_s = \frac{1}{2}\left[v_{si} + \sqrt{v_{si}^2 + \frac{13.59\, \mu_L}{\rho_L \sqrt{d_h}}}\right] \quad (C.613)$$

The wall friction loss term, which has been independently derived, is expressed as:

$$\tau_f = \frac{f\, \rho_L\, v_t^2}{2g_c\, d_h}\left[\frac{q_L + v_s\, A_p}{q_t + v_s\, A_p} + \delta\right] \quad (C.614)$$

The friction factor is obtained from Fig. 2.72 and is a function of the Reynolds number given by Eq. C.610 and the ϵ/d obtained from Fig. 2.73. The liquid distribution coefficient, δ, may be determined by the equation which meets the following conditions:

*The parallel work of Nicklen, Wilkes and Davidson (Ref. 28, Section 2.3) provided the basis for the extrapolation. They observed that bubble rise velocity was independent of N_b in the Reynolds number range of 9×10^3 to 1×10^5. Their correlation of bubble rise velocity was found comparable to Eq. C.79 when N_b was around 8×10^3. Thus, their results were incorporated into the above extrapolation.

Continuous liquid phase	v_t	Use equation
Water	<10	C.615
Water	>10	C.616
Oil	<10	C.617
Oil	>10	C.618

$$\delta = [(0.013 \log \mu_L)/d_h^{1.38}] - 0.681 + 0.232 \log v_t - 0.428 \log d_h \quad (C.615)$$

$$\delta = [(0.045 \log[\mu_L)/d_h^{0.799}] - 0.709 - 0.162 \log v_t - 0.888 \log d_h \quad (C.616)$$

$$\delta = [0.0127 \log[(\mu_L + 1)/d_h^{1.415}] - 0.284 + 0.167 \log v_t + 0.113 \log d_h \quad (C.617)$$

$$\delta = [0.0274 \log[(\mu_L + 1)/d_h^{1.371}] + 0.161 + 0.569 \log d_h - \log v_t \{[0.01 \log (\mu_L + 1)/d_h^{1.571}] + 0.397 + 0.63 \log d_h\} \quad (C.618)$$

But it is constrained by the following limits:

$$\delta \geq -0.065\, v_t \quad (C.619)$$

and when $v_t > 10$

$$\delta \geq -\frac{v_s A_p}{q_t + v_s A_p}\left(1 - \frac{\bar{\rho}}{\rho_L}\right) \quad (C.620)$$

(c) *Transition flow*

Duns and Ros approximated $\bar{\rho}$ and τ_f for transition flow. Their method is to first calculate these terms for both slug and mist flow, and then lineraly weight each term with respect to $\bar{v}_g$ and the limits of the transition zone $(L)_s$ and $(L)_M$. $(\bar{v})_g$, $(L)_M$, and $(L)_s$ are defined in Step 7. Thus the average density term would be:

$$\rho = \frac{(L)_M - \bar{v}_g}{(L)_M - (L)_s}\left[\bar{\rho}\right]_{slug} + \frac{\bar{v}_g - (L)_s}{(L)_M - (L)_s}\left[\bar{\rho}\right]_{mist} \quad (C.621)$$

The friction gradient term would be similarly weighted. A more accurate friction loss prediction is claimed if the gas volumetric flow rate for mist flow is taken as:

$$q_g = A_p\, (L)_M\, (\rho_L/g\,\sigma)^{-1/4} \quad (C.622)$$

(d) *Mist flow*

The average flowing density for mist flow is given in Eq. C.66. Since there is virtually no slip between the phases, H_g is:

$$H_g = 1/(1 + q_l/q_g) \quad \text{and} \quad \bar{\rho} = (1 - H_g)\,\rho_L + H_g\,\rho_g \quad (C.623)$$

Duns and Ros express the friction loss gradient as:

$$\tau_f = f\,\rho_g\, v_g^2/2g_c\, d_h \quad (C.624)$$

where v_g is the superficial gas velocity, and f is again obtained from Fig. 2.72, as a function of gas Reynolds number ($N_{Re} = 1488\,\rho_g d_h v_g/\mu_g$) and a correlated form of the Moody relative roughness factor ϵ/d that was developed by Duns and Ros. In their correlation, they limit ϵ/d to being no smaller than 10^{-3} but no greater than 0.5. Between these limits, ϵ/d is determined from Eq. C.665 if N_w is less than 0.005 and from Eq. C.626 if N_w is greater.

$$\epsilon/d = 34\,\sigma/(\rho_g v_g^2 d_h) \quad (C.625)$$

$$\epsilon/d = 174.8\,\sigma\,(N_w)^{0.302}/(\rho_g v_g^2 d_h) \quad (C.626)$$

where $N_w = 4.52 \times 10^{-7}\,(v_g \mu_L/\sigma)^2\,\rho_g/\rho_L$

(e) *Development of τ_f for slug flow*

A new method was developed to correlate the friction loss gradient for slug flow because neither Griffith and Wallis' nor Stanley's (an outgrowth of the Griffith and Wallis work) method proved accurate for the well conditions studied.* This new method accounted for the complex nature of friction loss in slug flow by the introduction of the correlated "liquid distribution" function, δ. (see Eq. C.614) which implicitly accounts for the following physical phenomena:

1) Liquid is distributed in three places: the slug, the film around the gas bubble, and in the gas bubble as entrained droplets; a change in this distribution will change the net friction losses.

2) The friction loss has essentially two contributions; one from the liquid slug and the other from the liquid film.

3) The slip velocity approaches zero as mist flow is approached.

9. Calculate Δh from the following equation

$$\Delta_p = \frac{1}{144}\left[\frac{\bar{\rho} + \tau_f}{1 - w_t\, q_g/4637\, A_p^2\, \bar{p}}\right]_k \Delta h \quad (C.627)$$

from which

$$\Delta h = 144\left[\frac{\Delta P\left(1 - \dfrac{w_t\, q_g}{4637\, A_p^2\, \bar{p}}\right)}{\bar{\rho} + \tau_f}\right] \quad (C.628)$$

10. Iterate, if necessary, starting with step 4 until the assumed Δh equals the calculated Δh.
11. Determine values of p and Z for that increment.
12. Repeat the procedure from step 3 until the sum of the Δh's equals the total length of the flow string.

C.62 EXAMPLE PROBLEM TO CALCULATE A PRESSURE TRAVERSE BY THE ORKISZEWSKI METHOD (SAME EXAMPLE PROBLEM GIVEN BY ORKISZEWSKI[22])

Given:

$q_o = 1{,}850$ bopd
GOR = 575
Water rate = 0
$\rho_o = 18.7°$API $\gamma_o = 0.942$)
Depth = 3,890 ft
$p_{wh} = 670$ psia
$d = 2.992$ in. $= 0.249$ ft
$p_{wf} = 1{,}500$ psi
Surface temp = 126°F
Bottom hole temp = 150°F
$\gamma_g = 0.75$
$A_p = 0.0488$ sq ft
Dead oil viscosity
at 100°F = 89 cp
at 210°F = 8.8 cp

1. Select the starting point as the wellhead.
2. The temperature gradient is noted in Fig. C.61.
3. Fix Δp at 100 psi

$$\bar{p}_k = p_{k-1} + \frac{\Delta p_k}{2} = 670 + \frac{100}{2} = 720 \text{ psia}$$

4. Assume a depth increment:

$$\Delta h = 540 \text{ ft}$$

$$\bar{h}_k = h_{k-1} + \frac{\Delta h_k}{2} = 0 + \frac{540}{2} = 270 \text{ ft}$$

5. From Fig. C.61 determine the average temperature of the increment to be 127.5°F.
6. Find and correct all fluid properties for temperature and pressure.

(a) $R_s = 115$ scf/bbl
$B_o = 1.073$ bbl/stk bbl
$\mu = 18$ cp
$Z = 0.875$

(b) The corrected volumetric flow rates are:

* The Griffith and Wallis data were taken from low flow velocity tests in which friction losses were minor and liquid entrainment was negligible.

$$q_o = 6.49 \times 10^{-5}\ q_o B_o = 6.49 \times 10^{-5}\ (1{,}850)(1.073)$$
$$= 0.129 \text{ cu ft/sec}$$

$$q_g = 3.27 \times 10^{-7}\ c\ q_o (R - R_s)\frac{(\bar{T} + 460)}{\bar{p}}$$

$$= 3.27 \times 10^{-7}\ (0.875)(1{,}850)(575\text{–}115)(587.5)/720$$
$$= 0.199 \text{ cu ft/sec}$$
$$q_t = 0.129 + 0.199 = 0.328 \text{ cu ft/sec}$$

(c) The corrected mass flow rates are:

$$w_L = q_o\ (4.05 \times 10^{-3}\ \gamma_o + 8.85 \times 10^{-7}\ \gamma_g R_s)$$
$$= 1{,}850\ [4.05 \times 10^{-3}\ (0.942) + 8.85 \times 10^{-7}\ (0.75)(115)]$$
$$= 7.20 \text{ lb/sec}$$
$$w_g = 8.85 \times 10^{-7}\ q_o \gamma_g (R - R_s)$$
$$= 8.85 \times 10^{-7}\ (1{,}850)(0.75)(575 - 115)$$
$$= 0.565 \text{ lb/sec}$$
$$w_t = 7.20 + 0.57 = 7.77 \text{ lb/sec}$$

(d) The corrected densities are:

$$\rho_L = w_L/q_L = 7.20/0.129 = 55.8 \text{ lb/cu ft}$$
$$\rho_g = w_g/q_g = 0.565/0.199 = 2.84 \text{ lb/cu ft}$$

7. Determine the type of flow regime. The variables described in Step 6 are calculated; they are then tested against the boundary limits to determine the flow regime.
   a) *Test variables:*

$$v_t = q_t/A_p = 0.328/0.0488 = 6.72 \text{ ft/sec}$$
$$q_g/q_t = 0.199/0.328 = 0.607$$
$$\bar{v}_g = 0.199\ [\sqrt[4]{0.534\ (55.8)}]/0.0488$$
$$= 9.53 \qquad \text{(C.61)}$$

   b) *Boundary limits:*

$$(L)_B = 1.071 - \frac{0.2218\ (6.72)^2}{0.249} \qquad \text{(C.62)}$$

$$(L)_B = -22. \text{ since } (L)_B \text{ must be } \geq 0.13$$

Therefore:

$$(L)_B = 0.13$$
$$(L)_s = 50 + 36\ (9.53)(0.129)/0.199$$
$$= 272$$

Since $q_g/q_t > (L)_B$ and $\bar{v}_g < (L)_s$, the fluids are in slug flow.

8. Determine the average density and friction loss gradient.
   a) Determine Reynolds number, bubble Reynolds number, and slip velocity (v_s).

$$N_{Re} = 1{,}488\ (55.8)(0.249)(6.72)/18$$
$$= 7{,}720 \qquad \text{(C.610)}$$

Since the slip velocity is a nonlinear correlation, iteration is necessary. Therefore assuming $v_s = 1.75$, bubble Reynolds number is

$$N_b = 1{,}488\ (55.8)(0.249)(1.75)/18$$
$$= 2{,}010$$

C_2 cannot be read from Fig. 2.75; therefore extrapolation of Eq. C.611 is used since $N_b < 3{,}000$.

$$v_s = [0.546 + 8.74 \times 10^{-6}\ (7{,}720)]\ \sqrt{32.2\ (0.249)}$$
$$= 1.74 \text{ ft/sec}$$

   b) Determine liquid distribution coefficient(δ) and friction factor (f). Eq. C.617 is used to evaluate δ since $v_t < 10$

$$\delta = \left[\frac{0.0127 \log (18 + 1)}{(0.249)^{1.415}}\right] - 0.284 + 0.167 \log 6.72 + 0.113 \log 0.249$$
$$= -0.097$$

Test limiting δ with Eq. C.619:

$$-0.097 \geq -0.065\ (6.72)$$
$$\geq -0.436$$

Therefore:

$$\delta = -0.097$$

ϵ/d, from Fig. 2.73, is 0.0006; with this value and the calculated N_{Re} of 7,720, a friction factor of 0.034 is read from Fig. 2.72.

   c) Evaluate $\bar{\rho}$ with Eq. C.68

$$\bar{\rho} = \frac{7.77 + 55.8\ (1.74)(0.0488)}{0.328 + 1.74\ (0.0488)} + (-0.097)(55.8)$$
$$= 24.9 \text{ lb/cu ft}$$

   d) Evaluate τ_f with Eq. C.614

$$\tau_f = \frac{0.034\ (55.8)(6.72)^2}{64.4\ (0.249)}\left[\frac{0.129 + 1.74\ (0.0488)}{0.328 + 1.74\ (0.0488)} - 0.097\right]$$
$$= 2.26 \text{ psi/ft}$$

9. The depth increment from Eq. (C.628)

$$\Delta h_1 = 144\left[\frac{\Delta p_1\left(1 - \frac{w_t q_g}{4637\ A_p^2\ \bar{p}}\right)}{\bar{\rho} + \tau_f}\right]_1$$
$$= 144\left[\frac{100\left(1 - \frac{7.77\ (0.199)}{4637\ (0.0488)^2(720)}\right)}{24.9 + 2.3}\right]_1$$
$$= 529 \text{ ft}$$

The true value of Δh_1 is near 529 ft. The calculation will converge very closely to this value even when the assumed Δh is off by ±10% of the assumed value (540 ft) because, under these well conditions, the pressure gradient is primarily controlled by the relatively temperature-insensitive density head. However, under those circumstances where the friction gradient, which is temperature sensitive, is significant, iteration would be necessary should the calculated value of Δh differ from the assumed value by ±10%.

10. The top of the next increment is fixed at 529 ft and 770 psi, and steps (1)–(10) are repeated for the new condition.
11. The procedure is continued until $\Sigma\ \Delta h$ is equal to the total depth.

C.63 EXAMPLE PROBLEM TO CALCULATE A PRESSURE TRAVERSE BY ORKISZEWSKI METHOD

This is the same problem that was worked by all other methods except that 1¼ in. tubing is used instead of 2 in.

1. Select the starting point as the 500 psig pressure.
2. The temperature at each point of pressure is given as 120°F at 500 psig and 150°F at 1,000 psig.
3. For this problem set the Δp at 1,000 − 500 = 500 psig.
4. It is not necessary to assume depth increment since the temperature at p_2 is known.
5. The average temperature of the increment is $\frac{120 + 150}{2} =$ 135°F.
6. The following calculations are made in order to complete step 7 to determine flow regime.
   (a) Average flow conditions are:

$$\Delta\bar{p} = \frac{500 + 1{,}000}{2} + 14.7 = 750 \text{ psig} = 764.7 \text{ psia.}$$

$$\bar{T} = 595°R$$
$$\bar{Z} = 0.91$$
$$\bar{R}_s = 92 \text{ scf/stk bbl}$$
$$\bar{B}_o = 1.06 \text{ bbl/stk bbl}$$

(b) The corrected volumetric flow rates are:

$$q_L = 6.49(10^{-5})(q_o B_o + q_w B_w)$$
$$= 6.49(10^{-5})(400(1.06) + 600) = 0.0665 \text{ cu. ft/sec}$$
$$q_g = 3.27(10^{-7})z(GLR - R_s)(\bar{T} + 460)q/\bar{p}$$
$$= 3.27(10^{-7})0.91(500 - 92)1{,}000(595)/764.7$$
$$= 0.0944 \text{ cu. ft/sec}$$
$$q_t = q_L + q_g = 0.1609 \text{ cu ft/sec}$$

(c) The corrected mass flow rates are:

$$w_L = 4.05(10^{-3})(q_o\gamma_o + q_w\gamma_w) + 8.85(10^{-7})q\gamma_g(R_s)$$
$$= 4.05(10^{-3})(400(0.9218) + 600)$$
$$+ 8.85(10^{-7})1{,}000(0.65)92 = 4.143 \text{ lb}_m/\text{sec}$$
$$w_g = 8.85(10^{-7})q\gamma_g(GLR - R_s)$$
$$= 8.85(10^{-7})1{,}000(0.65)(500 - 92) = 0.235 \text{ lb}_m/\text{sec}$$
$$w_t = w_L + w_g = 4.143 + 0.235 = 4.378 \text{ lb}_m/\text{sec}$$

(d) The corrected densities are:

$$\rho_L = w_L/q_L = 4.143/0.0665 = 62.2 \text{ lb}_m/\text{cu ft}$$
$$\rho_g = w_g/q_g = 0.235/0.0944 = 2.49 \text{ lb}_m/\text{cu ft}$$

7. Determine the type of flow regime:
(a) Test variables:

A_p = Flow area of tubing (calc. in Hagedorn & Brown method)

$$v_t = q_t/A_p = 0.1609/0.1036 = 15.46 \text{ ft/sec}$$
$$q_g/q_t = 0.0944/0.1609 = 0.586$$
$$\tilde{v}_g = 1.938\, v_{sL}\left(\frac{\rho_L}{\sigma_L}\right)$$

ρ_L = Value calculated in Hagedorn and Brown solution

$$v_{sL} = q_L/A_p = 0.0944/0.01036, \text{ then:}$$
$$\tilde{v}_g = 1.938(0.0944/0.01036)(62.2/54)^{1/4} = 18.15$$

(b) Boundary limits:

$$d_{hy} = 0.115 \text{ ft}$$
$$(L)_B = 1.071 - (0.2218\, v_t^2/d_{hy})$$
$$(L)_B = 1.071 - (0.2218(15.46)^2/0.115 = -460$$

Checking the limit of $(L)_B$ above, we must use $(L)_B = 0.13$.

$$(L)_S = 50 + 36v_g\, q_L/q_g$$
$$(L)_S = 50 + 36(18.15)(0.0665/0.0944) = 510$$

These two values indicate that the regime is *slug* flow.

8. Determine the average density and the friction loss gradient.
(a) Determination of the Reynolds number:

$$N_{Re} = 1488\, \rho_l v_t d_{hy}/\mu_l$$
$$N_{Re} = 1488(62.2)0.115(15.46)/7.162 = 22{,}900$$

Calculation of Bubble Reynolds number:

$$N_b = 1488\, \rho_l d_{hy} v_s/\mu_l$$

Since the slip velocity is a nonlinear correlation, iteration is necessary, therefore assuming $v_s = 1.50$

$$N_b = 1{,}488\ (62.2)\ 0.115\ (1.5)/7.162 = 2{,}220$$

because $N_b < 3{,}000$, then use Eq. C.611:

$$v_s = 0.546 + 8.74(10^{-6})\ 22{,}900\ [32.2(0.115)]^{1/2}$$
$$v_s = 1.44 \text{ ft/sec}$$

(b) Determination of liquid distribution coefficient, δ, and friction factor, f:
Eq. C.617 is used to evaluate δ since $v_t > 10$ and water is the principal liquid flow:

$$\delta = 0.0127 \log(8.178)/0.115^{1.415} - 0.284$$
$$+ 0.167 \log(15.46) + 0.113 \log(0.115)$$
$$= 0.117$$

Checking this value:

$$0.117 \geq -0.065(15.45)$$
$$0.117 > -1.01 \text{ therefore } \delta \text{ is O.K.}$$
$$\epsilon/d_{hy} = 7.52(10^{-5}) \text{ from Fig. 2.73}$$
$$f = 0.025 \text{ from Fig. 2.72}$$

(c) Evaluation of average flowing density, $\bar{\rho}$, from Eq. C.68:

$$\bar{\rho} = \frac{4.378 + 62.2(1.44)(0.01036)}{0.1609 + 1.44(0.1036)} + 0.117(62.2)$$
$$= 30.20 + 7.28 = 37.48 \text{ lb}_m/\text{cu ft}$$

(d) Calculation of wall friction loss term by use of Eq. C.614:

$$\tau_f = f_1\rho_L v_t^2 \left[\frac{q_L + v_s}{q_t + v_s A_p} + \delta\right]/64.4\ d_{hy}$$
$$= \frac{0.025(62.2)(15.45)^2}{64.4(0.115)}$$
$$\left[\frac{0.0665 + 144(0.01036)}{0.1609 + 1.44(0.01036)} + 0.117\right]$$
$$= 29.2 \text{ lb}_f/\text{sq ft}$$

9. Calculation of depth:

$$\Delta h = 144\left[\frac{500\left(1 - \frac{4.378(0.0944)}{4637(0.01036)(764.7)}\right)}{37.48 + 29.2}\right]$$
$$= 1{,}080 \text{ ft}$$

These calculations will vary slightly depending upon whether the flow regime is bubble, transition or mist.

10. Iteration is not necessary since we know the temperatures and could solve directly for Δh. The distance is found to be 2,100 ft for 2 in. tubing.

Appendix C.7

C.71 PROCEDURE FOR CALCULATING A PRESSURE TRAVERSE BY THE BEGGS AND BRILL METHOD

Starting with the equation:

$$\frac{\Delta p}{\Delta z} = \frac{\frac{g\,\rho_{tp}}{g_c}\sin\theta + \frac{f_{tp}G_m v_m}{2g_c d}}{1 - \frac{\rho_{tp} v_m v_{sg}}{g_c p}}$$

Solve the equation for the depth increment, Δz.

$$\Delta z = \frac{\Delta p\left[1 - \frac{\rho_{tp} v_m v_{sg}}{g_c\, p}\right]}{\frac{g\,\rho_{tp}}{g_c}\sin\theta + \frac{f_{tp}G_m v_m}{2\,g_c d}}$$

Start with a known pressure p_1, select a value for p_2 and assume a value Δz between the two points so that temperature $\overline{T}$, which is a function of depth, can be evaluated. Calculate the depth increment, which should coincide with the estimated value.

1. Calculate the average pressure and average depth between the two points.

$$\overline{p} = \frac{p_1 + p_2}{2} \qquad \overline{z} = \text{depth where } \overline{p}_1 \text{ occurs, } z + \frac{\Delta z}{2}$$

2. Determine the average temperature, $\overline{T}$ at the average depth. This value must be known from a temperature vs. depth survey.
3. From PVT analysis or appropriate correlations, calculate R_s, B_o, B_w, μ_o, μ_w, μ_g, σ_o, σ_w, and z_g at $\overline{T}$ and $\overline{p}$.
4. Calculate the specific gravity of the oil γ_o.

$$\gamma_o = \frac{141.5}{131.5 + API}$$

5. Calculate the liquid and gas densities at the average conditions of pressure and temperature.

$$\rho_L = \rho_o\left(\frac{1}{1+WOR}\right) + \rho_w\left(\frac{WOR}{1+WOR}\right) = \rho_o f_o + \rho_w f_w$$

$$\rho_o = \frac{350\,\gamma_o + 0.0764\,R_s\,\gamma_g}{5.615\,B_o}$$

$$\rho_w = \frac{350\,\gamma_w}{5.615\,B_w}$$

$$\rho_g = \frac{0.0764\,\gamma_g\,\overline{p}\,(520)}{(14.7)\,(\overline{T}+460)z_g}$$

6. Calculate the in situ gas and liquid flow rates.

$$q_g = \frac{3.27\times10^{-7}\,z_g q_o\,(R - R_s)\,(\overline{T}+460)}{\overline{p}}$$

$$q_L = 6.49\times10^{-5}\,(q_o B_o + q_w B_w)$$

7. Calculate the in situ superficial gas, liquid, and mixture velocities.

$$v_{sL} = q_L/Ap$$
$$v_{sg} = q_g/Ap$$
$$v_m = v_{sL} + v_{sg}$$

8. Calculate the liquid, gas, and total mass flux rates.

$$G_L = \rho_L\,v_{sL} \qquad G_g = \rho_g\,v_{sg}$$
$$G_m = G_L + G_g$$

9. Calculate the input liquid content (no-slip holdup).

$$\lambda = \frac{q_L}{q_L + q_g}$$

10. Calculate the Froude number, N_{FR}, the liquid viscosity, μ_L, the mixture viscosity, μ_m and the liquid surface tension, σ_L.

$$N_{FR} = \frac{v_m^2}{g\,d} \qquad \mu_L = \mu_o f_o + \mu_w f_w$$
$$\mu_m = (\mu_L\lambda + \mu_g(1-\lambda))(6.72\times10^{-4})$$
$$\sigma_L = \sigma_o f_o + \sigma_w f_w$$

11. Calculate the no-slip Reynolds number and the liquid velocity number.

$$(N_{Re})_{ns} = \frac{G_m d}{\mu_m} \qquad N_{Lv} = 1.938\,v_{sL}\left(\frac{\rho_L}{\sigma_L}\right)^{0.25}$$

12. To determine the flow pattern which would exist if flow were horizontal, calculate the correlating parameters, L_1, L_2, L_3 and L_4:

$$L_1 = 316\,\lambda^{0.302} \qquad L_2 = 0.0009252\,\lambda^{-2.4684}$$
$$L_3 = 0.10\,\lambda^{-1.4516} \qquad L_4 = 0.5\,\lambda^{-6.738}$$

13. Determine flow pattern using the following limits:
Segregated:
$\lambda < 0.01$ and $N_{FR} < L_1$
or $\lambda \geq 0.01$ and $N_{FR} < L_2$
Transition:
$\lambda \geq 0.01$ and $L_2 < N_{FR} \leq L_3$
Intermittent:
$0.01 \leq \lambda < 0.4$ and $L_3 < N_{FR} \leq L_1$
or $\lambda \geq 0.4$ and $L_3 < N_{FR} \leq L_4$
Distributed:
$\lambda < 0.4$ and $N_{FR} \geq L_1$
or $\lambda \geq 0.4$ and $N_{FR} > L_4$
14. Calculate the horizontal holdup, $H_L(O)$.

$H_L(O)\,\frac{a\lambda^b}{N_{FR}^c}$ where a, b, and c are determined for each flow pattern from the table:

Flow pattern	a	b	c
Segregated	0.98	0.4846	0.0868
Intermittent	0.845	0.5351	0.0173
Distributed	1.065	0.5824	0.0609

15. Calculate the inclination correction factor coefficient.

$C = (1-\lambda)\ln(d\lambda^e N_{Lv}^f N_{FR}^g)$ where d, e, f, and g are determined for each flow condition from the table:

Flow pattern	d	e	f	g
Segregated uphill	0.011	−3.768	3.539	−1.614
Intermittent uphill	2.96	0.305	−0.4473	0.0978
Distributed uphill	No Correction		C = 0	
All flow patterns downhill	4.70	−0.3692	0.1244	−0.5056

16. Calculate the liquid holdup inclination correction factor.

$\psi = 1 + C\,[\sin(1.8\theta) - 0.333\sin^3(1.8\theta)] = 1 + 0.3C$ for vertical well.

17. Calculate the liquid holdup and the two-phase density.

$$H_L(\theta) = H_L(O)\,\psi$$
$$\rho_{tp} = \rho_L H_L + \rho_g(1 - H_L)$$

18. Calculate the friction factor ratio.

$\frac{f_{tp}}{f_{ns}} = e^s$ where $S = [\ln(y)]/\{-0.0523 + 3.182\ln(y) - 0.8725[\ln(y)]^2 + 0.01853[\ln(y)]^4\}$,

and

$$y = \frac{\lambda}{[H_L(\theta)]^2}$$

S becomes unbounded at a point in the interval $1 < y < 1.2$; and for y in this interval, the function S is calculated from

$$S = \ln(2.2\,y - 1.2)$$

19. Calculate the no-slip friction factor.

$$f_{ns} = 1/[2 \log (N_{Rens}/(4.5223 \log N_{Rens} - 3.8215))]^2$$

or:

$$f_{ns} = 0.0056 + \frac{0.5}{(N_{Rens})^{0.32}}$$

20. Calculate the two-phase friction factor.

$$f_{tp} = f_{ns}\frac{f_{tp}}{f_{ns}}$$

21. Calculate Δz.
If the estimated and calculated values for Δz are not sufficiently close, the calculated value is taken as the new estimated value and the procedure is repeated until the values agree. A new pressure increment is then chosen and the process is continued until the sum of the Δz's is equal to the well depth.

C.72 EXAMPLE PROBLEM FOR CALCULATING A PRESSURE TRAVERSE BY THE BEGGS AND BRILL METHOD

The following problem is worked by the Beggs and Brill Method. In order to illustrate the procedure more clearly, the distance between the 500 psig and the 1,000 psig points will be determined. In actual practice much smaller pressure increments would be chosen for increased accuracy.

Given: $d = 1.995$ in.
$p_1 = 500$ psig
$p_2 = 1{,}000$ psig
$p_a = 14.7$ psia

Temperature $= 120 + 0.017z$

$\theta = 0°$ from vertical $= 90°$ from horizontal
$\gamma_g = 0.65$
$\gamma_w = 1.07$
$\gamma_o = 22°$ API
$q_o = 400$ stk bpd
$q_w = 600$ stk bpd (WOR = 1.5)
$\mu_g = 0.018$ cp (constant)
$\sigma_o = 30$ dynes/cm (constant)
$\sigma_w = 70$ dynes/cm (constant)
GLR = 500 scf/stk bbl

1. Estimate the distance between the 500 psig and 1,000 psig points to be 2,000 ft. Calculate the average pressure and average depth.

$$\bar{p} = \frac{500 + 1000}{2} + 14.7 = 764.7 \text{ psia}$$

$$\bar{z} = 0 + \frac{2000}{2} = 1{,}000 \text{ ft}$$

2. Determine $\bar{T}$.

$$\bar{T} = 120 + 0.017\,(1{,}000) = 137°\text{F}$$

3. From correlations, the following values are determined at 764.7 psia and 137°F.

$R_s = 92$ scf/stk bbl — $\mu_w = 0.63$ cp
$B_o = 1.063$ — $z_g = 0.91$
$\mu_o = 17$ cp — $B_w = 1.00$

4. Calculate the oil specific gravity.

$$\gamma_o = \frac{141.5}{131.5 + 22} = 0.922$$

5. Calculate the liquid and gas densities at 764.7 psia and 137°F.

$$\rho_o = \frac{(350)(0.922) + (0.0764)(0.65)(92)}{(5.615)(1.063)} = 54.07 \text{ lb}_m\text{/cu ft}$$

$$\rho_w = \frac{(350)(1.07)}{(5.615)(1)} = 66.70 \text{ lb}_m\text{/cu ft}$$

$$f_o = \frac{1}{1 + 1.5} = 0.4 \qquad f_w = \frac{1.5}{1 + 1.5} = 0.6$$

$$\rho_L = \rho_o f_o + \rho_w f_w = (54.07)(0.4) + (66.70)(0.6) = 61.65 \text{ lb}_m\text{/cu ft}$$

$$\rho_g = \frac{(0.0764)(0.65)(764.7)(520)}{(14.7)(137 + 460)(0.91)} = 2.473 \text{ lb}_m\text{/cu ft}$$

Calculate the gas and liquid in situ flow rates.

$$q_g = \frac{(3.27 \times 10^{-7})(0.91)(400)(1250 - 92)(137 + 460)}{764.7} = 0.1076 \text{ cu ft/sec}$$

$$q_L = (6.49 \times 10^{-7})\,[(400)(1.063) + (600)(1)] = 0.06653 \text{ cu ft/sec}$$

7. Calculate the in situ superficial liquid, gas, and mixture velocities.

$$Ap = \frac{\pi d^2}{4(144)} = \frac{\pi(1.995)^2}{4(144)} = 0.0217 \text{ sq ft}$$

$$v_{sL} = \frac{q_L}{Ap} = \frac{0.06653}{0.0217} = 3.066 \text{ ft/sec}$$

$$v_{sg} = \frac{q_g}{Ap} = \frac{0.1076}{0.0217} = 4.96 \text{ ft/sec}$$

$$v_m = v_{sL} + v_{sg} = 3.066 + 4.96 = 8.026 \text{ ft/sec}$$

8. Calculate the mass flux rates.

$G_L = \rho_L v_{sL} = (61.65)(3.066) = 189.02$ lb$_m$/sec-sq ft
$G_g = \rho_g v_{sg} = (2.473)(1.747) = 4.32$ lb$_m$/sec-sq ft
$G_m = G_L + G_g = 139.02 + 4.32 = 197.66$ lb$_m$/sec-sq ft

9. Calculate the no-slip holdup.

$$\lambda = \frac{q_L}{q_L + q_g} = \frac{0.06653}{0.06653 + 0.1076} = 0.382$$

10. Calculate the Froude number, the liquid viscosity, mixture viscosity, and liquid surface tension.

$$N_{FR} = \frac{v_m^2}{g\,d} = \frac{(4.813)^2\,(12)}{(32.2)(1.995)} = 4.327$$

$\mu_L = \mu_o f_o + \mu_w f_w = 17(0.4) + 0.63(0.6) = 7.18$
$\mu_m = (6.27 \times 10^{-4})(\mu_L\lambda + \mu_g(1 - \lambda))$
$= (6.27 \times 10^{-4})((7.18)(0.382) + (0.018)(0.618))$
$= 0.0017$ lb$_m$/ft-sec
$\sigma_L = \sigma_o f_o + \sigma_w f_w = 30(0.4) + 70(0.6) = 54$ dynes/cm

11. Calculate the no-slip Reynolds number and the liquid velocity number.

$$N_{Rens} = \frac{G_m\,d}{\mu_m} \quad \frac{(197.66)(1.995)}{(12)(0.0017)} = 19{,}330$$

$$N_{Lv} = 1.938\,v_{sL}\left(\frac{\rho_L}{\sigma_L}\right)^{0.25}$$

$$= 1.938\,(3.066)\left(\frac{61.65}{54}\right)^{0.25} = 6.142$$

12. Calculate L_1, L_2, L_3, L_4:

$L_1 = 316(0.382)^{0.302} = 236$
$L_2 = 0.0009252(0.382)^{-2.4684} = 0.010$
$L_3 = 0.10(0.382)^{-1.4516} = 0.404$
$L_4 = 0.5(0.382)^{-6.738} = 327$

13. Determine flow pattern:

Since $0.01 \leq \lambda < 0.4$ and $L_3 < N_{FR} \leq L_1$ flow pattern is intermittent.

14. Calculate horizontal holdup.

$$H_L(O) = \frac{0.845\ \lambda^{0.5351}}{N_{FR}^{0.0173}} = \frac{0.845\ (0.382)^{0.5351}}{(4.327)^{0.0173}} = 0.492$$

15. Calculate the inclination correction factor coefficient.

$C = (1 - \lambda)1n(2.96\ \lambda^{0.305}\ N_{Lv}^{-0.4473}\ N_{FR}^{0.0978})$
$= (1 - 0.382)\ \ln\ [(2.96)(.382)^{0.305}(6.142)^{-0.4473}(4.327)^{0.0978}]$
$= 0.076$

16. Calculate the liquid holdup inclination correction factor.

$$\psi = 1 + 0.3C = 1 + 0.3(0.076) = 1.023$$

17. Calculate the liquid holdup and two-phase density.

$H_L(60) = H_L(O)\psi = (0.492)(1.023) = 0.503$
$\rho_{tp} = \rho_L H_L + \rho_g(1 - H_L)$
$= (61.65)(0.503) + (2.473)(1 - 0.503)$
$= 32.24\ lb_m/cu\ ft$

18. Calculate the friction factor ratio.

$$y = \left[\frac{\lambda}{(H_L(90))^2}\right] = \left[\frac{0.382}{(0.503)^2}\right] = 1.510$$

$\ln y = 0.412$

$$\frac{f_{tp}}{f_{ns}} = EXP(0.412/(-0.0523 + 3.182(0.412) - 0.8725(0.412)^2 + 0.01853\ (0.412)^4))$$
$$= EXP\ (0.371) = 1.449$$

19. Calculate the no-slip friction factor.

$f_{ns} = 1/[2\ \log(N_{Rens}/(4.5223\ \log\ N_{Rens} - 3.8215))]^2$
$= 1/[2\ \log(19{,}330/(4.5223\ \log\ (19{,}330) - 3.8215))]^2$
$f_{ns} = 0.0261$

20. Calculate the two-phase friction factor.

$$f_{tp} = f_{ns}\frac{f_{tp}}{f_{ns}} = (0.0261)\ (1.449) = 0.0378$$

21. Calculate Δz.

$$\Delta z = \frac{\Delta p\left[1 - \frac{\rho_{tp}\ v_m\ v_{sg}}{g_c\ p}\right]}{\frac{g}{g_c}\rho_{tp}\ \sin\ \theta + \frac{f_{tp}\ G_m\ v_m}{2\ g_c\ d}}$$

$$\frac{(500)(144)\left[1 - \frac{(32.24)(8.026)(4.96)}{(32.17)(764.7)(144)}\right]}{(32.24)(1.000) + \frac{(0.0378)(197.66)(8.026)}{(2)(32.17)(0.1663)}}$$

$\Delta z = 1{,}901$ ft.

Appendix C.8

APPENDIX C.8

Fig. no.	Rate	Fluid	Tubing size
C.81	200 B/D	oil	1″
C.82	200	oil	1 1/4
C.83	"	"	1 1/2
C.84	"	"	1 3/4
C.85	100	"	2″
C.86	200	"	"
C.87	300	"	"
C.88	400	"	2″
C.89	600	"	"
C.90	800	"	"
C.91	1000	"	"
C.92	1500	"	"
C.93	800	50% water	2″
C.94	600	100% water	2″
C.95	1000	"	2″
C.96	1500	"	2″
C.97	50	oil	2 1/2″
C.98	100	"	"
C.99	200	"	"
C.100	300	"	"
C.101	400	"	"
C.102	500	"	"
C.103	600	"	"
C.104	800	"	"
C.105	1000	"	"
C.106	1500	"	"
C.107	2000	"	"
C.107 B	3000	100% oil	2 1/2″
C.108	600	50% water	2 1/2
C.109	800	100% water	"
C.110	1000	"	"
C.111	1500	"	"
C.112	1000	100% oil	3″
C.113	2000	"	"
C.114	3000	"	"
C.115	4000	"	"
C.116	1000	water	3″
C.117	1500	"	"
C.118	2000	"	"
C.119	3000	"	"
C.120	1000	oil	4″
C.121	2000	"	"
C.122	3000	"	"
C.123	4000	"	"
C.124	6000	"	"
C.125	8000	"	"
C.126	4000	50% oil	4″
C.127	1000	water	"
C.128	2000	"	"
C.129	4000	"	"
C.130	6000	"	"
C.131	8000	"	"

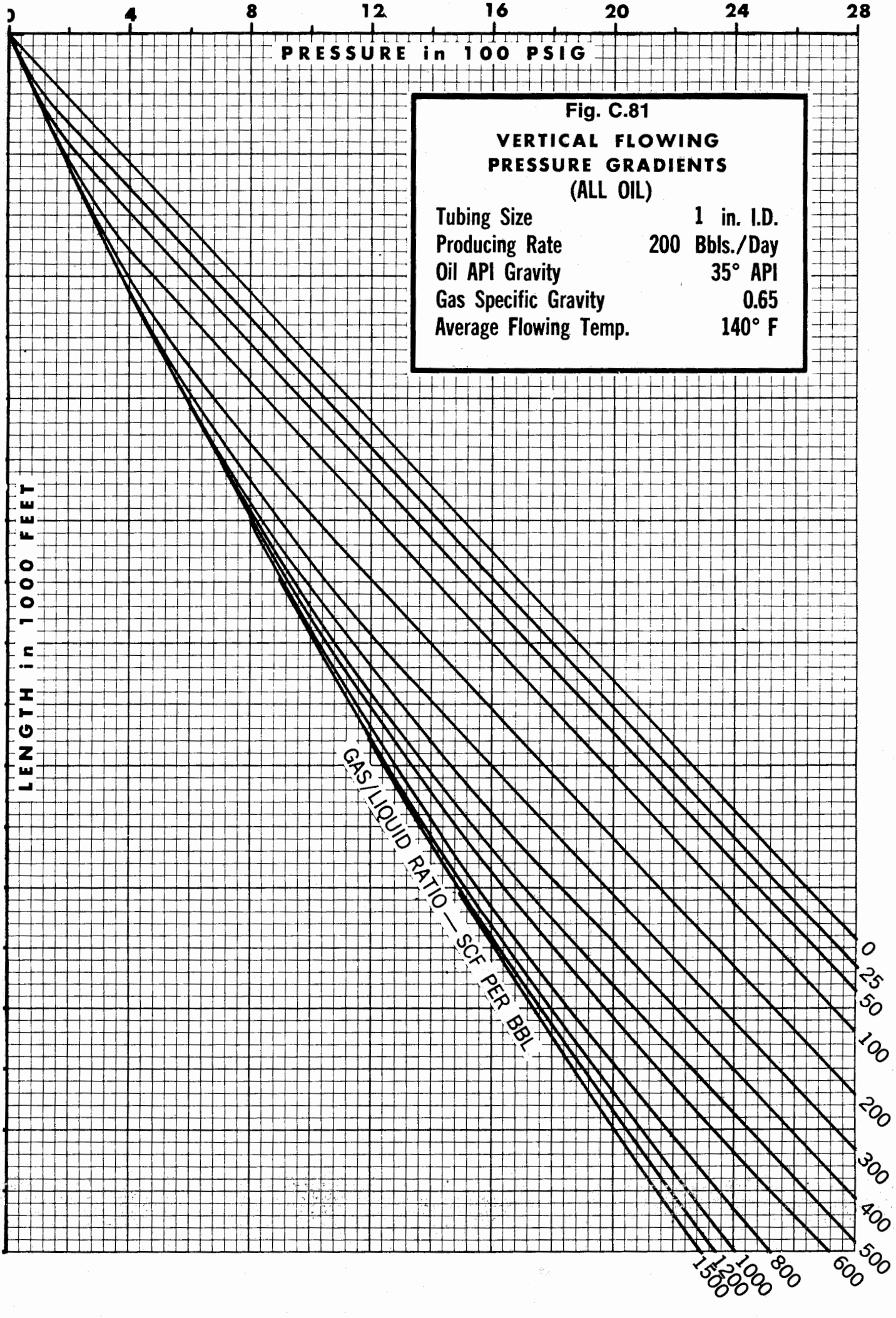
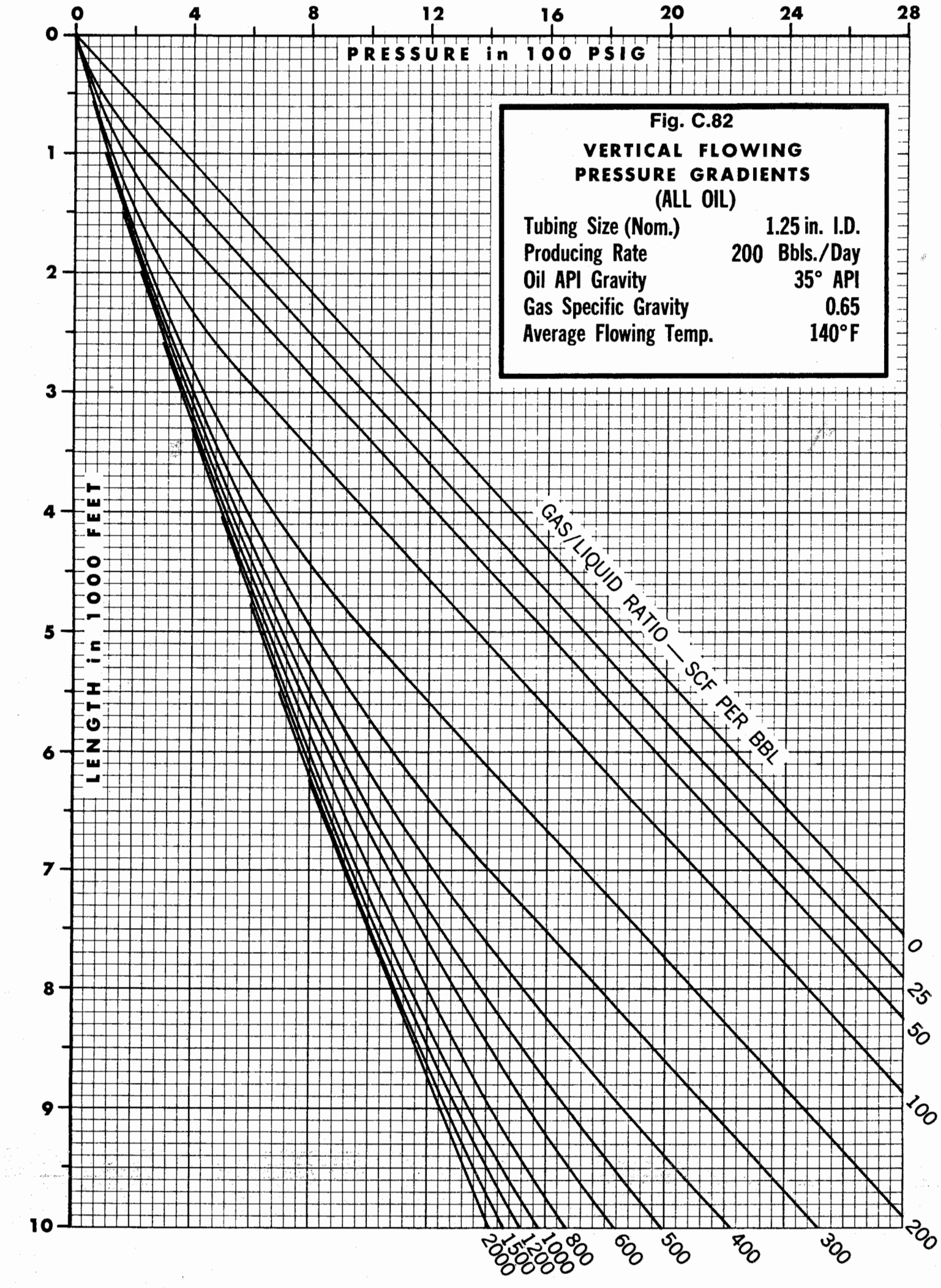


Fig. C.81

VERTICAL FLOWING PRESSURE GRADIENTS

(ALL OIL)

Tubing Size	1 in. I.D.
Producing Rate	200 Bbls./Day
Oil API Gravity	35° API
Gas Specific Gravity	0.65
Average Flowing Temp.	140° F


0
4
8
12
16
20
24
28
PRESSURE in 100 PSIG
0
1
2
3
4
5
6
7
8
9
10
LENGTH in 1000 FEET
Fig. C.82
VERTICAL FLOWING
PRESSURE GRADIENTS
(ALL OIL)
Tubing Size (Nom.) 1.25 in. I.D.
Producing Rate 200 Bbls./Day
Oil API Gravity 35° API
Gas Specific Gravity 0.65
Average Flowing Temp. 140°F
GAS/LIQUID RATIO — SCF PER BBL
0
25
50
100
200
300
400
500
600
800
1000
1200
1500
2000

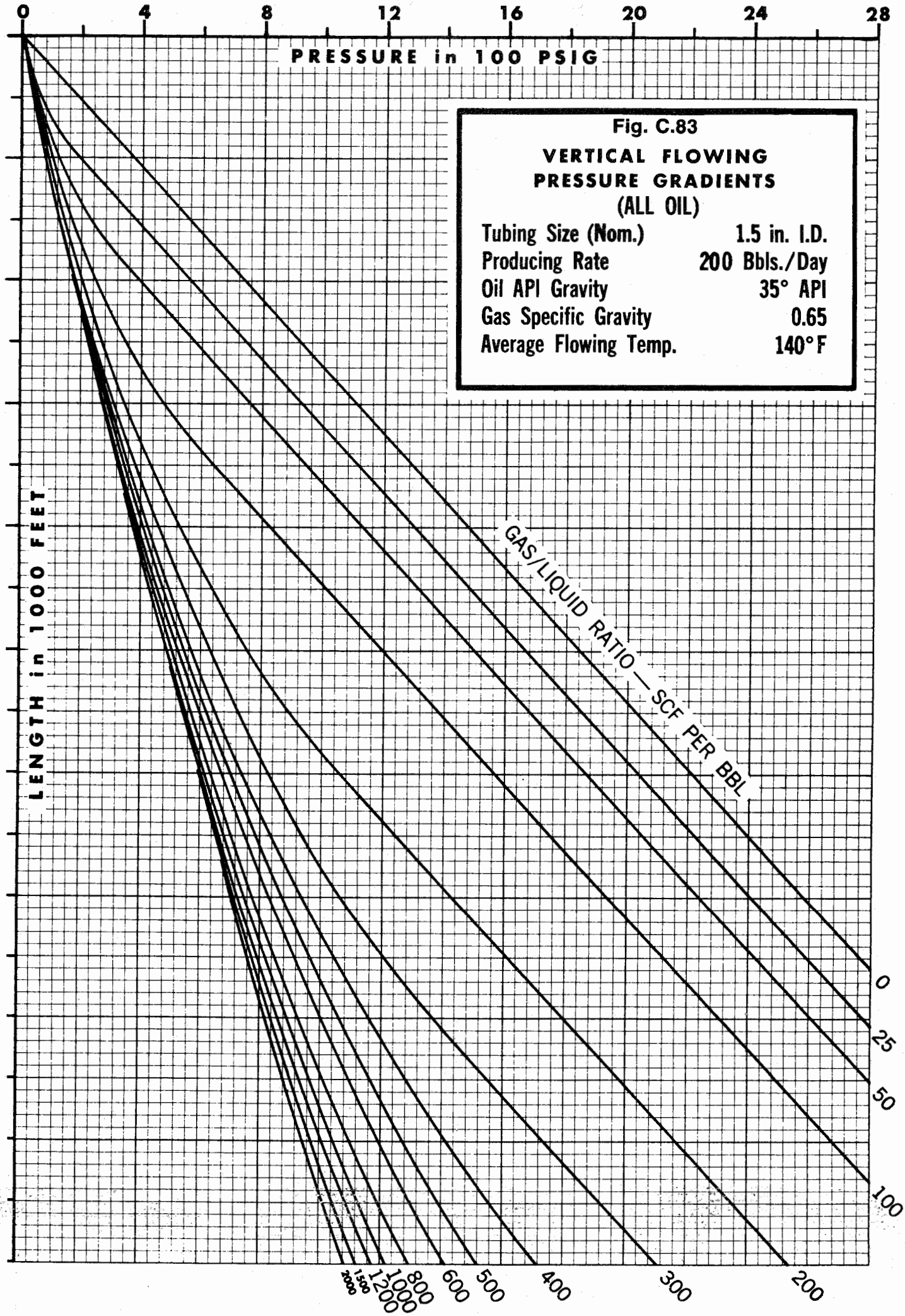


Fig. C.83

VERTICAL FLOWING PRESSURE GRADIENTS

(ALL OIL)

Tubing Size (Nom.)	1.5 in. I.D.
Producing Rate	200 Bbls./Day
Oil API Gravity	35° API
Gas Specific Gravity	0.65
Average Flowing Temp.	140° F

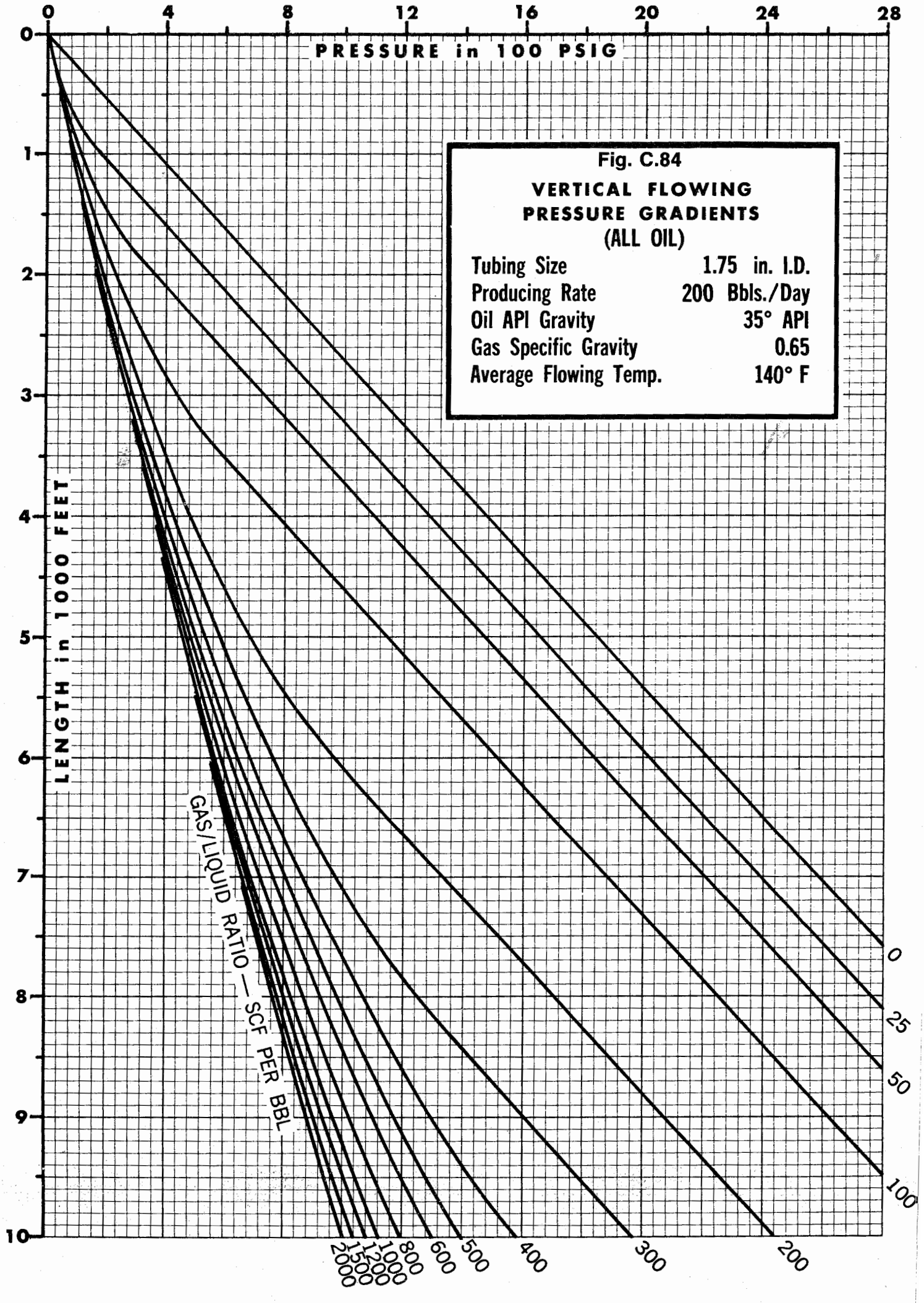


Fig. C.84
VERTICAL FLOWING
PRESSURE GRADIENTS
(ALL OIL)
Tubing Size 1.75 in. I.D.
Producing Rate 200 Bbls./Day
Oil API Gravity 35° API
Gas Specific Gravity 0.65
Average Flowing Temp. 140° F
PRESSURE in 100 PSIG
0
4
8
12
16
20
24
28
LENGTH in 1000 FEET
0
1
2
3
4
5
6
7
8
9
10
GAS/LIQUID RATIO — SCF PER BBL
0
25
50
100
200
300
400
500
600
800
1000
1200
1500
2000

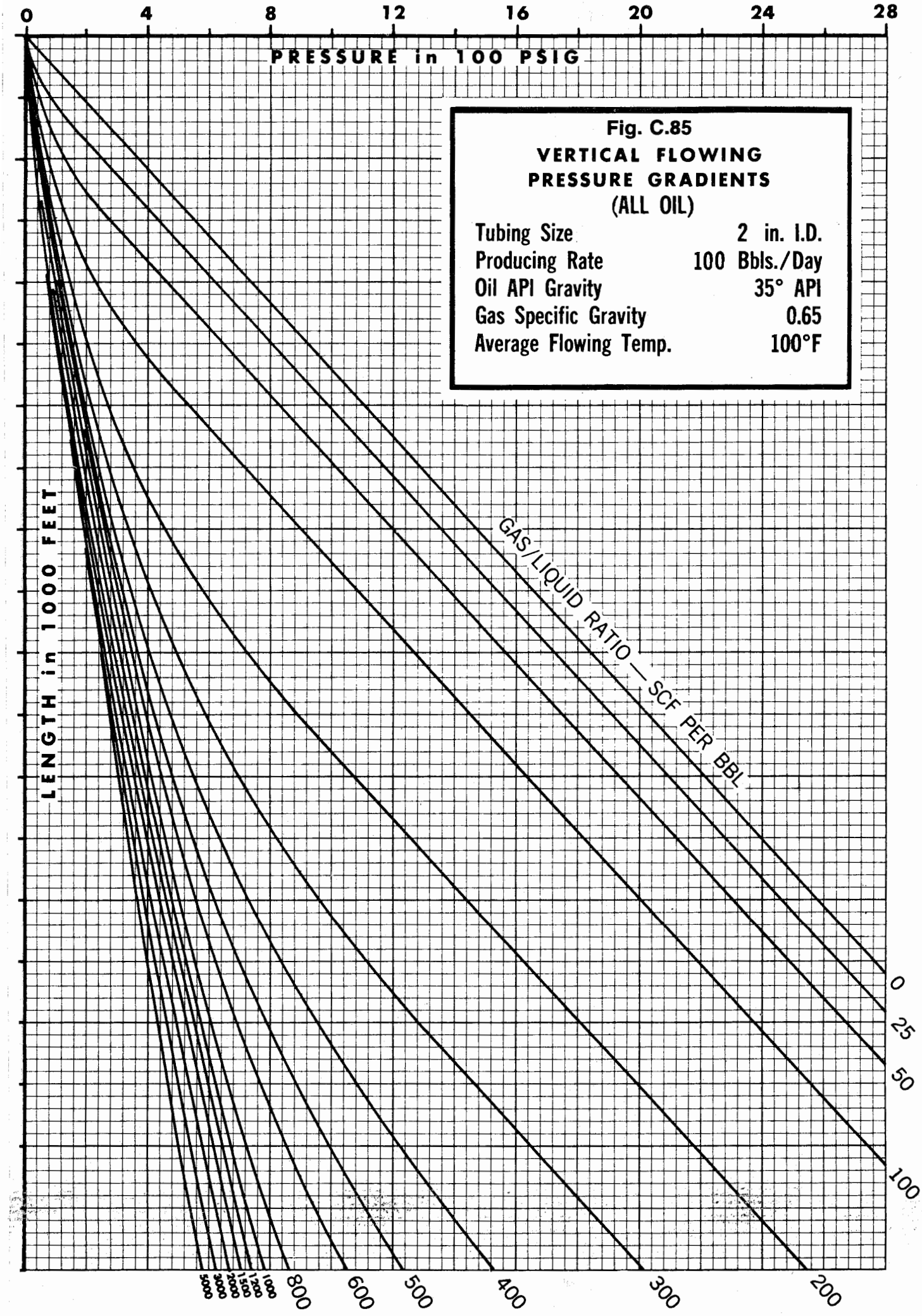
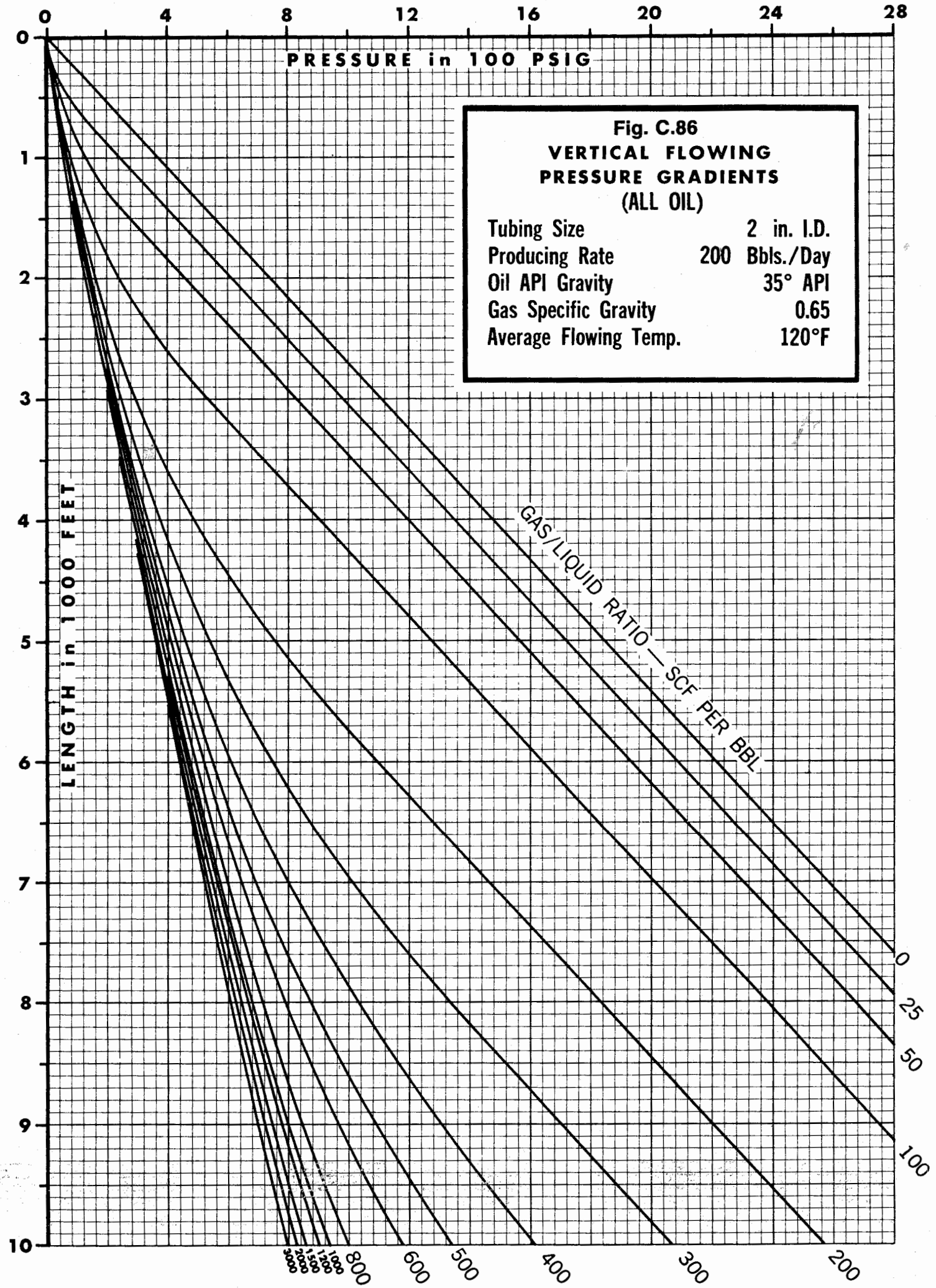


Fig. C.85
VERTICAL FLOWING PRESSURE GRADIENTS
(ALL OIL)

Tubing Size	2 in. I.D.
Producing Rate	100 Bbls./Day
Oil API Gravity	35° API
Gas Specific Gravity	0.65
Average Flowing Temp.	100°F


0
4
8
12
16
20
24
28
PRESSURE in 100 PSIG
0
1
2
3
4
5
6
7
8
9
10
LENGTH in 1000 FEET
Fig. C.86
VERTICAL FLOWING
PRESSURE GRADIENTS
(ALL OIL)
Tubing Size 2 in. I.D.
Producing Rate 200 Bbls./Day
Oil API Gravity 35° API
Gas Specific Gravity 0.65
Average Flowing Temp. 120°F
GAS/LIQUID RATIO — SCF PER BBL
0
25
50
100
200
300
400
500
600
800
1000
1200
1500
2000
3000

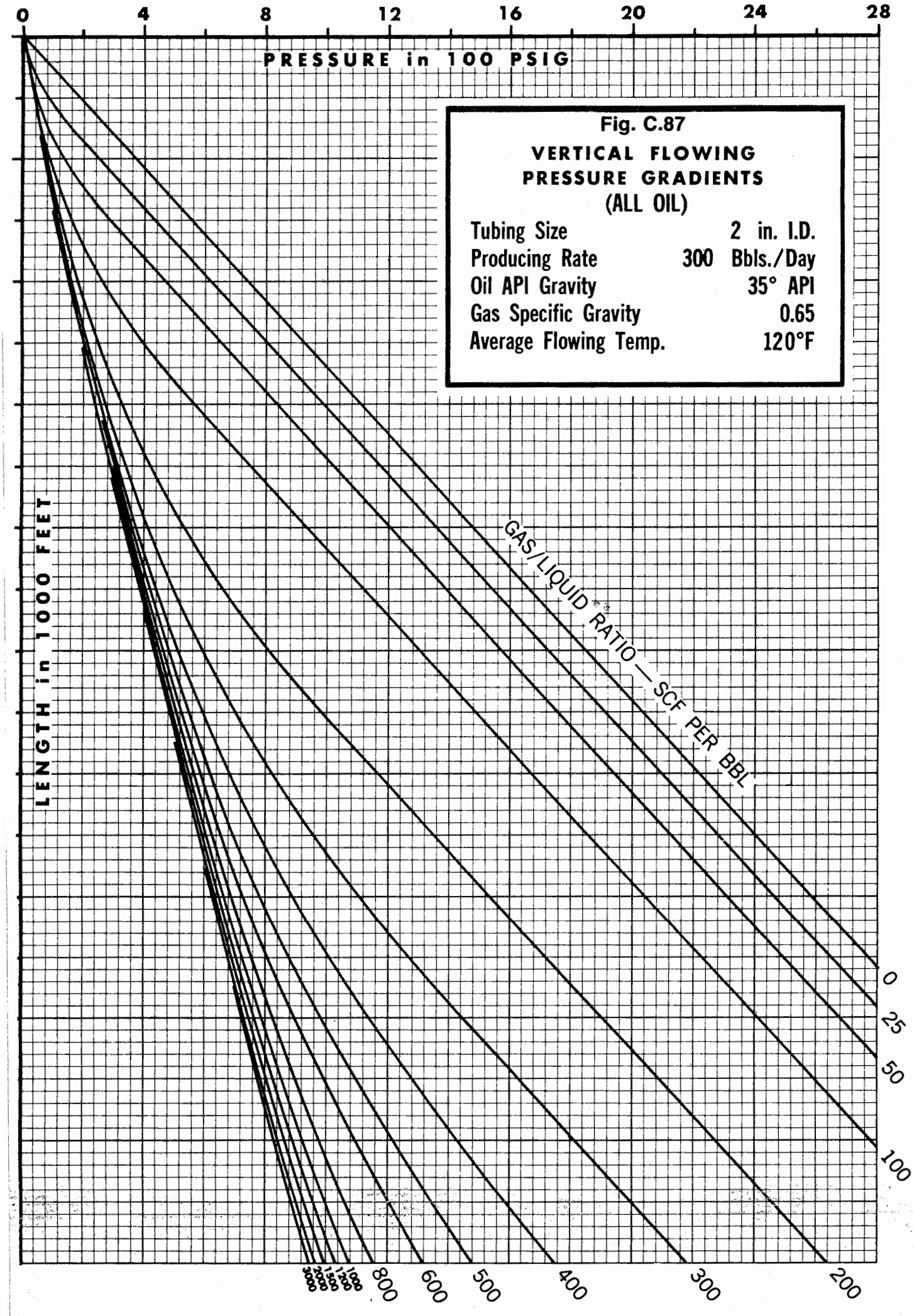
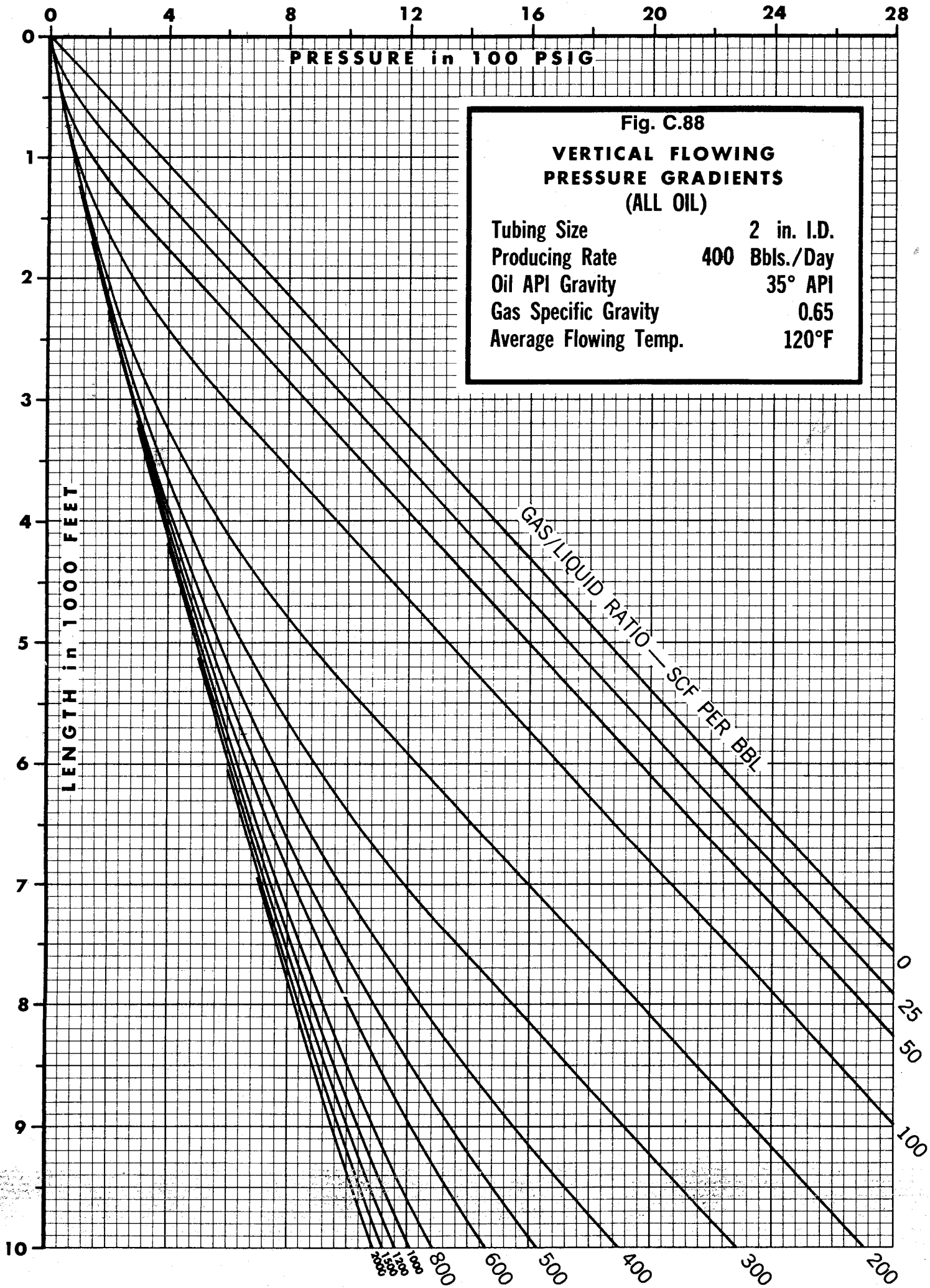


Fig. C.87
VERTICAL FLOWING
PRESSURE GRADIENTS
(ALL OIL)

Tubing Size	2 in. I.D.
Producing Rate	300 Bbls./Day
Oil API Gravity	35° API
Gas Specific Gravity	0.65
Average Flowing Temp.	120°F


0
4
8
12
16
20
24
28
PRESSURE in 100 PSIG
0
1
2
3
4
5
6
7
8
9
10
LENGTH in 1000 FEET
Fig. C.88
VERTICAL FLOWING
PRESSURE GRADIENTS
(ALL OIL)
Tubing Size 2 in. I.D.
Producing Rate 400 Bbls./Day
Oil API Gravity 35° API
Gas Specific Gravity 0.65
Average Flowing Temp. 120°F
GAS/LIQUID RATIO — SCF PER BBL
0
25
50
100
200
300
400
500
600
800
1000
1200
1500
2000

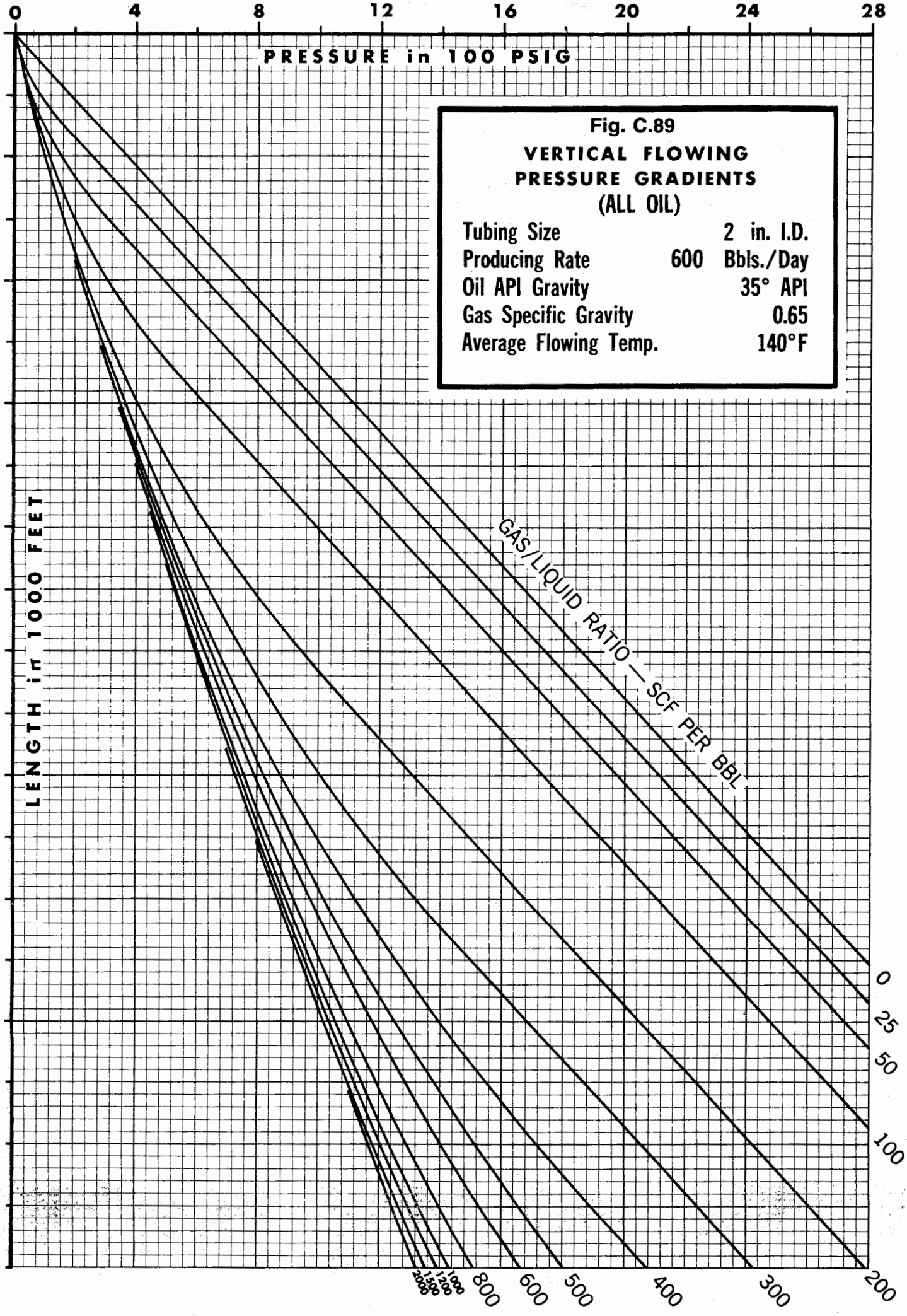
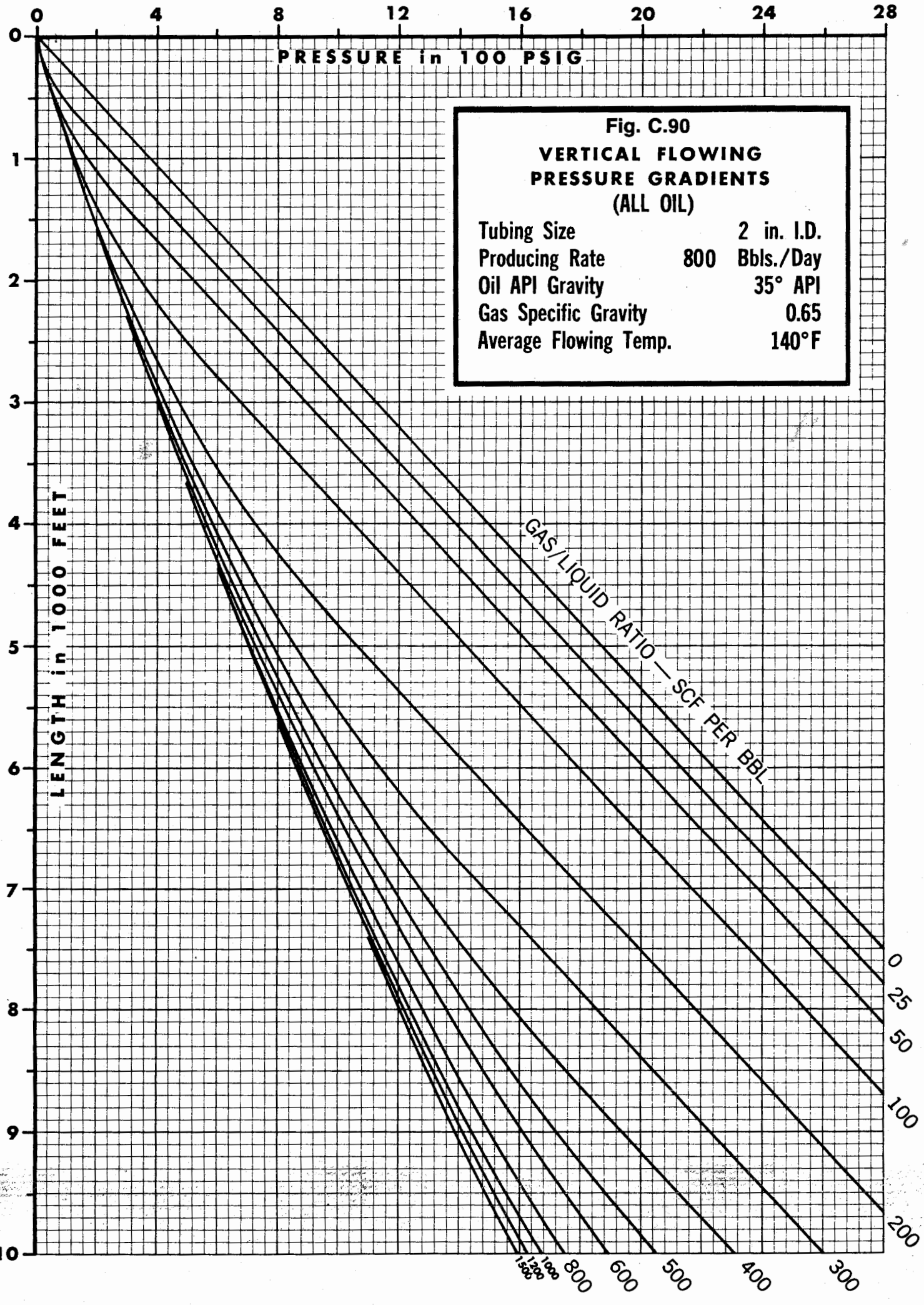


Fig. C.89

VERTICAL FLOWING PRESSURE GRADIENTS (ALL OIL)

Tubing Size	2 in. I.D.
Producing Rate	600 Bbls./Day
Oil API Gravity	35° API
Gas Specific Gravity	0.65
Average Flowing Temp.	140°F


0
4
8
12
16
20
24
28
PRESSURE in 100 PSIG
0
1
2
3
4
5
6
7
8
9
10
LENGTH in 1000 FEET
Fig. C.90
VERTICAL FLOWING
PRESSURE GRADIENTS
(ALL OIL)
Tubing Size 2 in. I.D.
Producing Rate 800 Bbls./Day
Oil API Gravity 35° API
Gas Specific Gravity 0.65
Average Flowing Temp. 140°F
GAS/LIQUID RATIO — SCF PER BBL
0
25
50
100
200
300
400
500
600
800
1000
1200
1500

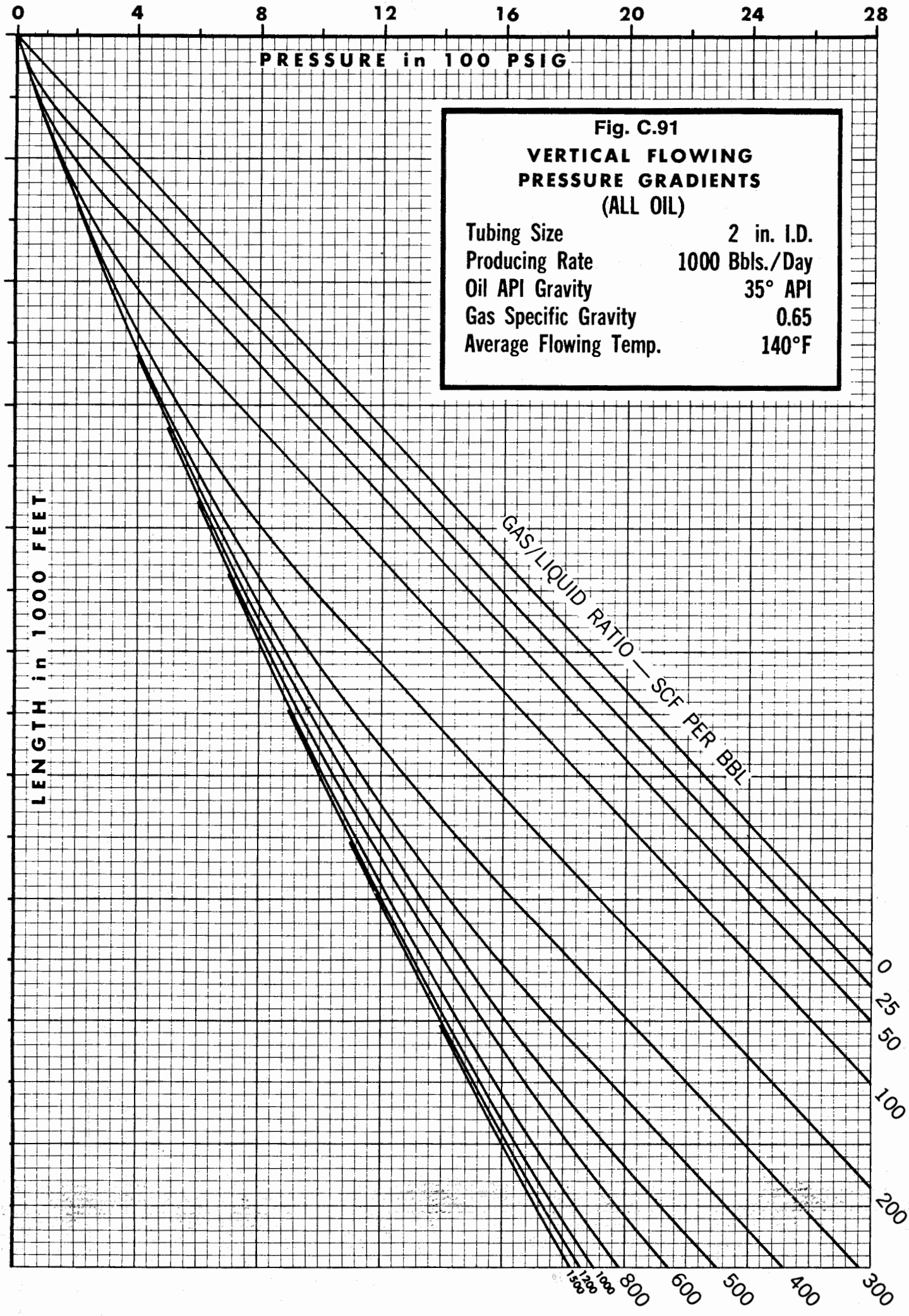
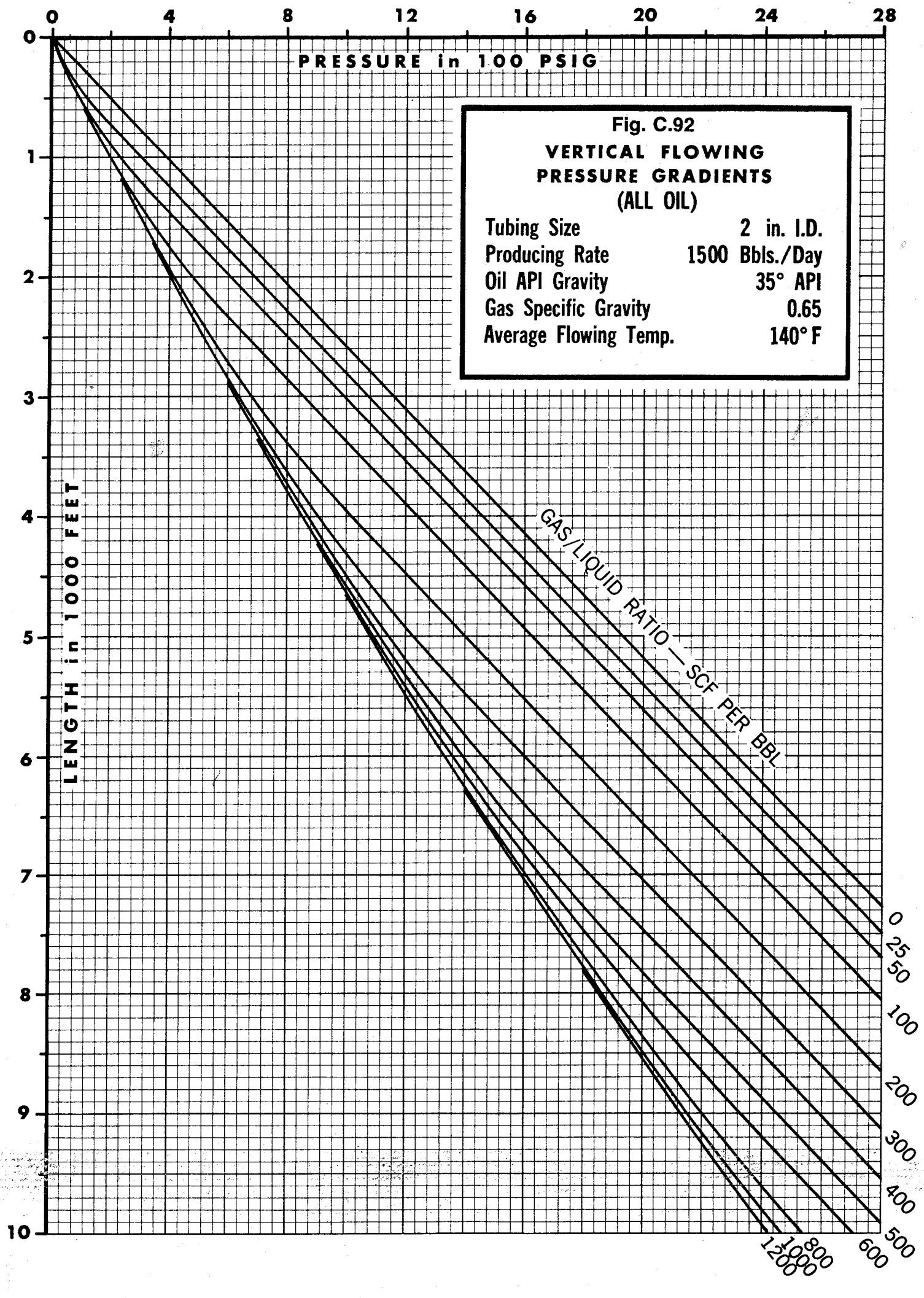


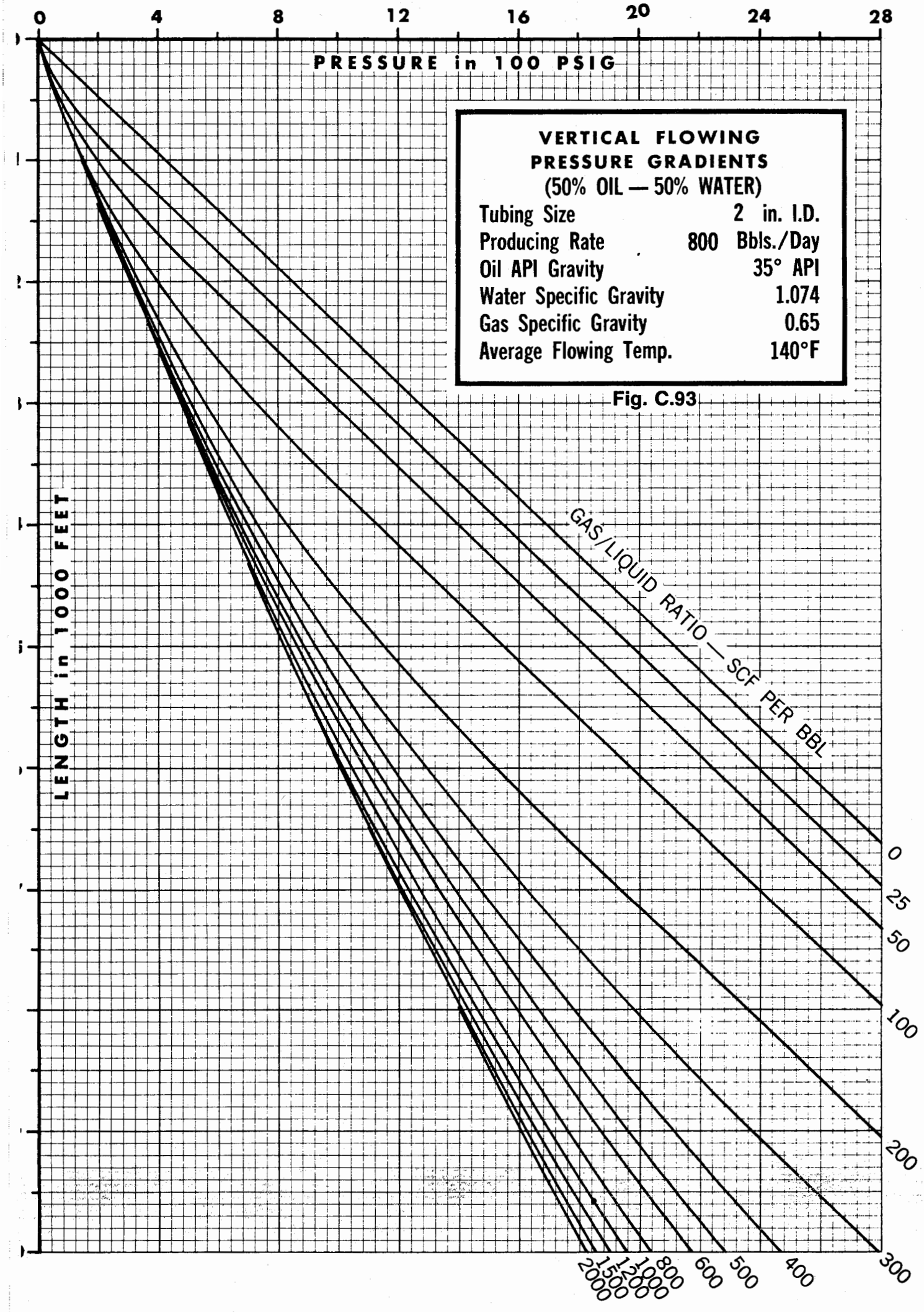
Fig. C.91

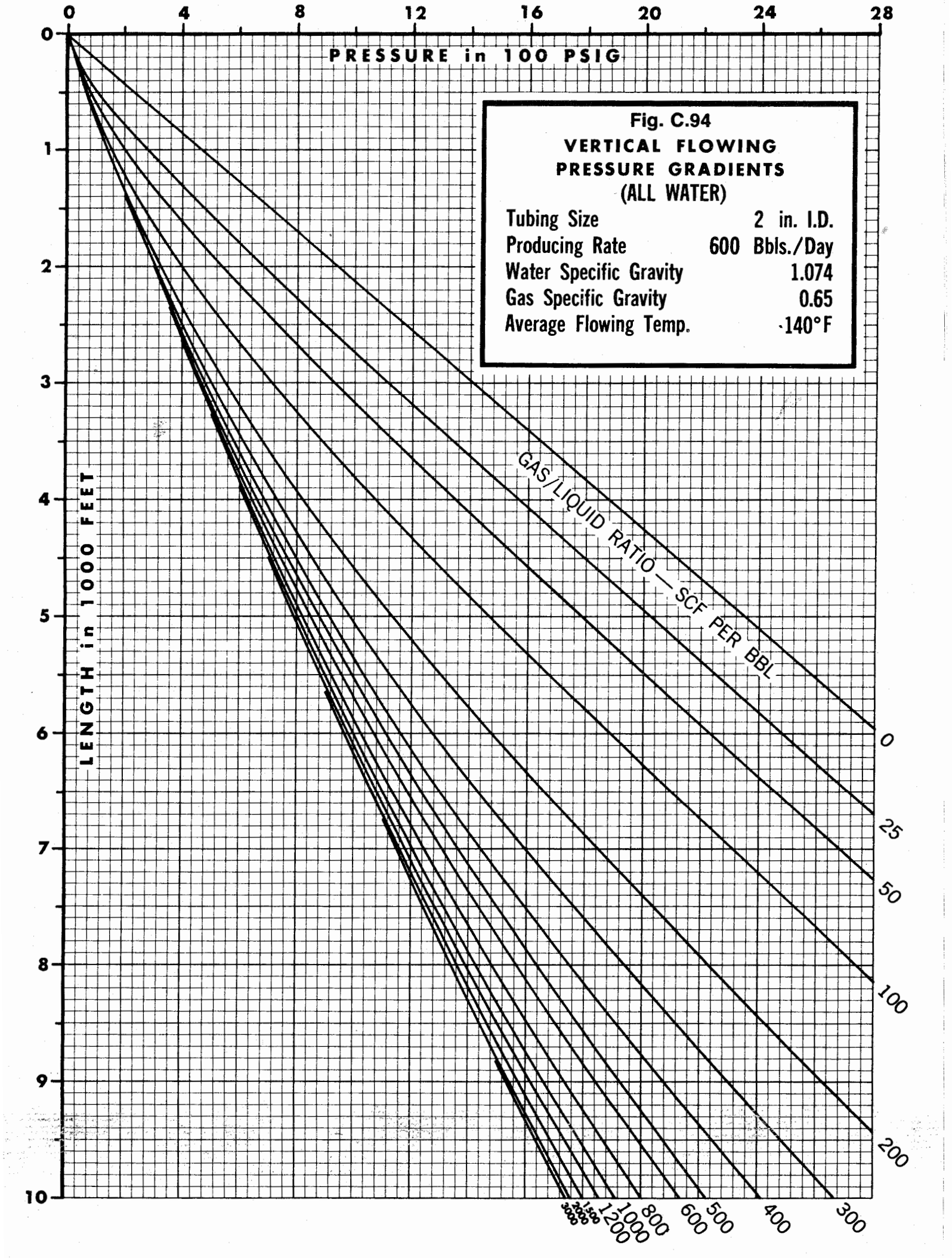
VERTICAL FLOWING PRESSURE GRADIENTS

(ALL OIL)

Tubing Size	2 in. I.D.
Producing Rate	1000 Bbls./Day
Oil API Gravity	35° API
Gas Specific Gravity	0.65
Average Flowing Temp.	140°F


0
4
8
12
16
20
24
28
PRESSURE in 100 PSIG
0
1
2
3
4
5
6
7
8
9
10
LENGTH in 1000 FEET
Fig. C.92
VERTICAL FLOWING
PRESSURE GRADIENTS
(ALL OIL)
Tubing Size 2 in. I.D.
Producing Rate 1500 Bbls./Day
Oil API Gravity 35° API
Gas Specific Gravity 0.65
Average Flowing Temp. 140° F
GAS/LIQUID RATIO — SCF PER BBL
0
25
50
100
200
300
400
500
600
800
1000
1200


0
4
8
12
16
20
24
28
PRESSURE in 100 PSIG
VERTICAL FLOWING
PRESSURE GRADIENTS
(50% OIL — 50% WATER)
Tubing Size 2 in. I.D.
Producing Rate 800 Bbls./Day
Oil API Gravity 35° API
Water Specific Gravity 1.074
Gas Specific Gravity 0.65
Average Flowing Temp. 140°F
Fig. C.93
LENGTH in 1000 FEET
GAS/LIQUID RATIO — SCF PER BBL
0
25
50
100
200
300
400
500
600
800
1000
1200
1500
2000


0
4
8
12
16
20
24
28
PRESSURE in 100 PSIG
0
1
2
3
4
5
6
7
8
9
10
LENGTH in 1000 FEET
Fig. C.94
VERTICAL FLOWING
PRESSURE GRADIENTS
(ALL WATER)
Tubing Size 2 in. I.D.
Producing Rate 600 Bbls./Day
Water Specific Gravity 1.074
Gas Specific Gravity 0.65
Average Flowing Temp. 140°F
GAS/LIQUID RATIO — SCF PER BBL
0
25
50
100
200
300
400
500
600
800
1000
1200

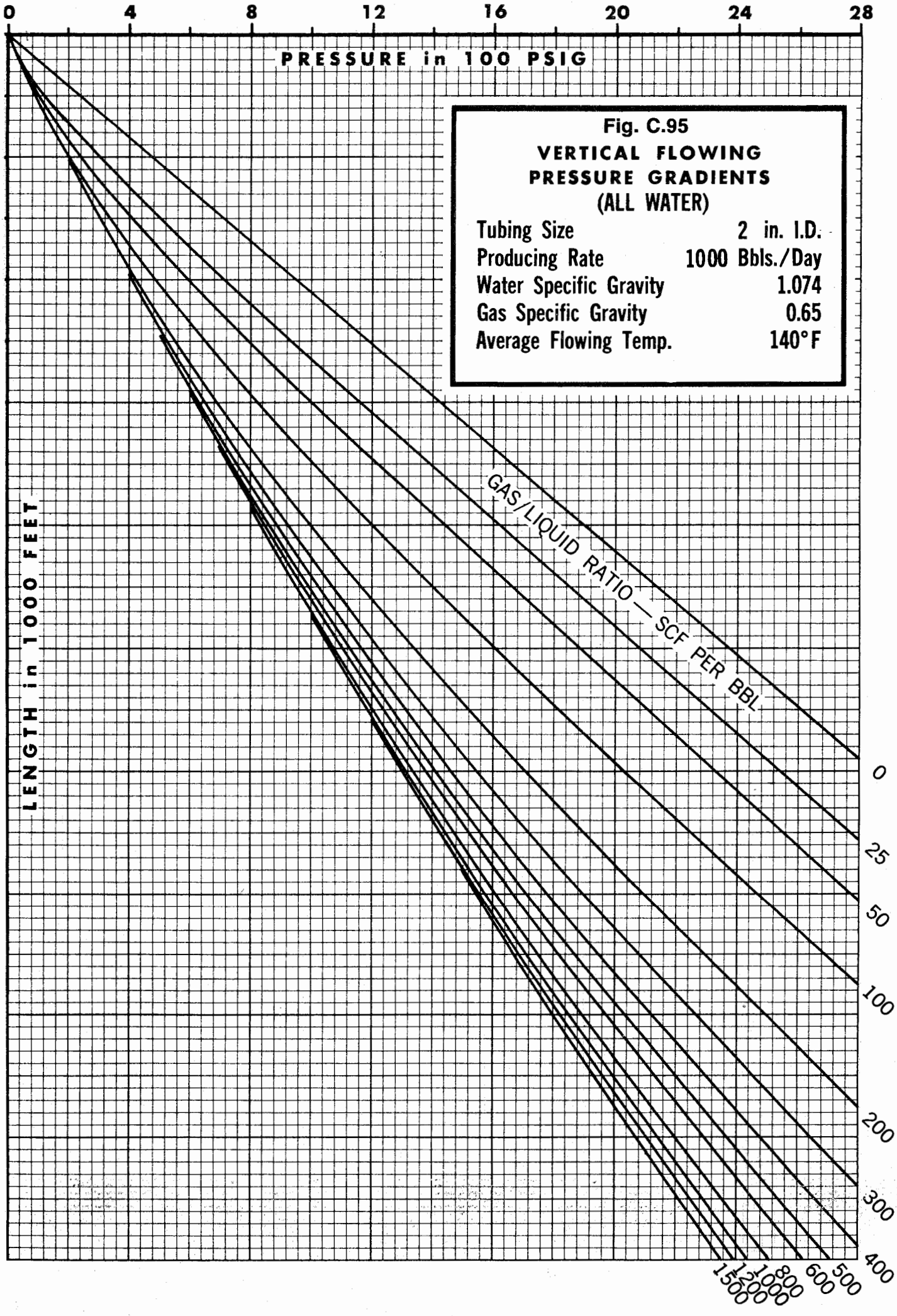


Fig. C.95

VERTICAL FLOWING PRESSURE GRADIENTS (ALL WATER)

Tubing Size	2 in. I.D.
Producing Rate	1000 Bbls./Day
Water Specific Gravity	1.074
Gas Specific Gravity	0.65
Average Flowing Temp.	140°F

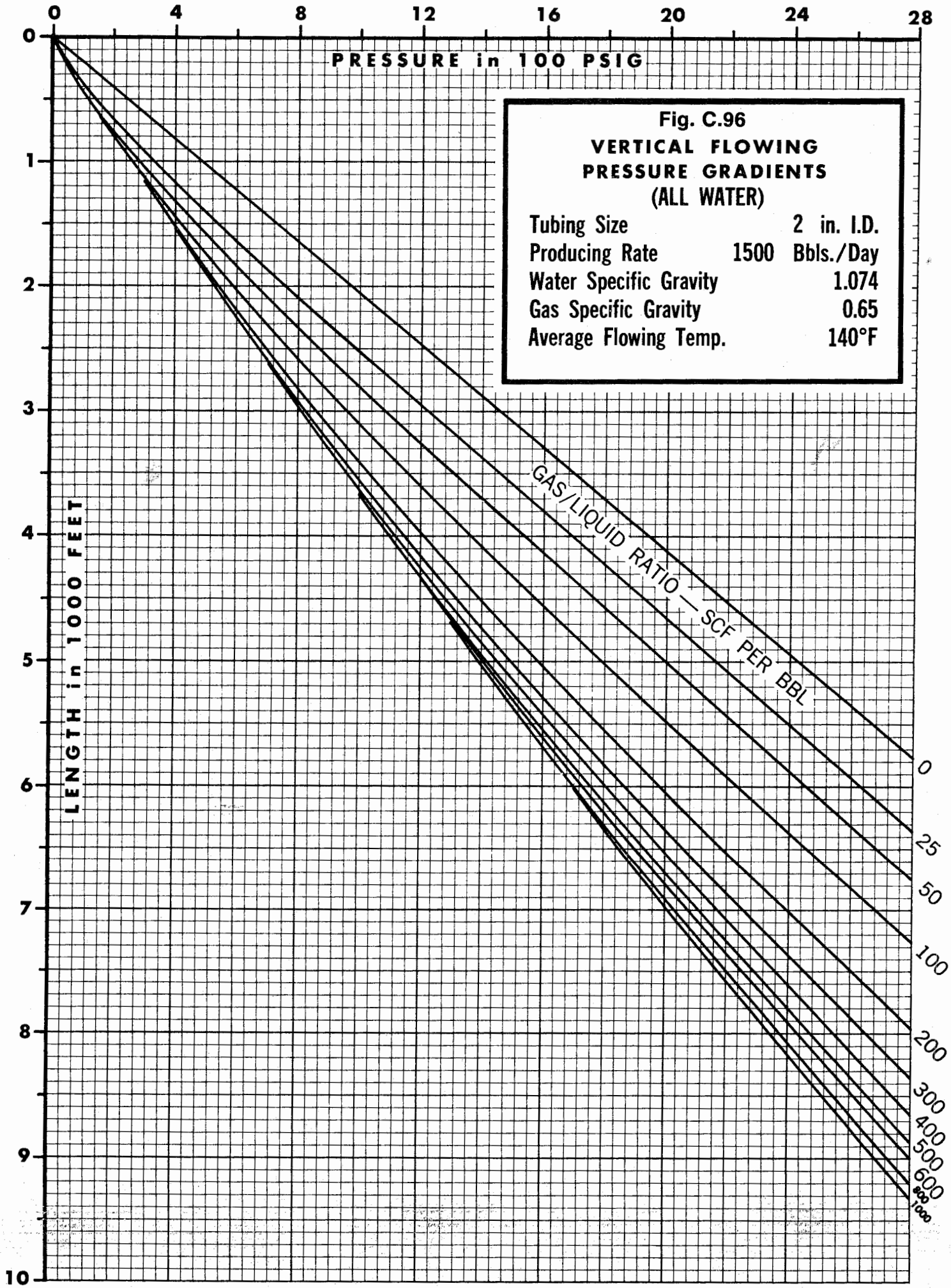
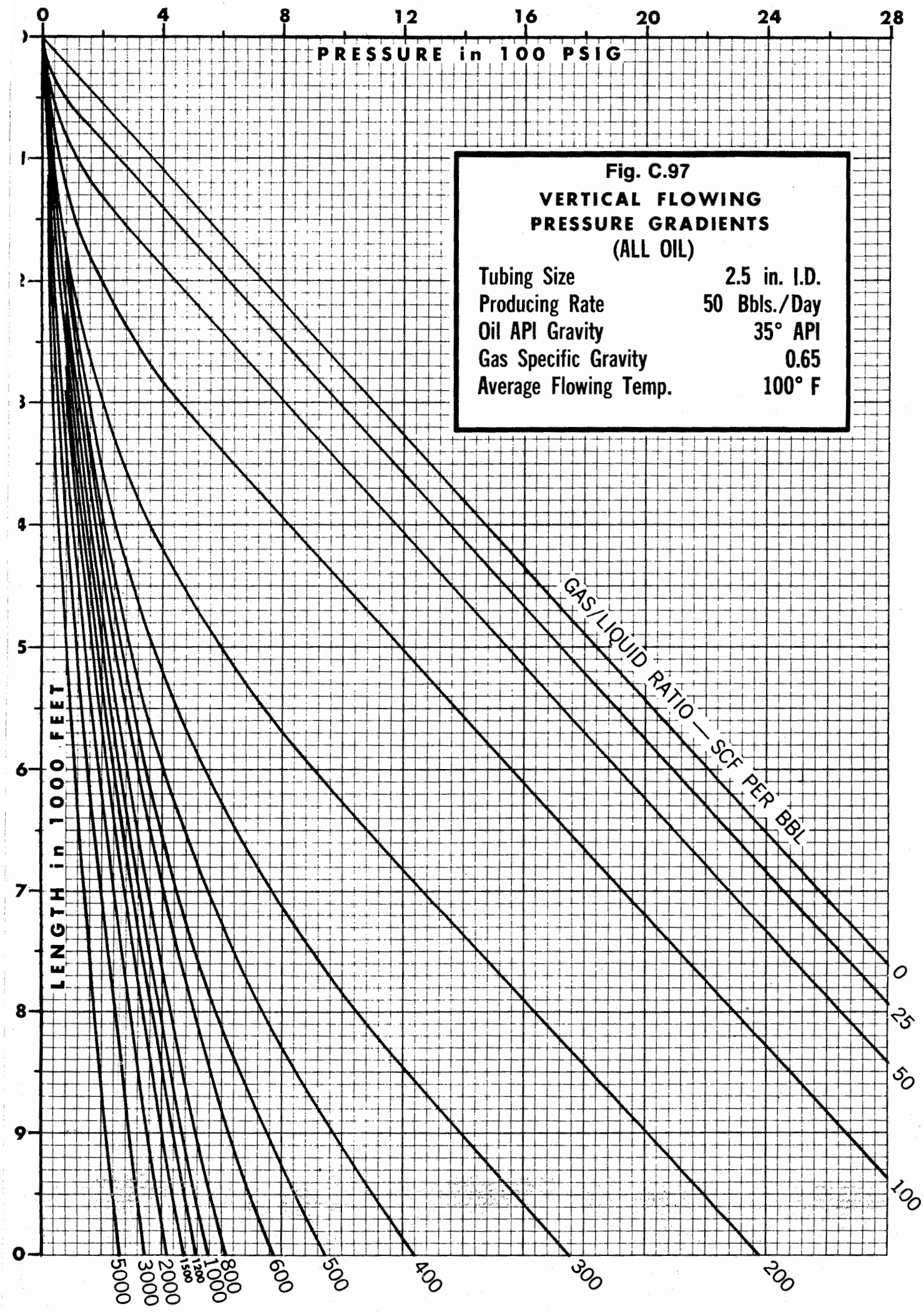


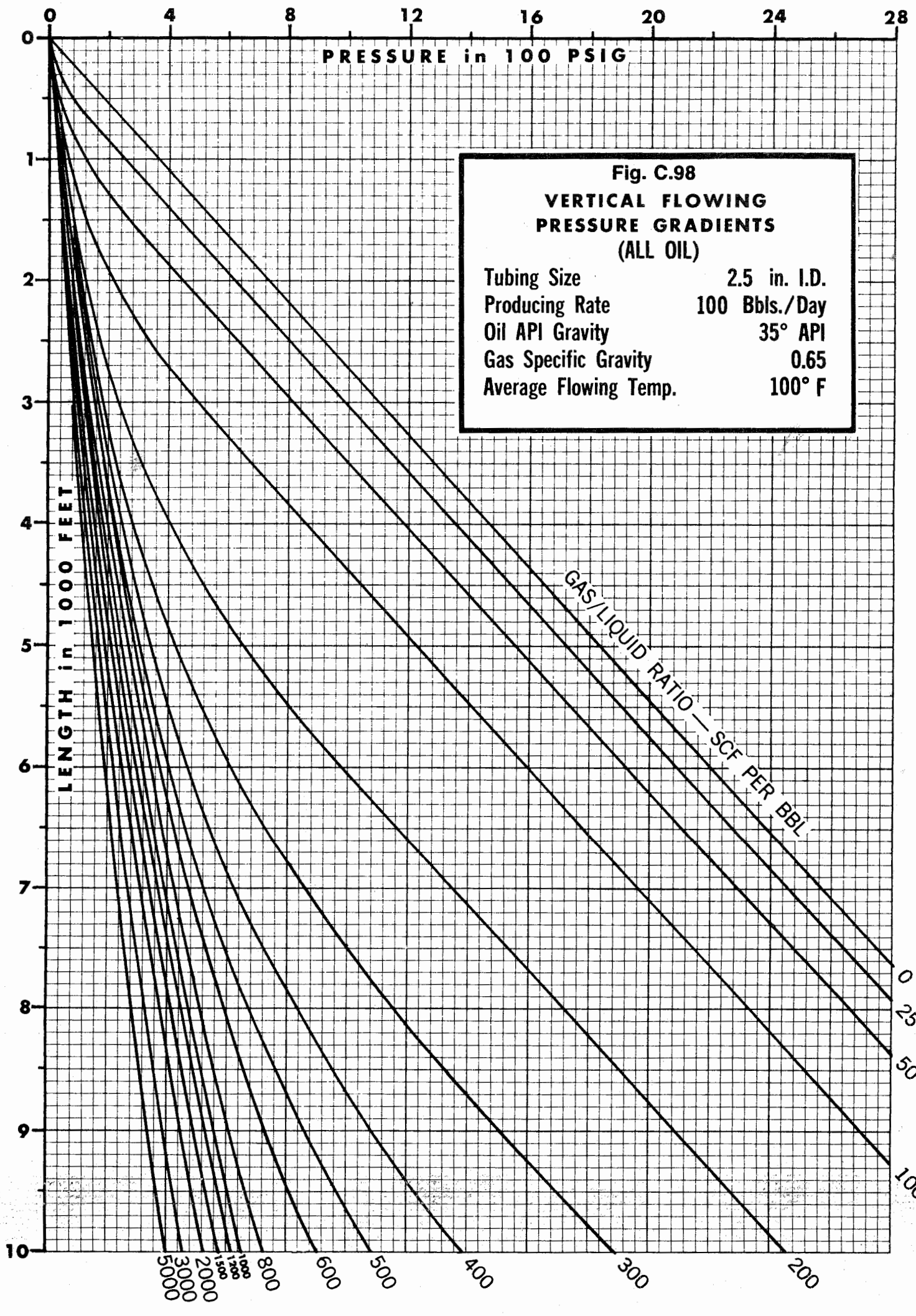
Fig. C.96

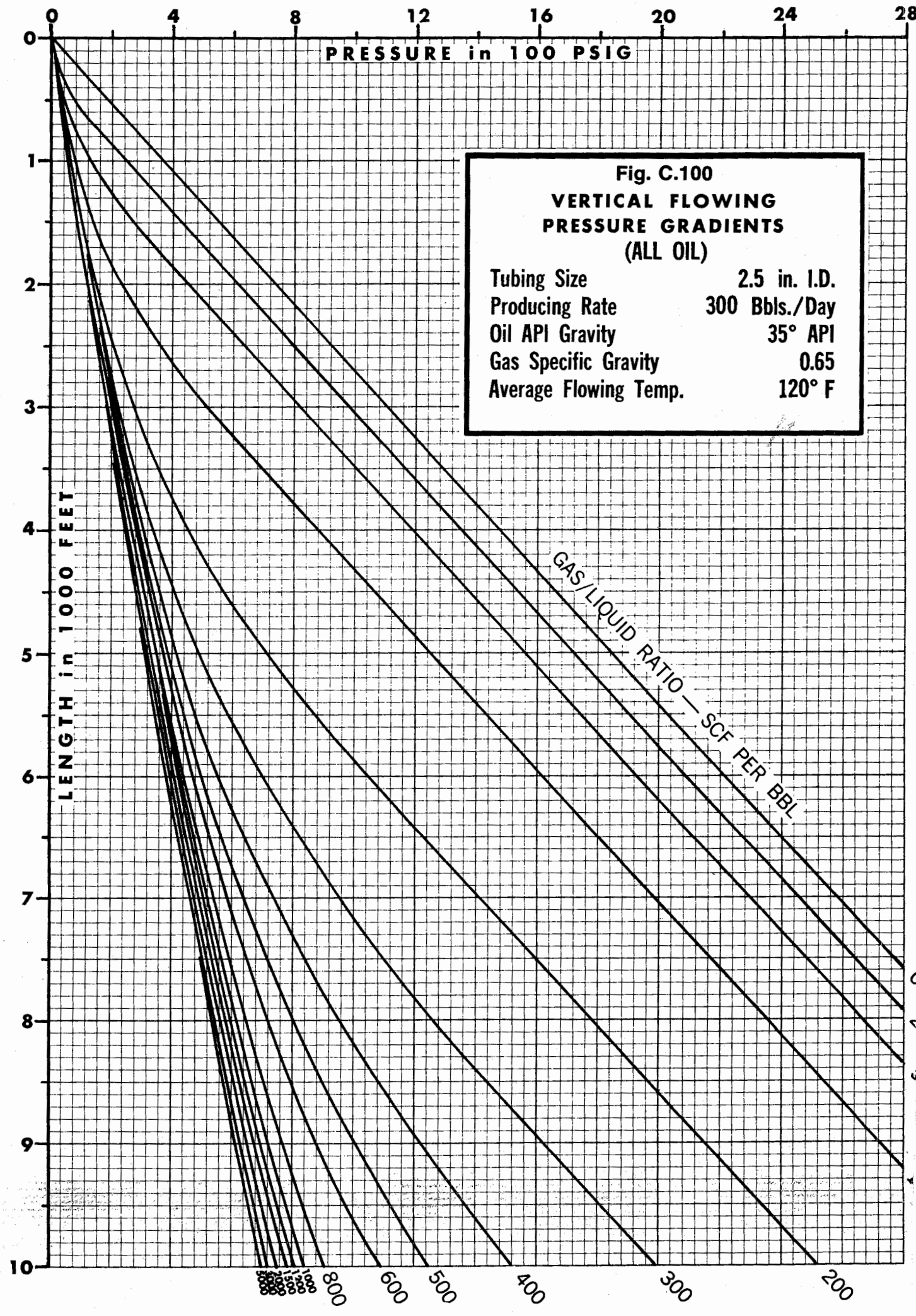
VERTICAL FLOWING PRESSURE GRADIENTS

(ALL WATER)

Tubing Size		2 in. I.D.
Producing Rate	1500	Bbls./Day
Water Specific Gravity		1.074
Gas Specific Gravity		0.65
Average Flowing Temp.		140°F


0
4
8
12
16
20
24
28
PRESSURE in 100 PSIG
Fig. C.97
VERTICAL FLOWING
PRESSURE GRADIENTS
(ALL OIL)
Tubing Size 2.5 in. I.D.
Producing Rate 50 Bbls./Day
Oil API Gravity 35° API
Gas Specific Gravity 0.65
Average Flowing Temp. 100° F
LENGTH in 1000 FEET
0
1
2
3
4
5
6
7
8
9
GAS/LIQUID RATIO — SCF PER BBL
0
25
50
100
200
300
400
500
600
800
1000
1200
1500
2000
3000
5000


PRESSURE in 100 PSIG
0
4
8
12
16
20
24
28
LENGTH in 1000 FEET
0
1
2
3
4
5
6
7
8
9
10
Fig. C.98
VERTICAL FLOWING
PRESSURE GRADIENTS
(ALL OIL)
Tubing Size 2.5 in. I.D.
Producing Rate 100 Bbls./Day
Oil API Gravity 35° API
Gas Specific Gravity 0.65
Average Flowing Temp. 100° F
GAS/LIQUID RATIO — SCF PER BBL
0
25
50
100
200
300
400
500
600
800
1000
1200
1500
2000
3000
5000


0
4
8
12
16
20
24
28
PRESSURE in 100 PSIG
0
1
2
3
4
5
6
7
8
9
10
LENGTH in 1000 FEET
Fig. C.100
VERTICAL FLOWING
PRESSURE GRADIENTS
(ALL OIL)
Tubing Size 2.5 in. I.D.
Producing Rate 300 Bbls./Day
Oil API Gravity 35° API
Gas Specific Gravity 0.65
Average Flowing Temp. 120° F
GAS/LIQUID RATIO — SCF PER BBL
800
600
500
400
300
200

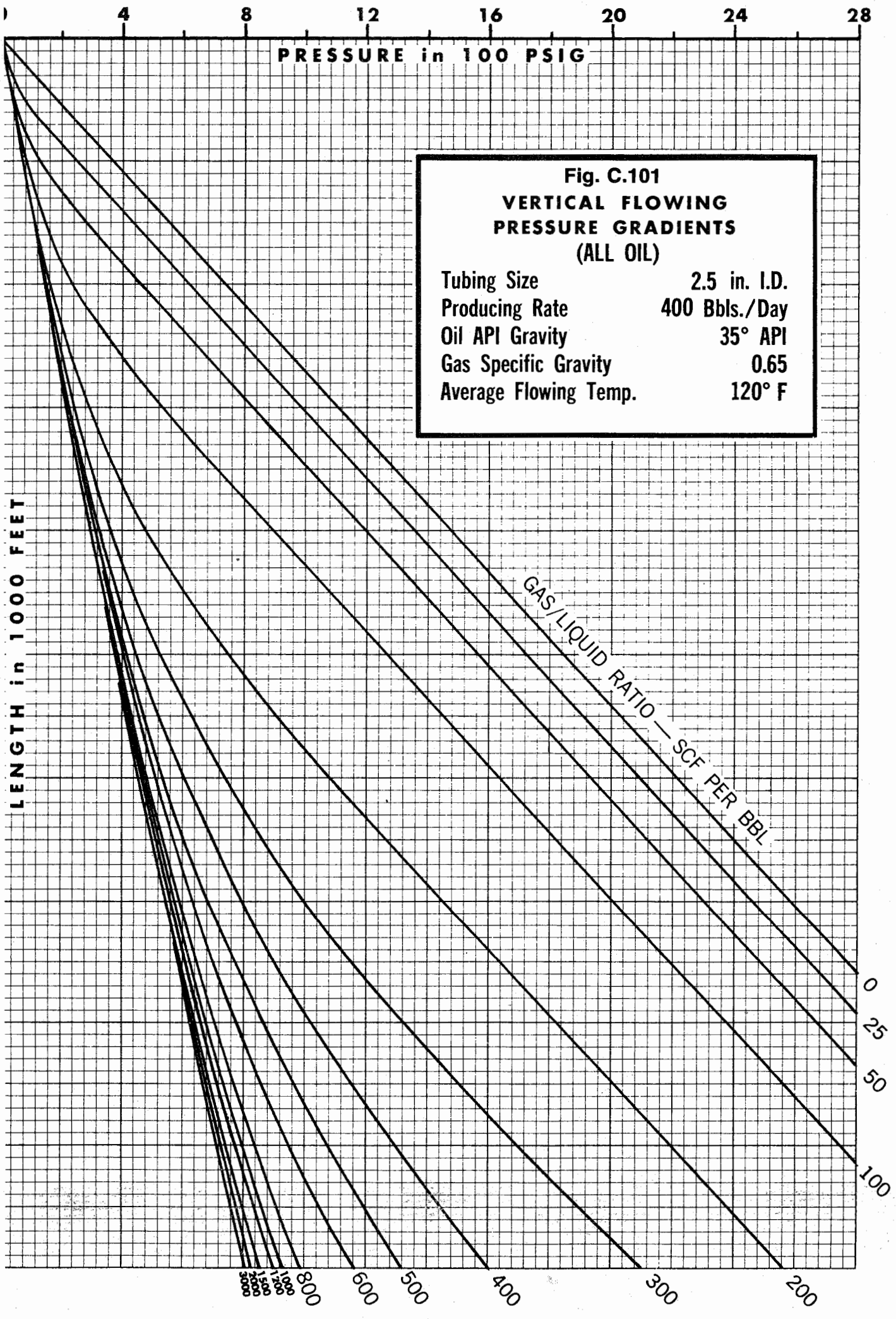
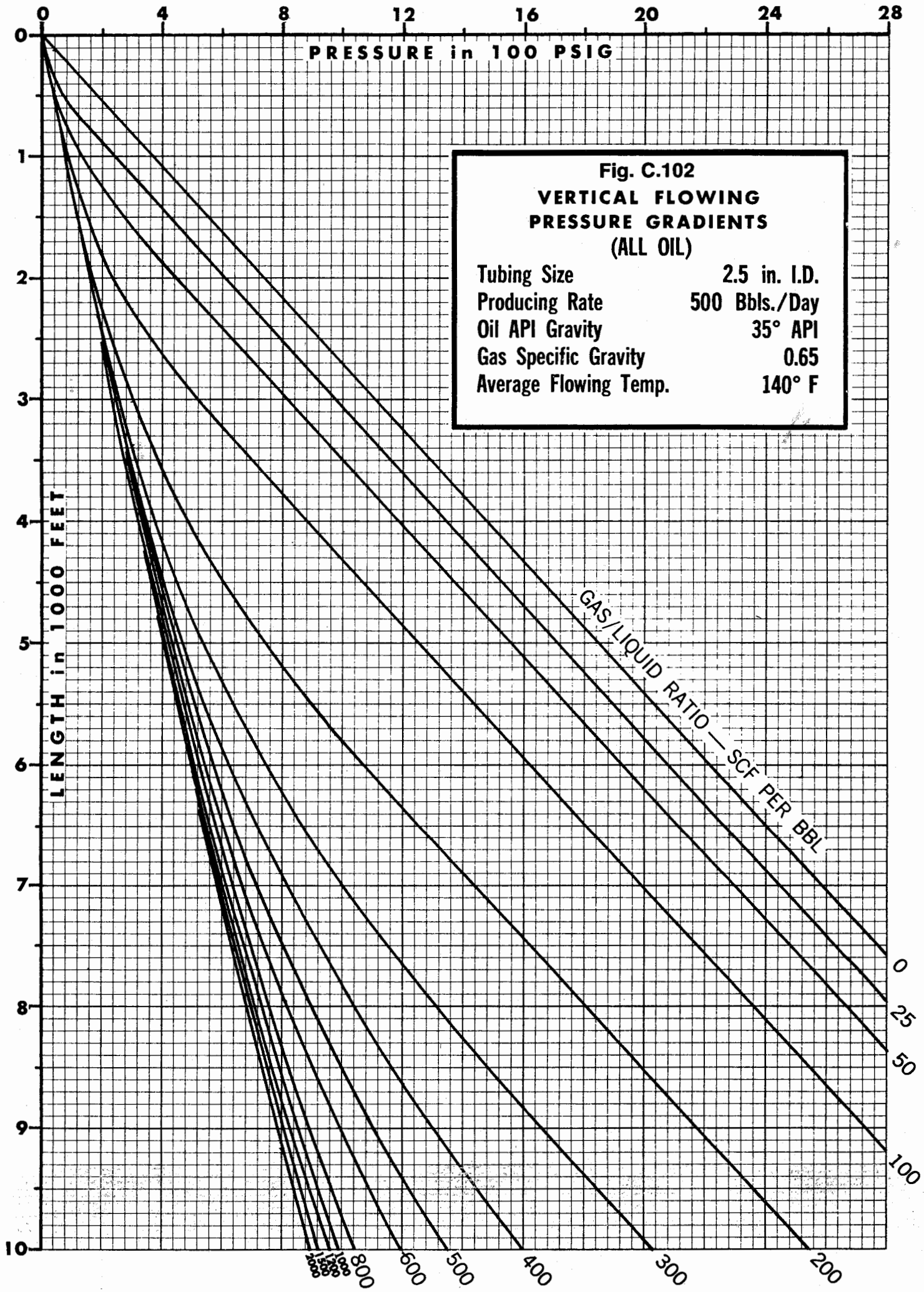


Fig. C.101
VERTICAL FLOWING PRESSURE GRADIENTS
(ALL OIL)

Tubing Size	2.5 in. I.D.
Producing Rate	400 Bbls./Day
Oil API Gravity	35° API
Gas Specific Gravity	0.65
Average Flowing Temp.	120° F


0
4
8
12
16
20
24
28
PRESSURE in 100 PSIG
0
1
2
3
4
5
6
7
8
9
10
LENGTH in 1000 FEET
Fig. C.102
VERTICAL FLOWING
PRESSURE GRADIENTS
(ALL OIL)
Tubing Size 2.5 in. I.D.
Producing Rate 500 Bbls./Day
Oil API Gravity 35° API
Gas Specific Gravity 0.65
Average Flowing Temp. 140° F
GAS/LIQUID RATIO — SCF PER BBL
0
25
50
100
200
300
400
500
600
800

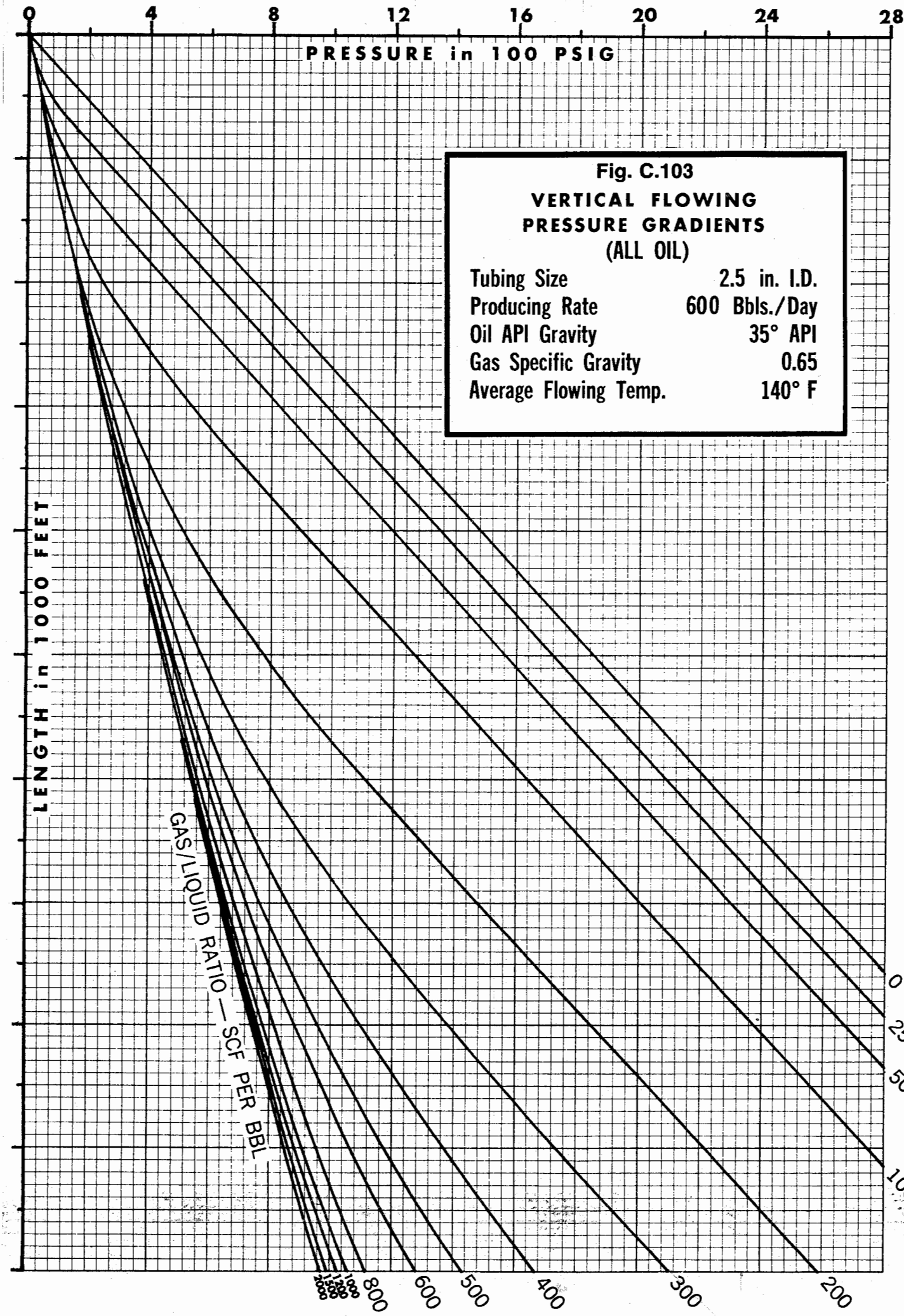
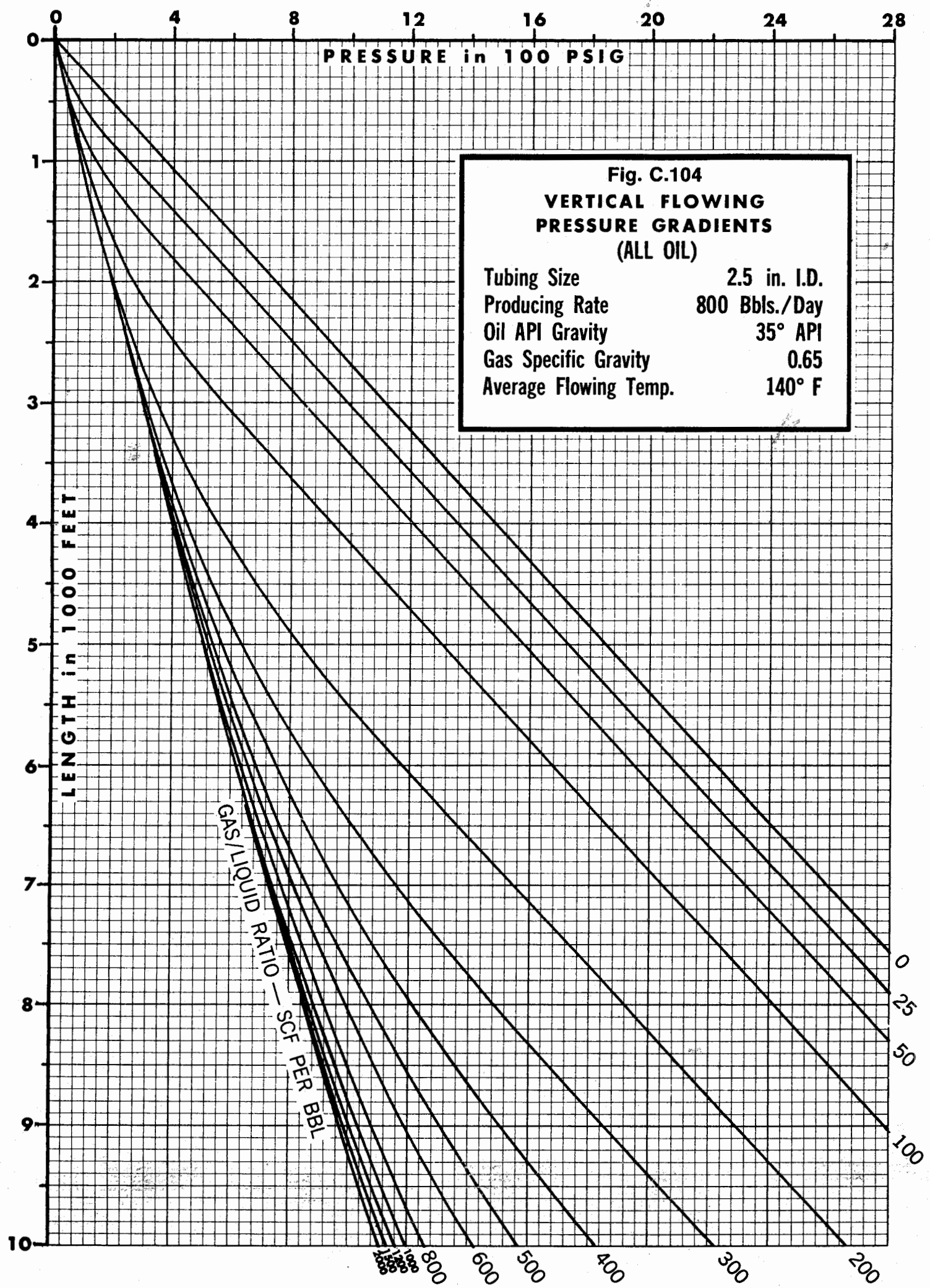


Fig. C.103

VERTICAL FLOWING PRESSURE GRADIENTS

(ALL OIL)

Tubing Size	2.5 in. I.D.
Producing Rate	600 Bbls./Day
Oil API Gravity	35° API
Gas Specific Gravity	0.65
Average Flowing Temp.	140° F


0
4
8
12
16
20
24
28
PRESSURE in 100 PSIG
0
1
2
3
4
5
6
7
8
9
10
LENGTH in 1000 FEET
Fig. C.104
VERTICAL FLOWING
PRESSURE GRADIENTS
(ALL OIL)
Tubing Size 2.5 in. I.D.
Producing Rate 800 Bbls./Day
Oil API Gravity 35° API
Gas Specific Gravity 0.65
Average Flowing Temp. 140° F
GAS/LIQUID RATIO — SCF PER BBL
0
25
50
100
200
300
400
500
600
800

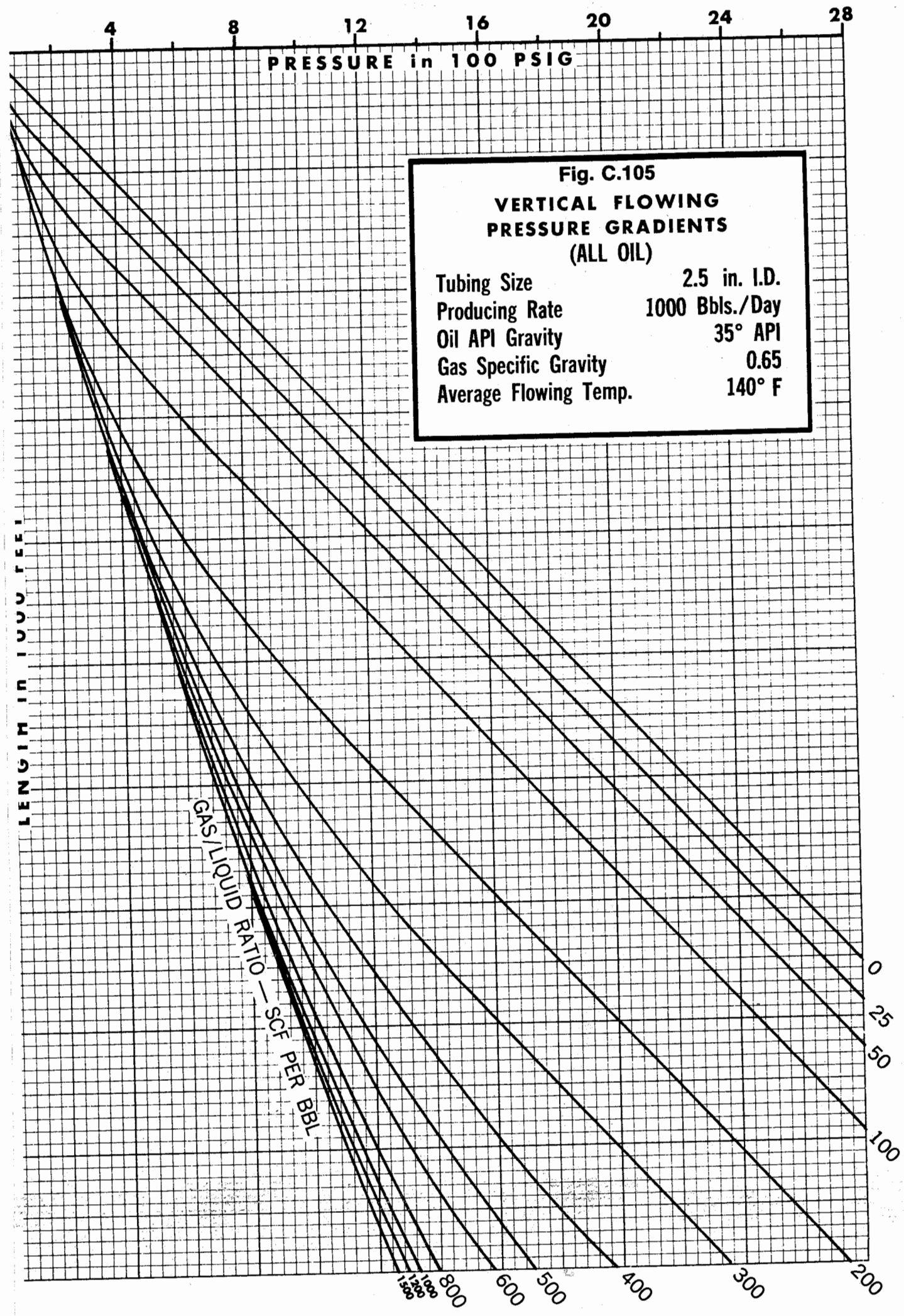
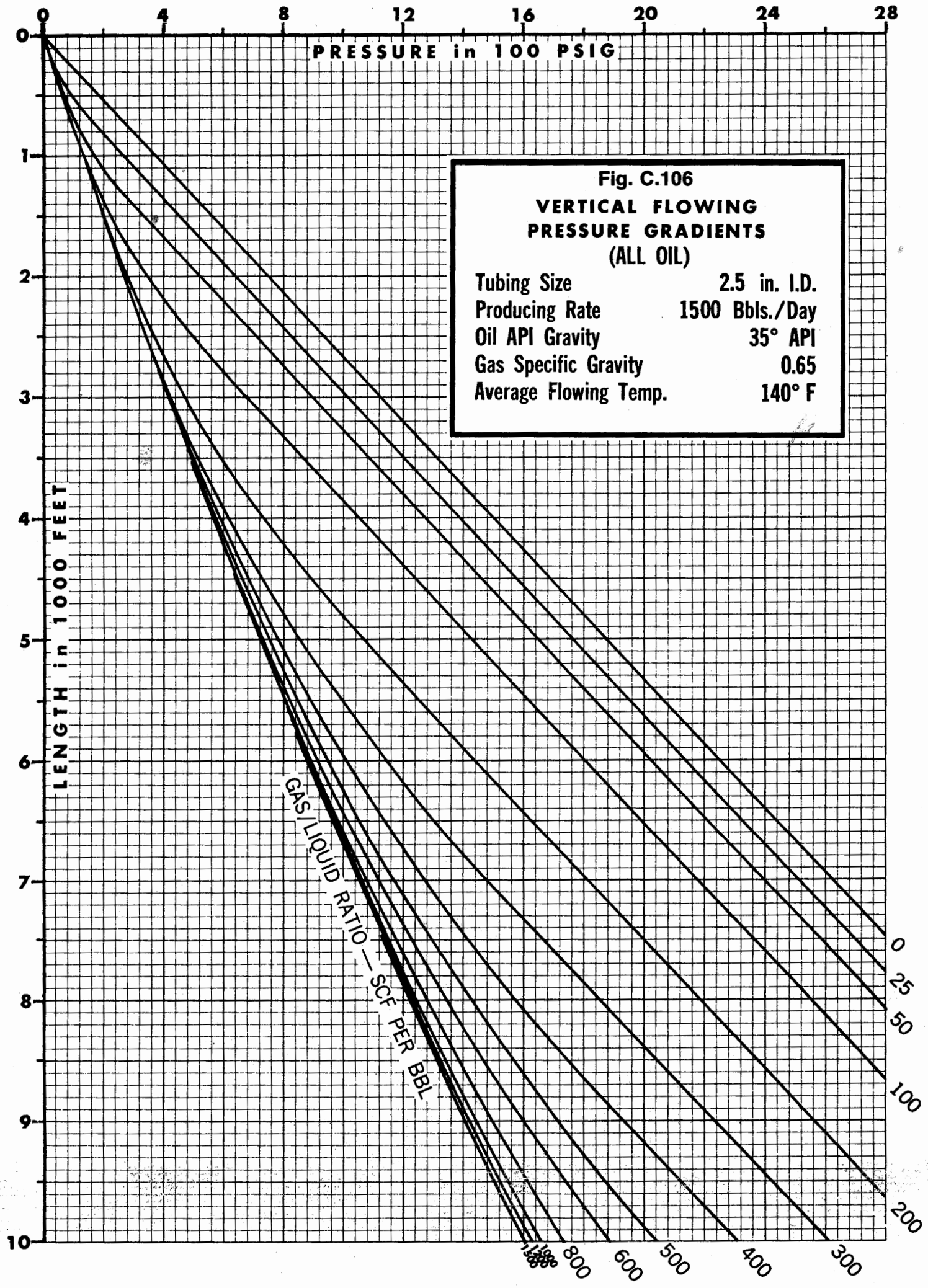


Fig. C.105

VERTICAL FLOWING PRESSURE GRADIENTS (ALL OIL)

Tubing Size	2.5 in. I.D.
Producing Rate	1000 Bbls./Day
Oil API Gravity	35° API
Gas Specific Gravity	0.65
Average Flowing Temp.	140° F


0
4
8
12
16
20
24
28
PRESSURE in 100 PSIG
0
1
2
3
4
5
6
7
8
9
10
LENGTH in 1000 FEET
Fig. C.106
VERTICAL FLOWING
PRESSURE GRADIENTS
(ALL OIL)
Tubing Size 2.5 in. I.D.
Producing Rate 1500 Bbls./Day
Oil API Gravity 35° API
Gas Specific Gravity 0.65
Average Flowing Temp. 140° F
GAS/LIQUID RATIO — SCF PER BBL
0
25
50
100
200
300
400
500
600
800

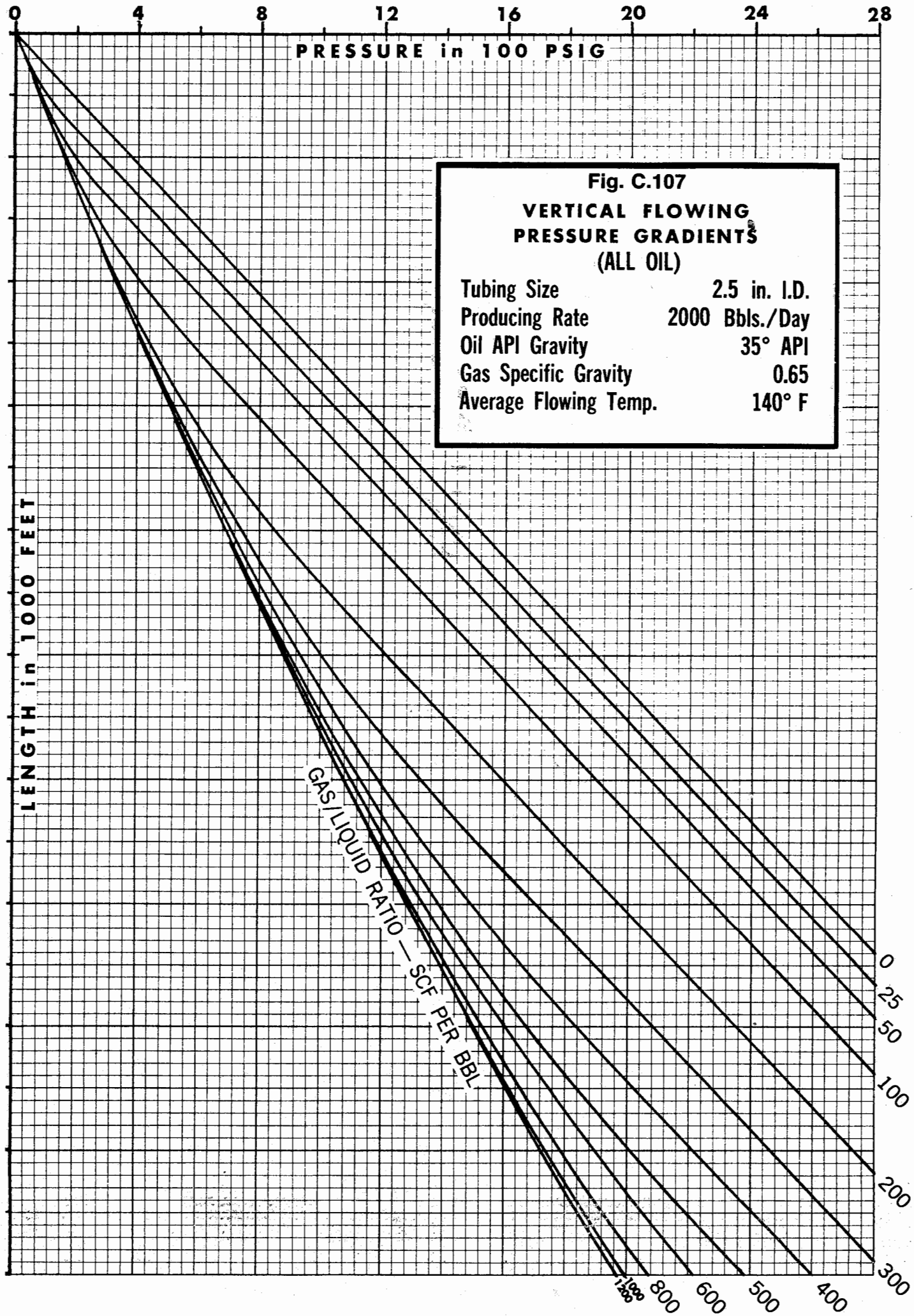
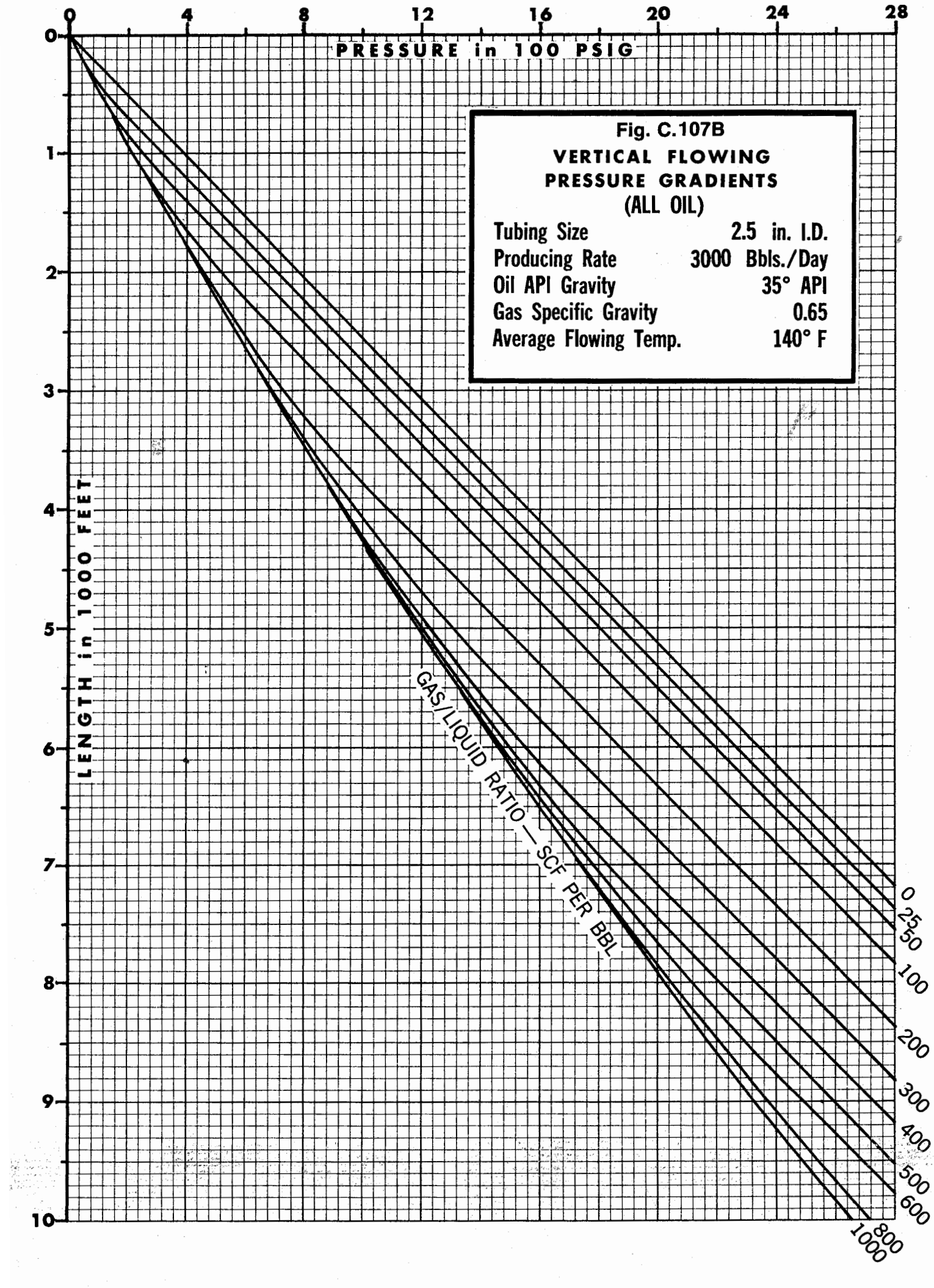


Fig. C.107

VERTICAL FLOWING PRESSURE GRADIENTS

(ALL OIL)

Tubing Size	2.5 in. I.D.
Producing Rate	2000 Bbls./Day
Oil API Gravity	35° API
Gas Specific Gravity	0.65
Average Flowing Temp.	140° F


0
4
8
12
16
20
24
28
PRESSURE in 100 PSIG
0
1
2
3
4
5
6
7
8
9
10
LENGTH in 1000 FEET
Fig. C.107B
VERTICAL FLOWING
PRESSURE GRADIENTS
(ALL OIL)
Tubing Size 2.5 in. I.D.
Producing Rate 3000 Bbls./Day
Oil API Gravity 35° API
Gas Specific Gravity 0.65
Average Flowing Temp. 140° F
GAS/LIQUID RATIO — SCF PER BBL
0
25
50
100
200
300
400
500
600
800
1000

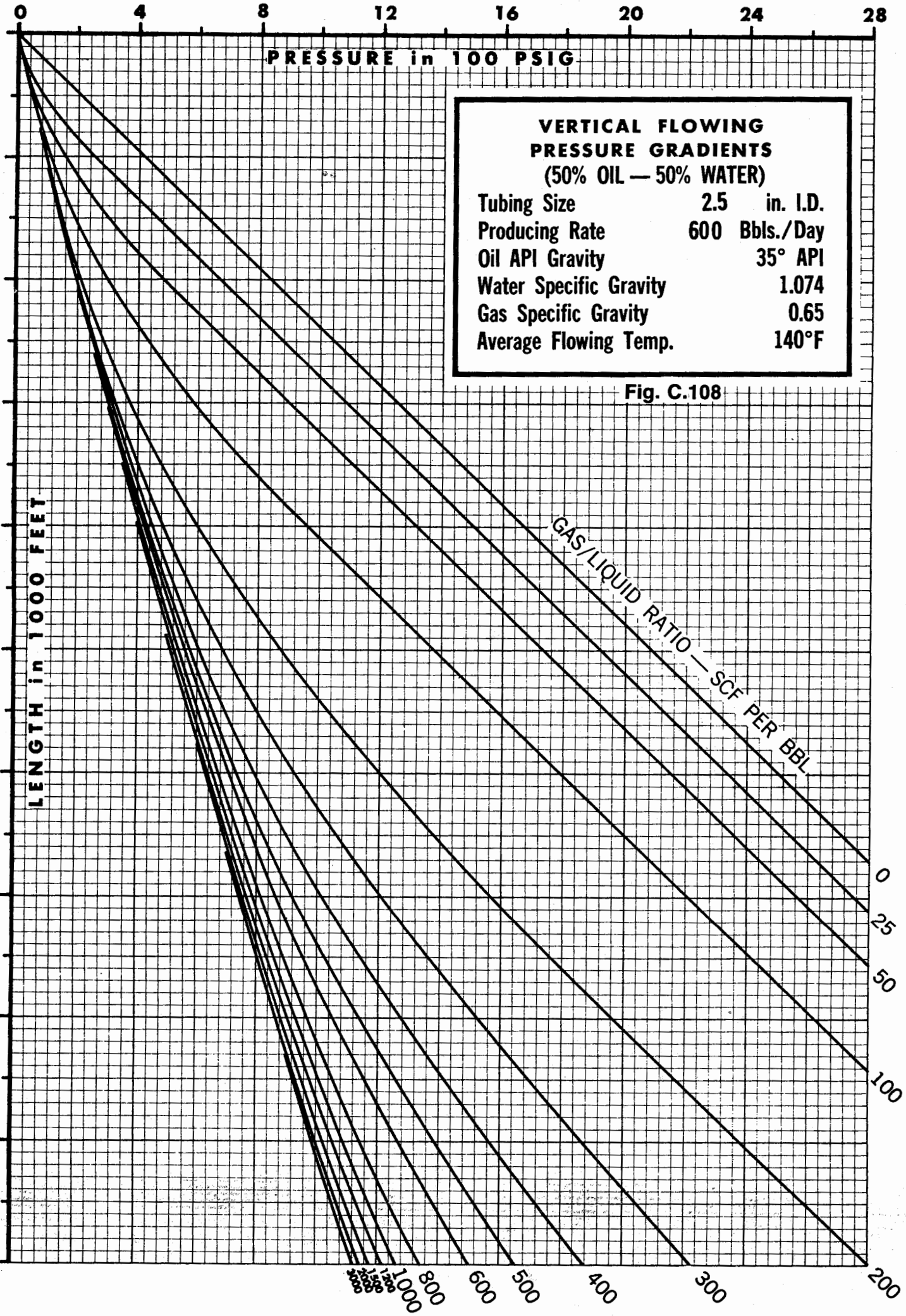
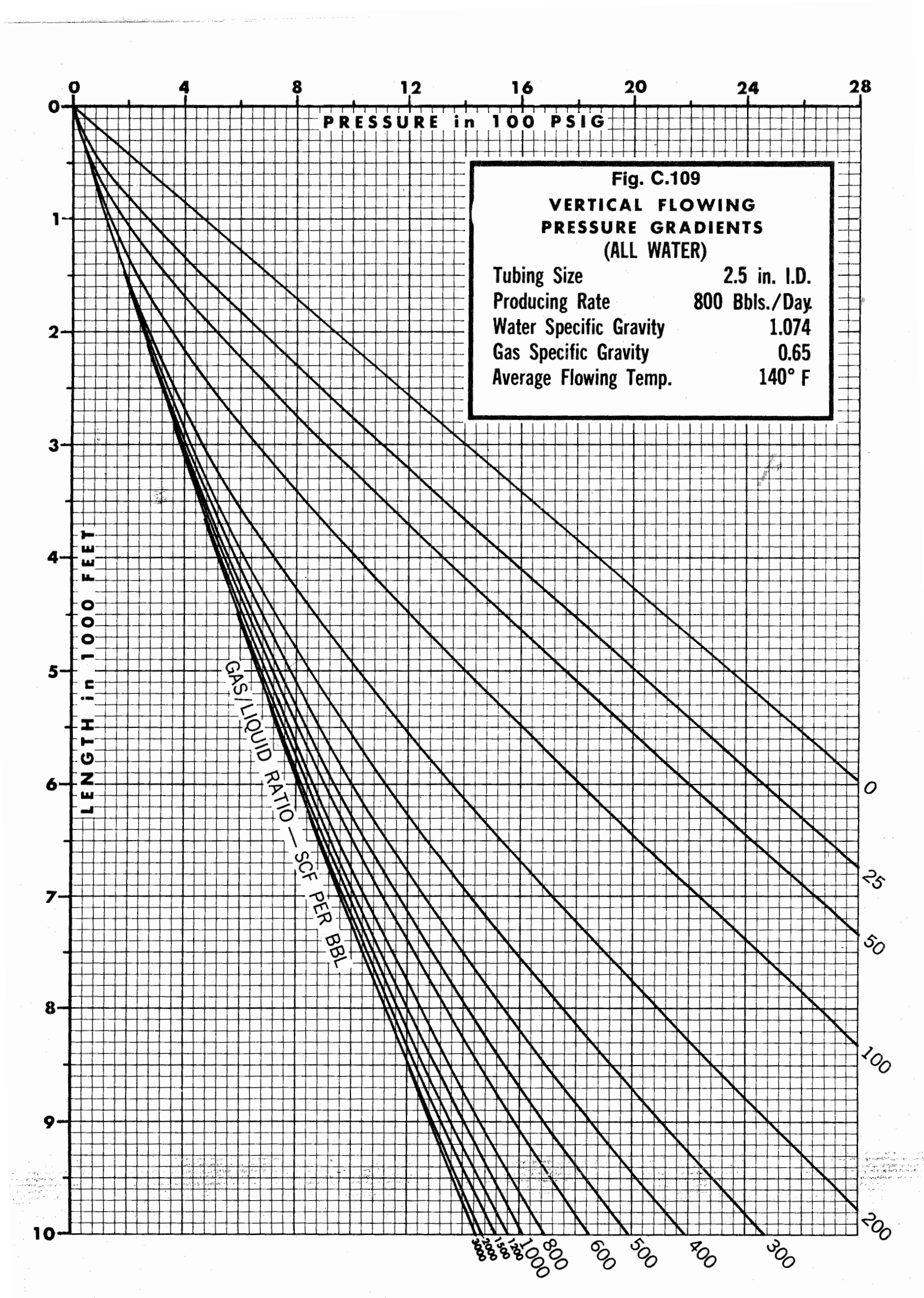
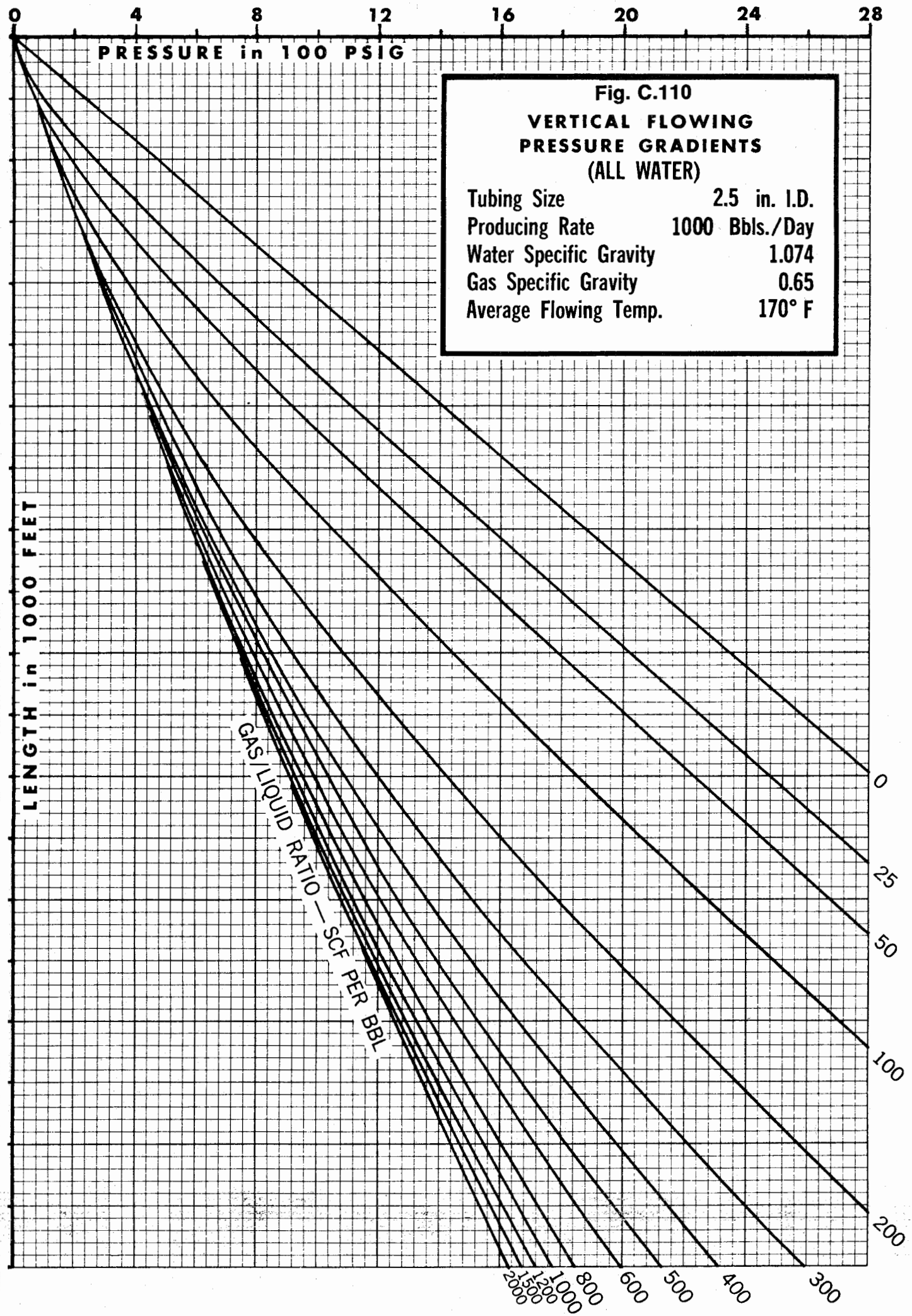
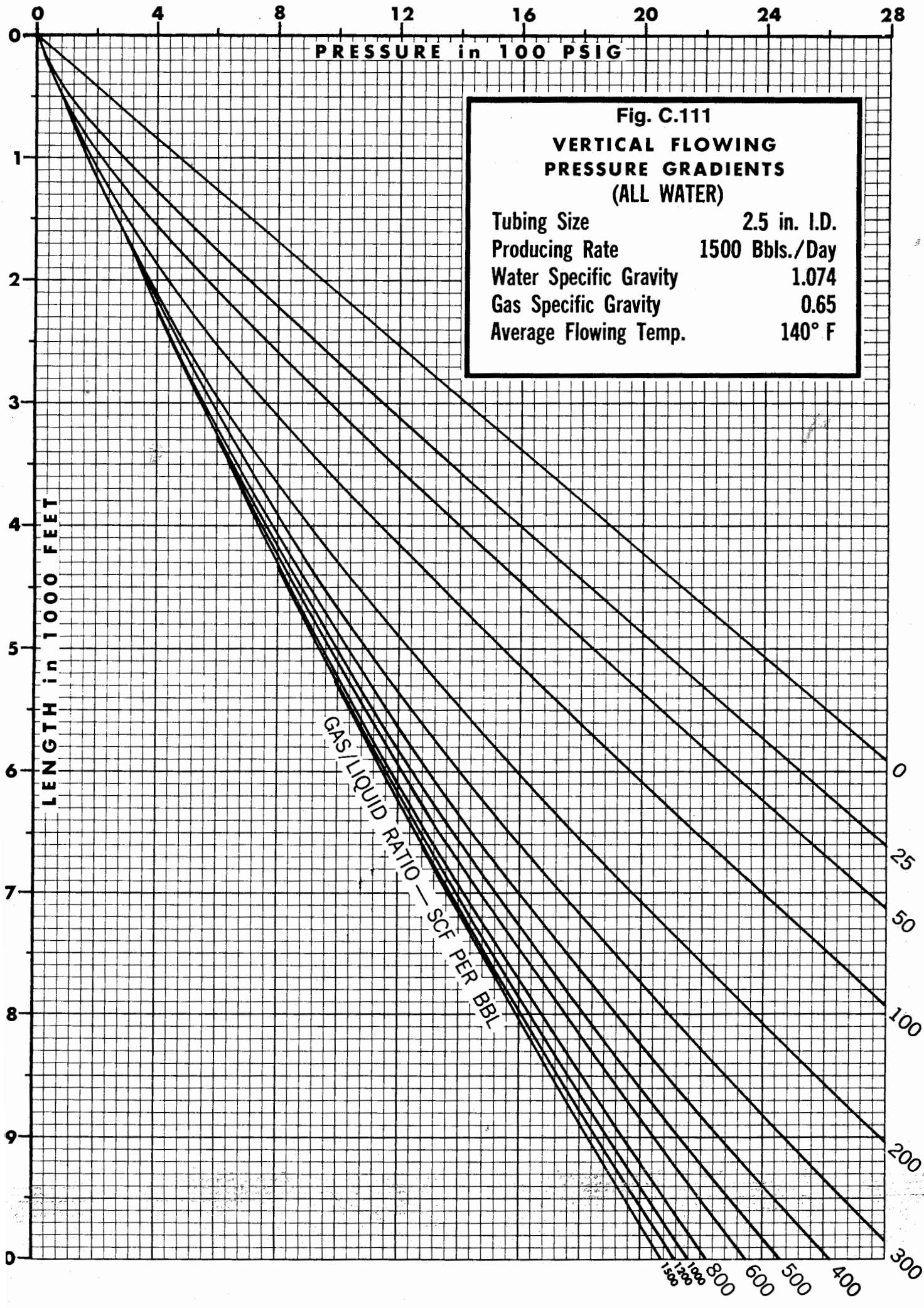
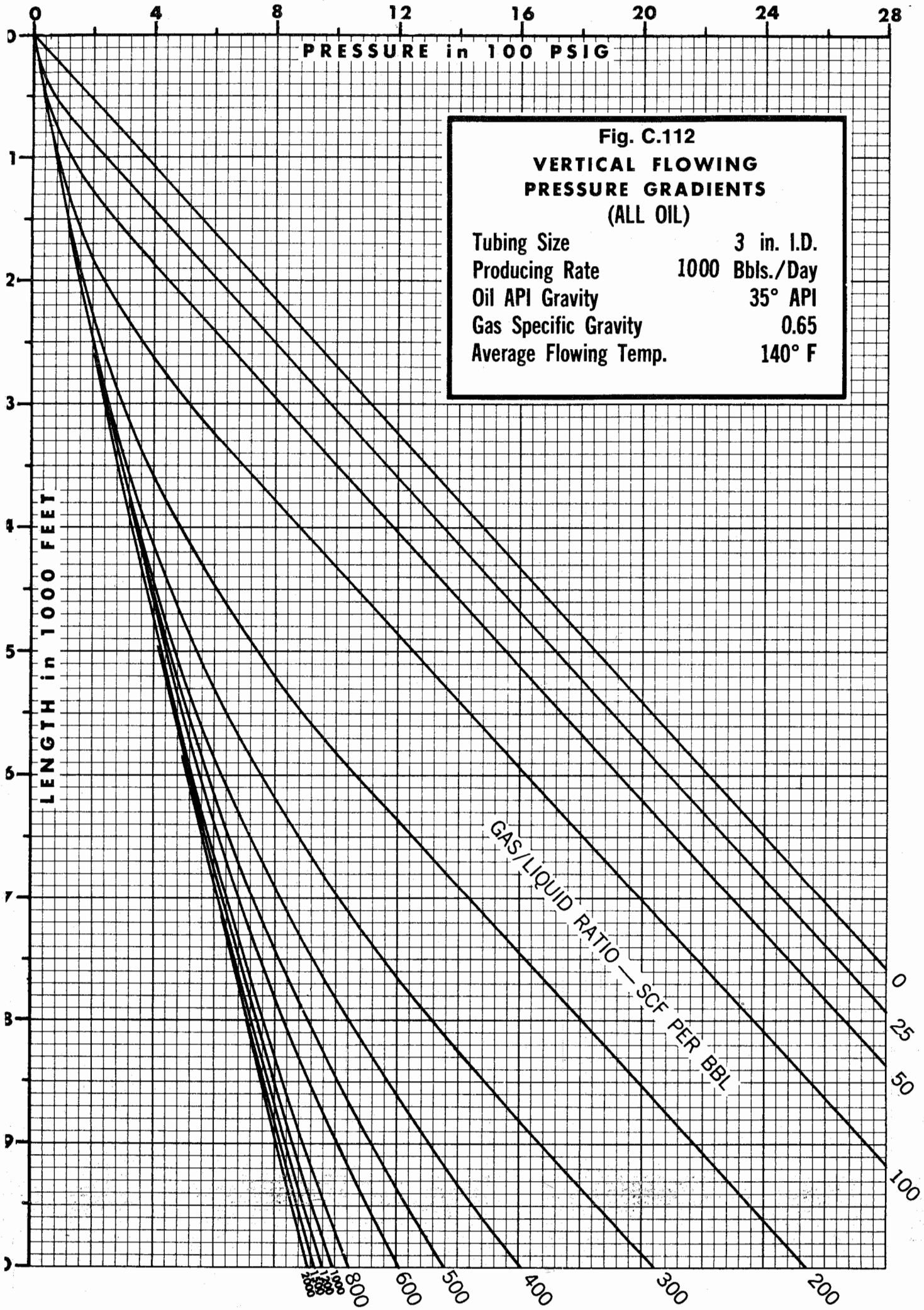

0
4
8
12
16
20
24
28
PRESSURE in 100 PSIG
LENGTH in 1000 FEET
VERTICAL FLOWING
PRESSURE GRADIENTS
(50% OIL — 50% WATER)
Tubing Size 2.5 in. I.D.
Producing Rate 600 Bbls./Day
Oil API Gravity 35° API
Water Specific Gravity 1.074
Gas Specific Gravity 0.65
Average Flowing Temp. 140°F
GAS/LIQUID RATIO — SCF PER BBL.
0
25
50
100
200
300
400
500
600
800
1000

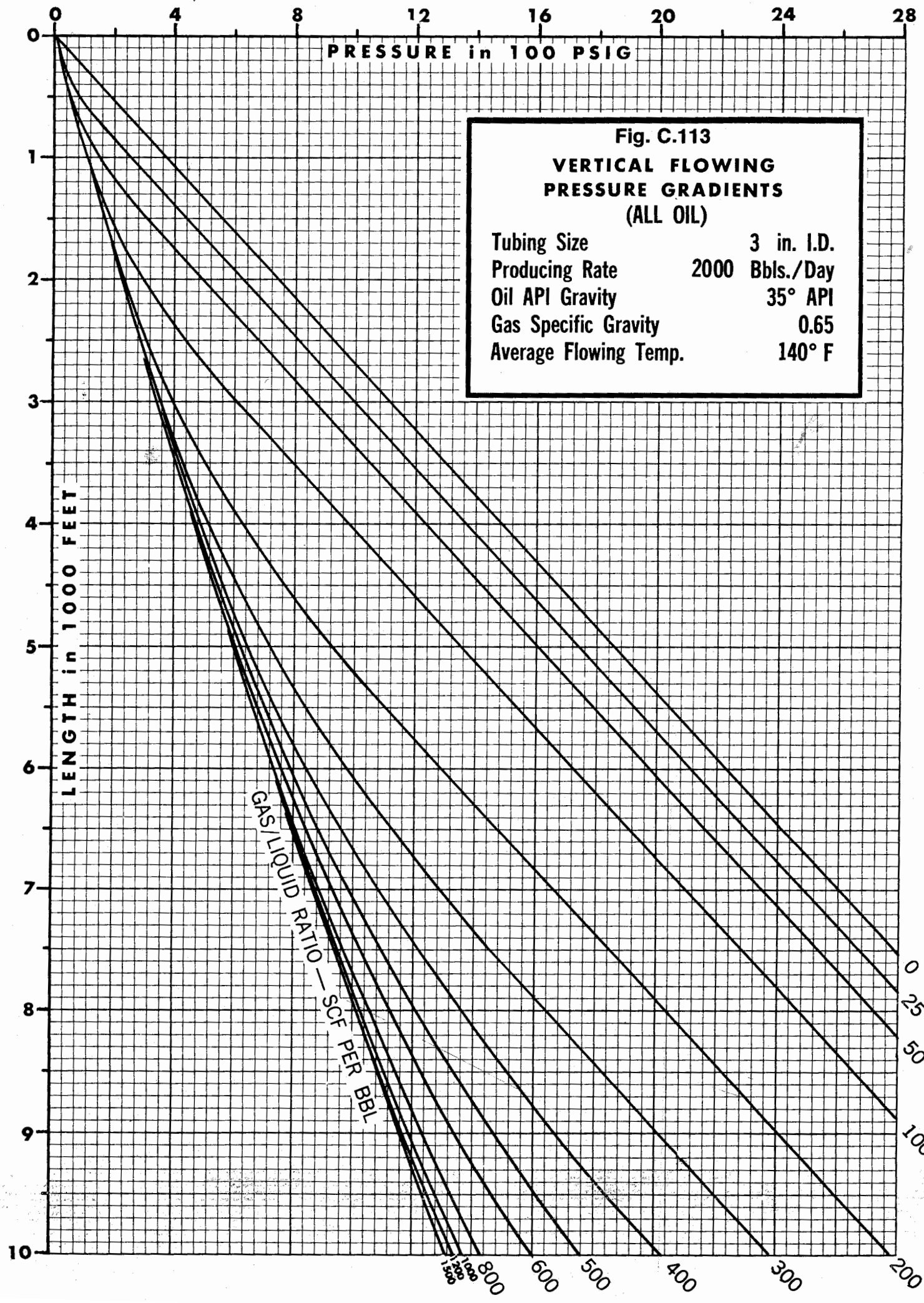
Fig. C.108

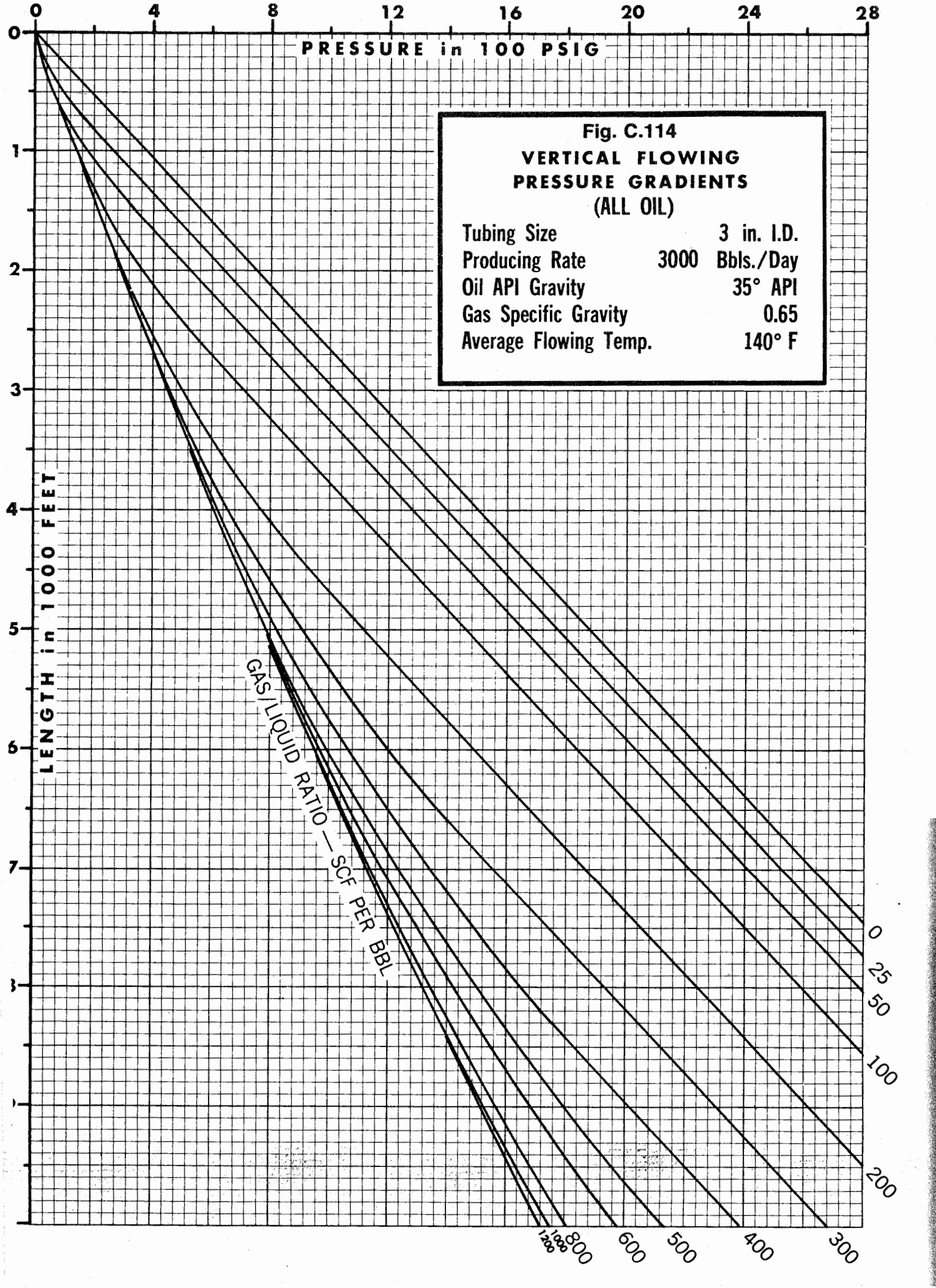

0
4
8
12
16
20
24
28
PRESSURE in 100 PSIG
0
1
2
3
4
5
6
7
8
9
10
LENGTH in 1000 FEET
GAS/LIQUID RATIO — SCF PER BBL
0
25
50
100
200
300
400
500
600
800
1000
1200
1500
2000
3000
Fig. C.109
VERTICAL FLOWING
PRESSURE GRADIENTS
(ALL WATER)
Tubing Size 2.5 in. I.D.
Producing Rate 800 Bbls./Day
Water Specific Gravity 1.074
Gas Specific Gravity 0.65
Average Flowing Temp. 140° F

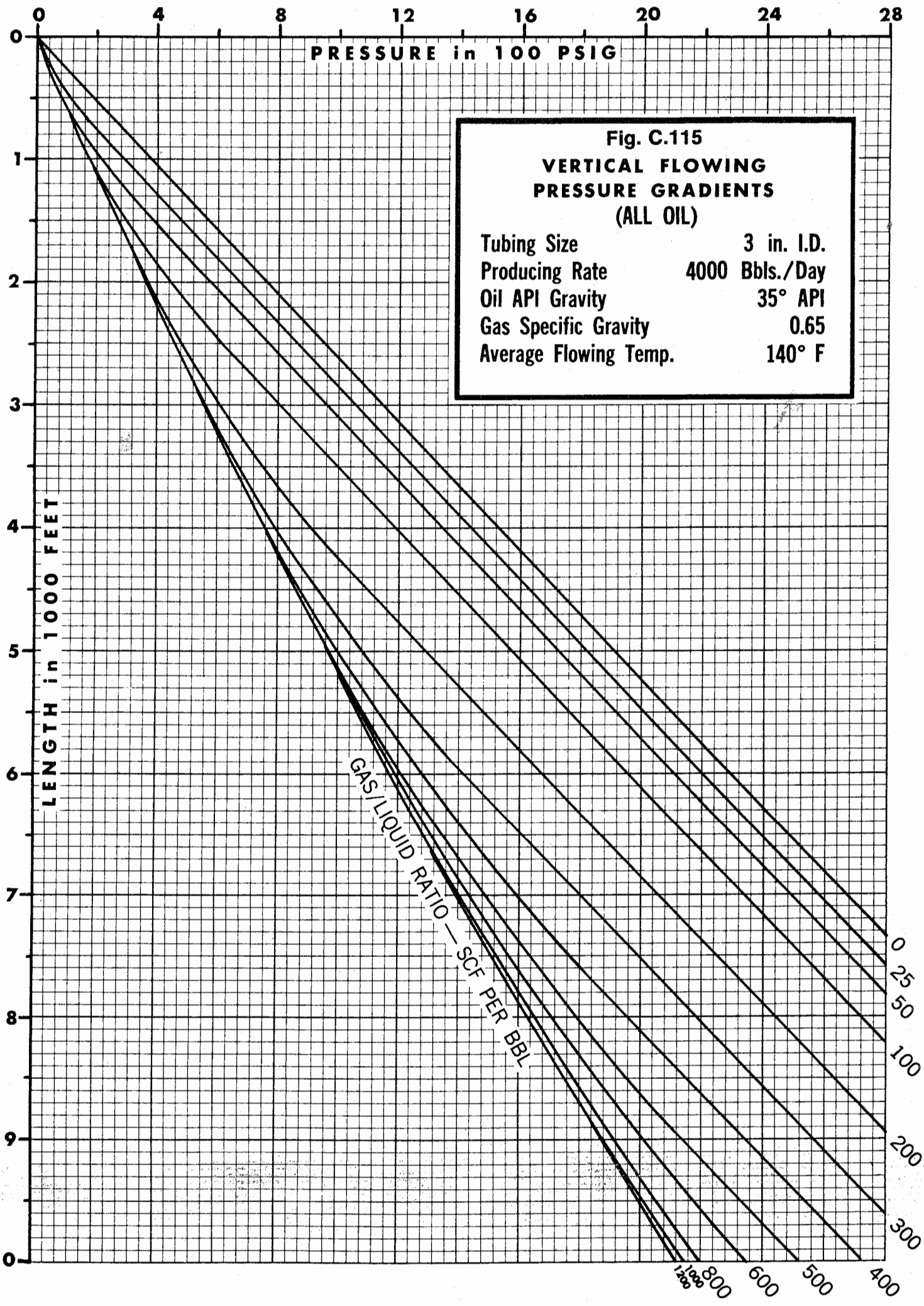

0
4
8
12
16
20
24
28
PRESSURE in 100 PSIG
Fig. C.110
VERTICAL FLOWING
PRESSURE GRADIENTS
(ALL WATER)
Tubing Size 2.5 in. I.D.
Producing Rate 1000 Bbls./Day
Water Specific Gravity 1.074
Gas Specific Gravity 0.65
Average Flowing Temp. 170° F
LENGTH in 1000 FEET
GAS/LIQUID RATIO — SCF PER BBL
0
25
50
100
200
300
400
500
600
800
1000
1200
1500
2000

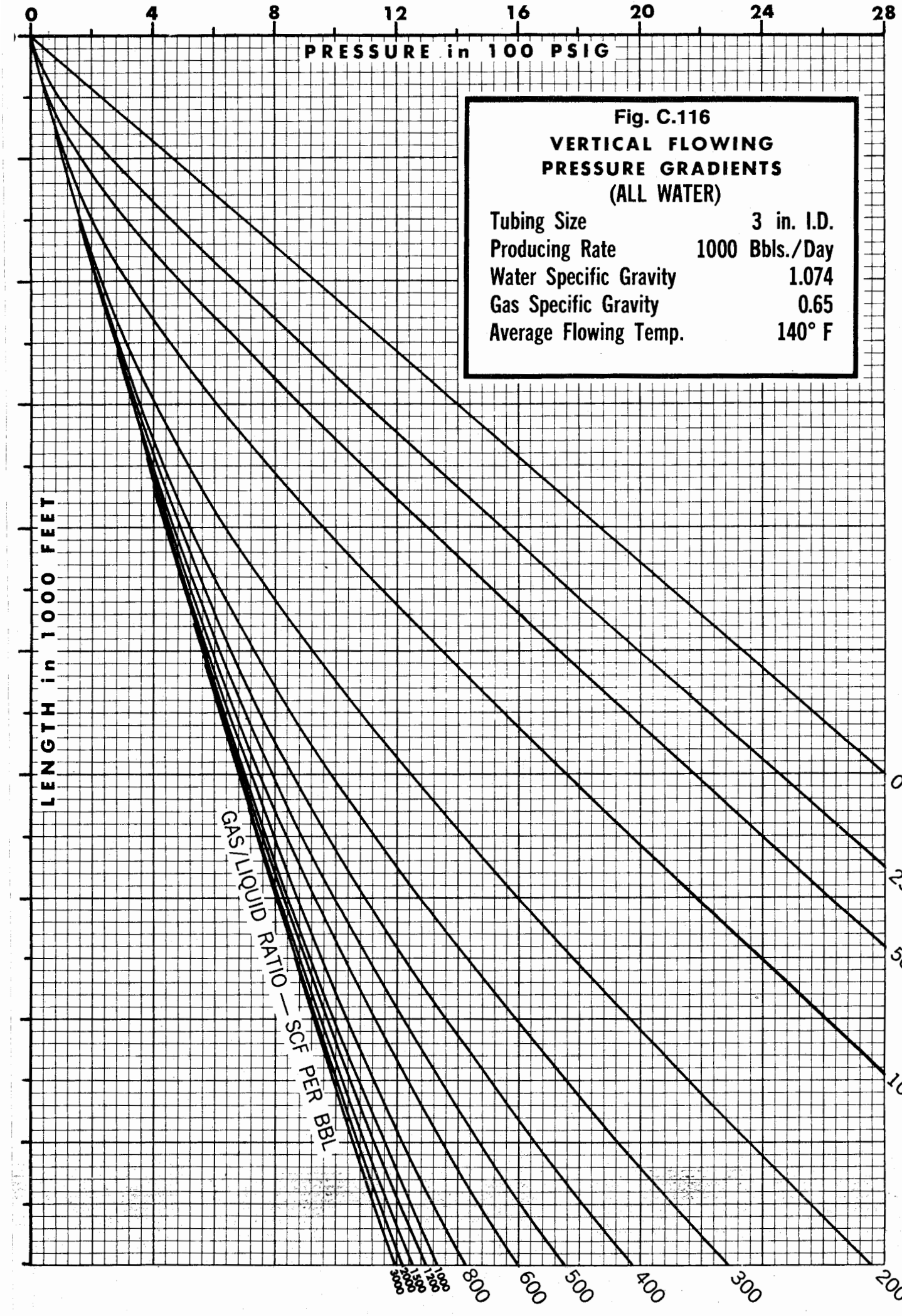

0
4
8
12
16
20
24
28
PRESSURE in 100 PSIG
0
1
2
3
4
5
6
7
8
9
0
LENGTH in 1000 FEET
Fig. C.111
VERTICAL FLOWING
PRESSURE GRADIENTS
(ALL WATER)
Tubing Size 2.5 in. I.D.
Producing Rate 1500 Bbls./Day
Water Specific Gravity 1.074
Gas Specific Gravity 0.65
Average Flowing Temp. 140° F
GAS/LIQUID RATIO — SCF PER BBL
0
25
50
100
200
300
400
500
600
800
1000
1200
1500

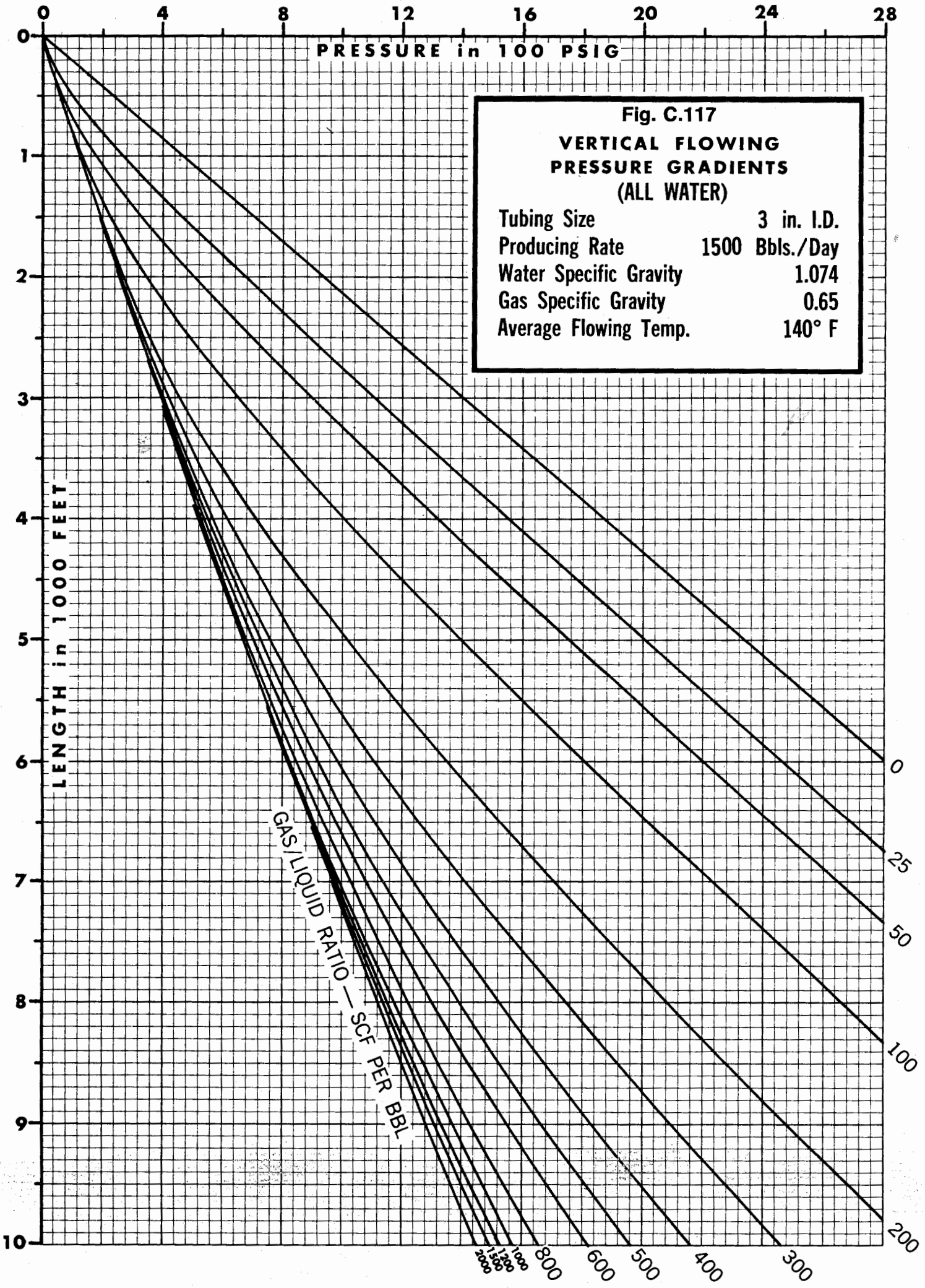

0
4
8
12
16
20
24
28
PRESSURE in 100 PSIG
LENGTH in 1000 FEET
Fig. C.112
VERTICAL FLOWING
PRESSURE GRADIENTS
(ALL OIL)
Tubing Size 3 in. I.D.
Producing Rate 1000 Bbls./Day
Oil API Gravity 35° API
Gas Specific Gravity 0.65
Average Flowing Temp. 140° F
GAS/LIQUID RATIO — SCF PER BBL
0
25
50
100
200
300
400
500
600
800


0
4
8
12
16
20
24
28
PRESSURE in 100 PSIG
0
1
2
3
4
5
6
7
8
9
10
LENGTH in 1000 FEET
Fig. C.113
VERTICAL FLOWING
PRESSURE GRADIENTS
(ALL OIL)
Tubing Size 3 in. I.D.
Producing Rate 2000 Bbls./Day
Oil API Gravity 35° API
Gas Specific Gravity 0.65
Average Flowing Temp. 140° F
GAS/LIQUID RATIO — SCF PER BBL
0
25
50
100
200
300
400
500
600
800
1000
1200
1500


0
4
8
12
16
20
24
28
PRESSURE in 100 PSIG
0
1
2
3
4
5
6
7
LENGTH in 1000 FEET
Fig. C.114
VERTICAL FLOWING
PRESSURE GRADIENTS
(ALL OIL)
Tubing Size 3 in. I.D.
Producing Rate 3000 Bbls./Day
Oil API Gravity 35° API
Gas Specific Gravity 0.65
Average Flowing Temp. 140° F
GAS/LIQUID RATIO — SCF PER BBL
0
25
50
100
200
300
400
500
600
800
1000
1200


0
4
8
12
16
20
24
28
PRESSURE in 100 PSIG
0
1
2
3
4
5
6
7
8
9
10
LENGTH in 1000 FEET
Fig. C.115
VERTICAL FLOWING
PRESSURE GRADIENTS
(ALL OIL)
Tubing Size 3 in. I.D.
Producing Rate 4000 Bbls./Day
Oil API Gravity 35° API
Gas Specific Gravity 0.65
Average Flowing Temp. 140° F
GAS/LIQUID RATIO — SCF PER BBL
0
25
50
100
200
300
400
500
600
800
1,000
1,200


0
4
8
12
16
20
24
28
PRESSURE in 100 PSIG
LENGTH in 1000 FEET
GAS/LIQUID RATIO — SCF PER BBL
0
25
50
100
200
300
400
500
600
800
1000
1200
1500
2000
3000
Fig. C.116
VERTICAL FLOWING
PRESSURE GRADIENTS
(ALL WATER)
Tubing Size 3 in. I.D.
Producing Rate 1000 Bbls./Day
Water Specific Gravity 1.074
Gas Specific Gravity 0.65
Average Flowing Temp. 140° F


0
4
8
12
16
20
24
28
PRESSURE in 100 PSIG
0
1
2
3
4
5
6
7
8
9
10
LENGTH in 1000 FEET
Fig. C.117
VERTICAL FLOWING
PRESSURE GRADIENTS
(ALL WATER)
Tubing Size 3 in. I.D.
Producing Rate 1500 Bbls./Day
Water Specific Gravity 1.074
Gas Specific Gravity 0.65
Average Flowing Temp. 140° F
GAS/LIQUID RATIO — SCF PER BBL
0
25
50
100
200
300
400
500
600
800
1000
1200
1500
2000

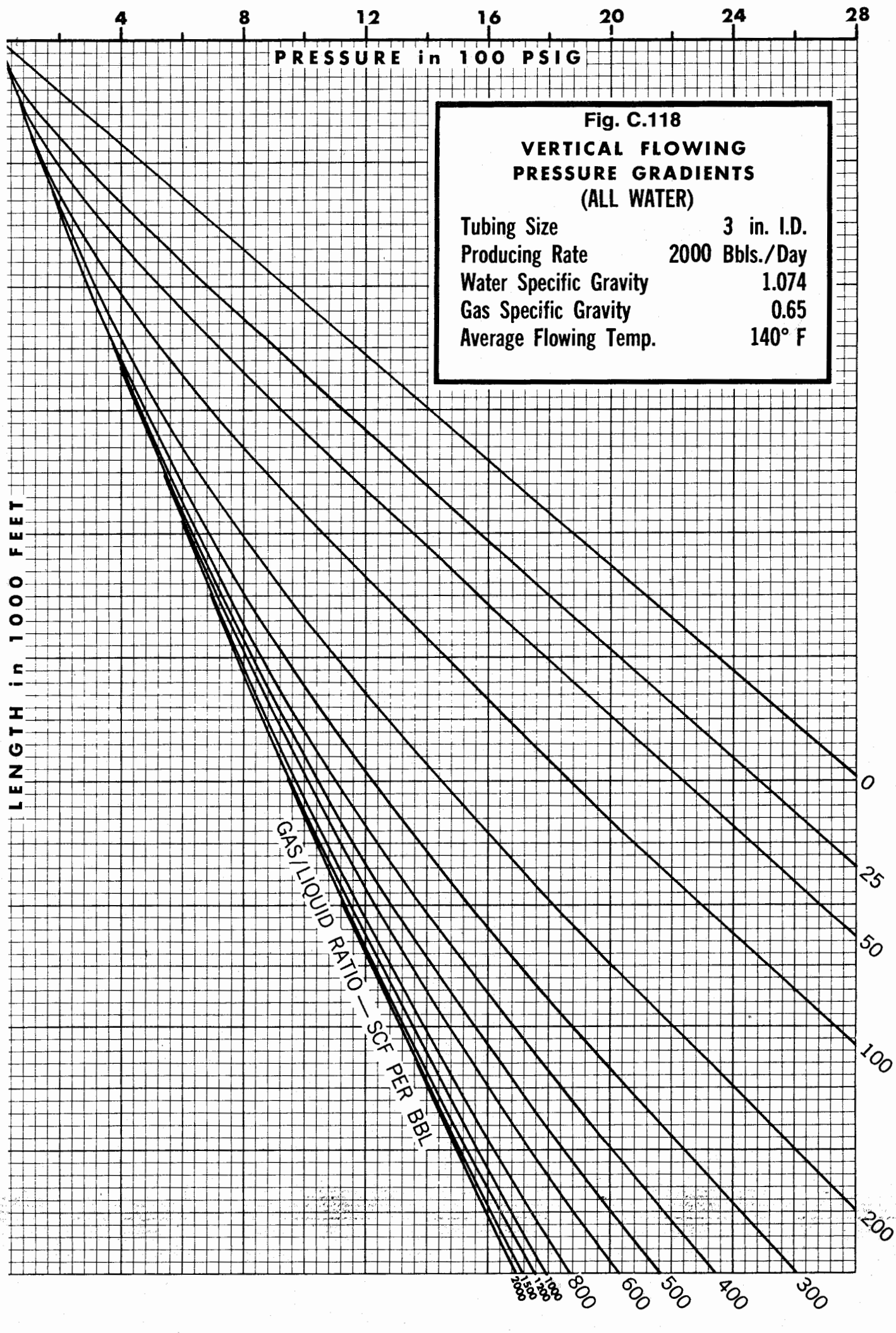
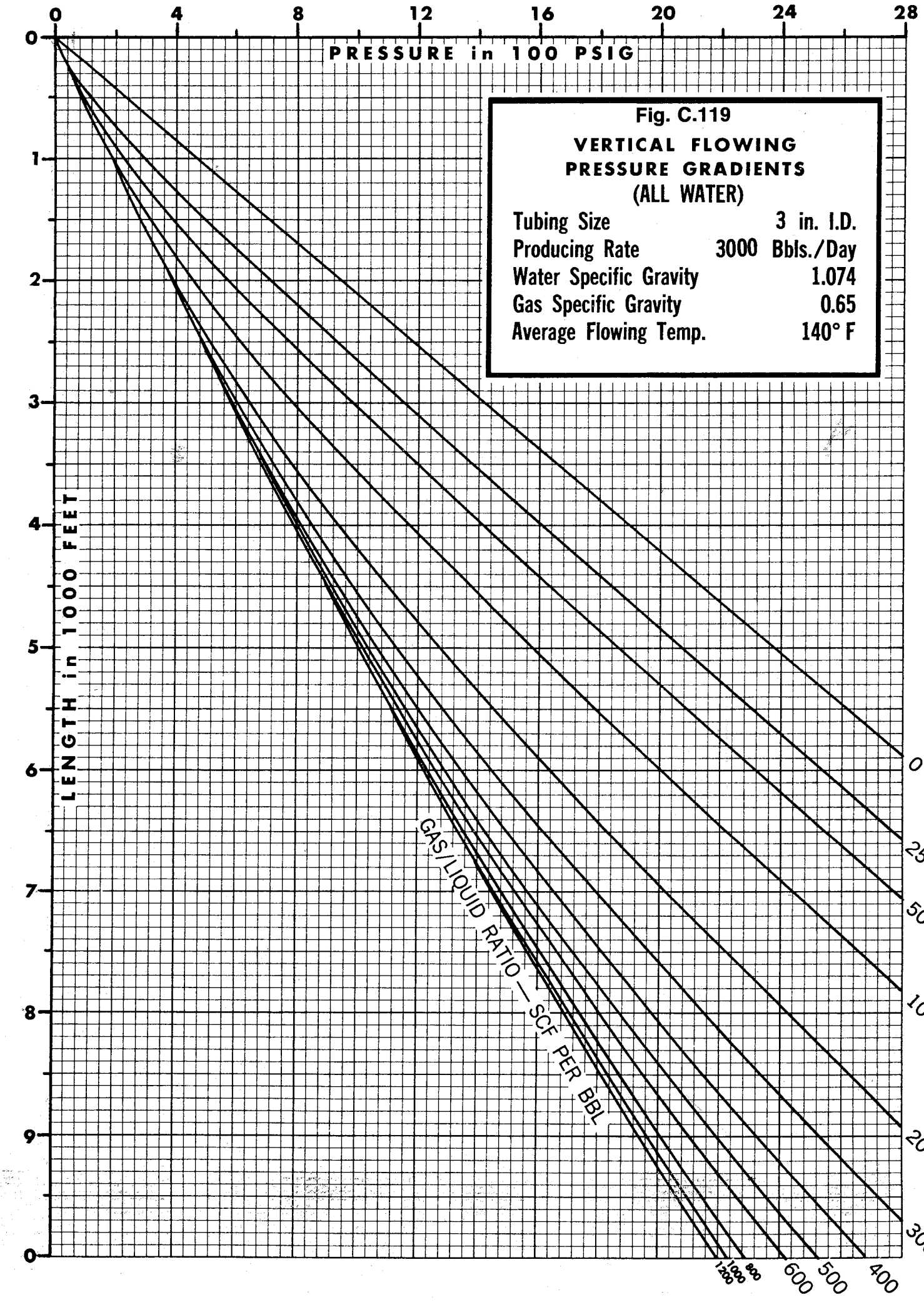
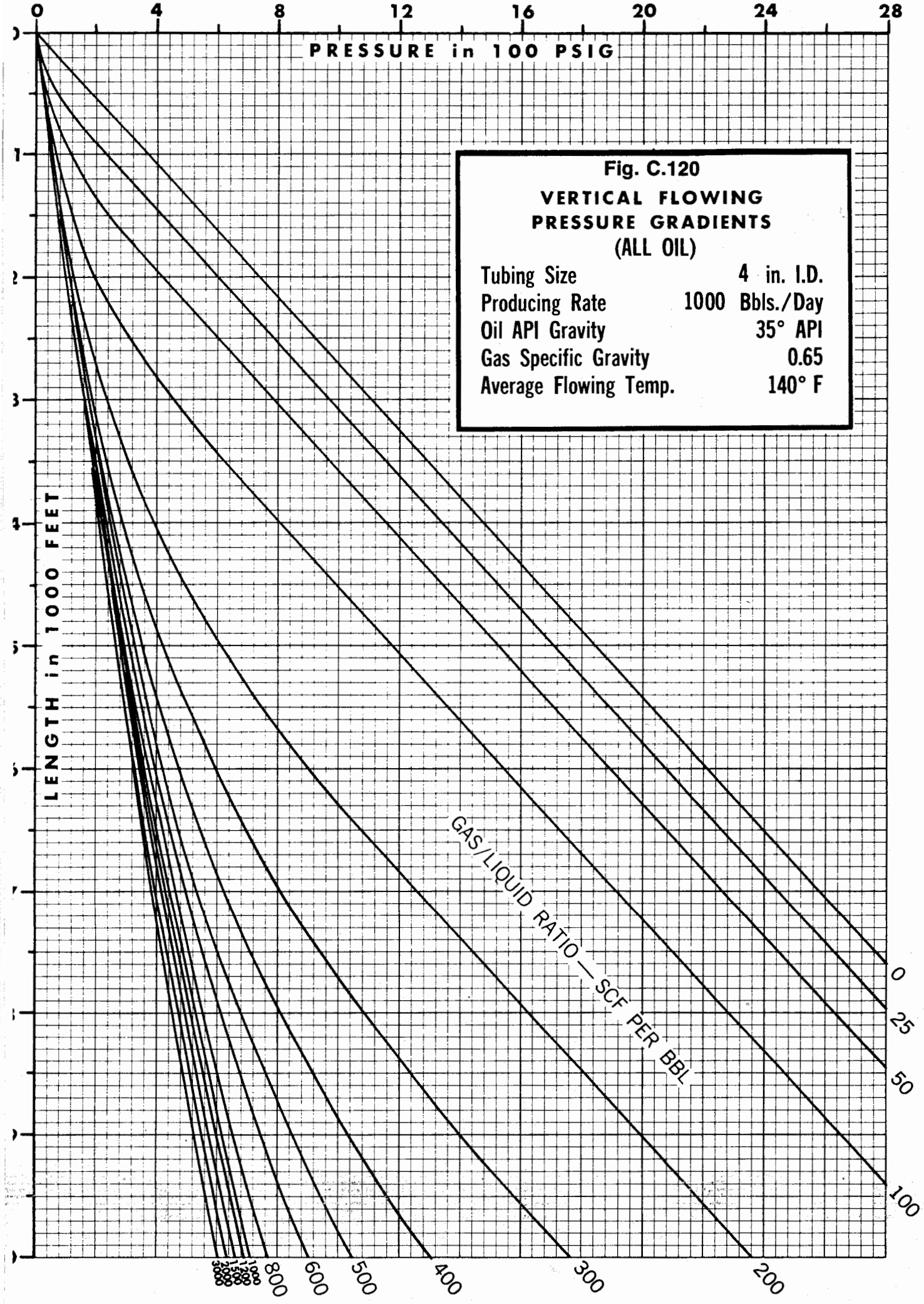


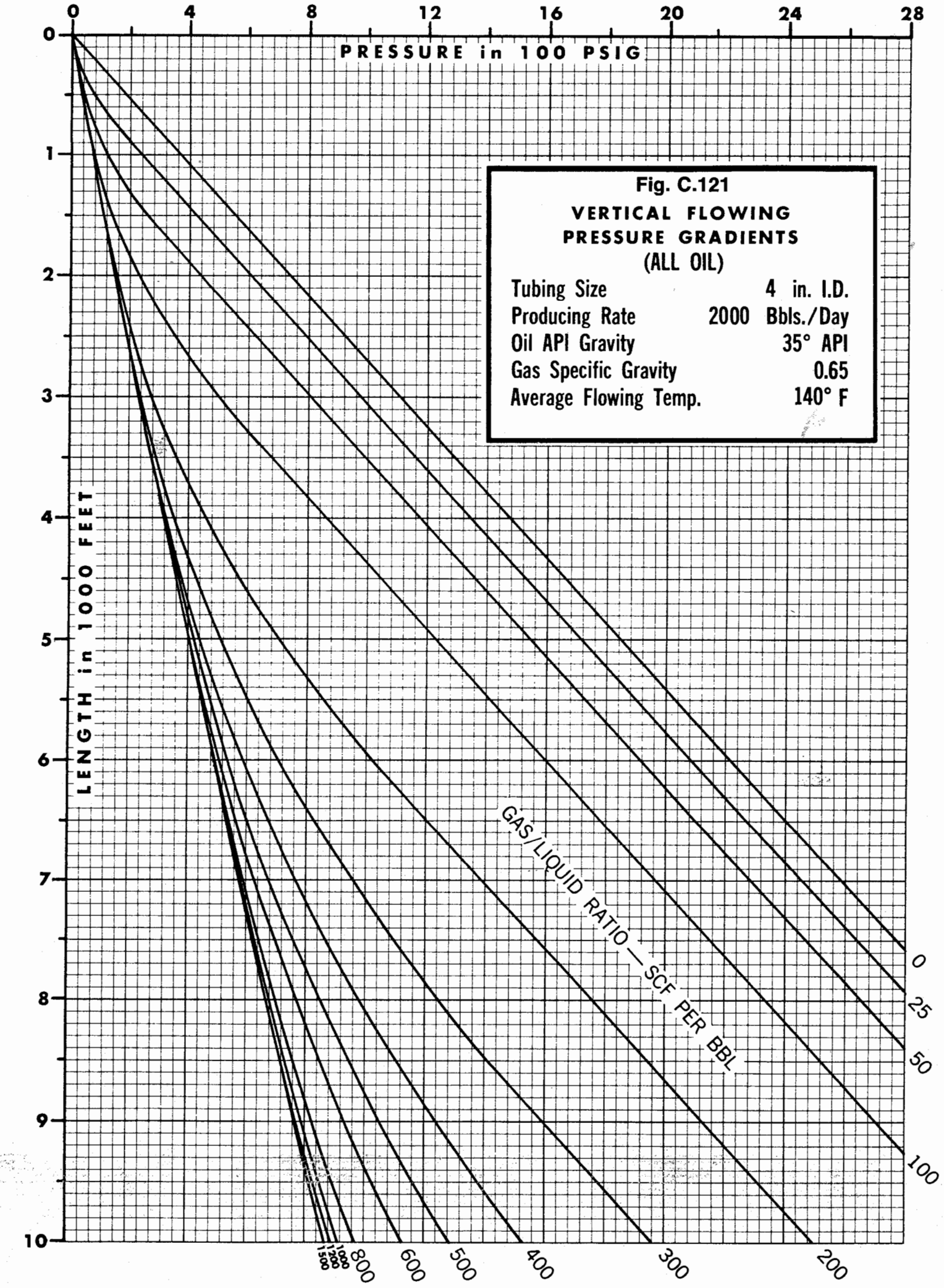
Fig. C.118

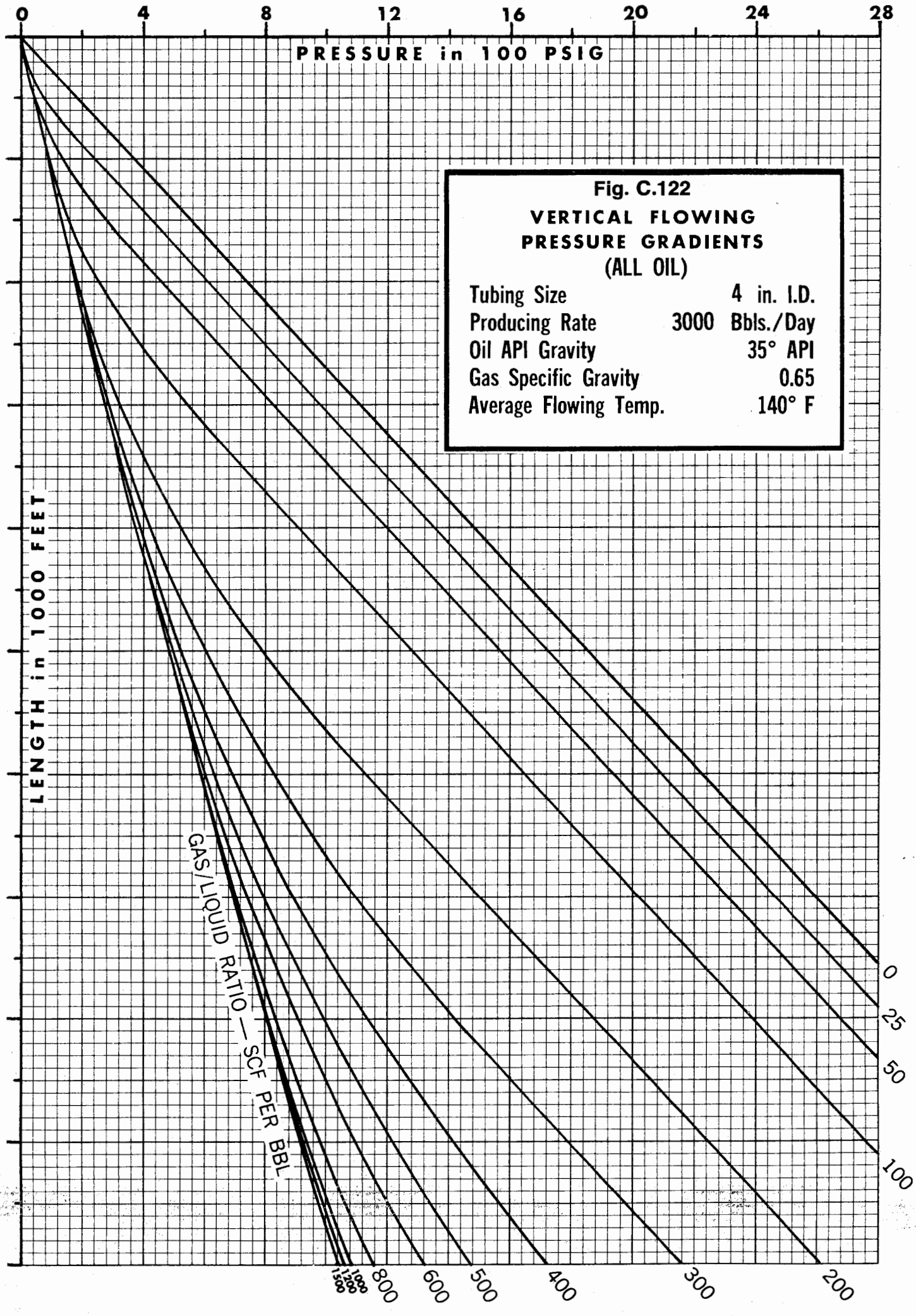
VERTICAL FLOWING PRESSURE GRADIENTS (ALL WATER)

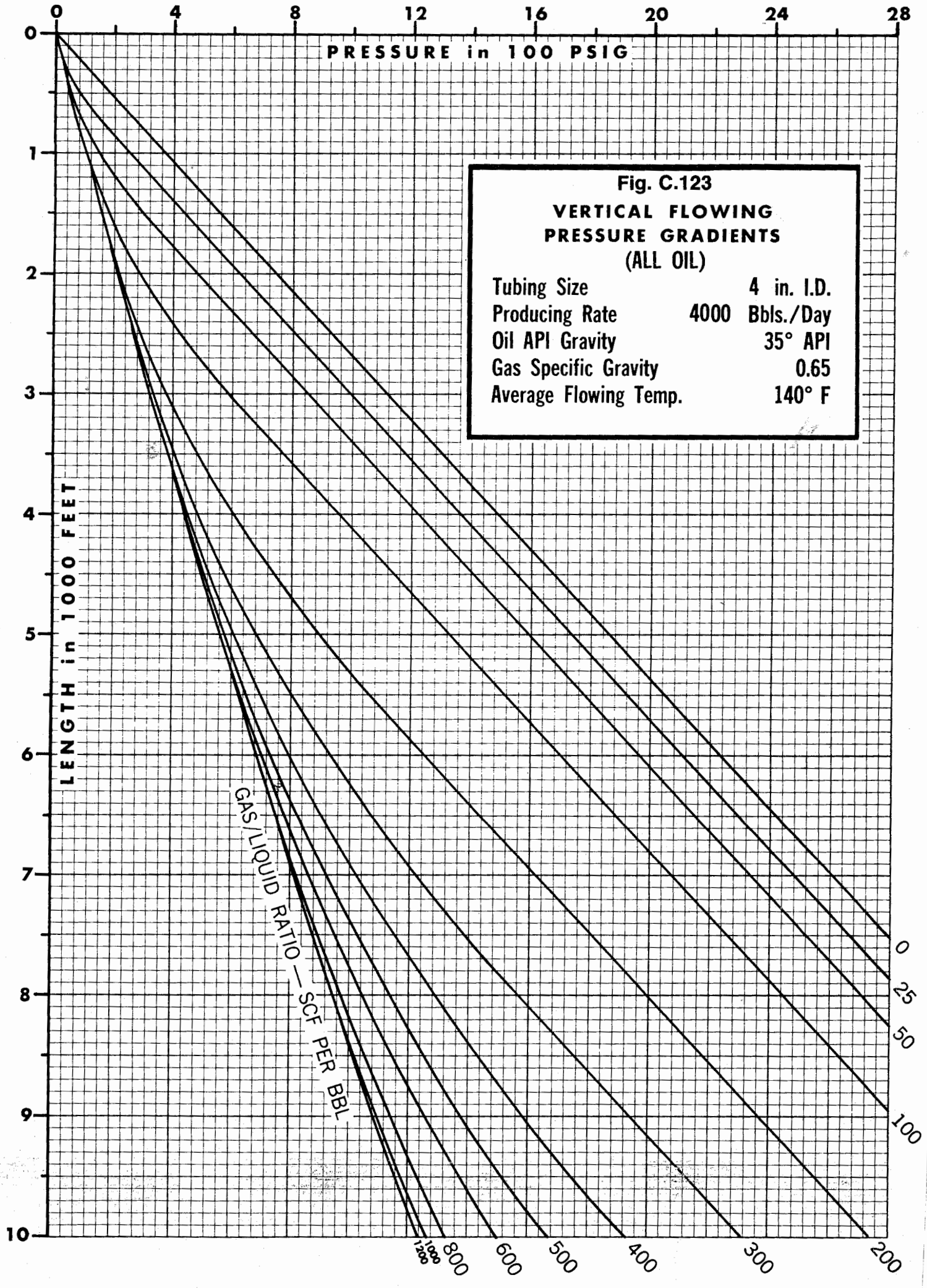
Tubing Size	3 in. I.D.
Producing Rate	2000 Bbls./Day
Water Specific Gravity	1.074
Gas Specific Gravity	0.65
Average Flowing Temp.	140° F

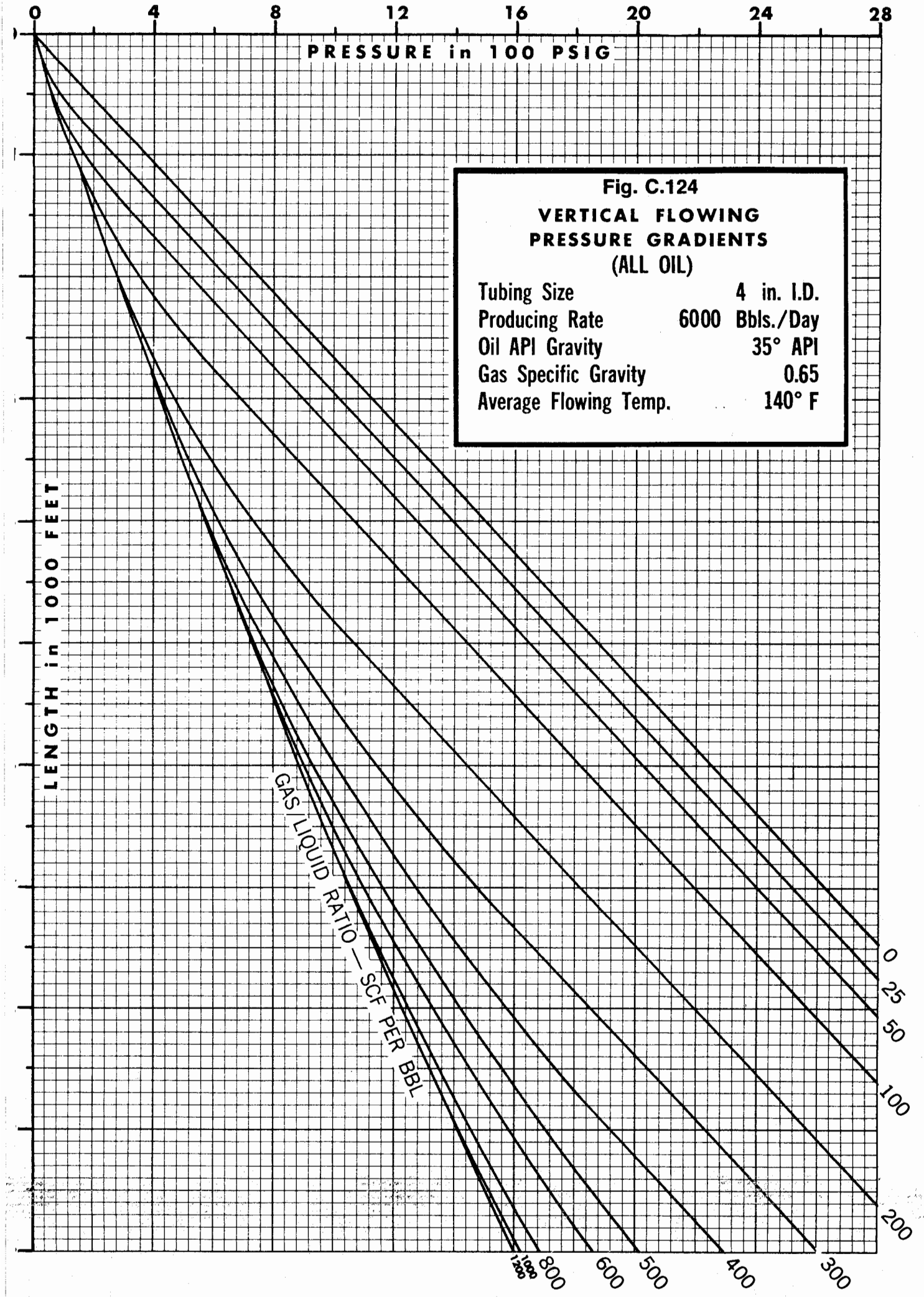

0
4
8
12
16
20
24
28
PRESSURE in 100 PSIG
0
1
2
3
4
5
6
7
8
9
0
LENGTH in 1000 FEET
Fig. C.119
VERTICAL FLOWING
PRESSURE GRADIENTS
(ALL WATER)
Tubing Size 3 in. I.D.
Producing Rate 3000 Bbls./Day
Water Specific Gravity 1.074
Gas Specific Gravity 0.65
Average Flowing Temp. 140° F
GAS/LIQUID RATIO — SCF PER BBL
0
25
50
100
200
300
400
500
600
800
1000
1200

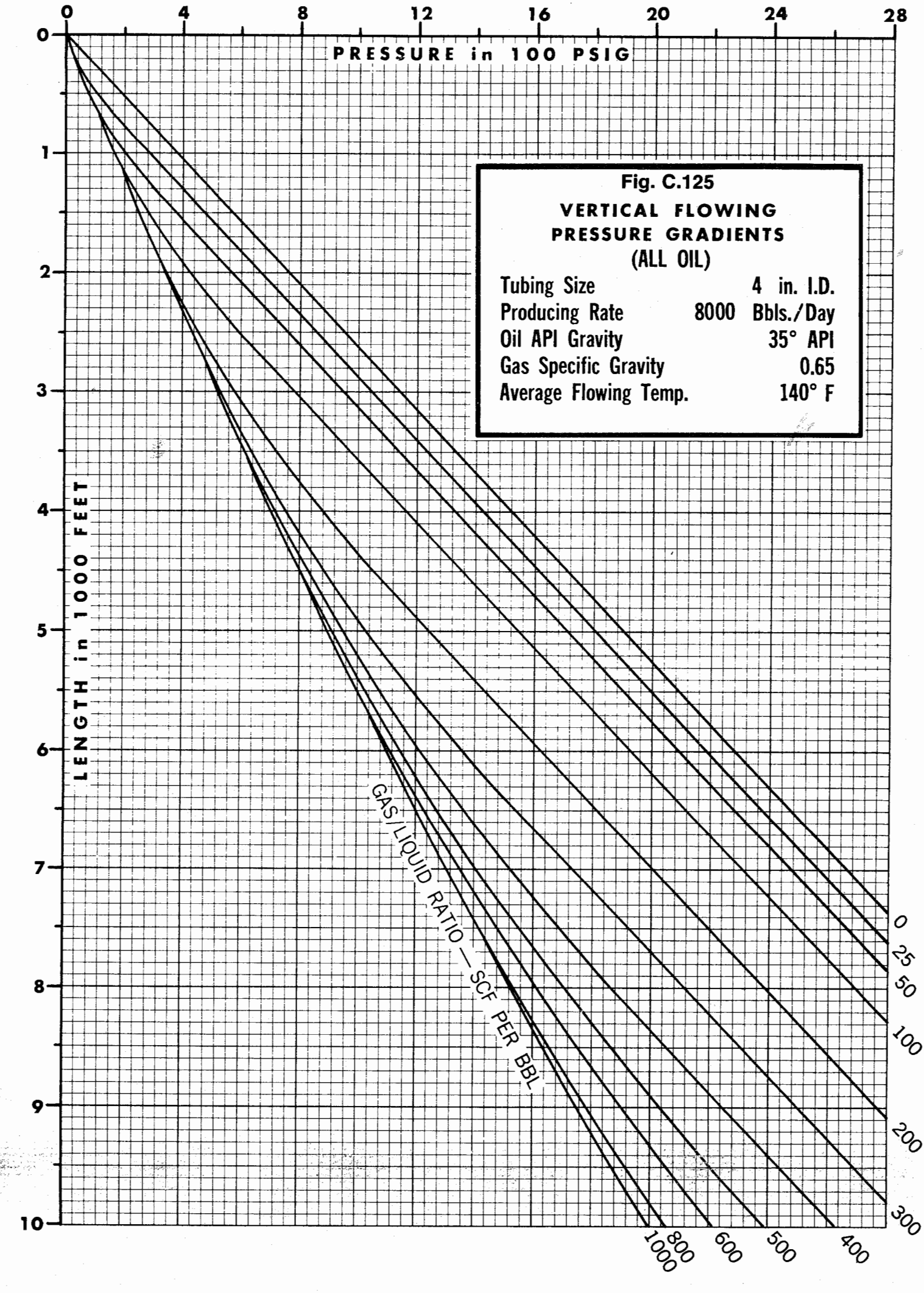

0
4
8
12
16
20
24
28
PRESSURE in 100 PSIG
LENGTH in 1000 FEET
GAS/LIQUID RATIO — SCF PER BBL
0
25
50
100
200
300
400
500
600
800
Fig. C.120
VERTICAL FLOWING
PRESSURE GRADIENTS
(ALL OIL)
Tubing Size 4 in. I.D.
Producing Rate 1000 Bbls./Day
Oil API Gravity 35° API
Gas Specific Gravity 0.65
Average Flowing Temp. 140° F


0
4
8
12
16
20
24
28
PRESSURE in 100 PSIG
0
1
2
3
4
5
6
7
8
9
10
LENGTH in 1000 FEET
Fig. C.121
VERTICAL FLOWING
PRESSURE GRADIENTS
(ALL OIL)
Tubing Size 4 in. I.D.
Producing Rate 2000 Bbls./Day
Oil API Gravity 35° API
Gas Specific Gravity 0.65
Average Flowing Temp. 140° F
GAS/LIQUID RATIO — SCF PER BBL
0
25
50
100
200
300
400
500
600
800
1000
1200
1500


0
4
8
12
16
20
24
28
PRESSURE in 100 PSIG
LENGTH in 1000 FEET
GAS/LIQUID RATIO — SCF PER BBL
0
25
50
100
200
300
400
500
600
800
1000
1250
1500
Fig. C.122
VERTICAL FLOWING
PRESSURE GRADIENTS
(ALL OIL)
Tubing Size 4 in. I.D.
Producing Rate 3000 Bbls./Day
Oil API Gravity 35° API
Gas Specific Gravity 0.65
Average Flowing Temp. 140° F


0
4
8
12
16
20
24
28
PRESSURE in 100 PSIG
0
1
2
3
4
5
6
7
8
9
10
LENGTH in 1000 FEET
Fig. C.123
VERTICAL FLOWING
PRESSURE GRADIENTS
(ALL OIL)
Tubing Size 4 in. I.D.
Producing Rate 4000 Bbls./Day
Oil API Gravity 35° API
Gas Specific Gravity 0.65
Average Flowing Temp. 140° F
GAS/LIQUID RATIO — SCF PER BBL
0
25
50
100
200
300
400
500
600
800
1000
1200


0
4
8
12
16
20
24
28
PRESSURE in 100 PSIG
LENGTH in 1000 FEET
GAS/LIQUID RATIO — SCF PER BBL
0
25
50
100
200
300
400
500
600
800
1000
1200
Fig. C.124
VERTICAL FLOWING
PRESSURE GRADIENTS
(ALL OIL)
Tubing Size 4 in. I.D.
Producing Rate 6000 Bbls./Day
Oil API Gravity 35° API
Gas Specific Gravity 0.65
Average Flowing Temp. 140° F


0
4
8
12
16
20
24
28
PRESSURE in 100 PSIG
0
1
2
3
4
5
6
7
8
9
10
LENGTH in 1000 FEET
Fig. C.125
VERTICAL FLOWING
PRESSURE GRADIENTS
(ALL OIL)
Tubing Size 4 in. I.D.
Producing Rate 8000 Bbls./Day
Oil API Gravity 35° API
Gas Specific Gravity 0.65
Average Flowing Temp. 140° F
GAS/LIQUID RATIO — SCF PER BBL
0
25
50
100
200
300
400
500
600
800
1000

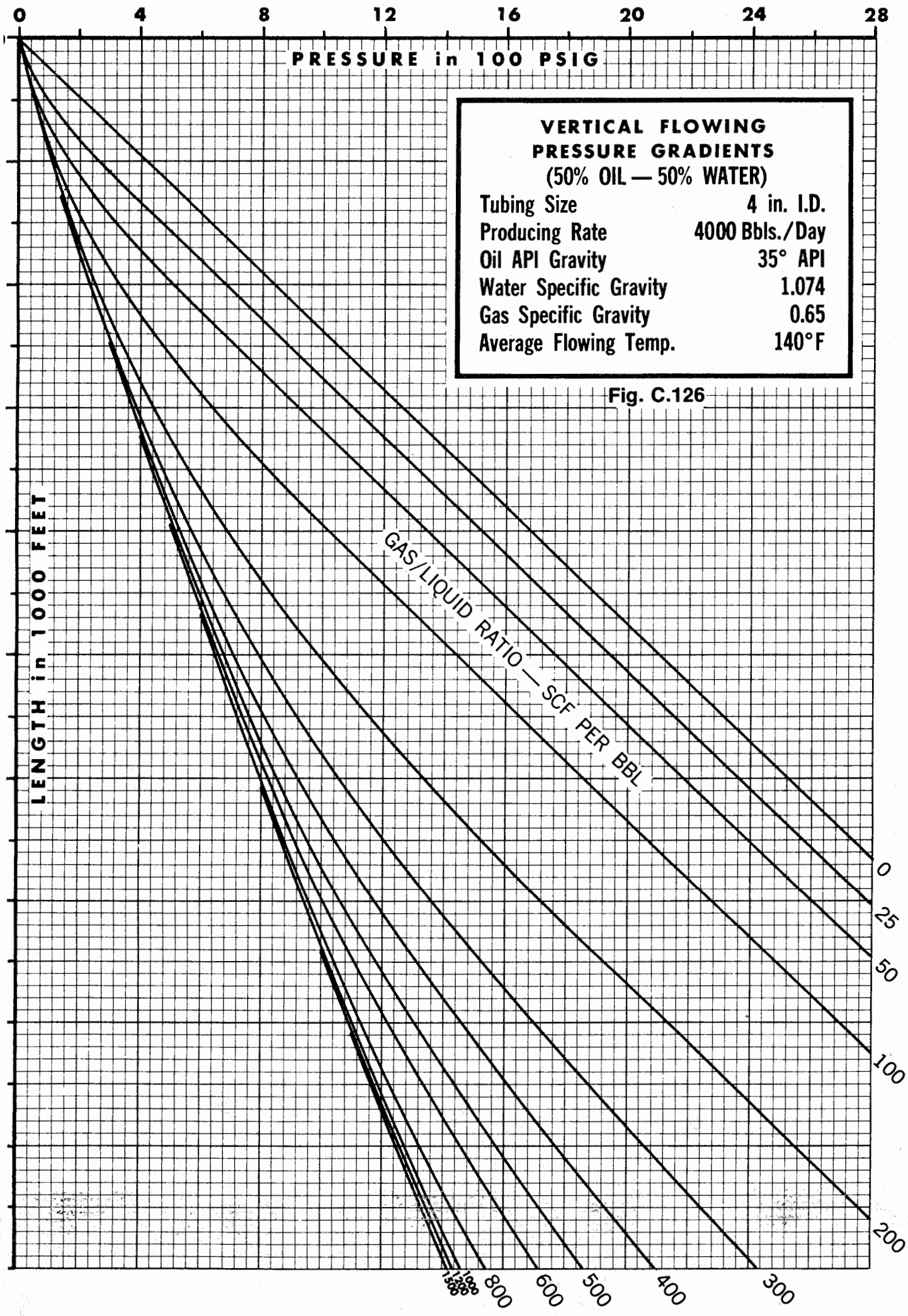
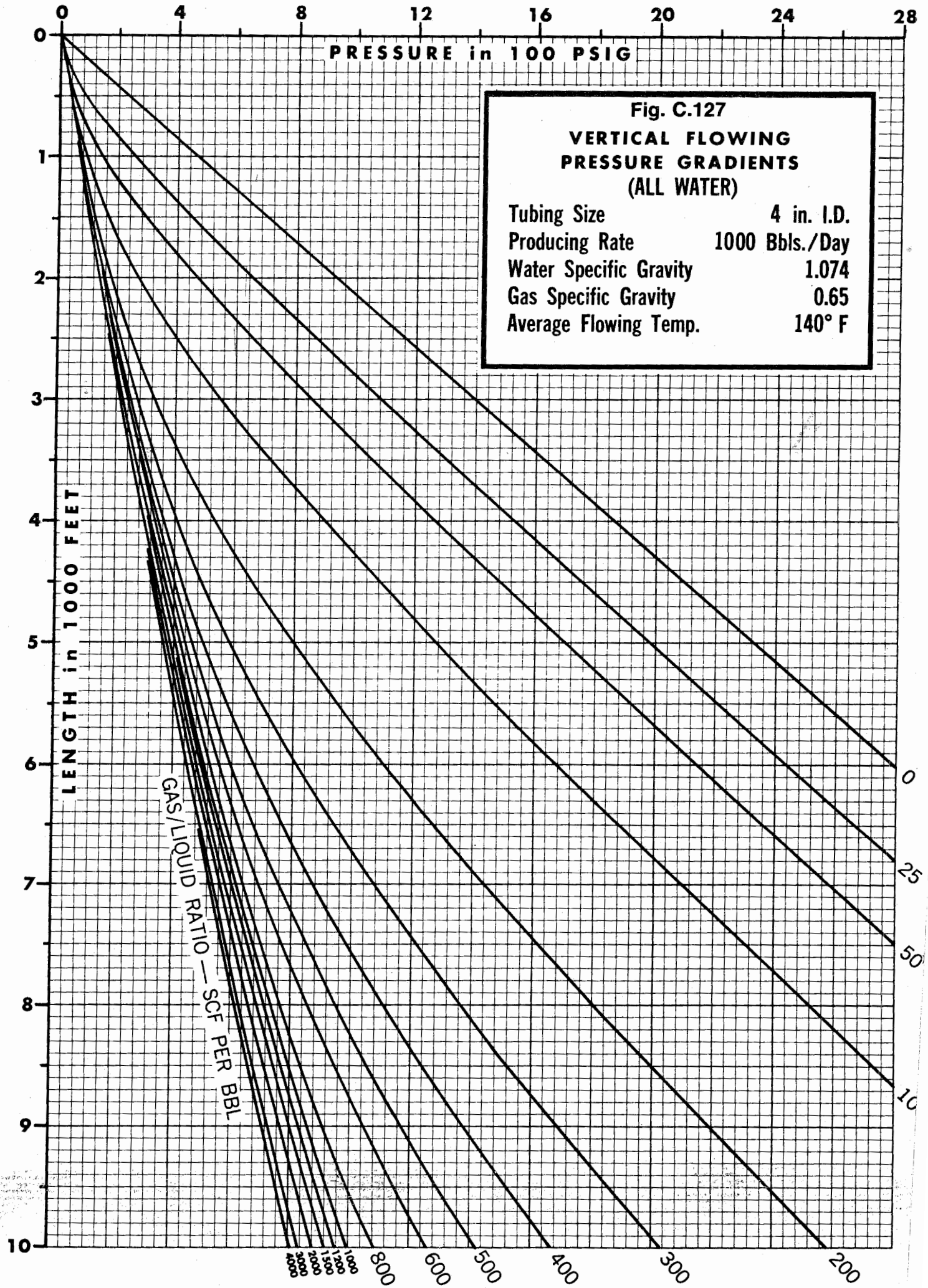

0
4
8
12
16
20
24
28
PRESSURE in 100 PSIG
VERTICAL FLOWING
PRESSURE GRADIENTS
(50% OIL — 50% WATER)
Tubing Size 4 in. I.D.
Producing Rate 4000 Bbls./Day
Oil API Gravity 35° API
Water Specific Gravity 1.074
Gas Specific Gravity 0.65
Average Flowing Temp. 140°F
LENGTH in 1000 FEET
GAS/LIQUID RATIO — SCF PER BBL
0
25
50
100
200
300
400
500
600
800

Fig. C.126


0
4
8
12
16
20
24
28
PRESSURE in 100 PSIG
0
1
2
3
4
5
6
7
8
9
10
LENGTH in 1000 FEET
Fig. C.127
VERTICAL FLOWING
PRESSURE GRADIENTS
(ALL WATER)
Tubing Size 4 in. I.D.
Producing Rate 1000 Bbls./Day
Water Specific Gravity 1.074
Gas Specific Gravity 0.65
Average Flowing Temp. 140° F
GAS/LIQUID RATIO — SCF PER BBL
0
25
50
100
200
300
400
500
600
800

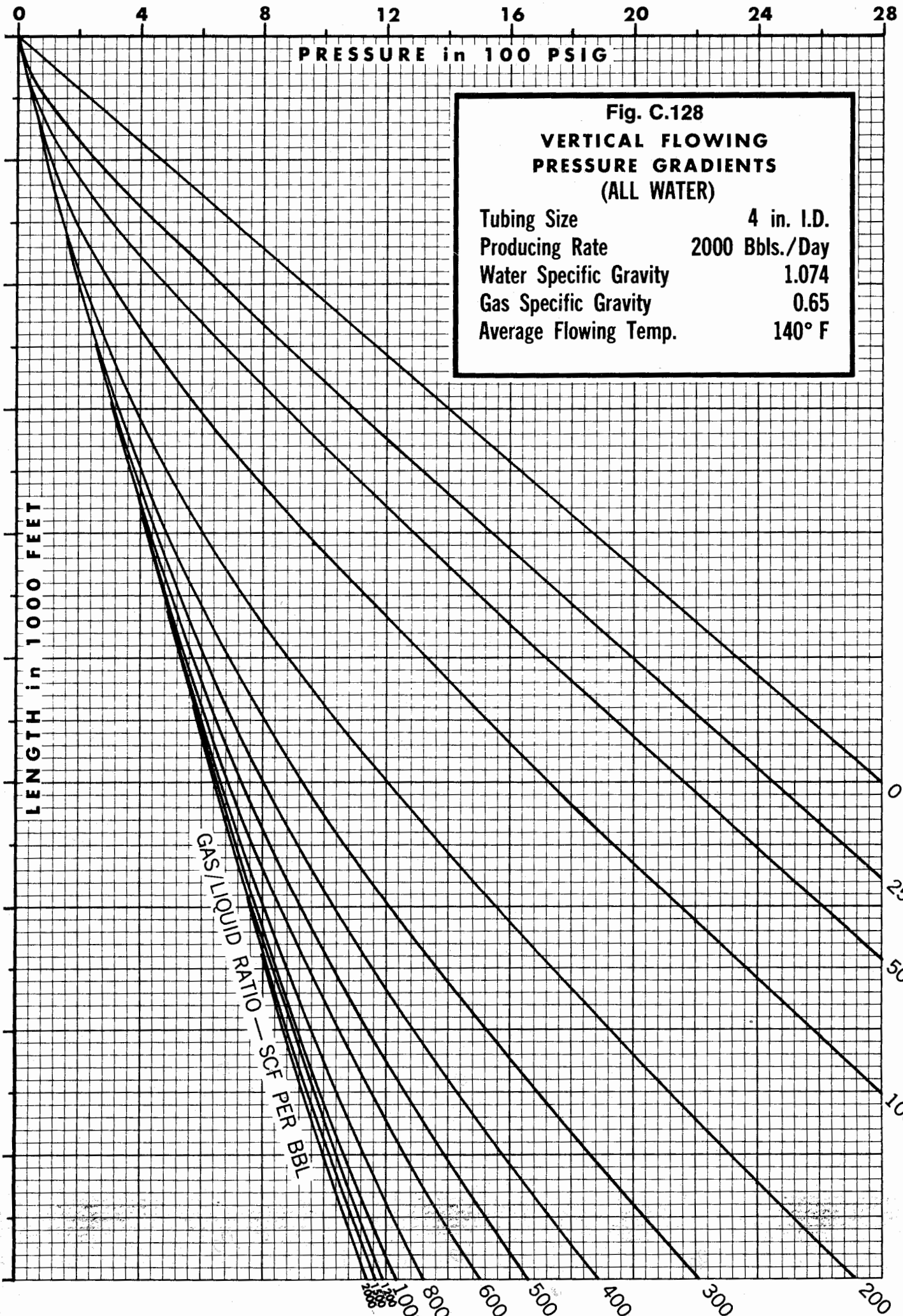


Fig. C.128
VERTICAL FLOWING PRESSURE GRADIENTS (ALL WATER)

Tubing Size	4 in. I.D.
Producing Rate	2000 Bbls./Day
Water Specific Gravity	1.074
Gas Specific Gravity	0.65
Average Flowing Temp.	140° F

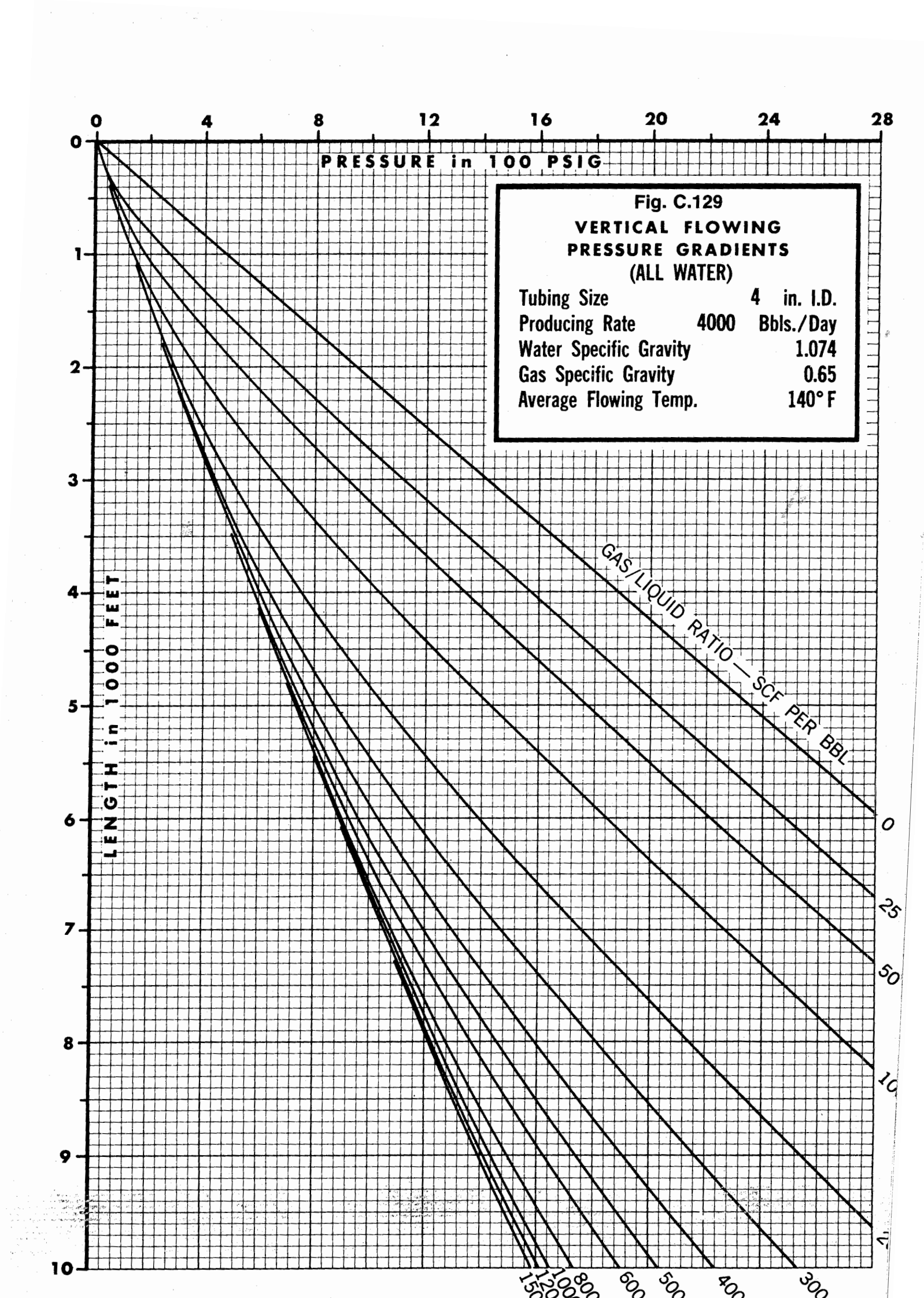


Fig. C.129
VERTICAL FLOWING
PRESSURE GRADIENTS
(ALL WATER)
Tubing Size 4 in. I.D.
Producing Rate 4000 Bbls./Day
Water Specific Gravity 1.074
Gas Specific Gravity 0.65
Average Flowing Temp. 140° F
PRESSURE in 100 PSIG
0
4
8
12
16
20
24
28
LENGTH in 1000 FEET
0
1
2
3
4
5
6
7
8
9
10
GAS/LIQUID RATIO — SCF PER BBL
0
25
50
100
200
300
400
500
600
800
1000
1200
1500

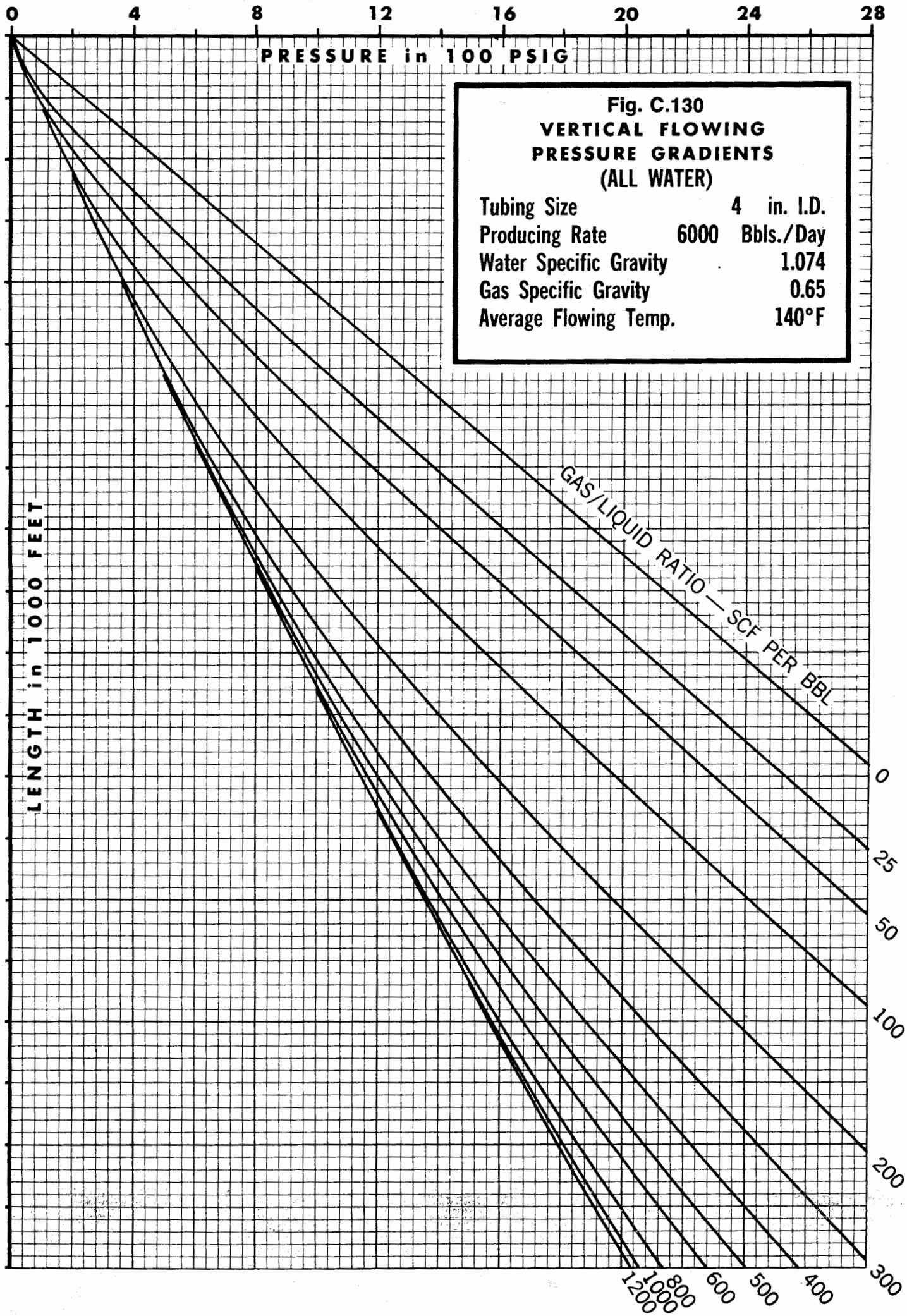
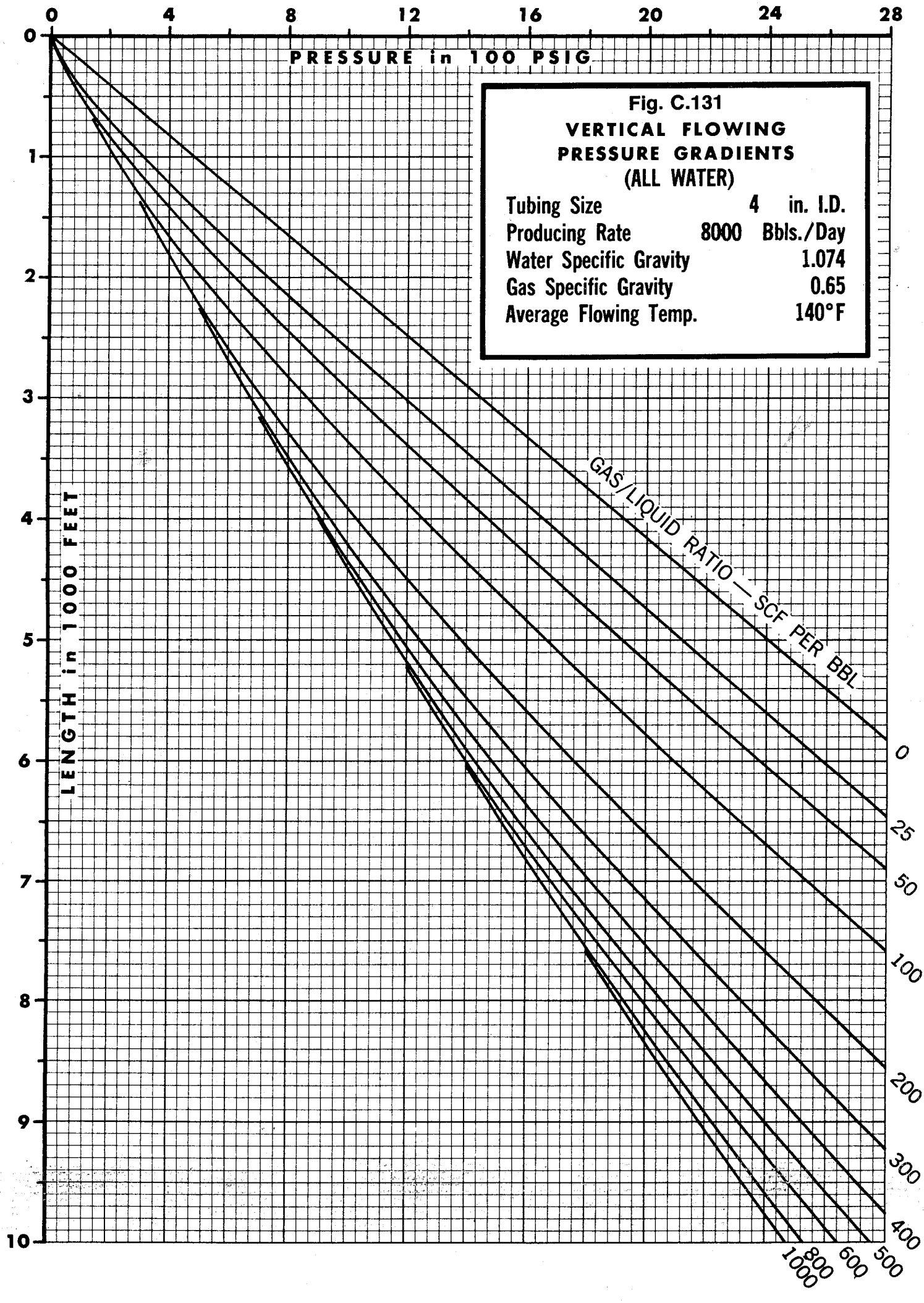


Fig. C.130
VERTICAL FLOWING PRESSURE GRADIENTS
(ALL WATER)

Tubing Size	4 in. I.D.
Producing Rate	6000 Bbls./Day
Water Specific Gravity	1.074
Gas Specific Gravity	0.65
Average Flowing Temp.	140°F


0
4
8
12
16
20
24
28
PRESSURE in 100 PSIG
0
1
2
3
4
5
6
7
8
9
10
LENGTH in 1000 FEET
Fig. C.131
VERTICAL FLOWING
PRESSURE GRADIENTS
(ALL WATER)
Tubing Size 4 in. I.D.
Producing Rate 8000 Bbls./Day
Water Specific Gravity 1.074
Gas Specific Gravity 0.65
Average Flowing Temp. 140°F
GAS/LIQUID RATIO — SCF PER BBL
0
25
50
100
200
300
400
500
600
800
1000

Appendix D

D.1 The Method of Baker
- D.11 Determination of Flow Pattern by Method of Baker
- D.12 Example Problem 1 to Determine Flow Pattern by Method of Baker
- D.13 Example Problem 2 to Find the Flow Pattern by Method of Baker

D.2 The Method of C. R. Knowles, *et al.*
- D.21 Flow Pattern Determination Using the Method of C. R. Knowles, *et al.*
- D.22 Example Problem to Determine Flow Pattern by the Method of C. R. Knowles, *et al.*

D.3 Lockhart and Martinelli
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- D.32 Procedure for Calculation of a Horizontal Pressure Traverse by the Method of Lockhart and Martinelli
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- D.41 Procedure to Determine Pressure Loss in Horizontal Lines by Method of Baker
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- D.63 Procedure for Selecting Size of Offshore Line to Bring Fluids to Onshore Facilities by Method of Dukler (Case I)

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- D.81 Mathematical Development of the Eaton, *et al.* Correlation
- D.82 Procedure for Calculation of a Pressure Traverse by the Method of Eaton, *et al.*
- D.83 Problem to Determine Pressure Loss in a 2 in. Line 1,500 ft Long by Method of Eaton, *et al.* (Kinetic Energy Neglected)
- D.84 Example Problem to Determine Pressure Loss by Method of Eaton, *et al.* (Same as Ex. D.33 by Lockhart and Martinelli) (Kinetic Energy Included)
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- D.86 Example Problem to Determine Line Size by Method of Eaton, *et al.*

D.9 Method of Beggs and Brill
- D.91 Procedure for Calculating a Pressure Traverse in a Horizontal Pipeline by the Method of Beggs and Brill
- D.92 Example Problem to Determine Pressure Loss in a 4 Inch Horizontal Line by Method of Beggs and Brill

D.10 Horizontal Flow Curves

Appendix D.1

D.11 DETERMINATION OF FLOW PATTERN BY METHOD OF BAKER

Procedure:

1. Compute the λ value

$$\lambda = \left[\left(\frac{\rho_g}{0.0764}\right)\left(\frac{\rho_L}{62.4}\right)\right]^{1/2}.$$

2. Compute ψ value

$$\psi = \frac{73}{\sigma L}\left[\mu_L\left(\frac{62.4}{\rho_L}\right)^2\right]^{1/3},$$

where σ_L = surface tension of liquid, dynes/cm.

3. Calculate the mass velocity of the liquid,

$$M_L = \frac{q_L(\text{bpd}) \times 5.61\left(\frac{\text{cu ft}}{\text{bbl}}\right) \times \rho_L(\text{lb/cu ft})}{24\ \frac{\text{hr}}{\text{day}}\ \text{A(sq ft)}}$$

where M_L = mass velocity of the liquid, $\frac{\text{lb}_m}{\text{sq ft hr}}$.

4. Calculate the mass velocity of the gas:

$$M_g = \frac{q_g(\text{scfd})\ \gamma_g\ 29}{24\ \frac{\text{hr}}{\text{day}} \times 379 \times \text{A sq ft}}$$

where M_g = mass velocity of the gas, $\frac{\text{lb}_m}{\text{hr} \times \text{sq ft}}$

5. Compute the value $\frac{M_g}{\lambda}$.

6. Compute the value $\frac{M_L\ \lambda\ \psi}{M_g}$.

7. Using the $\frac{M_g}{\lambda}$ and $\frac{M_L\ \lambda\ \psi}{M_g}$ values, from Fig. 2.130, find the flow pattern.

D.12 EXAMPLE PROBLEM 1 TO DETERMINE FLOW PATTERN BY METHOD OF BAKER

Given:

$L = 5$ miles	$P_2 = 700$ psia	$\overline{P} = 850$ psia
$P_1 = 1{,}000$ psia	$\mu_g = 0.02$ cp	
$g_g = 0.65$	$\mu_w = 1.0$ cp	
$\gamma_w = 1.07$	$\sigma_w = 70$ dynes/cm	
$q_L = 5{,}000$ bpd (water)	$d = 6$ in. (ID)	
$q_g = 6 \times 10^6$ scf/day		
$\overline{T} = 110°F$.		

1. Compute the λ value

$$\lambda = \left[\left(\frac{\rho_g}{0.0764}\right)\left(\frac{\rho_L}{62.4}\right)\right]^{1/2} = \left[\left(\frac{2.98}{0.0764}\right)\left(\frac{66.7}{62.4}\right)\right]^{1/2} = 6.46$$

$$\overline{\rho}_g = (0.65)(0.0764)\left(\frac{850}{14.7}\right)\left(\frac{520}{570}\right)\left(\frac{1}{0.88}\right) = 2.98\ \frac{\text{lb}}{\text{cu ft}}$$

$$\overline{\rho}_L = 1.07 \times 62.4\ \frac{\text{lb}}{\text{cu ft}} = 66.7\ \frac{\text{lb}}{\text{cu ft}}$$

2. Compute ψ value

$$\psi = \frac{73}{\sigma_L}\left[\mu_L\left(\frac{62.4}{\rho_L}\right)^2\right]^{0.34}$$

$$\psi = \frac{73}{70}\left[1.0\left(\frac{62.4}{66.7}\right)^2\right]^{0.34} = 1.047$$

3. Calculate M_L:

$$M_L = \frac{5{,}000(\text{bpd}) \times 5.61(\text{cu ft/bbl}) \times 66.7\ (\text{lb/cu ft})}{24(\text{hr/day}) \times 0.196(\text{sq ft})}$$

$$M_L = 398{,}000\ \text{lb}_m/\text{sq ft hr}$$

4. Calculate M_g:

$$M_g = \frac{q_L(\text{bpd}) \times \text{G/L} \times \gamma_g \times 29}{24(\text{hr/day}) \times 379 \times \text{A(sq ft)}}$$

$$M_g = \frac{6 \times 10^6 \times 0.65 \times 29}{24 \times 379 \times 0.196} = 63{,}300\ \frac{\text{lb}_m}{\text{sq ft hr}}$$

5. Compute the value $\frac{M_g}{\lambda}$

$$\frac{M_g}{\lambda} = \frac{76{,}000}{6.46} = 9{,}820$$

6. Compute the value $\frac{M_L\ \lambda\ \psi}{M_g}$:

$$\frac{M_L\ \lambda\ \psi}{M} = \frac{398{,}000 \times 6.46 \times 1.047}{63{,}300} = 42.6$$

7. From Fig. 2.130 with $\frac{M_g}{\lambda}$ and $\frac{M_L\ \lambda\ \psi}{M_g}$ values, the flow pattern is found to be *annular*.

D.13 EXAMPLE PROBLEM 2 TO FIND THE FLOW PATTERN BY METHOD OF BAKER

$L = 1{,}500$ ft	$\text{G/L} = 500$ scf/bbl
$p_1 = 1{,}500$ psia	$\overline{T} = 110°F$
$\gamma_g = 0.65$	$\mu_g = 0.02$
$\gamma_w = 1.07$	$\mu_w = 1.0$ cp
$q_L = 2{,}000$ B/D (water)	$\sigma_w = 70$ dynes/cm
$d = 2$ in. (I.D.)	

Solution:

1. Calculate the λ value

$$\lambda = \left[\left(\frac{\rho_g}{0.0764}\right)\left(\frac{\rho_L}{62.4}\right)\right]^{1/2}$$

$$\overline{\rho}_g = (0.65)(0.0764)\left(\frac{1500}{14.7}\right)\left(\frac{520}{570}\right)\left(\frac{1}{0.82}\right) = 5.63$$

$$\lambda = \left[\left(\frac{5.63}{0.0764}\right)\left(\frac{66.7}{62.4}\right)\right]^{1/2} = 8.88$$

2. Compute ψ value:

$\psi = 1.047$ (from previous calculation)

3. Calculate M_L:

$$M_L = \frac{2{,}000(\text{bpd}) \times 5.61(\text{cu ft/bbl}) \times 66.7(\text{lb/cu ft})}{24\left(\frac{\text{hr}}{\text{day}}\right) \times 0.0218\ (\text{sq ft})}$$

$$M_L = 1{,}440{,}000$$

4. Calculate M_g:

$$M_g = \frac{q_g \times \gamma_g \times 29}{24\ \frac{\text{hr}}{\text{day}} \times 379 \times \text{A(sq ft)}}$$

$$M_g = \frac{2{,}000(\text{bpd})\ (500\ \text{scf/bbl}) \times 0.65 \times 29}{24 \times 379 \times 0.0218\ \text{sq ft}} = 95{,}000$$

5. Compute the $\frac{M_g}{\lambda}$ value

$$\frac{M_g}{\lambda} = \frac{95{,}000}{8.88} = 10{,}580$$

6. Compute $\frac{M_L \lambda \psi}{M_g}$

$$\frac{M_L \lambda \psi}{M_g} = \frac{(1{,}440{,}000)(8.88)(1.047)}{95{,}000} = 142.5$$

7. From Fig. 2.130, using the calculated values of $\frac{M_g}{\lambda}$ and $\frac{M_L \lambda \psi}{M_g}$, the flow pattern is found to be *slug*.

Appendix D.2

D.21 FLOW PATTERN DETERMINATION USING THE METHOD OF C. R. KNOWLES, ET AL.

Procedure:

1. Calculate the mass flow rate of the liquid.

$$w_L = \frac{q_L(\text{bpd})\ 5.61(\text{cu ft/bbl})\ \rho_L(\text{lb/cu ft})}{86{,}400\left(\frac{\text{sec}}{\text{day}}\right)}$$

2. Calculate the mass flow rate of the gas.

$$w_L = \frac{\gamma_g \times q_g(\text{scfd})(0.0764)}{86{,}000\left(\frac{\text{sec}}{\text{day}}\right)}$$

3. Obtain w_T:

$$w_T = w_L + w_g$$

4. Determine the liquid holdup parameter:

$$\frac{(N_{LV})^{0.575}}{N_{gv}(N_d)^{0.0277}}\left(\frac{P}{P_a}\right)^{0.05}\left(\frac{N_L}{N_{LB}}\right)^{0.1},$$

and determine H_L from Fig. 2.136.

5. Calculate the liquid and gas velocities, v_L and v_g:

$$v_L = \frac{v_{sL}}{H_L}, \qquad v_g = \frac{v_{sg}}{(1 - H_L)}$$

6. Calculate the viscosity of the two phase mixture,

$$\mu_{TP} = \mu_L{}^{H_L}\,\mu_g{}^{(1-H_L)}.$$

7. Calculate the Reynold's function:

$$\frac{w_T\ H_L{}^2}{d_p\ \mu_{TP}},$$

where d_p is the pipe diameter.

8. Calculate the Weber function:

$$\frac{\rho_L\ v_L{}^2\ d_p\ H_L{}^{1/2}}{\sigma} + \frac{\rho_L\ v_s{}^2\ d_p\ (1 - H_L)^{1/2}}{\sigma}$$

where σ = surface tension, dynes/cm,
v_s = slip velocity, $= v_g - v_L$.

9. Find the gas density $\bar{\rho}_g$.
10. Using factors found in Steps 7 and 8, from Fig. 2.131 obtain the flow pattern.

D.22 EXAMPLE PROBLEM TO DETERMINE FLOW PATTERN BY THE METHOD OF C. R. KNOWLES, ET AL.

Example:

$L = 5$ miles	$\gamma_w = 1.07$	
$P_1 = 1{,}000$ psia	$P_2 = 700$ psia	$\bar{P} = 850$ psia
$\gamma_g = 0.65$	$\mu_g = 0.02$ cp	$\mu_w = 1.0$ cp
$q_L = 5{,}000$ bpd (water)	$\sigma_w = 70$ dynes/cm	
$d = 6$ in.	$q_g = 6{,}000{,}000$ scfd	
	$\bar{T} = 110°F$	

Procedure:

1. Calculate w_L, mass velocity of the liquid:

$$w_L = \frac{5{,}000\ (\text{bpd})\ 5.61\ (\text{cu ft/bbl})\ 66.7\ (\text{lb}_m/\text{cu ft})}{86{,}400\ \text{sec/day}}$$

$$= 21.6\ \text{lb}_m/\text{sec}$$

2. Calculate the mass velocity of the gas, w_g:

$$w_g = \frac{(0.65)(0.0764)(6 \times 10^6\ \text{scfd})}{86{,}400\ \text{sec/day}} = 3.16\ \frac{\text{lb}_m}{\text{sec}}$$

3. Obtain w_T:

$$w_T = w_L + w_g = 21.6 + 3.16 = 24.76\ \frac{\text{lb}_m}{\text{sec}}$$

4. Determine the liquid holdup H_L:

$$v_{sL} = \frac{q_L}{A_p} = \frac{5{,}000\ (\text{bpd})\ 5.61\ (\text{cu ft/bbl})}{86{,}400\left(\frac{\text{sec}}{\text{day}}\right)(0.1963)(\text{sq ft})} = 1.65\ \frac{\text{ft}}{\text{sec}}$$

$$q_g = 69.5\ \frac{\text{cu ft}}{\text{sec}}\left(\frac{14.7}{850}\right)\left(\frac{570}{520}\right)\left(\frac{0.92}{1}\right) = 0.824\ \frac{\text{cu ft}}{\text{sec}}$$

$$v_{sg} = \frac{0.824\ \text{cu ft/sec}}{0.1963\ (\text{sq ft})} = 4.2\ \text{ft/sec}$$

$$N_{LV} = 1.938 \times 1.65\ \sqrt[4]{\frac{66.7}{70}} = 3.16$$

$$N_{GV} = 1.938 \times 4.2\ \sqrt[4]{\frac{66.7}{70}} = 8.04$$

$$N_d = 120.872\ (0.1963)(0.974) = 23.1$$

$$N_L = 0.15726 \times 1 \times \sqrt[4]{\frac{1}{66.7(70)^3}} = 0.00228$$

$$\frac{(N_{LV})^{0.575}}{N_{GV}(N_d)^{0.0277}}\left(\frac{P}{P_a}\right)^{0.05}\left(\frac{N_L}{N_{LB}}\right)^{0.1}$$

$$= \frac{(3.16)^{0.575}}{(8.04)(23.1)^{0.0277}}\left(\frac{850}{14.7}\right)^{0.5}\left(\frac{0.00228}{0.00226}\right)^{0.1}$$

$$= \frac{1.94}{(8.04)(1.091)}\ (7.77) \times 1 = 1.72$$

$H_L = 0.94$ (from Fig. 2.136).

5. Liquid velocity $v_L = \dfrac{v_{sL}}{H_L} = \dfrac{1.65}{0.94} = 1.76$ ft/sec

Gas velocity $v_g = \dfrac{v_{sg}}{(1 - H_L)} = \dfrac{4.2}{0.06} = 70$ ft/sec

6. Calculate the viscosity of the two phase mixture:

$$\mu_{TP} = \mu_L{}^{H_L}\,\mu_g{}^{(1-H_L)}$$
$$\mu_{TP} = (1)^{0.94} \times (0.02)^{0.06} = 0.79$$

7. Calculate the Reynold's function:

$$\frac{w_T\ H_L{}^2}{d_p\ \mu_{TP}} = \frac{24.76\ (0.94)^2}{(0.5)(0.79)} = 55.7$$

8. Calculate the gas density $\bar{\rho}_g$:

$$\bar{\rho}_g = (0.65)(0.764)\left(\frac{850}{14.7}\right)\left(\frac{520}{570}\right)\left(\frac{1}{0.88}\right) = 2.98\ \frac{\text{lb}_m}{\text{cu ft}}$$

9. Calculate the Weber function:

$$\frac{\rho_L\ v_L{}^2\ d_p\ H_L{}^{1/2}}{\sigma} + \frac{\rho_g\ v_s{}^2\ d_p(1 - H_L)^{1/2}}{\sigma}$$

$$v_s = v_g - v_L \qquad v_s = 70 - 1.76 = 68.24\ \text{ft/sec}$$

$$\frac{66.7(\text{lb}_m/\text{ft}^3)(1.76)^2(\text{ft/sec})^2(0.1963)(0.94)^{1/2}}{70\ \text{dynes/cm}}$$

$$+ \frac{(2.98)^2(68.24)^2(\text{ft/sec})^2(0.1963)(0.06)^{1/2}}{70}$$

$$= \frac{(3.1)(66.7)(0.1963)(0.97)}{70} + \frac{(2.98)(4660)(0.1963)(0.245)}{70}$$

$$= 0.536 + 7.8 = 8.336$$

10. With the factors from steps 7 and 9, the flow pattern is found to be *laminar stratified* and *wave* (Fig. 2.131).

Appendix D.3: Lockhart and Martinelli

D.31 DERIVATION OF LOCKHART AND MARTINELLI'S FINAL EQUATION

The basic postulates of Lockhart and Martinelli are that:

(a) $\Delta p_L = \Delta p_g$
This is assumed to be true regardless of flow pattern.
(b) $Vol_L + Vol_g$ = total volume of pipe

Derivation:

Procedure 1:

(1) The static pressure drop due to liquid flow may be described by Fanning's equation:

$$\left(\frac{\Delta p}{\Delta L}\right)_L = \frac{2 f_L \rho_L v_L^2}{d_L g},$$

where d_L = hydraulic diameter of liquid flow pattern and is unknown.

(2) The static pressure drop due to gas flow may also be described by Fanning's equation:

$$\left(\frac{\Delta p}{\Delta L}\right)_g = \frac{2 f_g \rho_g v_g^2}{d_g g},$$

where: d_g = hydraulic diameter of gas flow pattern and is unknown.

As postulated:

$$\left(\frac{\Delta p}{\Delta L}\right)_L = \left(\frac{\Delta p}{\Delta L}\right)_g = \left(\frac{\Delta p}{\Delta L}\right)_{TP}.$$

For single phase fluid flow the flow area is the total area of the pipe.

(3) $A_p = \frac{\pi}{4} d_p^2$

For more complex cross-sectional areas such as a mixture of liquid and gas in a pipe, the following relations may be written:

(4) $A_L = \gamma \left(\frac{\pi}{4} d_L^2\right)$ (for liquid), and

(5) $A_g = \beta\left(\frac{\pi}{4} d_g^2\right)$ (for gas),

where γ and β are ratios of actual area of flow to the area of the total pipe.

(6) The velocity of the liquid can be written as:

$$v_L = \frac{q_L}{A_L} = \frac{w_L}{\rho_L A_L} = \frac{w_L}{\rho_L \gamma \left(\frac{\pi}{4} d_L\right)^2}$$

(7) The velocity of the gas can be written as:

$$v_g = \frac{q_g}{A_g} = \frac{w_g}{\rho_g A_g} = \frac{w_g}{\rho_g \beta\left(\frac{\pi}{4} dg^2\right)}$$

where v_L and v_g are relative velocities and γ and β are determined experimentally.

The friction factor for the liquid is expressed in the generalized Blasius form:

(8) $f_L = \frac{C_L}{(N_{Re_L})^n} = \frac{C_L}{\left(\frac{4}{\pi} \frac{w_L}{\gamma d_L \mu_L}\right)^n}$

Likewise, the friction factor for the gas expressed in the generalized Blasius form:

(9) $f_g = \frac{C_g}{(N_{R_{eg}})^m} = \frac{C_g}{\left(\frac{4}{\pi} \frac{w_g}{\beta d_g \mu_g}\right)^m}$

By substituting equations (6) and (8) into (1) and equation (7) and (9) into (2) and then set equal to each other:

$$\frac{C_L}{\left(\frac{4}{\pi} \frac{w_L}{\gamma d_L \mu_L}\right)^n} \frac{\rho_L}{g d_L} \frac{w_L^2}{\gamma^2 \left(\frac{\pi}{4}\right)^2 d_L^4 \rho_L^2} = \frac{C_g}{\left(\frac{4}{\pi} \frac{w_g}{\beta d_g \mu_g}\right)^m} \frac{\rho_g}{g d_g} \frac{w_g^2}{\beta^2 \left(\frac{\pi}{4}\right)^2 d_g^4 \rho_g^2}.$$

(10) Simplifying and solving for d:

$$\frac{d_L^{5-n}}{d_g^{5-m}} = \frac{C_L w_L^{2-n} \mu_L^n \rho_g \beta^{2-m}}{C_g w_g^{2-m} \mu_g^m \rho_L \gamma^{2-n}} \left(\frac{\pi}{4}\right)^{n-m}.$$

By substituting equations (6) and (8) into (1):

(11) $\left(\frac{\Delta p}{\Delta L}\right)_L = \left(\frac{\Delta p}{\Delta L}\right)_{TP} = \frac{2 C_L w_L^2}{\left(\frac{4 w_L}{\pi \gamma d_L \mu_L}\right)^n d_L^5 \gamma^2 \left(\frac{\pi}{4}\right)^2 g \rho_L}$

Multiply both numerator and demoninator by d_p^{5-n}.

(12) $\left(\frac{\Delta p}{\Delta L}\right)_{TP} \left[\frac{2\left(\frac{4}{\pi}\right)^{2-n} C_L \mu_L^n w_L^{2-n}}{\rho_L g d_p^{5-n}}\right] \gamma^{n-2} \left(\frac{d_p}{d_L}\right)^{5-n}$

The term in the brackets of equation (12) represents the pressure drop per unit length for the liquid flowing alone in the pipe. Therefore:

$$\left(\frac{\Delta p}{\Delta L}\right)_{TP} = \left(\frac{\Delta p}{\Delta L}\right)_L \gamma^{n-2} \left(\frac{d_p}{d_L}\right)^{5-n}$$

(13) Let $\sqrt{\frac{\left(\frac{\Delta p}{\Delta L}\right)_{TP}}{\left(\frac{\Delta p}{\Delta L}\right)_L}} = \phi_L = \gamma \frac{n-2}{2} \left(\frac{d_p}{d_L}\right)^{\frac{5-n}{2}}$

By generalizing equation (2):

(14) $\left(\frac{\Delta p}{\Delta L}\right)_{TP} = \left(\frac{\Delta p}{\Delta L}\right)_g \beta^{m-2} \left(\frac{d_p}{d_g}\right)^{5-m}$

(15) Let $\frac{\left(\frac{\Delta p}{\Delta L}\right)_{TP}}{\left(\frac{\Delta p}{\Delta L}\right)_g} = \phi_g = \beta^{\frac{m-2}{2}} \left(\frac{d_p}{d_g}\right)^{\frac{5-m}{2}}$

By applying the second postulate that the liquid volume plus the gas volume must be equal to the total pipe volume:

(16) $A_p = A_L + A_g$ or $\gamma \frac{\pi}{4} (d_L)^2 + \beta \frac{\pi}{4} (d_g)^2 = \frac{\pi}{4} (d_p)^2$

Therefore:

(17) $\gamma (d_L)^2 + \beta (d_g)^2 = d_p^2$

The fraction of the tube filled with liquid is known as holdup (H_L),

(18) or: $H_L = \frac{\gamma d_L^2}{d_p^2} = 1 - \beta\left(\frac{d_g}{d_p}\right)^2$

(19) or: $\beta = (1 - H_L) \left(\frac{d_p}{d_g}\right)^2$

The fraction of the pipe filled with gas is $1 - H_L$, or:

(20) $(1 - H_L) = \frac{\beta d_g^2}{d_p} = 1 - \gamma \left(\frac{d_L}{d_p}\right)^2$

or: $\gamma = (1 - H_L)\left(\frac{d_p}{d_L}\right)^2$.

Four variables have appeared:

$$\frac{d_L}{d_p}, \frac{d_g}{d_p}, \gamma, \text{ and } \beta$$

These are empirically related to ϕ, ϕ_g, H_L, $1 - H_L$. From equations (8) and (9):

(21) $N_{Re_L}{}^m = \left(\frac{4}{\pi} \frac{w_g}{d_p \mu_g}\right)^m$

(22) $N_{Re_L}{}^m = \left(\frac{4}{\pi} \frac{w_L}{d_p \mu_g}\right)^m$

By dividing equation (1) by equation (2) and making substitutions:

(23) $$\frac{\left(\frac{\Delta p}{\Delta L}\right)_L}{\left(\frac{\Delta_p}{\Delta L}\right)_g} = \frac{2\left[\frac{C_L}{\left(\frac{4}{\pi} \frac{w_L}{d_p \mu_L}\right)^n} \frac{\rho_L v_L^2}{d_g g}\right]}{2\left[\frac{C_g}{\left(\frac{4}{\pi} \frac{w_g}{\mu_g d_p}\right)^m} \frac{\rho_g v_g^2}{d_g g}\right]},$$

(24) $$= \left(\frac{4}{\pi d_p}\right)^{m-n} \frac{C_L w_g{}^m \mu_L{}^n \rho_L d_g v_L^2}{C_g w_L{}^n \mu_g{}^m \rho_g d_L v_g{}^2}.$$

Substituting for $\frac{v_L^2}{v_g^2}$ and assuming each fluid to be flowing alone:

(25) $$\frac{v_L^2}{v_g^2} = \frac{w_L^2}{\left(\frac{\pi}{4}\right)^2 d_p^4 \rho_L^2}\left[\frac{\left(\frac{\pi}{4}\right)^2 d_p^4 \rho_g^2}{w_g^2}\right] = \frac{w_L^2 \rho_g^2}{w_g^2 \rho_L^2}$$

Substituting (25) into (24):

(26) $$\frac{\left(\frac{\Delta p}{\Delta L}\right)_L}{\left(\frac{\Delta p}{\Delta L}\right)_g} = \left(\frac{4}{\pi d_p}\right)^{m-n} \frac{C_L w_L{}^{2-n} \rho_g \mu_L{}^n}{C_g w_g{}^{2-m} \rho_L \mu_g{}^m}$$

This equation (26) is the same as equation (20) in Lockhart and Martinelli's original paper.

The value of

(27) $$\left(\frac{4}{\pi d_p}\right)^{m-n} \frac{C_L w_L{}^{2-n} \rho_g \mu_L{}^n}{C_g w_g{}^{2-m} \rho_L \mu_g{}^m}$$

represents one more variable in addition to the other four, $\frac{d_L}{d_p}, \frac{d_g}{d_p}, \gamma$, and β as mentioned previously.

Rearranging the terms in equation (26):

(28) $$\frac{\left(\frac{\Delta p}{\Delta L}\right)_L}{\left(\frac{\Delta p}{\Delta L}\right)_g} = \frac{\left(\frac{4 w_g}{\pi d_p \mu_g}\right)^m C_L w_L{}^2 \rho_g}{\left(\frac{4 w_L}{\pi d_p \mu_L}\right)^n C_g w_g{}^2 \rho_L}$$

By substituting (21) and (22) into (28):

(29) $$\frac{\left(\frac{\Delta p}{\Delta L}\right)_L}{\left(\frac{\Delta p}{\Delta L}\right)_g} = \frac{(N_{Re_{gp}})^m C_L w_L{}^2 \rho_g}{(N_{Re_{Lp}})^n C_g w_g{}^2 \rho_L}.$$

This right-hand side of equation (29) is defined as X^2 by Lockhart and Martinelli.

Thus

(30) $$\frac{\left(\frac{\Delta p}{\Delta L}\right)_L}{\left(\frac{\Delta p}{\Delta L}\right)_g} = X^2.$$

D.32 PROCEDURE FOR CALCULATION OF A HORIZONTAL PRESSURE TRAVERSE BY THE METHOD OF LOCKHART AND MARTINELLI

The Lockhart and Martinelli method for calculation of a horizontal multiphase flow pressure traverse makes use of the following equations:

$$\left(\frac{\Delta p}{\Delta L}\right)_{TP} = \phi^2 \left(\frac{\Delta p}{\Delta L}\right)_L$$

or:

$$\left(\frac{\Delta p}{\Delta L}\right)_{TP} = \phi^2 \left(\frac{\Delta p}{\Delta L}\right)_g$$

As noted, the pressure loss for liquid flowing alone $\left(\frac{\Delta p}{\Delta L}\right)_L$ and gas flowing alone $\left(\frac{\Delta p}{\Delta L}\right)_g$ are both needed. These two pressure losses are calculated assuming each to be flowing in the pipe alone and that each phase is occupying the entire volume of the pipe.

Stepwise procedure

1. Calculate the pressure loss for the liquid, assuming it to be flowing alone in the pipe. Some form of the familiar Darcy equation may be used;

$$h = \frac{f L v^2}{d\ 2\ g_c}$$

where L = ft, v = ft/sec, d = ft, and h is expressed in feet of liquid.

This Darcy equation is valid for laminar or turbulent flow for any liquid in a horizontal pipe, except when extreme velocities cause the downstream pressure to fall to the vapor pressure of the liquid.

This equation may be written to express pressure drop in psi:

$$\Delta p = \frac{\rho f L v^2}{144\ d\ 2gc} = 0.001294 \frac{f \rho L v^2}{d^5}$$

where: ρ = lb$_m$/cu ft, d = ft, L = ft, Δp = psi, v = ft/sec

or:

$$\Delta p = 43.5 \frac{f L \rho q^2}{d^5}$$

where: v = ft/sec, q = cu ft/sec, d′ = in., ρ = lb$_m$/cu ft, L = ft, Δp = psi

Katz, *et al* (D.1) gave a formula for calculation of pressure loss:

$$\Delta p_L = \frac{34 \times 10^{-10} f q^2 L \rho_L}{g_c d^5}$$

ere: q = bpd
L = ft
d = ft
ρ_L = lb_m/cu ft
Δp_L = lb_f/sq ft

is obtained from the Moody diagram (Fig. 2.32). Note it Δp_L is in lb_f/sq ft and not in lb_f/sq in. as given in the ginal reference (D.1). To give Δp_L in psi this equation ould have a constant of 2.36×10^{-11} instead of 34×10^{-10}. As noted, all of these equations involve the determination an "f" (friction loss) factor.

f the flow is laminar ($N_{Re} < 2{,}000$), the friction factor y be determined from the equation:

$$f = \frac{64}{N_{Re}}$$

When the flow is turbulent ($N_{Re} > 4{,}000$), the friction tor then becomes dependent upon N_{Re} and the relative ughness ϵ/d. The roughness factor has a more decided ect on the friction factor in small pipe sizes. Although de of the same steel, small diameter pipes will have her friction factors than large diameter pipes.

The most widely used friction factor correlation is that of ody found in Fig. 2.32.

As noted from equations for liquid flow, the pressure loss ies inversely with the diameter to the fifth power, hence 1% reduction in diameter causes a 10% increase in pres- e loss.

n summary, for liquid flow the friction factor should be en from the Moody diagram with proper attention to the ughness of the pipe.

2. Calculate the pressure loss for the gas, assuming it be flowing alone in the pipe. Numerous formulas were en by Katz, et al (D.1) which may be used. One of these Weymouth's equation (D.3) as modified to include the mpressibility factor (D.4):

$$q = 3.22 \frac{T_{sc}}{p_{sc}} \left[\frac{(p_1^2 - p_2^2)\, d^5}{\gamma_g \overline{T} L f \overline{Z}} \right]^{0.5},$$

ere: q = gas flow rate in scf/hr
L = length of line, miles
d = internal diameter, in.
p = pressure, psia
γ_g = specific gravity of gas
$\overline{T}$ = average line temperature, °R
$\overline{Z}$ = average compressibility factor
f = friction factor.

e friction factor may also be taken from the Moody gram of Fig. 2.32.

Weymouth assumed that f could be expressed as:

$$f = \frac{0.032}{d^{1/3}}$$

king this substitution:

$$q = 18.062 \frac{T_{sc}}{p_{sc}} \left[\frac{(p_1^2 - p_2^2)\, d^{16/3}}{\gamma_g \overline{T} L \overline{Z}} \right]^{0.5}$$

substituting for T_{sc} and p_{sc} standard conditions and re- anging in a more useful form to solve for downstream ssure:

$$p_2 = \left[p_1^2 - \frac{q_g^2 \gamma_g \overline{T} \overline{Z} L}{639^2 d^{5.33}} \right]^{0.5}$$

The Panhandle formula may also be used and assumes t f can be expressed as:

$$\frac{1}{f} = 52 \left(\frac{\gamma_g q}{d} \right)^{0.1461}$$

This gives an equation as given by Katz (D.1).

$$q = 435.87\ E \left(\frac{T_{sc}}{p_{sc}} \right)^{1.07881} \left(\frac{p_1^2 - p_2^2}{L} \right)^{0.5394} \left(\frac{1}{\overline{T}} \right)^{0.5394} \times \left(\frac{1}{\gamma_g} \right)^{0.4606} d^{2.6182},$$

where: d = in.
q = gas flow rate, scfd.

For a roughness of 0.0006 in., Cullender and Brinkley (D.5) expressed f as:

$$f = 30.9208 \times 10^{-3} \frac{q^{-0.065}\, d^{-0.058}\, \gamma_g^{-0.065}}{\mu^{-0.065}},$$

where: q = mmcfd
d = ft

$$\mu = \frac{lb_m}{\text{ft-sec}}$$

3. Calculate the X parameter to be used in Fig. 2.134.

$$X = \sqrt{\frac{(\Delta p/\Delta L)_L}{(\Delta p/\Delta L)_g}} = \sqrt{\frac{(\Delta p)_L}{(\Delta p)_g}}$$

4. Determine a Reynolds number for both the gas and the liquid, assuming each to be flowing alone in the pipe.

$$N_{Re} = \frac{dv\rho}{\mu'}$$

Where: d = ft
v = ft/sec
ρ = lb_m/cu ft

$$\mu' = \frac{lb_m}{\text{ft-sec}}$$

More convenient forms of this equation are:

$$N_{Re} = 123.9 \frac{dv\rho}{\mu}$$

where: d = in.
v = ft/sec
ρ = lb_m/cu ft
μ = cp

or $$N_{Re} = 22{,}700 \frac{q\rho}{d\mu}$$

where: d = in.
q = cu ft/sec at flowing conditions
ρ = lb_m/cu ft
μ = cp

Additional forms of this equation may be found in Ref. D.6.

5. Determine the type of flow mechanism from the following table.

Reynolds Number		
Liquid	Gas	
> 2000	> 2000	Liquid turbulent gas turbulent
< 1000	> 2000	Liquid viscous gas turbulent
> 2000	< 1000	Liquid turbulent gas viscous
< 1000	< 1000	Liquid viscous gas viscous

6. From Fig. 2.134 select a value for the parameter ϕ.
7. Calculate the two-phase pressure loss from:

$$\left(\frac{\Delta p}{\Delta L} \right)_{TP} = \phi^2 \left(\frac{\Delta p}{\Delta L} \right)_L$$

or:

$$\left(\frac{\Delta p}{\Delta L} \right)_{TP} = \phi^2 \left(\frac{\Delta p}{\Delta L} \right)_g$$

D.33 EXAMPLE PROBLEM BY METHOD OF LOCKHART AND MARTINELLI TO DETERMINE DOWNSTREAM PRESSURE FOR A KNOWN UPSTREAM PRESSURE

The following data is given:

Pipe size = 2 in. ID
Length = 1,500 ft
Liquid rate = 2,000 bpd water
$\gamma_w = 1.07$
G/L = 1,000 scf/bbl
$\gamma_g = 0.65$
$p_u = 850$ psia
$\sigma_w = 66.7$ dynes/cm
$\mu_g = 0.015$ cp
$\overline{T} = 120°F$
$\mu_L = 1.0$ cp

1. Calculate the pressure loss assuming the liquid phase to be flowing alone.

$$\Delta p_L = \frac{2.36 \times 10^{-11} \, f \, q^2 \, L \, \rho_L}{g_c \, d^5}$$

Using $f = \frac{0.032}{d^{1/3}} = \frac{0.032}{(2)^{1/3}} = 0.0254$

$$\Delta p_L = \frac{(2.36 \times 10^{-11})(0.0254)(2{,}000)^2(1{,}500)(1.07 \times 62.4)}{(32.2)\,(0.1666)^5}$$

$$= 58.4 \text{ psi}$$

2. Calculate the pressure loss assuming the gas phase to be flowing alone in the pipe.

$$q = 639 \left[\frac{(p_1^2 - p_2^2)\, d^{16/3}}{\gamma_g \overline{T} L \overline{Z}_a}\right]^{0.5}$$

$$q = 83{,}400 \text{ scf/hr} = 639 \left[\frac{(p_1^2 - p_2^2)(2)^{16/3}}{(0.65)(580)\left(\frac{1{,}500}{5{,}280}\right)(0.92)}\right]^{0.5}$$

Solving for $p_1^2 - p_2^2$, we have:

$p_1^2 - p_2^2 = 41{,}500$
For p_1 of 850 psi, $p_2 = 825$ psi.

Therefore:

$$\Delta p_g = 25 \text{ psi.}$$

3. Calculate the X parameter to be used in Fig. 2.134 to obtain ϕ.

$$X = \sqrt{\frac{\Delta p_L}{\Delta p_g}} = \sqrt{\frac{58.4}{25}} = 1.53$$

4. Determine the Reynolds number for both the gas and the liquid phase assuming each to be flowing alone in the pipe.

(a) $N_{Re_L} = 123.9 \frac{dv\rho}{\mu}$

d = in.
v = ft/sec
ρ = lb_m/cu ft
μ = cp

$$v_{sL} = \frac{q_L\,(5.61)}{86{,}400\ A_p} =$$

$$v_{sL} = \frac{(2{,}000)(5.61)}{86{,}400(0.0218)(ft^2)}$$

$$= 5.97 \text{ ft/sec}$$

$$N_{Re_L} = \frac{(123.9)(2)(5.97)(66.7)}{1}$$

$N_{Re_L} = 98{,}670$

$N_{Re_L} > 2{,}000$, therefore flow is turbulent.

(b) $N_{Re_g} = 123.9 \frac{d\, v_{sg}\, \rho_g}{\mu_g}$

$$q_g = \frac{(1{,}000)(2{,}000)}{86{,}400} = 23.15 \text{ scf/sec}$$

$$q_g = (23.15)\left(\frac{14.7}{850}\right)\left(\frac{580}{520}\right)\left(\frac{0.92}{1}\right) = 0.412 \text{ cu ft/sec}$$

$$v_{sg} = \frac{0.412}{0.0218} = 18.9 \text{ ft/sec}$$

$$\overline{\rho}_g = (0.0764)(0.65)\left(\frac{520}{580}\right)\left(\frac{837.5}{14.7}\right)\left(\frac{1}{0.90}\right)$$

$$= 2.82 \text{ } lb_m/\text{cu ft}$$

$$N_{Re_g} = \frac{(123.9)(2)(18.9)(2.81)}{(0.015)} = 879{,}000$$

$N_{Re_g} = 879{,}000$

$N_{Re_g} > 2{,}000$, therefore gas is in turbulent flow.

5. From the Table of step 5 in the calculation procedure we note both gas and liquid to be in turbulent flow.
6. Select a value of ϕ from Fig. 2.134 to be 3.5 for the liquid.
7. Calculate the two-phase pressure loss.

$$\left(\frac{\Delta p}{\Delta L}\right)_{TP} = \phi^2 \left(\frac{\Delta p}{\Delta L}\right)_L$$

$$\left(\frac{\Delta p}{\Delta L}\right)_{TP} = (3.5)^2\,(58.4) = 715 \text{ psia}$$

Therefore:

$$p_2 = 850 - 715 = 135 \text{ psia}$$

Check by selecting a value from Fig. 2.134 for the gas, $\phi_g = 5.4$.

$$\left(\frac{\Delta p}{\Delta L}\right)_{TP} = \phi^2 \left(\frac{\Delta p}{\Delta L}\right)_g$$

$$\left(\frac{\Delta p}{\Delta L}\right)_{TP} = (5.4)^2\,(25) = 729 \text{ psia}$$

Therefore:

$$p_2 = 850 - 729 = 121 \text{ psia}$$

D.34 EXAMPLE PROBLEM TO DETERMINE NECESSARY LINE SIZE BY METHOD OF LOCKHART AND MARTINELLI (TRIAL AND ERROR PROCEDURE)

Given: L = 5 miles
Pressure upstream — 1,000 psia
Pressure downstream — 700 psia
q_L = 5,000 bpd (all water)
q_g = 6,000,000 scfd
$\overline{T}$ = 110°F
μ_g = 0.02 cp (constant)
μ_w = 1.0 cp

Find the necessary line size by the method of Lockhart and Martinelli.

For this particular problem first assume a line size, then calculate the pressure loss. If the pressure loss is then found to be near 300 psi (1,000 − 700), the assumption is valid. If the pressure loss exceeds 300 psi, the assumption is too small. If it is less than 300 psi, the assumption is too large. For this example, assume a line size of 6 in. ID.

Once the line size has been estimated, follow the standard procedure for calculating a pressure loss by the method of Lockhart and Martinelli.

1. Calculate the pressure loss for the liquid assuming it to be flowing alone in the pipe.

Using Baker's equation (Ref. 10, Section 2.4):

$$\Delta p_L = \frac{f L \, q^2 \, \rho_L}{181{,}916 \times d^5}$$

Find a value of N_{Re} to obtain f.

$$N_{Re_L} = \frac{11.05\ q_L\ \rho_L}{d\mu}$$

$$= \frac{(11.05)(5{,}000)(1.07)(8.33)}{(6)(1)} = 82{,}200$$

From Fig. 2.135 $f = 0.0047$
For 90% eff.: $f = 1.23(0.0047) = 0.0058$

$$\Delta p_L = \frac{(5.8 \times 10^{-3})(5)(5.28 \times 10^3)(0.25)(10^8)(1.07 \times 8.33)}{1.82 \times 10^5\ (0.6)^5 \times 10^5}$$

$= 24.5$ psia

Calculate the pressure loss assuming gas to be flowing alone.

$$\Delta p_g = \frac{f L q^2 \gamma_g \bar{T} \bar{Z}}{20{,}000\ d^5 \left(\frac{p_1 + p_2}{2}\right)}$$

Find N_{Re_g} in order to obtain a value for f.

$$N_{Re_g} = \frac{20.14\ q_g\ \gamma_g}{d\mu}$$

$$= \frac{(20.14)(6 \times 10^3)(0.65)}{6 \times 2 \times 10^{-2}} = 6.5 \times 10^5$$

$f = 0.0032$ (Fig. 2.135)

For 90% eff.: $f = (1.23)(0.0032) = 0.00344$

$$\Delta p_g = \frac{(3.44 \times 10^{-3})(5)(5{,}280)(36)(10^6)(0.65)(570)(0.88)}{10^4 \times 1.7 \times 10^3\ (0.6)^5\ (10^5)}$$

$\Delta p_g = 9.25$ psia

3. Calculate the X parameter.

$$X = \left(\frac{\Delta p_L}{\Delta p_g}\right)^{1/2} = \left(\frac{24.5}{9.25}\right)^{1/2} = 1.62$$

4. Note the Reynolds number of both the gas and liquid if flowing alone.

$$N_{Re_L} = 82.200$$
$$N_{Re_g} = 650{,}000$$

5. The type of flow mechanism is turbulent-turbulent.
6. From Fig. 2.134 a value of the ϕ parameter is:

$$\phi_{gtt} = 5.5, \qquad \phi_{Ltt} = 3.5$$

7. Calculate Δp_{TP}.

$$\Delta p_{TP} = \phi_{gtt}^2\ (\Delta p_L)$$
$$= (5.5)^2\ (9.25) = 280 \text{ psia}$$
$$\Delta p_{TP} = (3.5)^2\ (24.5) = 300 \text{ psia}$$

8. Check this pressure loss against the pressure loss desired. The allowable value was $1{,}000 - 700 = 300$ psi, so this is close enough. Recommend a 6 in. line.

Appendix D.4: Ovid Baker method

D.41 PROCEDURE TO DETERMINE PRESSURE LOSS IN HORIZONTAL LINES BY METHOD OF BAKER

Because of the change in flow pattern that can take place in any one line, it is better to work the problem choosing small pressure decrements rather than using an average pressure over the entire length of the line. Computer application allows small pressure decrements (10 – 100 psi) to be used without seriously lengthening the time of calculation.

1. Knowing P_1, assume P_2. Compute $\overline{P}$ and obtain $\overline{R}_s$, $\overline{B}_o$, $\overline{Z}$.
2. Calculate the Reynold's number for the liquid.

$$N_{Re_L} = \frac{11.05(\text{bpd})\ \rho_L(\text{lb}_m/\text{gal})}{d\mu}$$

3. Using the Reynold's number, obtain the friction factor from Fig. 2.135. Correct f for pipeline efficiency.
4. Calculate the pressure loss for the liquid flowing alone.

$$\left(\frac{\Delta P}{\Delta L}\right)_L = \frac{f'(\text{bpd})^2\ (\text{lb}_m/\text{gal})\ L'}{181{,}916\ d^5}$$

5. Calculate the Reynold's number for the gas.

$$N_{Re_g} = \frac{20.14\ (\text{Mcfd})\ \gamma_g}{d\mu}$$

6. Obtain the friction factor for the gas from Fig. 2.135. Correct f for pipe efficiency.
7. Calculate the pressure loss for the gas flowing alone.

$$\left(\frac{\Delta P}{\Delta L}\right)_g = \frac{f'(\text{Mcfd})^2\gamma_g\ \overline{T}\ \overline{Z}}{20{,}000\ d^5\ \overline{P}}$$

8. Calculate the X parameter.

$$X = \left[\frac{(\Delta P/\Delta L)_L}{(\Delta P/\Delta L)_g}\right]^{1/2}$$

9. Calculate λ.

$$\lambda = \left[\left(\frac{\rho_g}{.075}\right)\left(\frac{\rho_L}{62.4}\right)\right]^{1/2}$$

10. Calculate the mass velocity of the gas.

$$M_g = \frac{q_g\ \gamma_g\ (29)}{(24)(379)\ A_p}$$

11. Determine $\frac{M_g}{\lambda}$.
12. Calculate the mass velocity of the liquid in lb_m/hr sq ft.

$$M_L = \frac{q_L\ (5.61)\ \rho_L}{24\ A_p}$$

13. Calculate ψ.

$$\psi = \frac{73}{\sigma_L}\left[\mu_L\left(\frac{62.4}{\rho_L}\right)^2\right]^{1/3}$$

14. Calculate:

$$\frac{M_L\ \lambda\ \psi}{M_g}$$

15. Using $\frac{M_g}{\lambda}$ and $\frac{M_L\lambda\psi}{M_g}$ obtain the flow pattern from Fig. 2.130
16. Select the proper equation according to the flow pattern. The X parameter is the same as that of Lockhart and Martinelli and is $X = (\Delta P_L/\Delta P_g)^{1/2}$. These equations are:

(1) Froth or bubble flow $\phi_{gtt} = \dfrac{14.2\ X^{0.75}}{L^{0.1}}$

(2) Plug flow $\phi_{gtt} = \dfrac{27.315\ X^{0.855}}{L^{0.17}}$

(3) Stratified flow:

$$\phi_{gtt} = \frac{15{,}400\ X}{M_{vL}{}^{0.8}}$$

(4) For wave flow Baker proposed the equation given by Schneider, et al (Ref. 24, Section 2.4).

$$\Delta P_{TP} = \frac{f_{TP}\ M_{vg}{}^2\ L}{193.2\ d\ \rho_g}$$

where: f_{TP} = friction factor for two-phase wave flow
$= f_g\ \phi_g{}^2$
f_g = Fanning friction factor, gas flow
ϕ_g = Lockhart and Martinelli two-phase correlation factor
M_g = gas mass velocity, lb_m/sec/sq ft of total pipe cross section.

(5) Slug flow:

$$\phi_{gtt} = \frac{1{,}190\ X^{0.815}}{M_{vL}{}^{0.5}}$$

(6) Annular flow:

$$\phi_{gtt} = (4.8 - 0.3125\ d)\ X^{0.343-0.021d}$$

This equation should be used for pipe diameters less than 10 in. When d is greater than 10 in., always use 10 in. in the equation.

(7) Spray or dispersed flow:
a) For $X < 0.1$
$\phi_{gtt} = e^{0.16695\ \ln X + 1.01569}$
b) For $0.1 \le X \le 1.0$
$\phi_{gtt} = e^{0.34909\ \ln X + 1.43508}$
c) For $1.0 \le X \le 10$
$\phi_{gtt} = e^{0.61979\ \ln X + 1.43508}$
d) For $10 \le X \le 100$
$\phi_{gtt} = e^{0.79834\ \ln X + 1.02496}$

17. Calculate the two phase pressure loss.

$$(\Delta P)_{TP} = \phi_{gtt}{}^2\ (\Delta P)_g$$

18. Calculate ΔL.

$$\Delta L = \frac{p_1 - p_2}{\left(\frac{\Delta P}{\Delta L}\right)_{TP}}$$

For all pressure decrements the ΔL values are computed following steps 1 through 18. Then all the ΔL's are added until the total length of the line is obtained.

D.42 EXAMPLE PROBLEM TO DETERMINE PRESSURE LOSS IN 4 IN. AND 16 IN. LINES USING PRESSURE DECREMENTS BY METHOD OF BAKER

The following information is given concerning a horizontal multiphase flow problem:

Rate = 2,000 bpd
Average temperature = 120°F
Length of line = 3,000 ft
Gas-oil ratio = 1,000 scf/bbl
γ_g = 0.65
Upstream pressure = 500 psig
σ_o = 30 dynes/cm
μ_g = 0.02 cp (constant)
μ_o = 1 cp
ρ_o = 42°API

4 in. I.D. Line

1. Assume $\Delta p = 40$ psi, $P_2 = 460$ psig

$$P_{avg} = \frac{500 + 460}{2} = 480 \text{ psig} = 495 \text{ psia}$$

$\rho_g = 1.60$ lb$_m$/cu ft

$R_s = 120$ scf/bbl, $B_o = 1.070$, $\bar{Z} = 0.940$

2. Liquid: wt. dissolved gas per bbl $= \dfrac{(120)(1.07)(29)}{379}$

$= 9.8$ lb$_m$/bbl

$$\rho_L = \frac{(0.816)(62.4) + (9.8)/(5.61)}{1.070} = 49.2 \text{ lb}_m\text{/cu ft} = 6.59 \text{ lb}_m\text{/gal}$$

$$N_{ReL} = \frac{(11.05)(2000)(6.59)}{(4)\,(1)} = 36{,}400$$

3. $f' = 1.23\ (0.0055) = 0.00674$ (Fig. 2.135)

4. $\left(\dfrac{\Delta P}{\Delta L}\right)_L = \dfrac{(0.00674)(2000)^2(6.59)}{(181{,}916)\,(4)^5} = 9.52 \times 10^{-4}$ psi/ft

5. Gas: $N_{Reg} = \dfrac{(20.14)[2(1000 - 120)](0.65)}{(4)\,(.02)} = 288{,}000$

(f = 0.00375)(Fig. 2.135)

6. $f' = 1.23\ (0.00375) = 0.00461$

7. $\left(\dfrac{\Delta P}{\Delta L}\right)_g = \dfrac{(0.00461)[2(1000 - 120)]^2(0.65)(580)(0.940)}{(20{,}000)\,(4)^5\,(495)}$

$= 5.1 \times 10^{-4}$ psi/ft

8. $X = \left[\dfrac{(\Delta P/\Delta L)_L}{(\Delta P/\Delta L)_g}\right]^{1/2} = \left[\dfrac{9.52 \times 10^{-4}}{5.1 \times 10^{-4}}\right]^{1/2} = 1.36$

9. $\lambda = \left[\dfrac{\rho_g}{0.075}\dfrac{\rho_L}{62.3}\right]^{1/2} = \left[\dfrac{1.60}{0.075}\dfrac{49.2}{62.3}\right]^{1/2} = 4.10$

10. $M_g = \dfrac{(1.75 \times 10^6)(0.65)(29)}{(24)(379)(0.0874)} = 41{,}750$

11. $\dfrac{M_g}{\lambda} = \dfrac{41750}{4.10} = 10{,}180$

12. $M_L = \dfrac{(2000)(1.073)(5.61)(49.2)}{(24)(0.0874)} = 282{,}000$

13. $\psi = \dfrac{73}{\sigma_L}\left[\mu_L\left(\dfrac{62.4}{\rho_L}\right)^2\right]^{1/3} = \dfrac{73}{30}\left[\left(\dfrac{62.4}{49.2}\right)^2\right]^{1/3} = 2.86$

14. $\dfrac{M_L\,\lambda\,\psi}{M_g} = \dfrac{(282000)(4.10)(2.86)}{41750} = 79.3$

15. From Fig. 2.130, flow pattern is stratified.

16. $\phi_{gtt} = \dfrac{15{,}400\ X}{(L)^{0.8}} = \dfrac{(15{,}400)(1.36)}{(17{,}700)^{0.8}} = 8.38$

17. $\left(\dfrac{\Delta P}{\Delta L}\right)_{TP} = \left(\dfrac{\Delta P}{\Delta L}\right)_g \phi_{gtt} = (5.1 \times 10^{-4})(8.38)^2 = 3.58 \times 10^{-2}$ psi/ft

$\Delta P = (3000)(3.58 \times 10^{-2}) = 107.2$ psi

18. Repeat procedure using new assumed value of 107.2 psi.

16 in. ID line:

1. Assume $\Delta P = 10$ psi $P_2 = 490$ psig

$$P_{avg} = \frac{500 + 490}{2} = 495 \text{ psig} = 510 \text{ psia}$$

$R_s = 125$ scf/bbl $B_o = 1.073$ $\bar{Z} = 0.938$

2. $\rho_L = \dfrac{(0.816)(62.4) + 10.2/5.61}{1.073} = 49.1$ lb$_m$/cu ft $= 6.57$ lb$_m$/gal

$$N_{ReL} = \frac{(11.05)(2{,}000)(6.57)}{(16)\,(1)} = 9{,}075$$

3. $f' = 1.23\ (0.0080) = 0.00985$ (Fig. 2.135)

4. $\left(\dfrac{\Delta P}{\Delta L}\right)_L = \dfrac{(0.00985)(2{,}000)^2(6.57)}{(181{,}916)\,(16)^5} = 6.8 \times 10^{-7}$ psi/ft

5. $N_{Reg} = \dfrac{(20.14)[2(1{,}000 - 125)](0.65)}{(16)\,(0.02)} = 71{,}500$

6. $f' = 1.23\ (0.0048) = 0.0059$

7. $\left(\dfrac{\Delta P}{\Delta L}\right)_g = \dfrac{(0.0059)\ [2(1{,}000 - 125)]^2\ (0.65)(580)(0.938)}{(20{,}000)\ (16)^5\ (510)}$

$= 6.57 \times 10^{-7}$ psi/ft

8. $X = \left[\dfrac{(\Delta P/\Delta L)_L}{(\Delta P/\Delta L)_g}\right]^{1/2} = \left[\dfrac{6.8 \times 10^{-7}}{6.57 \times 10^{-7}}\right]^{1/2} = 1.017$

9. $\lambda = \left[\dfrac{1.60}{0.075}\dfrac{49.1}{62.3}\right]^{1/2} = 4.10$

10. $M_g = \dfrac{(1.75 \times 10^6)(0.65)(29)}{(24)(379)(1.394)} = 2{,}600$

11. $M_g/\lambda = \dfrac{2{,}600}{4.10} = 635$

12. $M_L = \dfrac{(2{,}000)(1.073)(5.61)(49.1)}{(24)\ (1.394)} = 17{,}700$

13. $\psi = 2.86$

14. $\dfrac{M_L\,\lambda\,\psi}{M_g} = \dfrac{(17{,}700)(4.10)(2.86)}{2{,}600} = 79.9$

15. From Fig. 2.130, flow pattern is stratified.

16. $\phi_{gtt} = \dfrac{15{,}400\ X}{(L)^{0.8}} = \dfrac{(15{,}400)\ (1.017)}{(17{,}700)^{0.8}} = 6.26$

17. $\left(\dfrac{\Delta P}{\Delta L}\right)_{TP} = (6.57 \times 10^{-7})(6.26)^2 = 2.58 \times 10^{-5}$ psi/ft

$\Delta P = (3{,}000)\ (2.58 \times 10^{-5}) = 0.0772$ psi (16″ I.D.)

(Pressure loss negligible in large line)

D.43 EXAMPLE PROBLEM TO DETERMINE LINE SIZE BY METHOD OF BAKER

(The diameter is first estimated and then checked.)

Given:

$q_o = 4{,}000$ bpd	$q_g = 100$ MMcfd
$\rho_o = 56$°API	$\gamma_g = 0.65$
$L = 30$ miles	$\mu_o = 1$ cp
$\bar{T} = 80$°F	$P_2 = 1{,}000$ psia
$P_1 = 1{,}500$ psia	$\beta_o = 1.27$

Initially a line size is estimated by the first three steps.

1. Obtain a value for d_L assuming liquid flowing alone:
Assume $d_L = 3$ in., $\rho_L = 45.0$ lb$_m$/cu ft $= 6.02$ lb$_m$/gal

$$N_{ReL} = \frac{(11.05)[(4{,}000)(1.27)](6.02)}{(3)\ (1.0)} = 113{,}000$$

From Fig. 2.135, $f_L = 0.0044$

$f_L' = 1.23\ (0.0044) = 0.0054$

$$d_L = \left[\frac{f_1 L' \,(bpd)^2 (lb_m/gal)}{181{,}916\,(P_1 - P_2)}\right]^{0.2}$$

$$= \left[\frac{(0.0054)\;[(30)(5280)]\;\{(4000)(1.27)\}^2(6.02)}{(181{,}900)(1500 - 1000)}\right]^{0.2}$$

$$= [(0.0054)(272{,}000)]^{0.2}$$

$$= 4.3 \text{ in.}$$

(This differs from the assumed value of 3 in. so solve again.) Correct for 4.3 in. diameter liquid line:

$$N_{ReL} = (113{,}000)\left(\frac{3}{4.3}\right) = 79{,}000$$

$f'_L = 1.23\ (0.0046) = 0.00565$ (Fig. 2.135)

$d_L = [(0.00565)(272{,}000)]^{0.2} = 4.35$ in.

2. Solve for diameter needed for gas flowing alone:
Mcfd $= 10^5 - 4\ (605) = 97{,}600 =$ free gas
Assume $d_g = 8$ in.

$$N_{Reg} = \frac{20.14(\text{Mcfd})\,\gamma_g}{d\mu}$$

$$= \frac{(20.14)(97{,}600)(0.65)}{(8)\ (0.02)} = 8 \times 10^6$$

$f'_g = 1.23\ (0.00265) = 0.00326$ (Fig. 2.135)

$$d_g = \left[\frac{f'L'\ (\text{Mcfd})\,\gamma_g\,T\,Z}{10{,}000(P_1 - P_2)\ (P_1 + P_2)}\right]^{0.2}$$

$$= \left[\frac{(0.00326)(30)(5{,}280)(97{,}600)^2(0.65)(540)(0.805)}{(104)(1500 - 1000)(1500 + 1000)}\right]^{0.2}$$

$$= [(0.00326)(3.4 \times 10^7)]^{0.2}$$

$$= 10.2 \text{ in.}$$

(This differs from the assumed value of 8 in. so solve again.) Correct for 10.2 in. diameter gas line:

$$N_{Reg} = (8 \times 10^6)\left(\frac{8}{10.2}\right) = 6.3 \times 10^6$$

f does not change (flat portion of curve); therefore, $d_g =$ 10.2 in.

3. $d_{TP} = d_l + d_g = 4.35 + 10.2 = 14.55$ in. (estimated value for d_{TP})
4. Using this value (14.55 in) for line diameter, calculate ΔP_{TP}.

$$N_{ReL} = \frac{(11.05)[(4{,}000)(1.27)](6.02)}{(14.55)\ (1.0)} = 23{,}200$$

$f'_L = 1.23\ (0.0062) = 0.0076$ (Fig. 2.135)

$$\Delta P_L = \frac{f'L'\ (bpd)^2(lb_m/gal)}{181{,}916\ d^5}$$

$$= \frac{(0.0076)[(30)(5{,}280)][(4{,}000)(1.27)]^2(6.02)}{(181{,}916)\ (14.55)^5}$$

$$= \frac{(0.0076)}{(6.52 \times 10^5)}(1.354 \times 10^8)$$

$$= 1.58 \text{ psi}$$

$$N_{Reg} = \frac{(20.14)(97{,}600)(0.65)}{(14.55)(0.02)} = 4.4 \times 10^6$$

$f_g' = 1.23\ (0.00265) = 0.00326$ (Fig. 2.135)

$$\Delta P_g = \frac{f'L'\ (\text{Mcfd})^2\gamma_g\,T\,Z}{20{,}000\ d^5\ P_{avg}}$$

$$= \frac{(0.00326)[(30)(5{,}280)](97{,}600)^2(0.65)(540)(0.805)}{(2 \times 10^4)\ (14.55)^2\ (1250)}$$

$$= \frac{(0.00326)}{(6.52 \times 10^5)}\ (1.70 \times 10^{10})$$

$$= 85.0 \text{ psi}$$

$$X = \left[\frac{\Delta P_L}{\Delta P_g}\right]^{1/2} = \left[\frac{1.58}{85.0}\right]^{1/2} = 0.136$$

$$A_p = \frac{\pi}{4}\left(\frac{14.55}{12}\right)^2 = 1.152 \text{ sq ft}$$

$$M_L = \frac{[(4{,}000)(1.27)](5.61)(45.0)}{(24)\ (1.152)} = 46{,}400\ \frac{lb_m}{\text{hr sq ft}}$$

$$M_g = \frac{(97.6 \times 10^6)(0.65)(29)}{(24)(379)(1.152)} = 176{,}000\ \frac{lb_m}{\text{hr sq ft}}$$

$\rho_g = 5.05\ lb_m/\text{cu ft}$

$$\lambda = \left[\left(\frac{5.05}{.075}\right)\left(\frac{45.0}{62.3}\right)\right]^{1/2} = 6.98$$

$$M_g/\lambda = \frac{176{,}000}{6.98} = 25{,}200$$

$$\psi = \left(\frac{73}{\sigma}\right)\left[\mu_o\left(\frac{62.3}{45.0}\right)^2\right]^{1/3}$$

$$= \left(\frac{73}{30}\right)\left[(1.0)\left(\frac{62.3}{45.0}\right)^2\right]^{1/3} = 3.03$$

$$\frac{M_L\,\lambda\,\psi}{M_g} = \frac{(46{,}400)(6.98)(3.03)}{176{,}000} = 5.55$$

From Fig. 2.130, flow regime is annular flow.

$$\phi_{gtt} = (4.8 - 0.3125\ d)\ X^{0.343 - 0.021d}$$

$$= [4.8 - (0.3125)(10.00)](0.136)^{[0.343 - (0.021)(10.00)]} \qquad (d > 10 \text{ in.})$$

$$= 1.675\ (0.136)^{0.133}$$

$$= 1.282$$

$$(\Delta P)_{TP} = \phi^2_{gtt}(\Delta P)_g$$

$$= (1.282)^2(85.0)$$

$$= 140 \text{ psi}$$

(Since a 500 psi drop is needed, the line size found by summing d_L and d_g is too large.)

$$\text{Try } d = 14.55\left(\frac{140}{500}\right)^{1/5} = 11.3 \text{ in.}$$

$$N_{ReL} = \frac{14.55}{11.3}\ (23{,}200) = 29{,}800$$

$f_L' = 1.23\ (0.0059) = 0.00725$ (Fig. 2.135)

$(d)^5 = (11.3)^5 = 1.84 \times 10^5$

$$(\Delta P)_L = \frac{(0.00725)}{(1.84 \times 10^5)}\ (1.35 \times 10^8) = 5.34 \text{ psi}$$

$$N_{Reg} = \frac{14.55}{11.3}\ (4.4 \times 10^6) = 5.66 \times 10^6$$

$f_g' = 1.23\ (0.0027) = 0.00332$ (Fig. 2.135)

$$(\Delta P)_g = \frac{(0.00332)}{(1.84 \times 10^5)}\ (1.70 \times 10^{10}) = 307 \text{ psi}$$

$$X = \left[\frac{5.34}{307}\right]^{1/2} = 0.132$$

Flow regime will not change; $\phi_{gtt} = 1.282$

$(\Delta P)_{TP} = (1.282)^2\ (307) = 505$ psi, close enough $(d > 10 \text{ in.})$

$d = 11.3$ inches

(Select standard line size 11 in. or greater.)

Appendix D.5: Method of Andrews, et al

D.51 PROCEDURE OF ANDREWS, ET AL. METHOD TO PREDICT PRESSURE LOSS IN HORIZONTAL TWO-PHASE FLOW (2 IN. LINE PIPE ONLY)

Procedure:

1. Knowing P_1 upstream, assume a ΔP value, fixing the downstream pressure P_2.
2. Obtain an average pressure and temperature over the pressure decrement, and at this condition calculate $\overline{w}_g$, $\overline{w}_L$, $\overline{\rho}_L$, $\overline{\rho}_g$, $\overline{v}_{sL}$, $\overline{v}_{sg}$, $\overline{v}_m$.
3. Calculate the abscissa of the holdup correlation and obtain the holdup from Fig. 2.136.

   The abscissa is:

$$\left(\frac{N_{LV}^{0.575}}{N_{GV}}\right)\left(\frac{P}{P_a}\right)^{0.05}$$

   Where: $\overline{N}_{LV} = 1.938\ \overline{v}_{sL}\left[\frac{\rho_L}{\sigma}\right]^{1/4}$

$$N_{GV} = 1.938\ \overline{v}_{sg}\left[\frac{\rho_L}{\sigma}\right]^{1/4}$$

4. Calculate the actual liquid velocity at flowing conditions.

$$v_L = \frac{v_{sL}}{H_L}$$

5. Obtain the Reynolds number for the liquid, and using this value find $f(K)^{0.1}$ from Fig. 2.137.

   Where: f = energy-loss factor
   K = ratio of liquid mass rate to total mass rate = $\frac{w_L}{w_T}$
   Compute f.
6. Calculate the liquid rate $\overline{q}_L$ and the gas rate $\overline{q}_g$.
7. Obtain the distance ΔL for assuming pressure drop.

$$\Delta L = \frac{2\ g_c\ d}{f\ \overline{v}_T^{\,2}}\left[\frac{144\ \Delta\ P\ (\overline{q}_L + \overline{q}_g)}{w_T}\right]$$

   New pressure drops should be assumed from the downstream pressure for the first interval to zero pressure.

   When the working pressure is lower than 150 psig, the kinetic energy term must be calculated and additional steps are added to this calculation:

   This kinetic energy term is computed as follows:

   a. At pressure P_2, calculate v_{sL2} and v_{sg2}.
   b. Obtain the abscissa of the holdup correlation. This term is:

$$\left[\frac{N_{LV}^{0.575}}{N_{GV}}\right]\left[\frac{P}{P_a}\right]^{0.05}$$

   From Fig. 2.136, obtain the holdup H_{L2} at this condition (P_2).

   c. Obtain v_{L_2} and v_{g_2}:

$$v_{L_2} = \frac{v_{sL_2}}{H_L} \qquad v_{g_2} = \frac{v_{sg_2}}{(1 - H_L)}$$

   Repeat steps a, b, and c for conditions at P_3.

   d. Calculate $(\Delta v)_L$ and $(\Delta v)_g$
   e. Calculate the distance for the pressure drop from P_2 to P_3.
8. The total distance will be equal to that obtained, adding the ΔX values due to each assumed pressure drop.

D.52 EXAMPLE PROBLEM BY METHOD OF ANDREWS, ET AL. TO DETERMINE PRESSURE LOSS

Given:

Rate	= 2,000 bpd − 42°API oil
Average temperature	= 120°F
Length of line	= 3,000 ft
Gas-oil ratio	= 1,000 scf/bbl
	$\gamma_g = 0.65$
	d = 2 in. ID
Upstream pressure	= 500 psig
	$\sigma_o = 30$ dynes/cm
	$\mu_g = 0.02$ cp (constant)
	$\mu_o = 1$ cp

1. Divide into two increments, from 500 to 100 psig, and from 100 to 0 psig, and find length of line for each. Neglect kinetic energy term in first increment ($P > 100$).

   First increment:

2. $\overline{P} = 314.7$ psia, $\quad \overline{w}_g = 1.07\ \frac{lb_m}{sec}$, $\quad \overline{w}_L = 6.69\ \frac{lb_m}{sec}$

$$\overline{\rho}_L = 49.1\ \frac{lb_m}{cu\ ft}, \qquad \overline{\rho}_g = 1.003\ \frac{lb_m}{cu\ ft}$$

$$\overline{v}_{sL} = 6.24\ \frac{ft}{sec}, \quad \overline{v}_{sg} = 48.9\ \frac{ft}{sec}$$

$$\overline{v}_m = \overline{v}_{sg} + \overline{v}_{sL} = 48.9 + 6.24 = 55.14\ \frac{ft}{sec}$$

3. $\overline{N}_{Lv} = 1.938\ \overline{v}_{sl}[\overline{\rho}_L/\sigma]^{1/4}$

$$= (1.938)(6.24)\ [49.1/30]^{1/4} = 13.65$$

$$\overline{N}_{Gv} = (1.938)(48.9)[49.1/30]^{1/4} = 107$$

$$\frac{N_{Lv}^{0.575}}{N_{Gv}}\left(\frac{P}{P_a}\right)^{0.05} = \frac{(13.65)^{0.575}}{107}\left(\frac{314.7}{14.7}\right)^{0.05}$$

$$= 0.049$$

   From Fig. 2.136, $H_L = 0.13$

4. $\overline{v}_L = \frac{v_{sL}}{H_L} = \frac{6.24}{0.13} = 48.0$ ft/sec

5. $N_{ReL} = 1{,}490\ \frac{\overline{\rho}_L\ \overline{v}_L\ d}{(\mu_L)^{0.1}}$

$$= 1{,}490\ \frac{(49.1)(48)(0.1667)}{(1.0)^{0.1}} = 586{,}000$$

   From Fig. 2.137, $f(K)^{0.1} = 0.015$

$$K = \frac{w_L}{w_g + w_L} = \frac{6.69}{7.76} = 0.862$$

$$f = \frac{0.015}{(0.862)^{0.1}} = 0.0152$$

6. $\overline{q}_L = \overline{v}_{sL}\ A_p = (6.24)(0.0218) = 0.136$ cu ft/sec

   $\overline{q}_g = (48.9)(0.0218) = 1.065$ cu ft/sec

7. $\Delta L = \frac{2\ g_c\ d}{f\ \overline{v}_m^{\,2}}\left[\frac{144\ \Delta\ p\ (\overline{q}_L + \overline{q}_g)}{w_T}\right]$

$$= \frac{(2)(32.2)(0.1667)}{0.0152\ (55.14)^2}\left[\frac{(144)(400)(0.136 + 1.065)}{7.76}\right]$$

$$= 2{,}063\ \text{ft}$$

Second increment:

$\overline{P} = 64.7$ psia, $\quad \overline{w}_g = 1.13 \frac{lb_m}{sec}$, $\quad \overline{w}_L = 6.63 \frac{lb_m}{sec}$

$\overline{\rho}_L = 50.4 \frac{lb_m}{cu\ ft}$, $\quad \overline{\rho}_g = 0.350 \frac{lb_m}{cu\ ft}$

$\overline{v}_{sL} = 6.04 \frac{ft}{sec}$, $\quad \overline{v}_{sg} = 148 \frac{ft}{sec}$

$\overline{v}_m = \overline{v}_{sL} + \overline{v}_{sg} = 6.04 + 148 = 154.04 \frac{ft}{sec}$

$\overline{N}_{Lv} = (1.938)(6.04)\ [50.4/30]^{1/4} = 13.3$

$\overline{N}_{Gv} = (1.938)(148)\ [50.4/30]^{1/4} = 326$

$$\frac{N_{Lv}^{0.575}}{N_{Gv}} \left(\frac{P}{P_a}\right)^{0.05} = \frac{(13.3)^{0.575}}{326} \left(\frac{64.7}{14.7}\right)^{0.05} = 0.0146$$

From Fig. 2.136, $H_L = 0.04$

$$\overline{v}_L = \frac{\overline{v}_{sL}}{H_L} = \frac{6.04}{0.04} = 151 \text{ ft/sec}$$

$$N_{ReL} = 1490 \frac{(50.4)(151)(0.1667)}{(1.0)^{0.1}} = 1.88 \times 10^6$$

From Fig. 2.137, $f(K)^{0.1} = 0.013$

$$K = \frac{6.63}{7.76} = 0.854$$

$$f = \frac{0.013}{(0.854)^{0.1}} = 0.0131$$

$\overline{q}_L = (6.04)(0.0218) = 0.1315$ cu ft/sec

$\overline{q}_g = (148)(0.0218) = 3.23$ cu ft/sec

Kinetic energy term:

a) At $P_2 = 100$ psig; $v_{sL_2} = 6.06$; $v_{sg_2} = 152.5$ ft/sec.

b) $N_{Lv_2} = (1.938)(6.06)\ [50/30]^{1/4} = 13.3$

$N_{Gv_2} = (1.938)(152.5)\ [50/30]^{1/4} = 335$

$$\frac{N_{Lv}^{0.575}}{N_{Gv}} \left(\frac{P}{P_a}\right)^{0.05} = \frac{(13.3)^{0.575}}{326} \left(\frac{114.7}{14.7}\right)^{0.05} = 0.015$$

From Fig. 2.136, $H_L = 0.04$.

c) $v_{L_2} = \frac{6.06}{0.04} = 151.5$ ft/sec

$v_{g_2} = \frac{152.5}{1 - 0.04} = 159$ ft/sec

At $P_3 = 0$ psig; $v_{sL_3} = 5.96$; $v_{sg_3} = 1{,}184$ ft/sec

$N_{Lv_3} = (1.938)(5.96)\ [51/30]^{1/4} = 13.2$

$N_{Gv_3} = (1.938)(1184)\ [51/30]^{1/4} = 2620$

$$\frac{N_{Lv}^{0.575}}{N_{Gv}} \left(\frac{P}{P_a}\right)^{0.05} = \frac{(13.2)^{0.575}}{2{,}620} \left(\frac{14.7}{14.7}\right)^{0.05} = 0.00168$$

From Fig. 2.136, $H_L = 0.02$,

$v_{L_3} = \frac{5.96}{0.02} = 298$ ft/sec

$v_{g_3} = \frac{1184}{1 - 0.02} = 1{,}210$ ft/sec

d) $(\Delta v)_1 = 298 - 152 = 146$ ft/sec

$(\Delta v)_g = 1{,}210 - 159 = 1{,}051$ ft sec

e) $$\Delta L = \frac{2\, g_c\, d}{f\, \overline{v}_m^{\,2}} \left[\frac{144\, \Delta p\, (q_L + q_g)}{w_T} - \frac{w_L}{w_T} \frac{(\Delta v_L)^2}{2\, g_c} - \frac{w_g}{w_T} \frac{(\Delta v_g)^2}{2\, g_c}\right]$$

$$= \frac{(64.35)(0.1667)}{(0.0131)(154.04)^2} \left[\frac{144(100)(0.1315 + 3.23)}{7.76} - \frac{6.63}{7.76} \frac{(146)^2}{64.35} - \frac{1.13}{7.76} \frac{(1210)^2}{64.35}\right]$$

$$= 3.45 \times 10^{-2}[6{,}240 - 284 - 3{,}320]$$

$$= 87.5 \text{ ft}$$

Total $\Delta L = 2{,}063 + 87.5 = 2{,}150.5$ ft, length for pressure to decrease to zero psig.

Therefore, the length of 3,000 ft cannot be reached with 500 psig upstream pressure and a rate of 2,000 bpd.

Appendix D.6

D.61 PROCEDURE TO CALCULATE THE PRESSURE LOSS IN A HORIZONTAL FLOW LINE FROM A KNOWN UPSTREAM PRESSURE BY METHOD OF DUKLER, ET AL. (CASE I)

1. Assume a downstream pressure which may be for the entire length of the line or for only a short distance. For example, if solving for the downstream pressure at the end of a 5 mile line, pressure decrements of 10 to 25 psi can be used and in particular if machine computation is available.
   Calculate the average pressure between P_1 and P_2.
2. Obtain values for $\bar{R}_s$, $\bar{B}_o$, $\bar{Z}$.
3. Calculate the volumetric flow rates of liquid and gas in cu ft/sec.

$$\bar{q}_L = \frac{q_L \times \bar{B}_o \times 5.61}{86{,}400} \qquad \bar{q}_g = \frac{q_L(GOR - \bar{R}_s)}{86{,}400} \frac{P_{sc}}{\bar{P}} \frac{\bar{T}}{T_{sc}} \bar{Z}$$

4. Calculate λ, the ratio of the volumetric flow rate of liquid to the total volumetric flow rate.

$$\lambda = \frac{\bar{q}_L}{\bar{q}_L + \bar{q}_g}$$

5. Calculate the total mass flow rate of liquid and gas, w_T.
   a) Calculate the liquid density.

$$\bar{\rho}_L = \frac{\gamma_L\,(62.4) + (0.0764)\,(\gamma_g)\,\bar{R}_s/5.61}{\bar{B}_o}$$

   b) Calculate the gas density.

$$\bar{\rho}_g = \gamma_g\,(0.0764)\left(\frac{520}{\bar{T}}\right)\left(\frac{\bar{p}}{14.7}\right)\left(\frac{1}{\bar{Z}}\right)$$

$$w_T = \bar{\rho}_L\,\bar{q}_L + \bar{\rho}_g\,\bar{q}_g$$

6. Calculate the total mass flux.

$$M_T = \frac{w_T}{A_p}$$

7. Calculate the density of the two-phase homogeneous mixture.

$$\rho_{TP} = \bar{\rho}_L\,\lambda + \bar{\rho}_g\,(1 - \lambda)$$

8. Calculate the viscosity of the two-phase mixture.

$$\bar{\mu}_{TP} = \bar{\mu}_L\,\lambda + \bar{\mu}_g\,(1 - \lambda)$$

9. Calculate the Reynolds number of the two-phase flow under conditions of no slip.

$$N_{Re_{TP}} = \frac{1{,}488\ w_T}{\frac{\pi}{4}\,d(ft)\,\bar{\mu}_{TP}(cp)}$$

10. Calculate the friction factor assuming homogeneous fluid.

$$f_{TP} = 0.00140 + \frac{0.125}{(N_{Re_{TP}})^{0.32}}$$

11. Calculate the frictional gradient.

$$\left(\frac{\Delta p}{\Delta L}\right)_f = \frac{\bar{2} f_{TP}\,M_T^2}{g_c\,\bar{\rho}_{TP}\,d}$$

12. Calculate the acceleration term.

$$\bar{a} = \frac{16\ w_T\ w_g\ \bar{P}}{d^4 P_1 P_2 \bar{\rho}_g \pi^2 g_c}$$

13. Calculate the total gradient.

$$\left(\frac{\Delta P}{\Delta L}\right)_t = \left(\frac{\Delta P}{\Delta L}\right)_f / (1 - \bar{a})$$

14. Calculate the total pressure drop.

$$\Delta P = L\left(\frac{\Delta P}{\Delta L}\right)$$

15. If pressure increments have been used, solve for ΔX corresponding to an assumed ΔP. Continue this procedure until $E\Delta X$ = length of line. This is a more accurate procedure because the average fluid properties are more representative over a short section of line.

D.62 EXAMPLE PROBLEM TO CALCULATE PRESSURE LOSS IN HORIZONTAL PIPE BY METHOD OF DUKLER, ET AL. (CASE I)

General information:

$q_L = 2{,}000$ stb bpd
$\rho_o = 42°$API
$\bar{T} = 120°$F
$L = 3{,}000$ ft
$GOR = 1{,}000$ scf/stk bbl
$\gamma_g = 0.65$ (Air = 1.0)
$p_1 = 500$ psig
$\sigma_o = 30$ dynes/cm (constant)
$\mu_g = 0.02$ centipoises (constant)
$\mu_o = 1$ centipoise (constant)
$d = 4.00$ in. (ID)
$\epsilon/d = 1.0 \times 10^{-4}$
$WOR = 0$

Required:

Calculate the downstream pressure p_2 by method of Dukler, et al.

1. Assume $\Delta p = 40$ psig and obtain the average pressure:

$$\bar{p} = \frac{500 + 460}{2} = 480 \text{ psig} = 495 \text{ psia}$$

2. Obtain $\bar{R}_s$, $\bar{B}_o$ and $\bar{Z}$

$\bar{R}_s = 120$ scf/bbl
$\bar{B}_o = 1.070$ bbl/stk bbl
$\bar{Z} = 0.940$

3. Calculate the volumetric flow rate of liquid $\bar{q}_L$ and gas $\bar{q}_g$:

$$\bar{q}_L = \frac{q_L \times \bar{B}_o \times 5.61}{86{,}400} = \frac{2000 \times 1.07 \times 5.61}{86{,}400}$$

$$= 0.139 \text{ cu ft/sec}$$

$$\bar{q}_g = \frac{[(GOR - \bar{R}_s) \times q_L]}{86{,}400}\left(\frac{p_{sc}}{\bar{p}}\right)\left(\frac{\bar{T}}{T_{sc}}\right)(\bar{z})$$

$$= \frac{[(1{,}000 - 120)\ (2{,}000)]}{86{,}400}\left(\frac{14.7}{495}\right)\left(\frac{580}{520}\right)(0.940)$$

$$= 0.635 \text{ cu ft/sec}$$

4. Calculate λ, the ratio of the volumetric flow rate of liquid to the total volumetric flow rate.

$$\lambda = \frac{\bar{q}_L}{\bar{q}_L + \bar{q}_g} = \frac{0.139}{0.139 + 0.635}$$

$$\lambda = 0.180$$

5. Calculate the total mass flow rate of liquid and gas, w_T:
   a) Calculate the liquid density:

$$\bar{\rho}_o = \frac{\gamma_o \times 62.4 + 0.0764 \times \gamma_g \times \bar{R}_s/5.61}{\bar{B}_o}$$

$$\bar{\rho}_o = \frac{0.816 \times 62.4 + 0.0764 \times 0.65 \times 120/5.61}{1.070}$$

$$= 48.4 \text{ lb}_m/\text{cu ft}$$

b) Calculate the gas density:

$$\bar{\rho}_g = \gamma_g \times \rho_{air_{sc}} \left(\frac{T_{sc}}{\bar{T}}\right)\left(\frac{\bar{p}}{p_{sc}}\right)\left(\frac{z_{sc}}{\bar{z}}\right)$$

$$= 0.65 \times 0.0764 \left(\frac{520}{570}\frac{495}{14.7}\right)\left(\frac{1}{0.940}\right)$$

$$= 1.05 \text{ lb}_m/\text{cu ft}$$

Total mass flow of liquid and gas:

$$w_T = \bar{\rho}_L \bar{q}_L + \bar{\rho}_g \bar{q}_g$$

$$= 48.4 \left(\frac{\text{lb}_m}{\text{cu ft}}\right) \times 0.139 \left(\frac{\text{cu ft}}{\text{sec}}\right) + 1.05 \left(\frac{\text{lb}_m}{\text{cu ft}}\right) \times 0.635 \left(\frac{\text{cu ft}}{\text{sec}}\right)$$

$$w_T = 7.40 \frac{\text{lb}_m}{\text{sec}}$$

6. Calculate the total mass velocity:

$$M_T = \frac{w_T}{A_p}$$

$$= \frac{7.40 \text{ (lb}_m/\text{sec)}}{0.0874 \text{ sq ft}} = 84.7 \frac{\text{lb}_m}{\text{sec-sq ft}}$$

7. Calculate the density of the two-phase homogeneous mixture:

$$\bar{\rho}_{TP} = \bar{\rho}_L \bar{\lambda} + \bar{\rho}_g (1 - \bar{\lambda})$$

$$= (48.4)(0.180) + (1.05)(1 - 0.180)$$

$$= 9.56 \frac{\text{lb}_m}{\text{cu ft}}$$

8. Calculate the viscosity of the two-phase mixture:

$$\bar{\mu}_{TP} = \bar{\mu}_L \times \bar{\lambda} + \bar{\mu}_g (1 - \bar{\lambda})$$

$$= (1)(0.180) + (0.02)(0.82) = 0.1964 \text{ cp}$$

9. Calculate the Reynolds number of two-phase flow under conditions of no slip:

$$N_{Re_{TP}} = \frac{1{,}488 \, w_T}{\frac{\pi}{4} d \, \mu_{TP}} = \frac{(1488)(7.40)}{\frac{\pi}{4}(0.333)(0.1964)} = 214{,}000$$

10. $f_{TP} = 0.00140 + \dfrac{0.125}{(214{,}000)^{0.32}} = 0.00334$

11. $\left(\dfrac{\Delta P}{\Delta L}\right)_f = \dfrac{2 f_{TP} M_T^2}{g_c \rho_{TP} d} = \dfrac{(2)(0.00334)(84.7)^2}{(32.2)(9.56)(0.333)} = 0.481 \dfrac{\text{lb}_f}{\text{cu ft}}$

$= 0.00334$ psi/ft

12. $a = \dfrac{(16) w_T w_g \bar{P}}{\pi^2 g_c d^4 P_1 P_2 \rho_g} = \dfrac{(16)(7.4)(0.666)(495)}{\pi^2 (32.2)(0.333)^4 (515)(475)(1.05)}$

$a = 0.0273$

13. $\left(\dfrac{\Delta P}{\Delta L}\right)_t = \dfrac{\left(\frac{\Delta P}{\Delta L}\right)_f}{(1 - a)} = \dfrac{0.00334}{0.9727} = 0.00335$

14. $\Delta P = (3000)(0.00335) = 10.33$ psi

Since this does not agree with ΔP assumed, another ΔP should be assumed and steps 1 through 14 recalculated.

D.63 PROCEDURE FOR SELECTING SIZE OF OFF-SHORE LINE TO BRING FLUIDS TO ON-SHORE FACILITIES BY METHOD OF DUKLER, CASE I

1. Calculate the volumetric flow rate of liquid ($\bar{q}_L$) and gas ($\bar{q}_g$).
2. Compute the ratio (λ) of the volumetric flow rate of liquid to the total volumetric flow rate.

$$\lambda = \frac{\bar{q}_L}{\bar{q}_L + \bar{q}_g}$$

3. Calculate the total mass flow rate of liquid and gas (w_T).

$$w_T = \bar{\rho}_L \bar{q}_L + \bar{\rho}_g \bar{q}_g$$

4. Assume a diameter (d) and compute the area of the pipe (A_p,ft^2).
5. Calculate the total mass velocity, M_T.

$$M_T = \frac{w_T}{A_p}$$

6. Calculate the density of two-phase homogeneous mixture (ρ_{TP}).

$$\rho_{TP} = \rho_L \lambda + \rho_g (1 - \lambda)$$

7. Calculate the viscosity of two-phase homogeneous mixture (μ_{TP}).

$$\mu_{TP} = \mu_L \lambda + \mu_g (1 - \lambda)$$

8. Calculate the Reynolds number of two-phase flow under conditions of no slip.

$$(N_{Re})_{TP} = \frac{1{,}488 \, w_T}{\frac{\pi}{4} d \, \mu_{TP}}$$

9. Compute the friction factor (f_{TP}).

$$f_{TP} = 0.00140 + \frac{0.125}{(N_{Re})_{TP})^{0.32}}$$

10. Calculate the frictional pressure loss $\left(\dfrac{\Delta p}{\Delta L}\right)_f$.

$$\left(\frac{\Delta p}{\Delta L}\right)_f = \frac{2 f_{TP} M_T^2}{g_c \rho_{TP} d}$$

11. Obtain the acceleration term.

$$a = \frac{16 \, w_T \, w_g \, \bar{P}}{\pi^2 g_c d^4 P_1 P_2 \bar{\rho}_g}$$

12. Calculate the total gradient.

$$\left(\frac{\Delta p}{\Delta L}\right)_t = \left(\frac{\Delta p}{\Delta L}\right)_f \Big/ 1 - a$$

13. Obtain the calculated pressure drop.

$$\Delta p = L \left(\frac{\Delta p}{\Delta L}\right)_t$$

If the Δp does not agree with the known pressure drop, another line size is assumed and the procedure repeated.

Appendix D.7

D.71 PROCEDURE TO CALCULATE THE PRESSURE LOSS IN A HORIZONTAL FLOW LINE FROM A KNOWN UPSTREAM PRESSURE BY METHOD OF DUKLER, CASE II.

1. Assume a downstream pressure and calculate the average pressure, $\overline{P}$.

$$\overline{P} = \frac{P_1 + P_2}{2}$$

2. Obtain values for $\overline{R}_s$, $\overline{B}_o$, $\overline{Z}$.
3. Calculate the volumetric flow rates of liquid and gas in cu ft/sec.

$$\overline{q}_L = \frac{q_L\,\overline{B}_o\,(5.61)}{86{,}400} \qquad \overline{q}_g = \frac{q_L\,(GOR - \overline{R}_s)}{86{,}400}\,\frac{P_{sc}}{P}\,\frac{\overline{T}}{T_{sc}}\,\overline{Z}$$

4. Calculate λ, the ratio of the volumetric flow rate of liquid to the total volumetric flow rate.

$$\lambda = \frac{\overline{q}_L}{\overline{q}_L + \overline{q}_g} = \frac{w_L/\rho_L}{w_L/\rho_L + w_g/\rho_g}$$

5. Calculate the liquid density.

$$\overline{\rho}_L = \frac{\gamma_L(62.4) + \gamma_g(0.0764)\,\overline{R}_s/5.61}{\overline{B}_o}$$

6. Calculate the gas density.

$$\overline{\rho}_g = \gamma_g\,(0.0764)\,\frac{\overline{P}}{P_{sc}}\,\frac{T_{sc}}{\overline{T}}\,\frac{1}{\overline{Z}}$$

7. Calculate the velocity of the mixture.

$$v_m = \frac{(q_L + q_g)(144)}{\frac{\pi d^2}{4}}$$

d = inches

8. Calculate a viscosity for the two-phase mixture.

$$\mu_{TP} = \mu_L\,\lambda + \mu_g\,(1 - \lambda)$$

9. Estimate a value for holdup, $\overline{H}_L$.
10. Calculate:

$$\rho_{TP} = \rho_l\left(\frac{\lambda^2}{\overline{H}_l}\right) + \rho_g\left(\frac{(1-\lambda)^2}{(1-\overline{H}_L)}\right)$$

11. Calculate the two-phase Reynolds number.

$$(N_{Re})_{TP} = \frac{d\,v_m\,\rho_{TP}}{\mu_{TP}}$$

12. With the no slip holdup λ from Step 4 and $(N_{Re})_{TP}$ from Step 11, go to Fig. 2.139 and read a value for $\overline{H}_L$.
13. Check $\overline{H}_L$ of Step 12 with the assumed value of Step 9 and if they agree within 5%, use the value of $\overline{H}_L$ as selected from Fig. 2.139. If they do not agree, repeat Steps 9 through 13 until they do agree within 5%.
14. From Fig. 2.140 read $\frac{f_{TP}}{f_o}$.
15. Calculate f_o:

$$f_o = 0.00140 + \frac{0.125}{((N_{Re})_{TP})^{0.32}}$$

16. Calculate f_{TP}.

$$f_{TP} = \frac{f_{TP}}{f_o} \times f_o = (\text{Step 14}) \times (\text{Step 15})$$

17. Calculate the pressure loss due to friction.

$$\Delta P_{friction} = \frac{2\,f_{TP}\,L\,\overline{v}_m^{\,2}\,\rho_{TP}}{12\,g_c\,d}$$

where: L = feet
v_m = ft/sec
ρ_{TP} = lb_m/cu ft
d = in.

This can be changed to calculate $\frac{\Delta P}{\Delta L}$ or to solve for ΔL over a certain pressure drop. If the upstream pressure (P_1) is known, a value of P_2 can be assumed and ΔL solved for directly, then $\Sigma\Delta L$ = length of line.

18. Acceleration pressure drop. This can be neglected in pipe lines but may need to be considered in process piping. Baker, et al. gave the following equation:

$$\Delta P_a = \frac{1}{144\,g_c\,A^2}\left\{\left[\frac{\rho_g\,q_g^{\,2}}{(1-H_L)} + \frac{\rho_L\,q_L^{\,2}}{H_L}\right]_{downstream} - \left[\frac{\rho_g\,q_g^{\,2}}{(1-H_L)} + \frac{\rho_L\,q_L^{\,2}}{H_L}\right]_{upstream}\right\}\cos\theta$$

θ = angle of pipe bend. For horizontal pipe, θ is unity. For transmission and production lines, neglect acceleration term.

19. Calculate total pressure loss.

$$\Delta P_{total} = \Delta P_{friction} + \Delta P_{acceleration}$$

20. If any elevation changes take place, add elevation component loss to Step 19.

D.72 CALCULATION OF PRESSURE DROP BY METHOD OF DUKLER, CASE II. (ASSUME P_2)

General information:

q_o = 7,140 bpd = 40,000 cu ft/day at $\overline{P}$
q_g = 2.569×10^6 scf/d = 105,600 cu ft/day at $\overline{P}$
ρ_o = 52.2 lb_m/cu ft (given)
ρ_g = 1.45 lb_m/cu ft (given)
μ_o = 4.2 cp
μ_g = 0.0105 cp
σ_o = 22.3 dynes/cm
d = 12 in. = 1 ft
L = 134,370 ft
P_1 = 424.7 psia
P_2 = 319.7 psia (Assumed)

1. $\overline{P} = \frac{P_1 + P_2}{2} = \frac{424.7 + 319.7}{2} = 372.2$ psia
2. $\overline{R}_s$, $\overline{B}_o$, and $\overline{Z}$ not required since data is given at $\overline{P}$.
3. $\overline{q}_L$ = 40,000 cu ft/D = 0.463 cu ft/sec
   $\overline{q}_g$ = 105,600 cu ft/D = 1.222 cu ft/sec
4. $\lambda = \frac{\overline{q}_L}{\overline{q}_L + \overline{q}_g} = \frac{40{,}000}{40{,}000 + 105{,}600} = 0.275$
5. $\overline{\rho}_o$ = 52.2 lb_m/cu ft (given)
6. $\overline{\rho}_g$ = 1.45 lb_m/cu ft (given)
7. $v_m = \frac{(q_L + q_g)(144)}{\frac{\pi d^2}{4}} = \frac{(0.463 + 1.222)(144)}{\frac{\pi(12)^2}{4}} = 2.14$ ft/sec
8. $\overline{\mu}_{TP} = \overline{\mu}_L\,\lambda + \overline{\mu}_g\,(1-\lambda) = (4.2)(0.275) + (0.0105)(0.725)$
   $\overline{\mu}_{TP}$ = 1.16 cp
   $\overline{\mu}_{TP}$ = (1.16)(0.000672) = 0.00078 lb_m/ft sec
9. Estimate a value for holdup. Assume $\overline{H}_L = 0.30$
10. $\rho_{TP} = (52.2)\left(\frac{0.275^2}{0.30}\right) + 1.45\left(\frac{0.75^2}{0.70}\right) = 13.17 + 1.163$
    = 14.333

11. $(N_{Re})_{TP} = \dfrac{d\ v_m\ \rho_{TP}}{\mu_{TP}} = \dfrac{(1)(2.14)(14.33)}{0.00078} = 39{,}300$
12. From Fig. 2.139, $H_L = 0.43$.
13. Compare Step 9 for $H_L = 0.30$ and calculated $H_L = 0.43$ and these are not close enough. Repeat Steps 9 through 13 with H_L assumed = 0.43. New calculated value of H_L from Fig. 2.139 = 0.45 with $(N_{Re})_{TP} = 29{,}200$. Therefore, use $H_L = 0.45$ and with new $\rho_{TP} = 10.63$ and new $(N_{Re})_{TP} = 29{,}200$.
14. From Fig. 2.140:

$$\frac{f_{TP}}{f_o} = 2.00$$

15. Calculate:

$$f_o = 0.00140 + \frac{0.125}{(29{,}200)^{0.32}} = 0.00140 + \frac{0.125}{31} = 0.00543$$

16. $f_{TP} = (2.00)(0.00543) = 0.01086$
17. $\Delta P_{friction} = \dfrac{2\ f_{TP}\ L\ \bar{v}_m^{\,2}\ \rho_{TP}}{12\ g_c\ d}$

$$= \frac{(2)(0.01086)(134{,}370)(2.14)^2(10.63)}{(12)(32.2)(12)}$$

$$= 30.64 \text{ psi}$$

18. Neglect acceleration.
19. Total pressure loss:

$$\Delta P_T = 30.64 + 0 = 30.64 \text{ psi}$$

20. $P_2 = 424.7 - 30.64 = 394.06$
21. Repeat procedure by assuming $P_2 = 394$ and compare results until the assumed value of P_2 = the calculated value.

Appendix D.8

1 MATHEMATICAL DEVELOPMENT OF THE EATON, ET AL CORRELATION

The basic energy balance equation in symbolic differential m based on one pound mass of flowing fluid is the same that for vertical flow.

$$144\ Vdp + \frac{g}{g_c}\,dh + \frac{vdv}{g_c} + \frac{g}{g_c}\,dW_f + \frac{g}{g_c}\,d\,W_e = 0 \qquad (D.81)$$

is has the units of $\frac{ft\ lb_f}{lb_m}$.

The second term which represents a change in elevation y be eliminated for horizontal flow, i.e., $g/g_c\ dh = 0$. The h and last term may be eliminated when no external rk is done by the fluid, i.e., $d\ W_e = 0$. Eq. D.81 then re- es to:

$$144\ Vdp + \frac{vdv}{g_c} + \frac{g}{g_c}\,dW_f = 0. \qquad (D.82)$$

For the flow of both a liquid and gaseous phase, an energy ance may be written for the respective quantities of each ase.

f a gas-liquid mixture flows between points x_1 and x_2, ving pressures of p_1 and p_2, respectively, the total mass w rate can be represented by w_L and w_g for flowing liquid l gas respectively. An energy balance for each phase is itten as follows:

the liquid phase:

$$144\ w_L\ V_L\ d_p + \frac{w_L\ v_L\ d\ v_L}{g_c} + \frac{g}{g_c}\,w_L\,dW_{fL} = 0. \qquad (D.83)$$

the gas phase:

$$144\ w_g\ V_g\ dp + \frac{w_g\ v_g\ dv_g}{g_c} + \frac{g}{g_c}\,w_g\,dW_{fg} = 0. \qquad (D.84)$$

adding Eq. D.83 and D.84 we obtain a total energy ance for both phases:

$$144\int_{p_1}^{p_2}[w_L\ V_L + w_g\ V_g]\,dp + \frac{1}{g_c}[w_L\ v_L\ dv_L + w_g\ v_g\ dv_g] + \frac{g}{g_c}[w_L\ dW_{fL} + w_g\ d\ W_{fg}] = 0 \qquad (D.85)$$

recalling that the two-phase mixture flows between x_1 d x_2, with pressures of p_1 and p_2, respectively, then Eq. 35 may be integrated as follows:

$$144\left[w_L\int_{p_1}^{p_2} V_L\,dp + w_g\int_{p_1}^{p_2} V_g\,dp\right] + \frac{w_L}{g_c}\int_{v_{L1}}^{v_{L2}} v_L\,dv_L + \frac{w_g}{g_c}\int_{v_{g1}}^{v_{g2}} v_g\,dv_g + \frac{g}{g_c}\int\left[w_L\,dW_{fL} + w_g\,dW_{fg}\right] = 0 \qquad (D.86)$$

The last integral of Eq. D.86 represents the sum of all eversible energy losses.

As a matter of convenience and simplicity, all energy ses are combined into one term, as follows:

$$\int w_T\,dW_{fT} \equiv \int[w_L\,dW_{fL} + w_g\,dW_{fg}] \qquad (D.87)$$

ere: $w_T = w_L + w_g$ (D.88)

By defining these multiphase energy losses by the Darcy- isbach equation:

$$w_T\frac{dW_{fT}}{dx} = \frac{f\,w_T\,\bar{v}_m^{\,2}}{2\,g\,d} \qquad (D.89)$$

By combining Eq. D.86, D.87 and D.89, we have:

$$144\int_{p_1}^{p_2}[w_L\ V_L + w_g\ V_g]\,dp + \frac{w_L}{g_c}\int_{v_{L1}}^{v_{L2}} v_L\,dv_L + \frac{w_g}{g_c}\int_{v_{g1}}^{v_{g2}} v_g\,dv_g + \int_{x_1}^{x_2}\frac{f\,w_T\,\bar{v}_m^{\,2}}{2\,g_c d}\,dx = 0 \qquad (D.810)$$

The last three terms of Eq. D.810 may be integrated directly to obtain:

$$144\left[w_L\int_{p_1}^{p_2} V_L\,dp + w_g\int_{p_1}^{p_2} V_g\,dp\right] + \frac{w_L}{g_c}\frac{\Delta v_L^{\,2}}{2} + \frac{w_g}{g_c}\frac{\Delta v_g^{\,2}}{2} + \frac{f w_T\,\bar{v}_m^{\,2}}{2\,g_c\,d}\,\Delta x = 0 \qquad (D.811)$$

Eq. D.811 contains two additional integrals that must be evaluated and a problem arises in that the functional relationship of V with p may not be known exactly. However, it can be approximated quite accurately by choosing small pressure decrements. The averaged integrated volumes of each phase between pressure points p_1 and p_2 are:

$$\bar{V}_L = \frac{\int_{p_2}^{p_1} V_L\,dp}{p_1 - p_2} = \frac{\int_{p_1}^{p_2} V_L\,dp}{p_1 - p_2} \qquad (D.812)$$

$$\bar{V}_g = \frac{\int_{p_2}^{p_1} V_g\,dp}{p_1 - p_2} = -\frac{\int_{p_1}^{p_2} V_g\,dp}{p_1 - p_2} \qquad (D.813)$$

The values of these integrals are approximated for small pressure decrements by:

$$\int_{p_1}^{p_2} V_L\,dp = -\bar{V}_L\,(p_1 - p_2) = -\bar{V}_L\,\Delta P \qquad (D.814)$$

$$\int_{p_1}^{p_2} V_g\,dp = -\bar{V}_g\,(p_1 - p_2) = -\bar{V}_g\,\Delta P \qquad (D.815)$$

All of the integrated average quantities are guaranteed to exist between pressure points (1) and (2) by the theorem of the mean for integrals.

By substituting Eqs. D.814 and D.815 into Eq. D.811:

$$-144\,[w_L\,\bar{V}_L + w_g\,\bar{V}_g]\,\Delta p + \frac{w_L\,\Delta v_L^{\,2} + w_g\,\Delta v_g^{\,2}}{2\,g_c} + \frac{f\,w_T\,\bar{v}_m^{\,2}}{2\,g_c\,d}\,\Delta x = 0 \qquad (D.616)$$

The integrated average specific volumes are related to the integrated average liquid and gas densities ρ_L and ρ_g, respectively, by:

$$\bar{\rho}_L = \frac{1}{\bar{V}_L} \text{ and } \bar{\rho}_g = \frac{1}{\bar{V}_g} \qquad (D.817)$$

By substituting back into Eq. D.816,

$$-144\left[\frac{w_L}{\bar{\rho}_L} + \frac{w_g}{\bar{\rho}_g}\right]\Delta p + \frac{w_L\,\Delta v_L^{\,2} + w_g\,\Delta v_g^{\,2}}{2\,g_c} + \frac{f\,w_T\,\bar{v}_m^{\,2}}{2\,g_c d}\,\Delta x = 0 \qquad (D.818)$$

This equation may be solved for Δx and gives:

$$\Delta x = \frac{2\,g_c d}{w_T\bar{v}_m^{\,2}\,f}\left[144\,\Delta p\left(\frac{w_L}{\bar{\rho}_L} + \frac{w_g}{\bar{\rho}_g}\right) - \frac{w_L\,\Delta v_L^{\,2} + w_g\,\Delta^2 v_g}{2\,g_c}\right] \qquad (D.819)$$

D.82 PROCEDURE FOR CALCULATION OF A PRESSURE TRAVERSE BY THE METHOD OF EATON, ET AL.

This solution is not a trial and error solution but does require the assumption of pressure points from which the

corresponding lengths are calculated. Assume that the upstream pressure is known as well as the length of pipeline, temperature, and fluid properties. The problem will be to solve for the downstream pressure, thus giving the pressure loss. The normal procedure will be to assume relatively small pressure decrements and calculate corresponding lengths. The accuracy of the method will depend to some extent on the magnitude of the pressure decrements with increasing accuracy occurring at the smaller pressure decrements.

Stepwise procedure

1. Knowing p_1, assume a value of p_2.
2. Knowing p_1, p_2, T_1, and T_2, determine a value for average pressure (p) and average temperature ($\bar{T}$).
3. Calculate or obtain a value for $\bar{v}_m$, w_L, w_g, $\bar{\rho}_L$, $\bar{\rho}_g$, σ and ρ_L, ρ_g, R_s, β_o, μ_g, μ_L at both conditions of pressure.
   If the given pressure is in the low region (less than 150 psia), the kinetic energy term must be computed, and therefore holdup. For pressures above 150 psia the kinetic energy term can be neglected and steps 4 and 5 are not required.
4. In order to determine holdup we use Fig. 2.144, which requires the evaluation of:

$$\frac{(N_{Lv})^{0.575}}{N_{gv}\,N_D^{0.0277}}\left(\frac{p}{p_b}\right)^{0.05}\left(\frac{N_L}{N_{LB}}\right)^{0.1} \text{ at both } p_1 \text{ and } p_2$$

where: $N_{Lv} = 1.938\, v_{sL}\sqrt[4]{\frac{\rho_L}{\sigma}}$

$N_{gv} = 1.938\, v_{sg}\sqrt[4]{\frac{\rho_L}{\sigma}}$

$N_d = 120.872\, d\sqrt{\frac{\rho_L}{\sigma}}$

$N_L = 0.15726\,\mu_L\sqrt[4]{\frac{1}{\rho_L\,\sigma^3}}$

$p/p_b = p/14.65$

$N_{LB} = \text{constant} = 0.00226$

Note that the solution gas and free gas must be determined in order to correctly evaluate v_{sL} and v_{sg}.

5. Obtain H_{L_1} and H_{L_2} from Fig. 2.144.
6. Evaluate v_{L_1}, v_{L_2}, Δv_L, v_{g_1}, v_{g_2}, Δv_g.
7. Obtain a value of the friction factor from Fig. 2.143. In order to do this, obtain a value for:

$$(GR)^{0.5}\left(\frac{d_B}{d}\right)^{1.25}\frac{w_T d}{\mu_g} \quad \text{For } \mu_g \text{ use units of } \frac{\text{lb}}{\text{ft-sec}}$$

$$\text{cp} \times 6.72 \times 10^{-4} = \frac{\text{lb}}{\text{ft-sec}}$$

Read a value for $f(LR)^{0.1}$ from Fig. 2.143. Knowing LR, calculate f.

8. Calculate Δx from Eq. D.819.

$$\Delta x = \frac{2\,g_c\,d'}{w_T \bar{v}_m^{\,2}\,f}\left[144\,\Delta p\left(\frac{w_L}{\bar{\rho}_L}+\frac{w_g}{\bar{\rho}_g}\right)-\frac{w_L\,\Delta v_L^{\,2}+w_G\,\Delta v_g^{\,2}}{2\,g_c}\right]$$

9. Starting with p_2 and x_2, assume a value for p_3 and repeat the procedure. Continue these calculations until reaching the full length of the system. This procedure lends itself quite readily to machine computation.

d = pipe diameter, in.
d_p = base pipe diameter, = 1.0 in. = 0.0833 ft
GR = ratio of gas mass rate to total mass rate
LR = ratio of liquid mass rate to total mass rate
p = pressure, psia
ρ = density, lb_m/cu ft
v = velocity, ft/sec
M_T = total mass flux = $\frac{w_T}{A_p}$
w_T = lb_m/sec
A_p = cross-sectional area of pipe, sq ft
v_m = velocity of mixture, ft/sec
Δp = pressure difference, psi
w_L = lb_m of liquid per sec
w_g = lb_m of gas per sec
v_L = velocity of liquid, ft/sec
v_g = velocity of gas, ft/sec
μ_g = viscosity of gas, $\frac{\text{lb}}{\text{ft-sec}}$

D.83 PROBLEM TO DETERMINE PRESSURE LOSS IN A 2 IN. LINE 1,500 FT LONG BY METHOD OF EATON, ET AL. (KINETIC ENERGY NEGLECTED)

Given:
L = 1,500 ft
p_1 = 1,500 psia
γ_g = 0.65
γ_w = 1.07
q_L = 2,000 bpd (all water)
d = 2 in. ID
$\bar{T}$ = 110°F
μ_g = 0.02
μ_w = 1.0 cp
σ_w = 70 dynes/cm
G/L ratio = 500 scf/bbl

Find the pressure at 1,500 ft by method of Eaton, et al.

The problem will be worked in this example by assuming one pressure decrement over the entire length to illustrate the procedure. However, greater accuracy would result if short pressure decrements were taken.

Follow the stepwise procedure as listed for working a problem by this method.

1. Assume p_2 = 1,375 psia
2. $\bar{p} = \frac{1{,}500 + 1{,}375}{2} = 1{,}437.5$ psia

   $\bar{T} = 110°F = 460 + 110 = 570°R$
3. Obtain values for v_m, ρ_L, ρ_g, R_s, B_o, σ, μ_g, μ_L, w_L, and w_g.

$$\bar{\rho}_g = (0.65)(0.0764)\left(\frac{1437.5}{14.7}\right)\left(\frac{520}{570}\right)\left(\frac{1}{0.85}\right) = 5.2\frac{lb_m}{ft^3}$$

$$\bar{\rho}_L = (1.07)(62.4) = 66.7\ lb_m/ft^3$$

$$w_L = \frac{(2{,}000)(5.65)(66.7)}{86{,}400} = 8.68\ lb_m/sec$$

$$w_g = \frac{(2{,}000)(500)(0.65)(0.0764)}{86{,}400} = 0.575\ lb_m/sec$$

$$w_T = 9.255\ lb_m/sec$$

For the high pressures and relatively low gas flow rates, the kinetic energy term can be neglected.

4. Determine holdup. This step is eliminated since the kinetic energy term is neglected.
5. Obtain H_{L1}, H_{L2} (not necessary when neglecting kinetic energy term).
6. Evaluate v_{L_1}, v_{L_2}, Δv_L, v_{g_1}, v_{g_2}, Δv_g – not necessary
7. Obtain a value for the friction factor.
   Evaluate

$$(GR)^{0.5}\left(\frac{d_B}{d}\right)^{1.25}\frac{M_T d}{\mu_g}$$

$$\bar{v}_{sL} = \frac{q_L}{Ap} = \frac{2000(5.61)}{86{,}400}\,\frac{1}{0.0218} = 5.97\ ft/sec$$

$$q_g = \frac{(2000)(500)}{86{,}400} = 11.57\ ft/sec$$

$$\bar{q}_g = 11.57\left(\frac{14.7}{1{,}437.5}\right)\left(\frac{570}{520}\right)(0.83) = 0.112 \text{ ft}^3/\text{sec}$$

$$\bar{v}_{sg} = \frac{0.112}{0.0218} = 5.15 \text{ ft/sec}$$

$$\bar{v}_m = \bar{v}_{sL} + \bar{v}_{sg} = 5.97 + 5.12 = 11.12 \text{ ft/sec}$$

$$(GR)^{0.5} = \left(\frac{w_g}{w_T}\right)^{0.5} = \left(\frac{0.575}{9.255}\right)^{0.5} = 0.25$$

$$\left(\frac{d_B}{d}\right)^{1.25} = \left(\frac{1}{2}\right)^{1.25} = 0.42$$

$$M_T = \left(\frac{w_T}{A_p}\right) = \frac{9.255}{0.0218} = 425 \text{ lb}_m/\text{sec-sq ft}$$

$$\frac{M_T d}{\mu_g} = \frac{(425)(0.1667)}{(0.02)(6.72 \times 10^{-4})} = 5.27 \times 10^6$$

Then

$$(GR)^{0.5}\left(\frac{d_B}{d}\right)^{1.25}\left(\frac{M_T d}{\mu_g}\right) = 5.5 \times 10^5$$

$f(LR)^{0.1} = 0.023$ (See Fig. 2.143)

$$(LR)^{0.1} = \frac{8.68}{9.255} \cong 1, \text{ therefore, } f = 0.023$$

Calculate the length increment:

$$L = \frac{2\,g_c d}{w_T \bar{v}_m{}^2 f}\left[144\,\Delta p\left(\frac{w_L}{\rho_L} + \frac{w_g}{\rho_g}\right)\right]$$

$$= \frac{(2)\,(32.2)(0.166)}{9.255\,(11.12)^2\,(10^2)\,(2.3 \times 10^{-2})}\left[144(125)\left(\frac{8.68}{66.7} + \frac{0.575}{5.2}\right)\right]$$

$L = 0.405\,[18{,}000\,(0.13 + 0.11)] = 1750$ ft

The problem requires pressure loss in 1,500 ft; therefore, assumed value of P_2 was too small.
A value of 1,400 psia is more in line and comes out very se to 1,500 ft.
This problem has been simplified somewhat by using water the liquid and assuming that no gas goes into solution and it the liquid volume remains constant. Also, neglecting kinetic energy term greatly reduces the number of calations.
The same problem will be computed assuming p_2 = 00 psi

.) Assume $p_2 = 1{,}400$ psi.

:) $\bar{p} = \dfrac{1{,}500 + 1{,}400}{2} = 1{,}450$ psi

$\bar{T} = 110°F = 460 + 110° = 570°R$
$Z = 0.81$

:) Obtain values for V_m, ρ_L, ρ_g, B_o, σ, μ_g, μ_L, w_L, and w_g

$$\bar{\rho}_g = (0.65)(0.0764)\left(\frac{1450}{14.7}\right)\left(\frac{520}{570}\right)\left(\frac{1}{0.81}\right) = 5.51\frac{\text{lb}}{\text{ft}^3}$$

$$\bar{\rho}_L = (1.07)\left(64.2\frac{\text{lb}}{\text{ft}^3}\right) = 66.7\frac{\text{lb}}{\text{ft}^3}$$

$$w_L = 8.68\frac{\text{lb}_m}{\text{sec}} \qquad w_g = 0.575\frac{\text{lb}_m}{\text{sec}} \qquad w_T = 9.255\frac{\text{lb}_m}{\text{sec}}$$

$$v_{sL_1} = v_{sL_2} = 5.97\frac{\text{ft}}{\text{sec}} \qquad v_{g_1} = 4.68\frac{\text{ft}}{\text{sec}}$$

$$q_{g_2} = \frac{500 \times 2000}{86{,}400}\left(\frac{14.7}{1400}\right)\left(\frac{570}{520}\right)\left(\frac{0.82}{1}\right) = 10.92 \times 10^{-2}\frac{\text{ft}^3}{\text{sec}}$$

$$v_{g_2} = \frac{10.92 \times 10^{-2} \text{ ft}^3/\text{sec}}{0.0218 \text{ ft}^2} = 5.02\frac{\text{ft}}{\text{sec}}$$

$$v_{m_1} = 5.97 + 4.68 = 10.65\frac{\text{ft}}{\text{sec}}$$

$$v_{m_2} = 5.97 + 5.02 = 10.99\frac{\text{ft}}{\text{sec}}$$

$$\bar{v}_m = \frac{10.65 + 10.99}{2} = 10.82\frac{\text{ft}}{\text{sec}}$$

$$\Delta x = \frac{2g_c d}{w_T v_m{}^2 f}\left[144\,\Delta p\left(\frac{w_L}{\bar{\rho}_L} + \frac{w_g}{\bar{\rho}_g}\right)\right]$$

$$\Delta x = \frac{2 \times 32.2 \times 2}{9.255\,(10.82)^2 0.0232 \times 12}\left[144 \times 100\left(\frac{8.68}{66.7} + \frac{0.575}{5.51}\right)\right]$$

$$\Delta x = \frac{131}{302}[3.37 \times 10^3] \text{ ft} = 1{,}462 \text{ ft}$$

(This is near enough to 1,500 ft)
Therefore, the pressure loss = 1,500 − 1,400
= 100 psig

D.84 EXAMPLE PROBLEM TO DETERMINE PRESSURE LOSS BY METHOD OF EATON, ET AL. (SAME AS EX. D.33 BY LOCKHART AND MARTINELLI) KINETIC ENERGY INCLUDED

Given:
$d = 2$ in.
$L = 1{,}500$ ft
$q_L = 2{,}000$ bpd water ($\gamma_w = 1.07$)
$G/L = 1{,}000$ scf/bbl
$\gamma_g = 0.65$
$P_1 = 850$ psia
$\sigma_w = 66.7$ dynes/cm
$\mu_g = 0.015$ cp
$\bar{T} = 120°F$
$\mu_L = 1.0$ cp

1. Given $P_1 = 850$ psia
   Assume $P_2 = 650$ psia at 1,500 ft

2. $\bar{P} = \dfrac{850 + 650}{2} = 750$ psia

   $T = 120°F$ (given)

3. (a) $\bar{\rho}_g = (0.65)(0.0764)\left(\frac{750}{14.7}\right)\left(\frac{520}{580}\right)\left(\frac{1}{0.90}\right) = 2.52\frac{\text{lb}_m}{\text{cu ft}}$

   (b) $\bar{\rho}_L = (1.07)(62.4) = 66.7$ lb$_m$/cu ft
   (c) $R_S = 0$ (assume zero for water)
   (d) $B_w = 1.00$ (assume no compressibility)
   (e) $\sigma_w = 66.7$ dynes/cm
   (f) $\mu_g = 0.015$ cp
   (g) $\mu_w = 1.0$ cp

   (h) $w_L = \dfrac{(2{,}000)(5.61)(66.7)}{86{,}400} = 8.68$ lb$_m$/sec

   (i) $w_g = \dfrac{(0.65)(0.0764)(1{,}000)(2{,}000)}{86{,}400} = 1.15$ lb$_m$/sec

   (j) $w_T = w_L + w_g = 8.68 + 1.15 = 9.83$ lb$_m$/sec

4. Determine the holdup at both P_1 and P_2.
   For H_L at $P_1 = 850$ psia we need to evaluate:

$$\frac{(N_{LV})^{0.575}}{N_{gv}\,N_d{}^{0.0277}}\frac{P^{0.05}}{P_b}\frac{N_L{}^{0.01}}{N_{LB}}$$

where: $N_{Lv} = 1.938\,v_{SL}\sqrt[4]{\dfrac{\rho_L}{\sigma}}$

$$N_{GV} = 1.938\ V_{SG}\sqrt[4]{\frac{\rho_L}{\sigma}}$$

$$N_d = 120.872d\sqrt{\frac{\rho_L}{\sigma}}$$

$$N_L = 0.15726\ \mu_L\sqrt[4]{\frac{1}{\rho_L\sigma^3}}$$

N_{LB} = constant = 0.00226

$$\frac{P}{P_b} = \frac{P}{14.7}$$

(a) Determine v_{sg} and v_{sL} at 850 psia

$$q_g = \frac{(1{,}000)(2{,}000)}{86{,}400} = 23.15 \text{ scf/sec}$$

$$q_g = (23.15)\left(\frac{14.7}{850}\right)\left(\frac{580}{520}\right)\left(\frac{0.92}{1}\right) = 0.412 \text{ cu ft/sec}$$

$$v_{sg} = \frac{0.412}{0.0218} = 18.9 \text{ ft/sec}$$

$$v_{sL} = \frac{q_L}{A_p}$$

$$= (2{,}000 \text{ bbl})\left(\frac{5.61 \text{ cu ft}}{\text{bbl}}\right)\left(\frac{\text{day}}{86{,}400 \text{ sec}}\right)\left(\frac{1}{0.0218 \text{ sq ft}}\right)$$

$$= 5.97 \text{ ft/sec}$$

$v_m = v_{sg} + v_{sL} = 5.97 + 18.90 = 24.87$ ft/sec

Determine dimensionless groups at 850 psia.

$$N_{LV} = 1.938\ V_{SL}\sqrt[4]{\frac{\rho_L}{\sigma}}$$

$$= (1.938)(5.97)\sqrt[4]{\frac{66.7}{66.7}} = 11.57$$

$$N_{gv} = 1.938\ v_{sg}\sqrt[4]{\frac{\rho_L}{\sigma}}$$

$$= (1.938)(18.90)\sqrt[4]{\frac{66.7}{66.7}} = 36.6$$

$$N_d = 120.872\ d\sqrt{\frac{\rho_L}{\sigma}}$$

$$= (120.872)(2/12)\ (1) = 20.1$$

$$N_L = 0.15726\ \mu_L\sqrt[4]{\frac{1}{\rho_L\sigma^3}}$$

$$= (0.15726)(1.0)\sqrt[4]{\frac{1}{(66.7)(66.7)^3}} = 0.00236$$

$$\frac{P}{P_b} = \frac{850}{14.7} = 57.7$$

Determine a value of:

$$\frac{(N_{LV})^{0.575}}{N_{gv}\ N_d^{0.0277}}\ \frac{P^{0.05}}{P_b}\ \frac{N_L^{0.1}}{N_{LB}} \text{ at 850 psia}$$

$$= \frac{(11.57)^{0.575}}{(36.6)\ (20.1)^{0.0277}}\ \frac{(57.7)^{0.05}}{}\ \frac{0.00236^{0.1}}{0.00226}$$

$$= 0.125$$

5. From Fig. 2.144, $H_L = 0.30$ = vol of pipe occupied by liquid.

$1 - H_L = 0.70$ = vol of pipe occupied by gas for 850 psia

For conditions of 650 psia:

$q_g = 23.15$ scf/sec

$$q_g = (23.15)\ \frac{14.7}{650}\ \frac{580}{520}\ \frac{0.912}{1} = 0.533 \text{ cu ft/sec}$$

$$v_{sg} = \frac{0.533}{0.0218} = 24.4 \text{ ft/sec}$$

$v_{sL} = 5.97$ ft/sec

$v_m = v_{sg} + v_{sL} = 5.97 + 24.4 = 30.37$ ft/sec

N_{Lv} = same = 11.57

$$N_{gv} = 1.938\ (24.4)\sqrt[4]{\frac{66.7}{66.7}} = 47.3$$

N_d = same = 20.1

N_L = same = 0.00236

$$\frac{P}{P_b} = \frac{650}{14.7} = 44.2$$

Determine:

$$\frac{(11.57)^{0.575}}{(47.3)\ (20.1)^{0.0277}}\ \frac{(44.2)^{0.05}}{}\ \frac{(0.00236)^{0.1}}{0.00226}$$

$$= \frac{(4.07)\ (1.21)}{(47.3)\ (1.087)} = 0.096$$

From Fig. 2.144, H_L at 650 psia = 0.27

$$1 - H_L = 0.73$$

6. Evaluate v_{L1}, v_{L2}, Δv_L, v_{g1}, v_{g2}, Δv_g

$$v_{L1} = (5.97)\ \frac{1}{0.3} = 19.9 \text{ ft/sec}$$

$$v_{L2} \quad (5.97)\ \frac{1}{0.27} = 22.1 \text{ ft/sec}$$

$\Delta v_L = 22.1 - 19.9 = 2.20$ ft/sec

$$v_{g1} = (18.9)\ \frac{1}{0.70} = 27 \text{ ft/sec}$$

$$v_{g2} = 24.4\ \frac{1}{0.73} = 33.5 \text{ ft/sec}$$

$\Delta v_g = 33.5 - 27 = 6.50$ ft/sec

7. To obtain a value of the friction factor, evaluate:

$$(GR)^{0.5}\ \frac{d_B^{1.25}}{d}\ \frac{M_T d}{\mu_g}$$

$$(GR)^{0.5} = \left(\frac{w_g}{w_T}\right)^{0.5} = \left(\frac{1.15}{9.83}\right)^{0.5} = 0.342$$

$$\left(\frac{d_B}{d}\right)^{1.25} = \left(\frac{0.08333}{0.1666}\right)^{1.25} = 0.42$$

$$M_T = \frac{w_T}{A_p} = \frac{9.83}{0.0218} = 450\ \frac{\text{lb}}{\text{sec-sq ft}}$$

$$\frac{M_T d}{\mu_g} = \frac{(450)(0.1666)}{(0.015)(6.72)(10^{-4})} = 7.45 \times 10^6$$

$$(0.342)(0.42)(7.45 \times 10^6) = 1.07 \times 10^6$$

From Fig. 2.143:

$$(LR)^{0.1} = 0.022$$

$$LR = \frac{w_L}{w_T} = \frac{8.68}{9.83} = 0.883$$

$$(0.883)^{0.1} = 0.99$$

$$f = \frac{0.022}{0.99} = 0.0222$$

8. Calculate ΔX

$$\overline{v}_m = \frac{v_{m850} + v_{m650}}{2} = \frac{24.87 + 30.37}{2} = 27.62$$

$$\Delta X = \frac{2\,g_c\,d}{w_T\,\bar{v}_m{}^2\,f}\left[144\,\Delta p\left(\frac{w_L}{\rho_L}+\frac{w_g}{\rho_g}\right)-\frac{m_L\,\Delta v_L{}^2+w_g\,\Delta v_g{}^2}{2\,g_c}\right]$$

$$\Delta X = \frac{(64.4)(0.1666)}{(9.83)(27.62)^2(0.0222)}\left[(144)(200)\frac{8.68}{66.7}+\frac{1.15}{2.52}-\frac{8.68(2.2)^2+(1.15)(6.5)^2}{(64.4)}\right]$$

can be noted the kinetic energy term is negligible for example and could have been neglected—its value is

$$\Delta X = (0.0644)\,(16{,}900) = 1{,}088 \text{ ft}$$

This compares to a length of 1,500 ft and, therefore, another assumption must be made giving a final answer of 580 psi.

Greater accuracy is obtained if closer increments are chosen. The following tabulation shows the results of 50 psi increments and shows 650 psia to occur at 1,468 ft which corresponds very closely to 1,500 ft. Closer increments may be chosen for a computer solution and 10 psi increments may be used for small pressure losses.

The problem may also be solved by choosing length increments and solving for ΔP. An advantage to this procedure is that the exact length may be selected in choosing the last increment.

Table showing greater accuracy by choosing 50 psi decrements

Pres.	850	800	750	700	650	600	550	500
v_g ft/sec	17.75	18.85	20.20	21.50	23.25	25.20	27.45	32.80
v_m ft/sec	24.72	25.81	27.40	28.80	30.70	32.75	35.00	40.60
ρ_g lb/cu ft	2.79	2.63	2.46	2.32	2.14	1.97	1.82	1.52
	850	800	750	700	650	600	550	
Interval	—	—	—	—	—	—	—	
	800	750	700	650	600	550	500	
$\bar{v}_m$	25.26	26.60	28.10	29.80	31.70	33.90	37.80	
ρ_g	2.71	2.55	2.35	2.23	2.06	1.90	1.67	
$\bar{v}_m{}^2$	638	707	790	880	1,000	1,140	1,430	
ΔX	400	375	358	335	314	295	262	
ΣΔX	400	775	1,133	1,468	1,782	2,077	3,339	

PROCEDURE FOR SELECTING LINE SIZE—EATON, ET AL. METHOD (NEGLECT ACCELERATION)

Obtain $\bar{P} = \frac{P_1 + P_2}{2}$

With $\bar{P}$ determine $\bar{R}_s$, $\bar{B}_o$, $\bar{Z}$.
With P_1 obtain R_{s1}, B_{o1}, Z_1.
With P_2 obtain R_{s2}, B_{o2}, Z_2

Check to see if $\bar{P} > 150$ psia and if so neglect kinetic energy term.

Assume a diameter d (in.).
Compute area A_p (ft²).

At P_1 compute:

$$q_{L1},\ \rho_{L1},\ v_{sL1},\ q_{g1},\ \rho_{g1},\ v_{sg1},\ v_{m1}$$

At P_2 compute:

$$q_{L2},\ \rho_{L2},\ v_{sL2},\ q_{g2},\ \rho_{g2},\ v_{sg2},\ v_{m2}$$

At $\bar{P}$ calculate $\bar{v}_{sL}$, $\bar{v}_{sg}$, $\bar{v}_m$, $\bar{v}_m{}'$, $\bar{w}_L$, $\bar{w}_g$, w_T:

$$v_m{}' = \tfrac{1}{4}\,(v_{m1} + 2\,v_m + v_{m2})$$

Compute GR:

$$GR = \frac{\bar{w}_g}{\bar{w}_L + \bar{w}_g}$$

Calculate $\frac{d_B}{d}$:

$$\frac{d_B}{d} = \frac{0.0833(\text{ft})}{d\ (\text{ft})}$$

Compute $M_T = \frac{w_T}{A_p}$.

From Fig. B.18 (Appendix) obtain μ_g.

Compute $(GR)^{0.5}\left(\frac{d_B}{d}\right)^{1.25}\frac{M_T d}{\mu_g}$.

(12) From Fig. 2.143 obtain $f(LR)^{0.1}$.

(13) $LR = \frac{\bar{w}_L}{\bar{w}_L + \bar{w}_g}$

(14) $f = \frac{f(LR)^{0.1}}{(LR)^{0.1}}$

(15) Calculate ΔL

$$\Delta L = \frac{2\,g_c\,d}{W_T + v_m{}'^2\,f}\left[144\,\Delta P\left(\frac{\bar{w}_L}{L}+\frac{\bar{w}_g}{g}\right)\right]$$

If ΔL calculated is the same as the actual ΔL, then the assumed d is the ture diameter, or the diameter may be solved for directly, where:

$$d = \frac{w_T\,\bar{v}_m{}^2\,f\,\Delta L}{2\,g_c\left[144\,(\Delta P)\left(\frac{\bar{w}_L}{\rho_L}+\frac{\bar{w}_g}{\rho_g}\right)\right]}$$

(16) Check d solved against the assumed d and if not close, repeat the procedure.

D.86 EXAMPLE PROBLEM TO DETERMINE LINE SIZE BY METHOD OF EATON, ET AL.

Given: It is known that 4000 bpd of 56°API distillate and 100,000,000 scf/day of 0.65 gravity gas are to be moved a distance of 30 miles. Viscosity of distillate = 1.0 cp at average line temperative of 80°F. The upstream pressure is 1,500 psia and it is necessary to reach the end of the line with 1,000 psia.

R_s at 1,500 psia = 750 scf/bbl
R_s at 1,000 psia = 475 scf/bbl
B_o at 1,500 psia = 1.34
B_o at 1,000 psia = 1.21

Determine the necessary line size.

1. $p = \dfrac{1{,}500 + 1{,}000}{2} = 1{,}250$ psia

2. $p > 150$ so neglect kinetic energy (H_L will not have to be determined)

3. Assume diameter = 13 in. $A_p = \dfrac{\pi}{4}\left(\dfrac{13}{12}\right)^2 = 0.920$ ft²

4. $\rho_{L1} = \dfrac{(0.755)\,(62.4) + (69.0/5.61)}{1.34} = 44.3$ lb$_m$/ft³

$q_{L1} = \dfrac{(4000)\,(1.34)\,(5.61)}{86{,}400} = 0.349$ ft³/sec

$v_{sL1} = (0.349)/(0.920) = 0.38$ ft/sec

$\rho_{g1} = (0.65)\,(0.0764)\left(\dfrac{1500}{14.7}\right)\left(\dfrac{520}{540}\right)\left(\dfrac{1}{0.775}\right)$
$= 6.29$ lb$_m$/ft³

$q_{g1} = \dfrac{[10^8 - (4{,}000)\,(750)]}{86{,}400}\left(\dfrac{14.7}{1500}\right)\left(\dfrac{540}{520}\right)(0.755)$
$= 8.64$ ft³/sec

$v_{sg1} = 8.64/(0.920) = 9.38$ ft/sec
$v_{m1} = 0.38 + 9.38 = 9.76$ ft/sec

5. $\rho_{L2} = \dfrac{(0.755)\,(62.4) + (45.9/5.61)}{1.21} = 45.6$ lb$_m$/ft³

$q_{L2} = \dfrac{(4000)\,(1.21)\,(5.61)}{86{,}400} = 0.315$ ft³/sec

$v_{sL2} = (0.315)/(0.920) = 0.342$ ft/sec

$\rho_{g2} = (0.65)\,(0.0764)\left(\dfrac{1{,}000}{14.7}\right)\left(\dfrac{520}{540}\right)\left(\dfrac{1}{0.840}\right)$
$= 3.88$ lb$_m$/ft³

$q_{g2} = \dfrac{[10^8 - (4{,}000)\,(475)]}{86{,}400}\left(\dfrac{14.7}{1000}\right)\left(\dfrac{540}{520}\right)(0.840)$
$= 14.55$ ft³/sec

$v_{sg2} = 14.55/(0.920) = 15.82$ ft/sec
$v_{m2} = 0.34 + 15.82 = 16.16$ ft/sec

6. For $\bar{P} = 1{,}250$ psia, $\bar{q}_L = 0.330$, $\bar{q}_g = 11.10$ ft³/sec:

$\bar{v}_{sl}' = (0.330)/(0.920) = 0.358$ ft/sec
$\bar{v}_{sg}' = 11.10/(0.920) = 12.08$ ft/sec
$\bar{v}_m' = 0.36 + 12.08 = 12.44$ ft/sec
$v_m = \frac{1}{4}\,(v_{m1} + 2\bar{v}_m' + v_{m2})$
$= \frac{1}{4}\,(9.76 + 2\,(12.44) + 16.16) = 12.70$ ft/sec

7. $GR = \dfrac{w_g}{w_T} = \dfrac{56.0}{14.85 + 56.0} = 0.791$

8. $\dfrac{d_B}{d} = \dfrac{1}{13} = 0.0769$

9. $M_T = w_T/A_p = \dfrac{14.85 + 56.0}{0.920} = 77.0\ \dfrac{\text{lb}_m}{\text{sec-ft}^2}$

10. $\mu_g = 0.02$

11. $(GR)^{0.5}\left(\dfrac{d_B}{d}\right)^{1.25}\dfrac{M_T d}{\mu_g}$
$= (0.791)^{0.5}\,(0.0769)^{1.25}\,\dfrac{(77.0)\,(13/12)}{(6.72 \times 10^{-4})\,(0.02)}$
$= 2.25 \times 10^5$

12. From Fig. 2.143, $f(LR)^{0.1} = 0.015$

13. $LR = \dfrac{14.85}{14.85 + 56.0} = 0.209$

14. $f = \dfrac{0.015}{(0.209)^{0.1}} = 0.0176$

15. $d = \dfrac{w_T \bar{v}_m^{\,2}\, f\, \Delta L}{2g_c\left[144\,(\Delta p)\left(\dfrac{w_L}{\rho_L} + \dfrac{w_g}{\rho_g}\right)\right]}$
$= \dfrac{(14.85 + 56.0)\,(12.7)^2\,(0.0176)\,(30)\,(5280)}{(2)\,(32.2)\left[(144)\,(500)\left(\dfrac{14.85}{45.0} + \dfrac{56.0}{5.05}\right)\right]}$
$= \dfrac{(70.8)\,(161.5)\,(0.0176)\,(30)\,(5280)}{(2)\,(32.2)\,[8.21 \times 10^5]}$
$= 0.601$ ft $= \underline{7.21 \text{ in.}}$

16a. *Second trial:*

Assume $d = 13\left(\dfrac{7.21}{13}\right)^{1/5} = (13)\,(0.864) = 11.55$ in.
$= 0.9628$ ft

$A_p = \frac{\pi}{4}\,(0.963)^2 = 0.728$ ft²

$\bar{v}_m = \dfrac{1}{4}\,(v_{m1} + 2\bar{v}_m{}^1 + v_{m2})$

$= \dfrac{1}{4}\,[(q_{L1} + q_{g1}) + 2(\bar{q}_L + \bar{q}_g) + (q_{L2} + q_{g2})]/A_p$

$= \dfrac{11.68}{0.728} = 16.04$ ft/sec

$\dfrac{d_B}{d} = \dfrac{1}{11.22} = 0.0890$

$M_T = w_T/A_p = \dfrac{70.8}{0.688} = 103$

$(GR)^{0.5}\left(\dfrac{d_B}{d}\right)^{1.25}\dfrac{M_T d}{\mu_g} = (0.791)^{0.5}\,(0.0890)^{1.25}\,\dfrac{(103)\,(0.936)}{(6.72 \times 10^{-4})\,(0.02)}$
$= 3.06 \times 10^5$

From Fig. 2.143, $f(LR)^{0.1} = 0.014$

$f = \dfrac{0.015}{(0.209)^{0.1}} = 0.0165$

$d = \dfrac{(70.8)\,(17.0)^2\,(0.0165)\,(30)\,(5280)}{(2)\,(32.2)[(144)\,(500)\left[\dfrac{14.85}{45.0} + \dfrac{56.0}{5.05}\right]}$

$= (0.212)\,(289)\,(0.0165)$
$= 1.01$ ft $= 12.12$ in.

16b. *Third trial:*

Assume $d = 11.22\left(\dfrac{12.12}{11.22}\right)^{1/5} = 11.45$ in. $= 0.950$ ft

$A_p = \dfrac{\pi}{4}\,(0.955)^2 = 0.708$ ft²

$\bar{v}_m = \dfrac{11.68}{0.716} = 16.3$ ft/sec

use $f = 0.0165$
$d = (0.212)\,(16.3)^2\,(0.0165)$
$= 0.930$ ft $= 11.2$ in.

Final $d = \dfrac{1}{2}(11.45 + 11.2) = \underline{11.32 \text{ in.}}$

Appendix D.9: Method of Beggs and Brill

1 PROCEDURE FOR CALCULATING A PRESSURE TRAVERSE IN A HORIZONTAL PIPELINE BY THE METHOD OF BEGGS AND BRILL

The pressure gradient equation is:

$$\frac{\Delta p}{\Delta z} = \frac{\frac{g}{g_c}\rho_{tp}\sin\theta + \frac{f_{tp}\,G_m\,v_m}{2g_c d}}{1 - \frac{\rho_{tp}\,v_m\,v_{sg}}{g_c p}}$$

ere: $\rho_{tp} = \rho_L H_L + \rho_g (1 - H_L)$ and $\sin\theta = 0$ for horizontal v.

Beginning with the known pressure p_1, estimate a value for pressure drop Δp.

Calculate the average pressure in the interval:

$$\bar{p} = p_1 + \frac{\Delta p}{2} \text{ if } p_1 \text{ is downstream pressure;}$$

$$\bar{p} = p_1 - \frac{\Delta p}{2} \text{ if } p_1 \text{ is upstream pressure.}$$

From PVT analysis or appropriate correlations, calculate: R_s, B_o, B_w, μ_o, μ_w, μ_g, σ_o, σ_w, and Z_g at $\bar{T}$ and $\bar{p}$.

Calculate the specific gravity of the oil γ_o

$$\gamma_o = \frac{141.5}{131.5 + API}$$

Calculate the liquid and gas densities at the average conditions of pressure and temperature:

$$\rho_L = \rho_o\left(\frac{1}{1+WOR}\right) + \rho_w\left(\frac{WOR}{1+WOR}\right) = \rho_o f_o + \rho_w f_w$$

$$\rho_o = \frac{(350\,\gamma_o + 0.0764\,R_s\,\gamma_g)}{5.615\,B_o}$$

$$\rho_w = \frac{350\,\gamma_w}{5.615\,B_w}$$

$$\rho_g = \frac{0.0764\,\gamma_g\,\bar{p}\,(520)}{(14.7)(\bar{T}+460)\,Z_g}$$

Calculate the in situ gas and liquid flow rates

$$q_g = \frac{3.27\times10^{-7}\,Z_g\,q_o\,(R - R_s)(\bar{T}+460)}{\bar{p}}$$

$$q_L = 6.49\times10^{-5}\,(q_o B_o + q_w B_w)$$

Calculate the in situ superficial gas, liquid, and mixture velocities:

$$v_{sL} = q_L/A_p$$
$$v_{sg} = q_g/A_p$$
$$v_m = v_{sL} + v_{sg}$$

Calculate the liquid, gas, and total mass flux rates:

$$G_L = \rho_L v_{sL} \qquad G_g = \rho_g v_{sg}$$
$$G_m = G_L + G_g$$

Calculate the input liquid content (no-slip holdup).

$$\lambda = \frac{q_L}{q_L + q_g}$$

Calculate the Froude number, N_{FR}, the liquid viscosity, the mixture viscosity, μ_m and the liquid surface tension, σ_L

$$N_{FR} = \frac{v_m^2}{gd} \qquad \mu_L = \mu_o f_o + \mu_w f_w$$
$$\mu_m = [(\mu_L\,\lambda + \mu_g(1-\lambda)](6.72\times10^{-4})$$
$$\sigma_L = \sigma_o f_o + \sigma_w f_w$$

11. Calculate the no-slip Reynolds number and the liquid velocity number.

$$N_{Rens} = \frac{G_m d}{\mu_m} \qquad N_{Lv} = 1.938\,v_{sL}\left(\frac{\rho_L}{\sigma_L}\right)^{0.25}$$

12. To determine the flow pattern which would exist if flow were horizontal, calculate the correlating parameters, L_1, L_2, L_3, and L_4.

$$L_1 = 316\,\lambda^{0.302} \qquad L_2 = 0.0009252\,\lambda^{-2.4684}$$
$$L_3 = 0.10\,\lambda^{-1.4516} \qquad L_4 = 0.5\,\lambda^{-6.738}$$

13. Determine flow pattern using the following limits:

Segregated:

$\lambda < 0.01$ and $N_{FR} < L_1$

or $\lambda \geq 0.01$ and $N_{FR} < L_2$

Transition:

$\lambda \geq 0.01$ and $L_2 < N_{FR} \leq L_3$

Intermittent:

$0.01 \leq \lambda < 0.4$ and $L_3 < N_{FR} \leq L_1$

or $\lambda \geq 0.4$ and $L_3 < N_{FR} \leq L_4$

Distributed:

$\lambda < 0.4$ and $N_{FR} \geq L_1$

or $\lambda \geq 0.4$ and $N_{FR} > L_4$

14. Calculate the horizontal holdup, H_L (O). If the flow pattern is transition, interpolation is required between segregated and intermittent values.

$H_L(O) = \frac{a\,\lambda^b}{N_{FR}^c}$ where a, b, and c are determined for each flow pattern from the table:

Flow pattern	a	b	c
Segregated	0.98	0.4846	0.0868
Intermittent	0.845	0.5351	0.0173
Distributed	1.065	0.5824	0.0609

15. Calculate the two-phase density.

$$p_{tp} = p_L H_L + p_g (1 - H_L)$$

16. Calculate the friction factor ratio.

$\frac{f_{tp}}{f_{ns}} = e^s$, where

$$S = [\ln(y)]/\{-0.0523 + 3.182\ln(y) - 0.8725[\ln(y)]^2 + 0.01853[\ln(y)]^4\}.$$

and

$$y = \frac{\lambda}{[H_L(\theta)]^2}$$

S becomes unbounded at a point in the interval $1 < y < 1.2$; and for y in this interval, the function S is calculated from:

$$S = \ln(2.2y - 1.2)$$

17. Calculate the no-slip friction factor,

$$f_{ns} = 1/[2\log(N_{Rens}/(4.5223\log N_{Rens} - 3.8215))]^2 \quad \text{or}$$

$$f_{ns} = 0.0056 + \frac{0.5}{(N_{Rens})^{0.32}}$$

18. Calculate the two-phase friction factor.

$$f_{tp} = f_{ns}\frac{f_{tp}}{f_{ns}}$$

19. Calculate
$$\Delta p = \frac{\Delta Z\left(\frac{f_{tp}\, G_m\, v_m}{2g_c d}\right)}{1 - \frac{\rho_{tp}\, v_m\, v_{sg}}{g_c\, p}}$$

20. If the pressure drop (or gain) estimated in Step 1 and that calculated in Step 21 are not sufficiently close, use the value calculated in Step 21 as the new estimated Δp and go back to Step 2. This procedure is repeated until the estimated and calculated values of Δp agree. The pressure at $z \pm \Delta z$ is then $p_1 \pm \Delta p$.

D.92 EXAMPLE PROBLEM TO DETERMINE PRESSURE LOSS IN A 4-IN. HORIZONTAL LINE BY THE METHOD OF BEGGS AND BRILL

Available data:

$q = 2{,}000$ stk bpd
$\bar{T} = 120°F$
Length of line = 3,000 ft
Gas oil ratio = 1,000 scf/stk bbl
$\gamma_g = 0.65$
$\gamma_o = 42°$API
Upstream pressure = 500 psig
$\sigma_o = 30$ dynes/cm
$\mu_o = 1$ cp
$\mu_g = 0.02$ cp

Calculate the downstream pressure

1. Estimate $\Delta p = 100$ psi

2. $\bar{P} = P_1 - \frac{\Delta p}{2} = 500 - 50 = 450$

3. $R_s = 100$, $B_o = 1.065$, $Z = 0.94$

4. $\gamma_o = \frac{141.5}{131.5 + 42} = 0.82$

5. $\rho_o = \frac{350\,(.82) + 0.0764\,(100)\,(.65)}{(5.615)\,(1.065)} = 48.82\ lb_m/ft^3$

$$\rho_g = \frac{(0.0764)\,(\gamma_g)\,(\bar{P})\,(520)}{(14.7)\,(T + 460)\,z_g} = \frac{0.0764\,(0.65)\,(464.7)\,(520)}{14.7\,(120 + 460)\,(0.94)} = 1.50\,\frac{lb_m}{ft^2}$$

6. $q_g = \frac{3.27 \times 10^{-7}\,(0.94)\,(2{,}000)\,(1{,}000 - 100)\,(580)}{464.7} = 0.69\ ft^3/sec$

$q_L = 6.49 \times 10^{-5}\,(2{,}000)\,(1.065) = 0.138\ ft^3/sec$

7. $A_p = \frac{\pi}{4}d^2 = 0.785\left(\frac{4}{12}\right)^2 = 0.0872\ ft^2$

$V_{sL} = \frac{q_L}{Ap} = \frac{0.138}{0.0872} = 1.58$ ft/sec

$V_{sg} = \frac{q_g}{A_p} = \frac{0.69}{0.0872} = 7.91$ ft/sec

$V_m = 1.58 + 7.91 = 9.49$ ft/sec

8. $G_L = \rho_L V_{sL} = 48.82\,(1.58) = 77.13$
$G_g = \rho_g V_{sg} = 1.50\,(7.91) = 11.87$
$G_m = G_L + G_g = 77.13 + 11.87 = 89\ lb_m/sec\text{-}ft^2$

9. $\lambda = \frac{q_L}{q_L + q_g} = \frac{0.138}{0.138 + 0.69} = 0.167$

10. $N_{FR} = \frac{(9.49)^2}{32.2\left(\frac{4}{12}\right)} = 8.39$

$\mu_m = 6.72 \times 10^{-4}\,(1(0.167) + 0.02\,(1 - 0.167) = 0.00012\,\frac{lb_m}{ft\text{-}sec}$

11. $N_{Rens} = \frac{89\,(0.333)}{0.00012} = 2.47 \times 10^5$

$N_{LV} = 1.938\,(1.58)\left(\frac{48.82}{30}\right)^{0.25} = 3.458$

12. $L_1 = 316\,(0.167)^{0.302} = 184$
$L_2 = 0.0009252\,(0.167)^{-2.4684}$
$L_2 = 0.077$
$L_3 = 0.10\,(0.167)^{-1.4156} = 1.26$
$L_4 = 0.5\,(0.167)^{-6.738} = 86{,}358$

13. Since $0.01 \leq \lambda \leq 0.4$ and $L_3 < N_{FR} \leq L_1$, flow pattern is intermittent.

14. $H_L = 0.845\,(0.167)^{0.5351}\,(8.39)^{-0.0173} = 0.313$

15. $\rho_{tp} = 48.82\,(0.313) + 1.50\,(1 - 0.313) = 16.31\ lb_m/ft^3$

16. $y = \frac{0.167}{(0.313)^2} = 1.704$ ln $y = 0.5333$

$$S = \frac{0.533}{-0.0523 + 3.182\,(0.533) - 0.8725\,(0.533)^2 + \sqrt{0.01853\,(0.533)^4}} = 1.367$$

$S = 0.381 \qquad \frac{f_{tp}}{f_{ns}} = e^s = 1.464$

17. $f_{ns} = 0.0056 + \frac{0.5}{(2.47 \times 10^5)^{0.32}} = 0.015$

18. $f_{tp} = 0.015\,(1.464) = 0.022$

19. $\Delta p = \frac{3000\left(\frac{0.022\,(89)\,(9.49)}{2(32.2)\,(0.333)}\right)}{1 - \frac{16.31(9.49)(7.91)}{32.2\,(464.7)\,(144)}} = \frac{2600}{0.99943} = 2{,}601\,\frac{lb_f}{ft^2}$

$\Delta p = \frac{2{,}601}{144} = 18$ psi

This is not sufficiently close to the estimated Δp in Step 1. A new estimate of $\Delta p = 18$ psi should be made and the calculations repeated.

Appendix D: References

D.1 Katz, et al., "Handbook of Natural Gas Engineering," McGraw-Hill Book Company, Inc., New York, 1959, pg. 320.

D.2 Moody, L. F., "Friction Factors for Pipe Flow," Trans ASME, 66:671 (1944).

D.3 Weymouth, T. R., "Problems in Natural Gas Engineering," Trans. ASME, 34:185 (1912).

D.4 Brown, G. G., et al., "Unit Operations," John Wiley & Sons, Inc., New York, 1950.

D.5 Cullender, M. H., and C. W. Brinkley, "Adaptation of the Relative Roughness Correlation of the Coefficient of Friction to the Flow of Natural Gas in Gas Well Casings," Report presented to Railroad Commission of Texas, Amarillo, November 9, 1950.

D.6 "Flow of Fluids Through Valves, Fittings, and Pipe," Technical Paper No. 410, Engineering Division, Crane Company, Chicago, 1957.

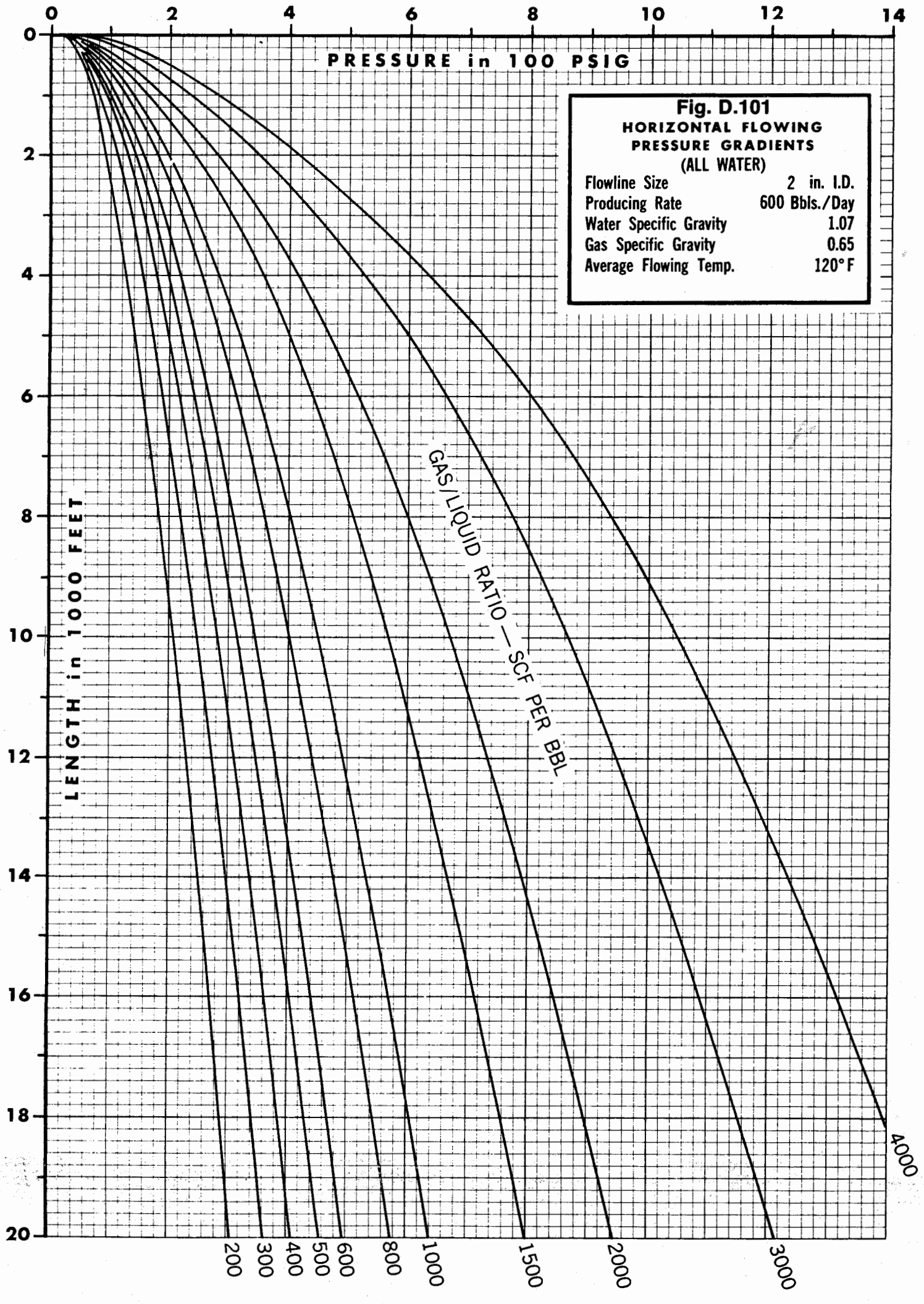


Fig. D.101
HORIZONTAL FLOWING PRESSURE GRADIENTS
(ALL WATER)

Flowline Size	2 in. I.D.
Producing Rate	600 Bbls./Day
Water Specific Gravity	1.07
Gas Specific Gravity	0.65
Average Flowing Temp.	120° F

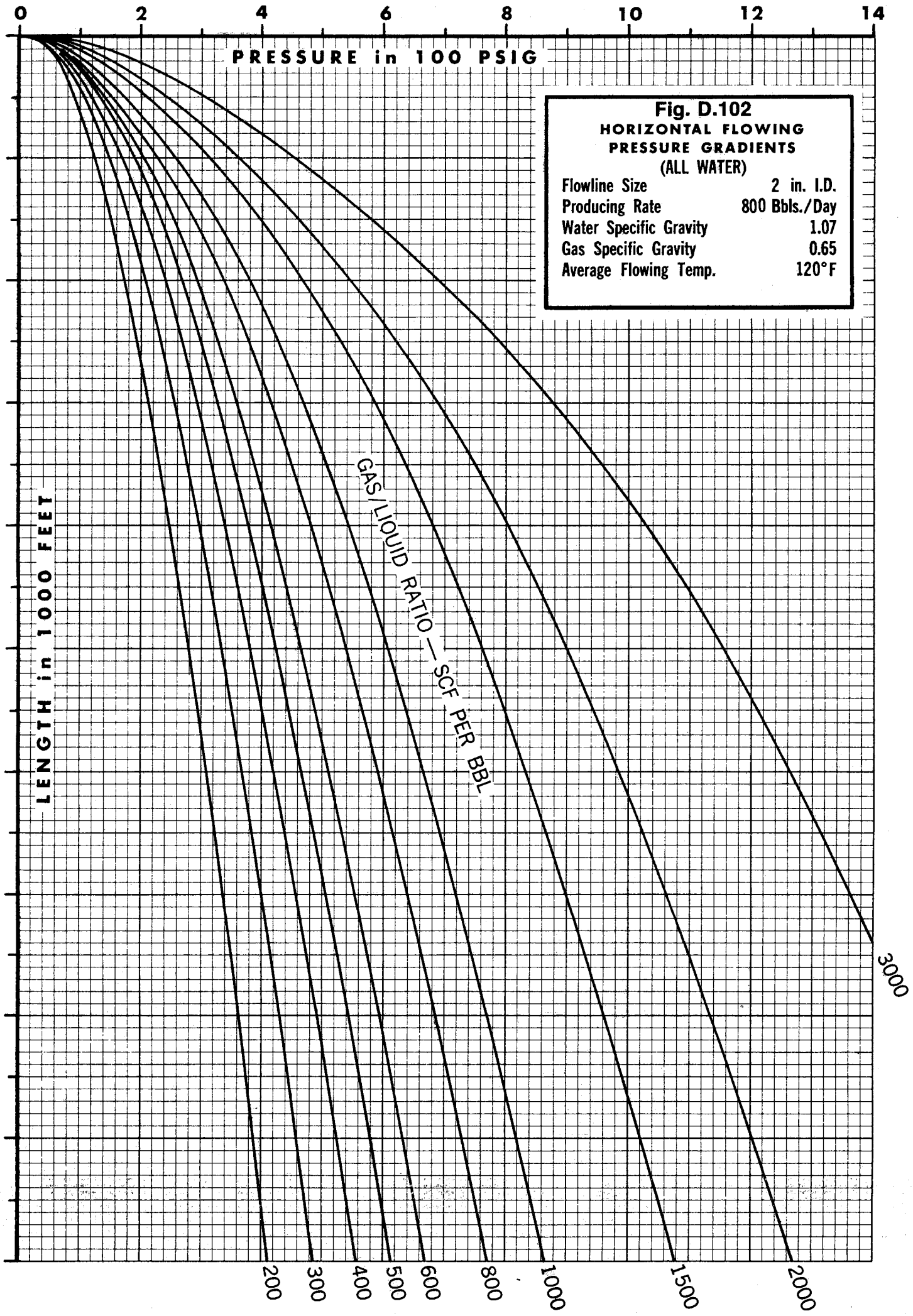
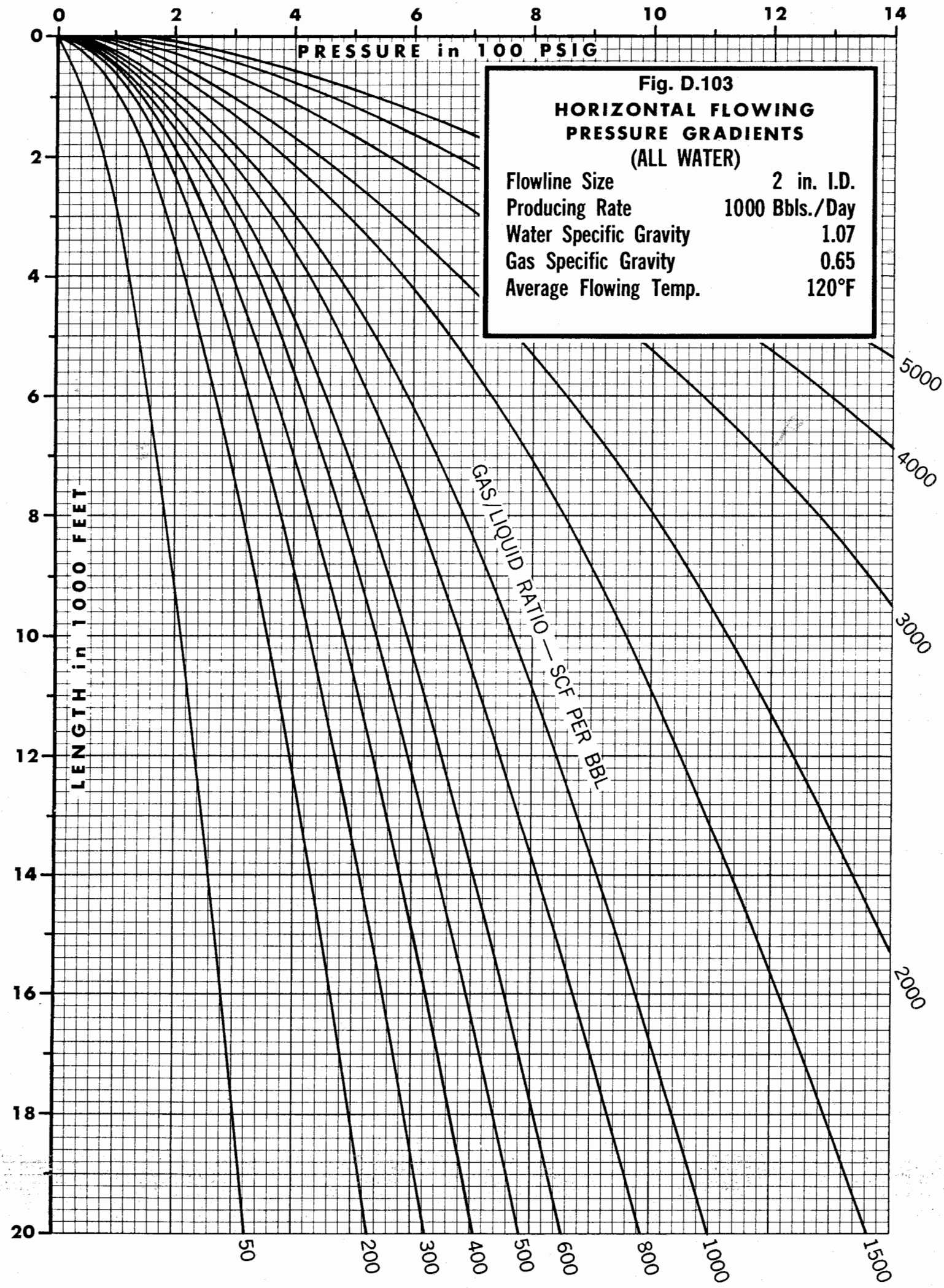


Fig. D.102
HORIZONTAL FLOWING PRESSURE GRADIENTS
(ALL WATER)

Flowline Size	2 in. I.D.
Producing Rate	800 Bbls./Day
Water Specific Gravity	1.07
Gas Specific Gravity	0.65
Average Flowing Temp.	120°F


0
2
4
6
8
10
12
14
PRESSURE in 100 PSIG
Fig. D.103
HORIZONTAL FLOWING
PRESSURE GRADIENTS
(ALL WATER)
Flowline Size 2 in. I.D.
Producing Rate 1000 Bbls./Day
Water Specific Gravity 1.07
Gas Specific Gravity 0.65
Average Flowing Temp. 120°F
LENGTH in 1000 FEET
0
2
4
6
8
10
12
14
16
18
20
GAS/LIQUID RATIO — SCF PER BBL
50
200
300
400
500
600
800
1000
1500
2000
3000
4000
5000

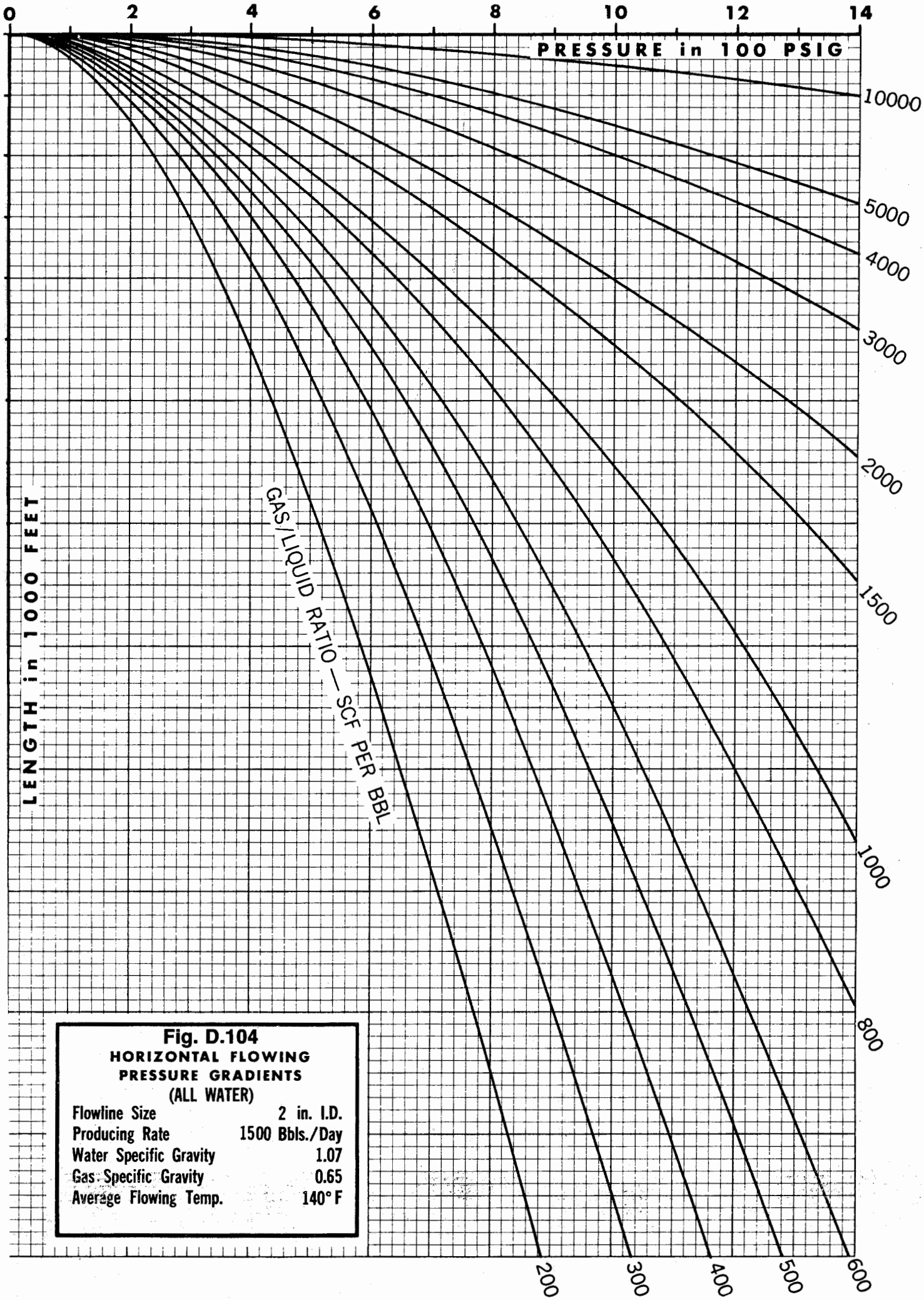


Fig. D.104
HORIZONTAL FLOWING PRESSURE GRADIENTS (ALL WATER)

Flowline Size	2 in. I.D.
Producing Rate	1500 Bbls./Day
Water Specific Gravity	1.07
Gas Specific Gravity	0.65
Average Flowing Temp.	140° F

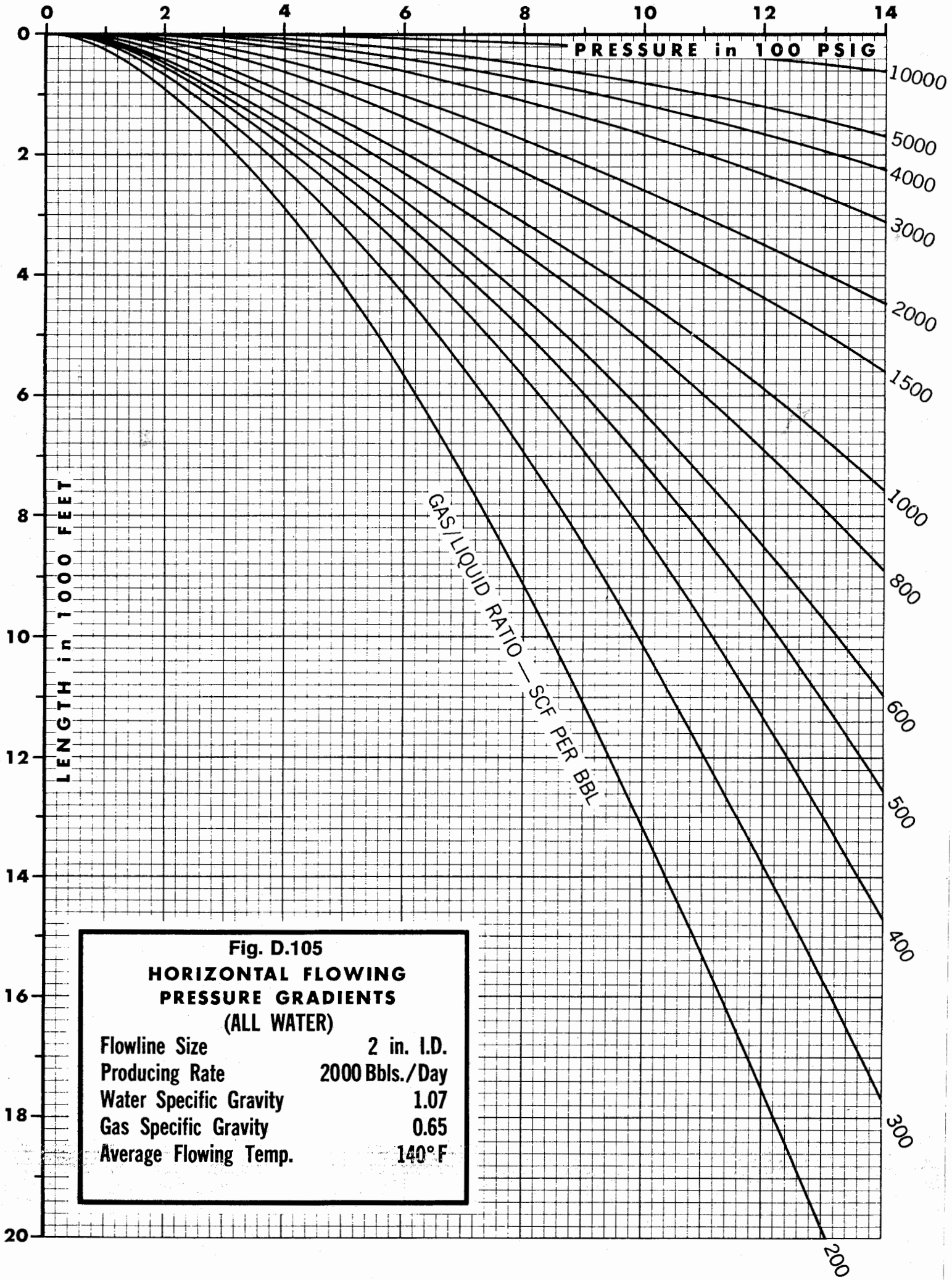


Fig. D.105

HORIZONTAL FLOWING PRESSURE GRADIENTS

(ALL WATER)

Flowline Size	2 in. I.D.
Producing Rate	2000 Bbls./Day
Water Specific Gravity	1.07
Gas Specific Gravity	0.65
Average Flowing Temp.	140° F

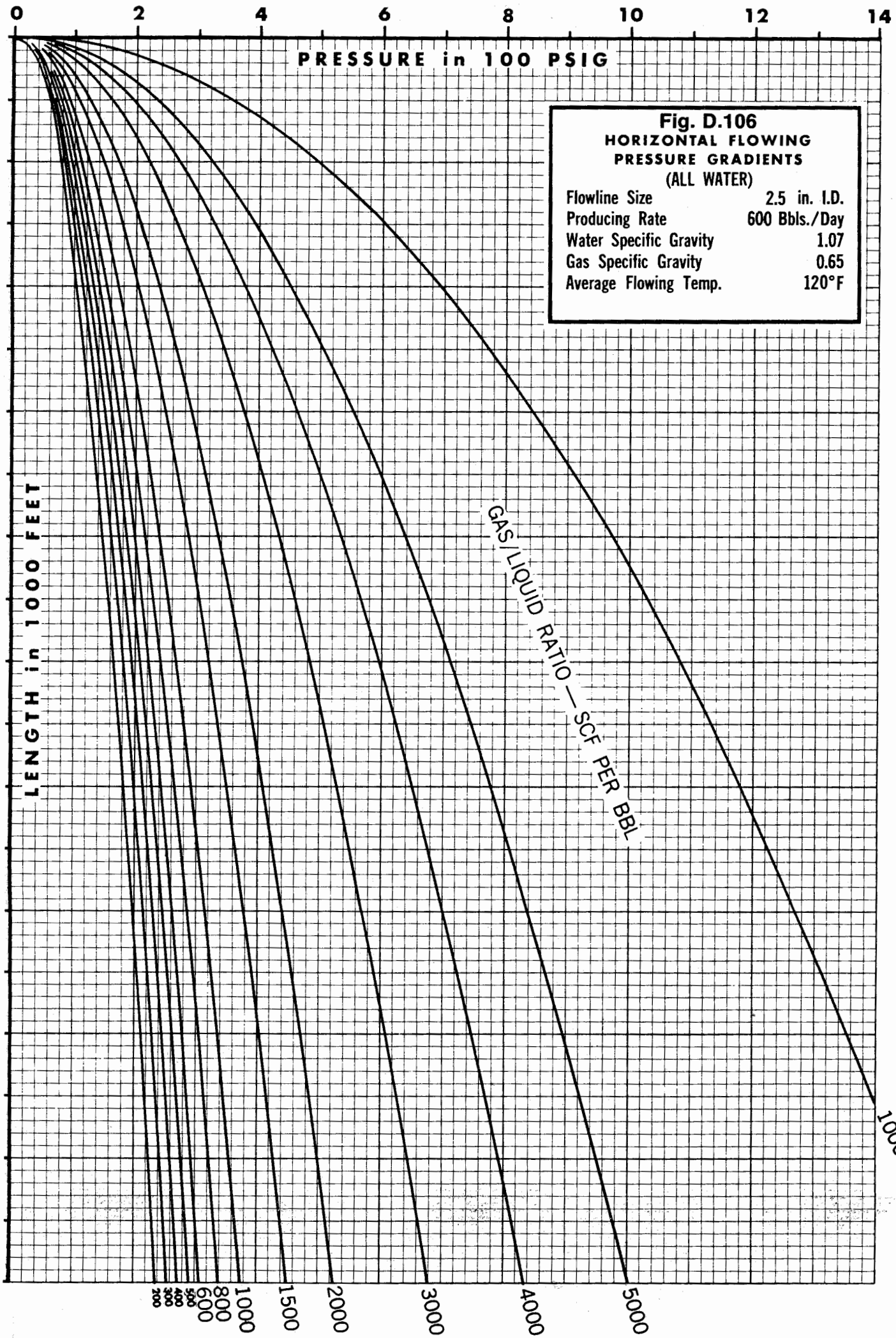
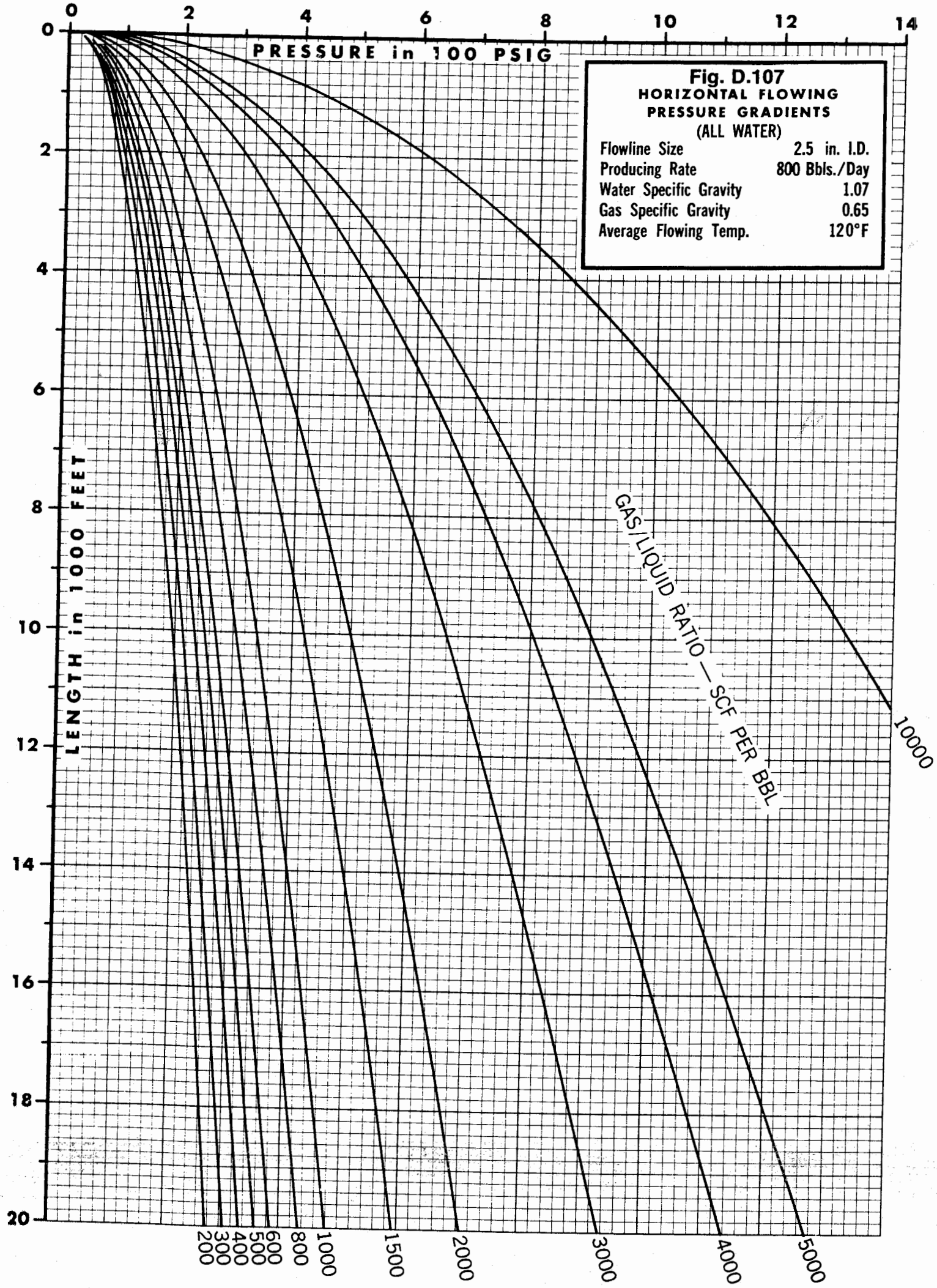
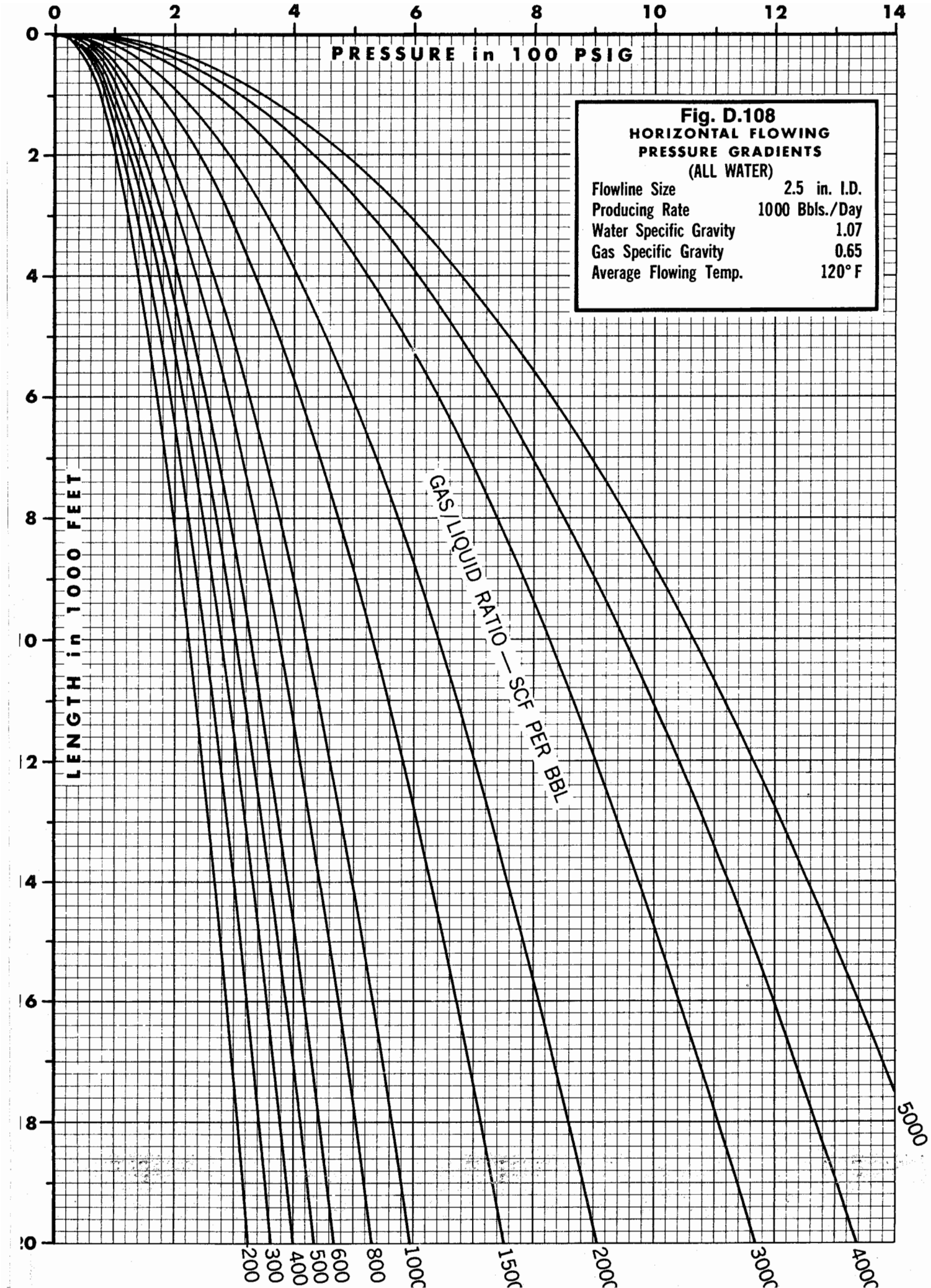


Fig. D.106
HORIZONTAL FLOWING PRESSURE GRADIENTS
(ALL WATER)

Flowline Size	2.5 in. I.D.
Producing Rate	600 Bbls./Day
Water Specific Gravity	1.07
Gas Specific Gravity	0.65
Average Flowing Temp.	120°F


PRESSURE in 100 PSIG
0
2
4
6
8
10
12
14
LENGTH in 1000 FEET
0
2
4
6
8
10
12
14
16
18
20
Fig. D.107
HORIZONTAL FLOWING
PRESSURE GRADIENTS
(ALL WATER)
Flowline Size 2.5 in. I.D.
Producing Rate 800 Bbls./Day
Water Specific Gravity 1.07
Gas Specific Gravity 0.65
Average Flowing Temp. 120°F
GAS/LIQUID RATIO — SCF PER BBL
200
300
400
500
600
800
1000
1500
2000
3000
4000
5000
10000


0
2
4
6
8
10
12
14
PRESSURE in 100 PSIG
LENGTH in 1000 FEET
GAS/LIQUID RATIO — SCF PER BBL
200
300
400
500
600
800
1000
1500
2000
3000
4000
5000
Fig. D.108
HORIZONTAL FLOWING
PRESSURE GRADIENTS
(ALL WATER)
Flowline Size 2.5 in. I.D.
Producing Rate 1000 Bbls./Day
Water Specific Gravity 1.07
Gas Specific Gravity 0.65
Average Flowing Temp. 120° F

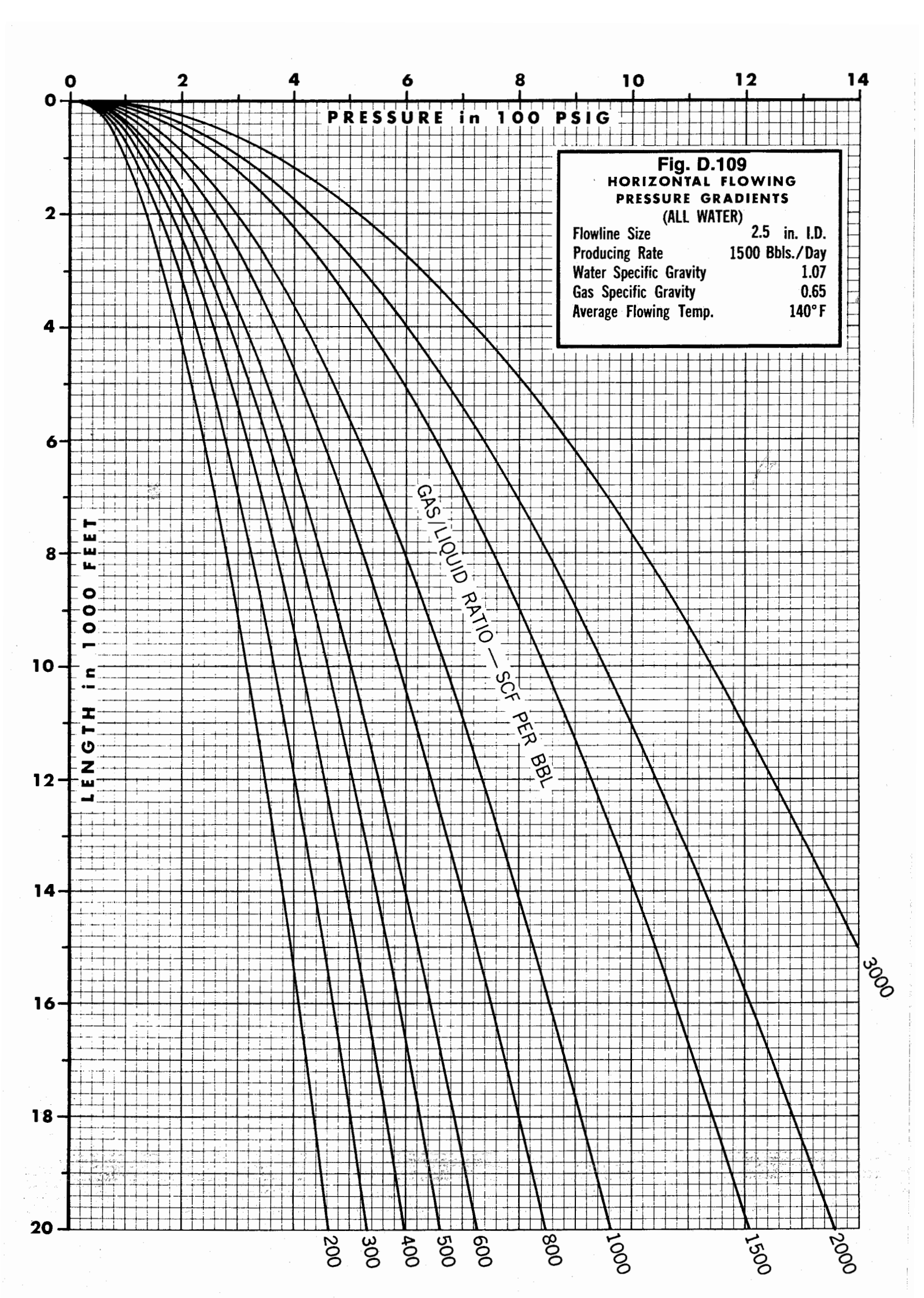


Fig. D.109
HORIZONTAL FLOWING
PRESSURE GRADIENTS
(ALL WATER)
Flowline Size 2.5 in. I.D.
Producing Rate 1500 Bbls./Day
Water Specific Gravity 1.07
Gas Specific Gravity 0.65
Average Flowing Temp. 140° F
PRESSURE in 100 PSIG
LENGTH in 1000 FEET
GAS/LIQUID RATIO — SCF PER BBL
0
2
4
6
8
10
12
14
16
18
20
200
300
400
500
600
800
1000
1500
2000
3000

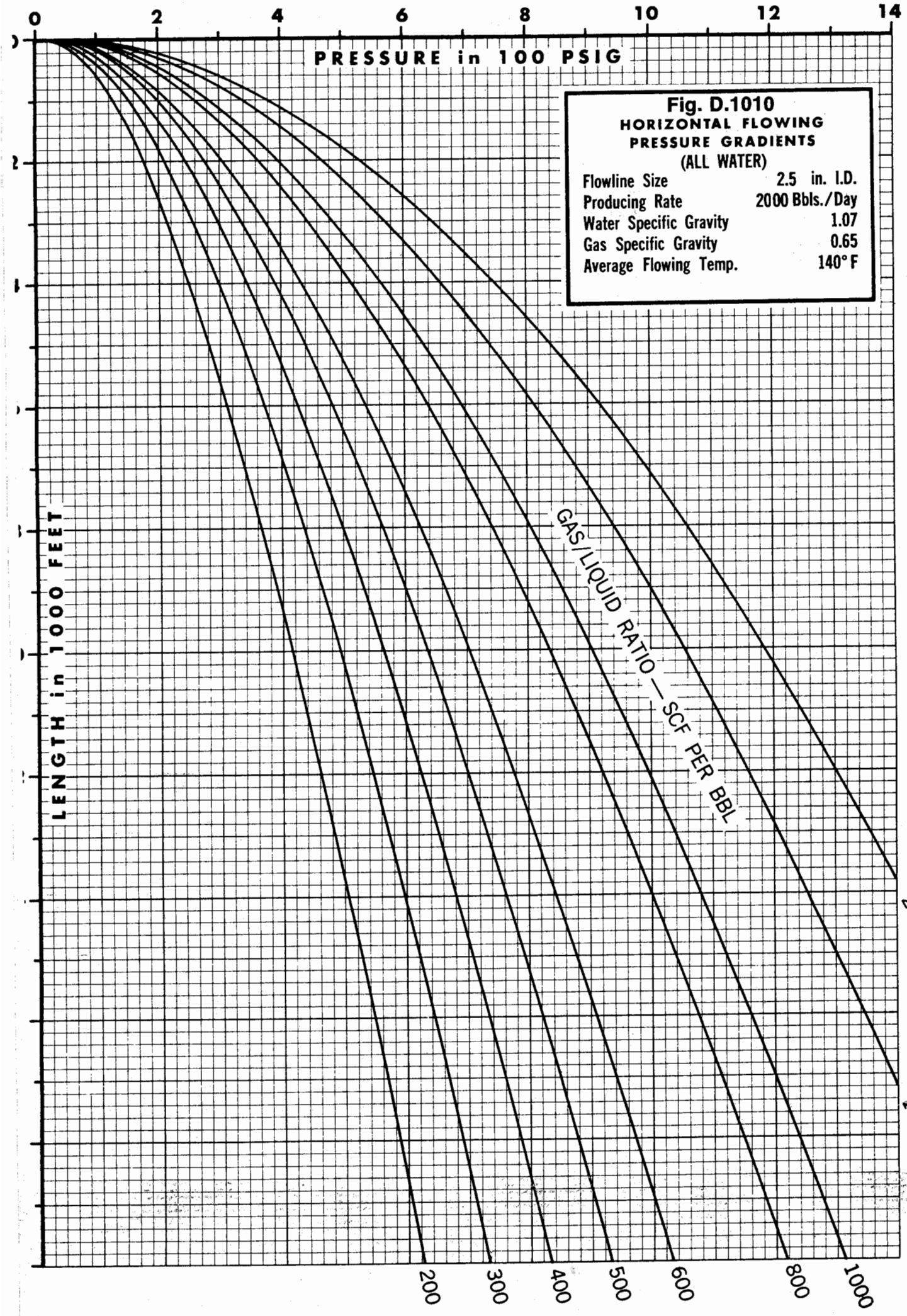


Fig. D.1010
HORIZONTAL FLOWING PRESSURE GRADIENTS
(ALL WATER)

Flowline Size	2.5 in. I.D.
Producing Rate	2000 Bbls./Day
Water Specific Gravity	1.07
Gas Specific Gravity	0.65
Average Flowing Temp.	140° F

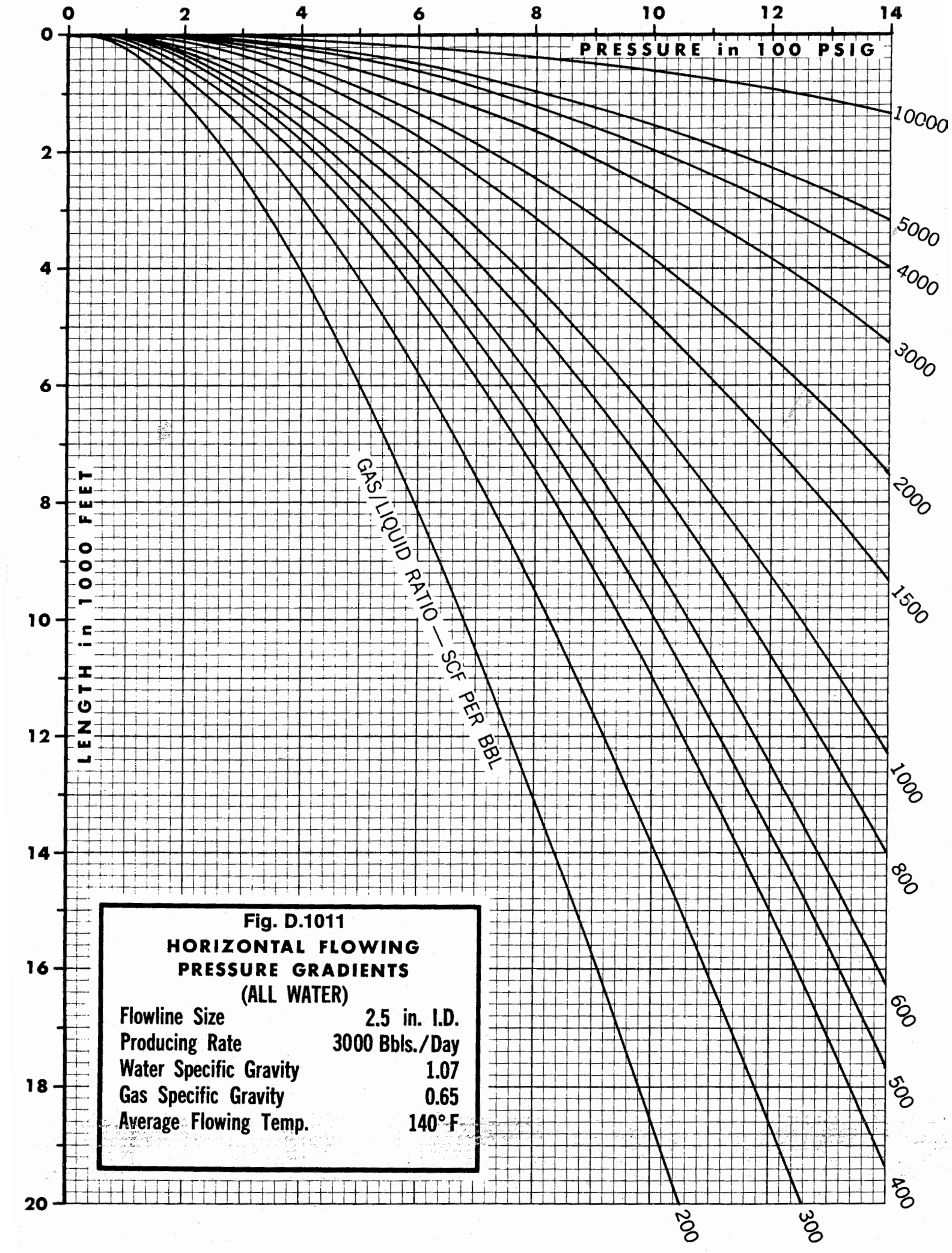


Fig. D.1011

HORIZONTAL FLOWING PRESSURE GRADIENTS (ALL WATER)

Flowline Size	2.5 in. I.D.
Producing Rate	3000 Bbls./Day
Water Specific Gravity	1.07
Gas Specific Gravity	0.65
Average Flowing Temp.	140° F

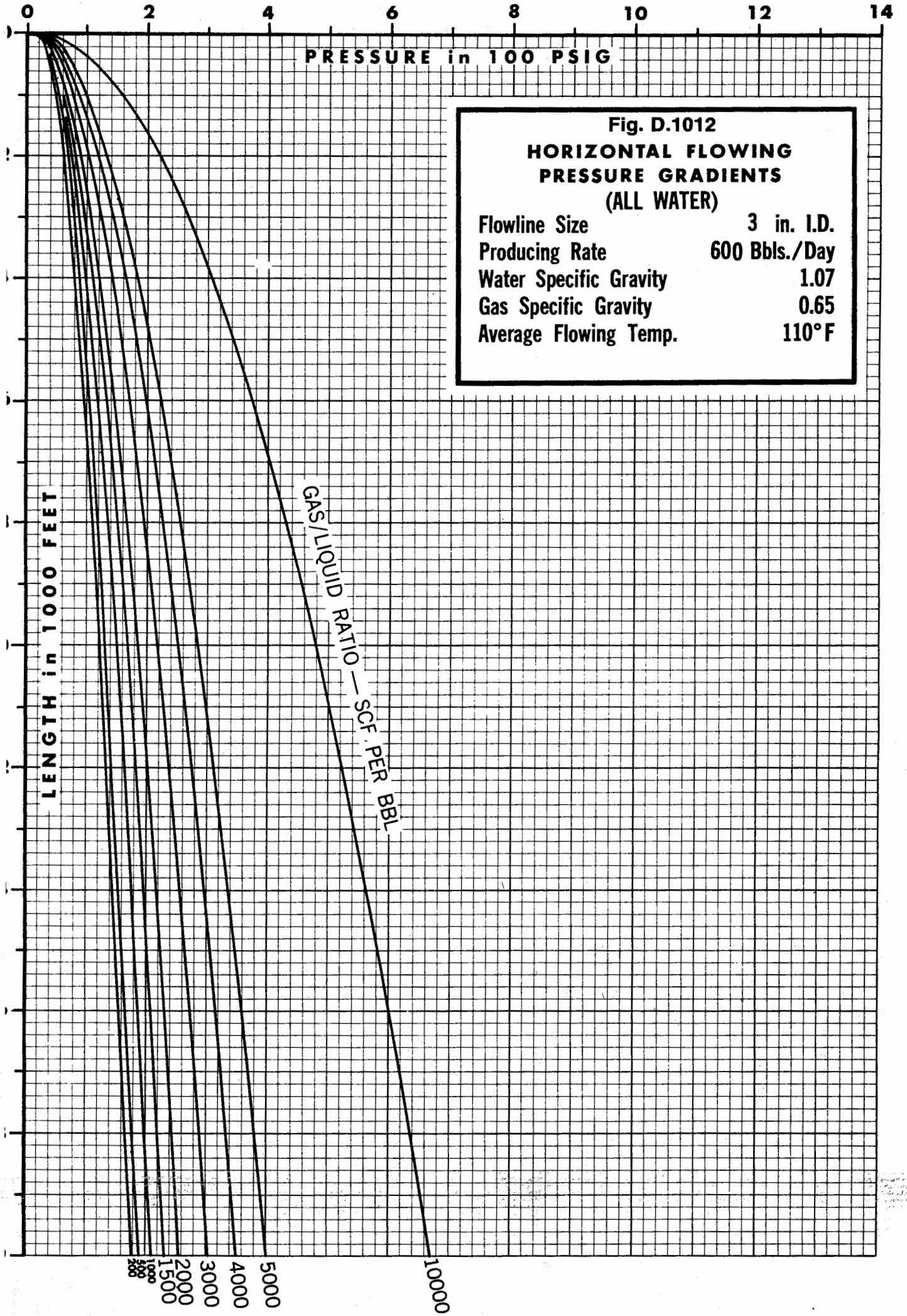


Fig. D.1012

HORIZONTAL FLOWING PRESSURE GRADIENTS

(ALL WATER)

Flowline Size	3 in. I.D.
Producing Rate	600 Bbls./Day
Water Specific Gravity	1.07
Gas Specific Gravity	0.65
Average Flowing Temp.	110° F

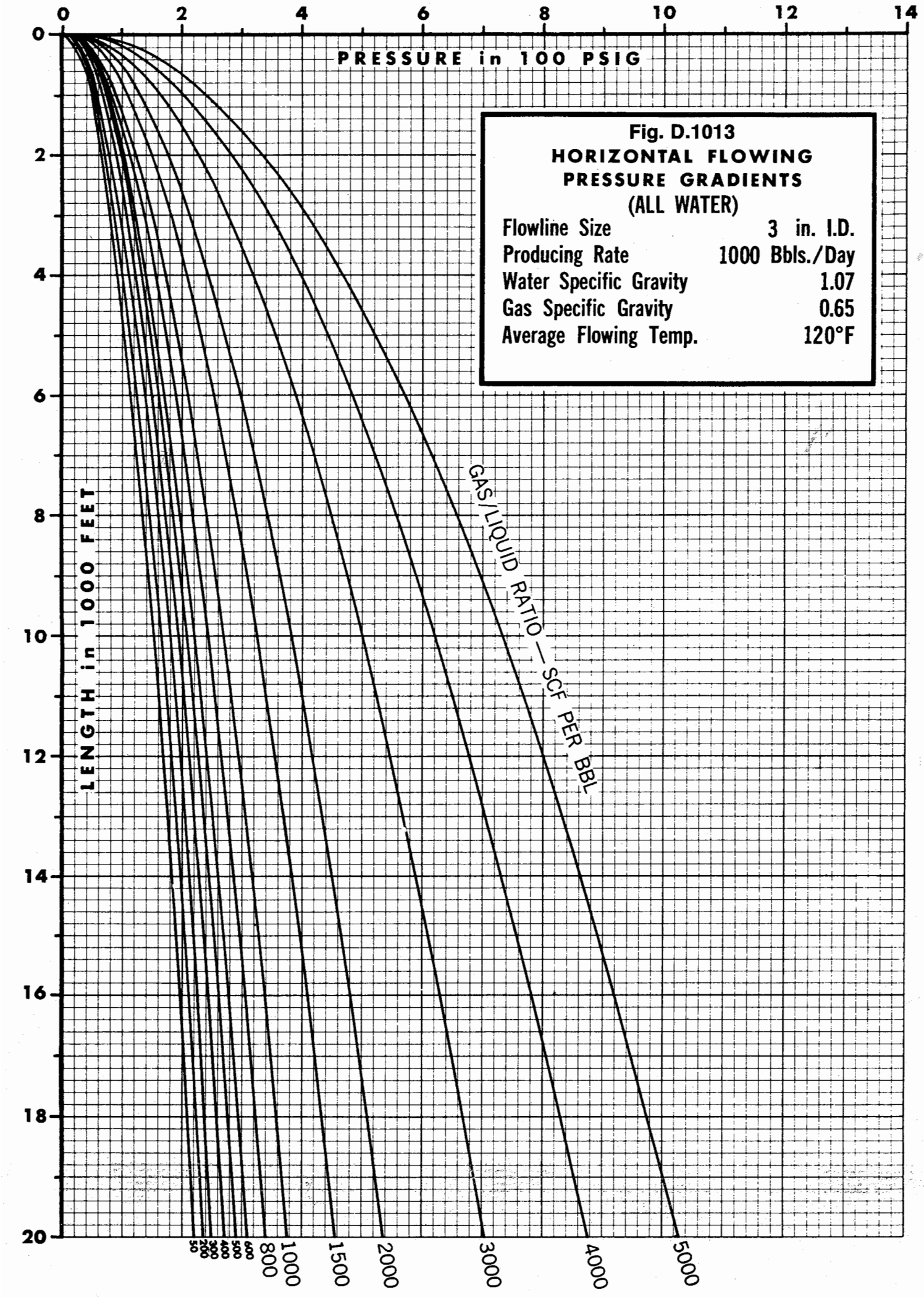


Fig. D.1013
HORIZONTAL FLOWING PRESSURE GRADIENTS
(ALL WATER)

Flowline Size	3 in. I.D.
Producing Rate	1000 Bbls./Day
Water Specific Gravity	1.07
Gas Specific Gravity	0.65
Average Flowing Temp.	120°F

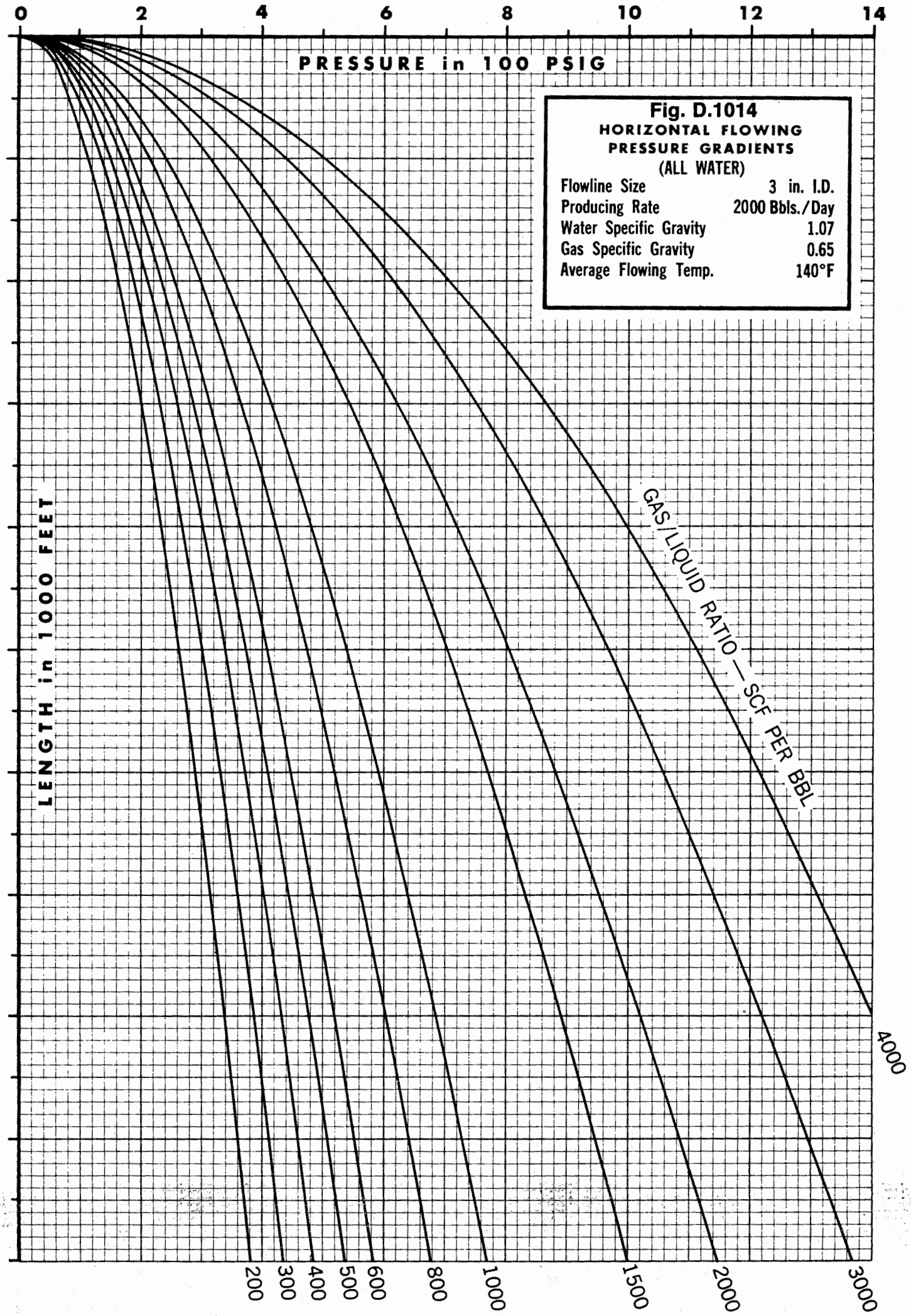
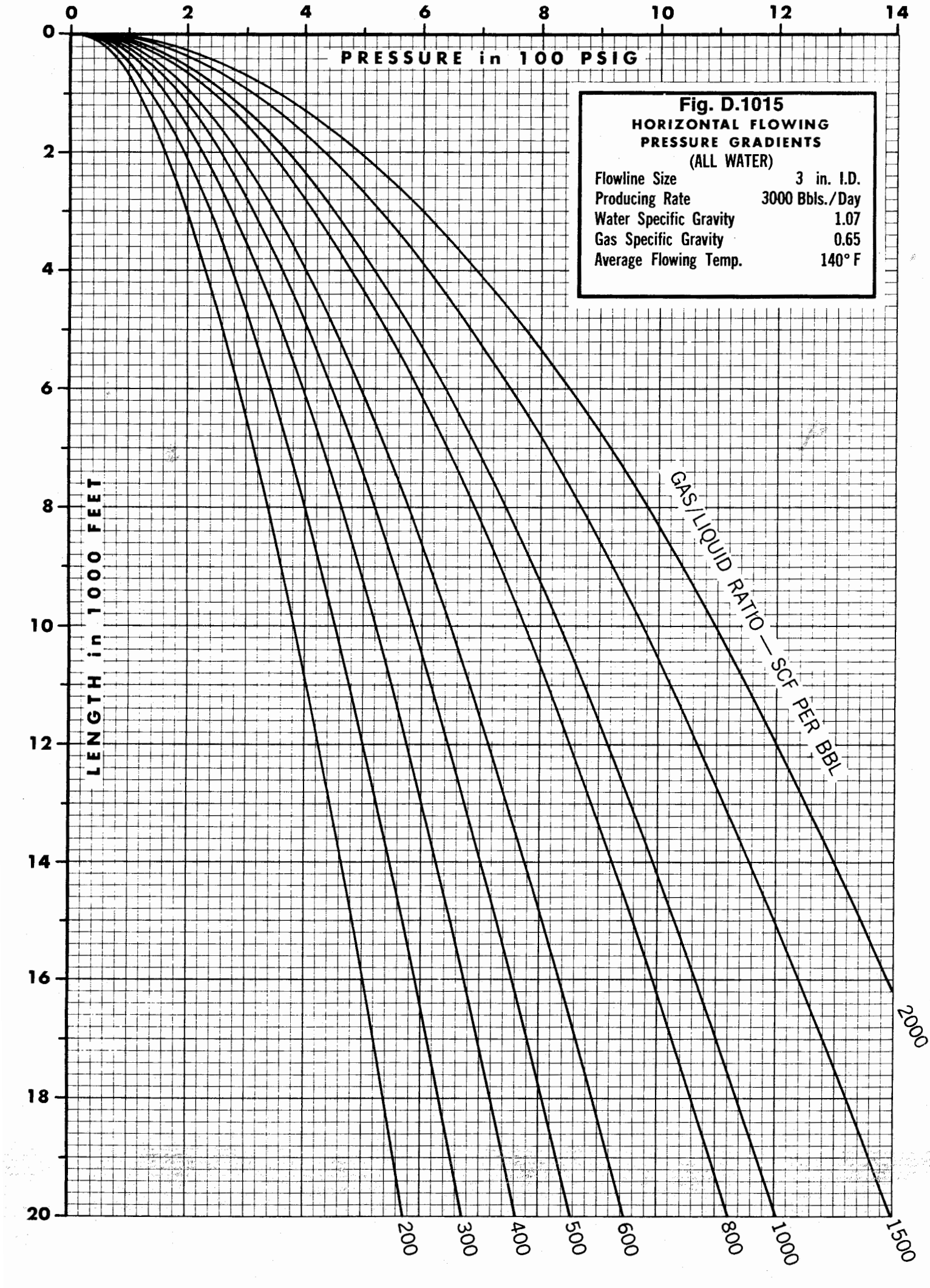


Fig. D.1014
HORIZONTAL FLOWING PRESSURE GRADIENTS
(ALL WATER)

Flowline Size	3 in. I.D.
Producing Rate	2000 Bbls./Day
Water Specific Gravity	1.07
Gas Specific Gravity	0.65
Average Flowing Temp.	140°F


0
2
4
6
8
10
12
14
PRESSURE in 100 PSIG
LENGTH in 1000 FEET
0
2
4
6
8
10
12
14
16
18
20
Fig. D.1015
HORIZONTAL FLOWING
PRESSURE GRADIENTS
(ALL WATER)
Flowline Size 3 in. I.D.
Producing Rate 3000 Bbls./Day
Water Specific Gravity 1.07
Gas Specific Gravity 0.65
Average Flowing Temp. 140° F
GAS/LIQUID RATIO — SCF PER BBL
200
300
400
500
600
800
1000
1500
2000

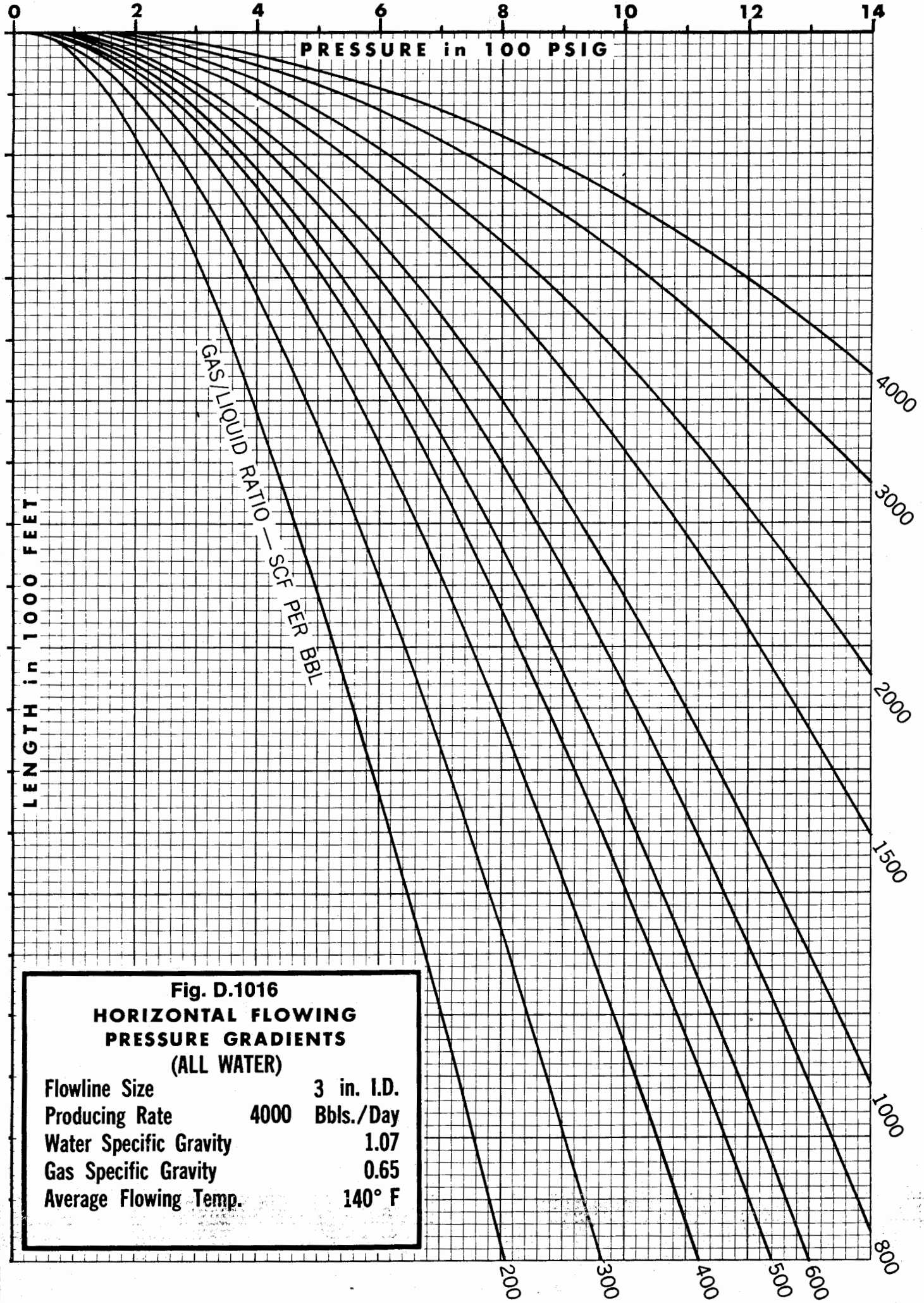


Fig. D.1016
HORIZONTAL FLOWING PRESSURE GRADIENTS
(ALL WATER)

Flowline Size		3 in. I.D.
Producing Rate	4000	Bbls./Day
Water Specific Gravity		1.07
Gas Specific Gravity		0.65
Average Flowing Temp.		140° F

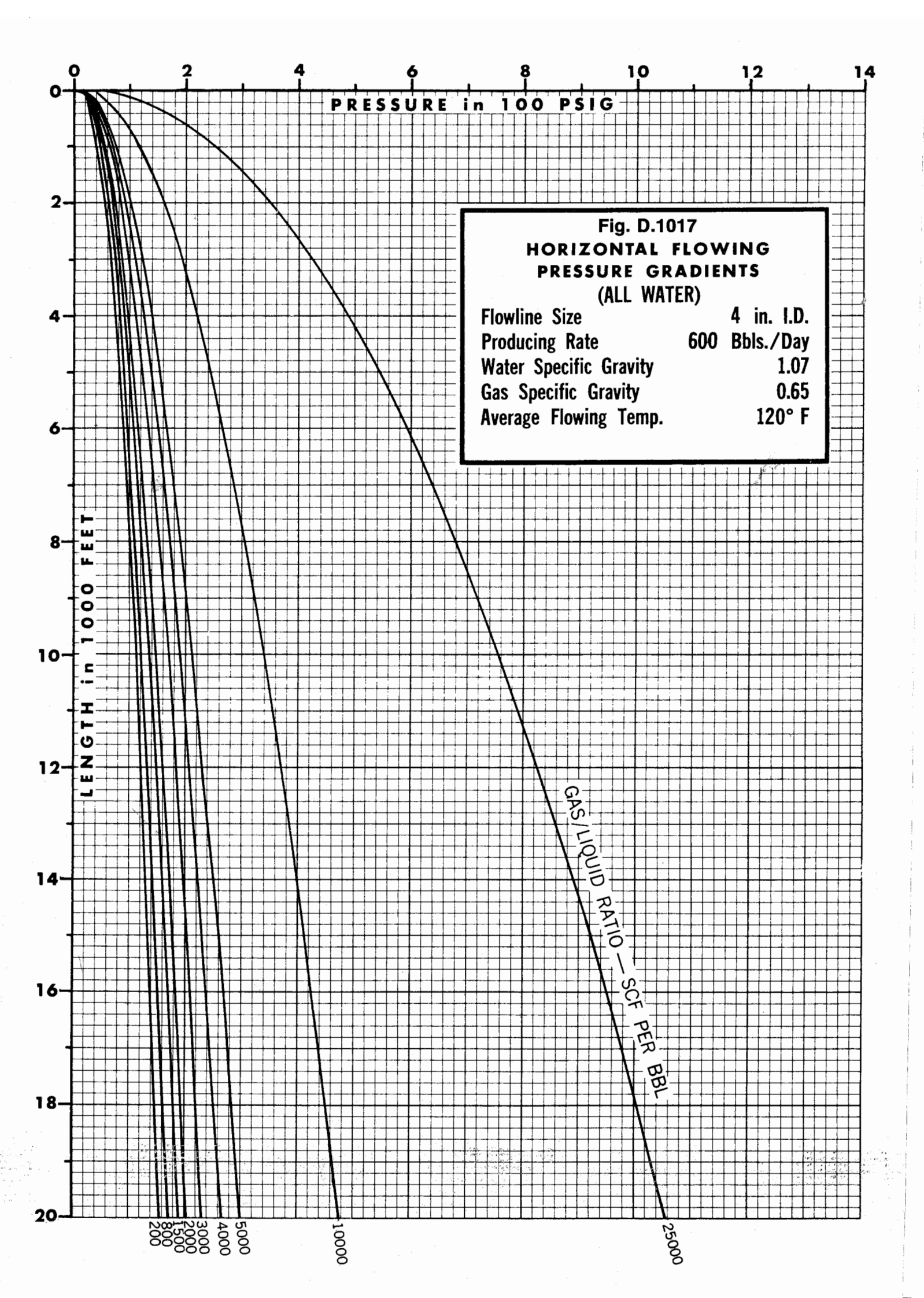


Fig. D.1017
HORIZONTAL FLOWING
PRESSURE GRADIENTS
(ALL WATER)
Flowline Size 4 in. I.D.
Producing Rate 600 Bbls./Day
Water Specific Gravity 1.07
Gas Specific Gravity 0.65
Average Flowing Temp. 120° F
PRESSURE in 100 PSIG
LENGTH in 1000 FEET
GAS/LIQUID RATIO — SCF PER BBL
200
800
1500
2000
3000
4000
5000
10000
25000

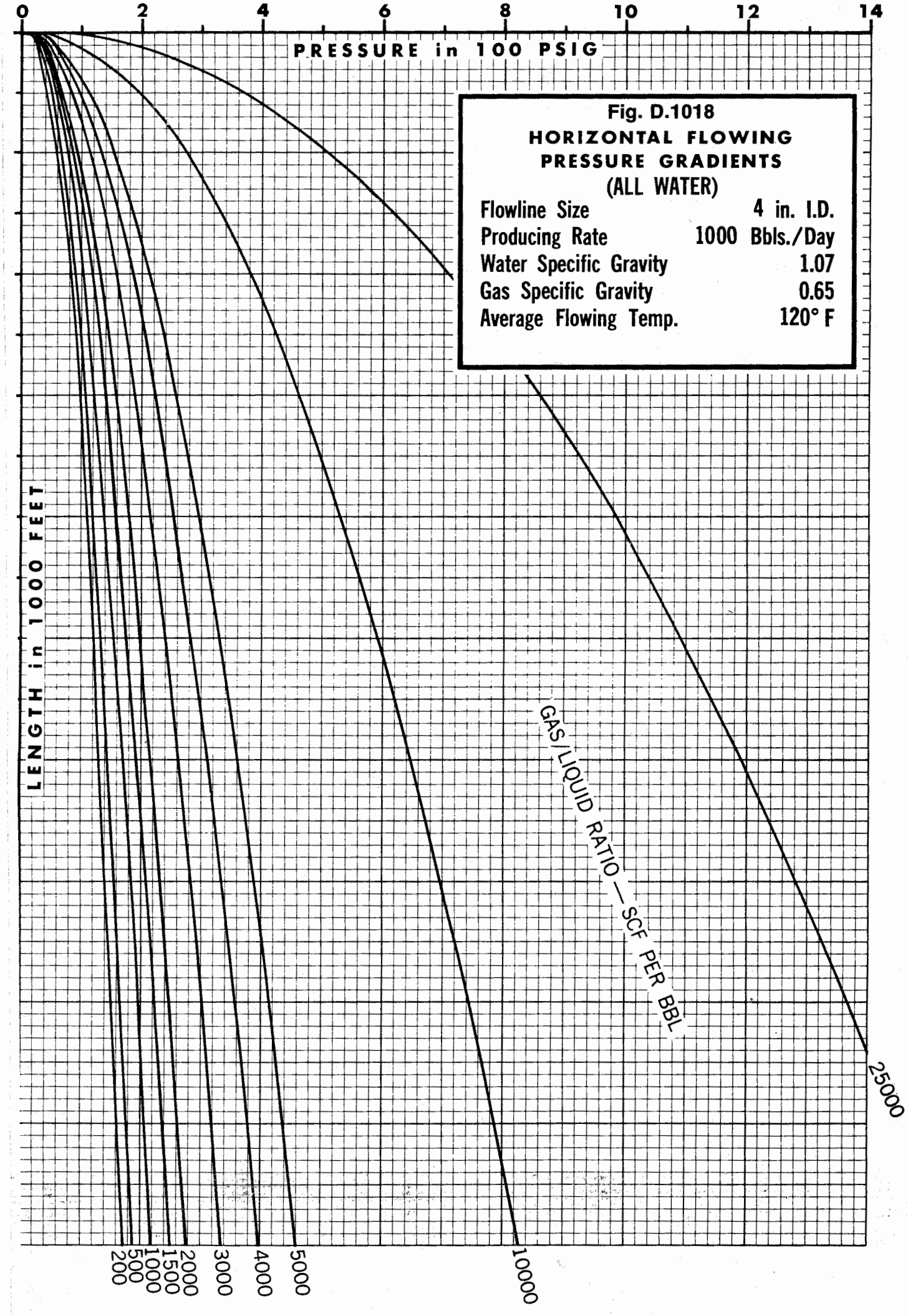


Fig. D.1018
HORIZONTAL FLOWING PRESSURE GRADIENTS
(ALL WATER)

Flowline Size	4 in. I.D.
Producing Rate	1000 Bbls./Day
Water Specific Gravity	1.07
Gas Specific Gravity	0.65
Average Flowing Temp.	120° F

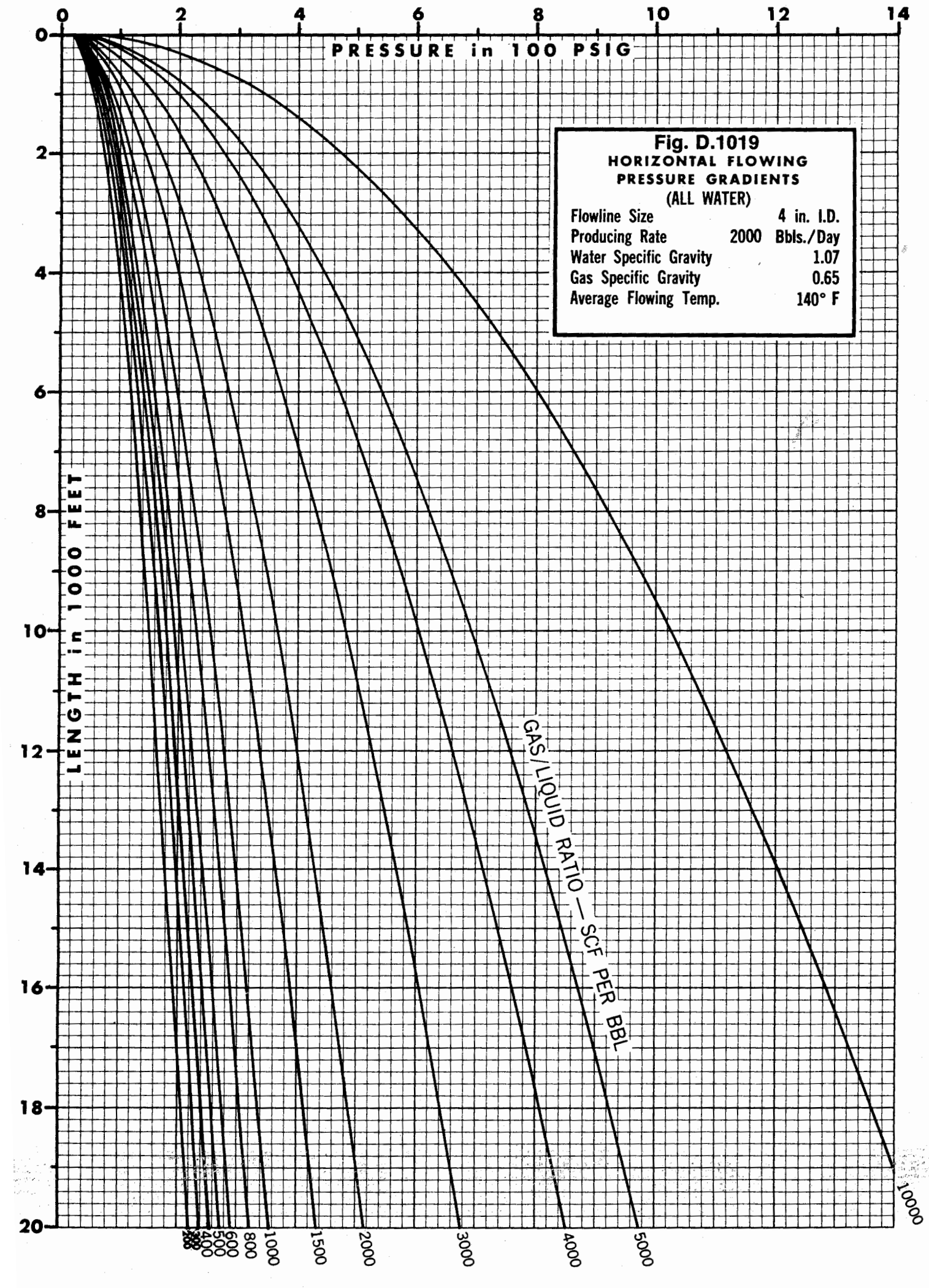


Fig. D.1019
HORIZONTAL FLOWING PRESSURE GRADIENTS
(ALL WATER)

Flowline Size	4 in. I.D.
Producing Rate	2000 Bbls./Day
Water Specific Gravity	1.07
Gas Specific Gravity	0.65
Average Flowing Temp.	140° F

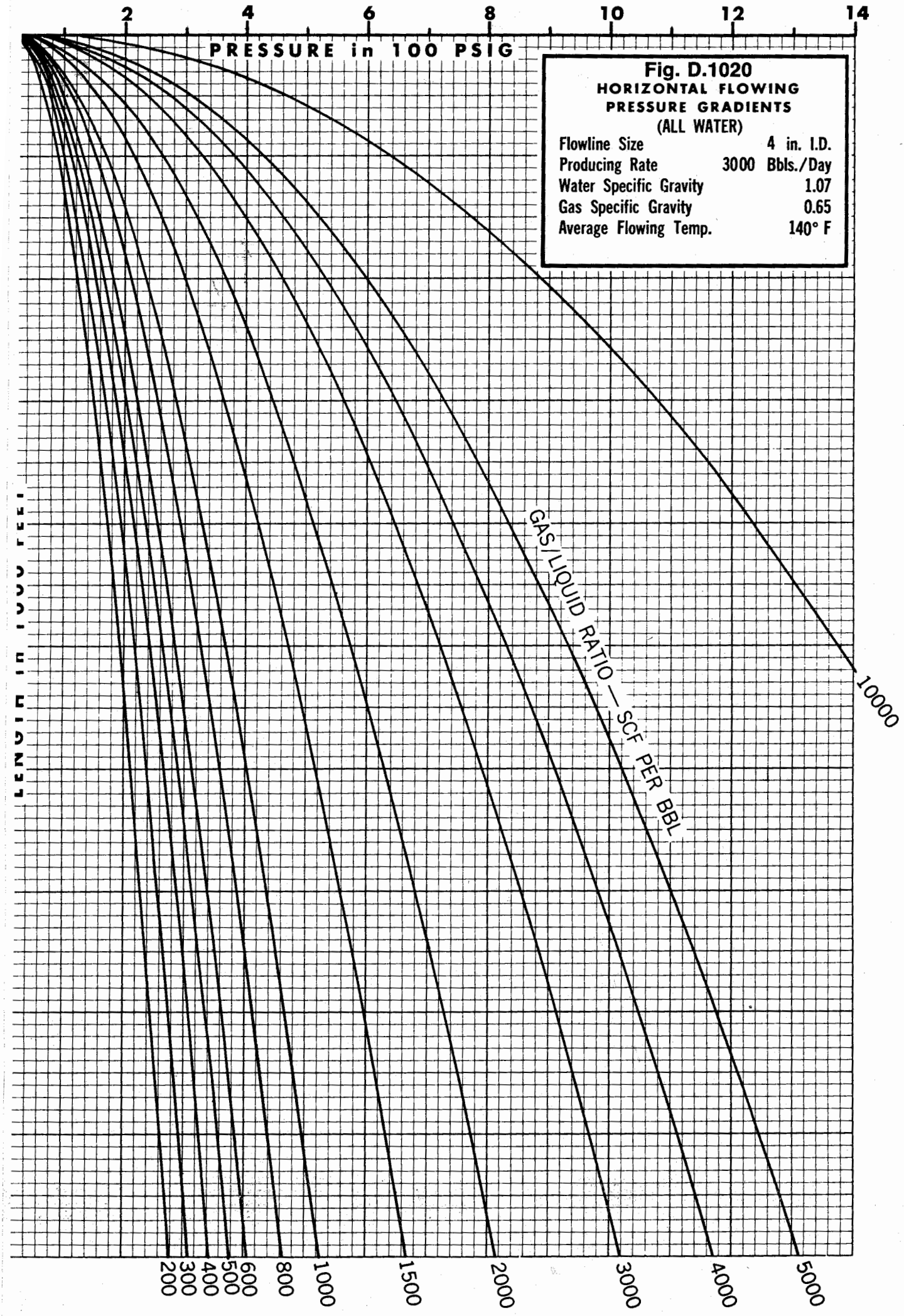


Fig. D.1020
HORIZONTAL FLOWING PRESSURE GRADIENTS
(ALL WATER)

Flowline Size	4 in. I.D.
Producing Rate	3000 Bbls./Day
Water Specific Gravity	1.07
Gas Specific Gravity	0.65
Average Flowing Temp.	140° F

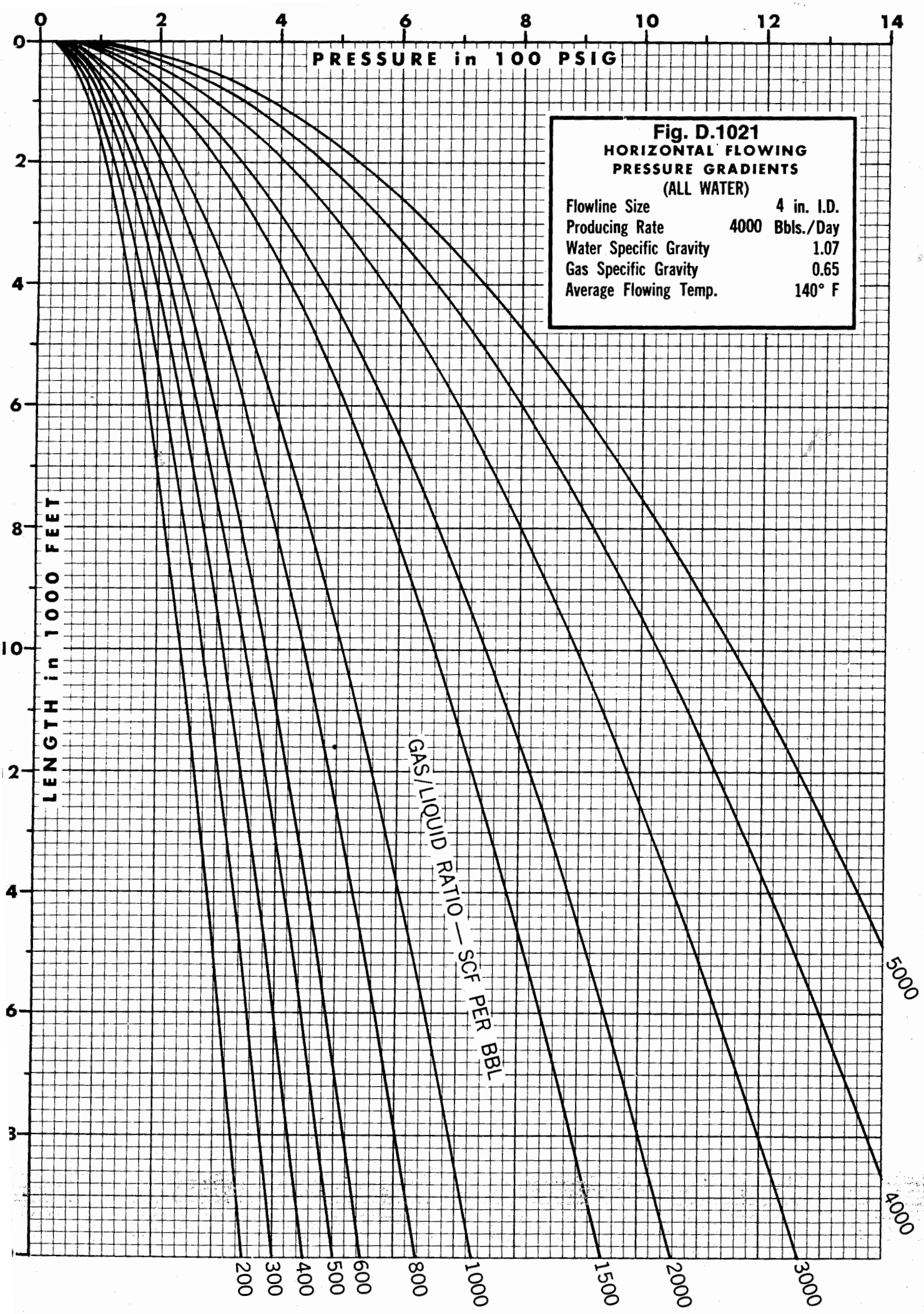
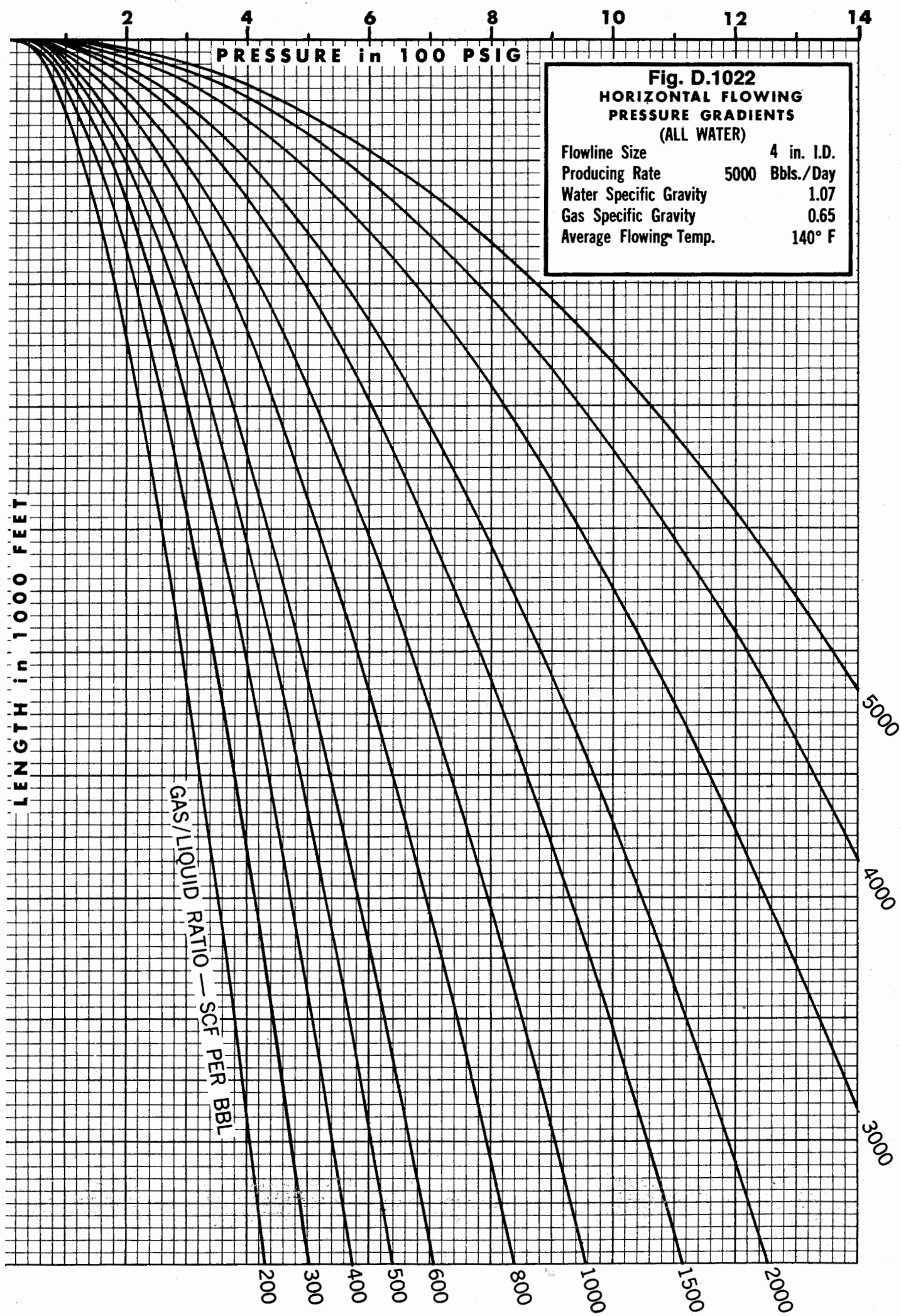


Fig. D.1021
HORIZONTAL FLOWING PRESSURE GRADIENTS
(ALL WATER)

Flowline Size	4 in. I.D.
Producing Rate	4000 Bbls./Day
Water Specific Gravity	1.07
Gas Specific Gravity	0.65
Average Flowing Temp.	140° F


2
4
6
8
10
12
14
PRESSURE in 100 PSIG
Fig. D.1022
HORIZONTAL FLOWING
PRESSURE GRADIENTS
(ALL WATER)
Flowline Size 4 in. I.D.
Producing Rate 5000 Bbls./Day
Water Specific Gravity 1.07
Gas Specific Gravity 0.65
Average Flowing Temp. 140° F
LENGTH in 1000 FEET
GAS/LIQUID RATIO — SCF PER BBL
200
300
400
500
600
800
1000
1500
2000
3000
4000
5000

Appendix E: Inclined flow

E.1 Flanigan's Method
- E.11 Flanigan's Calculation Procedure
- E.12 Example Problem for Determining Effect of Inclines by Flanigan's Method
- E.13 Example Problem for Selecting the Size of an Offshore Line to Bring Fluids to Onshore Facilities Including Hilly Terrain by Flanigan's Method
- E.14 Example Problem to Determine Size of Line Including Hilly Terrain by Combination Method of Eaton, *et al.* and Flanigan
- E.15 Example Problem to Determine Size of Line Including Hilly Terrain by Combination Method of Lockhart and Martinelli and Flanigan
- E.16 Example Problem to Determine Necessary Upstream Pressure for Hilly Terrain by Flanigan Method

E.2 Method of Baker
- E.21 Baker's Calculation Procedure for Inclined Flow
- E.22 Example Problem to Determine Line Size by Method of Baker Including Hilly Terrain (Same as Ex. E. 13)

E.3 Beggs and Brill Method
- E.31 Detailed Procedure for Calculating a Pressure Traverse in an Inclined Pipeline by the Method of Beggs and Brill
- E.32 Example Problem by Beggs and Brill Method

Appendix E.1: Flanigan's method

E.11 FLANIGAN'S CALCULATION PROCEDURE

The general procedure suggested by Flanigan to calculate the total pressure drop is as follows:

1. Calculate the superficial gas velocity, v_{sg}, in ft/sec by applying the formula:

$$v_{sg} = 31{,}194 \frac{q_g \overline{Z}}{d^2 \overline{p}} \left(\frac{\overline{T}}{T_{sc}}\right) \qquad (E.1)$$

where: q_g = gas flow rate, MMcfd
$\overline{Z}$ = compressibility factor
d = inside diameter of pipe, in.
$\overline{p}$ = average pressure in the pipeline section, psia.

2. Calculate the liquid to gas ratio, R, in bbl per MMcf of gas.
3. Calculate the abscissa, $v_{sg}/R^{0.32}$ and determine the Panhandle efficiency per cent from Fig. 2.160.
4. With the line efficiency, calculate the pressure drop due to friction, applying the Panhandle equation, or a similar one.

$$q_g = 0.43587 \left(\frac{T_o}{P_o}\right)^{1.07881} \left(\frac{P_1^2 - P_2^2}{ZTL}\right)^{0.5394} \left(\frac{d^{2.618}}{\gamma_g^{0.4606}}\right) \cdot E, \qquad (E.2)$$

where: d = pipe ID, in.
E = panhandle efficiency, %
L = line length, miles
P_1 = upstream pressure, psia
P_2 = downstream pressure, psia
q_g = gas flow rate, Mscfd
P_o = pressure base = 14.65 psia
T_o temperature base = 520°R
T = temperature of flowing gas, °R
γ_g = specific gravity of flowing gas
Z = supercompressibility factor

5. With the gas velocity value, obtain the elevation factor, H_L, from Fig. 2.161.
6. Calculate the summation of the rises, H, from a profile of the pipeline section.
7. Calculate the pressure drop due to elevation by the formula:

$$\Delta P = \frac{\rho_L H_L \Sigma H}{144} \qquad (5.2)$$

where: ρ_L = liquid density in lb_m/cu ft.

8. Calculate the total pressure drop by adding the friction component and the elevation component.

This procedure involves a trial and error calculation since a downstream pressure must be assumed in order to calculate the gas flow rate and the superficial gas velocity at the average pressure in the pipeline section. Note that the Panhandle equation is suggested here. Therefore, this solution for the friction loss is only suitable for high gas liquid ratios. I would suggest not using it for gas liquid ratios less than 40,000 scf/bbl. For a solution to the frictional loss for low GLR refer to Appendix D for other correlations.

E.12 EXAMPLE PROBLEM FOR DETERMINING EFFECT OF INCLINES BY FLANIGAN METHOD

The following data is given:

$d = 2$ in. ID
$L = 1{,}500$ ft
$q_L = 2{,}000$ bpd water
$\gamma_w = 1.07$
G/L = 1,000 scf/bbl
$\gamma_g = 0.65$
$P_u = 850$ psia
$\sigma_w = 66.7$ dynes/cm
$\mu_g = 0.015$ cp at $\overline{T} = 120°F$
$\mu_L = 1.0$ cp
$\rho_L = 66.7$ lb_m/cu ft

Hill no.	Height
1	20
2	15
3	10
4	160
5	100
6	55
	= 360

$$\overline{h} = \frac{360}{6} = 60$$

In this example the effect of hills only will be shown and the total pressure loss will not be calculated.

1. Calculate v_{sg}.

$$q_g = \frac{(1{,}000)(2{,}000)}{86{,}400} = 23.15 \text{ scf/sec}$$

Assume $p_d = 650$ psia
Therefore, $\overline{p} = 750$ psia
$\overline{z} = 0.90$

$$q_g = (23.15)\left(\frac{14.7}{750}\right)\left(\frac{580}{520}\right)\left(\frac{0.90}{1}\right) = 0.455 \text{ cu ft/sec}$$

$$v_{sg} = \frac{0.455}{0.0218} = 20.9 \text{ ft/sec}$$

2. From Fig. 2.161, $H_L = 0.13$.

$$\text{Effect of hills} = \frac{\rho_L H_L \Sigma H}{144}$$

$$= \frac{(6)(60)(0.13)(66.7)}{144} = 21.7 \text{ psi}$$

This shows the effect of hills to be 21.7 psi with an elevation factor of 0.13. The total pressure loss is found to be 21.7 psi plus the loss due to friction in the 1,500 feet.

To determine the frictional loss, refer to Appendix D.82 for Eaton, et al's method which gives $\frac{\Delta p}{\Delta L} = 200$ psia. Therefore the total loss is 21.7 + 200 = 221.7 psia or the downstream pressure is 850 − 221.7 = 628.3 psia.

E.13 EXAMPLE PROBLEM FOR SELECTING THE SIZE OF AN OFFSHORE LINE TO BRING FLUIDS TO ONSHORE FACILITIES INCLUDING HILLY TERRAIN BY FLANIGAN METHOD

An offshore line is to be installed from a central platform to onshore facilities.

Distance = 50 miles
Average temperature = 80°F
Oil Gravity = 35°API
$\sigma_o = 30$ dynes/cm
$\mu_o = 1$ cp
Flow rate = 25,000 bpd oil
500,000,000 scfd of gas

This line has the following slopes:

#1 — Rises 300 ft in 15 miles
#2 — Rises 20 ft in 100 ft and drops back 20 ft in 300 ft
#3 — Rises 60 ft in 400 ft and falls off 40 ft in 200 ft

The remainder of the line is horizontal.

The upstream pressure is 1,000 psia and the downstream pressure is 500 psia. Determine the line size needed by method of Flanigan:

1. Determine average pressure

$$= \frac{1{,}000 + 500}{2} = 750 \text{ psia}$$

2. Determine $\overline{R}_s$, $\overline{B}_o$, $\overline{Z}$, $\overline{\rho}_L$, $\overline{\rho}_g$

$\overline{R}_s = 165$ scf/bbl $\quad \overline{\rho}_L = 51.6$ lb/cu ft
$\overline{B}_o = 1.068$ $\quad \overline{\rho}_g = 2.78$ lb/cu ft
$\overline{Z} = 0.878$

3. Assume a pipe diameter.
Assume d = 26 in.
4. Calculate superficial gas velocity.

$$v_{sg} = (31{,}194)\frac{(500)(0.878)}{(26)^2(750)} = 26.95 \text{ ft/sec}$$

5. Calculate ratio of liquid rate to gas rate.

$$= \frac{25 \times 10^3}{5 \times 10^2} = 50 \text{ bbl/MMcf}$$

6. Calculate:

$$\frac{v_{sg}}{R^{0.32}} = \frac{26.95}{50^{0.32}} = 0.821$$

7. Determine efficiency from Fig. 2.160 = 80%.

8. Determine $(\Delta p_{TP})_h = \dfrac{\rho_L H_L \Sigma H}{144}$

H_L from Fig. 2.161 = 0.101.

$$(\Delta p_{TP})_h = \frac{(51.6)(0.101)(380)}{144} = 13.8 \text{ psi}$$

9. Determine upstream pressure after subtracting the the effect of hills = 1,000 − 13.8 = 986.2 psia.
10. Solve for "d" using upstream pressure of step 9. Given equation for Panhandle formula:

$$q_g = 435.9\left(\frac{T_{sL}}{p_{sc}}\right)^{1.0788}\left(\frac{p_1^2 - p_2^2}{\overline{Z}\,\overline{T}\,L}\right)^{0.5394}\frac{(d)^{2.618}}{(\gamma_g)^{0.4606}}E$$

Solving for d we have

$$d = \left[\frac{q_g}{435.9}\left(\frac{p_{sc}}{T_{sc}}\right)^{1.079}\left(\frac{\overline{Z}\,\overline{T}\,L}{p_1^2 - p_2^2}\right)^{0.5394}\left(\frac{\gamma_g}{E}\right)^{0.4606}\right]^{1/2.618}$$

$$d = \left[\left(\frac{5 \times 10^8}{435.9}\right)\left(\frac{14.7}{520}\right)^{1.079}\left(\frac{(6.878)(540)(50)}{(986.2)^2 - (500)^2}\right)^{0.5394}\frac{(0.65)^{0.4606}}{0.80}\right]^{1/2.618}$$

$= 23.7$ in.

11. Check the calculated value of "d" against the assumed value of "d." The assumed value was 26 in. and the calculated value 24 in.
12. If the calculated value and assumed value do not agree, assume another value of d and repeat the calculations.

Try d = 25 in.
$v_{sg} = 29.1$ ft/sec

$$\frac{v_{sg}}{R^{0.32}} = \frac{29.1}{3.5} = 8.3$$

E = 0.82
$H_L = 0.1$
$(\Delta p_{TP})_h = 13.65$

Upstream pressure necessary assuming no hills is 1,000 − 13.65 = 986.35. The calculated diameter is 23.5 in.
13. Check the assumptions against calculated for assumed d = 26 in. − calculated value = 23.7 in. and for assumed d = 25 in. − calculated values = 23.5 in. Interpolation = 23.6 in.
Answer = 24 in. diameter.

E.14 EXAMPLE PROBLEM TO DETERMINE SIZE OF LINE INCLUDING HILLY TERRAIN BY COMBINATION METHOD OF EATON, ET AL AND FLANIGAN

Given: (Same as Ex. E.13)

L = 50 mi.
$p_1 = 1{,}000$ psia
$p_2 = 500$ psia
$\overline{T} = 80°$F
$\gamma_o = 0.850$
$\sigma_o = 30$ dynes/cm
$\sigma_o = 1$ cp

Assume $\gamma_g = 0.65$

$q_L = 25{,}000$ bpd
$q_g = 5 \times 10^8$ scfd
$\Sigma H = 380$ ft

1. $\overline{p} = 750$ psia, $\overline{R}_s = 165$ scf/bbl, $\overline{B}_o = 1.068$, $\overline{Z} = 0.875$

$\overline{\rho}_L = 51.6$ lb_m/cu ft, $\overline{\rho}_g = 2.78$ lb_m/cu ft
$\overline{w}_L = 7.72 \times 10^6$ lb_m/day, $\overline{w}_g = 24.6 \times 10^6$ lb_m/day
At $p_1 = 1{,}000$ psia, $R_{s1} = 225$ scf/bbl, $B_{o1} = 1.091$, $Z_1 = 0.840$
At $p_2 = 500$ psia, $R_{s2} = 108$ scf/bbl, $B_{o2} = 1.045$, $Z_2 = 0.915$

2. Neglect kinetic energy term (P ≥ 500 psia)
3. Assume Δp due to hills is the same as that determined by Baker method

(13.6 psi); then $\Delta p_{TP} = 486.4$ psi.

4. Trial diameter: try d = 26 in.

$$A_p = \frac{\pi}{4}\left(\frac{26}{12}\right)^2 = 3.68 \text{ sq ft}$$

5. At $p_1 = 1{,}000$ psia,

$$q_{L_1} = \frac{(25{,}000)(1.091)(5.61)}{86{,}400} = 1.770 \text{ cu ft/sec}$$

$$v_{sL_1} = \frac{q_{L_1}}{A_p} = \frac{1.770}{3.68} = 0.481 \text{ ft/sec}$$

$$q_{g_1} = \frac{[5 \times 10^8 - (225)(25 \times 10^3)]}{86{,}400}\left(\frac{14.7}{1{,}000}\right)\left(\frac{540}{520}\right)(0.840)$$

$= 73.4$ cu ft/sec

$$v_{sg_1} = \frac{q_{g_1}}{A_p}\frac{73.4}{3.68} = 19.95 \text{ ft/sec}$$

$v_{m_1} = v_{sL_1} + v_{sg_1} = 0.48 + 19.95 = 20.43$ ft/sec

6. At $p_2 = 500$ psia:

$$q_{L_2} = \frac{(25{,}000)(1.045)(5.61)}{86{,}400} = 1.695 \text{ cu ft/sec}$$

$$v_{sL_2} = \frac{1.698}{3.68} = 0.461 \text{ ft/sec}$$

$$q_{g_2} = \frac{[5 \times 10^8 - (108)(25 \times 10^3)]}{86{,}400}\left(\frac{14.7}{500}\right)\left(\frac{540}{520}\right)(0.915)$$

$= 161.0$ cu ft/sec

$$v_{sg_2} = \frac{161.0}{3.68} = 43.75 \text{ ft/sec}$$

$v_{m_2} = 0.46 + 43.75 = 44.21$ ft/sec

7. at $\overline{p} = 750$ psia:

$$\overline{v}_{sL} = \frac{\overline{m}_1}{\overline{\rho}_1 A_p} = \frac{7.72 \times 10^6}{(51.6)(3.68)}\frac{1}{86{,}400} = 0.471 \text{ ft/sec}$$

$$\bar{v}_{sg} = \frac{\bar{m}_g}{\bar{\rho}_g A_p} = \frac{24.6 \times 10^6}{(2.78)(3.68)} \frac{1}{86{,}400} = 27.92 \text{ ft/sec}$$

$$\bar{v}_m = 0.47 + 27.92 = 28.39 \text{ ft/sec}$$

$$\bar{v}_m' = \frac{1}{4}(v_{m_1} + 2\bar{v}_m + v_{m_2})$$

$$= \frac{1}{4}(20.43 + 2(28.39) + 44.21) = 30.35 \text{ ft/sec}$$

8. $\dfrac{w_g}{w_T} = \dfrac{\bar{w}_g}{\bar{w}_L + \bar{w}_g} = \dfrac{24.6 \times 10^6}{32.3 \times 10^6} = 0.762$

9. $\dfrac{d_B}{d} = \dfrac{1}{26} = \dfrac{0.0833 \text{ ft}}{2.16 \text{ ft}} = 0.0385$

10. $M_t = \dfrac{w_t}{A_p} = \dfrac{32.3 \times 10^6}{(86{,}400)(3.68)} = 101.7 \dfrac{\text{lb}_m}{\text{sec-sq ft}}$

11. Use $\mu_g = 0.0118$ cp (Baker)

12. $\left(\dfrac{w_g}{w_t}\right)^{0.5}\left(\dfrac{d_B}{d}\right)^{1.25}\dfrac{M_t\, d}{\mu_g} = (0.762)^{0.5}(0.0385)^{1.25}\dfrac{(101.7)(26/12)}{(6.72 \times 10^{-4})(0.0118)}$

$= (0.873)(0.017)(2.78 \times 10^7)$

$= 4.12 \times 10^5$

13. From Fig. 2.143, $f\left(\dfrac{w_L}{w_t}\right)^{0.1} = 0.01$

14. $\dfrac{w_L}{w_t} = \dfrac{w_L}{\bar{w}_L + \bar{w}_g} = \dfrac{7.72 \times 10^6}{32.3 \times 10^6} = 0.238$

15. $f = \dfrac{0.01}{(0.238)^{0.1}} = 0.0115$

16. $\Delta L = \dfrac{2\, g_c\, d}{w_t\, \bar{v}_m^{\,2}\, f}\left[144\, \Delta p \left(\dfrac{w_L + w_g}{\bar{\rho}_l\, \bar{\rho}_g}\right) - \text{K.E.}\right]$

$= \dfrac{(2)(32.2)(26/12)}{(32.3 \times 10^6)(30.35)^2(0.0115)}$

$\left[144\,(486.4)\left(\dfrac{7.72 \times 10^6}{51.6} + \dfrac{24.6 \times 10^6}{2.78}\right)\right]$

$= 256{,}000$ ft

$= 48.6$ miles

Required ΔL is 50 miles.

Therefore, the correct answer is *26 in. diameter.*

E.15 EXAMPLE PROBLEM TO DETERMINE SIZE OF LINE INCLUDING HILLY TERRAIN BY COMBINATION METHOD OF LOCKHART AND MARTINELLI AND FLANIGAN.

1. Determine average pressure. (Same as Ex. E. 13)

$$\bar{p} = \frac{1{,}000 + 500}{2} = 750 \text{ psia}$$

2. Determine $\bar{R}_s$, $\bar{B}_o$, $\bar{Z}$, $\bar{\rho}_2$, $\bar{\rho}_g$, $\bar{W}_L$, $\bar{W}_g$

$\bar{R}_s = 165$ scf/bbl, $\bar{B}_o = 1.068$, $\bar{Z} = 0.878$

$\bar{\rho}_L = 51.6$ lb/cu ft

$\bar{\rho}_g = 2.78$ lb/cu ft

$\bar{w}_L = 7.72 \times 10^6$ lb_m/day

$\bar{w}_g = 24.6 \times 10^6$ lb_m/day

By assuming that the Blasius relation applies for the friction factor, a diameter may be solved for directly by the Lockhart and Martinelli method.

3. $X^2 = \dfrac{\left(\dfrac{\Delta p}{\Delta L}\right)_L}{(\Delta p/\Delta L)_g} = \left(\dfrac{\mu_L}{\mu_g}\right)^{1/4}\left(\dfrac{\rho_L}{\rho_g}\right)^{-1}\left(\dfrac{w_L}{w_g}\right)^{7/4}$

4. $\mu_g = 0.0118$ cp

5. $X = \left[\left(\dfrac{1.0}{.0118}\right)^{1/4}\left(\dfrac{51.6}{2.78}\right)^{-1}\left(\dfrac{7.72 \times 10^6}{24.6 \times 10^6}\right)^{7/4}\right]^{1/2} = 0.4$

$\phi_g = 2.5$

6. $\Delta p_g = \dfrac{\Delta p_{TP}}{\phi_g^2} = \dfrac{482.5}{(2.5)^2} = 77.3$ psi

Although the total Δ p is 500 psi, there is an effect of hills =

$$\frac{\rho_L H_L \Sigma H}{144} = \frac{(51.6)(0.101)(380)}{144} = 13.8 \text{ psi}$$

Therefore, the Δ p we are concerned with is 482.5 psi, which is (500 − 13.8).

7. $\Delta p_g = \dfrac{f_g L}{d}\dfrac{\rho_g v_{sg}^2}{2 g_c}$

By substituting and solving for d:

$$d = \left(\frac{f_g L w_g^2}{39.6 \rho_g \Delta p_g}\right)^{1/5}$$

8. Assume $f_g = 0.011$. $N_{Reg} = 7.5 \times 10^6$ $\dfrac{\epsilon}{d} = 5 \times 10^{-5}$

9. $d = \left[\dfrac{(0.011)\,[(50)(5{,}280)]\,\dfrac{24.6 \times 10^6}{86{,}400}}{39.6 \times 2.78\,(77.3 \times 144)}\right]^{1/5} = 2.68$ ft

$= 32.2$ in.

E.16 EXAMPLE PROBLEM TO DETERMINE NECESSARY UPSTREAM PRESSURE FOR HILLY TERRAIN BY FLANIGAN METHOD

Given:

A 3 in. ID flow line is 20 miles long and contains 6 hills of 30 degree slopes having a vertical rise of 100, 300, 600, 800, 1,000, and 2,000 ft each. This line is flowing 2,000 bpd of 35°API oil (σ_o = 32 dynes/cm, μ = 1.0 cp) at an average temperature of 105°F. In addition, it is flowing 2,000,000 scf/day of 0.65 gravity gas.

Calculate the upstream pressure for a downstream pressure of 60 psig. Use method of Flanigan.

This problem is one where the downstream pressure and line size are known. It is necessary to calculate the required upstream pressure.

1. Estimate the upstream pressure. Assume p_1 = 1,000 psig.
2. Determine an average pressure.

$$\bar{p} = \frac{1014.7 + 74.7}{2} = 545 \text{ psia}$$

3. Obtain R_s, β_o and $\bar{Z}$ for average conditions.

$\bar{R}_s = 106$ scf/bbl, $\beta_o = 1.054$, $Z = 0.925$

4. Calculate ρ_L

$$\rho_L = \frac{(62.4)(0.85) + (0.0764)(0.65)(106)/5.61}{1.054}$$

$= 51.5$ lb_m/cu ft

5. Determine superficial gas velocity.

$$v_{sg} = 31{,}194 \frac{q_g' \bar{Z}}{d^2 \bar{p}} \qquad q_g' = q_g\, 10^{-6}$$

$$= (31{,}194)\frac{(2)(0.925)}{(3)^2(545)} = 11.72 \text{ ft/sec}$$

6. Determine liquid to gas ratio.

$$R = \frac{2{,}000}{2} = 1{,}000$$

7. Determine value of $\frac{v_{sg}}{R^{0.32}}$.

$$\frac{11.72}{(1{,}000)^{0.32}} = \frac{11.72}{9.1} = 1.29$$

8. Select Panhandle Efficiency from Fig. 2.160 = 50%
9. Calculate pressure drop in line not accounting for hills.

$$q_g = 435.9 \left(\frac{T_{sc}}{p_{sc}}\right)^{1.079} \left(\frac{p_1^2 - p_2^2}{\overline{Z}\,\overline{T}\,L}\right)^{0.5394} \left(\frac{d^{2.618}}{\gamma_g^{0.4606}}\right) E$$

or:

$$p_1 = \left\{ p_2^2 + \overline{Z}\,\overline{T}\,L \left[\frac{q_g}{435.9}\left(\frac{p_{sc}}{T_{sc}}\right)^{1.079} \left(\frac{\gamma_g^{0.4606}}{d^{2.618}}\right) \frac{1}{E}\right]^{1/0.2344} \right\}^{1/2}$$

$$p_1 = \left\{ (74.7)^2 + (0.925)(565)(20) \left[\frac{2 \times 10^6}{435.9}\left(\frac{14.7}{520}\right)^{1.079} \left(\frac{(0.65)^{0.4606}}{(3)^{2.618}} \frac{1}{50}\right]^{1.86} \right\}^{1/2}$$

$p_1 = 1{,}491$ psia.

This represents horizontal loss assuming no hills.

10. Make a total summation of all hills.

$\Sigma H = 100 + 300 + 600 + 800 + 1{,}000 + 2{,}000 = 4{,}800$ ft

11. From Fig. 2.161, obtain an H_L factor = 0.205.
12. Calculate total loss due to hills.

$$\Delta p_h = \frac{\rho_L\, H_L\, \Sigma H}{144} = \frac{(51.5)(0.205)(4{,}800)}{144} = 352 \text{ psi}$$

13. Determine total pressure loss:

$$p_1 + \Delta p_h = 1{,}491 + 352 = 1{,}843 \text{ psia}$$

14. Check against original assumption. The assumed value of 1,000 is too low. Take as the next assumption the calculated value of 1,843 psi.
15. Repeat the procedure for the new assumed value.

$$\overline{p} = \frac{1{,}843 + 75}{2} = 959 \text{ psia}$$

$\overline{R}_s = 197$ scf/bbl
$\beta_o = 1.090$
$\overline{Z} = 0.877$
$\overline{\rho}_L = 50.9$ lb/cu ft
$v_{sg} = 6.34$ ft/sec

$$\frac{v_{sg}}{R^{0.32}} = 0.178$$

$E = 0.178$
$p_1 = 1{,}609$ psia
$H_L = 0.325$
$\Delta p_h = 550$
p total $= 1{,}609 + 550 = 2{,}159$ psia

The estimated value of 1,843 is still low. Another assumption of $p_1 = 2{,}300$ psia yields a value of 2,222 psia. An interpolation between 2,159 and 2,222 gives a final answer of 2,210 psia.

Appendix E.2: Method of Baker

E.21 BAKER'S CALCULATION PROCEDURE FOR INCLINED FLOW

Baker's general procedure to solve problems in which it is desired to calculate a pressure traverse by knowing the inlet pressure and all other parameters is as follows:

1. Assume an average pressure for the distance under consideration, P_{av}, psia.
2. Calculate the pressure drop as though only liquid is flowing, by means of the formula:

$$\Delta P_L = \frac{f\, L'\, q_L^2\, \rho_L}{181{,}916\, d^5}$$

where: ΔP_L = pressure drop for liquid flowing alone, psi
f = friction factor for the liquid (Fig. 2.135)
L' = distance, ft
q_L = liquid flow rate, bpd at flowing conditions
ρ_L = liquid density, lb_m/gal at flowing conditions
d = inside diameter of pipe, in.

3. Calculate the pressure drop as though only gas is flowing.

$$\Delta P_g = \frac{f\, L'\, q_g^2\, \gamma_g\, T\, Z}{20{,}000\, d^5\, \overline{P}}$$

where: ΔP_g = pressure drop for gas flowing alone, psi
f = friction factor for the gas (Fig. 2.135)
q_g = gas flow rate, Mcfd at flowing conditions
γ_g = specific gravity of the gas
T = flowing temperature, °R
Z = supercompressibility factor

4. Calculate the so-called X factor.

$$X = \sqrt{\frac{\Delta P_L}{\Delta P_g}}$$

5. Calculate the correction factor λ.

$$\lambda = \left[\left(\frac{\rho_g}{0.075}\right)\left(\frac{\rho_L}{62.3}\right)\right]^{0.5}$$

where: ρ_g = density of the gas, lb_m/cu ft
ρ_L = density of liquid, lb_m/cu ft

6. Calculate the correction factor ψ:

$$\psi = \left(\frac{73}{\sigma}\right)\left[\mu_L\left(\frac{62.3}{\rho_L}\right)^2\right]^{0.34}$$

where: σ = surface tension of the liquid, dynes/cm
μ_L = liquid viscosity, centipoise

7. Calculate the mass velocity of the liquid, M_L, in lb/hr per sq ft of pipe cross-section.
8. Calculate the mass velocity of the gas, M_g, in lb/hr per sq ft of pipe cross-section.
9. Calculate the value of the abcissa of the flow pattern map (Fig. 2.130.)

$$\text{Abcissa Value} = \frac{M_L\, \lambda\, \psi}{M_g}$$

10. Calculate the value of the ordinate of the flow pattern map.

$$M_g/\lambda. \text{ (Fig. 2.130)}$$

11. On the flow pattern map, Fig. 2.130, plot the above values. The flow pattern is indicated by the label on the area into which the plotted point falls.
12. Apply the equation for ϕ_{gtt} corresponding to the flow pattern. (Refer to Section 2.4).
13. Calculate the two-phase pressure drop by applying the equation

$$\Delta P_{TP} = \Delta P_g\, \phi_{gtt}^2.$$

14. Calculate the pressure drop due to elevations, as proposed by Flanigan.
15. Calculate the total two-phase pressure drop by adding the pressure drops as stated in Steps 13 and 14.
16. If the total pressure drop is not sufficiently close to the assumed pressure drop, make another assumption and repeat all steps until getting mentioned condition.

E.22 EXAMPLE PROBLEM TO DETERMINE LINE SIZE BY METHOD OF BAKER INCLUDING HILLY TERRAIN (SAME AS EX. E.13)

All data is given in Ex. E.13.

1. Assume d = 30 in. for 1st trial.

$$A_p = \frac{\pi}{4}(2.5)^2 = 4.90 \text{ sq ft}$$

$$v_{sg} = \frac{\overline{w}_g}{\overline{\rho}_g\, A_p} = \frac{24.6 \times 10^6}{(2.78)(4.90)}\,\frac{1}{86{,}400} = 20.9 \text{ ft/sec}$$

2. From Fig. 2.161, $H_L = 0.128$.

3. $(\Delta p_{TP})_h = \dfrac{\Sigma H\, H_L\, \rho_L}{144} = \dfrac{(380)(0.128)(51.6)}{144} = 17.5$ psi

4. $\Delta p_{TP} = 500 - 17.5 = 482.5$ psi

This is the pressure drop assuming a true horizontal section and will be the pressure loss to use in solving for the size line needed.

5. Solve for N_{ReL}.

$$N_{ReL} = \frac{(11.05)(1.068)(25 \times 10^3)(6.90)}{(30)(1.0)} = 67{,}600$$

6. From Fig. 2.135, f = 0.0048.
For 90% eff., f = (1.23)(0.0048) = 0.0059.

7. $\Delta p_L = \dfrac{f\,(q_L)^2\, \rho_L}{181{,}916\, d^5} = \dfrac{(0.0059)(2.64 \times 10^5)(2.67 \times 10^4)^2(6.90)}{(1.819 \times 10^5)(30)^5}$

$= 1.735$ psi

8. $N_{Reg} = \dfrac{20.14\,(q_g)\,\gamma_g}{d\,\mu_g}$

$= \dfrac{(20.14)(4.96 \times 10^5)(0.65)}{(30)\,(0.0118)} = 1.83 \times 10^7$

9. From Fig. 2.135, $f_g = 0.0023$.
For 90% eff., $f_g = (1.23)(0.0023) = 0.0028$.

10. $\Delta\, p_g = \dfrac{f\,(q_g)^2\, \gamma_g\, \overline{T}\, \overline{Z}}{20{,}000\, d^5\, \overline{p}}$

$= \dfrac{(0.0028)(2.64 \times 10^5)(4.96 \times 10^5)^2(0.65)(540)(0.878)}{(2 \times 10^4)(30)^5(750)}$

$= 153.8$ psi

11. $X = \sqrt{\dfrac{\Delta\, P_L}{\Delta\, p_g}} = \left(\dfrac{1.735}{153.8}\right)^{1/2} = 0.1063$

12. Determine flow regime.

(a) $\lambda = \left[\left(\dfrac{\rho_g}{0.0764}\right)\left(\dfrac{\rho_L}{62.4}\right)\right]^{1/2} = \left[\left(\dfrac{2.78}{0.0764}\right)\left(\dfrac{51.6}{62.4}\right)\right]^{1/2}$

$= 5.55$

(b) $$\psi = \frac{73}{\sigma_L}\left[\mu_L\left(\frac{62.4}{\rho_L}\right)^2\right]^{1/3}$$
$$= \frac{73}{30}\left[1.0\left(\frac{62.4}{51.6}\right)^2\right]^{1/3} = 2.75$$

(c) $$M_L = \frac{7.72 \times 10^6}{(24)\,(4.90)} = 6.56 \times 10^4 \frac{lb_m}{hr\text{-}ft^2}$$

(d) $$M_g = \frac{(24.6) \times 10^6}{(24)\,(4.90)} = 2.095 \times 10^5 \frac{lb_m}{hr\text{-}ft^2}$$

(e) $$\frac{M_g}{\lambda} = \frac{2.045 \times 10^5}{5.55} = 3.77 \times 10^4$$

(f) $$\frac{M_L\,\lambda\,\psi}{M_g} = \frac{(6.56 \times 10^4)(5.55)(2.75)}{(2.095 \times 10^5)} = 4.78$$

From Fig. 2.130 the flow regime is borderline between dispersed flow and annular flow, but shows to be annular.

13. $\phi_{gtt} = (4.8 - 0.3125\ d)\ X^{0.343\ -\ 0.021d}$

For d > 10 in. use 10 in. in the equation above. (Specified by Baker.)

$$\phi_{gtt} = 1.242$$

14. $\Delta p_{TP} = \phi_{gtt}^2\ \Delta p_g = (1.242)^2(153.8) = 237.5$ psia

15. Check this against the required drop of 482.5 psi, and it is noted that the trial line size of 30 in. is too large.

16. Select another diameter which can be approximated by:

$$d = 30\left(\frac{237.5}{482.5}\right)^{1/5} = 26 \text{ in.}$$

Repeating the same calculation, we find:

$A_p = 3.68$ ft²
$\overline{v}_{sg} = 27.9$ ft/sec
$H_L = 0.10$
$(\Delta p)_{TP_h} = 13.6$ psi
$\Delta p_{TP} = 486.4$ psi
$N_{Rel} = 78{,}000$
$f_L = 0.0058$
$\Delta p_L = 3.50$ psi
$N_{Reg} = 2.11 \times 10^7$
$f_g = 0.0028$
$\Delta p_g = 315.5$ psi
$X = 0.1054$
$\phi_{gtt} = 1.241$
$\Delta p_{TP} = 485$ psi

This agrees very closely with the required Δp of 486.4 psi and therefore the correct diameter is 26 in.

17. Select a line size of standard dimensions nearest to the calculated value. Circumstances will dictate whether or not a stock size smaller or larger than the calculated size will be used. Experience has shown that most correlations predict a lower size than needed. Therefore, a size larger than calculated is recommended.

Appendix E.3: Beggs and Brill method

31 DETAILED PROCEDURE FOR CALCULATING A PRESSURE TRAVERSE IN AN INCLINED PIPELINE BY THE METHOD OF BEGGS & BRILL

The pressure gradient equation is:

$$\frac{\Delta p}{\Delta z} = \frac{\frac{g}{g_c}\rho_{tp}\sin\theta + \frac{f_{tp}\, G_m\, v_m}{2g_c d}}{1 - \frac{\rho_{tp}\, v_m\, v_{sg}}{g_c p}}$$

here: $\rho_{tp} = \rho_L H_L + \rho_g (1 - H_L)$

Since liquid holdup, H_L, is a function of inclination angle, profile of the pipeline must be available. The traverse is ius calculated in sections, a section consisting of the length 'line where the angle is constant.

1. Beginning with the known pressure p_1, estimate a value for pressure drop (or pressure gain, if flow is downhill), Δp.
2. Calculate the average pressure in the interval:

$$\bar{p} = p_1 + \frac{\Delta p}{2} \text{ if } p_1 \text{ is downstream pressure;}$$

$$\bar{p} = p_1 - \frac{\Delta p}{2} \text{ if } p_1 \text{ is upstream pressure.}$$

3. From PVT analysis or appropriate correlations, calculate: R_s, B_o, B_w, μ_o, μ_w, μ_g, σ_o, σ_w, and Z_g at $\bar{T}$ and $\bar{p}$.
4. Calculate the specific gravity of the oil γ_o:

$$\gamma_o = \frac{141.5}{131.5 + API}$$

5. Calculate the liquid and gas densities at the average conditions of pressure and temperature:

$$\rho_L = \rho_o\left(\frac{1}{1+WOR}\right) + \rho_w\left(\frac{WOR}{1+WOR}\right) = \rho_o f_o + \rho_w f_w$$

$$\rho_o = \frac{(350\,\gamma_o + 0.0764\, R_s\, \gamma_g)}{5.615\, B_o}$$

$$\rho_w = \frac{350\,\gamma_w}{5.615\, B_w}$$

$$\rho_g = \frac{0.0764\,\gamma_g\,\bar{p}\,(520)}{(14.7)\,(\bar{T}+460)\,Z_g}$$

6. Calculate the in situ gas and liquid flow rates.

$$q_g = \frac{3.27\times10^{-7}\, Z_g\, q_o\, (R - R_s)\,(\bar{T}+460)}{\bar{p}}$$

$$q_L = 6.49\times10^{-5}\,(q_o\, B_o + q_w\, B_w)$$

7. Calculate the in situ superficial gas, liquid, and mixture velocities.

$$v_{sL} = q_L/Ap$$
$$v_{sg} = q_g/A_p$$
$$v_m = v_{sL} + v_{sg}$$

8. Calculate the liquid, gas, and total mass flux rates.

$$G_L = \rho_L\, v_{sL} \qquad G_g = \rho_g v_{sg}$$
$$G_m = G_L + G_g$$

9. Calculate the input liquid content (no-slip holdup).

$$\lambda = \frac{q_L}{q_L + q_g}$$

10. Calculate the Froude number, N_{FR}, the liquid viscosity, the mixture viscosity, μ_m and the liquid surface tension, σ_L

$$N_{FR} = \frac{v_m^2}{gd}$$

$$\mu_L = \mu_o f_o + \mu_w f_w$$
$$\mu_m = ((\mu_L\lambda + \mu_g\,(1-\lambda))(6.72\times10^{-4})$$
$$\sigma_L = \sigma_o f_o + \sigma_w f_w$$

11. Calculate the no-slip Reynolds number and the liquid velocity number.

$$N_{Rens} = \frac{G_m d}{\mu_m} \qquad N_{LV} = 1.938\, v_{sL}\left(\frac{\rho_L}{\sigma_L}\right)^{0.25}$$

12. To determine the flow pattern which would exist if flow were horizontal, calculate the correlating parameters, L_1, L_2, L_3, and L_4.

$$L_1 = 316\,\lambda^{0.302} \qquad L_2 = 0.0009252\,\lambda^{-2.4684}$$
$$L_3 = 0.10\,\lambda^{-1.4516} \qquad L_4 = 0.5\,\lambda^{-6.738}$$

13. Determine flow pattern using the following limits:

Segregated:

$\lambda < 0.01$ and $N_{FR} < L_1$
or $\lambda \geq 0.01$ and $N_{FR} < L_2$

Intermittent:

0.01 0.4 and L_3 N_{FR} L_1
or $\lambda \geq 0.4$ and $L_3 < N_{FR} \leq L_4$

Transition:

$\lambda \geq 0.01$ and $L_2 < N_{FR} \leq L_3$

Distributed:

0.4 and N_{FR} L_1
or $\lambda \geq 0.4$ and $N_{FR} > L_4$

14. Calculate the horizontal holdup, $H_L(O)$

$H_L(O) \equiv \frac{a\lambda^b}{N_{FR}^c}$ where a, b, and c are determined for each flow pattern from the table:

Flow pattern	a	b	c
Segregated	0.98	0.4846	0.0868
Intermittent	0.845	0.5351	0.0173
Distributed	1.065	0.5824	0.0609

15. Calculate the inclination correction factor coefficient.

$C = (1-\lambda)\ln(d\lambda^e N_{Lv}^f N_{FR}^g)$ where d, e, f, and g are determined for each flow condition from the table:

Flow pattern	d	e	f	g
Segregated uphill	0.011	−3.768	3.539	−1.614
Intermittent uphill	2.96	0.305	−0.4473	0.0978
Distributed uphill	No Correction		C = 0	
All flow patterns downhill	4.70	−0.3692	0.1244	−0.5056

16. Calculate the liquid holdup inclination correction factor.

$$\psi = 1 + C\,[\sin(1.8\theta) - 0.333\sin^3(1.8\theta)]$$

17. Calculate the liquid holdup and the two-phase density.

$$H_L(\theta) = H_L(O)\psi$$
$$\rho_{tp} = \rho_L H_L + \rho_g (1 - H_L)$$

18. Calculate the friction factor ratio.

$\frac{f_{tp}}{f_{ns}} = e^s$, where: S
$= [\ln(y)]/\{-0.0523 + 3.182 \ln(y) - 0.8725 [\ln(y)]^2 + 0.01853 [\ln(y)]^4\}$,

and:

$$y = \frac{\lambda}{[H_L(\theta)]^2}$$

S becomes unbounded at a point in the interval $1 < y < 1.2$; and for y in this interval, the function S is calculated from:

$$S = \ln(2.2y - 1.2)$$

19. Calculate the no-slip friction factor,

$$f_{ns} = 1/[2 \log(N_{Rens}/(4.5223 \log N_{Rens} - 3.8215))]^2$$

20. Calculate the two-phase friction factor.

$$f_{tp} = f_{ns} \frac{f_{tp}}{f_{ns}}$$

21. Calculate

$$\Delta p = \frac{\Delta z \left(\frac{g}{g_c} \rho_{tp} \sin\theta + \frac{f_{tp} G_m v_m}{2g_c d}\right)}{1 - \frac{\rho_{tp} v_m v_{sq}}{g_c p}}$$

22. If the pressure drop (or gain) estimated in Step 1 and that calculated in Step 21 are not sufficiently close, use the value calculated in Step 21 as the new estimated Δp and go back to Step 2. This procedure is repeated until the estimated and calculated values of Δp agree. The pressure at $z \pm \Delta z$ is then $p_1 \pm \Delta p$.
If flow is downhill, the pressure can either increase or decrease in the direction of flow, depending on the relative magnitudes of the hydrostatic and frictional pressure gradients. If a negative value is calculated for Δp in Step 21, this indicates a pressure gain in the flow direction.

E.32 EXAMPLE PROBLEM: CALCULATION OF PRESSURE CHANGE IN AN INCLINED PIPELINE BY THE METHOD OF BEGGS & BRILL

Available data:

$q_o = 7140$ stk bpd — Oil gravity = 20°API
$q_g = 2.57 \times 10^6$ scfd — $\gamma_g = 0.70$
$d = 12$ in. — GOR = 360 scf/stk bbl
$z = 1$ mile
$p_1 = 425$ psia (inlet)
$\theta = 3°$ from horizontal
$\bar{T} = 90°$ F.

Calculate the outlet pressure.

1. Estimate $\Delta p = 100$ psig.
2. Average pressure $p = p_1 - \frac{\Delta p}{2} = 425 - 50 = 375$ psia.
3. From appropriate correlations, find at 375 psia and 90° F:

$R_s = 55$ scf/stk bbl — $\sigma_o = 22.3$ dynes/cm
$B_o = 1.015$ — $z_g = 0.93$
$\mu_o = 4.2$ cp
$\mu_g = 0.0105$ cp

4. Calculate the oil specific gravity.

$$\gamma_o = \frac{141.5}{131.5 + API} = \frac{141.5}{131.5 + 20} = 0.93$$

5. Calculate the liquid and gas densities at $\bar{p}$ and $\bar{T}$.

$$\rho_o = \frac{350 \gamma_o + 0.0764 R_s \gamma_g}{5.615 B_o} = \frac{350(.93) + 0.0764(55)(0.70)}{5.615 (1.015)}$$

$\rho_o = 57.63$ lb$_m$/cu ft

$$\rho_g = \frac{0.0764 \gamma_g \bar{p} (520)}{(14.7)(T + 460) z_g} = \frac{0.0764 (0.70)(375)(520)}{14.7 (550)(0.93)}$$

$\rho_g = 1.39$ lb$_m$/cu ft

6. Calculate the in situ gas and liquid flow rates.

$$q_g = \frac{3.27 \times 10^{-7} z_g q_o (R - R_s)(T + 460)}{\bar{p}} = \frac{3.27 \times 10^{-7} (0.93)(7{,}140)(360 - 55)(550)}{375}$$

$q_g = 0.971$ cu ft/sec
$q_L = 6.49 \times 10^{-5} (q_o B_o) = 6.49 \times 10^{-5} (7{,}140)(1.015)$
$q_L = 0.470$ cu ft/sec

7. Calculate the in situ superficial gas, liquid, and mixture velocities.

$$v_{sL} = q_L/A_p = \frac{0.470(4)}{\pi (1)^2} = 0.598 \text{ ft/sec}$$

$$v_{sg} = q_g/A_p = \frac{0.971}{0.785} = 1.236 \text{ ft/sec}$$

$v_m = v_{sL} + v_{sg} = 0.598 + 1.236 = 1.834$ ft/sec

8. Calculate the liquid, gas, and total mass flux rates.

$G_L = \rho_L v_{sL} = (57.63)(0.598) = 34.49$ lb$_m$/sec-sq ft
$G_g = \rho_g v_{sg} = (1.39)(1.236) = 1.718$ lb$_m$/sec-sq ft
$G_m = G_L + G_g = 34.49 + 1.718 = 36.21$ lb$_m$/sec-sq ft

9. Calculate the input liquid content (no-slip holdup).

$$\lambda = \frac{q_L}{q_L + q_o} = \frac{0.470}{0.470 + 0.971} = 0.326$$

10. Calculate the Froude number, the liquid viscosity, the mixture viscosity, and the liquid surface tension.

$$N_{FR} = \frac{v_m^2}{gd} = \frac{(1.834)^2}{(32.17)(1)} = 0.1046$$

$\mu_m = 6.72 \times 10^{-4} (m_L \lambda + \mu_g (1 - \lambda))$
$= 6.72 \times 10^{-4} (4.2)(0.326) + 0.0105(0.674))$

$\mu_m = 0.00092$ lb$_m$/ft sec

$\sigma_L = \sigma_o = 22.3$ dynes/cm

11. Calculate the no-slip Reynolds number and the liquid viscosity number.

$$N_{Rens} = \frac{G_m d}{\mu_m} = \frac{(36.21)(1)}{0.00092} = 39{,}150$$

$$N_{Lv} = 1.938 v_{sL} \left(\frac{\rho_L}{\sigma_L}\right)^{0.25} = 1.938(0.598)\left(\frac{57.63}{22.3}\right)^{0.25}$$

$N_{Lv} = 1.469$

12. To determine the flow pattern which would exist if flow were horizontal, calculate L_1, L_2, L_3, and L_4.

$L_1 = 316(0.326)^{0.302} = 225$
$L_2 = 0.0009252(0.326)^{-2.4684} = 0.0147$
$L_3 = 0.10(0.326)^{-1.4516} = 0.509$ $L_4 = 0.5(0.326)^{-6.738} = 952$

13. Determine flow pattern. Since $\lambda > 0.04$ and $N_{fr} < L_2$, flow pattern is segregated.
14. Calculate the horizontal holdup.

$$H_L(O) = \frac{0.98\lambda^{0.4846}}{N_{FR}^{0.0868}} = \frac{0.98 (0.326)^{0.4846}}{(0.1046)^{0.0868}} = 0.693$$

15. Calculate the inclination correction factor coefficient.

$C = (1 - \lambda) \ln (0.011\lambda^{-3.768} N_{Lv}^{3.539} N_{FR}^{-1.614})$
$= (1 - 0.326)\ln(0.011(0.326)^{-3.768} (1.469)^{3.539}(0.1046)^{-1.614})$

$C = 3.18$

16. Calculate the liquid holdup inclination correction factor.

$$\psi = 1 + C\,[\sin(1.8\theta) - 0.333\sin^3(1.8\theta)]$$
$$= 1 + 3.18\,[\sin(1.8(3)) - 0.333\sin^3(5.4)]$$
$$= 1 + 3.18\,(0.094 - 0.0003)$$
$$= 1 + 3.18\,(0.094) = 1 + 0.299$$
$$\psi = 1.3$$

17. Calculate the liquid holdup.

$$H_L(3) = H_L(O)\,\psi = (0.693)(1.3) = 0.901$$
$$\rho_{tp} = \rho_L H_L + \rho_g(1 - H_L) = (57.63)(0.901) + (1.39)(0.099)$$
$$\rho_{tp} = 52.06\ lb_m/cu\ ft$$

18. Calculate the friction factor ratio.

$$y = \frac{\lambda}{H_L^2} = \frac{0.326}{(0.901)^2} = 0.402$$

$$\ln(y) = \ln(0.402) = -0.912$$

$$S = \frac{-0.912}{-0.0523 + 3.182(-0.912) - 0.8725(-0.912)^2 + 0.01853(-0.912)^4}$$

$$= \frac{-0.912}{-3.667}$$

$$S = 0.2487$$

$$\frac{f_{tp}}{f_{ns}} = e^{0.2487} = 1.282$$

19. Calculate the no-slip friction factor.

$$f_{ns} = 1/[2\log(N_{Rens}/(4.5223\log N_{Rens} - 3.8215))]^2$$

$$= \frac{1}{\left[2\log\left(\frac{39150}{4.5223\log(39150) - 3.8215}\right)\right]^2}$$

$$f_{ns} = 0.022$$

20. Calculate the two-phase friction factor.

$$f_{tp} = \frac{f_{tp}}{f_{ns}} f_{ns} = (1.282)(0.022) = 0.028$$

21. Calculate Δp.

$$\Delta p = \frac{\Delta z\left(\frac{g}{g_c}\rho_{tp}\sin\theta + \frac{f_{tp}G_m v_m}{2g_c d}\right)}{1 - \frac{\rho_{tp}\,v_m\,v_{sg}}{g_c\,p}}$$

$$= \frac{5280\left((52.06)(0.0523) + \frac{(0.028)(36.21)(1.834)}{(64.34)(1)}\right)}{1 - \frac{(52.06)(1.834)(1.236)}{(32.17)(375)(144)}}$$

$$\Delta p = \frac{5280\,(2.7516)}{0.9999(144)} = 102\ psi$$

This value is close to the estimated Δp, so the outlet pressure is $p_1 - \Delta p = 425 - 102 = 323$ psia.

Appendix F
Directional Well Problems

F.1 Ney and Fuentes Method
- F.11 Example Problem for Calculating Pressure Traverses in a Directional Well by the Generalized Hagedorn and Brown Method as Modified by Ney
- F.12 Example Problem for Calculating Pressure Traverse for a Directional Well by Use of Combined Vertical and Horizontal Correlation of Ney and Fuentes

F.2 Method of Beggs and Brill
- F.21 Detailed Procedure for Calculating a Pressure Traverse in a Directional Well by the Method of Beggs and Brill
- F.22 Example Problem for Calculating a Pressure Traverse in a Directional Well by the Method of Beggs and Brill

Appendix F: Directional Well Problem

F.1 NEY AND FUENTES METHOD

F.11 Example problem for calculating pressure traverses in a directional well by the generalized Hagedorn and Brown method as modified by Ney

Example of detailed procedure for calculating a pressure traverse for vertical and directional wells using the generalized method of Hagedorn and Brown as modified by Ney. This is the same problem worked for the vertical methods and all data is again given here:

$d = 1.995$ in.
$P_1 = 500$ psig
$P_2 = 1{,}000$ psig
$P_a = 14.7$ psia
$T_1 = 120°F$
$T_2 = 150°F$
$\gamma_g = 0.65$
$\gamma_w = 1.07$
$\rho_o = 22°API$
$q_o = 400$ stk bpd
$q_w = 600$ stk bpd (WOR = 1.5)
$\mu_g = 0.018$ cp (constant)
$\sigma_o = 30$ dynes/cm (assume constant)
$\sigma_w = 70$ dynes/cm (assume constant)
GLR = 500 scf/stk bbl
$\phi = 0$, 30°, and 45°

Required:

Distance between the 500 and 1,000 psig pressure points. The modified Hagedorn and Brown equation will be used and the cosine of the angle of deviation is included. This problem will be worked for angles of $\phi = 0$, $\phi = 30°$, and $\phi = 45°$. (angle from vertical)

It is necessary that the following equations be used:

$$144\frac{\Delta P}{\Delta L} = \rho_m \cos\phi + \frac{f\,w^2}{(2.9652)(10^{11})\,d^5\bar{\rho}_m} + \frac{\bar{\rho}_m}{2_g}\frac{\Delta(v_m^2)}{\Delta L}$$

or:

$$\Delta L = \frac{144\frac{\Delta P}{\Delta L} = \frac{\bar{\rho}_m}{2_g}\Delta(v_m^2)}{\bar{\rho}_m\cos\phi + \frac{f(w^2)}{(2.9652)(10^{11})\,d^5\,\bar{\rho}_m}}$$

The only difference between this equation and the Hagedorn and Brown equation is the inclusion of cosine ϕ. Using:

$$\Delta L = \frac{144\,\Delta P - \frac{\bar{\rho}_m}{2g_c}\Delta(v_m^2)}{\bar{\rho}_m(\cos\phi) + \frac{f\,w^2}{2.9652\times 10^{11}\,d^5\,\bar{\rho}_m}}$$

The calculation for three different inclination angles is carried out as follows:

A. Vertical well:

$$\phi = 0° \qquad \cos\phi = 1$$

In this case $\Delta L = \Delta h$

$$\Delta L = \frac{144\,(500) - \frac{35.7}{64.4}(66.3)}{(35.7)\,(1) + \frac{(0.021)\,(1.4332)\,(10)^{11}}{(2.9652)\,(10)^{11}\,(0.1663)^5\,(35.7)}}$$

$\Delta L = 1{,}870$ ft

B. Inclination angle 30°:

$$\phi = 30° \qquad \cos\phi = 0.86603$$

$$\Delta L = \frac{144\,(500) - \frac{35.7}{64.4}(66.3)}{(35.7)\,(0.86603) + \frac{(0.021)\,(1.4332)\,(10)^{11}}{(2.9652)\,(10)^{11}\,(0.1663)^5\,(35.7)}}$$

$\Delta L = 2{,}190$ ft

C. Inclination angle 45°:

$$\phi = 45° \qquad \cos\theta = 0.70711$$

$$\Delta L = \frac{144\,(500) - \frac{35.7}{64.4}(66.3)}{(35.7)\,(0.70711) + \frac{(0.021)\,(1.4332)\,(10)^{11}}{(2.9652)\,(10)^{11}\,(0.1663)^5\,(35.7)}}$$

$\Delta L = 2{,}672$ ft

As noted, we get the following answers:

1. For $\phi = 0°$ —— 1,870 ft
2. For $\phi = 30°$ —— 2,190 ft
3. For $\phi = 45°$ —— 2,672 ft

This is reasonable in that as the angle increases, the static gradient contributes less and less to the total pressure loss.

F.12 EXAMPLE PROBLEM FOR CALCULATING PRESSURE TRAVERSE FOR A DIRECTIONAL WELL BY USE OF COMBINED VERTICAL AND HORIZONTAL CORRELATION OF NEY AND FUENTES

USE OF COMBINED METHOD FOR THE WELL WITH INCLINATION ANGLE $\phi = 45°$

Procedure:

A. Calculate Δh by Hagedorn and Brown method assuming vertical well.

B. Calculate the true length of the pipe.

$$\Delta L = \frac{\Delta h}{\cos\phi} = \frac{1{,}870}{0.70711} = 2{,}643 \text{ ft}$$

C. Determine the difference between ΔL and Δh

$$\Delta L' = \Delta L - \Delta h = 2{,}643 - 1{,}870 = 773 \text{ ft}$$

D. Calculate the friction pressure drop corresponding to $\Delta L'$ by method of Eaton, et al.

Reference should be made to Appendix D.82 for complete details on the Eaton, et al procedure.

1. The Eaton equation is as follows:

$$\Delta L = \frac{2gcd}{w_t\,\bar{v}_m^{\,2} f}\left[144\,\Delta P\left(\frac{w_L}{\bar{\rho}_L} + \frac{w_g}{\bar{\rho}_g}\right)\right]$$

Calculation of liquid and gas mass flow rates:

$$w_L = \frac{(5.61)\,q_L\,\rho_L}{86{,}400} = 4.0944 \text{ lb}_m/\text{sec}$$

$$w_g = \frac{(q_L)\,(GLR)\,(\gamma_g)\,(0.0764)}{86{,}400} = 0.2874 \text{ lb}_m/\text{sec}$$

$$w_t = w_L + w_g = 4.3818 \text{ lb}_m/\text{sec}$$

Calculation of ratio of liquid mass rate to total mass rate:

$$LR = \frac{w_L}{w_t} = 0.9344$$

Calculation of ratio of gas mass rate to total mass rate:

$$GR = \frac{w_g}{w_t} = 0.0656$$

Calculation of total mass flux:

$$M_t = \frac{w_t}{A_t} = 201.9263 \text{ lb}_m\text{/sec-sq ft}$$

Calculation of correlating parameter for friction factor correlation:

$$(GR)^{0.5} = \left(\frac{d_B}{d}\right)^{1.25} \frac{M_t\, d}{\mu_g} = 2.98 \times 10^5$$

$$d_B = 0.08333 \text{ ft}$$

From Fig. 2.143, $f(LR)^{0.1} = 0.021$.

$$(LR)^{0.1} = 0.99326$$

$$f = \frac{0.021}{0.99326} = 0.0211$$

For $\Delta L' = 773$ ft, a pressure drop $\Delta P' = 15.5$ psi is assumed:

$$P_1 = 1{,}000 \text{ psig}$$
$$P_2 = 1{,}015.5 \text{ psig}$$
$$\bar{P} = 1{,}022.45 \text{ psia}$$
$$\bar{T} = 135°\text{F}$$
$$\bar{R}_s = 127 \text{ scf/bbl}$$
$$B_o = 1.077 \text{ bbl/stk bbl}$$
$$\bar{Z} = 0.885$$

Calculation of liquid and gas superficial velocities:

$$v_{sL} = 3.084 \text{ ft/sec}$$
$$v_{sg} = 3.484 \text{ ft/sec}$$
$$v_m = 6.568 \text{ ft/sec}$$

Calculation of liquid and gas densities:

$$\rho_L = 63.07 \text{ lb}_m\text{/cu ft}$$
$$\rho_g = 3.409 \text{ lb}_m\text{/cu ft}$$

$$144 \left[\frac{w_L}{\rho_L} + \frac{w_g}{\rho_g}\right] \Delta P = 313 \frac{\text{ft lb}_f}{\text{sec}}$$

$$\frac{f\, w_t\, v_m^2}{2\, g_c\, d} \Delta L' = 0.397\, \Delta L' \text{ lb}_f\text{/sec}$$

$$\Delta L' = \frac{313}{0.397} = 798 \text{ ft}$$

For $\Delta L' = 773$ ft with assumed $\Delta P' = 15.5$ psi, $\Delta P'$ was calculated to be 798 ft, which is close enough for this trial and error approach. Therefore, $\Delta P' = 15.5$ psi.

E. Calculation of total pressure drop corresponding to the true length ΔL of the inclined section:

$$\Delta P_t = \Delta P + \Delta P' = 500 + 15.5 = 515.5 \text{ psi}$$

Appendix F.2: Method of Beggs and Brill

F.21 DETAILED PROCEDURE FOR CALCULATING A PRESSURE TRAVERSE IN A DIRECTIONAL WELL BY THE METHOD OF BEGGS & BRILL

Starting with the equation:

$$\frac{\Delta p}{\Delta z} = \frac{\frac{g}{g_c}\rho_{tp}\sin\theta + \frac{f_{tp}G_m v_m}{2g_c d}}{1 - \frac{\rho_{tp} v_m v_{sg}}{g_c p}}$$

Solve the equation for the depth increment, Δz.

$$\Delta z = \frac{\Delta p\left[1 - \frac{\rho_{tp} v_m v_{sg}}{g_c p}\right]}{\frac{g}{g_c}\rho_{tp}\sin\theta + \frac{f_{tp}G_m v_m}{2 g_c d}}$$

Start with a known pressure p_1, select a value for p_2 and assume a value Δz between the two points so that temperature $\overline{T}$, and inclination angle θ, which are functions of depth, can be evaluated. Calculate the depth increment, which should coincide with the estimated value.

1. Calculate the average pressure and average depth between the two points.

$$\overline{p} \frac{p_1 + p_2}{2} \qquad \overline{z} = \text{depth where } p_1 \text{ occurs, } z + \frac{\Delta z}{2}$$

2. Determine the average temperature, $\overline{T}$ and average inclination angle, θ, at the average depth. These values must be known from temperature vs. depth and angle vs. depth surveys.
3. From PVT analysis or appropriate correlations, calculate R_s, B_o, B_w, μ_o, μ_w, μ_g, σ_o, σ_w, and z_g at $\overline{T}$ and $\overline{p}$.
4. Calculate the specific gravity of the oil γ_o.

$$\gamma_o = \frac{141.5}{131.5 + API}$$

5. Calculate the liquid and gas densities at the average conditions of pressure and temperature.

$$\rho_L = \rho_o\left(\frac{1}{1+WOR}\right) + \rho_w\left(\frac{WOR}{1+WOR}\right) = \rho_o f_o + \rho_w f_w$$

$$\rho_o = \frac{360\,\gamma_o + 0.0764\,R_s\,\gamma_g}{5.615\,B_o}$$

$$\rho_w = \frac{350\,\gamma_w}{5.615\,B_w}$$

$$\rho_g = \frac{0.0764\,\gamma_g\,\overline{p}\,(520)}{(14.7)\,(\overline{T}+460)z_g}$$

6. Calculate the in situ gas and liquid flow rates.

$$q_g = \frac{3.27\times10^{-7} z_g q_o\,(R - R_s)\,(\overline{T}+460)}{\overline{p}}$$

$$q_L = 6.49\times10^{-5}\,(q_o B_o + q_w B_w)$$

7. Calculate the in situ superficial gas, liquid, and mixture velocities.

$$v_{sL} = q_L/A_p$$
$$v_{sg} = g_g/A_p$$
$$v_m = v_{sL} + v_{sg}$$

8. Calculate the liquid, gas, and total mass flux rates.

$$G_L = \rho_L v_{sL} \qquad G_g = \rho_g v_{sg}$$
$$G_m = G_L + G_g$$

9. Calculate the input liquid content (no-slip holdup).

$$\lambda = \frac{q_L}{q_L + q_g}$$

10. Calculate the Froude number, N_{FR}, the liquid viscosity, μ_L the mixture viscosity, μ_m and the liquid surface tension, σ_L.

$$N_{FR} = \frac{v_m^2}{g\,d} \qquad \begin{aligned}\mu_L &= \mu_o f_o + \mu_w f_w\\ \mu_m &= (\mu_L\lambda + \mu_g(1-\lambda))(6.72\times10^{-4})\\ \sigma_L &= \sigma_o f_o + \sigma_w f_w\end{aligned}$$

11. Calculate the no-slip Reynolds number and the liquid velocity number.

$$N_{Rens} = \frac{G_m d}{\mu_m} \qquad N_{Lv} = 1.938\,v_{sL}\left(\frac{\rho_L}{\sigma_L}\right)^{0.25}$$

12. To determine the flow pattern which would exist if flow were horizontal, calculate the correlating parameters, L_1, L_2, L_3 and L_4.

$$L_1 = 316\lambda^{0.302} \qquad L_2 = 0.0009252\lambda^{-2.4684}$$
$$L_3 = 0.10\lambda^{-1.4516} \qquad L_4 = 0.5\lambda^{-6.738}$$

13. Determine flow pattern using the following limits:

Segregated:

$\lambda < 0.01$ and $N_{FR} < L_1$
or $\lambda \geq 0.01$ and $N_{FR} < L_2$

Transition:

$\lambda \geq 0.01$ and $L_2 < N_{FR} \leq L_2$

Intermittent:

$0.01 \leq \lambda < 0.4$ and $L_3 < N_{FR} \leq L_1$
or $\lambda \geq 0.4$ and $L_3 < N_{FR} \leq L_4$

Distributed:

$\lambda < 0.4$ and $N_{FR} \geq L_1$
or $\lambda \geq 0.4$ and $N_{FR} > L_4$

14. Calculate the horizontal holdup, $H_L(O)$.

$$H_L(O) = \frac{a\lambda^b}{N_{FR}^c}$$ where a, b, and c are determined for each flow pattern from the table:

Flow pattern	a	b	c
Segregated	0.98	0.4846	0.0868
Intermittent	0.845	0.5351	0.0173
Distributed	1.065	0.5824	0.0609

15. Calculate the inclination correction factor coefficient.

$C = (1-\lambda)\ln(d\lambda^e N_{Lv}^f N_{FR}^g)$ where d, e, f, and g are determined for each flow condition from the table:

Flow pattern	d	e	f	g
Segregated uphill	0.011	−3.768	3.539	−1.614
Intermittent uphill	2.96	0.305	−0.4473	0.0978
Distributed uphill	No Correction		C = 0	
All flow patterns downhill	4.70	−0.3692	0.1244	−0.5056

16. Calculate the liquid holdup inclination correction factor.

$$\psi = 1 + c\,[\sin(1.8\theta) - 0.333\sin^3(1.8\theta)]$$

17. Calculate the liquid holdup and the two-phase density.

$$H_L(\theta) = H_L(O)\,\psi$$
$$\rho_{tp} = \rho_L H_L + \rho_g(1 - H_L)$$

18. Calculate the friction factor ratio.

$$\frac{f_{tp}}{f_{ns}} = e^{s}, \quad \text{where} \quad S = [\ln(y)]/\{-0.0523 + 3.182\ln(y) - 0.8725[\ln(y)]^2 + 0.01853[\ln(y)]^4\},$$

and:

$$y = \frac{\lambda}{[H_L(\theta)]^2}$$

S becomes unbounded at a point in the interval $1 < y < 1.2$; and for y in this interval, the function S is calculated from

$$S = \ln(2.2\,y - 1.2).$$

19. Calculate the no-slip friction factor.

$$f_{ns} = 1/[2\log(N_{Rens}/(4.5223\log N_{Rens} - 3.8215))]^2$$

20. Calculate the two-phase friction factor.

$$f_{tp} = f_{ns}\frac{f_{tp}}{f_{ns}}$$

21. Calculate Δz.

If the estimated and calculated values for Δz are not sufficiently close, the calculated value is taken as the new estimated value and the procedure is repeated until the values agree. A new pressure increment is then chosen and the process is continued until the sum of the Δz's is equal to the well depth.

F.22 EXAMPLE PROBLEM FOR CALCULATING A PRESSURE TRAVERSE IN A DIRECTIONAL WELL BY THE METHOD OF BEGGS & BRILL

The following problem is worked by the Beggs method for a well inclined at 30° from the vertical. In order to illustrate the procedure more clearly, the distance between the 500 psig and the 1,000 psig points will be determined. In actual practice much smaller pressure increments would be chosen for increased accuracy.

Given:
$d = 1.995$ in.
$p_1 = 500$ psig
$p_2 = 1{,}000$ psig
$p_a = 14.7$ psia
Temperature $= 120 + 0.017z$
$\theta = 30°$ from vertical $= 60°$ from horizontal
$\gamma_g = 0.65$
$\gamma_w = 1.07$
$\rho_o = 22°$ API
$q_o = 400$ stk bpd
$q_w = 600$ stk bpd (WOR = 1.5)
$\mu_g = 0.018$ cp (constant)
$\sigma_o = 30$ dynes/cm (constant)
$\sigma_w = 70$ dynes/cm (constant)
GLR = 500 scf/stk bbl

1. Estimate the distance between the 500 psig and 1,000 psig points to be 2,000 ft. Calculate the average pressure and average depth.

$$\bar{p} = \frac{500 + 1{,}000}{2} + 14.7 = 764.7 \text{ psia}$$

$$\bar{z} = 0 + \frac{2{,}000}{2} = 1{,}000 \text{ ft}$$

2. Determine $\bar{T}$ and θ.

$\bar{T} = 120 + 0.017\,(1{,}000) = 137°F$
$\theta = 30°$ from vertical $= 60°$ from horizontal

3. From correlations, the following values are determined at 764.7 psia and 137°F.

$R_s = 92$ scf/stk bbl
$B_o = 1.063$
$\mu_o = 17$ cp
$\mu_w = 0.63$ cp
$z_g = 0.91$
$B_w = 1.00$

4. Calculate the oil specific gravity.

$$\gamma_o = \frac{141.5}{131.5 + 22} = 0.922$$

5. Calculate the liquid and gas densities at 764.7 psia and 137°F.

$$\rho_o = \frac{(350)(0.922) + (0.0764)(0.65)(92)}{(5.615)(1.063)} = 54.07 \text{ lb}_m/\text{cu ft}$$

$$\rho_w = \frac{(350)(1.07)}{(5.615)(1)} = 66.70 \text{ lb}_m/\text{cu ft}$$

$$f_o = \frac{1}{1 + 1.5} = 0.4 \qquad f_w = \frac{1.5}{1 + 1.5} = 0.6$$

$$\rho_L = \rho_o f_o + \rho_w f_w = (54.07)(0.4) + (66.70)(0.6) = 61.65 \text{ lb}_m/\text{cu ft}$$

$$\rho_g = \frac{(0.0764)(0.65)(764.7)(520)}{(14.7)(137 + 460)(0.91)} = 2.473 \text{ lb}_m/\text{cu ft}$$

6. Calculate the gas and liquid in situ flow rates.

$$q_g = \frac{(3.27 \times 10^{-7})(0.91)(400)(1{,}250 - 92)(137 + 460)}{764.7} = 0.1076 \text{ cu ft/sec}$$

$$q_L = (6.49 \times 10^{-7})\,[(400)(1.063) + (600)(1)] = 0.06653 \text{ cu ft/sec}$$

7. Calculate the in situ superficial liquid, gas, and mixture velocities.

$$A_p = \frac{\pi d^2}{4(144)} = \frac{\pi(1.995)^2}{4(144)} = 0.0217 \text{ sq ft}$$

$$v_{sL} = \frac{q_L}{A_p} = \frac{0.06653}{0.0217} = 3.066 \text{ ft/sec}$$

$$v_{sg} = \frac{q_g}{A_p} = \frac{0.1076}{0.0217} = 4.96 \text{ ft/sec}$$

$$v_m = v_{sL} + v_{sg} = 3.066 + 4.96 = 8.026 \text{ ft/sec}$$

8. Calculate the mass flux rates.

$G_L = \rho_L v_{sL} = (61.65)(3.066) = 189.02 \text{ lb}_m/\text{sec-sq ft}$
$G_g = \rho_g v_{sg} = (2.473)(4.96) = 12.27 \text{ lb}_m/\text{sec-sq ft}$
$G_m = G_L + G_g = 139.02 + 12.27 = 201.29 \text{ lb}_m/\text{sec-sq ft}$

9. Calculate the no-slip holdup.

$$\lambda = \frac{q_L}{q_L + q_g} = \frac{0.06653}{0.06653 + 0.1076} = 0.382$$

10. Calculate the Froude number, the liquid viscosity, mixture viscosity and liquid surface tension.

$$N_{FR} = \frac{v_m^2}{g\,d} = \frac{(4.813)^2(12)}{(32.2)(1.995)} = 4.327$$

$\mu_L = \mu_o f_o + \mu_w f_w = 17(0.4) + 0.63(0.6) = 7.18$
$\mu_m = (6.27 \times 10^{-4})(\mu_L\lambda + \mu_g(1 - \lambda))$
$= (6.27 \times 10^{-4})((7.18)(0.382) + (0.018)(0.618))$
$= 0.0017 \text{ lb}_m/\text{ft-sec}$
$\sigma_L = \sigma_o f_o + \sigma_w f_w = 30(0.4) + 70(0.6) = 54 \text{ dynes/cm}$

11. Calculate the no-slip Reynolds number and the liquid velocity number.

$$N_{Rens} = \frac{G_m\,d}{\mu_m} = \frac{(201.29)(1.995)}{(12)(0.0017)} = 19680$$

$$N_{Lv} = 1.938\ v_{sL}\left(\frac{\rho_L}{\sigma_L}\right)^{0.25} = 1.938\ (3.066)\left(\frac{61.65}{54}\right)^{0.25} = 6.142$$

12. Calculate L_1, L_2, L_3 and L_4.

$$L_1 = 316(0.382)^{0.302} = 236$$
$$L_2 = 0.0009252(0.382)^{-2.4684} = 0.010$$
$$L_3 = 0.10(0.382)^{-1.4516} = 0.404$$
$$L_4 = 0.5(0.382)^{-6.738} = 327$$

13. Determine the flow pattern. Since $0.01 \leq \lambda < 0.4$ and $L_3 < N_{FR} \leq L_1$, flow pattern is intermittent.
14. Calculate horizontal holdup.

$$H_L(O) = \frac{0.845\ \lambda^{0.5351}}{N_{FR}^{0.0173}} = \frac{0.845\ (0.382)^{0.5351}}{(4.327)^{0.0173}} = 0.492$$

15. Calculate the inclination correction factor coefficient.

$$C = (1-\lambda)\ \ln\ (2.96\ \lambda^{0.305}\ N_{Lv}^{-0.4473} N_{FR}^{0.0978})$$
$$= (1-0.382)\ \ln\ (2.96(0.382)^{0.305}\ (6.142)^{-0.4473}\ (4.327)^{0.0978})$$
$$= 0.076$$

16. Calculate the liquid holdup inclination correction factor.

$$\psi = 1 + C\ [\sin(1.8\theta) - 0.333\ \sin^3\ (1.8\theta)]$$
$$= 1 + (0.076)\ [\sin((1.8)(60)) - 0.333\ \sin^3\ ((1.8)(60))]$$
$$= 1.051$$

17. Calculate the liquid holdup and two-phase density.

$$H_L(60) = H_L(O)\psi = (0.492)(1.051) = 0.517$$
$$\rho_{tp} = \rho_L H_L + \rho_g(1 - H_L) = (61.65)(0.517) + (2.473)(1 - 0.517)$$
$$= 33.07\ lb_m/cu\ ft$$

18. Calculate the friction factor ratio.

$$y = \left[\frac{\lambda}{(H_L(60))^2}\right] = \left[\frac{0.382}{(0.517)^2}\right] = 1.429$$

$$\ln y = 0.357$$

$$\frac{f_{tp}}{f_{ns}} = EXP\ (0.357/(-0.0523 + 3.182(0.357) - 0.8725(0.327)^2 + 0.01853\ (0.357)^4))$$
$$= EXP\ (0.367) = 1.443$$

19. Calculate the no-slip friction factor.

$$f_{ns} = 1/[2\ \log\ (N_{Rens}/(4.5223\ \log\ N_{Rens} - 3.8215))]^2$$
$$= 1/[2\ \log\ (19680/(4.5223\ \log\ (19680) - 3.8215))]^2$$
$$f_{ns} = 0.0261$$

20. Calculate the two-phase friction factor.

$$f_{tp} = f_{ns}\frac{f_{tp}}{f_{ns}} = (0.0261)(1.443) = 0.0377$$

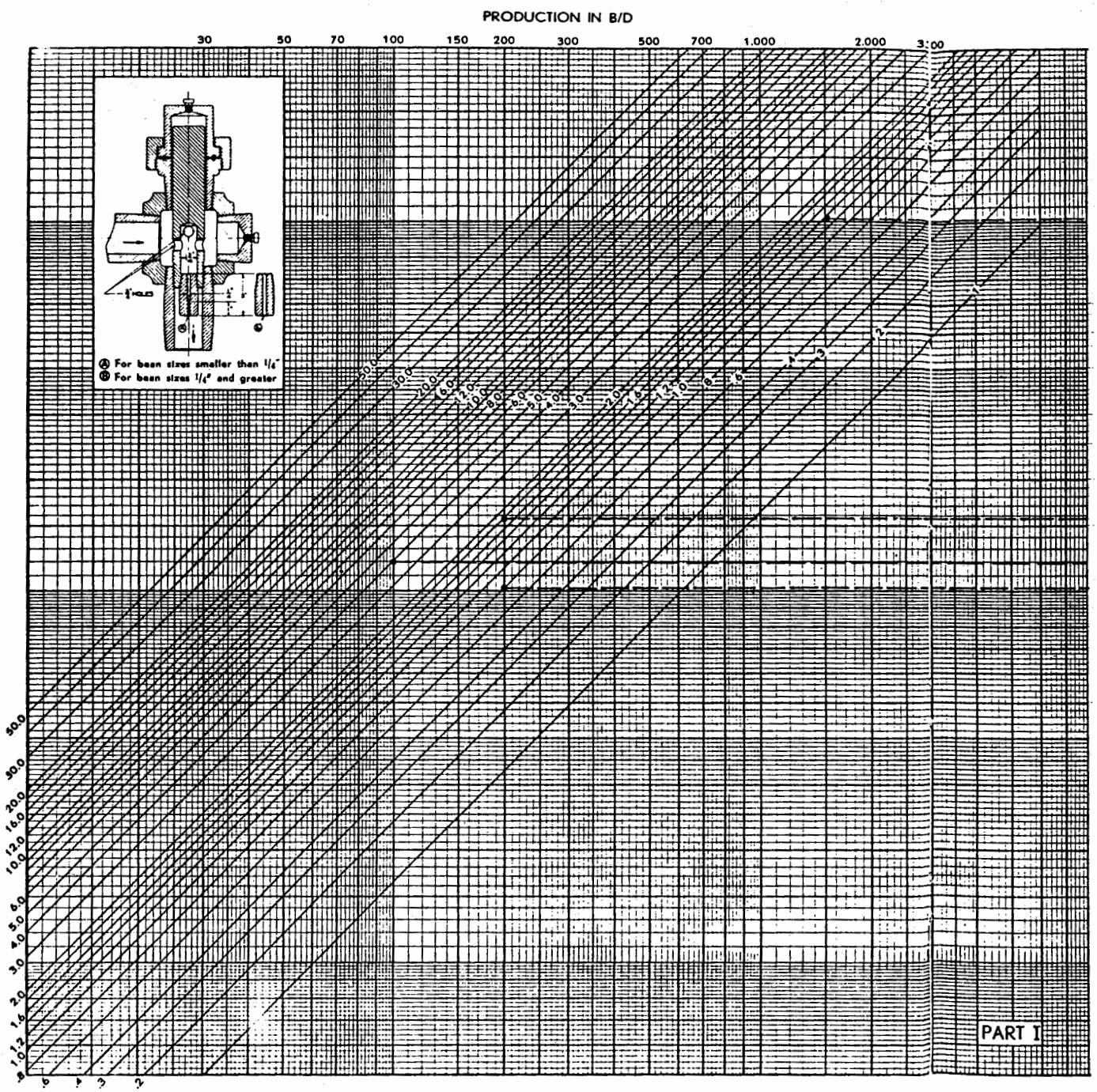
21. Calculate Δz.

$$\Delta z = \frac{\Delta p\left[1 - \frac{\rho_{tp}\ v_m\ v_{sg}}{g_c\ p}\right]}{\frac{g}{g_c}\rho_{tp}\ \sin\ \theta + \frac{f_{tp}\ G_m\ v_m}{2\ g_c\ d}}$$
$$= \frac{(500)(144)\left[1 - \frac{(33.07)(8.026)(4.96)}{(32.17)(764.7)(144)}\right]}{(33.07)(0.866) + \frac{(0.0377)(197.66)(8.026)}{(2)(32.17)(0.1663)}}$$

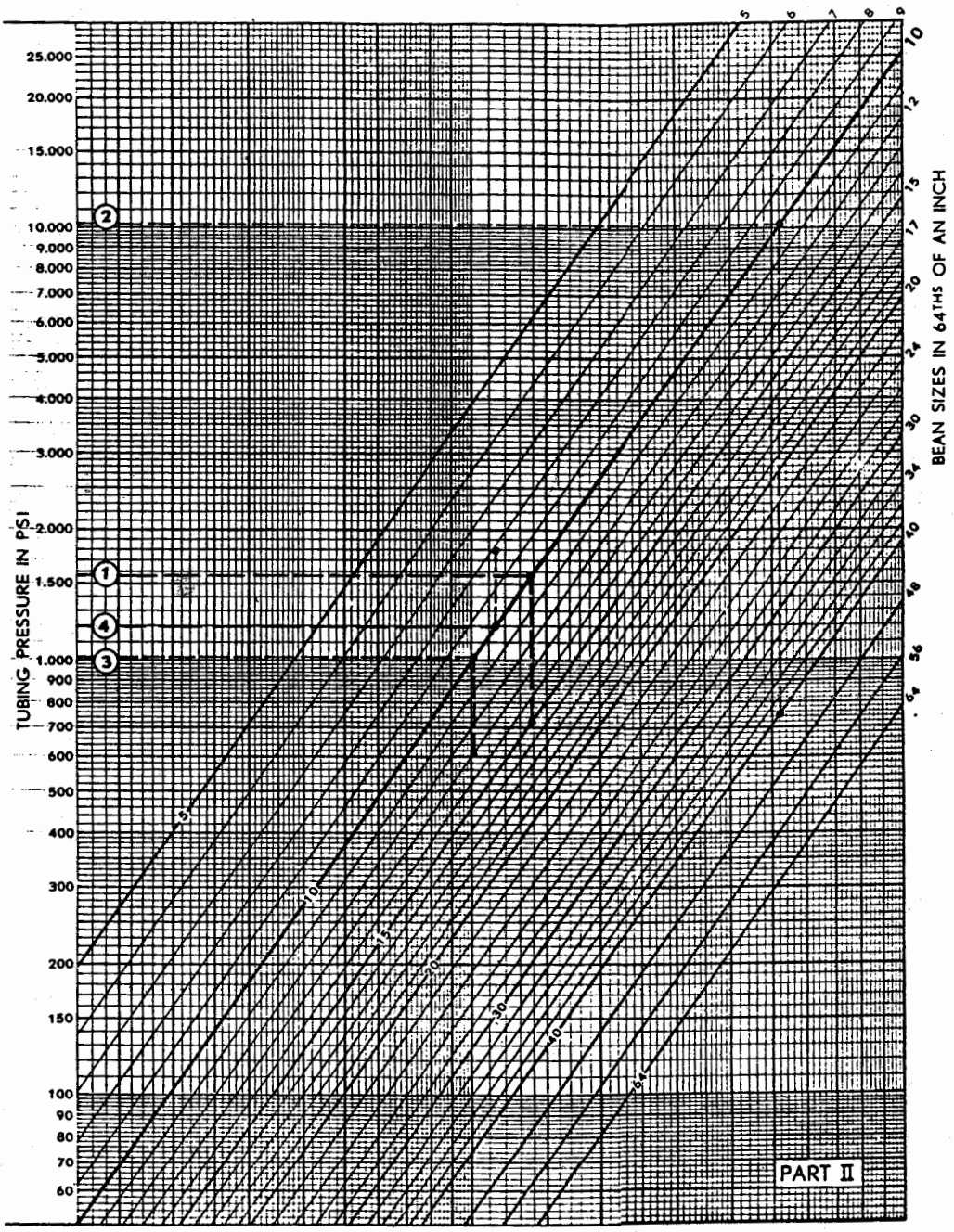
$$\Delta z = 2{,}450\ ft.$$

This value is not close enough to the estimated value of 2,000 ft. Therefore, the new estimated distance between the 500 and 1,000 psig pressure points would be taken as 2,450 ft and the procedure repeated from Step 2. Once agreement is reached another pressure increment is chosen and the entire procedure repeated until the well depth is reached.

Appendix G



G.1 Bean Performance Chart (after Gilbert).


TUBING PRESSURE IN PSI
25.000
20.000
15.000
10.000
9.000
8.000
7.000
6.000
5.000
4.000
3.000
2.000
1.500
1.000
900
800
700
600
500
400
300
200
150
100
90
80
70
60
BEAN SIZES IN 64THS OF AN INCH
5
6
7
8
9
10
12
15
17
20
24
30
34
40
48
56
64
1
2
3
4
PART II

Explanation of Figure G-1

USE OF BEAN-PERFORMANCE CHART (AFTER GILBERT[2])

A graphical means of estimating the bean performance is given in Fig. G-1. Four variables are considered, and when three of these variables are known, the fourth may be estimated. In construction, Part I of the chart represents the performance of a 10/64 in. bean and Part II is simply a means of correcting results for other bean sizes. In solving for any one of the four variables, enter Part I if both the barrels per day and the gas-liquid ratio are known; or enter Part II of the chart if the tubing pressure and bean size are known. The further procedure in solving for each variable may be best explained by examples.

1. ESTIMATE GAS-LIQUID RATIO OR THE GAS RATE

   Suppose we have a well equipped with a 15/64 in. bean operating with a tubing pressure of 720 psi at 200 bbl per day. No differential recording meter has been installed and an estimate of the gas production is desired.
   SOLUTION: First find in Part II of the chart, the intersection of the line for the 15 bean at 720 psi; go vertically to the slanting 10-bean line and then horizontally to the vertical 200 bbl per day line in Part I. At this point the gas-liquid ratio is about 1.8 Mcf per bbl, from which the total gas is 360 Mcf per day. (200×1.8).

2. ESTIMATE PRODUCTION RATE

   After a bean change, a new well which showed a ratio of 1,400 cu ft per bbl on the previous gage, is flowing through a 40 bean with 750 psi tubing pressure. What is the probable production rate?
   SOLUTION: Enter Part II at the intersection of the 40-bean and 750-psi lines and go vertically to the 10 line; then go horizontally to 1.4 Mcf per bbl—the estimated production rate being found to be 1,530 bbl per day.

3. ESTIMATE TUBING PRESSURE

   Establish the performance of a 13 bean for 0.8 Mcf per bbl.
   SOLUTION: Since the formula indicates that there is a straight-line relationship between tubing pressure and barrels per day, only one point need be established. Thus, going horizontally from the intersection in Part I of the 0.8 ratio and, say, 200 bbl per day, to the 10-bean line in Part II and thence vertically to the 13-bean line, the pressure is found to be 605 psi and for any production rate with a 13 bean and 0.8 ratio, the tubing pressure in psi is equal to the barrels per day multiplied by 605/200 or 3.03.

4. ESTIMATE BEAN SIZE

   A well has been operating for an extended period at 200 bbl per day and 4.0 Mcf per bbl ratio. It is desired to reduce the rate to 100 bbl per day and the tubing outlet performance curve indicates that the tubing pressure at this rate will be 1,800 psi. What size bean should be used?
   SOLUTION: Enter Part I and go horizontally from the intersection of the 100 bbl per day and 4.0 ratio lines to the 10-bean line in Part II and thence vertically to the horizontal 1,800-psi line. An 8/64 in. bean should be used.

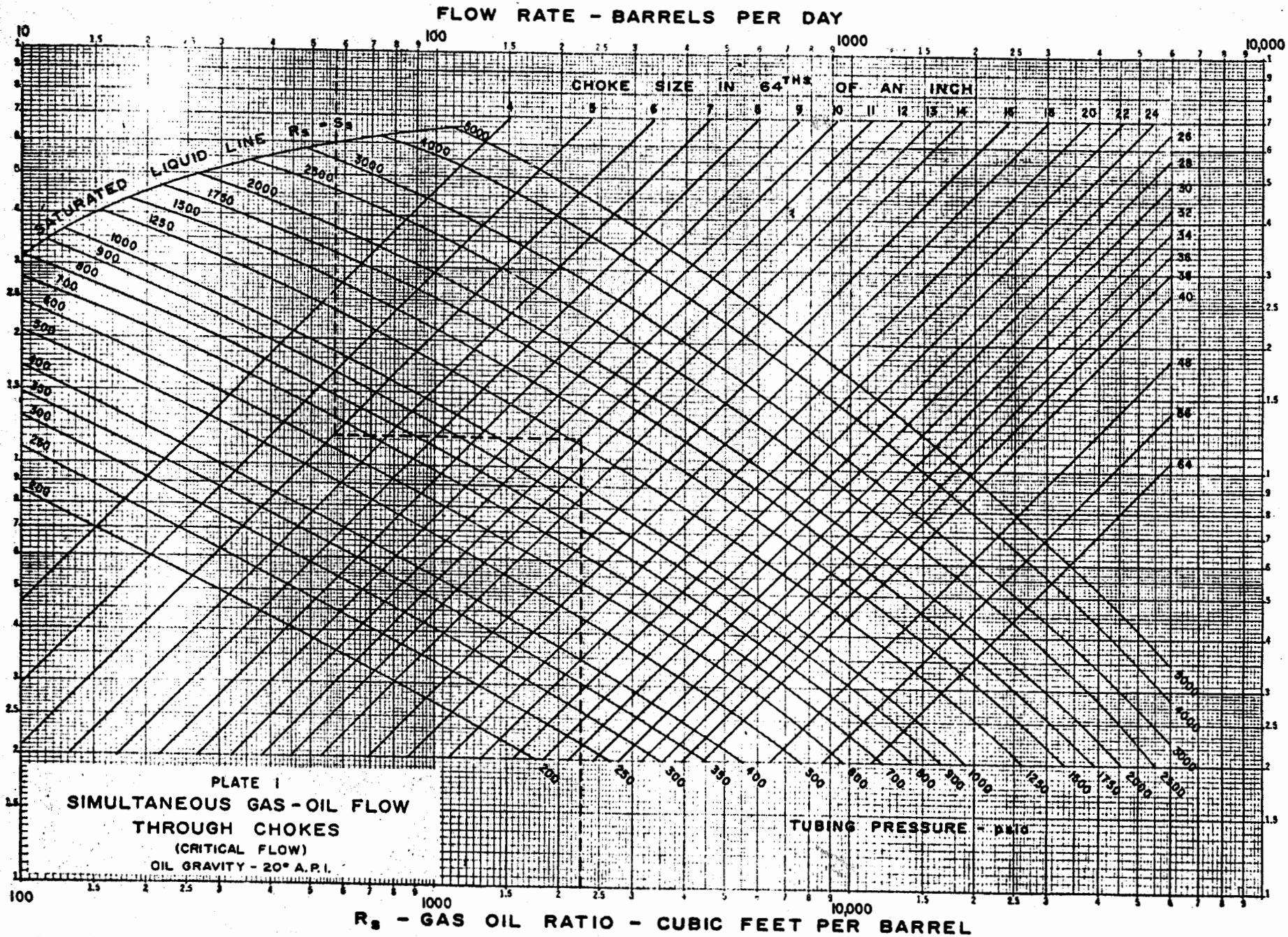


Fig. G.2 Bean Performance Chart (after Poettmann and Beck).

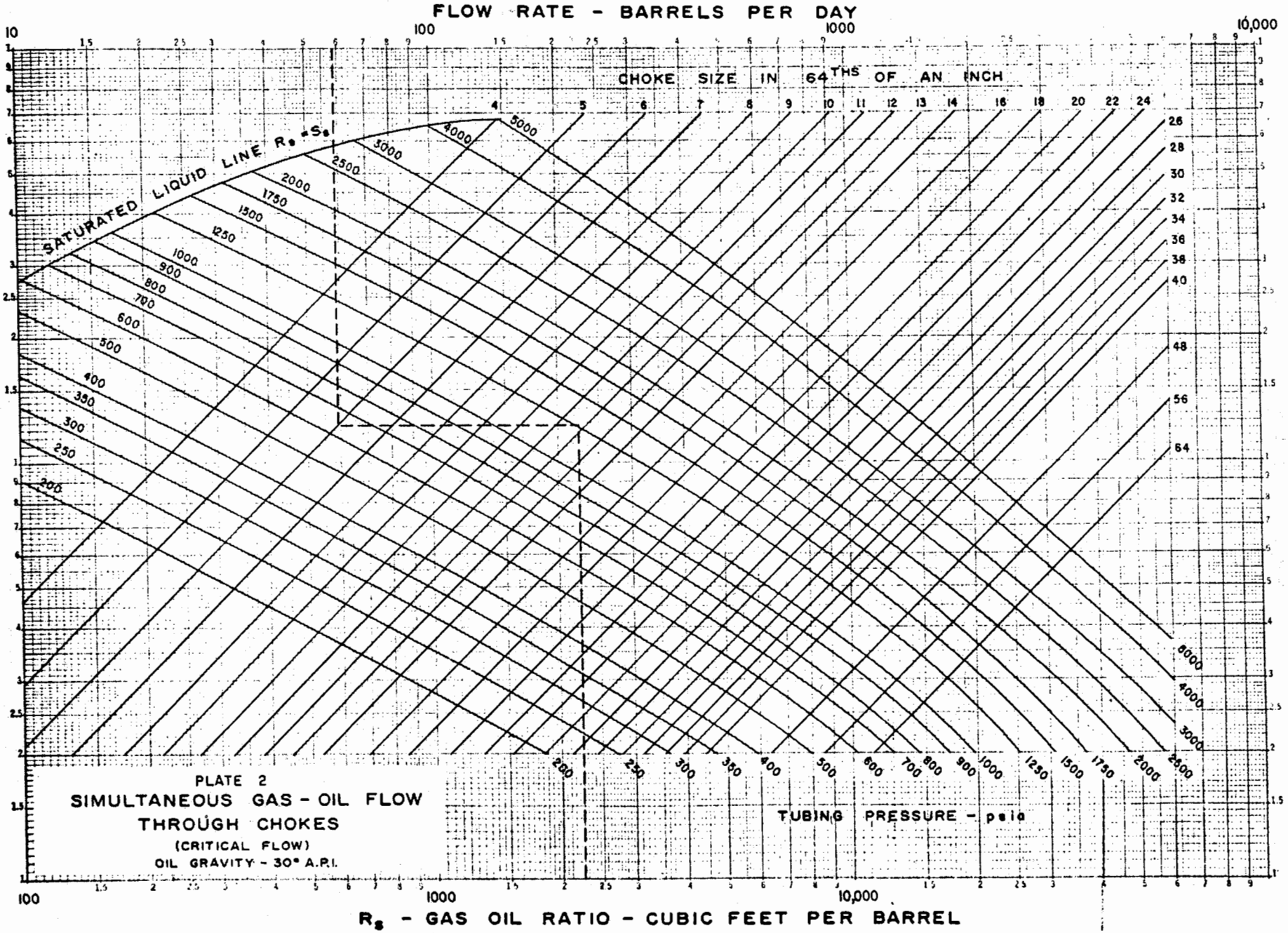


Fig. G.3 Bean Performance Chart (after Poettmann and Beck).

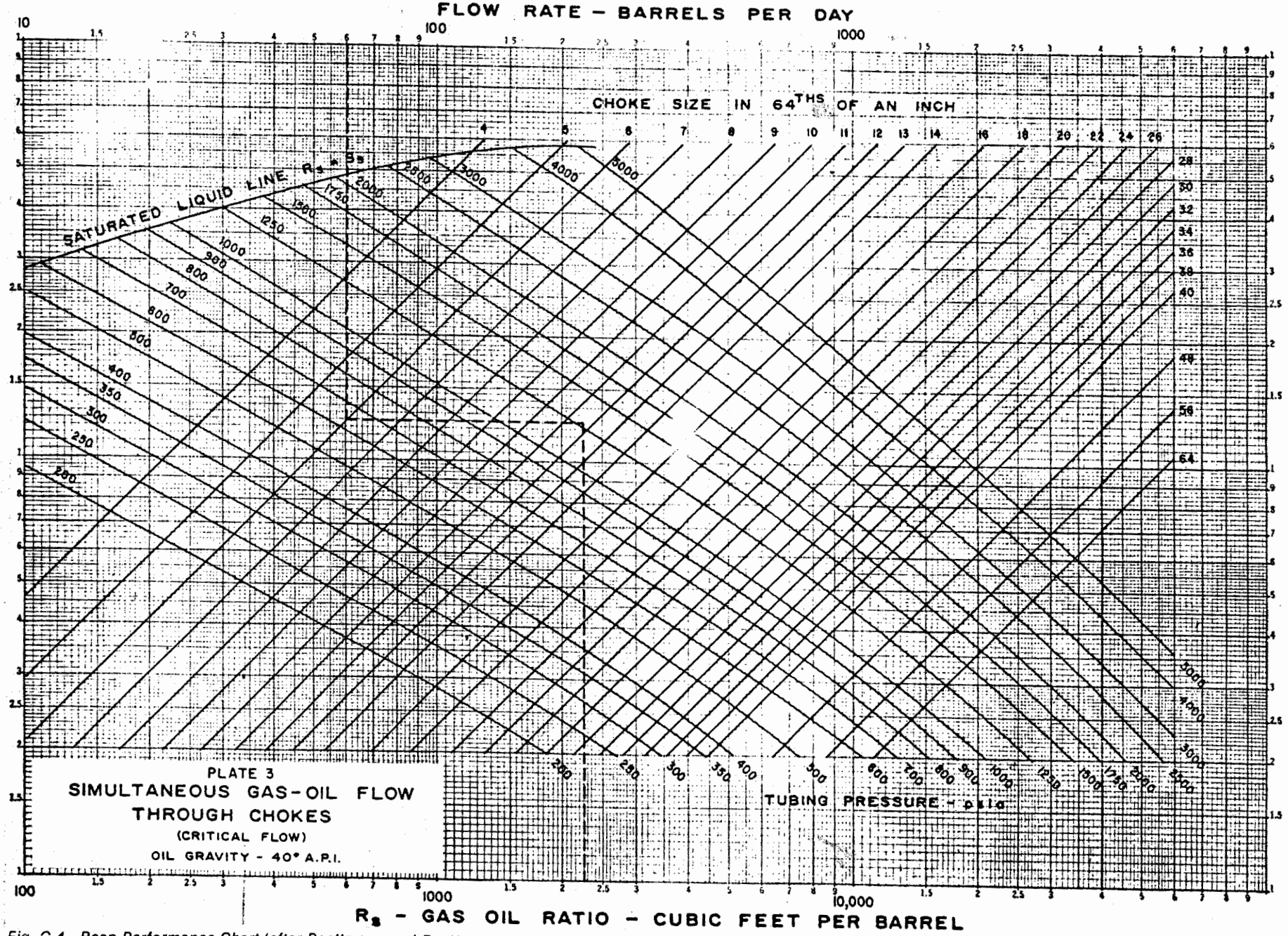


Fig. G.4 Bean Performance Chart (after Poettmann and Beck).

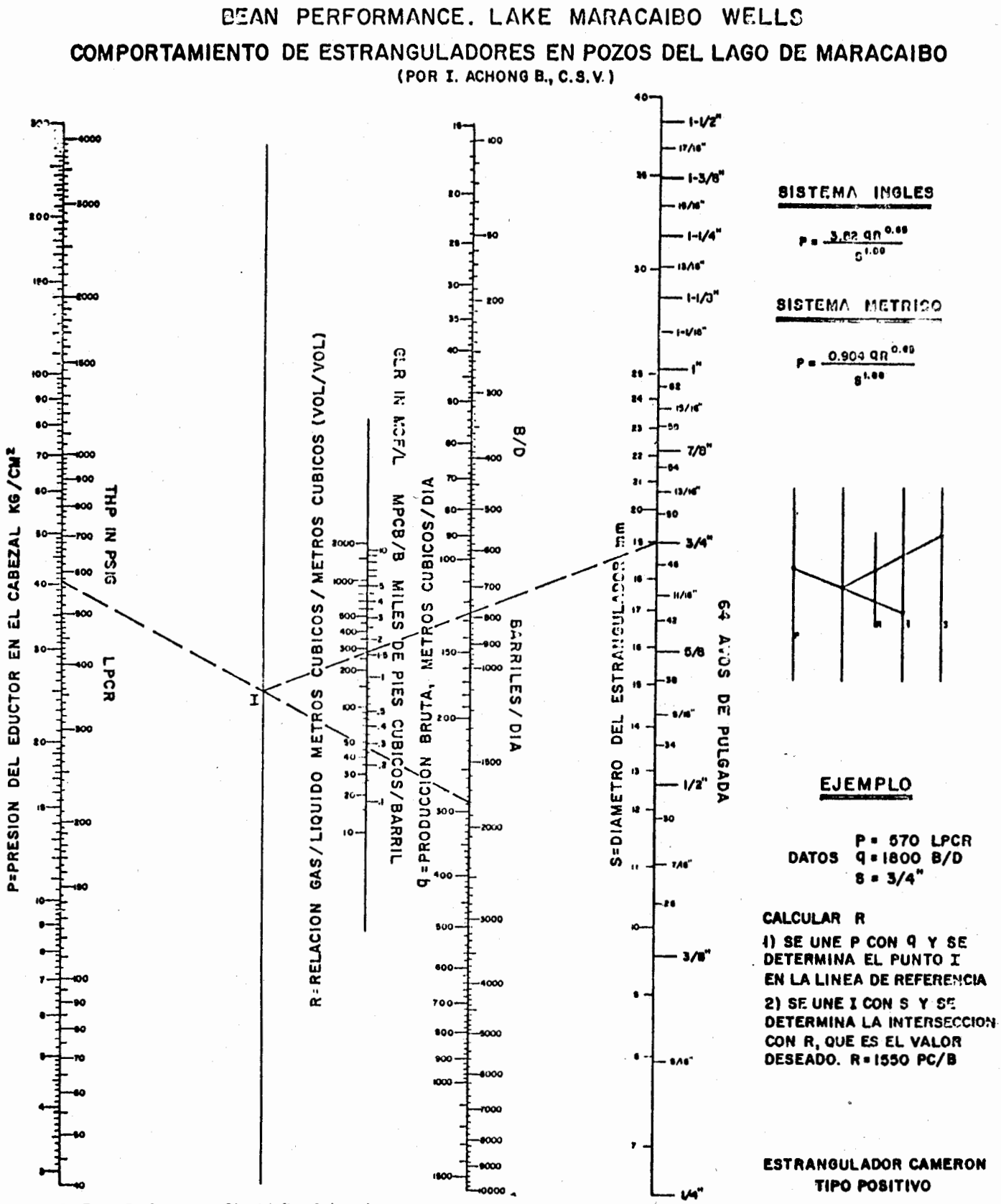
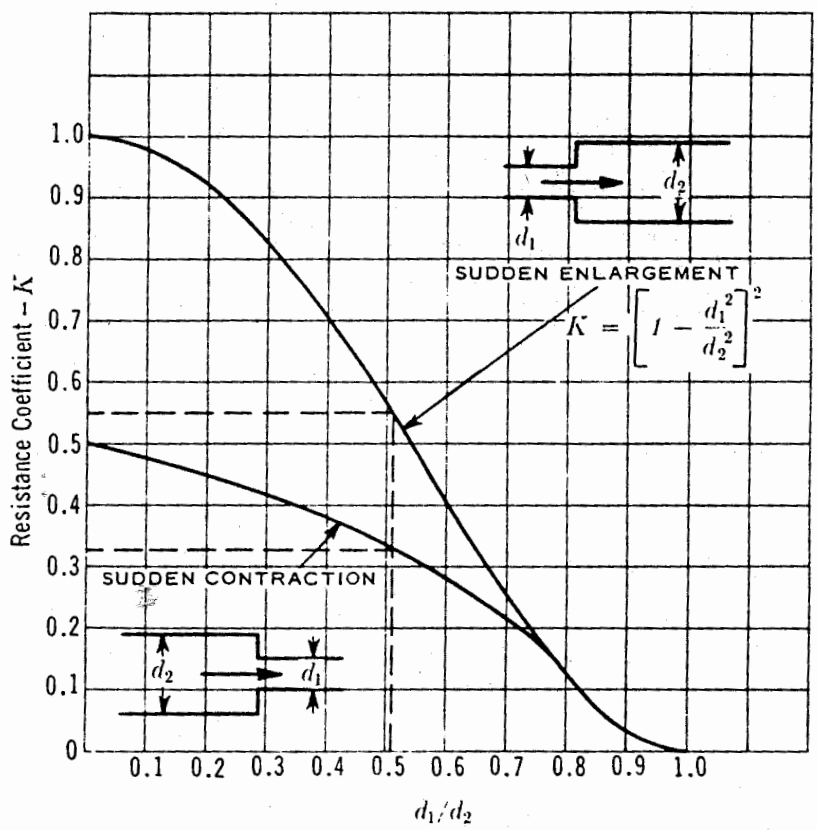


Fig. G.5 Bean Performance Chart (after Achong).

Resistance Due to Sudden Enlargements and Contractions[20]



Sudden enlargement: The resistance coefficient K for a sudden enlargement from 6-inch Schedule 40 pipe to 12-inch Schedule 40 pipe is **0.55**, based on the 6-inch pipe size.

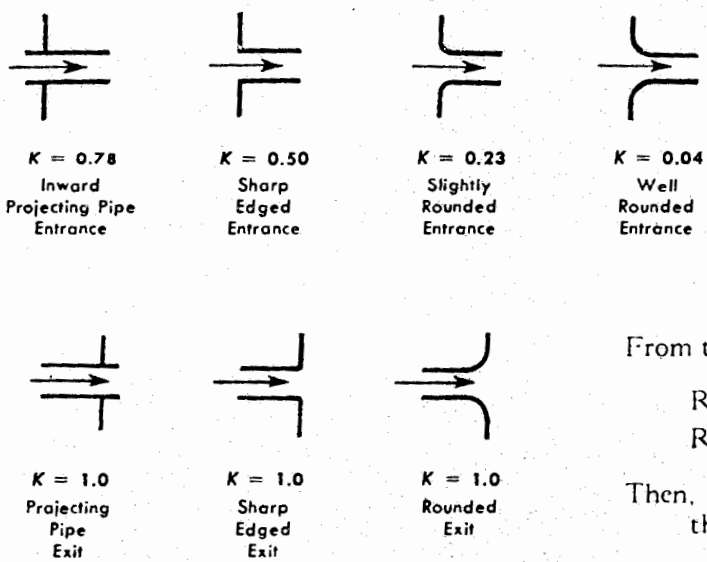
$$\frac{d_1}{d_2} = \frac{6.065}{11.938} = 0.51$$

Sudden contraction: The resistance coefficient K for a sudden contraction from 12-inch Schedule 40 pipe to 6-inch Schedule 40 pipe is **0.33**, based on the 6-inch pipe size.

$$\frac{d_1}{d_2} = \frac{6.065}{11.938} = 0.51$$

Note: The values for the resistance coefficient, K, are based on velocity in the small pipe. To determine K values in terms of the greater diameter, multiply the chart values by $(d_2/d_1)^4$.

Resistance Due to Pipe Entrance and Exit



Problem: Determine the total resistance coefficient for a pipe one diameter long having a sharp edged entrance and a sharp edged exit.

Solution: The resistance of pipe one diameter long is small and can be neglected ($K = f\,L/D$).

From the diagrams, note:

Resistance for a sharp edged entrance = 0.5
Resistance for a sharp edged exit = 1.0

Then,
the total resistance, K, for the pipe = **1.5**

Fig. G.6 Resistance in pipe (after Crane).

Resistance of 90 Degree Bends[21]

The chart at the right shows the resistance of 90 degree bends to the flow of fluids in terms of equivalent lengths of straight pipe.

Resistance of bends greater than 90 degrees is found using the formula:

$$\frac{L}{D} = R_t + (n-1)\left(R_l + \frac{R_b}{2}\right)$$

n = total number of 90° bends in coil
R_t = total resistance due to one 90° bend, in L/D
R_l = resistance due to length of one 90° bend, in L/D
R_b = bend resistance due to one 90° bend, in L/D

Problem: Determine the equivalent lengths in pipe diameters of a 90 degree bend and a 270 degree bend having a relative radius of 12.

Solution: Referring to the "Total Resistance" curve, the equivalent length for a 90 degree bend is **34.5** pipe diameters.

The equivalent length of a 270 degree bend is:

$$L/D = 34.5 + (3-1)\,[18.7 + (15.8 \div 2)]$$
$$L/D = \mathbf{87.7} \text{ pipe diameters}$$

Note: This loss is less than the sum of losses through three 90 degree bends separated by tangents. For "resistance of bends theory", see page 2-12.

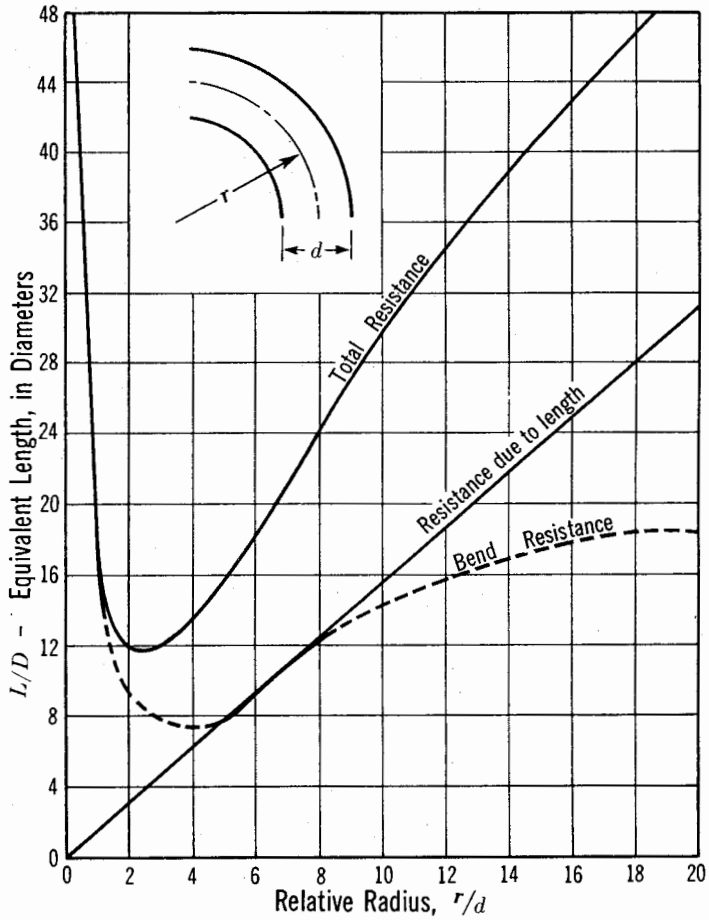


Chart for Resistance of 90 Degree Bends

From *Pressure Losses for Fluid Flow in 90 Degree Pipe Bends* by K. H. Beij. Courtesy of Journal of Research of National Bureau of Standards, Vol. 21, July, 1938.

Resistance of Miter Bends[4]

The chart at the lower right shows the resistance of miter bends to the flow of fluids. The chart is based on data published by the American Society of Mechanical Engineers (ASME).

Problem: Determine the equivalent length in pipe diameters of a 40 degree miter bend.

Solution: Referring to the "Total Resistance" curve in the chart, the equivalent length is 12 pipe diameters.

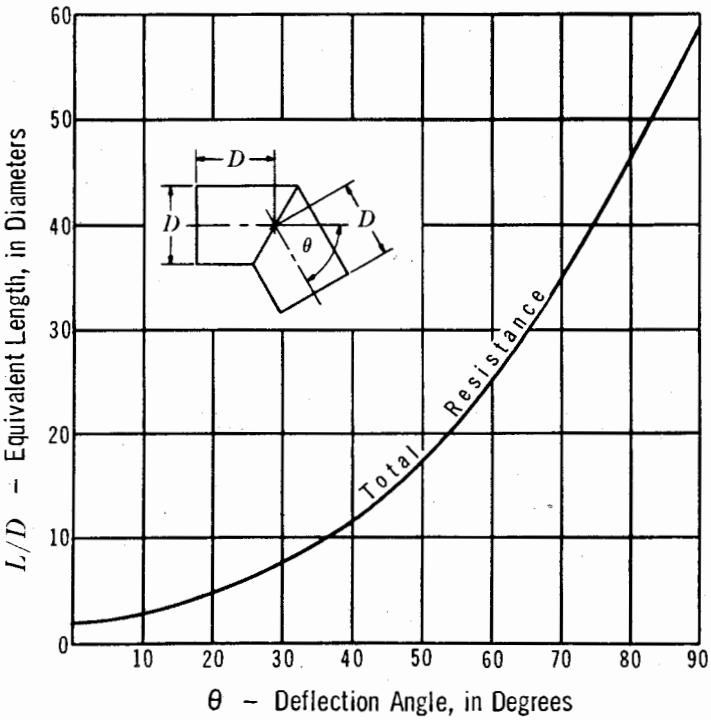


Chart for Resistance of Miter Bends

Fig. G.7 Resistance of bends (after Crane).

Schedule (Thickness) of Steel Pipe Used in Obtaining Resistance Of Valves and Fittings of Various Pressure Classes by Test*

Valve or Fitting ASA Pressure Classification (Steam Rating)		Schedule No. of Pipe (Thickness)
250-Pound and Lower		Schedule 40
300-Pound to 600-Pound		Schedule 80
900-Pound		Schedule 120
1500-Pound		Schedule 160
2500-Pound	Sizes ½ to 6-inch	xx (Double Extra Strong)
	Sizes 8-inch and larger	Schedule 160

*These schedule numbers have been arbitrarily selected only for the purpose of identifying the various pressure classes of valves and fittings with specific pipe dimensions for the interpretation of flow test data; they should not be construed as a recommendation for installation purposes.

Representative Equivalent Length‡ in Pipe Diameters (*L/D*) Of Various Valves and Fittings

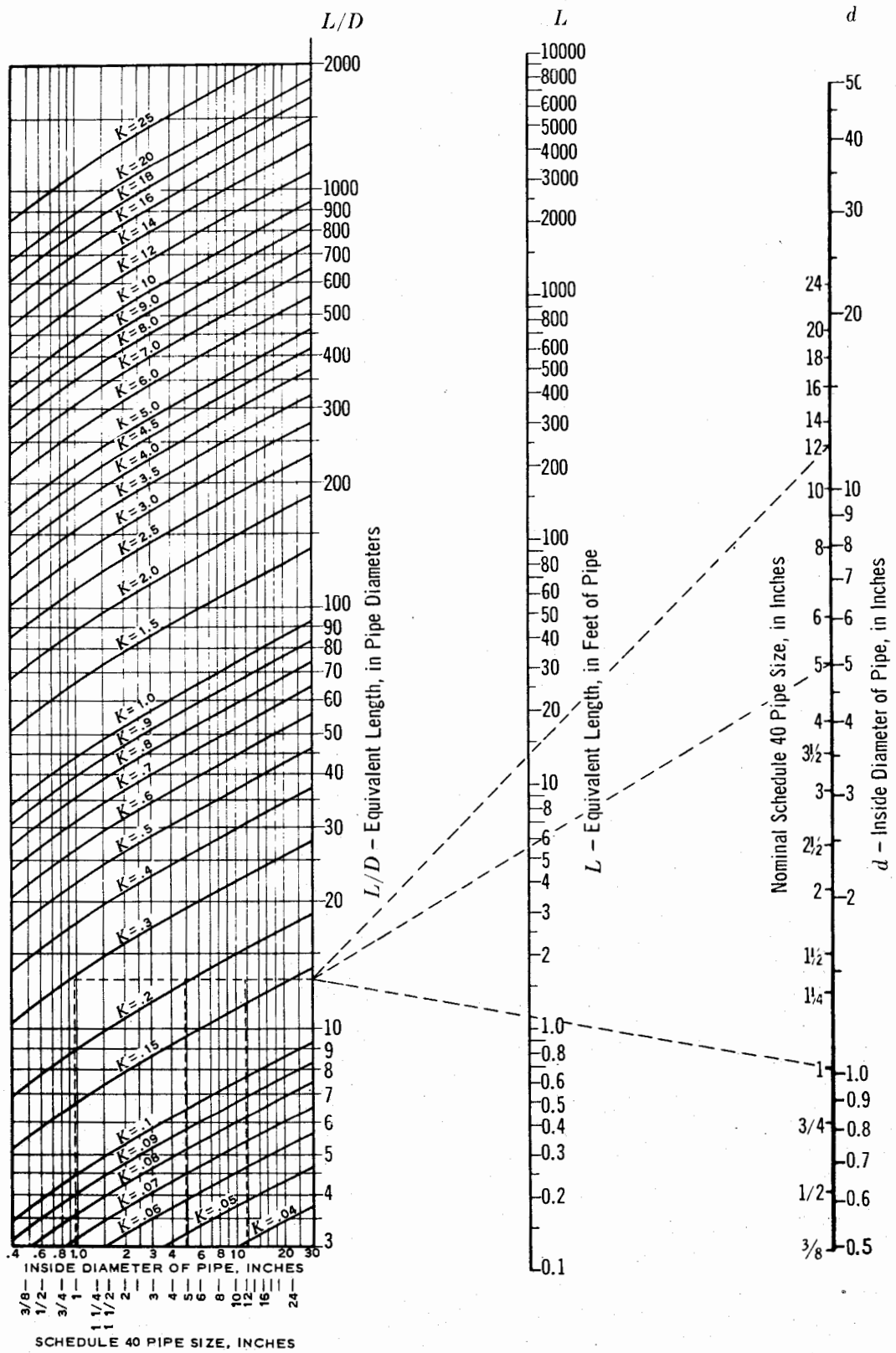
Description of Product				Equivalent Length In Pipe Diameters (*L/D*)
Globe Valves	Conventional	With no obstruction in flat, bevel, or plug type seat	Fully open	340
		With wing or pin guided disc	Fully open	450
	Y-Pattern	(No obstruction in flat, bevel, or plug type seat)		
		– With stem 60 degrees from run of pipe line	Fully open	175
		– With stem 45 degrees from run of pipe line	Fully open	145
Angle Valves	Conventional	With no obstruction in flat, bevel, or plug type seat	Fully open	145
		With wing or pin guided disc	Fully open	200
Gate Valves	Conventional Wedge Disc, Double Disc, or Plug Disc		Fully open	13
			Three-quarters open	35
			One-half open	160
			One-quarter open	900
	Pulp Stock		Fully open	17
			Three-quarters open	50
			One-half open	260
			One-quarter open	1200
	Conduit Pipe Line		Fully open	3**
Check Valves	Conventional Swing		0.5†... Fully open	135
	Clearway Swing		0.5†... Fully open	50
	Globe Lift or Stop		2.0†... Fully open	Same as Globe
	Angle Lift or Stop		2.0†... Fully open	Same as Angle
	In-Line Ball	2.5 vertical and 0.25 horizontal	†... Fully open	150
Foot Valves with Strainer		With poppet lift-type disc	0.3†... Fully open	420
		With leather-hinged disc	0.4†... Fully open	75
Butterfly Valves (8-inch and larger)			Fully open	40
Cocks	Straight-Through	Rectangular plug port area equal to 100% of pipe area	Fully open	18
	Three-Way	Rectangular plug port area equal to 80% of pipe area (fully open)	Flow straight through	44
			Flow through branch	140
Fittings	90 Degree Standard Elbow			30
	45 Degree Standard Elbow			16
	90 Degree Long Radius Elbow			20
	90 Degree Street Elbow			50
	45 Degree Street Elbow			26
	Square Corner Elbow			57
	Standard Tee	With flow through run		20
		With flow through branch		60
	Close Pattern Return Bend			50
Pipe	90 Degree Pipe Bends			See Page A-27
	Miter Bends			See Page A-27
	Sudden Enlargements and Contractions			See Page A-26
	Entrance and Exit Losses			See Page A-26

**Exact equivalent length is equal to the length between flange faces or welding ends.

†Minimum calculated pressure drop (psi) across valve to provide sufficient flow to lift disc fully.

‡For limitations, see page 2-11. For effect of end connections, see page 2-10.

Fig. G.8 Equivalent lengths (after Crane).

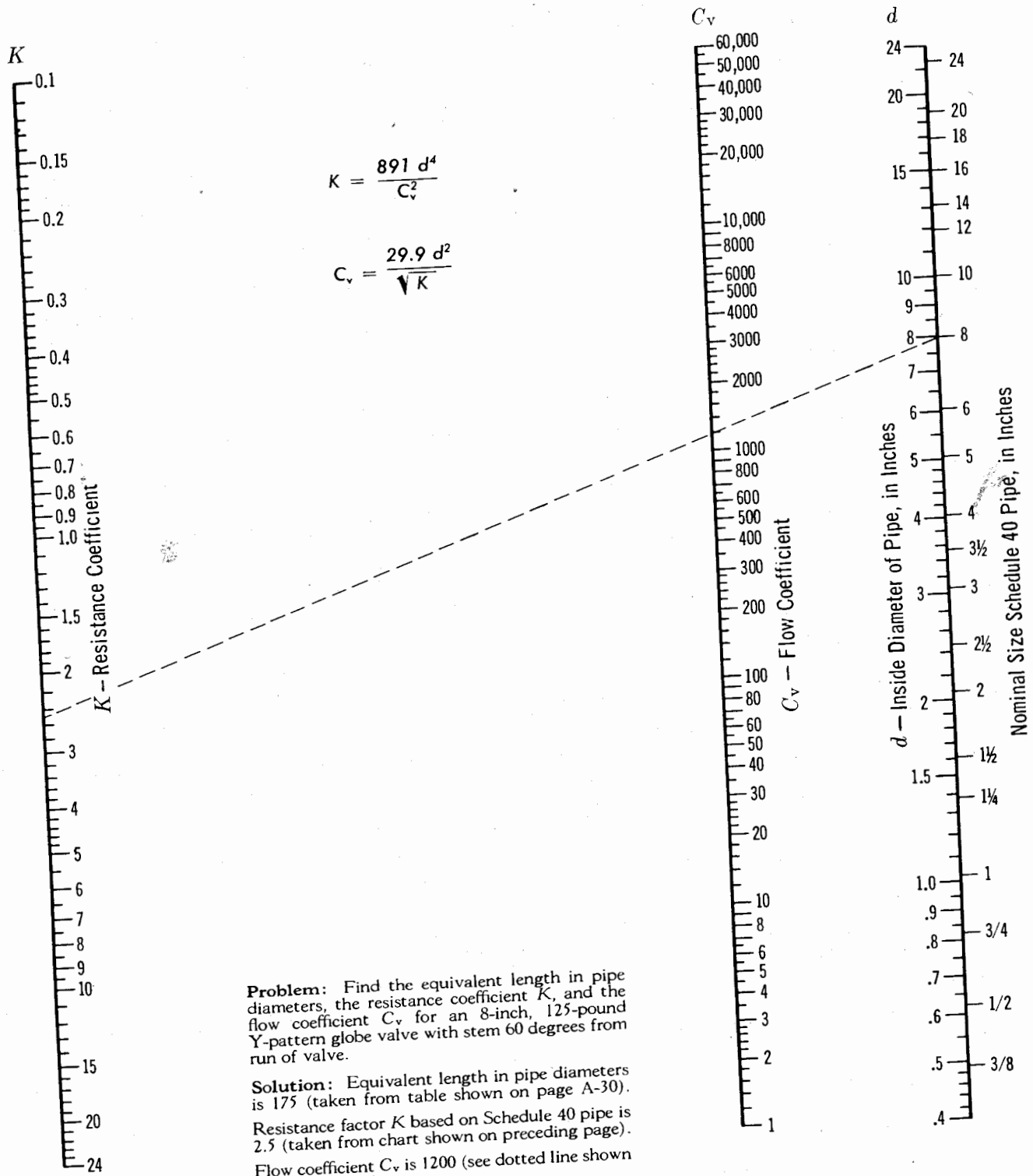


Problem: Find the equivalent length in pipe diameters and feet of Schedule 40 pipe, and the resistance factor K for 1, 5, and 12-inch fully-opened gate valves.

Solution

Valve Size	1″	5″	12″	*Refer to*
Equivalent length, pipe diameters	13	13	13	Page A-30
Equivalent length, feet of Sched. 40 pipe	1.1	5.5	13	Dotted lines on chart.
Resist. factor K, based on Sched. 40 pipe	0.30	0.20	0.17	

Fig. G.9 Equivalent lengths L and L/D and resistance coefficient K (after Crane).



Problem: Find the equivalent length in pipe diameters, the resistance coefficient K, and the flow coefficient C_v for an 8-inch, 125-pound Y-pattern globe valve with stem 60 degrees from run of valve.

Solution: Equivalent length in pipe diameters is 175 (taken from table shown on page A-30).

Resistance factor K based on Schedule 40 pipe is 2.5 (taken from chart shown on preceding page).

Flow coefficient C_v is 1200 (see dotted line shown on chart above).

Fig. G.10 Equivalents of resistance coefficient K and flow coefficient C (after Crane).

Appendix H

Year	Factor for Year	Equal Payments	5% Per Yr. Decline	10% Per Yr. Decline	15% Per Yr. Decline	20% Per Yr. Decline	30% Per Yr. Decline	40% Per Yr. Decline	50% Per Yr. Decline
1	.97590	.97590	.97590	.97590	.97590	.97590	.97590	.97590	.97590
2	.92943	.95266	.95326	.95389	.95455	.95525	.95676	.95847	.96041
3	.88517	.93017	.93172	.93335	.93506	.93687	.94075	.94501	.94966
4	.84302	.90838	.91122	.91420	.91732	.92059	.92751	.93489	.94255
5	.80288	.88728	.89171	.89636	.90122	.90625	.91672	.92747	.93805
6	.76464	.86684	.87316	.87976	.88662	.89367	.90803	.92215	.93529
7	.72823	.84704	.85550	.86433	.87343	.88269	.90112	.91843	.93366
8	.69355	.82785	.83870	.84999	.86154	.87316	.89567	.91587	.93272
9	.66053	.80926	.82271	.83667	.85085	.86492	.89144	.91414	.93219
10	.62907	.79124	.80751	.82432	.84126	.85783	.88817	.91298	.93189
11	.59912	.77378	.79304	.81288	.83267	.85175	.88567	.91222	.93173
12	.57059	.75684	.77928	.80228	.82500	.84656	.88377	.91172	.93164
13	.54342	.74043	.76618	.79248	.81817	.84215	.88235	.91140	.93159
14	.51754	.72451	.75372	.78342	.81209	.83842	.88128	.91119	.93157
15	.49290	.70907	.74187	.77505	.80670	.83527	.88048	.91106	.93156
16	.46942	.69409	.73060	.76733	.80192	.83262	.87990	.91098	.93155
17	.44707	.67956	.71988	.76020	.79771	.83040	.87946	.91093	.93154
18	.42578	.66546	.70968	.75364	.79398	.82855	.87915	.91089	.93154
19	.40551	.65178	.69998	.74760	.79071	.82700	.87892	.91087	.93154
20	.38620	.63850	.69075	.74204	.78783	.82571	.87875	.91086	.93154
21	.36781	.62561	.68197	.73693	.78531	.82465	.87862	.91085	.93154
22	.35029	.61309	.67362	.73224	.78309	.82377	.87854	.91085	.93154
23	.33361	.60094	.66568	.72793	.78116	.82304	.87847	.91084	.93154
24	.31773	.58914	.65813	.72398	.77947	.82244	.87843	.91084	.93154
25	.30260	.57768	.65094	.72036	.77800	.82195	.87839	.91084	.93154
26	.28819	.56655	.64411	.71705	.77672	.82154	.87837	.91084	.93154
27	.27446	.55573	.63762	.71401	.77560	.82121	.87835	.91084	.93154
28	.26139	.54522	.63144	.71123	.77463	.82094	.87834	.91084	.93154
29	.24895	.53500	.62556	.70869	.77379	.82072	.87833	.91084	.93154
30	.23709	.52507	.61997	.70637	.77306	.82054	.87833	.91084	.93154
31	.22580	.51542	.61466	.70426	.77243	.82039	.87832	.91084	.93154
32	.21505	.50603	.60961	.70232	.77189	.82027	.87832	.91084	.93154
33	.20481	.49690	.60480	.70056	.77142	.82017	.87832	.91084	.93154
34	.19506	.48802	.60023	.69895	.77101	.82009	.87831	.91084	.93154
35	.18577	.47939	.59589	.69749	.77066	.82003	.87831	.91084	.93154
36	.17692	.47099	.59176	.69616	.77036	.81998	.87831	.91084	.93154
37	.16850	.46281	.58783	.69494	.77010	.81993	.87831	.91084	.93154
38	.16047	.45485	.58409	.69384	.76987	.81990	.87831	.91084	.93154
39	.15283	.44711	.58054	.69284	.76968	.81987	.87831	.91084	.93154
40	.14555	.43957	.57717	.69192	.76951	.81985	.87831	.91084	.93154
41	.13862	.43223	.57396	.69109	.76937	.81983	.87831	.91084	.93154
42	.13202	.42508	.57091	.69034	.76925	.81982	.87831	.91084	.93154
43	.12573	.41812	.56801	.68966	.76914	.81981	.87831	.91084	.93154
44	.11975	.41134	.56525	.68904	.76905	.81980	.87831	.91084	.93154
45	.11404	.40473	.56262	.68848	.76898	.81979	.87831	.91084	.93154
46	.10861	.39830	.56013	.68797	.76891	.81978	.87831	.91084	.93154
47	.10344	.39202	.55776	.68750	.76885	.81978	.87831	.91084	.93154
48	.09852	.38591	.55551	.68708	.76881	.81977	.87831	.91084	.93154
49	.09383	.37995	.55337	.68670	.76876	.81977	.87831	.91084	.93154
50	.08936	.37414	.55133	.68636	.76873	.81977	.87831	.91084	.93154

Fig. H.1 Present worth discount factors at 5% for income received in one payment at middle of year.

Year	Factor for Year	Equal Payments	5% Per Yr. Decline	10% Per Yr. Decline	15% Per Yr. Decline	20% Per Yr. Decline	30% Per Yr. Decline	40% Per Yr. Decline	50% Per Yr. Decline
1	.97129	.97129	.97129	.97129	.97129	.97129	.97129	.97129	.97129
2	.91631	.94380	.94450	.94524	.94603	.94685	.94865	.95067	.95296
3	.86444	.91734	.91917	.92109	.92311	.92524	.92981	.93483	.94031
4	.81551	.89189	.89521	.89871	.90238	.90620	.91433	.92299	.93199
5	.76935	.86738	.87256	.87799	.88365	.88953	.90178	.91435	.92675
6	.72580	.84378	.85112	.85881	.86678	.87499	.89172	.90820	.92356
7	.68472	.82106	.83085	.84107	.85162	.86236	.88376	.90391	.92168
8	.64596	.79917	.81167	.82469	.83803	.85146	.87752	.90097	.92060
9	.60940	.77809	.79352	.80956	.82587	.84208	.87269	.89899	.91999
10	.57490	.75777	.77635	.79560	.81501	.83404	.86898	.89768	.91965
11	.54236	.73818	.76011	.78273	.80534	.82719	.86616	.89681	.91947
12	.51166	.71931	.74473	.77088	.79675	.82137	.86403	.89625	.91937
13	.48270	.70111	.73019	.75996	.78913	.81644	.86243	.89589	.91931
14	.45538	.68356	.71642	.74993	.78238	.81229	.86124	.89566	.91928
15	.42960	.66662	.70339	.74070	.77642	.80880	.86035	.89552	.91927
16	.40528	.65029	.69105	.73222	.77117	.80588	.85970	.89543	.91926
17	.38234	.63453	.67938	.72444	.76655	.80344	.85923	.89537	.91926
18	.36070	.61932	.66833	.71730	.76249	.80141	.85888	.89533	.91925
19	.34028	.60463	.65786	.71076	.75893	.79972	.85862	.89531	.91925
20	.32102	.59045	.64796	.70477	.75581	.79833	.85844	.89529	.91925
21	.30285	.57676	.63858	.69928	.75309	.79718	.85831	.89529	.91925
22	.28571	.56353	.62969	.69426	.75071	.79622	.85821	.89528	.91925
23	.26954	.55074	.62128	.68967	.74864	.79544	.85814	.89528	.91925
24	.25428	.53839	.61332	.68548	.74684	.79480	.85809	.89528	.91925
25	.23989	.52645	.60577	.68165	.74527	.79428	.85806	.89527	.91925
26	.22631	.51491	.59863	.67815	.74391	.79384	.85803	.89527	.91925
27	.21350	.50374	.59186	.67497	.74274	.79349	.85801	.89527	.91925
28	.20141	.49295	.58544	.67206	.74172	.79321	.85800	.89527	.91925
29	.19001	.48250	.57937	.66941	.74083	.79297	.85799	.89527	.91925
30	.17926	.47239	.57361	.66700	.74007	.79278	.85799	.89527	.91925
31	.16911	.46261	.56816	.66481	.73941	.79263	.85798	.89527	.91925
32	.15954	.45314	.56299	.66281	.73885	.79250	.85798	.89527	.91925
33	.15051	.44397	.55810	.66100	.73836	.79240	.85797	.89527	.91925
34	.14199	.43509	.55346	.65935	.73794	.79232	.85797	.89527	.91925
35	.13395	.42648	.54906	.65785	.73758	.79225	.85797	.89527	.91925
36	.12637	.41815	.54489	.65649	.73726	.79220	.85797	.89527	.91925
37	.11922	.41007	.54094	.65525	.73700	.79215	.85797	.89527	.91925
38	.11247	.40223	.53720	.65413	.73677	.79212	.85797	.89527	.91925
39	.10610	.39464	.53365	.65311	.73657	.79209	.85797	.89527	.91925
40	.10010	.38728	.53028	.65219	.73640	.79207	.85797	.89527	.91925
41	.09443	.38014	.52709	.65136	.73626	.79205	.85797	.89527	.91925
42	.08909	.37321	.52407	.65060	.73613	.79203	.85797	.89527	.91925
43	.08404	.36648	.52120	.64991	.73603	.79202	.85797	.89527	.91925
44	.07929	.35995	.51848	.64929	.73593	.79201	.85797	.89527	.91925
45	.07480	.35362	.51590	.64873	.73586	.79200	.85797	.89527	.91925
46	.07056	.34746	.51346	.64822	.73579	.79200	.85797	.89527	.91925
47	.06657	.34149	.51114	.64776	.73573	.79199	.85797	.89527	.91925
48	.06280	.33568	.50894	.64735	.73568	.79199	.85797	.89527	.91925
49	.05925	.33004	.50685	.64697	.73564	.79198	.85797	.89527	.91925
50	.05589	.32456	.50488	.64663	.73561	.79198	.85797	.89527	.91925

Fig. H.2 Present worth discount factors at 6% for income received in one payment at middle of year.

Year	Factor for Year	Equal Payments	5% Per Yr. Decline	10% Per Yr. Decline	15% Per Yr. Decline	20% Per Yr. Decline	30% Per Yr. Decline	40% Per Yr. Decline	50% Per Yr. Decline
1	.96900	.96900	.96900	.96900	.96900	.96900	.96900	.96900	.96900
2	.90986	.93943	.94019	.94099	.94183	.94272	.94465	.94683	.94929
3	.85433	.91107	.91303	.91509	.91726	.91953	.92444	.92984	.93572
4	.80219	.88385	.88741	.89116	.89508	.89918	.90789	.91717	.92682
5	.75323	.85772	.86325	.86906	.87511	.88140	.89450	.90795	.92122
6	.70726	.83264	.84047	.84867	.85718	.86593	.88380	.90140	.91783
7	.66409	.80857	.81898	.82986	.84110	.85254	.87535	.89685	.91583
8	.62356	.78544	.79871	.81254	.82672	.84100	.86875	.89373	.91468
9	.58550	.76322	.77958	.79658	.81389	.83110	.86364	.89164	.91404
10	.54977	.74188	.76153	.78190	.80246	.82264	.85973	.89026	.91368
11	.51621	.72136	.74450	.76840	.79231	.81544	.85676	.88935	.91349
12	.48471	.70164	.72843	.75600	.78331	.80934	.85452	.88876	.91338
13	.45512	.68268	.71325	.74460	.77534	.80419	.85285	.88838	.91333
14	.42735	.66444	.69893	.73415	.76831	.79985	.85160	.88814	.91330
15	.40126	.64690	.68541	.72456	.76211	.79622	.85068	.88799	.91328
16	.37677	.63001	.67264	.71577	.75666	.79318	.85001	.88789	.91327
17	.35378	.61376	.66058	.70772	.75187	.79065	.84951	.88783	.91327
18	.33219	.59812	.64919	.70035	.74767	.78855	.84915	.88779	.91327
19	.31191	.58306	.63843	.69361	.74399	.78681	.84889	.88777	.91327
20	.29288	.56855	.62827	.68744	.74078	.78537	.84869	.88776	.91326
21	.27500	.55457	.61866	.68181	.73798	.78418	.84856	.88775	.91326
22	.25822	.54110	.60959	.67667	.73555	.78320	.84846	.88774	.91326
23	.24246	.52811	.60102	.67198	.73342	.78240	.84839	.88774	.91326
24	.22766	.51560	.59291	.66770	.73158	.78174	.84834	.88774	.91326
25	.21376	.50352	.58525	.66380	.72998	.78120	.84830	.88773	.91326
26	.20072	.49188	.57801	.66025	.72860	.78076	.84827	.88773	.91326
27	.18847	.48064	.57116	.65701	.72740	.78040	.84826	.88773	.91326
28	.17696	.46979	.56469	.65406	.72636	.78011	.84824	.88773	.91326
29	.16616	.45932	.55857	.65139	.72546	.77987	.84823	.88773	.91326
30	.15602	.44921	.55278	.64895	.72469	.77968	.84823	.88773	.91326
31	.14650	.43945	.54730	.64673	.72403	.77952	.84822	.88773	.91326
32	.13756	.43001	.54212	.64472	.72345	.77939	.84822	.88773	.91326
33	.12916	.42090	.53722	.64290	.72296	.77929	.84822	.88773	.91326
34	.12128	.41209	.53258	.64124	.72253	.77921	.84821	.88773	.91326
35	.11388	.40357	.52819	.63973	.72217	.77914	.84821	.88773	.91326
36	.10693	.39533	.52404	.63837	.72185	.77909	.84821	.88773	.91326
37	.10040	.38735	.52011	.63713	.72159	.77904	.84821	.88773	.91326
38	.09427	.37964	.51638	.63601	.72136	.77901	.84821	.88773	.91326
39	.08852	.37218	.51286	.63500	.72116	.77898	.84821	.88773	.91326
40	.08312	.36495	.50953	.63408	.72099	.77895	.84821	.88773	.91326
41	.07804	.35795	.50637	.63324	.72084	.77894	.84821	.88773	.91326
42	.07328	.35117	.50338	.63249	.72072	.77892	.84821	.88773	.91326
43	.06881	.34461	.50055	.63181	.72061	.77891	.84821	.88773	.91326
44	.06461	.33824	.49786	.63119	.72052	.77890	.84821	.88773	.91326
45	.06067	.33208	.49532	.63063	.72044	.77889	.84821	.88773	.91326
46	.05696	.32610	.49292	.63013	.72038	.77888	.84821	.88773	.91326
47	.05349	.32030	.49064	.62967	.72032	.77888	.84821	.88773	.91326
48	.05022	.31467	.48848	.62926	.72027	.77888	.84821	.88773	.91326
49	.04716	.30921	.48643	.62889	.72023	.77887	.84821	.88773	.91326
50	.04428	.30391	.48449	.62855	.72020	.77887	.84821	.88773	.91326

Fig. H.3 Present worth discount factors at 6½% for income received in one payment at middle of year.

Year	Factor for Year	Equal Payments	5% Per Yr. Decline	10% Per Yr. Decline	15% Per Yr. Decline	20% Per Yr. Decline	30% Per Yr. Decline	40% Per Yr. Decline	50% Per Yr. Decline
1	.96674	.96674	.96674	.96674	.96674	.96674	.96674	.96674	.96674
2	.90349	.93511	.93593	.93678	.93768	.93863	.94069	.94302	.94566
3	.84439	.90487	.90696	.90916	.91148	.91391	.91915	.92490	.93119
4	.78914	.87594	.87973	.88372	.88790	.89227	.90154	.91143	.92172
5	.73752	.84826	.85413	.86030	.86673	.87341	.88734	.90165	.91578
6	.68927	.82176	.83005	.83874	.84777	.85706	.87602	.89472	.91218
7	.64418	.79639	.80741	.81892	.83082	.84293	.86710	.88991	.91007
8	.60203	.77209	.78610	.80071	.81570	.83079	.86016	.88663	.90886
9	.56265	.74882	.76606	.78398	.80224	.82040	.85479	.88443	.90819
10	.52584	.72652	.74719	.76863	.79028	.81154	.85070	.88298	.90781
11	.49144	.70515	.72943	.75454	.77968	.80402	.84759	.88203	.90761
12	.45929	.68466	.71272	.74163	.77030	.79766	.84525	.88142	.90750
13	.42924	.66502	.69698	.72980	.76203	.79231	.84351	.88102	.90744
14	.40116	.64617	.68216	.71897	.75473	.78781	.84222	.88077	.90741
15	.37492	.62809	.66820	.70906	.74832	.78404	.84126	.88061	.90739
16	.35039	.61073	.65505	.69999	.74268	.78090	.84056	.88051	.90738
17	.32747	.59407	.64266	.69171	.73775	.77829	.84005	.88045	.90738
18	.30604	.57807	.63099	.68414	.73343	.77613	.83967	.88041	.90738
19	.28602	.56270	.61999	.67723	.72966	.77433	.83940	.88039	.90738
20	.26731	.54793	.60961	.67093	.72637	.77286	.83921	.88037	.90738
21	.24982	.53373	.59983	.66518	.72350	.77164	.83906	.88036	.90738
22	.23348	.52008	.59061	.65994	.72101	.77064	.83896	.88036	.90738
23	.21821	.50696	.58191	.65517	.71885	.76982	.83889	.88035	.90738
24	.20393	.49433	.57371	.65082	.71697	.76915	.83884	.88035	.90738
25	.19059	.48218	.56597	.64687	.71535	.76860	.83880	.88035	.90738
26	.17812	.47049	.55867	.64327	.71394	.76815	.83877	.88035	.90738
27	.16647	.45923	.55177	.64000	.71272	.76779	.83876	.88035	.90738
28	.15558	.44838	.54527	.63702	.71167	.76749	.83874	.88035	.90738
29	.14540	.43794	.53912	.63432	.71077	.76725	.83873	.88035	.90738
30	.13589	.42787	.53332	.63187	.70999	.76705	.83873	.88035	.90738
31	.12700	.41816	.52784	.62965	.70932	.76689	.83872	.88035	.90738
32	.11869	.40880	.52267	.62763	.70874	.76677	.83872	.88035	.90738
33	.11092	.39978	.51778	.62580	.70824	.76666	.83872	.88035	.90738
34	.10367	.39107	.51317	.62414	.70782	.76658	.83871	.88035	.90738
35	.09689	.38266	.50880	.62263	.70745	.76651	.83871	.88035	.90738
36	.09055	.37455	.50468	.62127	.70714	.76646	.83871	.88035	.90738
37	.08462	.36671	.50078	.62004	.70687	.76641	.83871	.88035	.90738
38	.07909	.35914	.49710	.61892	.70664	.76638	.83871	.88035	.90738
39	.07391	.35183	.49361	.61791	.70644	.76635	.83871	.88035	.90738
40	.06908	.34476	.49032	.61699	.70627	.76632	.83871	.88035	.90738
41	.06456	.33793	.48720	.61617	.70612	.76630	.83871	.88035	.90738
42	.06034	.33132	.48425	.61542	.70600	.76629	.83871	.88035	.90738
43	.05639	.32492	.48146	.61474	.70589	.76628	.83871	.88035	.90738
44	.05270	.31874	.47883	.61413	.70580	.76627	.83871	.88035	.90738
45	.04925	.31275	.47633	.61358	.70573	.76626	.83871	.88035	.90738
46	.04603	.30695	.47397	.61308	.70566	.76625	.83871	.88035	.90738
47	.04302	.30133	.47173	.61263	.70560	.76625	.83871	.88035	.90738
48	.04020	.29589	.46961	.61222	.70556	.76624	.83871	.88035	.90738
49	.03757	.29062	.46761	.61185	.70551	.76624	.83871	.88035	.90738
50	.03512	.28551	.46571	.61152	.70548	.76624	.83871	.88035	.90738

Fig. H.4 Present worth discount factors at 7% for income received in one payment at middle of year.

Year	Factor for Year	Equal Payments	5% Per Yr. Decline	10% Per Yr. Decline	15% Per Yr. Decline	20% Per Yr. Decline	30% Per Yr. Decline	40% Per Yr. Decline	50% Per Yr. Decline
1	.96449	.96449	.96449	.96449	.96449	.96449	.96449	.96449	.96449
2	.89720	.93084	.93170	.93261	.93357	.93458	.93678	.93925	.94206
3	.83460	.89876	.90098	.90332	.90577	.90836	.91392	.92003	.92671
4	.77637	.86816	.87218	.87641	.88084	.88546	.89529	.90577	.91668
5	.72221	.83897	.84518	.85170	.85851	.86557	.88031	.89545	.91041
6	.67182	.81111	.81986	.82903	.83856	.84836	.86839	.88816	.90662
7	.62495	.78452	.79612	.80824	.82077	.83354	.85903	.88310	.90440
8	.58135	.75912	.77384	.78919	.80495	.82083	.85175	.87967	.90314
9	.54079	.73486	.75293	.77173	.79090	.80998	.84614	.87737	.90243
10	.50306	.71168	.73331	.75575	.77844	.80075	.84187	.87585	.90204
11	.46796	.68953	.71488	.74113	.76743	.79293	.83864	.87486	.90183
12	.43531	.66834	.69759	.72775	.75771	.78633	.83621	.87422	.90171
13	.40494	.64808	.68134	.71553	.74915	.78079	.83440	.87381	.90165
14	.37669	.62870	.66608	.70436	.74162	.77614	.83306	.87355	.90162
15	.35041	.61014	.65174	.69417	.73501	.77226	.83208	.87339	.90160
16	.32596	.59238	.63826	.68486	.72923	.76903	.83135	.87328	.90159
17	.30322	.57537	.62559	.67637	.72416	.76634	.83083	.87322	.90159
18	.28207	.55908	.61367	.66864	.71974	.76412	.83044	.87318	.90159
19	.26239	.54346	.60247	.66159	.71588	.76229	.83016	.87315	.90159
20	.24408	.52849	.59193	.65517	.71252	.76078	.82996	.87314	.90159
21	.22705	.51414	.58201	.64932	.70961	.75953	.82982	.87313	.90158
22	.21121	.50037	.57267	.64400	.70707	.75852	.82972	.87312	.90158
23	.19648	.48716	.56389	.63917	.70488	.75768	.82964	.87312	.90158
24	.18277	.47447	.55562	.63477	.70297	.75700	.82959	.87312	.90158
25	.17002	.46230	.54783	.63078	.70133	.75644	.82955	.87312	.90158
26	.15816	.45060	.54049	.62715	.69990	.75599	.82952	.87311	.90158
27	.14712	.43936	.53357	.62386	.69868	.75562	.82950	.87311	.90158
28	.13686	.42855	.52706	.62087	.69762	.75532	.82949	.87311	.90158
29	.12731	.41817	.52092	.61816	.69671	.75508	.82948	.87311	.90158
30	.11843	.40818	.51513	.61570	.69592	.75488	.82947	.87311	.90158
31	.11016	.39856	.50967	.61347	.69525	.75472	.82947	.87311	.90158
32	.10248	.38931	.50452	.61145	.69467	.75459	.82947	.87311	.90158
33	.09533	.38040	.49966	.60963	.69417	.75449	.82946	.87311	.90158
34	.08868	.37182	.49508	.60797	.69374	.75440	.82946	.87311	.90158
35	.08249	.36355	.49075	.60647	.69338	.75433	.82946	.87311	.90158
36	.07674	.35559	.48667	.60511	.69306	.75428	.82946	.87311	.90158
37	.07138	.34791	.48282	.60389	.69279	.75423	.82946	.87311	.90158
38	.06640	.34050	.47918	.60278	.69256	.75420	.82946	.87311	.90158
39	.06177	.33335	.47574	.60177	.69237	.75417	.82946	.87311	.90158
40	.05746	.32645	.47250	.60087	.69220	.75415	.82946	.87311	.90158
41	.05345	.31980	.46943	.60005	.69205	.75413	.82946	.87311	.90158
42	.04972	.31337	.46653	.59930	.69193	.75411	.82946	.87311	.90158
43	.04625	.30715	.46379	.59864	.69182	.75410	.82946	.87311	.90158
44	.04303	.30115	.46120	.59803	.69173	.75409	.82946	.87311	.90158
45	.04002	.29535	.45876	.59748	.69166	.75408	.82946	.87311	.90158
46	.03723	.28974	.45644	.59699	.69159	.75408	.82946	.87311	.90158
47	.03463	.28431	.45425	.59655	.69154	.75407	.82946	.87311	.90158
48	.03222	.27906	.45218	.59615	.69149	.75407	.82946	.87311	.90158
49	.02997	.27397	.45022	.59578	.69145	.75407	.82946	.87311	.90158
50	.02788	.26905	.44837	.59546	.69141	.75406	.82946	.87311	.90158

Fig. H.5 Present worth discount factors at 7½% for income received in one payment at middle of year.

Year	Factor for Year	Equal Payments	5% Per Yr. Decline	10% Per Yr. Decline	15% Per Yr. Decline	20% Per Yr. Decline	30% Per Yr. Decline	40% Per Yr. Decline	50% Per Yr. Decline
1	.96225	.96225	.96225	.96225	.96225	.96225	.96225	.96225	.96225
2	.89097	.92661	.92753	.92849	.92950	.93057	.93290	.93552	.93849
3	.82497	.89273	.89508	.89755	.90014	.90287	.90875	.91522	.92227
4	.76387	.86052	.86476	.86921	.87388	.87876	.88913	.90019	.91171
5	.70728	.82987	.83641	.84327	.85043	.85787	.87339	.88935	.90512
6	.65489	.80071	.80990	.81953	.82954	.83984	.86090	.88170	.90115
7	.60638	.77295	.78510	.79781	.81096	.82435	.85111	.87641	.89883
8	.56146	.74651	.76190	.77797	.79446	.81110	.84352	.87283	.89750
9	.51987	.72133	.74019	.75983	.77986	.79982	.83769	.87043	.89676
10	.48136	.69733	.71986	.74327	.76695	.79024	.83325	.86885	.89636
11	.44571	.67446	.70083	.72815	.75555	.78215	.82990	.86783	.89614
12	.41269	.65264	.68300	.71435	.74552	.77533	.82739	.86716	.89602
13	.38212	.63183	.66630	.70177	.73670	.76961	.82552	.86674	.89596
14	.35382	.61198	.65064	.69030	.72896	.76483	.82414	.86647	.89592
15	.32761	.59302	.63596	.67985	.72218	.76084	.82313	.86630	.89591
16	.30334	.57491	.62220	.67034	.71625	.75753	.82238	.86620	.89590
17	.28087	.55762	.60929	.66168	.71108	.75479	.82184	.86613	.89589
18	.26007	.54109	.59718	.65380	.70656	.75252	.82145	.86609	.89589
19	.24080	.52528	.58581	.64663	.70264	.75065	.82116	.86607	.89589
20	.22297	.51017	.57514	.64011	.69922	.74911	.82096	.86605	.89589
21	.20645	.49570	.56512	.63419	.69626	.74785	.82081	.86604	.89589
22	.19116	.48186	.55571	.62882	.69369	.74681	.82071	.86603	.89589
23	.17700	.46861	.54686	.62393	.69147	.74596	.82063	.86603	.89589
24	.16389	.45591	.53855	.61950	.68955	.74527	.82058	.86603	.89589
25	.15175	.44374	.53073	.61548	.68789	.74471	.82054	.86603	.89589
26	.14051	.43208	.52339	.61184	.68645	.74425	.82051	.86603	.89589
27	.13010	.42089	.51647	.60853	.68522	.74388	.82049	.86603	.89589
28	.12046	.41016	.50997	.60554	.68416	.74358	.82048	.86603	.89589
29	.11154	.39987	.50385	.60283	.68324	.74333	.82047	.86603	.89589
30	.10328	.38998	.49809	.60037	.68245	.74314	.82046	.86603	.89589
31	.09563	.38049	.49266	.59814	.68178	.74298	.82046	.86603	.89589
32	.08854	.37136	.48755	.59613	.68120	.74285	.82045	.86603	.89589
33	.08198	.36259	.48274	.59431	.68070	.74274	.82045	.86603	.89589
34	.07591	.35416	.47820	.59266	.68027	.74266	.82045	.86603	.89589
35	.07029	.34605	.47393	.59117	.67991	.74259	.82045	.86603	.89589
36	.06508	.33825	.46989	.58982	.67959	.74253	.82045	.86603	.89589
37	.06026	.33073	.46609	.58861	.67932	.74249	.82045	.86603	.89589
38	.05580	.32350	.46251	.58751	.67910	.74245	.82045	.86603	.89589
39	.05166	.31653	.45912	.58651	.67890	.74242	.82045	.86603	.89589
40	.04784	.30981	.45593	.58561	.67873	.74240	.82045	.86603	.89589
41	.04429	.30333	.45292	.58480	.67859	.74238	.82045	.86603	.89589
42	.04101	.29709	.45008	.58407	.67847	.74237	.82045	.86603	.89589
43	.03797	.29106	.44739	.58341	.67836	.74236	.82045	.86603	.89589
44	.03516	.28525	.44485	.58281	.67827	.74235	.82045	.86603	.89589
45	.03256	.27963	.44246	.58227	.67820	.74234	.82045	.86603	.89589
46	.03015	.27421	.44019	.58179	.67813	.74233	.82045	.86603	.89589
47	.02791	.26897	.43805	.58135	.67808	.74233	.82045	.86603	.89589
48	.02584	.26390	.43603	.58096	.67803	.74232	.82045	.86603	.89589
49	.02393	.25901	.43412	.58060	.67799	.74232	.82045	.86603	.89589
50	.02216	.25427	.43231	.58028	.67796	.74232	.82045	.86603	.89589

Fig. H.6 Present worth discount factors at 8% for income received in one payment at middle of year.

Year	Factor for Year	Equal Payments	5% Per Yr. Decline	10% Per Yr. Decline	15% Per Yr. Decline	20% Per Yr. Decline	30% Per Yr. Decline	40% Per Yr. Decline	50% Per Yr. Decline
1	.96003	.96003	.96003	.96003	.96003	.96003	.96003	.96003	.96003
2	.88482	.92243	.92339	.92441	.92547	.92660	.92906	.93183	.93496
3	.81550	.88678	.88926	.89186	.89459	.89746	.90365	.91046	.91790
4	.75162	.85299	.85745	.86213	.86704	.87217	.88307	.89469	.90681
5	.69273	.82094	.82779	.83499	.84250	.85030	.86659	.88334	.89990
6	.63846	.79053	.80014	.81022	.82070	.83149	.85355	.87535	.89575
7	.58845	.76166	.77435	.78763	.80136	.81536	.84335	.86984	.89333
8	.54235	.73425	.75028	.76703	.78424	.80160	.83546	.86611	.89196
9	.49986	.70820	.72782	.74826	.76911	.78991	.82941	.86363	.89119
10	.46070	.68345	.70684	.73115	.75577	.78001	.82482	.86199	.89077
11	.42461	.65992	.68724	.71558	.74403	.77166	.82136	.86093	.89054
12	.39134	.63754	.66894	.70140	.73370	.76464	.81877	.86025	.89042
13	.36069	.61624	.65182	.68849	.72465	.75877	.81685	.85981	.89036
14	.33243	.59597	.63582	.67676	.71672	.75387	.81544	.85954	.89032
15	.30639	.57666	.62085	.66609	.70979	.74979	.81440	.85936	.89030
16	.28238	.55827	.60685	.65639	.70374	.74640	.81364	.85926	.89029
17	.26026	.54074	.59374	.64758	.69847	.74360	.81308	.85919	.89029
18	.23987	.52403	.58147	.63958	.69388	.74129	.81268	.85915	.89029
19	.22108	.50808	.56997	.63232	.68990	.73939	.81239	.85912	.89029
20	.20376	.49287	.55920	.62573	.68644	.73783	.81218	.85910	.89029
21	.18780	.47834	.54911	.61975	.68344	.73655	.81203	.85909	.89029
22	.17309	.46446	.53964	.61433	.68084	.73550	.81193	.85909	.89029
23	.15953	.45121	.53076	.60941	.67860	.73465	.81185	.85908	.89029
24	.14703	.43853	.52244	.60496	.67666	.73395	.81180	.85908	.89028
25	.13551	.42641	.51462	.60093	.67499	.73338	.81176	.85908	.89028
26	.12489	.41481	.50728	.59727	.67355	.73292	.81173	.85908	.89028
27	.11511	.40371	.50039	.59397	.67231	.73255	.81171	.85908	.89028
28	.10609	.39308	.49391	.59097	.67125	.73224	.81170	.85908	.89028
29	.09778	.38290	.48782	.58826	.67033	.73200	.81169	.85908	.89028
30	.09012	.37314	.48210	.58581	.66954	.73180	.81168	.85908	.89028
31	.08306	.36378	.47672	.58360	.66887	.73164	.81167	.85908	.89028
32	.07655	.35481	.47166	.58159	.66829	.73151	.81167	.85908	.89028
33	.07056	.34620	.46690	.57978	.66779	.73140	.81167	.85908	.89028
34	.06503	.33793	.46242	.57815	.66737	.73132	.81167	.85908	.89028
35	.05993	.32998	.45820	.57667	.66700	.73125	.81167	.85908	.89028
36	.05524	.32235	.45423	.57533	.66669	.73120	.81166	.85908	.89028
37	.05091	.31501	.45049	.57413	.66642	.73115	.81166	.85908	.89028
38	.04692	.30796	.44696	.57304	.66620	.73112	.81166	.85908	.89028
39	.04325	.30117	.44364	.57206	.66600	.73109	.81166	.85908	.89028
40	.03986	.29464	.44050	.57117	.66583	.73106	.81166	.85908	.89028
41	.03674	.28835	.43755	.57037	.66569	.73105	.81166	.85908	.89028
42	.03386	.28229	.43476	.56965	.66557	.73103	.81166	.85908	.89028
43	.03121	.27645	.43213	.56899	.66547	.73102	.81166	.85908	.89028
44	.02876	.27082	.42965	.56841	.66538	.73101	.81166	.85908	.89028
45	.02651	.26539	.42731	.56788	.66530	.73100	.81166	.85908	.89028
46	.02443	.26015	.42509	.56740	.66524	.73100	.81166	.85908	.89028
47	.02252	.25510	.42300	.56697	.66519	.73099	.81166	.85908	.89028
48	.02075	.25022	.42103	.56658	.66514	.73099	.81166	.85908	.89028
49	.01913	.24550	.41917	.56623	.66510	.73098	.81166	.85908	.89028
50	.01763	.24094	.41740	.56591	.66507	.73098	.81166	.85908	.89028

Fig. H.7 Present worth discount factors at 8½% for income received in one payment at middle of year.

Year	Factor for Year	Equal Payments	5% Per Yr. Decline	10% Per Yr. Decline	15% Per Yr. Decline	20% Per Yr. Decline	30% Per Yr. Decline	40% Per Yr. Decline	50% Per Yr. Decline
1	.95783	.95783	.95783	.95783	.95783	.95783	.95783	.95783	.95783
2	.87874	.91828	.91930	.92036	.92149	.92268	.92526	.92817	.93146
3	.80618	.88092	.88351	.88624	.88910	.89212	.89862	.90576	.91357
4	.73962	.84559	.85025	.85516	.86030	.86567	.87709	.88927	.90197
5	.67855	.81218	.81934	.82686	.83471	.84287	.85990	.87743	.89476
6	.62252	.78057	.79060	.80111	.81204	.82330	.84633	.86911	.89044
7	.57112	.75065	.76386	.77768	.79198	.80657	.83575	.86339	.88793
8	.52396	.72232	.73897	.75637	.77426	.79233	.82757	.85952	.88650
9	.48070	.69547	.71580	.73700	.75865	.78025	.82132	.85695	.88571
10	.44101	.67002	.69422	.71940	.74491	.77005	.81658	.85526	.88527
11	.40460	.64589	.67411	.70340	.73284	.76146	.81302	.85417	.88504
12	.37119	.62300	.65537	.68887	.72225	.75426	.81036	.85347	.88491
13	.34054	.60127	.63789	.67568	.71299	.74825	.80839	.85302	.88484
14	.31242	.58064	.62159	.66371	.70489	.74323	.80694	.85274	.88481
15	.28663	.56104	.60637	.65284	.69783	.73907	.80588	.85256	.88479
16	.26296	.54241	.59216	.64299	.69167	.73562	.80510	.85245	.88478
17	.24125	.52469	.57889	.63406	.68632	.73278	.80454	.85238	.88478
18	.22133	.50784	.56649	.62596	.68167	.73043	.80413	.85234	.88477
19	.20305	.49180	.55489	.61862	.67763	.72850	.80384	.85231	.88477
20	.18629	.47652	.54405	.61197	.67414	.72692	.80363	.85230	.88477
21	.17091	.46197	.53391	.60595	.67111	.72563	.80348	.85229	.88477
22	.15679	.44810	.52442	.60050	.66850	.72457	.80337	.85228	.88477
23	.14385	.43487	.51553	.59556	.66624	.72371	.80329	.85228	.88477
24	.13197	.42225	.50720	.59110	.66429	.72301	.80323	.85227	.88477
25	.12107	.41020	.49940	.58706	.66261	.72243	.80320	.85227	.88477
26	.11108	.39870	.49209	.58341	.66117	.72197	.80317	.85227	.88477
27	.10191	.38771	.48523	.58010	.65993	.72160	.80315	.85227	.88477
28	.09349	.37720	.47880	.57712	.65886	.72129	.80313	.85227	.88477
29	.08577	.36715	.47276	.57442	.65794	.72105	.80312	.85227	.88477
30	.07869	.35753	.46709	.57198	.65716	.72085	.80312	.85227	.88477
31	.07219	.34833	.46177	.56978	.65648	.72069	.80311	.85227	.88477
32	.06623	.33951	.45677	.56779	.65591	.72056	.80311	.85227	.88477
33	.06076	.33107	.45207	.56599	.65541	.72045	.80311	.85227	.88477
34	.05575	.32297	.44765	.56437	.65499	.72037	.80310	.85227	.88477
35	.05114	.31520	.44349	.56291	.65463	.72030	.80310	.85227	.88477
36	.04692	.30775	.43958	.56159	.65432	.72024	.80310	.85227	.88477
37	.04305	.30060	.43590	.56039	.65405	.72020	.80310	.85227	.88477
38	.03949	.29372	.43244	.55932	.65383	.72017	.80310	.85227	.88477
39	.03623	.28712	.42917	.55835	.65363	.72014	.80310	.85227	.88477
40	.03324	.28078	.42610	.55747	.65347	.72011	.80310	.85227	.88477
41	.03049	.27467	.42321	.55668	.65333	.72010	.80310	.85227	.88477
42	.02798	.26880	.42048	.55597	.65321	.72008	.80310	.85227	.88477
43	.02567	.26314	.41790	.55533	.65311	.72007	.80310	.85227	.88477
44	.02355	.25770	.41548	.55475	.65302	.72006	.80310	.85227	.88477
45	.02160	.25245	.41319	.55423	.65295	.72005	.80310	.85227	.88477
46	.01982	.24739	.41103	.55376	.65288	.72005	.80310	.85227	.88477
47	.01818	.24252	.40899	.55333	.65283	.72004	.80310	.85227	.88477
48	.01668	.23781	.40707	.55295	.65278	.72004	.80310	.85227	.88477
49	.01530	.23327	.40525	.55261	.65274	.72003	.80310	.85227	.88477
50	.01404	.22889	.40353	.55230	.65271	.72003	.80310	.85227	.88477

Fig. H.8 Present worth discount factors at 9% for income received in one payment at middle of year.

Year	Factor for Year	Equal Payments	5% Per Yr. Decline	10% Per Yr. Decline	15% Per Yr. Decline	20% Per Yr. Decline	30% Per Yr. Decline	40% Per Yr. Decline	50% Per Yr. Decline
1	.95564	.95564	.95564	.95564	.95564	.95564	.95564	.95564	.95564
2	.87273	.91418	.91525	.91636	.91754	.91879	.92150	.92455	.92800
3	.79701	.87513	.87784	.88069	.88369	.88685	.89364	.90112	.90929
4	.72786	.83831	.84318	.84829	.85366	.85927	.87120	.88392	.89719
5	.66472	.80359	.81105	.81888	.82707	.83557	.85332	.87160	.88969
6	.60705	.77083	.78126	.79219	.80356	.81527	.83925	.86297	.88521
7	.55438	.73991	.75361	.76796	.78281	.79796	.82829	.85704	.88260
8	.50628	.71071	.72796	.74599	.76453	.78326	.81985	.85305	.88113
9	.46236	.68311	.70413	.72606	.74846	.77082	.81340	.85040	.88031
10	.42225	.65703	.68199	.70798	.73434	.76034	.80853	.84866	.87986
11	.38561	.63235	.66141	.69160	.72198	.75154	.80488	.84754	.87962
12	.35216	.60900	.64228	.67676	.71115	.74417	.80215	.84682	.87949
13	.32161	.58690	.62448	.66331	.70170	.73803	.80014	.84636	.87942
14	.29370	.56595	.60791	.65113	.69345	.73292	.79866	.84607	.87939
15	.26822	.54611	.59247	.64010	.68627	.72868	.79757	.84589	.87937
16	.24495	.52728	.57809	.63011	.68002	.72518	.79678	.84578	.87936
17	.22370	.50943	.56469	.62107	.67460	.72229	.79621	.84571	.87935
18	.20429	.49247	.55219	.61289	.66989	.71991	.79579	.84566	.87935
19	.18657	.47637	.54053	.60550	.66582	.71796	.79550	.84564	.87935
20	.17038	.46107	.52964	.59880	.66229	.71637	.79528	.84562	.87935
21	.15560	.44653	.51948	.59275	.65925	.71506	.79513	.84561	.87935
22	.14210	.43269	.50998	.58728	.65662	.71400	.79502	.84560	.87935
23	.12977	.41952	.50110	.58234	.65435	.71313	.79494	.84560	.87935
24	.11851	.40698	.49279	.57787	.65240	.71242	.79489	.84560	.87935
25	.10823	.39503	.48502	.57384	.65072	.71185	.79485	.84560	.87935
26	.09884	.38364	.47775	.57019	.64927	.71139	.79482	.84560	.87935
27	.09027	.37277	.47094	.56690	.64803	.71101	.79480	.84559	.87935
28	.08244	.36240	.46456	.56393	.64697	.71071	.79478	.84559	.87935
29	.07528	.35250	.45858	.56124	.64605	.71046	.79477	.84559	.87935
30	.06875	.34304	.45297	.55882	.64527	.71026	.79477	.84559	.87935
31	.06279	.33400	.44771	.55664	.64460	.71010	.79476	.84559	.87935
32	.05734	.32536	.44278	.55466	.64402	.70997	.79476	.84559	.87935
33	.05236	.31708	.43814	.55288	.64353	.70987	.79476	.84559	.87935
34	.04782	.30916	.43379	.55128	.64311	.70978	.79476	.84559	.87935
35	.04367	.30158	.42970	.54983	.64275	.70971	.79475	.84559	.87935
36	.03988	.29431	.42586	.54852	.64245	.70966	.79475	.84559	.87935
37	.03642	.28734	.42224	.54735	.64218	.70962	.79475	.84559	.87935
38	.03326	.28065	.41884	.54628	.64196	.70958	.79475	.84559	.87935
39	.03038	.27424	.41565	.54533	.64177	.70955	.79475	.84559	.87935
40	.02774	.26807	.41263	.54446	.64161	.70953	.79475	.84559	.87935
41	.02534	.26215	.40980	.54369	.64147	.70951	.79475	.84559	.87935
42	.02314	.25646	.40713	.54299	.64135	.70950	.79475	.84559	.87935
43	.02113	.25099	.40461	.54235	.64125	.70949	.79475	.84559	.87935
44	.01930	.24572	.40224	.54179	.64116	.70948	.79475	.84559	.87935
45	.01762	.24065	.40001	.54127	.64109	.70947	.79475	.84559	.87935
46	.01609	.23577	.39790	.54081	.64102	.70946	.79475	.84559	.87935
47	.01470	.23107	.39591	.54039	.64097	.70946	.79475	.84559	.87935
48	.01342	.22654	.39404	.54002	.64093	.70945	.79475	.84559	.87935
49	.01226	.22216	.39226	.53968	.64089	.70945	.79475	.84559	.87935
50	.01119	.21794	.39059	.53938	.64085	.70945	.79475	.84559	.87935

Fig. H.9 Present worth discount factors at 9½% for income received in one payment at middle of year.

Year	Factor for Year	Equal Payments	5% Per Yr. Decline	10% Per Yr. Decline	15% Per Yr. Decline	20% Per Yr. Decline	30% Per Yr. Decline	40% Per Yr. Decline	50% Per Yr. Decline
1	.95346	.95346	.95346	.95346	.95346	.95346	.95346	.95346	.95346
2	.86678	.91012	.91123	.91240	.91364	.91494	.91777	.92096	.92457
3	.78799	.86941	.87224	.87522	.87835	.88164	.88873	.89653	.90506
4	.71635	.83115	.83621	.84154	.84713	.85297	.86539	.87865	.89248
5	.65123	.79516	.80291	.81105	.81955	.82839	.84685	.86587	.88470
6	.59203	.76131	.77211	.78345	.79524	.80740	.83229	.85693	.88005
7	.53820	.72943	.74361	.75847	.77384	.78954	.82097	.85081	.87736
8	.48928	.69941	.71723	.73586	.75503	.77440	.81228	.84669	.87584
9	.44480	.67112	.69279	.71541	.73853	.76163	.80566	.84397	.87499
10	.40436	.64445	.67014	.69690	.72407	.75089	.80066	.84218	.87453
11	.36760	.61928	.64913	.68017	.71143	.74188	.79691	.84103	.87428
12	.33418	.59552	.62964	.66504	.70039	.73436	.79413	.84029	.87415
13	.30380	.57308	.61155	.65136	.69076	.72810	.79207	.83983	.87408
14	.27618	.55187	.59475	.63899	.68238	.72290	.79056	.83953	.87405
15	.25108	.53182	.57914	.62782	.67510	.71860	.78946	.83935	.87403
16	.22825	.51285	.56462	.61772	.66877	.71505	.78866	.83923	.87402
17	.20750	.49489	.55112	.60860	.66329	.71213	.78808	.83916	.87401
18	.18864	.47787	.53854	.60036	.65854	.70973	.78766	.83912	.87401
19	.17149	.46175	.52684	.59292	.65444	.70776	.78736	.83909	.87401
20	.15590	.44646	.51593	.58620	.65089	.70615	.78714	.83907	.87401
21	.14173	.43194	.50575	.58013	.64783	.70484	.78699	.83906	.87401
22	.12884	.41817	.49627	.57465	.64519	.70377	.78688	.83906	.87401
23	.11713	.40508	.48741	.56971	.64292	.70289	.78680	.83905	.87401
24	.10648	.39264	.47914	.56525	.64096	.70219	.78674	.83905	.87401
25	.09680	.38080	.47142	.56122	.63928	.70161	.78670	.83905	.87401
26	.08800	.36954	.46420	.55759	.63784	.70115	.78667	.83905	.87401
27	.08000	.35882	.45745	.55431	.63660	.70077	.78665	.83905	.87401
28	.07273	.34860	.45113	.55136	.63554	.70047	.78664	.83905	.87401
29	.06612	.33886	.44521	.54869	.63463	.70022	.78663	.83905	.87401
30	.06011	.32957	.43967	.54629	.63385	.70002	.78662	.83905	.87401
31	.05464	.32070	.43448	.54412	.63318	.69986	.78662	.83905	.87401
32	.04967	.31223	.42962	.54217	.63261	.69973	.78661	.83905	.87401
33	.04516	.30414	.42505	.54041	.63212	.69963	.78661	.83905	.87401
34	.04105	.29640	.42077	.53882	.63170	.69955	.78661	.83905	.87401
35	.03732	.28900	.41675	.53739	.63135	.69948	.78661	.83905	.87401
36	.03393	.28191	.41298	.53610	.63104	.69943	.78661	.83905	.87401
37	.03084	.27513	.40943	.53494	.63078	.69938	.78661	.83905	.87401
38	.02804	.26862	.40610	.53389	.63056	.69935	.78661	.83905	.87401
39	.02549	.26239	.40296	.53295	.63037	.69932	.78661	.83905	.87401
40	.02317	.25641	.40002	.53210	.63021	.69930	.78661	.83905	.87401
41	.02107	.25067	.39724	.53133	.63007	.69928	.78661	.83905	.87401
42	.01915	.24516	.39463	.53064	.62996	.69926	.78661	.83905	.87401
43	.01741	.23986	.39217	.53002	.62986	.69925	.78661	.83905	.87401
44	.01583	.23477	.38986	.52946	.62977	.69924	.78661	.83905	.87401
45	.01439	.22987	.38768	.52896	.62970	.69924	.78661	.83905	.87401
46	.01308	.22516	.38562	.52850	.62964	.69923	.78661	.83905	.87401
47	.01189	.22062	.38368	.52810	.62959	.69923	.78661	.83905	.87401
48	.01081	.21625	.38185	.52773	.62954	.69922	.78661	.83905	.87401
49	.00983	.21204	.38013	.52740	.62950	.69922	.78661	.83905	.87401
50	.00893	.20797	.37850	.52710	.62947	.69922	.78661	.83905	.87401

Fig. H.10 Present worth discount factors at 10% for income received in one payment at middle of year.

Year	Factor for Year	Equal Payments	5% Per Yr. Decline	10% Per Yr. Decline	15% Per Yr. Decline	20% Per Yr. Decline	30% Per Yr. Decline	40% Per Yr. Decline	50% Per Yr. Decline
1	.94916	.94916	.94916	.94916	.94916	.94916	.94916	.94916	.94916
2	.85510	.90213	.90333	.90460	.90594	.90735	.91043	.91389	.91780
3	.77036	.85820	.86126	.86448	.86786	.87142	.87909	.88752	.89674
4	.69402	.81716	.82261	.82834	.83436	.84065	.85403	.86831	.88323
5	.62524	.77877	.78708	.79580	.80492	.81440	.83422	.85465	.87490
6	.56328	.74286	.75439	.76650	.77910	.79210	.81874	.84514	.86996
7	.50746	.70923	.72431	.74011	.75648	.77322	.80676	.83866	.86710
8	.45717	.67772	.69659	.71635	.73670	.75729	.79760	.83432	.86550
9	.41186	.64818	.67105	.69495	.71942	.74390	.79065	.83145	.86461
10	.37105	.62047	.64749	.67569	.70435	.73269	.78542	.82958	.86412
11	.33428	.59445	.62574	.65834	.69123	.72333	.78152	.82838	.86387
12	.30115	.57001	.60566	.64272	.67981	.71554	.77863	.82761	.86373
13	.27131	.54703	.58710	.62865	.66990	.70908	.77650	.82713	.86366
14	.24442	.52542	.56993	.61599	.66130	.70373	.77494	.82682	.86362
15	.22020	.50507	.55404	.60459	.65384	.69933	.77381	.82663	.86360
16	.19838	.48590	.53933	.59432	.64740	.69570	.77299	.82652	.86359
17	.17872	.46783	.52569	.58508	.64183	.69272	.77239	.82644	.86358
18	.16101	.45079	.51304	.57676	.63702	.69028	.77197	.82640	.86358
19	.14505	.43470	.50130	.56927	.63287	.68829	.77166	.82637	.86358
20	.13068	.41949	.49040	.56252	.62930	.68666	.77144	.82635	.86358
21	.11773	.40512	.48027	.55645	.62622	.68534	.77128	.82634	.86358
22	.10606	.39153	.47085	.55098	.62358	.68426	.77117	.82634	.86358
23	.09555	.37866	.46209	.54606	.62130	.68339	.77109	.82633	.86358
24	.08608	.36647	.45393	.54163	.61935	.68268	.77104	.82633	.86358
25	.07755	.35491	.44632	.53764	.61768	.68211	.77100	.82633	.86358
26	.06987	.34395	.43923	.53405	.61625	.68164	.77097	.82633	.86358
27	.06294	.33354	.43262	.53082	.61502	.68127	.77095	.82633	.86358
28	.05670	.32366	.42644	.52791	.61397	.68097	.77094	.82633	.86358
29	.05109	.31426	.42068	.52529	.61307	.68072	.77093	.82633	.86358
30	.04602	.30532	.41529	.52294	.61230	.68053	.77092	.82633	.86358
31	.04146	.29680	.41025	.52081	.61164	.68037	.77091	.82633	.86358
32	.03735	.28870	.40553	.51890	.61108	.68024	.77091	.82633	.86358
33	.03365	.28097	.40112	.51719	.61060	.68014	.77091	.82633	.86358
34	.03032	.27360	.39699	.51564	.61019	.68005	.77091	.82633	.86358
35	.02731	.26656	.39311	.51424	.60984	.67999	.77090	.82633	.86358
36	.02461	.25984	.38948	.51299	.60954	.67994	.77090	.82633	.86358
37	.02217	.25341	.38607	.51186	.60929	.67989	.77090	.82633	.86358
38	.01997	.24727	.38287	.51085	.60907	.67986	.77090	.82633	.86358
39	.01799	.24139	.37987	.50993	.60888	.67983	.77090	.82633	.86358
40	.01621	.23576	.37704	.50911	.60873	.67981	.77090	.82633	.86358
41	.01460	.23037	.37439	.50837	.60859	.67979	.77090	.82633	.86358
42	.01316	.22520	.37190	.50770	.60848	.67978	.77090	.82633	.86358
43	.01185	.22024	.36955	.50710	.60838	.67977	.77090	.82633	.86358
44	.01068	.21547	.36734	.50656	.60830	.67976	.77090	.82633	.86358
45	.00962	.21090	.36526	.50607	.60823	.67975	.77090	.82633	.86358
46	.00867	.20650	.36330	.50564	.60817	.67974	.77090	.82633	.86358
47	.00781	.20227	.36146	.50524	.60812	.67974	.77090	.82633	.86358
48	.00703	.19821	.35972	.50489	.60807	.67973	.77090	.82633	.86358
49	.00634	.19429	.35808	.50457	.60804	.67973	.77090	.82633	.86358
50	.00571	.19052	.35654	.50428	.60801	.67973	.77090	.82633	.86358

Fig. H.11 Present worth discount factors at 11% for income received in one payment at middle of year.

Year	Factor for Year	Equal Payments	5% Per Yr. Decline	10% Per Yr. Decline	15% Per Yr. Decline	20% Per Yr. Decline	30% Per Yr. Decline	40% Per Yr. Decline	50% Per Yr. Decline
1	.94491	.94491	.94491	.94491	.94491	.94491	.94491	.94491	.94491
2	.84367	.89429	.89559	.89696	.89840	.89992	.90322	.90695	.91116
3	.75328	.84729	.85056	.85401	.85764	.86145	.86967	.87872	.88861
4	.67257	.80361	.80943	.81555	.82197	.82869	.84298	.85826	.87421
5	.60051	.76299	.77182	.78110	.79080	.80089	.82199	.84377	.86538
6	.53617	.72518	.73740	.75023	.76359	.77738	.80566	.83373	.86015
7	.47872	.68998	.70588	.72257	.73987	.75756	.79308	.82692	.85715
8	.42743	.65716	.67700	.69779	.71922	.74093	.78350	.82237	.85546
9	.38163	.62654	.65050	.67557	.70126	.72700	.77625	.81938	.85454
10	.34074	.59796	.62617	.65565	.68567	.71538	.77083	.81744	.85403
11	.30424	.57126	.60382	.63780	.67214	.70573	.76679	.81619	.85377
12	.27164	.54629	.58327	.62178	.66042	.69772	.76382	.81540	.85362
13	.24254	.52293	.56435	.60742	.65028	.69110	.76163	.81490	.85355
14	.21655	.50104	.54693	.59454	.64151	.68564	.76003	.81459	.85351
15	.19335	.48053	.53086	.58298	.63394	.68115	.75888	.81439	.85349
16	.17263	.46129	.51604	.57261	.62741	.67747	.75804	.81427	.85348
17	.15414	.44322	.50235	.56330	.62178	.67446	.75743	.81420	.85347
18	.13762	.42624	.48970	.55495	.61694	.67199	.75700	.81415	.85347
19	.12288	.41027	.47800	.54745	.61277	.66999	.75669	.81412	.85347
20	.10971	.39525	.46717	.54072	.60919	.66835	.75647	.81410	.85347
21	.09796	.38109	.45714	.53468	.60612	.66703	.75631	.81409	.85347
22	.08746	.36774	.44783	.52925	.60348	.66595	.75620	.81409	.85347
23	.07809	.35515	.43920	.52437	.60122	.66508	.75612	.81408	.85347
24	.06972	.34326	.43118	.52000	.59929	.66437	.75606	.81408	.85347
25	.06225	.33202	.42372	.51606	.59763	.66380	.75602	.81408	.85347
26	.05558	.32138	.41679	.51253	.59621	.66334	.75600	.81408	.85347
27	.04963	.31132	.41034	.50935	.59500	.66297	.75598	.81408	.85347
28	.04431	.30178	.40432	.50650	.59396	.66267	.75596	.81408	.85347
29	.03956	.29274	.39872	.50393	.59307	.66242	.75595	.81408	.85347
30	.03532	.28416	.39349	.50163	.59231	.66223	.75595	.81408	.85347
31	.03154	.27601	.38861	.49956	.59167	.66207	.75594	.81408	.85347
32	.02816	.26827	.38406	.49770	.59112	.66195	.75594	.81408	.85347
33	.02514	.26090	.37980	.49602	.59065	.66185	.75593	.81408	.85347
34	.02245	.25389	.37581	.49452	.59025	.66177	.75593	.81408	.85347
35	.02004	.24720	.37208	.49316	.58990	.66170	.75593	.81408	.85347
36	.01790	.24083	.36859	.49195	.58961	.66165	.75593	.81408	.85347
37	.01598	.23476	.36532	.49085	.58936	.66161	.75593	.81408	.85347
38	.01427	.22895	.36225	.48987	.58915	.66157	.75593	.81408	.85347
39	.01274	.22341	.35937	.48898	.58897	.66155	.75593	.81408	.85347
40	.01137	.21811	.35667	.48819	.58882	.66152	.75593	.81408	.85347
41	.01015	.21304	.35414	.48747	.58869	.66151	.75593	.81408	.85347
42	.00907	.20818	.35175	.48682	.58858	.66149	.75593	.81408	.85347
43	.00810	.20353	.34951	.48625	.58848	.66148	.75593	.81408	.85347
44	.00723	.19907	.34741	.48572	.58840	.66147	.75593	.81408	.85347
45	.00645	.19479	.34543	.48526	.58833	.66147	.75593	.81408	.85347
46	.00576	.19068	.34356	.48483	.58828	.66146	.75593	.81408	.85347
47	.00514	.18673	.34180	.48445	.58823	.66146	.75593	.81408	.85347
48	.00459	.18294	.34015	.48411	.58818	.66145	.75593	.81408	.85347
	.00410	.17929	.33859	.48381	.58815	.66145	.75593	.81408	.85347
	.00366	.17577	.33712	.48353	.58812	.66145	.75593	.81408	.85347

Fig. H.12 Present worth discount factors at 12% for income received in one payment at middle of year.

Year	Factor for Year	Equal Payments	5% Per Yr. Decline	10% Per Yr. Decline	15% Per Yr. Decline	20% Per Yr. Decline	30% Per Yr. Decline	40% Per Yr. Decline	50% Per Yr. Decline
1	.93659	.93659	.93659	.93659	.93659	.93659	.93659	.93659	.93659
2	.82157	.87908	.88055	.88210	.88374	.88547	.88922	.89345	.89825
3	.72067	.82627	.82997	.83385	.83794	.84224	.85151	.86172	.87288
4	.63217	.77775	.78425	.79110	.79828	.80581	.82181	.83893	.85683
5	.55453	.73311	.74290	.75320	.76398	.77519	.79867	.82295	.84708
6	.48643	.69199	.70544	.71958	.73432	.74954	.78083	.81197	.84136
7	.42670	.65409	.67148	.68974	.70870	.72812	.76721	.80457	.83809
8	.37429	.61912	.64065	.66325	.68660	.71029	.75690	.79967	.83627
9	.32833	.58681	.61263	.63972	.66754	.69549	.74918	.79647	.83528
10	.28801	.55693	.58714	.61880	.65112	.68323	.74344	.79441	.83474
11	.25264	.52927	.56391	.60019	.63699	.67312	.73919	.79309	.83446
12	.22161	.50363	.54273	.58363	.62484	.66479	.73608	.79226	.83431
13	.19440	.47984	.52340	.56889	.61439	.65795	.73381	.79174	.83423
14	.17052	.45775	.50572	.55577	.60542	.65234	.73216	.79142	.83419
15	.14958	.43720	.48954	.54406	.59772	.64776	.73097	.79121	.83417
16	.13121	.41808	.47471	.53363	.59111	.64402	.73011	.79109	.83416
17	.11510	.40026	.46111	.52432	.58546	.64097	.72950	.79101	.83415
18	.10096	.38363	.44862	.51602	.58061	.63850	.72906	.79097	.83415
19	.08856	.36810	.43714	.50860	.57646	.63649	.72874	.79094	.83415
20	.07769	.35358	.42656	.50197	.57291	.63486	.72852	.79092	.83415
21	.06815	.33999	.41682	.49605	.56988	.63354	.72836	.79091	.83415
22	.05978	.32725	.40783	.49075	.56728	.63247	.72825	.79090	.83415
23	.05244	.31530	.39953	.48602	.56507	.63161	.72817	.79090	.83415
24	.04600	.30408	.39186	.48178	.56318	.63092	.72812	.79090	.83415
25	.04035	.29353	.38476	.47799	.56156	.63036	.72808	.79090	.83415
26	.03539	.28360	.37818	.47459	.56018	.62990	.72805	.79090	.83415
27	.03105	.27425	.37208	.47155	.55901	.62954	.72803	.79090	.83415
28	.02723	.26543	.36641	.46882	.55801	.62925	.72802	.79090	.83415
29	.02389	.25710	.36115	.46638	.55715	.62901	.72801	.79090	.83415
30	.02096	.24923	.35626	.46419	.55643	.62883	.72800	.79089	.83415
31	.01838	.24178	.35170	.46222	.55581	.62868	.72799	.79089	.83415
32	.01612	.23473	.34746	.46046	.55528	.62855	.72799	.79089	.83415
33	.01414	.22804	.34350	.45888	.55483	.62846	.72799	.79089	.83415
34	.01241	.22170	.33981	.45746	.55445	.62838	.72799	.79089	.83415
35	.01088	.21568	.33636	.45618	.55412	.62832	.72799	.79089	.83415
36	.00955	.20995	.33314	.45504	.55384	.62827	.72798	.79089	.83415
37	.00837	.20450	.33013	.45401	.55361	.62823	.72798	.79089	.83415
38	.00735	.19932	.32731	.45309	.55341	.62819	.72798	.79089	.83415
39	.00644	.19437	.32467	.45226	.55323	.62817	.72798	.79089	.83415
40	.00565	.18965	.32219	.45152	.55309	.62815	.72798	.79089	.83415
41	.00496	.18515	.31987	.45085	.55297	.62813	.72798	.79089	.83415
42	.00435	.18084	.31769	.45025	.55286	.62812	.72798	.79089	.83415
43	.00382	.17673	.31564	.44971	.55277	.62811	.72798	.79089	.83415
44	.00335	.17279	.31372	.44922	.55269	.62810	.72798	.79089	.83415
45	.00294	.16901	.31192	.44878	.55263	.62809	.72798	.79089	.83415
46	.00258	.16539	.31022	.44839	.55258	.62809	.72798	.79089	.83415
47	.00226	.16192	.30862	.44804	.55253	.62808	.72798	.79089	.83415
48	.00198	.15859	.30712	.44772	.55249	.62808	.72798	.79089	.83415
49	.00174	.15539	.30570	.44744	.55245	.62807	.72798	.79089	.83415
50	.00152	.15231	.30436	.44718	.55243	.62807	.72798	.79089	.83415

Fig. H.13 Present worth discount factors at 14% for income received in one payment at middle of year.

Year	Factor for Year	Equal Payments	5% Per Yr. Decline	10% Per Yr. Decline	15% Per Yr. Decline	20% Per Yr. Decline	30% Per Yr. Decline	40% Per Yr. Decline	50% Per Yr. Decline
1	.92848	.92848	.92848	.92848	.92848	.92848	.92848	.92848	.92848
2	.80041	.86444	.86609	.86781	.86964	.87156	.87574	.88045	.88579
3	.69001	.80630	.81038	.81467	.81919	.82394	.83419	.84547	.85782
4	.59484	.75343	.76056	.76807	.77595	.78420	.80178	.82059	.84029
5	.51279	.70530	.71596	.72717	.73891	.75113	.77675	.80329	.82972
6	.44206	.66143	.67596	.69124	.70719	.72368	.75763	.79151	.82357
7	.38109	.62138	.64003	.65964	.68004	.70095	.74315	.78363	.82009
8	.32852	.58477	.60771	.63184	.65680	.68218	.73228	.77844	.81816
9	.28321	.55127	.57860	.60734	.63693	.66672	.72418	.77508	.81711
10	.24415	.52055	.55234	.58573	.61994	.65401	.71820	.77293	.81655
11	.21047	.49237	.52860	.56667	.60541	.64359	.71381	.77156	.81625
12	.18144	.46645	.50712	.54982	.59301	.63507	.71061	.77070	.81610
13	.15641	.44261	.48765	.53492	.58241	.62810	.70829	.77017	.81602
14	.13484	.42062	.46997	.52174	.57336	.62243	.70661	.76984	.81598
15	.11624	.40033	.45390	.51005	.56564	.61782	.70540	.76963	.81596
16	.10021	.38157	.43927	.49970	.55906	.61407	.70454	.76951	.81595
17	.08639	.36421	.42592	.49050	.55344	.61103	.70392	.76943	.81594
18	.07447	.34811	.41373	.48234	.54864	.60857	.70348	.76938	.81594
19	.06420	.33317	.40259	.47508	.54456	.60658	.70317	.76935	.81594
20	.05534	.31928	.39237	.46863	.54108	.60497	.70294	.76934	.81593
21	.04771	.30635	.38300	.46288	.53811	.60368	.70279	.76933	.81593
22	.04113	.29429	.37440	.45776	.53559	.60263	.70268	.76932	.81593
23	.03546	.28304	.36648	.45320	.53343	.60179	.70260	.76932	.81593
24	.03057	.27252	.35919	.44913	.53160	.60111	.70254	.76931	.81593
25	.02635	.26267	.35247	.44550	.53004	.60057	.70250	.76931	.81593
26	.02272	.25344	.34626	.44225	.52871	.60013	.70248	.76931	.81593
27	.01958	.24478	.34052	.43935	.52758	.59978	.70246	.76931	.81593
28	.01688	.23664	.33520	.43676	.52662	.59949	.70244	.76931	.81593
29	.01455	.22898	.33027	.43444	.52580	.59927	.70244	.76931	.81593
30	.01255	.22177	.32570	.43237	.52511	.59908	.70243	.76931	.81593
31	.01082	.21496	.32146	.43051	.52451	.59894	.70242	.76931	.81593
32	.00932	.20854	.31751	.42885	.52401	.59882	.70242	.76931	.81593
33	.00804	.20246	.31384	.42735	.52358	.59873	.70242	.76931	.81593
34	.00693	.19671	.31042	.42602	.52322	.59865	.70242	.76931	.81593
35	.00597	.19126	.30723	.42482	.52291	.59859	.70242	.76931	.81593
36	.00515	.18609	.30425	.42375	.52264	.59854	.70241	.76931	.81593
37	.00444	.18118	.30146	.42278	.52242	.59851	.70241	.76931	.81593
38	.00383	.17651	.29886	.42192	.52223	.59848	.70241	.76931	.81593
39	.00330	.17207	.29643	.42114	.52206	.59845	.70241	.76931	.81593
40	.00284	.16784	.29415	.42044	.52193	.59843	.70241	.76931	.81593
41	.00245	.16381	.29202	.41982	.52181	.59842	.70241	.76931	.81593
42	.00211	.15996	.29001	.41925	.52171	.59840	.70241	.76931	.81593
43	.00182	.15628	.28814	.41875	.52163	.59839	.70241	.76931	.81593
44	.00157	.15276	.28637	.41829	.52155	.59838	.70241	.76931	.81593
45	.00135	.14940	.28472	.41789	.52149	.59838	.70241	.76931	.81593
46	.00117	.14618	.28316	.41752	.52144	.59837	.70241	.76931	.81593
47	.00101	.14309	.28170	.41719	.52140	.59837	.70241	.76931	.81593
48	.00087	.14013	.28032	.41689	.52136	.59836	.70241	.76931	.81593
49	.00075	.13728	.27902	.41663	.52133	.59836	.70241	.76931	.81593
50	.00064	.13455	.27780	.41639	.52130	.59836	.70241	.76931	.81593

Fig. H.14 Present worth discount factors at 16% for income received in one payment at middle of year.

Year	Factor for Year	Equal Payments	5% Per Yr. Decline	10% Per Yr. Decline	15% Per Yr. Decline	20% Per Yr. Decline	30% Per Yr. Decline	40% Per Yr. Decline	50% Per Yr. Decline
1	.92057	.92057	.92057	.92057	.92057	.92057	.92057	.92057	.92057
2	.78015	.85036	.85216	.85406	.85605	.85816	.86275	.86791	.87377
3	.66114	.78729	.79173	.79640	.80131	.80649	.81764	.82994	.84339
4	.56029	.73054	.73824	.74635	.75486	.76378	.78279	.80317	.82452
5	.47482	.67940	.69082	.70284	.71545	.72858	.75613	.78471	.81324
6	.40239	.63323	.64869	.66498	.68199	.69960	.73592	.77224	.80672
7	.34101	.59148	.61121	.63198	.65360	.67581	.72073	.76396	.80305
8	.28899	.55367	.57778	.60317	.62950	.65632	.70941	.75855	.80103
9	.24491	.51936	.54792	.57800	.60904	.64037	.70104	.75506	.79994
10	.20755	.48818	.52119	.55596	.59168	.62736	.69489	.75284	.79937
11	.17589	.45979	.49721	.53665	.57693	.61675	.69040	.75144	.79906
12	.14906	.43390	.47567	.51970	.56441	.60812	.68715	.75057	.79890
13	.12632	.41024	.45628	.50480	.55377	.60112	.68479	.75002	.79882
14	.10705	.38858	.43878	.49169	.54475	.59543	.68310	.74969	.79878
15	.09072	.36872	.42297	.48014	.53708	.59083	.68189	.74948	.79876
16	.07688	.35048	.40865	.46995	.53056	.58711	.68103	.74935	.79875
17	.06515	.33370	.39566	.46095	.52503	.58411	.68041	.74928	.79874
18	.05522	.31823	.38385	.45299	.52033	.58168	.67997	.74923	.79874
19	.04679	.30394	.37310	.44594	.51634	.57973	.67967	.74920	.79874
20	.03966	.29073	.36329	.43969	.51295	.57815	.67945	.74918	.79873
21	.03361	.27848	.35433	.43415	.51006	.57688	.67929	.74917	.79873
22	.02848	.26712	.34613	.42922	.50762	.57586	.67918	.74917	.79873
23	.02414	.25656	.33861	.42485	.50553	.57504	.67911	.74916	.79873
24	.02045	.24672	.33170	.42095	.50377	.57439	.67905	.74916	.79873
25	.01733	.23754	.32535	.41748	.50227	.57386	.67901	.74916	.79873
26	.01469	.22897	.31950	.41439	.50099	.57344	.67899	.74916	.79873
27	.01245	.22095	.31410	.41163	.49990	.57310	.67897	.74916	.79873
28	.01055	.21344	.30912	.40917	.49898	.57282	.67895	.74916	.79873
29	.00894	.20639	.30451	.40698	.49820	.57260	.67895	.74916	.79873
30	.00758	.19976	.30024	.40501	.49753	.57243	.67894	.74916	.79873
31	.00642	.19352	.29628	.40325	.49697	.57229	.67893	.74916	.79873
32	.00544	.18765	.29260	.40168	.49649	.57218	.67893	.74916	.79873
33	.00461	.18210	.28918	.40028	.49608	.57209	.67893	.74916	.79873
34	.00391	.17686	.28600	.39902	.49573	.57201	.67893	.74916	.79873
35	.00331	.17190	.28304	.39789	.49544	.57196	.67893	.74916	.79873
36	.00281	.16720	.28027	.39687	.49518	.57191	.67893	.74916	.79873
37	.00238	.16275	.27769	.39597	.49497	.57187	.67893	.74916	.79873
38	.00202	.15852	.27528	.39515	.49479	.57184	.67892	.74916	.79873
39	.00171	.15450	.27303	.39442	.49464	.57182	.67892	.74916	.79873
40	.00145	.15067	.27092	.39377	.49450	.57180	.67892	.74916	.79873
41	.00123	.14703	.26895	.39318	.49439	.57179	.67892	.74916	.79873
42	.00104	.14355	.26710	.39265	.49430	.57177	.67892	.74916	.79873
43	.00088	.14023	.26536	.39218	.49422	.57176	.67892	.74916	.79873
44	.00075	.13706	.26374	.39175	.49415	.57176	.67892	.74916	.79873
45	.00063	.13403	.26221	.39137	.49409	.57175	.67892	.74916	.79873
46	.00054	.13113	.26077	.39103	.49404	.57175	.67892	.74916	.79873
47	.00045	.12835	.25942	.39072	.49400	.57174	.67892	.74916	.79873
48	.00039	.12568	.25815	.39044	.49396	.57174	.67892	.74916	.79873
49	.00033	.12312	.25695	.39019	.49393	.57174	.67892	.74916	.79873
50	.00028	.12067	.25583	.38997	.49391	.57173	.67892	.74916	.79873

Fig. H.15 Present worth discount factors at 18% for income received in one payment at middle of year.

Year	Factor for Year	Equal Payments	5% Per Yr. Decline	10% Per Yr. Decline	15% Per Yr. Decline	20% Per Yr. Decline	30% Per Yr. Decline	40% Per Yr. Decline	50% Per Yr. Decline
1	.91287	.91287	.91287	.91287	.91287	.91287	.91287	.91287	.91287
2	.76073	.83680	.83875	.84080	.84297	.84525	.85022	.85582	.86216
3	.63394	.76918	.77395	.77897	.78426	.78982	.80183	.81506	.82955
4	.52828	.70895	.71717	.72583	.73493	.74446	.76479	.78660	.80947
5	.44023	.65521	.66732	.68007	.69345	.70739	.73669	.76713	.79756
6	.36686	.60715	.62344	.64060	.65855	.67715	.71555	.75407	.79072
7	.30572	.56409	.58473	.60649	.62917	.65251	.69979	.74546	.78690
8	.25477	.52542	.55050	.57695	.60442	.63246	.68812	.73987	.78482
9	.21230	.49063	.52016	.55133	.58357	.61618	.67955	.73629	.78370
10	.17692	.45926	.49320	.52906	.56597	.60297	.67329	.73402	.78310
11	.14743	.43091	.46920	.50966	.55113	.59227	.66874	.73260	.78279
12	.12286	.40524	.44777	.49275	.53860	.58361	.66546	.73171	.78263
13	.10238	.38195	.42859	.47797	.52801	.57661	.66310	.73116	.78255
14	.08532	.36076	.41139	.46502	.51906	.57096	.66141	.73082	.78250
15	.07110	.34145	.39593	.45368	.51150	.56640	.66020	.73062	.78248
16	.05925	.32381	.38200	.44371	.50509	.56273	.65934	.73049	.78247
17	.04938	.30767	.36942	.43494	.49968	.55977	.65873	.73041	.78247
18	.04115	.29286	.35804	.42721	.49509	.55739	.65830	.73037	.78246
19	.03429	.27925	.34771	.42039	.49121	.55548	.65799	.73034	.78246
20	.02857	.26672	.33833	.41437	.48791	.55394	.65778	.73032	.78246
21	.02381	.25515	.32978	.40904	.48512	.55271	.65763	.73031	.78246
22	.01984	.24446	.32197	.40431	.48276	.55172	.65752	.73031	.78246
23	.01654	.23455	.31484	.40012	.48075	.55093	.65744	.73030	.78246
24	.01378	.22535	.30831	.39640	.47905	.55029	.65739	.73030	.78246
25	.01148	.21679	.30231	.39309	.47761	.54978	.65735	.73030	.78246
26	.00957	.20882	.29680	.39015	.47638	.54937	.65733	.73030	.78246
27	.00797	.20138	.29172	.38753	.47534	.54904	.65731	.73030	.78246
28	.00665	.19443	.28704	.38519	.47446	.54878	.65730	.73030	.78246
29	.00554	.18792	.28271	.38311	.47371	.54857	.65729	.73030	.78246
30	.00461	.18181	.27871	.38124	.47307	.54840	.65728	.73030	.78246
31	.00385	.17606	.27501	.37958	.47253	.54826	.65728	.73030	.78246
32	.00320	.17066	.27157	.37809	.47207	.54816	.65727	.73030	.78246
33	.00267	.16557	.26838	.37676	.47168	.54807	.65727	.73030	.78246
34	.00223	.16077	.26541	.37557	.47135	.54800	.65727	.73030	.78246
35	.00185	.15623	.26265	.37451	.47107	.54794	.65727	.73030	.78246
36	.00155	.15193	.26007	.37355	.47083	.54790	.65727	.73030	.78246
37	.00129	.14786	.25767	.37270	.47063	.54786	.65727	.73030	.78246
38	.00107	.14400	.25543	.37193	.47045	.54784	.65727	.73030	.78246
39	.00089	.14033	.25333	.37124	.47031	.54781	.65727	.73030	.78246
40	.00075	.13684	.25137	.37062	.47018	.54780	.65727	.73030	.78246
41	.00062	.13352	.24954	.37007	.47008	.54778	.65727	.73030	.78246
42	.00052	.13035	.24782	.36957	.46999	.54777	.65727	.73030	.78246
43	.00043	.12733	.24621	.36912	.46991	.54776	.65727	.73030	.78246
44	.00036	.12444	.24469	.36872	.46984	.54775	.65727	.73030	.78246
45	.00030	.12168	.24327	.36836	.46979	.54775	.65727	.73030	.78246
46	.00025	.11904	.24194	.36804	.46974	.54774	.65727	.73030	.78246
47	.00021	.11651	.24069	.36775	.46970	.54774	.65727	.73030	.78246
48	.00017	.11409	.23951	.36749	.46967	.54773	.65727	.73030	.78246
49	.00014	.11177	.23840	.36725	.46964	.54773	.65727	.73030	.78246
50	.00012	.10953	.23735	.36704	.46962	.54773	.65727	.73030	.78246

Fig. H.16 Present worth discount factors at 20% for income received in one payment at middle of year.

Year	Factor for Year	Equal Payments	5% Per Yr. Decline	10% Per Yr. Decline	15% Per Yr. Decline	20% Per Yr. Decline	30% Per Yr. Decline	40% Per Yr. Decline	50% Per Yr. Decline
1	.87706	.87706	.87706	.87706	.87706	.87706	.87706	.87706	.87706
2	.67466	.77586	.77845	.78119	.78406	.78710	.79372	.80116	.80959
3	.51897	.69023	.69636	.70281	.70961	.71677	.73224	.74933	.76807
4	.39921	.61747	.62768	.63845	.64979	.66169	.68715	.71457	.74348
5	.30708	.55540	.56997	.58536	.60155	.61849	.65424	.69167	.72941
6	.23622	.50220	.52122	.54136	.56251	.58453	.63035	.67681	.72158
7	.18171	.45641	.47986	.50472	.53081	.55781	.61310	.66730	.71733
8	.13977	.41683	.44458	.47408	.50496	.53674	.60069	.66129	.71506
9	.10752	.38247	.41434	.44832	.48382	.52011	.59180	.65754	.71387
10	.08271	.35249	.38829	.42657	.46647	.50695	.58546	.65520	.71326
11	.06362	.32623	.36575	.40813	.45218	.49654	.58095	.65377	.71294
12	.04894	.30312	.34615	.39242	.44038	.48828	.57775	.65289	.71278
13	.03765	.28270	.32902	.37898	.43060	.48173	.57548	.65235	.71269
14	.02896	.26458	.31399	.36745	.42249	.47652	.57388	.65203	.71265
15	.02228	.24842	.30074	.35750	.41573	.47238	.57275	.65183	.71263
16	.01713	.23397	.28900	.34890	.41008	.46908	.57196	.65171	.71262
17	.01318	.22098	.27857	.34144	.40537	.46646	.57140	.65164	.71261
18	.01014	.20927	.26926	.33494	.40141	.46436	.57101	.65159	.71261
19	.00780	.19866	.26092	.32926	.39809	.46269	.57073	.65157	.71261
20	.00600	.18903	.25342	.32429	.39530	.46136	.57054	.65155	.71261
21	.00461	.18025	.24666	.31992	.39295	.46030	.57041	.65154	.71261
22	.00355	.17222	.24054	.31608	.39097	.45945	.57031	.65154	.71261
23	.00273	.16485	.23499	.31270	.38930	.45877	.57024	.65153	.71261
24	.00210	.15807	.22993	.30971	.38789	.45823	.57020	.65153	.71261
25	.00162	.15181	.22532	.30706	.38670	.45780	.57016	.65153	.71261
26	.00124	.14602	.22110	.30471	.38569	.45745	.57014	.65153	.71261
27	.00096	.14064	.21723	.30263	.38484	.45717	.57013	.65153	.71261
28	.00074	.13565	.21367	.30078	.38411	.45695	.57011	.65153	.71261
29	.00057	.13099	.21040	.29913	.38350	.45678	.57011	.65153	.71261
30	.00044	.12664	.20738	.29766	.38298	.45664	.57010	.65153	.71261
31	.00033	.12256	.20459	.29635	.38254	.45652	.57010	.65153	.71261
32	.00026	.11874	.20201	.29518	.38217	.45643	.57009	.65153	.71261
33	.00020	.11515	.19961	.29413	.38185	.45636	.57009	.65153	.71261
34	.00015	.11177	.19739	.29320	.38158	.45630	.57009	.65153	.71261
35	.00012	.10858	.19532	.29236	.38135	.45626	.57009	.65153	.71261
36	.00009	.10556	.19339	.29161	.38116	.45622	.57009	.65153	.71261
37	.00007	.10271	.19160	.29094	.38099	.45619	.57009	.65153	.71261
38	.00005	.10001	.18993	.29034	.38085	.45616	.57009	.65153	.71261
39	.00004	.09745	.18836	.28980	.38073	.45615	.57009	.65153	.71261
40	.00003	.09501	.18690	.28932	.38063	.45613	.57009	.65153	.71261
41	.00002	.09270	.18553	.28889	.38054	.45612	.57009	.65153	.71261
42	.00002	.09049	.18425	.28850	.38047	.45611	.57009	.65153	.71261
43	.00001	.08838	.18305	.28815	.38041	.45610	.57009	.65153	.71261
44	.00001	.08638	.18192	.28784	.38036	.45609	.57009	.65153	.71261
45	.00001	.08446	.18087	.28755	.38031	.45609	.57009	.65153	.71261
46	.00001	.08262	.17987	.28730	.38027	.45609	.57009	.65153	.71261
47	.00001	.08086	.17894	.28707	.38024	.45608	.57009	.65153	.71261
48	.00000	.07918	.17806	.28687	.38021	.45608	.57009	.65153	.71261
49	.00000	.07756	.17724	.28669	.38019	.45608	.57009	.65153	.71261
50	.00000	.07601	.17646	.28652	.38017	.45608	.57009	.65153	.71261

Fig. H.17 Present worth discount factors at 30% for income received in one payment at middle of year.

Year	Factor for Year	Equal Payments	5% Per Yr. Decline	10% Per Yr. Decline	15% Per Yr. Decline	20% Per Yr. Decline	30% Per Yr. Decline	40% Per Yr. Decline	50% Per Yr. Decline
1	.84515	.84515	.84515	.84515	.84515	.84515	.84515	.84515	.84515
2	.60368	.72442	.72751	.73077	.73421	.73783	.74572	.75460	.76466
3	.43120	.62668	.63376	.64123	.64911	.65741	.67535	.69520	.71703
4	.30800	.54701	.55848	.57059	.58337	.59680	.62561	.65677	.68976
5	.22000	.48161	.49754	.51442	.53222	.55089	.59049	.63222	.67460
6	.15714	.42753	.44783	.46940	.49214	.51592	.56573	.61672	.66639
7	.11225	.38249	.40694	.43302	.46051	.48914	.54828	.60703	.66203
8	.08018	.34470	.37304	.40338	.43537	.46853	.53601	.60103	.65975
9	.05727	.31276	.34471	.37906	.41526	.45259	.52738	.59734	.65857
10	.04091	.28558	.32085	.35895	.39906	.44021	.52132	.59508	.65796
11	.02922	.26227	.30061	.34219	.38595	.43055	.51707	.59371	.65766
12	.02087	.24216	.28330	.32814	.37526	.42300	.51408	.59288	.65750
13	.01491	.22467	.26840	.31628	.36652	.41706	.51199	.59237	.65742
14	.01065	.20939	.25549	.30621	.35932	.41239	.51052	.59207	.65738
15	.00761	.19594	.24422	.29760	.35338	.40870	.50950	.59188	.65736
16	.00543	.18403	.23434	.29022	.34846	.40578	.50878	.59177	.65735
17	.00388	.17343	.22563	.28385	.34436	.40346	.50827	.59171	.65735
18	.00277	.16395	.21790	.27834	.34095	.40162	.50792	.59167	.65734
19	.00198	.15543	.21101	.27354	.33809	.40016	.50767	.59164	.65734
20	.00141	.14773	.20485	.26936	.33569	.39900	.50750	.59163	.65734
21	.00101	.14074	.19931	.26569	.33368	.39807	.50738	.59162	.65734
22	.00072	.13437	.19431	.26248	.33199	.39734	.50729	.59162	.65734
23	.00052	.12855	.18978	.25965	.33056	.39675	.50723	.59161	.65734
24	.00037	.12321	.18567	.25715	.32936	.39628	.50719	.59161	.65734
25	.00026	.11830	.18192	.25494	.32834	.39590	.50716	.59161	.65734
26	.00019	.11375	.17850	.25299	.32748	.39560	.50714	.59161	.65734
27	.00013	.10954	.17537	.25125	.32675	.39536	.50713	.59161	.65734
28	.00010	.10564	.17249	.24971	.32614	.39517	.50712	.59161	.65734
29	.00007	.10200	.16984	.24834	.32562	.39502	.50711	.59161	.65734
30	.00005	.09860	.16740	.24712	.32518	.39489	.50710	.59161	.65734
31	.00003	.09542	.16514	.24603	.32480	.39480	.50710	.59161	.65734
32	.00002	.09244	.16305	.24506	.32448	.39472	.50710	.59161	.65734
33	.00002	.08964	.16112	.24419	.32421	.39466	.50710	.59161	.65734
34	.00001	.08700	.15932	.24341	.32399	.39461	.50710	.59161	.65734
35	.00001	.08451	.15765	.24272	.32379	.39457	.50709	.59161	.65734
36	.00001	.08217	.15610	.24210	.32363	.39453	.50709	.59161	.65734
37	.00000	.07995	.15465	.24154	.32349	.39451	.50709	.59161	.65734
38	.00000	.07784	.15330	.24104	.32337	.39449	.50709	.59161	.65734
39	.00000	.07585	.15204	.24059	.32327	.39447	.50709	.59161	.65734
40	.00000	.07395	.15086	.24019	.32318	.39446	.50709	.59161	.65734
41	.00000	.07215	.14975	.23983	.32311	.39445	.50709	.59161	.65734
42	.00000	.07043	.14872	.23951	.32305	.39444	.50709	.59161	.65734
43	.00000	.06879	.14775	.23922	.32299	.39443	.50709	.59161	.65734
44	.00000	.06723	.14684	.23896	.32295	.39443	.50709	.59161	.65734
45	.00000	.06573	.14599	.23873	.32291	.39442	.50709	.59161	.65734
46	.00000	.06431	.14518	.23852	.32288	.39442	.50709	.59161	.65734
47	.00000	.06294	.14443	.23833	.32285	.39442	.50709	.59161	.65734
48	.00000	.06163	.14372	.23816	.32283	.39441	.50709	.59161	.65734
49	.00000	.06037	.14306	.23801	.32281	.39441	.50709	.59161	.65734
50	.00000	.05916	.14243	.23787	.32279	.39441	.50709	.59161	.65734

Fig. H.18 Present worth discount factors at 40% for income received in one payment at middle of year.

Year	Factor for Year	Equal Payments	5% Per Yr. Decline	10% Per Yr. Decline	15% Per Yr. Decline	20% Per Yr. Decline	30% Per Yr. Decline	40% Per Yr. Decline	50% Per Yr. Decline
1	.81650	.81650	.81650	.81650	.81650	.81650	.81650	.81650	.81650
2	.54433	.68041	.68390	.68758	.69145	.69553	.70443	.71443	.72577
3	.36289	.57457	.58234	.59053	.59917	.60828	.62801	.64986	.67393
4	.24192	.49141	.50367	.51663	.53032	.54474	.57573	.60937	.64513
5	.16128	.42538	.44203	.45970	.47838	.49802	.53985	.58418	.62953
6	.10752	.37241	.39317	.41532	.43875	.46333	.51514	.56863	.62124
7	.07168	.32945	.35400	.38031	.40819	.43735	.49809	.55909	.61691
8	.04779	.29424	.32224	.35239	.38436	.41772	.48628	.55327	.61468
9	.03186	.26509	.29619	.32986	.36561	.40276	.47809	.54973	.61354
10	.02124	.24070	.27459	.31150	.35072	.39129	.47240	.54758	.61296
11	.01416	.22011	.25651	.29640	.33878	.38243	.46844	.54629	.61267
12	.00944	.20255	.24123	.28385	.32914	.37555	.46568	.54551	.61252
13	.00629	.18745	.22818	.27334	.32131	.37018	.46375	.54504	.61245
14	.00420	.17436	.21696	.26447	.31490	.36597	.46240	.54476	.61241
15	.00280	.16293	.20723	.25693	.30962	.36266	.46147	.54459	.61239
16	.00186	.15286	.19873	.25048	.30527	.36005	.46081	.54448	.61238
17	.00124	.14394	.19126	.24494	.30165	.35798	.46035	.54442	.61238
18	.00083	.13599	.18466	.24015	.29864	.35634	.46003	.54439	.61237
19	.00055	.12886	.17879	.23599	.29613	.35504	.45980	.54436	.61237
20	.00037	.12244	.17354	.23237	.29403	.35401	.45965	.54435	.61237
21	.00025	.11662	.16883	.22920	.29226	.35318	.45954	.54434	.61237
22	.00016	.11133	.16458	.22642	.29078	.35253	.45946	.54434	.61237
23	.00011	.10649	.16074	.22397	.28952	.35200	.45941	.54434	.61237
24	.00007	.10206	.15726	.22182	.28847	.35159	.45937	.54433	.61237
25	.00005	.09798	.15408	.21991	.28758	.35125	.45934	.54433	.61237
26	.00003	.09421	.15118	.21822	.28683	.35099	.45932	.54433	.61237
27	.00002	.09072	.14852	.21673	.28619	.35078	.45931	.54433	.61237
28	.00001	.08748	.14608	.21540	.28565	.35061	.45930	.54433	.61237
29	.00001	.08446	.14384	.21421	.28519	.35047	.45929	.54433	.61237
30	.00001	.08165	.14177	.21316	.28481	.35036	.45929	.54433	.61237
31	.00000	.07902	.13986	.21222	.28448	.35027	.45929	.54433	.61237
32	.00000	.07655	.13809	.21138	.28420	.35020	.45928	.54433	.61237
33	.00000	.07423	.13645	.21063	.28396	.35015	.45928	.54433	.61237
34	.00000	.07204	.13493	.20996	.28376	.35010	.45928	.54433	.61237
35	.00000	.06999	.13352	.20936	.28359	.35007	.45928	.54433	.61237
36	.00000	.06804	.13220	.20883	.28345	.35004	.45928	.54433	.61237
37	.00000	.06620	.13097	.20835	.28333	.35002	.45928	.54433	.61237
38	.00000	.06446	.12983	.20792	.28322	.35000	.45928	.54433	.61237
39	.00000	.06281	.12876	.20753	.28313	.34999	.45928	.54433	.61237
40	.00000	.06124	.12776	.20719	.28306	.34997	.45928	.54433	.61237
41	.00000	.05974	.12682	.20688	.28299	.34996	.45928	.54433	.61237
42	.00000	.05832	.12595	.20660	.28294	.34996	.45928	.54433	.61237
43	.00000	.05696	.12513	.20635	.28289	.34995	.45928	.54433	.61237
44	.00000	.05567	.12436	.20612	.28286	.34995	.45928	.54433	.61237
45	.00000	.05443	.12363	.20592	.28282	.34994	.45928	.54433	.61237
46	.00000	.05325	.12296	.20574	.28279	.34994	.45928	.54433	.61237
47	.00000	.05212	.12232	.20558	.28277	.34994	.45928	.54433	.61237
48	.00000	.05103	.12172	.20543	.28275	.34993	.45928	.54433	.61237
49	.00000	.04999	.12115	.20530	.28273	.34993	.45928	.54433	.61237
50	.00000	.04899	.12062	.20518	.28272	.34993	.45928	.54433	.61237

Fig. H.19 Present worth discount factors at 50% for income received in one payment at middle of year.